

# Fiscal Challenges to Monetary Dominance\*

Olivier Jeanne and Hou Wang  
Johns Hopkins University\*\*

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## Abstract

We study the interaction between government debt and monetary policy in a dynamic stochastic equilibrium model in which the government may default. The central bank has a low inflation mandate and the government normally follows (but may deviate from) a fiscal rule that ensures solvency conditional on low inflation. The government may confront the central bank with a choice between increasing seigniorage or letting the government default (fiscal challenge to monetary dominance). We show that a central bank that is more willing to provide a monetary backstop to the government debt may reduce the government's incentives to challenge monetary dominance, and thus lower the risk of inflation in equilibrium. This is less likely to be true in a currency union where countries can externalize the cost of seigniorage to other members.

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\*\*Corresponding author: Olivier Jeanne, Department of Economics, Johns Hopkins University, 3400 N.Charles Street, Baltimore MD 21218; email: ojeanne@jhu.edu

# 1 Introduction

Based on the fiscal fundamentals, it might come as a surprise that government debt problems arose in the euro area rather than in other parts of the world. Figure 1 shows the ratio of the primary balance to GDP (on the horizontal axis) and the ratio of net government debt to GDP (on the vertical axis) for the euro area, the US, Japan and the UK in 2009.<sup>1</sup> The fiscal fundamentals were bad everywhere, but they were worse in the US, Japan and the UK than in the euro area on average. Figure 2 shows that the fiscal fundamentals of the US, the UK or Japan were comparable to those of Greece, Portugal, Ireland or Spain, the euro area economies that were the most affected by the crisis.

Several differences between the euro area and the rest of the world can explain this puzzle. The euro area deprives its members from certain margins of flexibility, such as exporting their way out of low growth by depreciating their currencies. Euro area countries do not enjoy the benefits of issuing a reserve currency to the same extent as the US, and most of them cannot rely on a high domestic saving rate to the same extent as Japan. Finally—and this is the difference that we focus on in this paper—the relationship between monetary policy and fiscal policy is not the same in the euro area as elsewhere.

The euro area was explicitly designed to minimize the risk of monetization of government debts, that is, to enforce the maximum degree of “monetary dominance” (Sargent and Wallace, 1981). The risk of monetization is perhaps not zero, because it is not certain what the European Central Bank (ECB) would do in a large-scale government debt rollover crisis that would threaten the existence of the euro. But it is certainly more likely that the monetary authorities would let the fiscal authorities default in the euro area than elsewhere.<sup>2</sup>

If this is what makes the euro area special, then in order to study the European debt crisis one would need a theoretical framework in which mon-

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<sup>1</sup>The data come from the World Economic Outlook (October 2011). The data for the euro area are the GDP-weighted average of the 11 largest euro area economies.

<sup>2</sup>Martin Feldstein puts it in this way in his recent piece “The French Don’t Get It”: “When interest and principal on British government debt come due, the British government can always create additional pounds to meet those obligations. By contrast, the French government and the French central bank cannot create euros.” Project Syndicate at <http://www.project-syndicate.org/commentary/feldstein43/English>

etary dominance can be challenged, and the monetary authorities have a choice between monetizing government debt or letting the government default. We present a DSGE model with these features below.

The model has somewhat unconventional implications, but it sheds an interesting light on the current debates on European policies. This debate often takes the form of an opposition between two seemingly irreconcilable views. The first view is that the interest rate spreads associated with the threat of default may be a normal and even desirable feature of the equilibrium, to the extent that they give euro area governments incentives to keep their fiscal house in order.

The second view is that the spreads are harmful, and that their presence in the euro area (and not elsewhere) comes from the ECB's failure to play its role of "lender of last resort" (De Grauwe, 2011). According to that view, the spreads reflect a vicious circle in government debt dynamics and market expectations—with high spreads leading to exploding debts, which in turn justifies the expectation of a default. By standing ready to buy government debt at the right price, the ECB could ensure that the economy stays in the good equilibrium with low interest rates. And like in the Diamond-Dybvig model, the commitment to lend would imply that lending-in-last-resort is not necessary in equilibrium.

One theme of this paper is that those two views are a bit too simple. On the one hand, the proponents of the second view make their lives too easy by simply assuming that government solvency would be ensured by low spreads. It is true that, other things equal, lowering spreads to zero would reduce the probability of default, but this does not mean that the probability of default would be reduced to zero. And a positive residual probability of default implies that the monetary authorities might be called to "lend in last resort" to an insolvent government in equilibrium. Debt monetization, thus, is not a purely out-of-equilibrium risk, it may be a real risk that has to be weighted against the benefits from low spreads.

On the other hand, it is not obvious either that high spreads necessarily provide the appropriate fiscal incentives. They could as well encourage the fiscal authorities to challenge monetary dominance by making the dynamics of debt unsustainable. As a result, high spreads might actually make inflation more (not less) likely, as we will show in this paper. In the long run, it might be necessary to accept a small risk of debt monetization (occurring, say, once every century on average) in order to establish a relationship between fiscal policy and monetary policy that is sustainable—i.e., one that does not

generate a government debt crisis every ten years.

**Relationship to the literature.** The distinction between monetary dominance and fiscal dominance was originally made by Sargent and Wallace (1981). There is monetary dominance when the monetary authorities are entirely focused on controlling inflation, whereas the fiscal authorities adjust fiscal policy to stay solvent conditional on an exogenous flow of seigniorage. Fiscal dominance, conversely, occurs when monetary policy is subject to the constraint of providing enough seigniorage to the government to ensure solvency. This distinction appears under different guises in the literature that looks at monetary and fiscal policy rules in recursive models. In Leeper’s (1991) terminology, monetary dominance corresponds to the case where monetary policy is “active” and fiscal policy is “passive”. In the analysis of Woodford (2003), monetary dominance results when the monetary rule follows the “Taylor principle” and the fiscal rule is “locally Ricardian”.

The approach in this paper is related to other contributions that explore the grey area between pure fiscal dominance and pure monetary dominance. For example, Davig and Leeper (2007) study an environment in which the monetary policy rule switches between an active stance and a passive stance. Davig, Leeper and Walker (2010) use a rational expectations framework to assess the implications of rising debt in an environment with a “fiscal limit”, i.e., a point where the government no longer has the ability to finance higher debt levels by increasing taxes, so that either a fiscal adjustment or inflation must occur to stabilize debt. Those papers, however, do not consider *default* as an alternative to fiscal adjustment or inflation.

Other papers introduce the possibility of government default in dynamic optimizing models of monetary and fiscal policy. Uribe (2006) makes the point that if fiscal and monetary policy are both “active”, then the only way that the government can satisfy its intertemporal budget constraint is by sometimes defaulting. He shows that the equilibrium behavior of default and rates and risk premiums may be quite sensitive to the specification of the monetary rule. Bi (2011) presents an intertemporal optimizing model in which default is an alternative to fiscal consolidation. However, Uribe’s model does not have fiscal adjustments and Bi’s model does not have monetary policy. This paper, by contrast, embeds the three options of default, inflation and fiscal adjustment in the context of a single framework.

Our modeling approach is close to the dynamic optimizing models of default that have been developed in the sovereign debt literature (Arellano (2008), Aguiar and Gopinath (2006)). The paper is related to more recent

contributions that have analyzed the possible role of a monetary backstop when there are self-fulfilling roll-over crises on government debt (Aguiar et al. (2013), Corsetti and Dedola (2012)). Themes similar to this paper are developed, in the context of more simple two-period models, in Jeanne (2012) and Gourinchas and Jeanne (2012).

## 2 Model

The economy is composed of four sectors: households, firms, the banking sector and the government.

### 2.1 Households

We consider an economy with a government and an infinitely-lived representative consumer. There is one homogeneous good. The consumer's intertemporal utility is given by

$$U_t = \sum_{s \geq t} (1+r)^{-(s-t)} \left[ c_s - \frac{\ell_s^{1+1/\sigma}}{1+1/\sigma} + v(m_s) \right], \quad (1)$$

where  $\ell_s$  is labor supply at time  $s$ ,  $m_s = M_s/P_s$  is real money balances, and  $v(m)$  is the utility of real money balances. Note that the consumer is risk-neutral, implying that the riskless interest rate is equal to  $r$ .<sup>3</sup>

The consumer's budget constraint is

$$c_t + q_t b_{t+1} + x_t m_{t-1} = (1 - \tau_t) w_t \ell_t + (1 - \delta_t h_t) b_t - \delta_t \gamma^d + (1 - \phi_t) z_t, \quad (2)$$

where  $b_{t+1}$  denotes the holdings of one-period government real bonds (promises to pay one unit of good in the next period) at the end of period  $t$ ;  $q_t$  is the price of a bond issued at time  $t$ ;  $\delta_t$  is a dummy variable for government default;  $h_t$  is the haircut conditional on a default at time  $t$ ;  $\tau_t$  is a tax on labor income;  $w_t$  is the real wage;  $\gamma^d$  is the cost of a default; and the last terms on the left-hand side and the right-hand side are respectively the opportunity cost of holding money and the profit received from the banking sector (to be

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<sup>3</sup>Risk aversion would introduce a pure risk premium in the price of defaultable debt. Arellano (2008) has shown that this term is small and does not significantly influence debt dynamics unless one assumes a very large level of risk aversion.

explained in a moment when we present the assumptions about the banking sector). The assumption that government debt is real, rather than nominal, is not essential (see section ??).

## 2.2 Firms

The consumption good is produced by perfectly competitive firms that use labor as their production input,

$$y_t = \theta_t \ell_t - \delta_t \gamma^d. \quad (3)$$

A government default reduces output by  $\gamma^d$  and this cost is simply passed along to the consumers (see equation (2)). The assumption that a default reduces output captures in reduced form the negative impact of the government default on the productive sector.<sup>4</sup> This assumption has become standard in the DSGE literature on sovereign default (Arellano (2008); Aguiar and Gopinath (2006)).<sup>5</sup>

The only source of uncertainty in the model is the stochastic productivity parameter  $\theta_t$ , which is governed by the AR(1) process,

$$\theta_t - \theta^* = \rho(\theta_{t-1} - \theta^*) + \varepsilon_t, \quad (4)$$

where  $\varepsilon_t$  is an i.i.d. shock. The average level of productivity is normalized to 1 ( $\theta^* = 1$ ).

## 2.3 The banking sector

The banking sector issues money and holds claims on households. Money in the model should be interpreted as bank deposits—a broad money aggregates such as M2. To simplify, we will represent the banking sector (including the central bank) as one single entity. The behavior of the consolidating

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<sup>4</sup>This could occur for example because of the financial disruption induced by government default. See Bolton and Jeanne (2011) for a micro-founded model in which government debt is used as a collateral in private financial contracts and default reduces the efficiency of resource allocation in the private sector. See Borensztein and Panizza (2009) for a review of the evidence on the cost of a government default.

<sup>5</sup>These authors assume that a default reduces productivity. It is more convenient here to assume that the default cost enters additively in the production function in order to make the cost of a default more comparable to the cost of inflation and financial repression.

banking sector is determined by the central bank through monetary policy and banking regulation. We will call these policies "central bank policy".

The banking sector creates deposits by lending them to households. The representative household obtains a level of real deposits  $m_{t-1}$  in period  $t-1$  by borrowing it from the banking sector at the real interest rate  $r$ . The banking sector pays a nominal interest rate  $i_{t-1}^m$  on the deposits between period  $t-1$  and period  $t$ . As mentioned above, variable  $m_t$  represents a broad money aggregate, some components of which yield a return. In period  $t$  the representative household must repay  $(1+r)m_{t-1}$  and holds a level of real deposits equal to  $m_{t-1}(1+i_{t-1}^m)/(1+\pi_t)$ , where  $\pi_t$  is the rate of inflation between  $t-1$  and  $t$ . The opportunity cost of holding money, thus, is the spread between the real interest rate at which households borrow deposits and the real return that they receive on these deposits,

$$x_t = 1 + r - \frac{1 + i_{t-1}^m}{1 + \pi_t}.$$

The cost of holding money for households is also the profit of the banking sector,

$$z_t = x_t m_{t-1}. \tag{5}$$

A share  $\phi_t$  of this profit is distributed to the government as seigniorage and taxation of banking profits. The rest is distributed to households, which explains why the last term on the right-hand side of (2) is multiplied by  $1 - \phi_t$ .

The inflation rate,  $\pi_t$ , the nominal interest rate on deposits,  $i_t^m$ , and the share of the banking profit that is distributed to households,  $\phi_t$ , are determined by various policies. The inflation rate is determined by monetary policy. The central bank has a mandate to implement a low inflation rate  $\pi^*$ , but it may occasionally deviate from the target, as we will see below.

The nominal interest rate on the deposits,  $i_t^m$ , and is determined by banking policy. This interest rate is equal to the marginal return that private banks receive on their assets if there is perfect competition for deposits in the banking sector. However, banking regulation may impose a ceiling on the interest rate paid on deposits and lower  $i_t^m$ . This allows banks to earn a spread on their deposits.

The share of the banking profit that goes to the government,  $\phi_t$ , is determined by banking and tax policies. For example, an increase in the private banks' cash reserve requirement reduces their profits and raises the profit of

the central bank if the reserves are unremunerated. The central bank's profit is transferred to the government. An increase in  $\phi_t$  could also be achieved by raising the tax rate on bank profits, or nationalizing the banks.

For the purpose of our analysis, we do not need to explain in detail the policy instruments that are involved in the determination of  $i_t^m$  and  $\phi_t$ . These variables capture in reduced form everything we need to know about how the banking sector contributes to government revenues. We shall assume, furthermore, that these variables change infrequently. Most of the time, the economy is in a financially liberalized regime in which households receive a high return on bank deposits because of competition between banks, as well as a large share of the bank profits. There could be, however, a switch to a regime of financial repression in which households receive a low return on deposits and a low share of banking profits. The assumptions regarding this aspect of the model will be presented in more detail in section 2.5. We will call  $\phi_t z_t$  "seigniorage" but it is important to keep in mind that this is an extended definition of seigniorage that includes all the revenues that the government extracts from the banking sector, possibly through financial repression. (In particular, seigniorage is not limited to the creation of base money by the central bank, as it is often defined in the literature).

## 2.4 Government

The government issues debt and taxes labor to finance its expenditures. The government has an exogenous stream of expenditures,  $g$ , so that its budget constraint can be written,

$$g + (1 - \delta_t h_t) b_t = q_t b_{t+1} + \tau_t w_t \ell_t + \phi_t z_t. \quad (6)$$

We assume for simplicity that the level of government expenditures is exogenous and constant. This assumption is not realistic but the dynamics of debt are determined by the primary balance, which we will model in a more realistic way (see below).

We assume that in the event of default, government debt is reduced by an exogenous amount  $\Delta b$ . The haircut, thus, is given by<sup>6</sup>

$$h_t = \frac{\Delta b}{b_t}. \quad (7)$$

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<sup>6</sup>A more common alternative assumption is that the haircut is constant. The assumption that debt is reduced by a fixed amount is convenient here because it makes default and inflation easier to compare.

We assume that the government follows a fiscal policy rule that gives the primary balance as a linear function of debt and productivity,

$$\tau_t w_t \ell_t - g = \alpha_0 + \alpha_b b_t + \alpha_\theta (\theta_t - \theta^*). \quad (8)$$

The left-hand-side is the government's primary balance conditional on no seigniorage. If  $\alpha_b$  and  $\alpha_\theta$  are two positive coefficients, this equation says that the primary balance increases with debt as well as with productivity which is a realistic representation of the behavior of the primary balance in many economies.

The fiscal rule (8) implicitly defines the tax rate as a function of productivity and debt. To see this, note that the linearity of the production function implies that the real wage is equal to labor productivity,  $w_t = \theta_t$ . The first-order condition for labor supply then implies  $\ell_t = [(1 - \tau_t)\theta_t]^\sigma$ . Using this expression and  $w_t = \theta_t$  to substitute out  $w_t \ell_t$  in (8) gives an equation linking  $\theta_t$ ,  $b_t$  and  $\tau_t$ . Solving for  $\tau_t$  then defines the tax rate as a function of the state,

$$\tau_t = \tau(\theta_t, b_t). \quad (9)$$

Equation (8) is a passive fiscal rule a la Leeper (1991) in the case where there is no risk of default in government debt. If there is no default risk ( $\delta_t = 0$ ), the price of government bonds is imply equal to the inverse of the discount factor,  $q_t = (1 + r)^{-1}$ . Assuming no seigniorage ( $\phi_t z_t = 0$ ) and using (8) to substitute out the primary balance from the government's budget constraint (6) gives,

$$b_{t+1} - b^* = (1 + r) [(1 - \alpha_b)(b_t - b^*) - \alpha_\theta (\theta_t - \theta^*)], \quad (10)$$

where  $b^* = \alpha_0 / (\alpha_b - r / (1 + r))$  is the target level of debt. Government debt follows stationary fluctuations around  $b^*$  if  $(1 + r)(1 - \alpha_b) < 1$ , that is if the responsiveness of the primary balance to the level of debt is large enough,

$$\alpha_b > \frac{r}{1 + r}. \quad (11)$$

If this condition is met, the government sets the tax rate in such a way that it stays solvent conditional on zero seigniorage. That is, equation (8) defines a passive fiscal rule in the sense of Leeper (1991) (or a locally Ricardian one in the sense of Woodford (2003)). Note however that equation (10) does not apply in the general case where the price of government bonds is lower than  $(1 + r)^{-1}$  because of a risk of government default.

## 2.5 Fiscal challenges

If the government always followed rule (8), there would be no default in equilibrium. In order to have default as a possible equilibrium outcome, we assume that the government can deviate from the fiscal rule in the following way. At the beginning of any period  $t$ , the government can declare that rule (8) will be applied using a level of debt  $b_t - \Delta b$  instead of  $b_t$  on the right-hand-side of the equation.<sup>7</sup> In other words, the government announces that it will raise enough taxes to repay  $b_t - \Delta b$  instead of  $b_t$  conditional on zero seigniorage. It follows that in the absence of an intervention by the central bank to raise seigniorage, the government cannot roll over its debt and must default in period  $t$ . Making this announcement is what we call a “fiscal challenge to monetary dominance”.

The outcome of a fiscal challenge depends on how the central bank responds to it. We assume that the central bank has a mandate to keep inflation equal to a low target rate. The central bank implements its mandate unless there is a fiscal challenge to monetary dominance, in which case the central bank follows one of two policies: either the central bank sticks to its mandate and lets the government default, or it increases inflation and seigniorage in order to avoid a government default (in this case, we will say that the central bank provides a “monetary backstop” to government debt). If there is a default, the government’s debt is reduced from  $b_t$  to  $b_t - \Delta b$  and the inflation rate remains equal to the target. If the central bank saves the government from a default, the economy enters a new regime that is characterized by higher inflation and a higher level of seigniorage. The higher seigniorage pays for  $\Delta b$  so that the government does not default.

More formally, we assume that in the low-inflation regime, the central bank keeps the inflation rate equal to a target  $\pi^*$ , the nominal interest rate on money balances to a fixed level  $i^*$ , and that the government does not get any of the profit of the banking sector ( $\phi^* = 0$ ).<sup>8</sup> If the central bank saves the government from a default in period  $t$ , the inflation rate between period

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<sup>7</sup>We assume that such a deviation is possible only if the debt level  $b_t$  is larger than  $\Delta b$ . Otherwise reducing debt by  $\Delta b$  could imply a negative debt level.

<sup>8</sup>Seigniorage conventionally defined is low in advanced economies. Seigniorage as we define it here (i.e. including the taxation of the part of private banks’ profits that comes from their ability to pay low interest rate on their liabilities) is higher, although difficult to estimate. It would not change the essence of our results to assume that the government receives a small seigniorage stream in normal times, the important assumption being that seigniorage is significantly increased when the central bank provides a monetary backstop.

$t$  and period  $t + 1$  is raised to a higher level  $\tilde{\pi}$ , while the interest rate on money is lowered to  $i_t^m = 0$  and all the banking profits are transferred to the government in the following period ( $\phi_{t+1} = 1$ ). Note the timing of the increase in inflation: the inflation rate is raised between period  $t$  and period  $t + 1$ , not between  $t - 1$  and period  $t$ . Thus, there is no surprise in the period- $t$  nominal price level. This assumption can be justified by the fact the price level is unlikely to jump in the short run because of nominal stickiness (which is left out of the model).<sup>9</sup>

For simplicity we assume that the inflation rate is raised for only one period when the central bank prevents a government default. Backstopping government debt in period  $t$  raises seigniorage for one period, in period  $t + 1$ . We denote by  $\tilde{z}$  the fiscal revenue produced by seigniorage in period  $t + 1$ . We assume that this extra revenue is sufficient to pay  $\Delta b$ , the amount of debt on which the government wants to default in period  $t$ ,

$$\Delta b = \frac{\tilde{z}}{1 + r}. \quad (12)$$

That is, seigniorage is increased by exactly the amount that is required to avoid a default in period  $t$ .<sup>10</sup> This requires the inflation rate  $\tilde{\pi}$  to be set at a certain level that will be derived in the following section.

The response of the central bank, when there is a fiscal challenge to monetary dominance, depends on the strength of the inflation objective in its mandate. As discussed in the introduction, the probability that the central bank let the government default seems higher in some settings (such as the euro area) than in others—although even in the euro area, this probability may be lower than 100 percent. Since we are interested in the behavior of the central bank insofar as it is determined by the “constitutional” features of the monetary framework (that have been built in the system *ex ante* rather than optimized upon period by period), it is natural to treat the likelihood that the central bank deviates from its low-inflation mandate as exogenous during the time period under consideration. Thus, we assume that conditional on a fiscal challenge, there is a probability  $\mu$  that the central bank avoids a government default by providing a monetary backstop, and a probability  $1 - \mu$  that it

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<sup>9</sup>This assumption plays an important role if government debt is nominal, see section ??.

<sup>10</sup>We assume that  $\tilde{z}$  is earmarked to repay  $(1+r)\Delta b$  in period  $t+1$ . Thus the government can roll over  $b_t - \Delta b$  in period  $t$  by applying the fiscal rule (8) to the reduced amount of debt.

lets the government default. Parameter  $\mu$  is an exogenous parameter that measures the central bank's propensity to backstop government debt when challenged by the government. This probability is taken as exogenous for now, but we will discuss later the case where the central bank maximizes welfare.

We assume that the government is benevolent, i.e., that it challenges the central bank only when this raises the welfare of the representative consumer. The welfare benefit from a fiscal challenge comes from the fact that it reduces the taxation of labor income and the associated distortion. The cost of a fiscal challenge is the welfare cost of a default or the welfare cost of higher inflation, depending on the response of the central bank. The government does not know, when it decides to challenge the central bank, whether this will lead to a default or to inflation. It simply expects to be saved from a default by the central bank with probability  $\mu$ .

To sum up, the sequence of events and decisions that take place in each period of the low-inflation regime is depicted in figure 3. First, the government decides whether to challenge the central bank or not. If the government does not challenge the central bank, it applies the fiscal rule (8) and rolls over its debt. If the government challenges the central bank, nature decides whether the central bank rescues the government from a default or not. There are three possible outcomes in equilibrium: (i) a successful rollover of the government's debt; (ii) a default; and (iii) inflation.

The sequence depicted in figure 3 assumes that the government can roll over its debt if it implements the fiscal rule (8). This is true (provided that  $\alpha_b$  is large enough) if the government does not pay a default premium on its debt ( $q = (1 + r)^{-1}$ ). But this is not necessarily the case if there is a risk of default on the government's debt. Then the government may be unable to roll over its debt because the investors are unwilling to buy the new debt or are willing to buy it at a price that is too low. That is, a debt rollover crisis may occur in equilibrium because of the government's inability to roll over its debt rather than an unwillingness to apply the fiscal rule. We assume in this case that the central bank behaves in the same way as when it is faced with an opportunistic fiscal challenge: it increases seigniorage with probability  $\mu$  and lets the government default with probability  $1 - \mu$ .

## 2.6 Equilibrium

We now derive the equilibrium conditions for a recursive equilibrium. The state at the beginning of period  $t$  is composed of two variables: the level of productivity  $\theta_t$ , and the level of government debt  $b_t$ . The state can be in three sets,  $S_1$ ,  $S_2$  or  $S_3$ . The set  $S_1$  contains the states in which the government does not challenge the central bank, implements the fiscal rule (8) and rolls over its debt without defaulting. The set  $S_2$  contains the states in which the government can roll over its debt but chooses to deviate from the fiscal rule and challenge the central bank. Finally, the set  $S_3$  contains the states in which the government is unable to roll over its debt and challenges the central bank out of necessity rather than choice. These sets are endogenous in equilibrium.

We first present a few important equilibrium relationships. By adding up the budget constraints (2), (5) and (6) we obtain that private consumption is equal to output minus government consumption

$$c_t = y_t - g. \quad (13)$$

It follows that the consumer's utility flow from consumption and labor can be written as,

$$c_t - \frac{\ell_t^{1+1/\sigma}}{1+1/\sigma} = \underbrace{\theta_t^{1+\sigma} \left[ (1-\tau_t)^\sigma - \frac{(1-\tau_t)^{1+\sigma}}{1+1/\sigma} \right]}_{u(\theta_t, b_t)} - g. \quad (14)$$

Function  $u(\theta_t, b_t)$  is a reduced form for the flow utility from consumption and labor. It can be written as a function of the debt level,  $b_t$ , by substituting out the tax rate using equation (9). The flow utility is increasing with productivity  $\theta_t$  and decreasing with  $b_t$  because higher debt implies a higher level of taxation of labor income, which distorts the equilibrium away from the first best.

We then derive the equilibrium conditions for money demand. The first-order condition for money demand is,

$$v'(m_t) = \frac{x_t}{1+r} = 1 - \frac{1+i_t^m}{(1+r)(1+\pi_{t+1})}. \quad (15)$$

Note that there is no expectation term in this equation because the inflation rate between  $t$  and  $t+1$ ,  $\pi_{t+1}$ , is known at time  $t$ .

The level of real money holdings can take two values in equilibrium. In the low-inflation regime, it is equal to the level  $m^*$  that satisfies equation (15) with  $i_t^m = i^*$  and  $\pi_{t+1} = \pi^*$ . If the central bank backstops government debt, it is equal to the level  $\tilde{m}$  that satisfies equation (15) with  $i_t^m = 0$  and  $\pi_{t+1} = \tilde{\pi}$ . When there is a switch to the high-inflation regime, thus, real money balances decrease from  $m^*$  to  $\tilde{m}$  satisfying,

$$v'(\tilde{m}) = 1 - \frac{1}{(1+r)(1+\tilde{\pi})}. \quad (16)$$

This increases the next-period level of seigniorage from zero to  $\tilde{z} = (1+r)\tilde{m}v'(\tilde{m})$ , so that equation (12) can be written,

$$\Delta b = \tilde{m}v'(\tilde{m}). \quad (17)$$

We assume that there exists a solution  $\tilde{m}$  to this equation.<sup>11</sup> Given  $\tilde{m}$ , the level of inflation in the high-inflation regime,  $\tilde{\pi}$ , can be derived from equation (16).

We now introduce notations for the continuation values of the consumer's optimization problem for the different regimes and decision nodes. Time subscripts are omitted to alleviate notations and we denote next-period variables with primes. We denote by  $V^n(\theta, b)$  the continuation value associated with the government rolling over its debt and having *no* default and *no* inflation; by  $V^d(\theta, b)$  the value associated with government *default*, and by  $V^i(\theta, b)$  the value associated with high *inflation* and no default. Further, we denote by  $V^c(\theta, b)$  the continuation value associated with a fiscal challenge to monetary dominance (before the central bank's response is known) and by  $V(\theta, b)$  the unconditional continuation value at the beginning of the period.

The continuation value for the case with no default and no inflation satisfies,

$$V^n(\theta, b) = u(\theta, b) + v(m^*) + \frac{1}{1+r}EV(\theta', b'),$$

where  $u(\theta, b)$  is the reduced-form flow utility from consumption and labor derived above. The transition equation for productivity is (4) and the transition equation for debt will be written later (equation (24)).

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<sup>11</sup>If there are multiple solutions we choose the highest level of  $\tilde{m}$  on the efficient branch of the seigniorage Laffer curve.

The continuation value under a default is the same as with no default,<sup>12</sup> except that government debt is reduced by  $\Delta b$  and the domestic consumer pays the cost of default  $\gamma^d$ ,

$$V^d(\theta, b) = V^n(\theta, b - \Delta b) - \gamma^d. \quad (18)$$

The continuation value when the central bank raises inflation to avoid a default is the same as with no default,<sup>13</sup> except that debt is reduced by  $\Delta b$  and the utility from real money is  $v(\tilde{m})$  rather than  $v(m^*)$ ,

$$V^i(\theta, b) = u(\theta, b - \Delta b) + v(\tilde{m}) + \frac{1}{1+r}EV(\theta', b').$$

This can also be written,

$$V^i(\theta, b) = V^n(\theta, b - \Delta b) - \gamma^i, \quad (19)$$

where the welfare cost of high inflation,  $\gamma^i$ , is given by,

$$\gamma^i = v(m^*) - v(\tilde{m}). \quad (20)$$

Comparing equations (18) and (19) shows the similarity between inflation and a default. Inflation has exactly the same effect on debt as a default (it reduces debt by  $\Delta b$ ) and it has an additive welfare cost too. This cost is the reduction in the utility of real money holdings coming from the fact that money is taxed by inflation and financial repression.

A fiscal challenge is followed by inflation with probability  $\mu$  and by a default with probability  $1 - \mu$ , so that,

$$V^c(\theta, b) = (1 - \mu)V^d(\theta, b) + \mu V^i(\theta, b). \quad (21)$$

If the government can roll over its debt by applying the fiscal rule, (that is, if the state is not in  $S_3$ ), then it chooses to challenge monetary dominance

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<sup>12</sup>Note that it is assumed that the government can roll over its debt when debt is reduced to  $b - \Delta b$ . This is generally the case in our simulations, but when this is not true, we assume that debt is reduced by the smallest amount that allows debt to be rolled over, i.e., debt is reduced by the smallest  $\psi\Delta b \geq \Delta b$  such that  $b - \psi\Delta b$  can be rolled over. The default cost is scaled proportionately, i.e., it is set to  $\psi\gamma^d$ .

<sup>13</sup>If raising inflation to  $\tilde{\pi}$  is not sufficient to avoid a default because the government cannot roll over  $b - \Delta b$ , then we assume that the government defaults and we increase  $\Delta b$  in the way explained in the previous footnote.

if and only if this increases the welfare of the representative consumer. Thus we have,

$$V(\theta, b) = \max [V^n(\theta, b), V^c(\theta, b)] \text{ for } (\theta, b) \notin S_3. \quad (22)$$

Finally, the government may be unable to roll over its debt even when it applies the fiscal rule because the price at which it can sell bonds is too low. The equilibrium bond price is equal to the discount factor times 1 minus the probability of a default times the haircut,

$$q(\theta, b') = \frac{1 - \Pr [(\theta', b') \in S_2 \cup S_3] (1 - \mu)h'}{1 + r}, \quad (23)$$

where  $h' = \Delta b/b'$  is the haircut if there is a default in the next period. The probability of a default is the probability that the next-period state is in  $S_2$  or  $S_3$  times the probability that the government will not be rescued from a default by the central bank. Note that if the government can count on the monetary backstop with certainty ( $\mu = 1$ ), then  $q(\theta, b')$  is equal to  $(1 + r)^{-1}$ , that is, there is no default premium on government debt. There is a positive default premium if the government cannot count on the monetary backstop with certainty ( $\mu$  is smaller than 1), but the sign of the variation of  $q$  with  $\mu$  is a priori ambiguous since the sets  $S_2$  and  $S_3$  are both endogenous to  $\mu$  in equilibrium.

Under fiscal rule (8) the budget constraint of a non-defaulting government can be written,

$$b = q(\theta, b')b' + \alpha_0 + \alpha_b b + \alpha_\theta (\theta - \theta^*). \quad (24)$$

This is the transition equation for government debt, since it implicitly defines  $b'$  in function of  $\theta$  and  $b$ . However, it may be impossible to find a value for  $b'$  that satisfies this budget constraint because  $\max_{b'} b'q(\theta, b')$  is too low. In this case, there is a debt rollover crisis caused by the government's inability to roll over its debt rather than by a deviation from the fiscal rule. This is when the state  $(\theta, b)$  belongs to the set  $S_3$ . If the state is in  $S_3$ ,  $V^n(\theta, b)$  is not defined and the continuation value is the same as under a fiscal challenge,

$$V(\theta, b) = V^c(\theta, b) \text{ for } (\theta, b) \in S_3.$$

### 3 Quantitative Results

Our benchmark calibration is presented in sections 3.1 and 3.2. We then present the main properties of the equilibrium under our benchmark calibra-

tion, and focus on particular on how the equilibrium depends on the probability of monetary backstop. Section 3.4 shows how the equilibrium changes when the cost of inflation can be externalized to other countries, which can be interpreted as the case of a currency union. Section ?? discusses extensions of the model.

### 3.1 Calibration

We calibrate the model using European data. The parameter values for the benchmark calibration are reported in table 1. A period is a year. The household discount rate is equal to 2 percent. The elasticity of labor supply is set to 0.2, which is consistent with various estimates in the microeconomic empirical literature based on US data (see Keane, 2011). Bargain, Orsini and Peichl (2012) review and compare the existing estimates of labor supply elasticities in Europe and in the US and find little difference between the two.

The productivity is assumed to follow an AR(1) process with mean  $\theta^* = 1$ . We estimate the stochastic process of productivity using quarterly HP-detrended data on hourly labor productivity and then compounding to obtain the autocorrelation coefficient and the volatility of productivity shocks at the annual frequency (see table 2).

The fiscal variables are calibrated to match their real-world counterparts. The level of government expenditure is set to 0.3. The implied tax rate when productivity is at its average level is  $\tau = 0.324$ , and the implied level of output is  $y = 0.925$ , so that the ratio of government expenditures to GDP,  $g/y$ , is equal to 32.4 percent.<sup>14</sup> The debt target is set to 0.56, or 60 percent of GDP for the average level of productivity, consistent with the Stability and Growth Pact.<sup>15</sup>

The coefficients in the fiscal rule (8) have been estimated by regressing the ratio of the primary balance to GDP on the ratio of government debt to GDP and on detrended productivity for euro area countries. A pooled OLS

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<sup>14</sup>The implied tax rate satisfies  $\tau(1 - \tau)^\sigma = g$  and the implied level of output is  $y = (1 - \tau)^\sigma$ . The numbers are slightly different if one adds the cost of servicing the debt to the government expenditures.

<sup>15</sup>Sixty percent of GDP is a ceiling rather than a target for government debt in the Stability and Growth Pact. However, assuming a target of 60 percent seems consistent with the observed behavior of euro area governments. The average debt-to-GDP ratio for euro area countries over the period 2000-08 (before the crisis) was 60.3 percent.

regression using annual data for the 17 euro area countries gives,

$$pb_t = 0.355 + 0.000 \cdot d_t + 0.475^{***} \cdot prod_t$$

(0.006)                      (0.166)

No.obs=254     $R^2 = 0.0315$

where  $pb$  is the ratio of the primary balance to GDP in percent,  $d$  is the ratio of government debt to GDP in percent, and  $prod$  is detrended log productivity in percent. The coefficient on productivity implies that a one-percent positive productivity shock increases the primary balance by about one half of a percentage point of GDP. Thus we set  $\alpha_\theta$  to 0.5 in our benchmark calibration. The coefficient on debt is not significantly different from zero. There is little evidence, therefore, that the condition for a passive fiscal rule ( $\alpha_b > 0.02$  by equation (11)) is satisfied in the data. However the model requires this condition to be satisfied for the debt-to-GDP ratio to be stationary. Thus, we set  $\alpha_b$  to 0.03, a value that is low but ensures stationarity (conditional on no default premium) in the benchmark calibration.

A default is assumed to reduce debt by 0.4, i.e., 43.2 percent of average GDP. The 2012 Greek debt restructuring reduced the face value of privately-held Greek government bonds by 57 percent of GDP, but it reduced total government debt by only about 30 percent of GDP because of the simultaneous increase in official crisis loans to the Greek government. The value that we set for  $\Delta b$  is between the two.

We set the cost of a default to  $\gamma^d = 0.055$ , approximately 5.5 percent of average GDP. The empirical literature does not give us precise estimates for the cost of a government default, in part because the direction of the causality in the negative association that is observed between default and growth is unresolved. In their review of the costs of *sovereign* defaults (that is, on governments' debt owned by foreign creditors), Borensztein and Panizza (2009) find that growth falls by 2.6 percentage points in the first year of a default episode. They do not find statistically significant evidence that output catches up to the pre-default trend in the following years, suggesting that the intertemporal cost of default might be substantially higher than the cost observed in the first year of the default.<sup>16</sup> Reinhart and Rogoff (2011) focus on domestic government debt defaults rather than sovereign defaults. They

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<sup>16</sup>Borensztein and Panizza control for a number of determinants of growth, including banking crises. This would tend to bias downwards their estimates of the cost of a default if banking crises are caused, or made more likely, by sovereign defaults.

find that, in the year of the default, GDP is 8 percent lower than GDP four years before the default, but that there is a sharper recovery after domestic defaults than after sovereign defaults. Given the difficulties involved in estimating the cost of default, we feel that this cost could be plausibly argued to be anywhere between 0 and 15 percent of GDP. As we will see, the value that we choose for  $\gamma^d$  in our benchmark calibration leads the government to default for realistic levels of debt (about 200 percent of GDP in the presence of a monetary backstop).

Our benchmark calibration assumes that the cost of high inflation and financial repression is the same as the cost of a default ( $\gamma^i = \gamma^d$ ). As we argue in the following section, the literature does not give us more precise estimates for the cost of high inflation and financial repression than for the cost of default and there are reasons to believe that these two costs have the same order of magnitude. Assuming that the two costs are equal is a way of focusing the attention on the key difference between default and inflation (in this model), which is that the risk of default generates a default premium whereas the risk of inflation does not.<sup>17</sup> From an analytical point this assumption is a natural starting point to discuss alternatives.

### 3.2 Cost of inflation and financial repression

The cost of high inflation and financial repression is equal to the welfare cost from reducing real money balances (see equation (20)). The utility of real money balances,  $v(m)$ , can be specified and calibrated by reference to the empirical literature that estimates the demand for money. In this literature, the demand for money is typically estimated by regressing the logarithm of real money balances on the opportunity cost of holding money and other determinants of the demand for money such as GDP or wealth. In our model the only determinant of the demand for real money balances is the opportunity cost of holding money (see equation (15)) so that the equation for money demand can be written,

$$\log\left(\frac{m_t}{m^*}\right) = -\xi x_t,$$

where  $m^*$  is the satiation level of money holdings, and  $\xi$  is the semi-elasticity of money demand with respect to the opportunity cost of holding money.

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<sup>17</sup>The risk of inflation does not generate an inflation premium on one-period nominal debt because inflation is assumed to be known one period ahead in this model.

Using equation (15) one then has  $v'(m) = -\log(m/m^*)/[(1+r)\xi]$ , which can be solved to give a closed-form expression for the utility of real money balances,

$$v(m) = v(m^*) - \frac{1}{(1+r)\xi} \left[ m^* - m + m \log \left( \frac{m}{m^*} \right) \right].$$

We can use this specification to compute seigniorage, as well as the welfare cost of inflation and financial repression in the inflationary regime.

We set the value of  $m^*$  to 1 so that M2 is close to 100 percent of average GDP in the normal regime, consistent with the levels observed in the euro area over the last ten years. The inflation rate and nominal interest rate on bank deposits are respectively set to 2 percent and 4 percent in the low-inflation regime, so that real money holdings are equal to the satiation level. The implied inflation rate,  $\tilde{\pi}$ , and welfare cost of high inflation,  $\gamma^i$ , can then be computed using equations (16) and (17) given the value of the semi-elasticity of the demand for broad money,  $\xi$ .

The empirical literature on money demand has produced a range of estimates for the semi-elasticity of money demand. We restrict the attention here on the literature that focused on the euro area. In an early study that has influenced the ECB's subsequent efforts to estimate money demand in the euro area, Calza, Gerdesmeier and Levy (2001) measured the opportunity cost of holding broad money (M3) as the spread between the 3-month rate and the rate of return on M3 assets, which is close to the definition of  $x$  in our model. Their estimate for the semi-elasticity of money demand was  $\xi = 0.86$  for the period 1980-1999. Money demand seems to have changed with the creation of the euro, however, and more recent studies have found a range of estimates for  $\xi$ , depending on the specification of the model. On one hand, Setzer and Wolff (2012) find lower estimates for  $\xi$  (close to 0.2) over 2001-2008 when using the long-term interest rate as the opportunity cost of holding money. On the other hand, Dreger and Wolters (2010) find that a large semi-elasticity of money demand (between 5 and 6) if the opportunity cost of holding money is measured by the inflation rate. Estimates for the semi-elasticity of money demand vary widely depending on how the opportunity cost of holding money is measured, as well as the other variables included in the money demand equation.

Given these uncertainties about the value of semi-elasticity of money demand, it may be prudent to look how the welfare cost inflation depends on  $\xi$  when this parameter takes values in a relatively wide range. Figure

4 shows the variations of the welfare cost of high inflation,  $\gamma^i$ , when  $\xi$  is between 0.1 and 3. Unsurprisingly, the welfare cost of inflation increases with the elasticity of money demand. The line on the left-hand side of the figure corresponds to the case where inflation is raised for one period (as assumed in the model). We observe that if  $\xi = 0.5$  the welfare cost of inflation is close to 0.05, i.e., it is of the same order of magnitude as the welfare cost of a default. We also observe that the line stops when the semi-elasticity of money demand exceeds 0.9 because in this case the maximum level of seigniorage is lower than  $\Delta b = 0.4$ . This is somewhat problematic since some estimates for the semi-elasticity of the demand for broad money are larger than 1.

Thus we also look at the case where the central bank raises seigniorage for a number of years  $n$  that is larger than 1. Since the welfare cost of raising inflation is a convex function of the inflation rate, spreading the inflation tax over several years reduces the welfare cost of inflation, measured as the present discounted value of the utility gap,  $v(m^* - v(\tilde{m}))$ . Figure 4 shows the variations of the welfare cost of inflation with  $\xi$  when inflation is high during 2 or 3 years instead of just one year. When inflation is raised for three years, the welfare cost of high inflation varies between 0.05 and 0.1 when the semi-elasticity of money demand is between 1 and 2.

Overall, it is difficult to pinpoint a precise estimate for the welfare cost of inflation and financial repression. But it is not difficult to calibrate the model in such a way that this welfare cost is of the same order of magnitude as the cost of a default. Thus, we start by assuming that the two costs are the same and will then explore the more general case where they are different.

### 3.3 Benchmark results

The model is solved numerically using the same iteration approach as in Aguiar and Gopinath (2006). The AR(1) process for productivity is approximated by a 51-state Markov process following the method of Tauchen and Hussey (1991).

First, we look at the region of the state space for which the government challenges the central bank. The government tends to challenge the central bank when the level of debt,  $b$ , exceeds a threshold because the welfare gain from reducing taxes on labor income is increasing with the level of these taxes, whereas the welfare cost from default or inflation is fixed. Thus, in the same way as one can define a "default frontier" in dsge models of sovereign debt (Arellano, 2008), one can define a frontier in the state space such that the

government challenges the central bank if and only if the state  $(\theta, b)$  is above the frontier. This is what we call the "challenge frontier". Figure 5 shows this frontier for different values of  $\mu$  under our benchmark calibration.

Figure 5 shows the challenge frontier for three different values of  $\mu$ : 1, 0.9 and 0.8. When  $\mu$  is equal to 1 or 0.9 the challenge frontier is almost exactly the same and it is close to  $b = 2$ , that is, the government starts to deviate from the fiscal rule and challenge the central bank when its debt exceeds 200 percent of GDP. If  $\mu = 0.8$ , the challenge frontier is much lower, at  $b = \Delta b = 0.4$ , that is, the government starts to challenge the central bank when its debt exceeds 40 percent of GDP. The challenge frontier cannot be lower than that since by assumption, the government cannot challenge the central bank if its debt is lower than  $\Delta b$  (an assumption that we have made to prevent post-default debt from being negative).<sup>18</sup> The challenge frontier for values of  $\mu$  lower than 0.8 is the same as when  $\mu = 0.8$ .

The fact that the challenge frontier shifts down so abruptly when  $\mu$  decreases from 0.9 to 0.8 may suggest a discontinuity. To see whether this is the case, we define  $\bar{b}(\mu)$  as the debt threshold above which the government challenges the central bank when productivity is at its average level ( $\theta = \theta^*$ ). In principle the debt threshold also depends on  $\theta$  but as one can see in figure 5, the slope of the challenge frontier is close to zero, meaning that the government's decision to challenge the central bank depends on the level of debt rather than on the level of productivity.<sup>19</sup>

Figure 6 reports the variation of  $\bar{b}$  with  $\mu$ . We indeed observe a discontinuity when  $\mu$  falls below a threshold of 0.83. Then the government starts to challenge the central bank for much lower levels of debt (in fact whenever this is possible). In other words, the government's willingness to apply the fiscal rule (8) is heavily conditioned by the central bank's willingness to provide a monetary backstop with a high probability.

The intuition behind this result involves the interest rate at which the government can roll over its debt. For  $\mu = 1$ , the government does not

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<sup>18</sup>This begs the question of how the equilibrium looks like when we allow debt to be reduced by less than  $\Delta b$  when it is lower than this level. We have studied a variant of the model in which post-default debt is reduced to zero if  $b < \Delta b$ . Then the challenge frontier goes all the way down to  $b = 0$  when  $\mu$  is reduced.

<sup>19</sup>Closer inspection shows that the challenge frontier is positively sloped if it is above  $\Delta b$ . The reason is that if productivity is lower it takes a higher tax rate on labor income to achieve the same primary balance, so that the benefit from challenging the central bank is larger. But this effect is small so that the slope of the frontier is close to zero.

pay a default risk premium and is always able to roll over its debt at the riskless interest rate. Thus, it challenges the central bank opportunistically when the cost of default (or inflation) is lower than the benefit from lowering distortionary taxes conditional on the assurance that it is and will always be possible to borrow at the riskless interest rate. Since the cost of default, at 5.5 percent of GDP, is rather substantial, this occurs only for relatively high levels of debt, about 200 percent of GDP.

For lower levels of  $\mu$ , however, the government may have to pay a default risk premium in order to roll over its debt. If productivity and the primary balance are low, debt is bound to increase and (if debt is just below the challenge frontier) the government is very likely to challenge the central bank in the following period, which may provoke a default. Thus the government has a choice between rolling over its debt at a high interest rate today and probably challenging the central bank tomorrow, or challenging the central bank today. If it is very likely that the government is going to deviate from the fiscal rule anyway, then it is optimal to do it today at a lower level of debt than tomorrow when the debt has been increased by the high interest rate at which it was rolled over. Thus, a low  $\mu$  tends to shift the challenge frontier down in the state space for low levels of  $\theta$ . For high levels of  $\theta$  there is a risk that  $\theta$  will fall so that the government has to pay a default premium too, and the same logic applies. This shifts down the whole challenge frontier.

The discontinuity in the dependence of the challenge frontier on  $\mu$  comes from the fact that the government’s decision to challenge the central bank—rather than rolling over its debt at a high interest rate—is largely independent of the level of debt. Once  $\mu$  is low enough to make the government unwilling to roll over its debt at a high interest rate, then the challenge frontier cannot stay “in between”  $b = 2$  and  $b = 0.4$ . If it did, then the government would roll over its debt at a high interest rate when  $b$  is just below the frontier—a contradiction. Thus, when the equilibrium in which the government rolls over its debt without challenging the central bank starts to unravel, it unravels completely.

Figure 7 then shows the variations with  $\mu$  of the unconditional frequency of fiscal challenges, inflation and default. We construct this figure based on model simulations over 50,000 periods for different values of  $\mu$ . Figure 7 gives rise to several interesting observations. First, the frequency of fiscal challenges decreases with the probability of a monetary backstop, to (almost) zero when  $\mu$  is larger than 0.83. When  $\mu$  exceeds 0.83, the debt threshold for a fiscal challenge is about 200 percent, a level that is almost never reached

if the government follows the fiscal rule and rolls over its debt at the riskless interest rate.<sup>20</sup>

If there is no monetary backstop at all ( $\mu = 0$ ) the unconditional frequency of a fiscal challenge is 14 percent, i.e., there is a fiscal challenge every 7.1 years on average and it leads every time to a default. The frequency of fiscal challenges decreases continuously with the probability of monetary backstop, and falls to about 3 percent when  $\mu = 0.5$ . Recall that in this case the government challenges the central banks as soon as its debt exceeds  $\Delta b$ , i.e., 40 percent of average GDP. But in equilibrium debt stays most of the time below this level as it is occasionally reduced by a default and the fiscal rule implies that it increases toward the target  $b^*$  very slowly (because  $\alpha_b$  is small). If  $\mu$  is higher, the government pays a lower interest rate before defaulting and post-default debt is thus lower.

Our last observation is that the risk of inflation is maximized for intermediate values of  $\mu$ . Uncertainty about the monetary backstop combines the worst of both worlds from the point of inflation. It makes the government prone to challenge the central bank and at the same time makes it possible for these challenges to be followed by inflation.

### 3.4 Currency union

So far, we have considered an economy with one fiscal authority and one monetary authority. This is a realistic assumption for most countries, not for countries that are in a monetary union. In a monetary union, less inflation is required at the level of the union to rescue a given member country from a default. The welfare cost of extracting seigniorage is diluted across the members of the union. This changes the incentives and may make individual members more likely to deviate from the fiscal rule.

We capture the case of a currency union in tractable way by making the following assumption. The seigniorage revenue,  $\tilde{z}$ , is the same as before but it is extracted from a union whose size is  $n$  times the size of the country

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<sup>20</sup>If  $\mu = 1$ , the risk of an inflationary episode is not mathematically zero, it is the probability that debt exceeds the threshold  $\bar{b}$ . If the objective is to have a strictly zero risk of inflation, the only way of achieving this objective is to set  $\mu$  to 0. Unlike in models of lending-in-last-resort with multiple equilibria, a monetary backstop does not provide a completely free lunch—the backstop has to be provided with a nonzero probability in equilibrium.

under consideration. The level of money demand under high inflation rate, thus, satisfies

$$\frac{\Delta b}{n} = \tilde{m}v'(\tilde{m}).$$

This equation replaces equation (17), which corresponds to the special case where  $n$  is equal to 1. The same level of seigniorage revenue can be extracted with lower inflation and a lower welfare loss if  $n$  is larger than 1.

We set the semi-elasticity of money demand,  $\xi$ , to 0.489, the level such that the cost of inflation is equal to the cost of default ( $\gamma^i = \gamma^d$ ) when there is one single fiscal authority. We then compute the cost of inflation when the number of fiscal authorities,  $n$ , increases. For  $n = 2$ , the cost of inflation for a given country is  $\gamma^i = 0.0115$ , i.e., it is about one fifth of the cost of high inflation when there is one country. If there are three countries ( $n = 3$ ), the cost of inflation is  $\gamma^i = 0.0048$ , i.e., one half of a percent of average GDP. The cost of high inflation for a given country decreases more than proportionately with the number of countries because the marginal cost of raising fiscal revenue through the inflation tax is increasing with the level of revenue to be raised.

How do the results change when the number of countries,  $n$ , is larger than 1? The answer is, quite a lot. Our numerical simulations show that as soon as the number of countries is 2 or larger, the government starts to challenge the central bank as soon as debt increases above  $\Delta b$ . The reason is that the low cost of raising inflation (given that this cost is externalized to other countries) makes the monetary backstop an attractive option at a much lower level of debt. As a result, the debt threshold above which the government challenges the central bank is  $\bar{b} = \Delta b$  irrespective of  $\mu$  (see figure 8).

Thus, providing a monetary backstop may be much more destabilizing in a currency union than in more conventional monetary arrangements because this gives member countries incentives to free-ride on union-wide seigniorage. This may explain why it may be more important to avoid a monetary backstop in the euro area than, say, in the US, the UK or Japan. However, the absence of a monetary backstop is also destabilizing because it generates a default premium in the government debt market. This may explain why there is a government debt crisis in the euro area and not in the US, the UK or Japan. A currency union, thus, puts the central bank between a rock and a hard place when it comes to the decision of whether or not to provide a monetary backstop to governments' debts.

### 3.5 Extensions [to be completed]

The government's debt could be nominal instead of real. This would not change our results at all because we have assumed that the expected inflation rate remains equal to  $\pi^*$  in the low-inflation regime. The monetary backstop does not reduce debt by inflating it away through an inflationary surprise but by increasing seigniorage revenue over time.

The results would be very different obviously if debt were nominal and the monetary backstop operated through an inflationary surprise. Then the interest rate on nominal debt would bear an inflation premium and so would give rise to the same problems as the default premium on defaultable real debt.

Debt could be real and have a maturity longer than one period. This should not change the essence of our results if all the debt principal is accelerated in a default. However, if debt were long-term *and* nominal there would be an inflation premium even under our assumption that there is no inflationary surprise at a one-period horizon. The implication is probably that the government would endogenously choose to borrow at short maturities when there is a monetary backstop.

## 4 Conclusion

We have presented a model in which a fiscal authority can challenge the monetary authority's dominance by deviating from a passive fiscal rule, following which the monetary authority can respond either by letting the fiscal authority default or by providing a monetary backstop. We have shown that the monetary authorities' insistence to enforce monetary dominance may paradoxically induce the fiscal authorities to challenge this dominance more aggressively, because the alternative is to pay default premia that make the debt dynamics unsustainable. If the fiscal rule is calibrated by reference with European data, we find that government debt is sustainable only if the central bank provides a monetary backstop with a very high probability. However, debt sustainability also requires the cost of the monetary backstop to be high for the deviating fiscal authority (not much lower than the cost of a default), a condition that may not be satisfied in a currency union. The model provides a possible explanation for the fact that a government debt crisis has

occurred in the euro area in spite of the fact that the fiscal fundamentals were a priori not much worse there than in the US, the UK or Japan.

Our model captures the incentives effects of monetary policy on fiscal policy in the sense that the government decides to deviate from the fiscal rule or not taking into account the probability of a “bailout” by the monetary authorities. But there are other ways of looking at the incentives problem.

In particular, there could be a choice made about the fiscal rule itself. The target debt level could be lower, and the primary balance could be more responsive to the debt level, or less responsive to the level of productivity. In other words, one could use the model to address the question of the fiscal rules that make debt sustainable given the absence of monetary backstop. Our calibrated model suggests that such rules are likely to be different from the observed behavior of European fiscal authorities.

Table 1: Benchmark calibration

Households	
Discount rate	$r = 0.02$
Labor supply elasticity	$\sigma = 0.2$
Firms	
Productivity process	$\rho = 0.15$ $stde(\epsilon) = 1.30\%$
Government	
Government expenditure	$g = 0.3$
Debt target	$b^* = 0.56$
Responsiveness of primary balance to debt	$\alpha_b = 0.03$
Responsiveness of primary balance to productivity	$\alpha_\theta = 0.5$
Debt Crisis	
Debt reduction in a default	$\Delta b = 0.4$
Cost of default	$\gamma^d = 0.055$
Cost of inflation and financial repression	$\gamma^i = \gamma^d$

Note: The results are from the regression  $\log(\theta_t) = \rho_q \log(\theta_{t-1}) + \varepsilon_{qt}$ , where  $\theta_t$  is the HP-detrended hourly labor productivity at the quarterly frequency. The autocorrelation coefficient at the annual frequency is then computed as  $\rho = \rho_q^4$  and the standard error at the annual frequency is computed as  $stde(\varepsilon) = stde(\varepsilon_q) \sqrt{1 + \rho_q^2 + \rho_q^4 + \rho_q^6}$ . (Source: ECB, ESA National Accounts)

Table 2: Productivity process

Country	$\rho$	stde( $\varepsilon$ ), %
France (1980-2012)	0.43	0.68
Germany (1991-2012)	0.18	0.95
Ireland (1998-2012)	0.03	1.53
Italy (1992-2012)	0.25	0.93
Portugal (1995-2012)	0.07	1.10
Spain (2000-2012)	0.00	0.55
Average	0.15	0.96
17 Euro Countries (2000-2012)	0.13	1.75

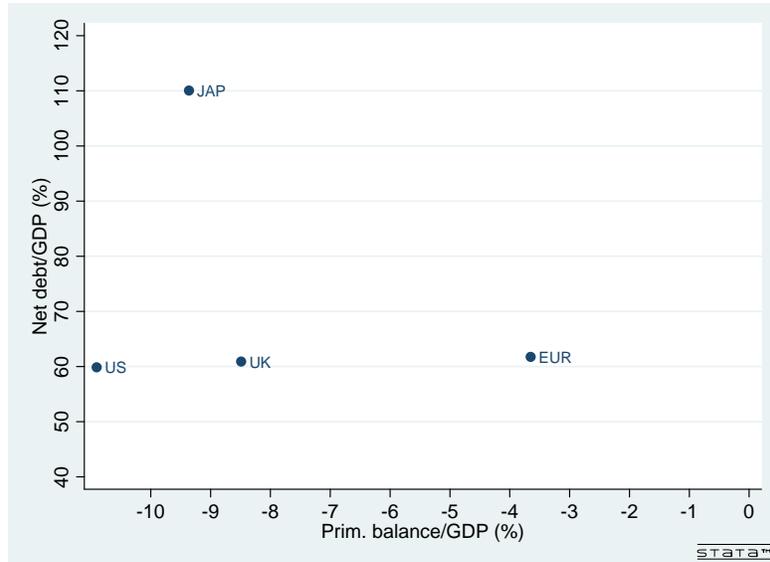


Figure 1: Fiscal fundamentals in advanced economies (2009, WEO)

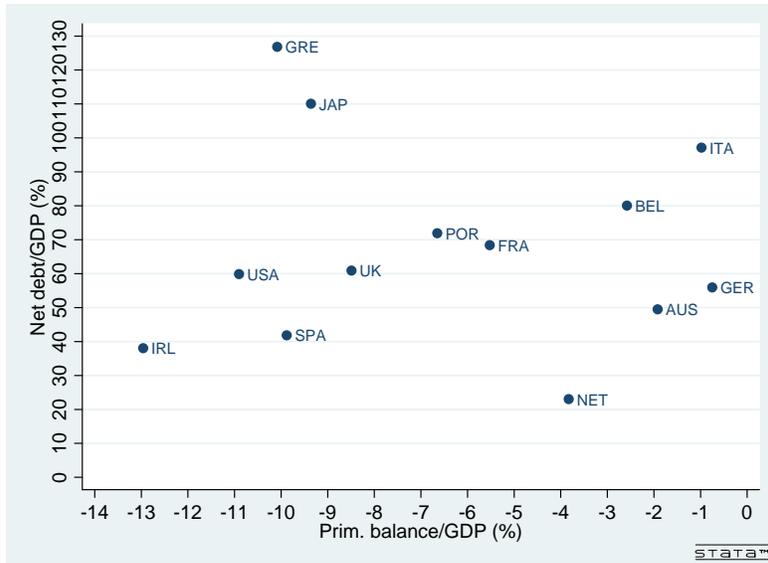


Figure 2: Fiscal fundamentals in advanced economies (2009, WEO)

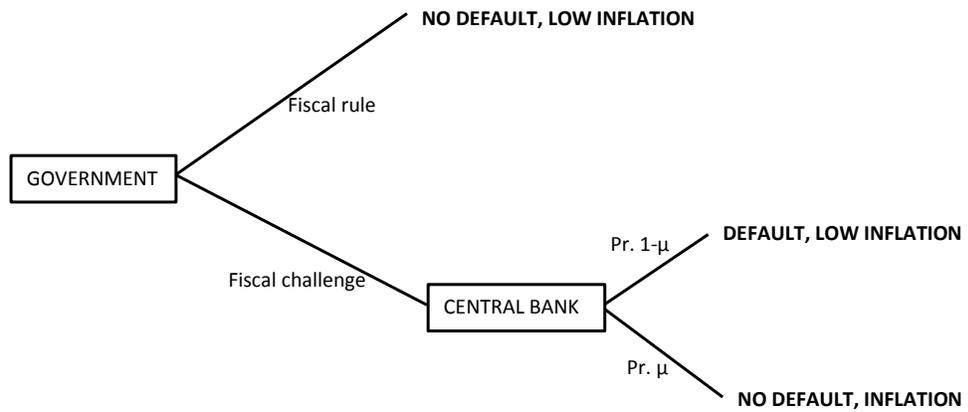


Figure 3: Sequence of events and actions

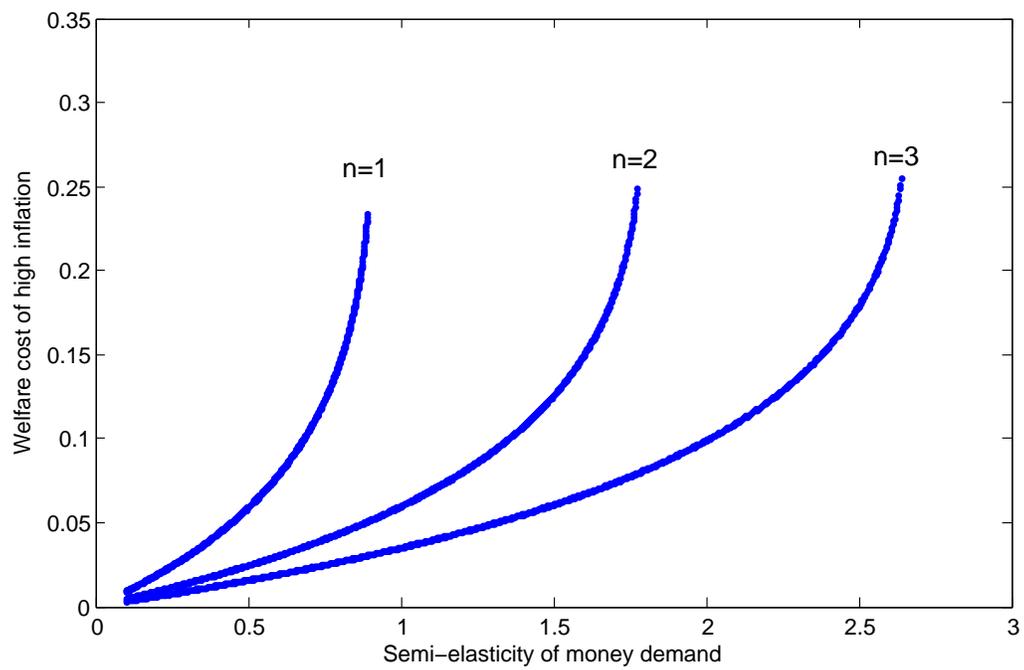


Figure 4: Variation of welfare cost of inflation ( $\gamma^i$ ) with semi-elasticity of money demand ( $\xi$ )

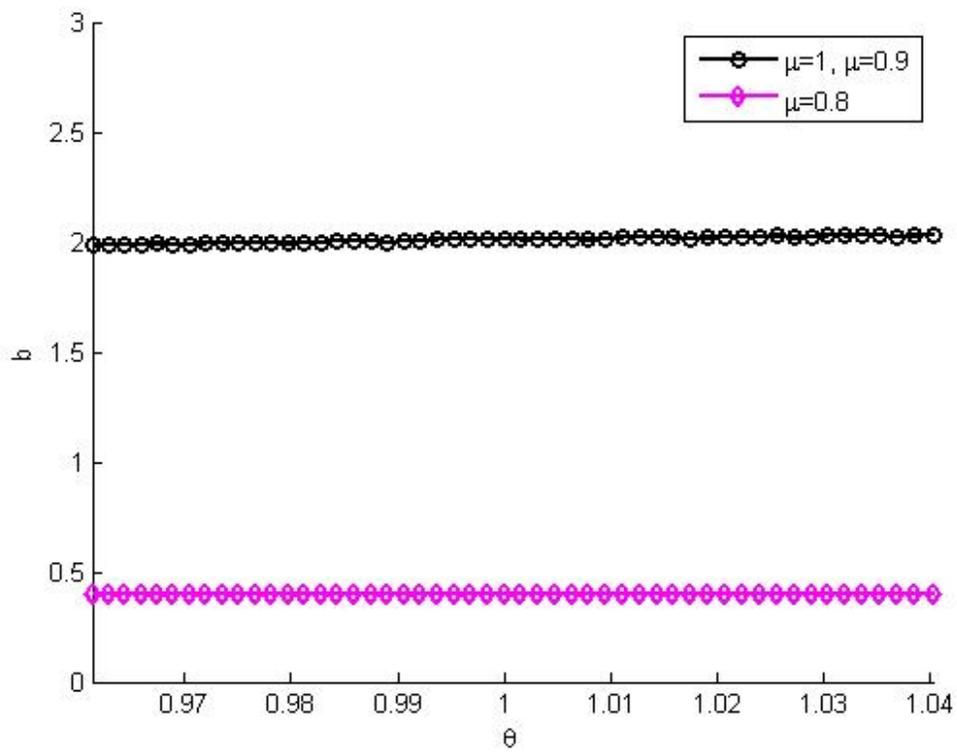


Figure 5: Challenge frontier in the state space

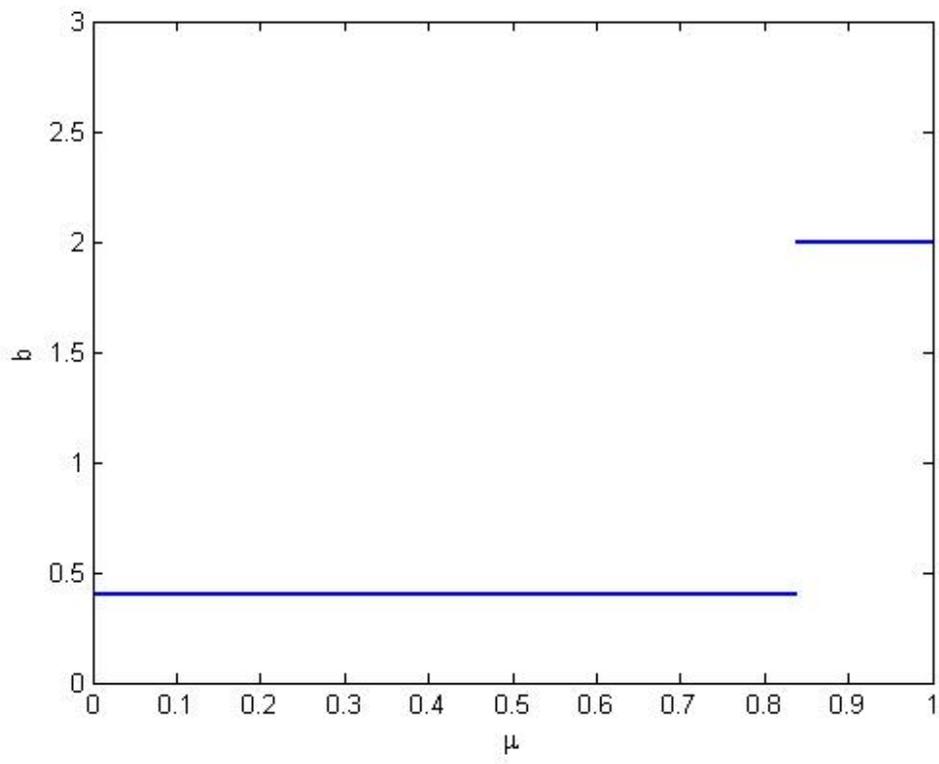


Figure 6: Variation of challenge debt threshold  $\bar{b}$  with  $\mu$

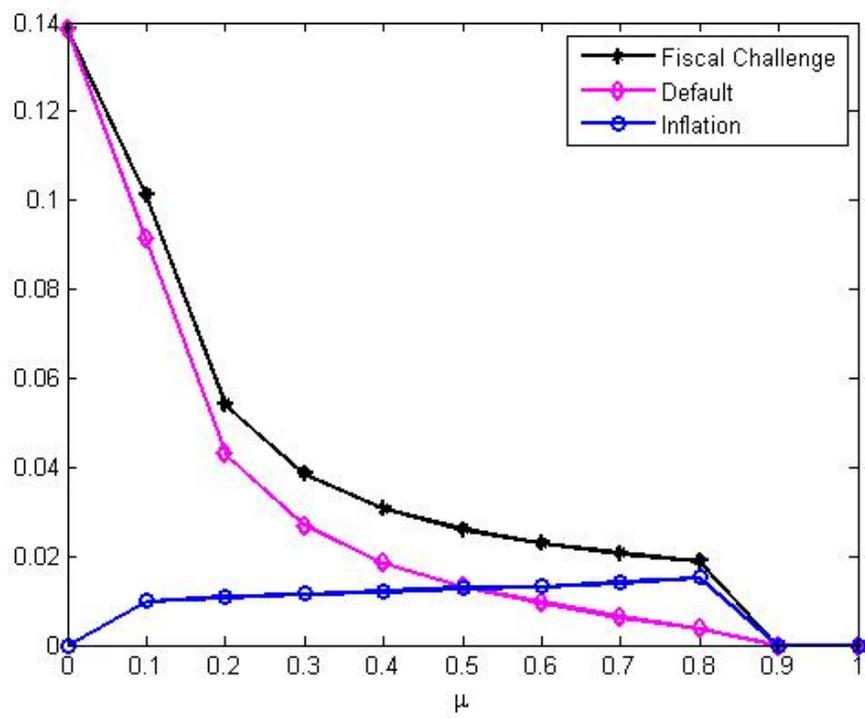


Figure 7: Frequency of fiscal challenge, default and inflation

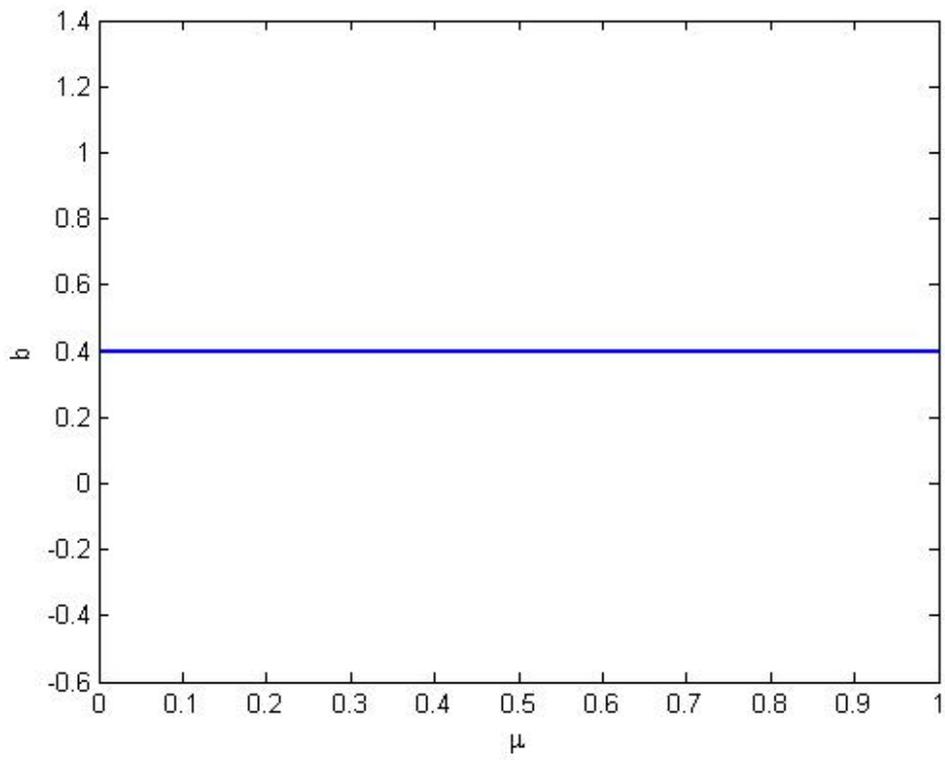


Figure 8: Variation of challenge debt threshold  $\bar{b}$  with  $\mu$  in a currency union with two countries or more

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