

On gender gaps and self-fulfilling expectations: An alternative approach based on paid-for training*

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ABSTRACT

This paper presents a model of self-fulfilling expectations by firms and households which generates multiplicity of equilibria in pay and housework time allocation for ex-ante identical spouses. Multiplicity arises from statistical discrimination exerted by firms in the provision of paid-for training to workers, rather than from incentive problems in the labor market. Employers' beliefs about differences in spouses' reactions to housework shocks lead to symmetric (ungendered) and asymmetric (gendered) equilibria. We find that: (i) the ungendered equilibrium tends to prevail as aggregate productivity in the economy increases (regardless of the generosity of family aid policies), (ii) the ungendered equilibrium could yield higher welfare under some scenarios, and (iii) gender-neutral job subsidies are more effective than gender-targeted ones in removing the gendered equilibrium.

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1. Introduction

The recent literature on gender differences in the labor market has focused mainly on two issues. The first one deals with the mechanisms (e.g. adverse selection) that can amplify initial differences in preferences or productivity across genders leading to sizeable gender gaps in wages, labor force participation or working hours. These models help us understand to what extent changes in gender gaps have been due to changes in fundamentals, including differences in the pattern of endogenous accumulation of human capital by men and women.¹ The second one is related to the channels explaining persistence in gender gaps, even when exogenous differences across the sexes are of little importance or even absent.² These studies tend to rely upon the existence of incentive problems in the labor market (e.g., moral hazard) leading to self-fulfilling prophecies about differences in gender roles without assuming any initial comparative advantages. Our paper falls into this second strand of the literature by considering an alternative mechanism based on statistical discrimination by firms in the provision of paid-for training to male and female workers. This yields multiplicity of equilibria in gender gaps even in the absence of moral hazard.³

The basic idea in models of gender gaps relying upon self-fulfilling prophecies is that employers' beliefs about women's lower attachment to the labor market lead to wage differentials in favour of men. Hence, since women expect to face a lower expected opportunity cost, they end up devoting more time to housework, validating in this way firms' beliefs. For example, Albanesi and Olivetti (2009) propose a model where firms are subject to incentive compatible constraints due to their imperfect monitoring of workers' effort (a *moral hazard* problem) and of hours of housework (an *adverse selection* problem). As a result, different types of labor contracts are offered to men and women. In a similar vein, Lommerud and Vagstad (2007) deal with a model where firms allocate workers to fast and slow track jobs, the former requiring a fixed investment cost. If women have traditionally exerted primary major responsibility at

¹ See, *inter alia*, Erosa *et al.* (2010), Flabbi (2010) and Gayle and Golan (2010).

² See, *inter alia*, Francois (1998), Engineer and Welling (1999), Albanesi and Olivetti (2009), Lommerud and Vagstad (2007).

³ Following the seminal work by Arrow and Phelps, there is a large literature on statistical discrimination leading to asymmetric treatment in equilibrium of *ex ante* identical groups. In particular, our model deals with some of the issues raised earlier by Moro and Norman (2003, 2004), namely, the interaction between an informational externality and general equilibrium effects. However, whereas their cross-group externalities stem from the marginal productivity in market work of one group being affected by the size of another group (say the ratio of black and white workers in a given occupation), ours relies upon household decisions on the division of housework interacting with firms' decision on training.

home and wages are non-contractible, they will predominantly follow a “mommy track” in equilibrium.

While incentive problems - often used to derive additional predictions on the structure of wages to be confronted with the data – provide a useful modelling device, they could be somewhat restrictive. For instance, concerning the difficulty of perfectly monitoring effort, pay gaps should be negligible for routine tasks performed by less-skilled employees for whom effort and output should be easily observable. Yet, even after removing differences in observable characteristics and in the overall wage dispersion, there is evidence about substantial gender gaps in hourly wages in these categories, (see, e.g., Blau and Kahn, 2000, Bassanini and Saint Martin, 2008, and de la Rica *et al.* 2008).

Our paper contributes to this literature by proposing a novel mechanism which does not rely on incentive problems. We consider a two-period model in which firms initially offer paid-for training to workers, taking as given the division of housework between spouses. In the second period, spouses decide on how to split the certain and uncertain parts of housework, where the latter is captured by the realization of a shock affecting household tasks during this period (e.g., unexpected need of housework or events that require parental leave, etc.). Both decisions interact because the housework decision determines whether a spouse works or quits after receiving training and the firm chooses the amount of training without observing the household’s housework allocation. Statistical discrimination arises in this setup as a result of firms’ prior beliefs about which spouse is more likely to quit the job. Under the key assumptions that wages are predetermined with respect to housework shocks and that the household does not re-optimize after the shock is revealed, asymmetric beliefs will induce differences in the provision of training across spouses and, as a result, they will lead to gender wage differences. Further, since spouses also form beliefs about their relative wages when allocating housework, this mechanism leads to self-fulfilling prophecies and therefore to a multiplicity of equilibria. Indeed, under some assumptions about the distribution of household shocks, the disutility of housework and the degree of diminishing returns to training, two types of equilibria arise: (i) an *ungendered* equilibrium, with a fully egalitarian division of housework and equal pay, and (ii) a *gendered* equilibrium, where one of the spouses earns a higher wage and devotes less time to housework.

Our model is able to generate some novel predictions about the relationship between the division of housework and the gender wage gap. First, we find that, under

plausible conditions, the gendered equilibrium tends to vanish in economies with higher aggregate productivity (i.e., where training results in larger market productivity gains). Secondly, as regards the role of policies in reducing gender gaps, we find that, under our assumptions, gender-neutral policies tend to be more effective than gender-based policies. In particular, when a *gendered* equilibrium prevails, we show that job subsidies targeted at women can backfire by shifting the economy to an even more unequal equilibrium.⁴ Lastly, in contrast to most existing work using incentive problems (see, e.g., the discussion in Lommerud and Vagstad, 2007), we find that welfare could be higher in the symmetric than in the asymmetric equilibrium. Notice that the converse result is often found in the literature because an asymmetric equilibrium promotes some form of “efficient specialization” in the labor market. Although this channel is also present in our model, under the rather plausible assumption that the disutility of housework is convex and minimized under an even split of these tasks between *ex-ante* identical spouses, the latter effect may dominate in some scenarios (notably in *high-productivity* economies).

Finally, it is important to stress that there could be other mechanisms at play which are somewhat akin to training. For example, there is abundant evidence about the over-representation of women (with the same observable characteristics than men) in college degrees with lower market returns (despite performing better than men in high school) which has been found to explain a substantial part of the gender wage gap (see, e.g., Machin and Puhani, 2003).⁵ Although paid-for training differs from education (or employee-paid training) in that the former is paid by the firm and the latter by the individual, it could well be the case that women choose less demanding degrees than men anticipating firms’ reactions to their higher probability of future career interruptions. Notwithstanding, we opt to focus on paid-for training not only because it provides as a simpler device to capture the interaction between employers’ and households’ decisions, but also because there is ample empirical about a lower intensity of on-the-job training for women than for men even in full-time jobs (see, *inter alia*, Altonji and Spletzer, 1991, Barron *et al.*, 1993, Royalty, 1996, for the US, de la Rica *et al.*, 2008, for Spain, and Puhani and Sonderhof, 2010, for Germany).

⁴ Moro and Norman (2003) analyze a model of racial statistical discrimination with human capital investments where they also find that affirmative action may result in higher wage inequality across racial groups, in the spirit of Coate and Loury’s (1993) seminal work on the effects of this type of policies.

⁵ For example, the fraction of female undergraduates in humanities degrees is much higher than in engineering or hard sciences (see Hunt, 2010).

The rest of the paper is organised as follows. Section 2 lays out the model. Section 3 discusses the properties of the different equilibria. Section 4 deals with welfare analysis. Section 5 analyses the effects of using different policies to eliminate the asymmetric equilibrium. Section 6 provides empirical evidence on some of the main predictions of the model using micro-data from a time-use survey in Spain. Finally, section 7 concludes. Further data descriptions and some algebraic derivations are relegated to three Appendices.

2. Modelling gender gaps

2.1 *The basic setup: A training model*

To account for the joint presence of gender gaps in wages and housework, we build a model of firms' and households' decisions which relies on the substantial available evidence about firms providing specific paid-for training to their workers (OECD, 2003). To simplify the analysis, we ignore issues about the provision of this type of training in frictional labor markets (Acemoglu and Pischke, 1998) and simply take this fact as given.⁶

The basic setup is as follows. *Ex ante* identical men and women live for two unit-length periods (1 and 2). Each gender represents half of the overall population whose mass is also set equal to unity. In period 1, firms are randomly matched with just one worker of either gender who is assumed to be single. The firm then invests an amount of (specific) training on the worker, τ , where the cost of training is assumed to be linear and equal to τ . For simplicity, we assume that workers are not paid while being trained. Finally, there is free entry of firms in period 1 until the expected profits from training workers are driven down to zero.

At the start of period 2, individuals of each gender form couples (exogenously) and decide how to split the household chores on the basis of their relative wages, where s and $(1 - s)$ denote the shares of housework performed by women and men, respectively. These household tasks in period 2 are assumed to involve two components which require the same dedication: one which is known with certainty and another which is uncertain (stochastic). One major assumption is that, once these shares are agreed, the

⁶ Our results would not be affected if the firm pays for only part of the training and the worker pays for the rest. What is crucial is simply that the employer engages in statistical discrimination when choosing how much training the firm finances.

firm will not internalize the response of the household since wages are predetermined. Thus, it cannot affect the time allocation decision of the spouses once the household shock is realized. At the start of period 2, before the shock occurs and taking as *given* the wages offered by firms, spouses choose the division of housework regarding both housework components which they do not change later. Accordingly, workers will always get trained in period 1 and they will not quit in period 2 insofar as the value of staying with the firm is higher than the outside option. Next, the disutility shock is realized, participation decisions made, and production takes place at the predetermined wage by those who remain in employment. If the worker quits, no wage is paid.

2.2 Firms' decisions

To solve for the wage and the amount of paid-for training, we proceed by backward induction, first considering decisions in period 2 and later in period 1. Under the assumption that the wage offer is made before the disutility shock ω_i is realized, firms will choose the wage W_i to maximize expected gross profit in period 2, Π_i , taking into account that the worker may quit after being trained. This leads to the following optimization problem:

$$\max_{W_i} \Pi_i = \max_{W_i} \int_0^{W_i} [a(\tau_i) - W_i] \frac{1}{\varepsilon_i} d\omega = \max_{W_i} \frac{a(\tau_i)W_i - W_i^2}{\varepsilon_i}, \quad i = f, m \quad (1)$$

where the integral in the middle term of (1) captures the firm's expected gross profit when the worker does not quit. Hence, the first-order condition (hereafter, f.o.c.) with respect to the wage implies that the wage paid in equilibrium, W_i^* , satisfies:⁸

$$W_i^*(\tau) = \frac{a(\tau)}{2}. \quad (2)$$

which, in turn, leads to the following expected gross profit in period 2:

$$\Pi(\tau_i) = \left[a(\tau) - \frac{a(\tau)}{2} \right] \frac{W_i^*}{\varepsilon_i} = \frac{a(\tau)^2}{4\varepsilon_i}, \quad (3)$$

where the term W_i^* / ε_i captures the probability of not quitting, i.e., $\Pr(\omega_i \leq W_i)$.

Once the firm's decision on the wage (conditional on training) has been established in (2), we move back to period 1 when the firm chooses the level of training.

⁸ This is just the average of the worker's productivity and the outside wage, which is assumed to be zero. The weight $\frac{1}{2}$ in the wage is due to the choice of the uniform distribution. Alternative distributions will give rise to a weighted average with unequal weights.

Since we are assuming free entry, the zero-profit condition pins down the equilibrium level of training in period 1, τ^* , which is given by:⁹

$$\Pi(\tau^*) - \tau^* = 0. \quad (4)$$

Hence, under the functional form assumed for $a(\tau)$, τ^* is chosen to be:

$$\tau_i^* = \left(\frac{\beta^2}{4\varepsilon_i} \right)^{\frac{1}{1-\alpha}}, \quad (5)$$

which, after being replaced into (2), yields the optimal wage:

$$W_i^* = \left(\frac{\beta^2}{4\varepsilon_i^\alpha} \right)^{\frac{1}{2(1-\alpha)}}. \quad (6)$$

Equations (5) and (6) imply that, as the size of the disutility shock becomes larger (i.e., as ε_i increases), workers face a higher probability of quitting in period 2 and, since this reduces expected profits, firms respond by lowering the amount of paid-for training and therefore wages. Note that our assumption that $0 < \alpha < 1$ plays a crucial role in this result. If $\alpha \geq 1$ (i.e., if there were *weak* diminishing returns in training) then the firm would respond to a higher probability of quitting by *increasing* the amount of training, raising the wage to offset the higher expected value of the shock. Our assumption of strong diminishing returns to training prevents this rather counterintuitive outcome.

From (6), the probability of working ($P_i^* = \Pr(\omega_i \leq W_i^*) = W_i^* / \varepsilon_i$) and the expected wage ($P_i^* W_i^* = W_i^{*2} / \varepsilon_i$) are given respectively by:

$$P_i^* = \left(\frac{\beta^2}{4\varepsilon_i^{2-\alpha}} \right)^{\frac{1}{2(1-\alpha)}} \quad (7)$$

$$P_i^* W_i^* = \left(\frac{\beta^2}{4\varepsilon_i} \right)^{\frac{1}{1-\alpha}}. \quad (8)$$

As before, a larger value of ε_i results in lower participation and a lower expected wage since $\alpha \in (0, 1)$.

Further, the following assumption is needed:

⁹ Alternatively, we could have allowed for positive profits and have the firm maximize profits with respect to training. Equilibrium training would be lower, but all the comparative statics would remain unchanged.

Assumption 1: Let $b_i \equiv (\beta^2 / 4\varepsilon_i)^{1/1-\alpha}$ and $b_1 \equiv (\beta^2 / 4\varepsilon)^{1/1-\alpha}$. We suppose that b_i verifies $b_i \leq \varepsilon_i$ for $i = m, f$ and that $b_1^{1-\alpha/\alpha} < 1/2$.

The inequality $b_i \leq \varepsilon_i$ ensures that the unit length of period 2 is not exceeded in the labor market, namely, that $P_i^* \leq 1$ for both types of agents. Notice that, since this inequality can be rewritten as $\beta^2 \leq 4\varepsilon_i^{1-\alpha}$, it requires the productivity parameter (β) to be not too large relative to the size of the shock; otherwise, the resulting wage would be sufficiently high (relative to the individual shock) to lead to a corner solution for participation in the labor market. As will be seen later, the second inequality, $b_1^{1-\alpha/\alpha} < 1/2$, ensures that an interior solution exists in the household decision problem. Finally, since this inequality is equivalent to $\beta^2 \leq \varepsilon 2^{2-\alpha}$, it again requires that β is not too large.

3. Household division of labor and multiplicity of equilibria

3.1 Household division of labor

The next step is to endogenize the household's housework allocation decision at the beginning of period 2 (once couples have been formed). We assume that there is a household good to be produced by the spouses, and that this good provides a fixed utility level denoted by \bar{u} . As mentioned earlier, the production of this good involves two utility costs. Part of the cost is perfectly known in advance, while the remaining component is uncertain (stochastic) and depends on the uniformly distributed disutility shock. To provide a specific example, suppose that the household good consists of raising a given number of children. Children have to be collected from school and ferried to their after-school activities every day, imposing a (known) utility cost to the parent in charge of this task, irrespective of whether he/she is employed or not. Additionally, there are shocks, such as a child falling sick and needing to stay home with a carer.¹⁰ The latter impose an opportunity cost only if the parent is working since they imply a reduction in the (monetary) utility derived from market work.

In line with our assumption of ex-ante identical individuals, we consider a

¹⁰ Note that the random shock need not be solely related to the presence of children in the household. Other examples could be the need to stay at home to undertake unexpected house repair, etc.

unitary household, so that there is full-income sharing and no bargaining between its members. Hence, on the basis of their expected wages, the spouses decide jointly on how to split the responsibility for production of the household good by choosing the fraction $s \in [0,1]$ of the total household chores allocated to the wife and $(1-s) \in [0,1]$ to the husband which maximize the household's utility function given by:

$$V^H = \bar{u} + Y(s) - [c(s) + c(1-s)], \quad (9)$$

where $c(s)$ and $c(1-s)$ are the disutility costs of housework incurred by the woman and the man, respectively.¹¹ These costs are assumed to be increasing and convex in the amount of housework undertaken, i.e. $c' > 0$ and $c'' > 0$. Expected net household income, $Y(s)$, is a function of the allocation of housework, given by:

$$Y(s) = \int_0^{W_m/(1-s)} \frac{W_m - (1-s)\omega}{\varepsilon} d\omega + \int_0^{W_f/s} \frac{W_f - s\omega}{\varepsilon} d\omega \quad (10)$$

where (10) also captures the fact that a woman (man) will quit the firm if $W_f < s\omega$ (respectively, $W_m < (1-s)\omega$) and that the monetary loss induced by the shock is only borne by the individual when employed.

Hence, because the division of housework takes place before ω is realized, partners choose s to maximize (9) subject to (10). Actual income and hence actual utility will however differ across households depending on the realization of the shock. Recall that a key assumption is that there is no change in s after the shock takes place. Under this assumption, the f.o.c. and the s.o.c. for a maximum yield:

$$\frac{\partial V^H}{\partial s} = Y'(s) - c'(s) + c'(1-s) = 0, \quad (11)$$

$$\frac{\partial^2 V^H}{\partial s^2} = Y''(s) - c''(s) - c''(1-s) < 0 \quad (12)$$

To understand the determinants of the household decision, it is convenient to consider separately the two components (besides \bar{u}) in V^H : expected income and the disutility of housework. As regards the former, integration of (10) yields:

$$Y(s) = \frac{1}{2\varepsilon} \left[\frac{W_m^2}{(1-s)} + \frac{W_f^2}{s} \right]. \quad (13)$$

It is straightforward to verify that $Y(s)$ is a U-shaped function of s implying that

¹¹ The assumption we are making here is that time not spent in the labour market is not necessarily leisure since it may be devoted to household production. See Rapoport et al. (2010) on the implications of such a formulation.

expected household income is maximized when the household member with the highest wage undertakes a sufficiently low amount of housework to never withdraw from the labor market.¹² This is the traditional division of labour emphasized by Becker (1985, 1991) as a result of comparative advantage. In our alternative setup, however, the intuition is simply that the expected income loss is largest when both spouses bear some of the cost because a large shock may imply that both individuals quit employment. Hence, there is a *participation effect* which leads to specialization since expected household income is maximized when the household member with the lower wage bears the whole shock since this ensures full labor market participation of at least one of them.

The second element of V^H is the convex cost of housework by each of the spouses- the last bracketed term in (9) - which has an *equalizing effect* as total disutility is minimized when housework is evenly split between husband and wife ($s = 0.5$). This result is due to the symmetry in the way in which we model the costs of housework (i.e., no comparative advantage of either gender in home production), together with our assumption of convex costs.

Thus, the choice of s is driven by the trade-off between full specialization and equal share of housework.¹³ In order to examine the interaction of these two forces, we adopt in the sequel a specific functional form for the cost function that yields analytically tractable solutions. Yet, in Appendix 1 we show that the same characterization of equilibria applies to any cost function satisfying our previous assumptions. Our assumed cost function for the woman takes the form $c(s) = 0.5 s/(1-s)$, which is increasing and convex in her share of housework, s , and tends to infinity when she bears the entire burden, i.e. as $s \rightarrow 1$. Likewise, the husband's cost function is given by $c(1-s) = 0.5 (1-s)/s$, which now tends to infinity if he bears the entire burden, i.e., as $(1-s) \rightarrow 1$. The key feature of these cost functions is that they tend to infinity when one of the household members fully specializes in housework, i.e., as $s=0$ or 1 . As a result, they will always lead to interior solutions in which

¹² For example, the man would have the highest wage when $1-s = W_m/\varepsilon$. Hence, expected household income would be given by $Y = W_m + 0.5 W_f^2/(\varepsilon - W_m)$, where the first term is the (certain) male income and the second term is the expected female income.

¹³ The importance of the convexity assumption in the disutility function can be seen by inspecting the s.o.c. in (12). Since income is a convex function of s , the only way in which $\partial^2 V^H / \partial s^2$ can be negative is if $c''(s)$ is positive. That is, with concave costs of housework, there would be no interior maximum and the overall disutility would be minimized when one agent performs all housework, reinforcing the tendency toward specialisation of household members induced by income maximization.

marketplace and housework activities are always combined. We adopt this assumption to mimic the evidence drawn from time-use surveys in developed countries where strictly positive housework shares are reported by both partners (see Section 6). Notice that, since the cost functions go to infinity when a corner solution is chosen, household income, given by (13), will also become unbounded in this case, so as to yield a properly defined household's maximization problem leading to interior solutions.

Using these cost functions, the expected utility of the household becomes:

$$V^H = \bar{u} + \frac{1}{2\varepsilon} \left[\frac{W_m^2}{(1-s)} + \frac{W_f^2}{s} \right] - \frac{1}{2} \left[\frac{1-s}{s} + \frac{s}{1-s} \right]. \quad (14)$$

Maximizing with respect to s yields the f. o. c.:

$$\frac{1}{2\varepsilon} \left[\frac{W_m^2}{(1-s)^2} - \frac{W_f^2}{s^2} \right] + \frac{1}{2} \left[\frac{1}{s^2} - \frac{1}{(1-s)^2} \right] = 0, \quad (15)$$

which implies that the equilibrium share of housework, denoted by s^* , is determined by equating the marginal rates of substitution between marketplace and household work:

$$\left(\frac{1-s^*}{s^*} \right)^2 = \frac{1 - \frac{W_m^2}{\varepsilon}}{1 - \frac{W_f^2}{\varepsilon}}. \quad (16)$$

It can be easily shown that, for s^* to be a maximum, we require $W_i^2 / \varepsilon < 1$ which, using (6), is ensured by Assumption 1. As a result, $ds^* / dW_f < 0$ and $ds^* / dW_m > 0$, implying that a higher female (male) wage leads to a reduction (increase) of the female housework share. Moreover, when wages are equalised, i.e., $W_f = W_m$, then $s^* = 1 - s^* = 0.5$. Note that this result is due to the symmetry assumption in the way in which we model the costs of housework (i.e., no comparative advantage of either gender) which leads to total cost minimization when housework is evenly split. Lastly, since the household takes wages as given, the household's decision in (16) leads to the following proposition:

Proposition 1: *Under Assumption 1, for given relative wages, an increase in the support of the household disutility shock (ε) decreases (increases) s^* whenever $W_m > W_f$ ($W_m < W_f$).*

The intuition for this result stems again from the first bracketed term in (14): the higher is the upper bound of the shock, the lower is expected income and, as a result, the spouses will prefer to share housework more evenly in order to maximize V^H . However, as shown below, the effect of ε on s will change its sign once we combine households' and firms' decisions. This different effect of ε on s in the two setups will become one of the testable implications of the model in the empirical section.

3.2 Multiplicity of equilibria

Firms' and households' decisions are given by equations (6) and (16). In equilibrium, expectations are fulfilled and hence the values of wages and housework shares are jointly determined as the solution of the following system of equations:

$$W_f = \left(\frac{\beta^2}{4\varepsilon^\alpha s^\alpha} \right)^{\frac{1}{2(1-\alpha)}}, \quad (\text{E.1})$$

$$W_m = \left(\frac{\beta^2}{4\varepsilon^\alpha (1-s)^\alpha} \right)^{\frac{1}{2(1-\alpha)}} \quad (\text{E.2})$$

$$\left(\frac{1-s^*}{s^*} \right)^2 = \frac{1 - \frac{W_m^2}{\varepsilon}}{1 - \frac{W_f^2}{\varepsilon}}, \quad (\text{E.3})$$

The insight for why the model generates multiplicity of equilibria can be easily grasped by this system. Equations (E.1) and (E.2) represent the decision of firms in terms of wages (training) to their beliefs about how household partners split housework, whereas (E.3) represents the household's decision on housework shares based upon their beliefs on wages offered by firms. It is clear that, if $s^* = 0.5$, this becomes an equilibrium with both equal wages and housework shares. Yet, as will be show in detail below, under our assumptions, there is another equilibrium where $s^* \neq 0.5$. The intuition is simply that if, for example, firms believe that women are going to undertake a larger share of housework, i.e. $s^* > 0.5$, they will offer them less training. This will result in lower female wages, implying that women face a lower opportunity cost of carrying out housework than men and will hence induce the couple to allocate a larger share of housework to women. As a result, firm's expectations will be fulfilled. The converse will hold if firms expected $s^* < 0.5$, in which case men would undertake more household chores and receive lower wages than women.

To analyse these equilibrium configurations, it is useful to substitute (E.1) and

(E.2) into (E.3), so that the f.o.c. in (16) can be rewritten as:

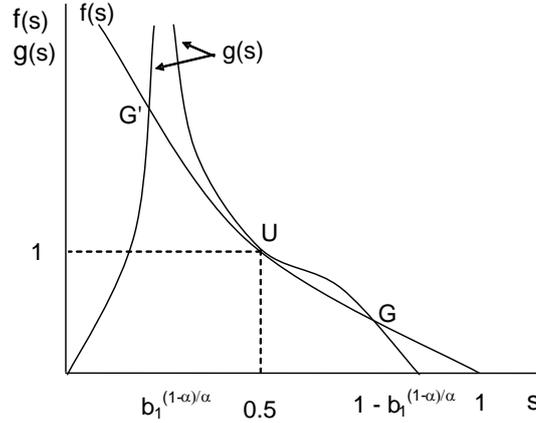
$$\left(\frac{1-s^*}{s^*}\right)^2 = \frac{1-b_1(1-s^*)^{\frac{\alpha}{1-\alpha}}}{1-b_1(s^*)^{\frac{\alpha}{1-\alpha}}}, \quad (17)$$

where $b_1 \equiv (\beta^2 / 4\varepsilon)^{1/1-\alpha}$. From (E.1) and (E.2), we can also define the gender wage and participation gaps as follows:

$$w = \frac{W_m}{W_f} = \left(\frac{s}{1-s}\right)^{\frac{\alpha}{2(1-\alpha)}}, \quad p = \frac{P_m}{P_f} = \left(\frac{s}{1-s}\right)^{\frac{2-\alpha}{2(1-\alpha)}}. \quad (18)$$

To solve for s in (17), it is convenient to think of the LHS and RHS of (17) as the functions $f(s)$ and $g(s)$, respectively, whose intersection results in the equilibrium allocation of the housework share, s^* . On the one hand, $f(s)$ (which is a monotonically increasing transformation of the disutility cost of housework for men) is decreasing and convex with a vertical asymptote at $s=0$, such that $f(1)=0$ and $f(0.5)=1$. On the other, under Assumption 1, $g(s)$ is increasing in the range $s \in [0, b_1^{(1-\alpha)/\alpha})$ and decreasing when $s \in (b_1^{\frac{1-\alpha}{\alpha}}, 1)$, with two vertical asymptotes at $s = b_1^{\frac{1-\alpha}{\alpha}}$ and $s = 1$, such that $g(0) = 0$, $g(0.5) = 1$ and $g\left(1 - b_1^{\frac{1-\alpha}{\alpha}}\right) = 0$. Lastly, $g(s)$ has an inflection point within the range $s \in \left(b_1^{\frac{1-\alpha}{\alpha}}, 1 - b_1^{\frac{1-\alpha}{\alpha}}\right)$. Notice that $g(\cdot)$ is non-monotonic because expected household income is a U-shaped function of s : when men bear a high share ($s < 0.5$) expected income is higher the lower is s , but when women bear a higher share (i.e. $s > 0.5$) expected income is increasing in s and maximized when there is full specialisation.

The intersections of $f(s)$ and $g(s)$ are depicted in Figure 1. The vertical axis represents the inverse of the wage gap in the LHS of (18), so that a value above below 0.5 implies $W_f < W_m$. As can be seen, there are three values of s that satisfy (17). In one of them, $s_1^* = 0.5$, while in the other two we have $s_2^* \in \left(0.5, 1 - b_1^{\frac{1-\alpha}{\alpha}}\right)$ and $s_3^* \in \left(0, b_1^{\frac{1-\alpha}{\alpha}}\right)$. As pointed out earlier, corner solutions are ruled out by our assumption that disutility becomes infinite under complete specialization in housework.

FIGURE 1: Gendered and ungendered equilibria

Due to our assumption of *ex-ante* symmetry across genders, two possible asymmetric equilibria exist: one in which women bear a greater housework share and get a lower wage (G), and another in which the same outcomes apply to men (G'). The two equilibria are actually symmetric in the sense that $s_3^* = 1 - s_2^*$. In the sequel, we will solely focus on case where women carry out a disproportionate share of the household chores, so that $s \in \left(b_1^{\frac{1-\alpha}{\alpha}}, 1 \right) \equiv S$ becomes the permitted domain of $g(s)$.¹⁴ This assumption therefore restricts the analysis to two possible interior equilibria, labelled respectively as the *gendered* equilibrium (denoted by G), where $s_G^* > 0.5$, and the *ungendered* equilibrium (denoted by U) where $s_U^* = 0.5$. Likewise, the gender wage gaps in these two equilibria are labelled as w_G^* and w_U^* . The following result summarises this discussion:

¹⁴ The equilibrium in which males undertake a greater share of housework than women is, in terms of our model, as likely as the one in which women do a greater share. However, such a situation is not found neither in the data on wages and housework shares in Tables 1 and 2 for Spain nor for other countries reported in the Multinational Time Use Survey (MTUS). The likely reason is that historical patterns of comparative advantage had led women to specialise in home production (see Alesina *et al.* 2012), making it difficult for an economy to jump to an equilibrium with the opposite pattern of specialisation. Properly ruling out this equilibrium would require modelling the way in which beliefs about gender roles are formed, which is beyond the scope of this paper.

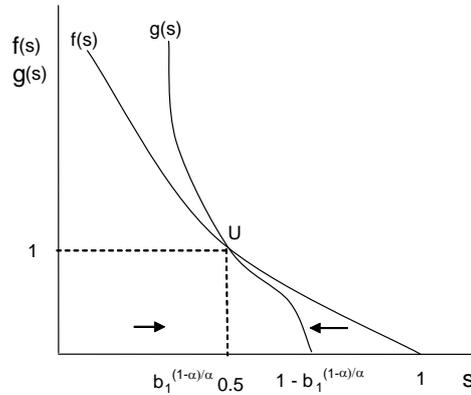
Proposition 2: *Under Assumption 1 and with $s \in S$, there are two equilibrium solutions for the female share of housework and the wage gap: (i) an ungendered solution with $s_U^* = 0.5$ and $w_U^* = 1$ and (ii) a gendered solution with $s_G^* \in (0.5, 1 - b_1^{(1-\alpha)/\alpha})$ and $w_G^* > 1$.*

3.3 The effect of changes in aggregate productivity level on equilibria

Inspection of (17) and Figure 1 indicates that the system (E.1)-(E.3) may exhibit a unique equilibrium. Indeed, the existence of multiple equilibria crucially depends on the size of the b_1 parameter. In effect, as b_1 increases, the range $s \in (b_1^{(1-\alpha)/\alpha}, 1 - b_1^{(1-\alpha)/\alpha})$ shrinks and, as a result, $g(s)$ becomes steeper. This shifts the G-equilibrium to the left, leading to a more even division of housework and a lower wage gap.

As depicted in Figure 2, there will be a unique U-equilibrium for sufficiently high values of b_1 . Since $b_1 = (\beta^2 / 4\varepsilon)^{\frac{1}{1-\alpha}}$, changes in its value will be driven by changes in β and ε , with $\partial b_1 / \partial \beta > 0$ and $\partial b_1 / \partial \varepsilon < 0$.

FIGURE 2: The effect of an increase in b_1 on gender gaps



The previous discussion can be summarised in the following proposition:

Proposition 3: *Under Assumption 1 and $s \in S$, it holds that:*

- (i) *An increase in the aggregate productivity parameter β leads to lower equilibrium gender gaps. Moreover, economies with a sufficiently high value of β will exhibit a unique ungendered equilibrium.*
- (ii) *An increase in the upper bound ε of the household disutility shock increases the equilibrium gender gaps.*

To understand the intuition behind Proposition 3, note that the only effect of an increase in β is to raise wages. The reason why productivity matters is that it leads to an income effect. Recall the trade-off faced by a household between expected income and housework disutility: the former effect implies that income is higher with full specialization ($s=1$), while the latter effect induces an even allocation of housework ($s=0.5$). When wages are low (β is small), the household is less willing to forgo expected income in order to reduce the utility cost. Hence, if firms offer different wages, housework will be unevenly allocated. By contrast, when wages are high (β is large), the opposite holds, leading to a lower s_G^* . If wages are sufficiently high, the disutility effect dominates, making the housework division more and more equal across genders.¹⁵ Yet, if s is (close to) 0.5, then firms will pay similar wages to men and women. Hence the G-equilibrium cannot exist.

Concerning the effect of ε , notice that it increases unambiguously the equilibrium gender gaps, in sharp contrast to the opposite result obtained in (16) about the household decision for given wages, where a larger value of ε lowered the gaps in the gendered equilibrium. The intuition for this contrasting result is that an increase in ε operates in the same way as a decrease in β in the full solution of the model, and thus it leads to a higher gender gap.

In sum, productivity plays a crucial role in determining the equilibrium gender gaps in wages and time allocation. Interestingly, this result by itself suggests that, abstracting from the differences in the generosity of family-aid policies, rich countries/regions could exhibit the lower gender gaps than poor countries/regions solely due to their higher aggregate productivity.

4. Welfare analysis

In order to compare welfare in the two equilibria, we consider the unitary-household problem faced by a social planner who, conditioning on the firm's optimal training and wage decisions, is allowed to choose the allocation of housework internalizing its effect on training and therefore on wages. Since firms make zero expected profits due to the free-entry assumption, aggregate welfare in this economy, V^W , is simply equal to the

¹⁵ To see this simply let $b_1 \rightarrow \infty$ in equation (17), which makes its RHS equal to 1, implying that $s=0.5$ for all wages.

welfare of the representative household where, in contrast to (9), the choice of s is now allowed to affect wages through the amount of training provided by firms. Substituting (E.1) and (E.2) into (14), yields the following unitary-household's welfare function:

$$V^W(s) = \bar{u} + \frac{1}{2} \left[b_1(1-s)^{\frac{1}{1-\alpha}} + b_1 s^{\frac{1}{1-\alpha}} \right] - \frac{1}{2} \left[\frac{1-s}{s} + \frac{s}{1-s} \right], \quad (19)$$

We can now examine which of the two equilibria results in a higher level of welfare by substituting (17) into (19). This yields:

$$V^W(s^*) = 1 + \bar{u} - \frac{1 - b_1 s^{*\frac{\alpha}{1-\alpha}}}{2(s^*)^2}. \quad (20)$$

Then, differentiation of (20) implies that:

$$\frac{dV^W(s^*)}{ds^*} = \frac{1}{2s^{*3}} \left[2 - \frac{2-\alpha}{1-\alpha} b_1 s^{*\frac{\alpha}{1-\alpha}} \right],$$

which may be positive or negative depending on the sign of the bracketed term. Hence, it is ambiguous whether welfare in the unitary-household problem is higher in the G-equilibrium or in the U-equilibrium, the reason being once more the trade-off between full specialisation and equal sharing of housework.

As before, the level of productivity β is a key parameter determining which effect dominates. Given that b_1 is increasing in β , s_G^* will decrease with the productivity level. Hence, $dV^W(s^*)/ds^* < 0$ for sufficiently high values of β . This implies higher welfare in the U-equilibrium since $s_G^* > s_U^* = 0.5$.

This finding differs from the results in models that rely on incentive problems, where it has been generally found that specialization results in higher welfare.¹⁶ The difference lies in both the symmetry in preferences and the fact that we assume an increasing and convex cost of housework for both spouses. Moreover, our analysis has the implication that the nature of the efficient equilibrium may change over time as the productivity parameter grows. Initially, when β is low, specialization delivers higher welfare. Yet, as productivity grows, the opportunity cost of sharing housework falls and

¹⁶ See Lommerud and Vagstad (2007) for a discussion of welfare in this type of models. In the statistical discrimination literature, however, there are examples where discrimination leads to lower welfare. For example, this is so in Coate and Loury (1993) because the discriminated group invests less than optimally in human capital. This is also the case in the racial discrimination model with exogenous posted wages proposed by Lang et al. (2005).

the U-equilibrium becomes more efficient.¹⁷

5. Policies

5.1 Affirmative actions

We next discuss which kind of gender policies could shift the economy from the G-equilibrium to the U-equilibrium. The literature on this issue has focused on two specific policies: affirmative action and subsidised family aid. In our setup, affirmative action would take the form of a law that prevents firms from engaging in statistical discrimination and offering differential training to men and women. Since men and women receive now the same amount of training, (2) implies that they also receive identical wages leading to equal sharing of housework. Hence, the only possible equilibrium is $s = 0.5$, implying that it is optimal for the firm to offer the same amount of training to the household partners. In other words, since the reason for the existence of the U-equilibrium is a coordination problem, affirmative action will coordinate firms and households on the U-equilibrium in which firms would choose not to differentiate between genders even if they could.

There is an extensive debate on the effects of affirmative action policies. As discussed earlier, Coate and Loury (1993) have shown that an exogenous increase in the hiring probability reduces the educational effort of the minority. However, Moro and Norman (2003) find that this result depends crucially on assuming that the marginal product of labor is constant for each type of workers. By contrast, when the marginal products depend on the relative supply of the two groups, equilibrium effects imply that the wage changes resulting from affirmative action policies may induce minority workers to increase, rather than decrease, their educational investment. Our analysis illustrates how, even in the absence of these externalities, targeted policies towards statistically discriminated groups can have different effects.

One problem with affirmative action policies is that they may be difficult to implement in many instances. This would be the case if the training that individuals receive is imperfectly observed by the policymaker or when there are differences in the ‘quality’ of training provided by firms. In this case, the standard tool left to affect the equilibrium is subsidized family aid whose effects are analyzed in the next subsections.

¹⁷ For alternative analyses of how exogenous changes in productivity affect gender differences in the labour market, see Olivetti (2006) and Albanesi and Olivetti (2009).

5.2 Subsidised family aid

5.2.1 Gender-based vs. Gender-neutral family aid

Consider the introduction of government-funded family aid subsidy. To start with, suppose that it is targeted on working women and that this subsidy, κ , is proportional to the female wage in period 2. Thus, women will receive an income equal to $W_f(1+\kappa)$, where $0 < \kappa < 1$, so that they will not quit in period 2 if $W_f(1+\kappa) - \omega \geq 0$, whereas men will work if $W_m - \omega \geq 0$. For the time being, we neglect the issue of how the targeted subsidy is financed which will be addressed at the end of this section.

The analysis of firms' decisions, taking household decisions as given, is the same as in section 2.2, but this time with the upper limit of the integral for women in (1) changed from W_f to $W_f(1+\kappa)$. This yields the following optimal amount of training and wage chosen by the firm:

$$\tau_f^{\kappa*} = \left(\frac{(1+\kappa)\beta^2}{4\varepsilon_f} \right)^{\frac{1}{1-\alpha}}, \quad W_f^{\kappa*} = \frac{\beta}{2} (\tau_f^{\kappa*})^{\alpha/2}, \quad (21)$$

where the superscript κ is used to denote the equilibrium values under subsidies. Male workers are offered the training level and wage derived in (5) and (6). Note that the total income of women in period 2, $Y_f^{\kappa*}$, is now given by:

$$Y_f^{\kappa*} = (1+\kappa)W_f^{\kappa*} = \frac{\beta(1+\kappa)}{2} (\tau_f^{\kappa*})^{\alpha/2}. \quad (22)$$

Not surprisingly, women fare better in the labor market when they are subsidised to stay in the job since $\tau_f^{\kappa*} > \tau_f^*$ and $W_f^{\kappa*} > W_f^*$,¹⁸ despite the fact that, for $\kappa < (\varepsilon_f - \varepsilon_m)/\varepsilon_m$ (i.e. if the subsidy is not too large), they will still receive less training and lower wages than men, that is, $\tau_f^{\kappa*} < \tau_m^*$ and $W_f^{\kappa*} < W_m^*$.¹⁹

Equations (21) and (22) imply that the corresponding participation and wage gaps would be lower with than without subsidies. However, this result changes once the division of housework is endogenized. In this case, each household chooses s to maximize the expected net utility given by:

¹⁸ They may even get higher gross wages than men if the subsidy is sufficiently large but we ignore this possibility in the sequel.

¹⁹ In equilibrium, since $\varepsilon_f = s\varepsilon$ and $\varepsilon_m = (1-s)\varepsilon$, this condition becomes $\kappa < (2s-1)/(1-s)$.

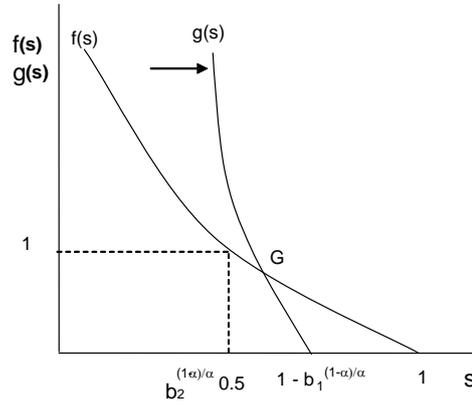
$$V^{H\kappa} = \bar{u} - 2 + \frac{1}{2\mathcal{E}} \left[\frac{W_m^2}{(1-s)} + \frac{Y_f^{\kappa^2}}{s} \right] - \frac{1}{2} \left[\frac{1-s}{s} + \frac{s}{1-s} \right]. \quad (23)$$

The resulting f. o. c., once we have substituted for wages, yields the new equilibrium relationship:

$$\left(\frac{1-s^{\kappa^*}}{s^{\kappa^*}} \right)^2 = \frac{1-b_1(1-s^{\kappa^*})^{\frac{-\alpha}{1-\alpha}}}{1-b_2s^{*\kappa^{\frac{-\alpha}{1-\alpha}}}}, \quad (24)$$

where $b_2 = b_1(1+\kappa)^{\frac{2-\alpha}{1-\alpha}} > b_1$. The LHS of equation (24) is the same as in (17), while the RHS tilts upwards and takes a value greater than 1 when $s=0.5$. The new equilibrium is depicted in Figure 3.

FIGURE 3: The effect of a subsidy to women on gender gaps



The following proposition summarizes the main result of this section.

Proposition 4: *Under Assumption 1 and with $s \in S$, a wage subsidy targeted to female workers leads to a gendered equilibrium with $s^{\kappa^*} \in (0.5, 1)$, so that gender gaps in wages and housework are larger than in the absence of the subsidy.*

The remarkable feature of (24) is that, with the subsidy in place, the U-equilibrium with $s = 0.5$ no longer exists. In other words, a gender-based subsidy policy only yields the G-equilibrium since the asymmetry in income induced by the subsidy prevents a symmetric equilibrium. In effect, suppose that households set $s=0.5$. Then, women have a lower probability of quitting than men (the combination of the same

shock plus the subsidy) which implies that firms will offer more training and a higher gross wage to them. Yet, if female wages are different from men's, then $s=0.5$ cannot be a solution to the household's problem. Hence, the U-equilibrium no longer exists. Moreover, it can be easily shown that the new G-equilibrium in Figure 3 lies to the right of the initial one in Figure 1, leading to a higher equilibrium value of s .²⁰ Thus, gender-based job subsidies can backfire.

The intuition behind this seemingly puzzling result relies once again the trade-off faced by the household. Because the subsidy increases the probability of female labor participation, the unitary household can now afford to raise the probability of male participation by reducing men's housework share. This result shares the spirit of the analysis of affirmative action policies in Coate and Loury (1993) where it is argued that an exogenous increase in the hiring probability faced by a minority would increase the educational gap. Similarly, in our framework the exogenous increase in the probability of participation of women reduces their commitment to the labor market.

By contrast, consider now an alternative policy which offers the same subsidy to men and women. Following the same reasoning as above, this would yield the equilibrium relationship:

$$\left(\frac{1 - s^{\kappa^*}}{s^{\kappa^*}} \right)^2 = \frac{1 - b_2(1 - s^{\kappa^*})^{\frac{-\alpha}{1-\alpha}}}{1 - b_2 s^{*\kappa^* \frac{-\alpha}{1-\alpha}}}, \quad (25)$$

which again will narrow the range of values of $s \in S$ for which the RHS of (25) is positive, since $b_2 > b_1$. The first implication is that the subsidy shifts the G-equilibrium to the left, reducing the value of s_G^* . Moreover, if the subsidy is high enough (i.e. for sufficiently large values of b_3), equation (25) will yield a unique U-equilibrium, as depicted in Figure 5. Once more, the above-mentioned trade-off faced by the household explains this result. The subsidy effectively increases expected income and hence reduces the opportunity cost of sharing housework. If the increase in income is large enough, the household will simply minimize the disutility associated with housework and choose an even allocation of domestic chores.

²⁰ To show this, denote the LHS of (24) as a function of s by $g_\kappa(s)$. Then differentiating $g_\kappa(s)$ with respect to s in a neighbourhood of its crossing with the LHS of (24), given by $f(s)$, one gets that $g'_\kappa(s)$ becomes more negative (steeper) for a higher value of κ . Hence, since $f(s)$ is the same as in the case where $\kappa = 0$, the new gendered equilibrium must be to the right of the equilibrium without subsidies.

The same reasoning as in (24) implies that the U-equilibrium becomes more likely in (25). Whether it is a unique equilibrium or not hinges on the size of b_2 , which in turn depend upon κ and β (for given values of α and ε).

Note that our policy of gender-based family aid can be seen as being equivalent to a lower marginal tax rate for women, as it reduces the gap between the take-home pay of men and women. Our result hence differs from the argument made by Alesina *et al.* (2011) who propose different taxation for men and women. In their reasoning, the asymmetry across genders arises from women having a higher elasticity of labour supply than men. Thus, according to the Ramsey principle of optimal taxation, the former should have lower taxes than the latter. Our model differs from their analysis in two crucial dimensions. First, in Alesina *et al.* the key element is the different elasticity of labor supply across genders. In contrast, in our setup the two types of individuals have the same labor-supply elasticities (which is equal to 1, see equation (7)). This means that we are shutting down the channel examined by these authors, despite it being empirically relevant. The second difference is that in their model there are exogenous differences across genders, either because of differences in bargaining power or comparative advantage in home production. Because men and women differ, so do the elasticities and hence the welfare effects of taxation. In our setup, the two types of individuals are ex ante identical, and yet a gendered equilibrium is possible. What our analysis shows is that when there are no ex ante differences and the gendered equilibrium is due only to expectations, differential tax treatment does not help reduce gender gaps. Of course, if we were to impose a different bargaining power across household members, then the policy implications would favour gender-based taxes and subsidies.

Moreover, the generality of the result by Alesina *et al.* has been questioned by Guner *et al.* (2011) who show, in their calibration of the effects of tax changes in the U.S, that, although gender-based taxes improve welfare and are preferred by a majority of households, welfare gains are higher under a proportional, gender-neutral income tax. Interestingly, our reasoning in favour of neutral-gender subsidies also echoes some of Saint-Paul (2007)'s arguments against gender-based taxation.

5.2.2 Financing of the subsidy

We next consider the financing of the subsidy. One possibility would be to finance the wage subsidy by taxing men. However, our analysis in the previous section indicates that such policy, by introducing an asymmetry in the RHS of (25), eliminates the U-equilibrium. In other words, taxing men would imply that the only equilibrium that exists is one with unequal wages and an unequal division of labor.

An alternative mode of financing the subsidy is to tax firms. Specifically, we suppose that they are taxed for their training expenditures in period 1 at a proportional rate t .²¹ Under a balanced budget, this implies that $t(\tau_f + \tau_m) = \kappa(W_f P_f + W_m P_m)$. In this tax-subsidy scheme, denoted by the superscript TS , participation is given by $P_i^{TS*} = (1 + \kappa)W_i^{T*} / \varepsilon_i$, and firms offer the wage $W_i^{TS*}(\tau_i) = a(\tau_i)/2$, implying that gross expected profits become now:

$$\Pi(\tau_i) = \frac{a(\tau_i)^2}{4\varepsilon_i}(1 + \kappa), \quad (26)$$

while the zero-profit condition for firms yields:

$$\Pi(\tau_i) - (1 + t)\tau_i = 0. \quad (27)$$

Noticing that we can write $\Pi(\tau_i) = P_i W_i$, this condition is simply equivalent to $\tau_i(1 + t) = P_i W_i$, which can be replaced into the budget constraint to obtain the equilibrium relation between the tax and the subsidy rates, i.e., $t = \kappa/(1 - \kappa)$. The zero-profit condition, together with this value of t , yields the optimal level of training:

$$\tau_i^{TS*} = \left[\frac{\beta^2}{4\varepsilon_i} \frac{1 + \kappa}{1 + t} \right]^{\frac{1}{1-\alpha}} = \left[\frac{\beta^2}{4\varepsilon_i} (1 - \kappa^2) \right]^{\frac{1}{1-\alpha}}. \quad (28)$$

Equation (28) implies lower training and wages than without subsidies as a result of the labor tax paid by firms. Participation, given by $P_i^{TS*} = (1 + \kappa)W_i^{TS*} / \varepsilon_i$, may be higher or lower than under laissez-faire due to the opposite effects of the subsidy and the lower wage. The former tends to increase participation while the latter tends to reduce it.

As regards the household decision on s , a similar argument as before yields the following f.o.c.:

²¹ We have also examined the case where the tax is lump-sum in the first period. This case yields similar results though the calculations are somewhat more cumbersome.

$$\left(\frac{1 - s^{TS^*}}{s^{TS^*}} \right)^2 = \frac{1 - b_3 (1 - s^{TS^*})^{\frac{-\alpha}{1-\alpha}}}{1 - b_3 s^{TS^* \frac{-\alpha}{1-\alpha}}}, \quad (29)$$

where $b_3 \equiv b_1 h(\kappa)$ with $h(\kappa) \equiv [(1-\kappa)^\alpha (1+\kappa)]^{\frac{1}{1-\alpha}}$. Then, $h(0) = 1$ and $h'(\kappa) > 0$ if and only if $\kappa < (1-\alpha)/(1+\alpha)$. Thus, for not too high values of κ , $h(\kappa)$ is increasing and therefore $b_3 > b_1$. Hence, this tax-subsidy scheme makes the $g(s)$ function steeper, implying that the equilibrium value of s will decrease and, potentially, a unique U-equilibrium could be achieved. Indeed, for the G-equilibrium to disappear, we need that there is a unique intersection, which will be the case if $1 - b_3^{(1-\alpha)/\alpha} \leq 0.5$, that is, if $(1+\kappa)(1-\kappa)^\alpha \geq 2^{3-2\alpha} \varepsilon / \beta^2$. Hence, the following result holds.

Proposition 5: *Under Assumption 1 and with $s \in S$, if κ is not too large, i.e., $(1+\kappa)(1-\kappa)^\alpha \geq 2^{3-2\alpha} \varepsilon / \beta^2$, an equal wage subsidy to male and female workers financed through a proportional tax on training expenditures by firms in period 1 will reduce gender gaps and may even lead to an ungendered equilibrium with $s^{TS^*} = 0.5$.*

The intuition for this result again relies on the two conflicting effects affecting participation: a direct effect from the subsidy which tends to increase participation, and an indirect one, operating through the reduction in training induced by the tax paid by firms, which tends to reduce participation. The condition $(1-\alpha)/(1+\alpha) > \kappa$ is easy to interpret since, from (29), a low value of α implies a low elasticity of training with respect to the subsidy. This means that the wage does not decrease by much, implying that the direct effect dominates and leads to higher expected income for any given division of housework. As in section 3.3, a higher income implies that couples can afford to reduce the disutility of housework, thereby choosing an even split.

6. Some empirical micro evidence

6.1. Data and descriptive statistics

In this section we use data from a time use survey to test some relevant implications of our model regarding households' decision on housework allocation both for given wages and in the full solution of the model. The main reason for neglecting the joint

estimation of these decisions with firms' decisions on paid-for training is that we are not aware of the availability of either time-use surveys reporting information on workers' paid-for training or firms' databases on training reporting time use of their workers.

In particular, we use micro data drawn from the Spanish Time Use Survey (STUS) 2002-03 (<http://www.ine.es/prensa/np333.pdf>) which is part of the Multinational Time Use Survey (MTUS). This data set contains harmonized information on the use of time by households living in a variety of European countries. Specifically, the STUS it reports data on how much time each individual devotes to a wide range of activities (41 in total) on a representative day. For each 10-minute interval (and during 24 hours), respondents are required to keep a diary recording which are their primary and secondary activities during this period of time. These are coded according to a list provided in Table A1 in Appendix.1. Housework time is defined as the number of minutes reported in the diary that each individual devotes to categories AV7 (housework) as primary activity. Likewise, this definition can be extended to include time devoted to childcare (housework plus childcare), in which case AV7 and AV11 are lumped together. The spouses' shares of household work are therefore computed for each of these two definitions.

In addition, STUS provides information on basic demographic and labor-market characteristics of the respondents, including wages. We use this survey because, in contrast to the surveys for other countries, it also contains information about on the availability of family-aid subsidies, domestic service and the region of residence of the households, all of which are relevant variables in our analysis.

We restrict our sample to two-earner couples with full-time jobs,²² who belong to the 25-64 age bracket and report complete information on housework share, wages and the remaining controls. This gives rise to a sample of 2915 couples. Further, as will be discussed further below, to mimic our assumption of ex-ante identical individuals, we will use age and education differences between spouses as additional controls in the two regression models. In this way, our results can be interpreted as comparing spouses with the same age and educational attainment.

In Table 1 we present descriptive statistics about net hourly wages (computed as the ratio between reported net monthly wages and (four times) weekly working hours)

²² By focusing exclusively on full-time workers, hourly wages (times the number of fixed working hours) correspond to the income each spouse brings to the household. Otherwise, if their supply of working hours was freely chosen, like in part-time jobs, labour income would become endogenous.

and some demographics (age, education levels and presence of children) in STUS.

As can be observed, the raw hourly wage gap is 22 log-points on average. Men tend to be two year older than women whereas the latter exhibit slightly higher educational attainments. The fraction of childless households is 57% reflecting the low fertility rate in Spain at the beginning of the 2000s. Lastly, 26% of the households in our sample have domestic service and 4% report receiving some form of family-aid subsidies.

TABLE 1

Descriptive Statistics (Demographic and Labor-Market Characteristics)
Full -time working couples (25-64 years of age)

	Mean	S.d.
Wages		
Hourly Wage, Husband	8.34	8.07
Hourly Wage, Wife	6.51	6.43
Average Log Wage Gap (H-W)	0.22	0.20
Education		
% Primary Education, Husband	0.11	0.31
% Primary Education, Wife	0.09	0.32
% Secondary Educ., Husband	0.52	0.37
% Secondary Educ. Wife	0.49	0.36
% University Educ. Husband	0.37	0.31
% University Educ. Wife	0.40	0.29
Age		
Average Age, Husband	42.9	40.7
Average Age, Wife	40.6	40.3
Children		
% Couples with no child	0.57	0.40
% couples with child <5 years	0.12	0.22
% couples with child >5 years	0.31	0.38
Household aid^(*)		
% with family aid income	0.04	--
% with domestic service	0.26	--
No. of couples.	2915	

Source: STUS 2002-2003. (*) denotes the percentage of couples who receive some type of state-funded family aid/ have domestic service.

Table 2 reports the female shares (s) using the two above-mentioned definitions of housework, which are equal to 80% and 76%, respectively. By age and education, the shares are lower for younger and more educated women.

TABLE 2

Average Female Housework Share

Share	Housework Duties	Housework & Childcare
Average	0.80 (0.28)	0.76 (0.26)
Less-ed.	0.82 (0.27)	0.78 (0.27)
Highly-ed.	0.75 (0.30)	0.70 (0.27)
25-30	0.74 (0.34)	0.70 (0.31)
31-40	0.78 (0.29)	0.72 (0.25)
41-50	0.81 (0.26)	0.79 (0.26)
51-64	0.87 (0.23)	0.87 (0.23)

Source: STUS (2002-2003).

Note: Standard errors in parentheses. The definition of less- educated couples is that both spouses have less than a college degree, while highly- educated couples are those where both have a college degree.

6.2. Testable implications

Our empirical application focuses on the estimation of equations (16) and (17), describing how spouses allocate housework time for given wages and in the full solution of the model, respectively. To obtain an estimable regression models, we log-linearize (16) and (17) around a generic (possibly gendered) equilibrium value. As shown in Appendix 3, these approximations yield:

$$\ln\left(\frac{s}{1-s}\right) = \theta_0 + \theta_1(\ln W_m - \ln W_f) + \theta_2 \ln W_f + \theta_3 \ln \varepsilon, \quad (30)$$

for (16),²³ and

$$\ln\left(\frac{s}{1-s}\right) = \phi_0 + \phi_1 \ln \beta + \phi_2 \ln \varepsilon, \quad (31)$$

for (17), where the logit transformation of the dependent variable in both equations is always feasible since all individuals in our sample report values of s which differ from either 0 or 1. Further, under the plausible assumption that wages are taken as parametric by the household in (16), both equations can be consistently estimated by OLS (with heteroskedasticity-robust standard errors).

The *first* set of testable implications relates to the signs and relative sizes of some of

²³ Notice that $\ln \beta$ does appear in (30) since we are conditioning on given wages and it is only through wages that productivity affects housework shares.

the parameters in (30) and (31). As shown in Appendix 3, the coefficients on the wage gap and on the female wage in (30) satisfy the following restrictions: $\theta_1 > \theta_2 > 0$ as long as $W_m > W_f$. Accordingly, the impact of the male wage (given by θ_1) on the relative share is positive, i.e., a rise in W_m increases s , whereas the corresponding impact of the female wage (given by $\theta_2 - \theta_1$) is negative, i.e., a rise in W_f decreases s . Moreover, insofar as there is a unitary-household decision model with equal weights, θ_2 is predicted to be smaller in economies where gender gaps are closer to the *ungendered* equilibrium. Interestingly, if men were to have larger bargaining power than women, we also show in Appendix 3 that the restriction $\theta_1 > \theta_2$ need not hold. Thus, testing whether $\theta_1 > \theta_2$ can be interpreted as a first-pass check on whether asymmetric bargaining plays a relevant role in explaining the gap. Next, according to Propositions 1 and 3, the coefficient on $\ln \varepsilon$ should be negative for given wages and positive in the full- solution model, that is, $\theta_3 < 0$ in (30) and $\phi_2 > 0$ in (31). Finally, according to Proposition 3, another key implication of the model is that higher aggregate productivity reduces the gender gap, so that $\phi_1 > 0$ in the full-solution model.

A *second* testable prediction relates to comparing the sizes of the estimated coefficients in (30) and (31) for the two above-mentioned definitions of housework. Since it is plausible that disutility shocks are likely to be more frequent in households with children, we would expect the estimated coefficients to be more sizeable for the definition of housework that contains childcare.

Before discussing the results, there are two issues to be addressed. First, given that our model assumes individuals' characteristics to be ex-ante identical (including education), we add a set of additional controls to (30) and (31) which includes the age gap between husband and wife plus two dummy variables capturing differences in educational attainments.²⁴ Secondly, there is the issue of how to measure $\ln \beta$ in (31) and $\ln \varepsilon$ in (30) and (31). As regards $\ln \beta$, we proxy aggregate productivity by means of a dummy variable of whether the household lives in a high-productivity region (living in a low productivity region is the reference category). Using indexes of regional labor productivity in 2002-03, this indicator (*Dummy rich region*) takes a value of 1 for

²⁴ These dummy variables are defined as follow: (i) both spouses have a college degree (High-ed.), and (ii) both spouses have less than a college degree (Low-ed.). The reference category is only one of the spouses has a college degree (Mixed-ed.).

couples living in one of the five regions with the highest GDP per employee (Balearic Islands, Cataluña, Madrid, Navarra and the Basque Country) out of the seventeen regions in which Spain is divided, and 0 otherwise. With regard to $\ln \varepsilon$, individual heterogeneity in the upper bound of the household shock is captured by children age status (household without children are the reference category) plus a dummy variable on the availability of domestic help (*Dummy domestic service*). Finally, we also include an indicator variable on whether the household receives family-aid subsidies (*Dummy family aid*) which, lacking information on its nature, we interpret it as being non-gender targeted.

6.3. Results

OLS estimates of the coefficients in specifications (30) and (31) are presented in the first two and the last two columns of Table 3, respectively.

Regarding the first set of predictions, the estimates reported for (30) show that there is highly significant response of the relative housework share with respect to the female wage, pointing out to the existence of a gendered equilibrium.²⁵ Moreover, the finding that the estimated coefficient on the wage gap (θ_1) is significantly larger than the coefficient on the female wage (θ_2) yields some support for the symmetric household decision model. Further, in agreement with Proposition 3 and 4, the estimated coefficients on the dummies of having children are negative in (30), when wages are taken as given, whereas they are positive (though much less statistically significant) in (31), when the full-solution model is considered. Yet, the estimated coefficient on the dummy variable of availability of domestic service, which is also negative and significant in (30), does not become positive in (31) although it is insignificant. Another important finding is that the coefficient on the rich-region dummy, capturing aggregate productivity, is negative and highly significant in (31), as predicted by our model. Finally, the evidence on family-aid effects is inconclusive since, though the coefficient on this dummy is statistically insignificant. One potential reason for this inconclusive finding is that some items of family-aid in Spain is means-

²⁵ The estimated coefficients in Table 3 can be used to compute the percentage- points change in the female housework share, s , corresponding to a change of x % in each of the spouses' wages. For example, using the definition of housework which includes childcare, the coefficient on the male wage is 0.31. Thus, $\partial s / \partial \ln W_m = 0.31 \times [\partial s / \partial \log(s/1-s)] = 0.31 \times s(1-s)$. Using the average value of s in Table 2 (0.76) an increase of 10% in the husband' s wage yields a rise of 0.56 percentage points in s , while a 10% increase in the wife' s wage leads to a reduction of 0.28 percentage points in s .

tested and is hence capturing the fact that the household is low income.²⁶ Since family-aid would tend to reduce the female share but lower income tends to increase it, the two offsetting effects may cancel out. To control for this, we also added interaction terms of the family-aid dummy and the rich-poor regional dummies (not reported) but the results remain inconclusive.

TABLE 3
Household's Decision on Housework Time Allocation (STUS).
Dependent Variable: $\ln[s / (1-s)]$

	Eq. (30) (for given wages)		Eq. (31) (full sol.)	
	Housework Duties	Housework & Childcare	Housework Duties	Housework & Childcare
Log. Wage Gap	0.25*** (0.10)	0.31*** (0.09)	---	---
Log. Fem Wage	0.09*** (0.03)	0.17*** (0.06)	---	---
(H-W) Age gap.	0.03*** (0.01)	0.03*** (0.01)	0.02** (0.01)	0.03*** (0.01)
High-ed. couple.	-0.05 (0.08)	-0.09 (0.06)	-0.06 (0.05)	-0.07 (0.05)
Less-ed. couples.	0.25*** (0.10)	0.38*** (0.11)	0.21** (0.09)	0.27*** (0.10)
Children<5 yrs.	-0.23*** (0.09)	-0.28** (0.11)	0.13** (0.07)	0.17* (0.09)
Children>5 yrs.	-0.09 (0.07)	-0.15** (0.08)	0.05 (0.06)	0.11 (0.07)
Dummy rich regions	---	---	-0.13*** (0.04)	-0.15*** (0.06)
Dummy domestic service	-0.09** (0.04)	-0.11*** (0.04)	-0.06 (0.05)	-0.08 (0.06)
Dummy family aid	-0.04 (0.04)	-0.02 (0.05)	-0.07 (0.05)	-0.07 (0.05)
R-squared	0.09	0.16	0.09	0.14
N.obs. (couples)	2915	2915	2915	2915

Note: OLS estimates with heteroskedasticity-robust standard errors. *, **, *** mean significantly different from zero at 10%, 5% and 1% levels, respectively. Reference categories are Mixed-ed. couples and no children. Additional controls are the interactions of family-aid dummy and rich/poor regional dummies.

As regards the second testable implication, we find that, as expected, the estimated impacts of the different covariates tend to be larger for the extended of housework definition.

²⁶ Family aid in Spain consists of: (i) 1,200 euro per year for working mothers with children less than 3 years old, (ii) 2,000-6,500 euro per year for multiple births or adoptions (2 to 4); 25 euro per month for adopted/handicapped children; and (iv) a lump-sum subsidy of 450 euro for third born or adopted child and successive. Aid related to (i) and (ii) is not means tested while (iii) and (iv) are subject to an annual income threshold of 9,100 euro. This information is drawn from the Spanish Ministry of Health and Social Services (2002) (www.msc.es).

Additional specifications of (30) have been tried without altering the previous main qualitative results. For example, neither the inclusion of quadratic terms in the partners' wages (to account for previous evidence on the existence of a convex effect in the impact of relative earnings on the relative housework due to social norms; see, e.g., Bittman *et al.*, 2003) - nor interactions of the education dummies and the age gap with the wages led to statistically significant coefficients on those terms.

7. Conclusions

We have proposed a model of self-fulfilling prophecies in which statistical discrimination results in both wage and housework time differences across *ex ante* generically identical individuals, except for gender. In contrast to a large strand of this literature, our model does not rely on either moral hazard due to unobservable effort, efficiency wages in some sectors or adverse selection problems. In our setup, employers would provide identical training to *ex ante* equally-able men and women in the absence of uncertainty. However, under uncertainty, they form different expectations about the burden of the housework shocks that each spouse would face. If firms believe that, say, women are more likely to quit than men when shocks arise, they will offer them less training leading to a gender wage gap. Conversely, when couples make decisions about the division of housework, women would undertake more housework than men if they expect their wages to be lower, validating in this way both sets of beliefs.

The model gives rise to two types of equilibria -*gendered* and *ungendered*- leading to several novel policy implications which are harder to obtain in other type of models: (i) higher aggregate productivity tends to eliminate the gendered equilibrium; (ii) welfare in the symmetric equilibrium can be greater than in the asymmetric one; (iii) a gender-targeted policy (e.g., wage subsidies targeted to married women) may not only fail to achieve a symmetric equilibrium but could also worsen the prior gender gap, and (iv) a gender-neutral subsidy (i.e., targeted to both members of the couple) could be more efficient in achieving an ungendered equilibrium, and that such policy works better in more productive economies.

Empirical evidence using micro data from a time-use survey for Spain yields some support to of our main theoretical predictions concerning the relationship between the hourly wages and the sharing of housework of two full-time earner couples. However, more empirical work is clearly needed in order to test other implications,

notably the effect of alternative tax-subsidy policies whose effects cannot be identified with the datasets at hand.

Finally, our model could also be used to interpret the time profiles of gender wage gaps. In our set-up, men and women have the same (zero) wage when entering the labor market, but differential training implies faster wage growth for males. This result is in line with recent evidence that shows that there are no gender wage differences at entry level but a gap appears shortly afterwards and grows up to at least age 40-45 (see Manning and Swaffield, 2008). This literature has stressed the role played by on-the-job learning and out-of-employment spells in explaining lower wage growth for women, but our paper suggests that firms' statistical discrimination in training may be another relevant mechanism. This issue remains high in our future research agenda.

Appendix 1: The household's decision under a general cost function

Let us rewrite the f.o.c. for the household as given by equation (11) as $Y'(s^*) = C'(s^*)$, where $C(s) \equiv c(s) + c(1-s)$ is the total disutility incurred by the household, and its first derivative is $C'(s) = c'(s) - c'(1-s)$. The function $Y'(s)$ is characterized by $Y'(1) = \infty = -Y'(0)$, and $Y''(s) > 0$, implying that $Y'(s)$ is an increasing function that goes from minus infinity to plus infinity. Also, $Y'(\hat{s}) = 0$, with $\hat{s} \equiv [1 + W_m/W_f]^{-1}$ which is less than 0.5 whenever $W_m > W_f$ and equal to $\frac{1}{2}$ if $W_m = W_f$. This implies that $Y'(0.5) > 0$ insofar as there is a wage gap in favour of men, while for equal wages $Y'(0.5) = 0$. Also, it can be shown that $Y'(s)$ is first concave and then convex. Turning now to $C'(s)$, we have $C'(0.5) = 0$ and $C''(s) = c''(s) + c''(1-s) > 0$. Hence, $C'(s)$ is an increasing function that takes the value 0 at $s=0.5$. Lastly, we have that $C'''(s) = c'''(s) - c'''(1-s) = -C'''(1-s)$, implying that if $C'(s)$ is concave for $s=x$, then it will be convex for $s=1-x$, and vice versa.

Assuming a solution to the household problem exists, and since both $Y'(s)$ and $C'(s)$ are increasing, the functions may cross once or several times. Consider first the case of a wage gap in favour of men. Then, all the solutions for the f.o.c. that yield $s^* < 0.5$ cannot be a global maximum. To see this consider a solution given by $s^* = x_1 < 0.5$, so that household utility is given by $V^H(x_1) = Y(x_1) - [c(x_1) + c(1-x_1)]$. Now consider the utility achieved under a division of labor where $s = x_2 = 1 - x_1 > 0.5$, so that $V^H(x_2) = Y(x_2) - [c(1-x_1) + c(x_1)]$. The disutility term is the same in both cases, but, as long as $W_m > W_f$, income is higher when females do a higher share of housework, i.e. $Y(x_2) > Y(x_1)$ implying that $V^H(x_2) > V^H(x_1)$. Thus, $s^* = x_1$ cannot maximize household utility and any solution to the household problem must yield a value of $s^* > 0.5$. It is possible, however, that there exists more than one solution satisfying $s^* > 0.5$. In this case, only one of them is a global maximum, and that is the allocation chosen by the household.

In the case of equal wages, it holds that $Y'(0.5) = C'(0.5) = 0$, implying that an equal division of housework is a solution to the f.o.c. Whether this is a maximum or a minimum depends on the relative concavity of the functions $Y(s)$ and $C(s)$ and hence on specific functional forms.

Appendix 2

Table A1 – List of Activities coded in STUS (and in MTUS)

MTUS Variable Name	Variable Label	MTUS Variable Name	Variable Label
AV1	Paid work	AV21	Walking
AV2	Paid work at home	AV22	Religious activities
AV3	Paid work, second job	AV23	Civic activities
AV4	School, classes	AV24	Cinema or theatre
AV5	Travel to/from work	AV25	Dances or parties
AV6	Cook, wash up	AV26	Social clubs
AV7	Housework	AV27	Pubs
AV8	Odd jobs	AV28	Restaurants
AV9	Gardening	AV29	Visit friends at their homes
AV10	Shopping	AV30	Listen to radio
AV11	Childcare	AV31	Watch television or video
AV12	Domestic travel	AV32	Listen to records, tapes, cds
AV13	Dress/personal care	AV33	Study, homework
AV14	Consume personal services	AV34	Read books
AV15	Meals and snacks	AV35	Read papers, magazines
AV16	Sleep	AV36	Relax
AV17	Free time travel	AV37	Conversation
AV18	Excursions	AV38	Entertain friends at home
AV19	Active sports participation	AV39	Knit, sew
AV20	Passive sports participation	AV40	Other leisure
		AV41	Unclassified or missing activities

The two housework share definitions used in the empirical analysis are:

- 1) AV7: Housework, which includes the following activities: Washing clothes, hanging washing out to dry, bringing it in, Ironing clothes, Making, changing beds, Making, changing beds, Dusting, hovering, vacuum cleaning, general tidying, Outdoor cleaning, Other manual domestic work, Housework elsewhere unspecified, Putting shopping away.
- 2) AV7+AV11, where AV11 includes the following activities: Feeding and food preparation for babies and children, Washing, changing babies and children, Putting children and babies to bed or getting them up, Babysitting (i.e. other people's children), Other care of babies, Medical care of babies and children, Reading to, or playing with babies and children, Helping children with homework, Supervising children, Care of children and babies elsewhere unspecified.

Appendix 3: Log-linearization of household's time allocation decision

In order to log-linearize the function $f(X)$, with $X > 0$, around a reference value, \bar{X} , we use the expression $f(X) \approx f(\bar{X})[1 + \eta x]$, where $x \equiv \ln X - \ln \bar{X}$ and $\eta \equiv [(df(\bar{X})/\partial \bar{X}) \cdot (\bar{X} / f(\bar{X}))]$.

Then, $\ln f(X) \approx \ln f(\bar{X}) + \ln[1 + \eta x] \approx \ln f(\bar{X}) + \eta x$. Let us consider the inverse of (16):

$$\left(\frac{s}{1-s}\right)^2 = \frac{1-a_f}{1-a_m}, \quad (\text{A.1})$$

where $a_i = W_i^2 / \varepsilon, (i = f, m)$. Then, using the previous result, log-linearization of (A.1) around the reference values $(s^* / 1 - s^*)$ and a_i^* and omitting constant terms for simplicity yields

$$\ln\left(\frac{s}{1-s}\right) = 0.5 \left(\frac{a_m^*}{1-a_m^*} (\ln a_m - \ln a_f) + \frac{a_m^* - a_f^*}{(1-a_m^*)(1-a_f^*)} \ln a_f \right). \quad (\text{A.2})$$

Since $\ln a_i = 2 \ln W_i - \ln \varepsilon$, we get equation (30) in the main text where

$$\theta_1 = \frac{a_m^*}{(1-a_m^*)} > 0, \quad \theta_2 = \frac{(a_m^* - a_f^*)}{(1-a_f^*)(1-a_m^*)} > 0, \quad \text{and} \quad \theta_3 = -0.5 \theta_2 < 0.$$

Under Assumption 1 (*i.e.*, $a_i^* < 1$), and with $s \in S$ so that $a_m^* > a_f^*$, it can be easily checked that $\theta_1 > \theta_2 > 0$ since $\theta_1 / \theta_2 = [(a_m^* - a_m^* a_f^*) / (a_m^* - a_f^*)] > 1$. Further, since θ_2 is proportional to $(a_m^* - a_f^*)$, it should be smaller for countries with gender gaps closer to the ungendered equilibrium, where $a_m^* = a_f^*$. Using (E.1) and (E.2), to replace wages in (16) in terms of $\ln \varepsilon$ and $\ln \beta$, a similar procedure can be used to obtain equation (31).

Lastly, let us consider the case where the bargaining weights of the spouses in the household's expected utility are different. For simplicity, let us assume that, instead of being 0.5 as in (14), the male and female weights now become $0.5(1 + \eta)$ and $0.5(1 - \eta)$, respectively, with $0 < \eta < 1$. Defining $\xi = (1 + \eta) / (1 - \eta) > 1$, the corresponding f.o.c. becomes in this case:

$$\left(\frac{1-s^*}{s^*}\right)^2 = \frac{1 - \xi(W_f^2 / \varepsilon)a_m}{\xi - (W_f^2 / \varepsilon)},$$

which can be log-linearized to yield:

$$\ln\left(\frac{s}{1-s}\right) = 0.5 \left(\frac{a_m^* \xi}{1-a_m^* \xi} (\ln a_m - \ln a_f) + \frac{a_m^* \xi^2 - a_f^*}{(1-a_m^* \xi)(\xi - a_f^*)} \ln a_f \right). \quad (\text{A.3})$$

It can be checked that, in contrast to the symmetric bargaining case, $\eta = 0$, the coefficient on the female wage need not be smaller than the coefficient on the wage gap.

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