Optimal Sovereign Default

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Introduction

- Provide a normative & quantitative analysis of sovereign default:

  When is it optimal for a sovereign to default?

- Contribution: determine ex-ante optimal default policies

  => **Ramsey policy with full commitment**

Introduction

- **Dominant view:** sovereign default (SD) is inefficient

  Policy discussions:

  ex-post costs associated with SD => default 'inefficient'

  private plans in disarray, financial & economic collapse

  Academic view:

  Limited commitment lit.: ex-post efficient but ex-ante inefficient

  Full commitment lit.: full repayment assumed & no SD

Introduction

- Important economic role for SD: resource transfer in times of scarcity

  SD *ex-ante efficient* if other insurance more costly or unavailable

- Government bond markets incomplete:

  partial repayment can be optimal under commitment

Introduction

Grossman & Van Huyck (AER 1988): ‘excusable default’

1. endowment economy with ex. incomplete gov. bond markets
2. determine optimal default policy when default is cost-free
3. can it be implemented as a reputational equilibrium?

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Contribution here:

1. prod. economy with **endogenously incomplete** gov. bond markets
2. opt. def. policies for **positive ex-post/default costs** (plausible)
   $=>$ *discontinuously affected* opt. default policies

Introduction

- Contribution relative to Ramsey policy literature:

  Aiyagari et al. (2002), Angeletos (2002), Chari et al. (1991), Sims (2001), Schmitt-Grohe & Uribe (2004), Adam (2011)

  Assumes market incompleteness and full repayment

- Full repayment inconsistent with optimality for empirically plausible levels of 'default costs': welfare gains of 1-2% of cons.

- Source of welfare gain: (1) relaxed borrowing limits, (2) increased efficiency of investment, (3) more efficient risk sharing .

Introduction

- Quantitative analysis:

  Default tends to be suboptimal following BC cycle-sized shocks, unless country close to maximally sustainable net foreign asset position.

- Full repayment assumption:

  'Reasonable' approx. to opt. repayment decisions

  An exact approximation only for very high levels of default costs

- Default optimal following occurrence of a disaster shock

  (Barro and Jin (2011)

  Optimal even if far from maximal net foreign debt position

  & even if sufficient resources for repayment available!

Outline of Remaining Talk

1. Introduce the model & formulate Ramsey problem:
   assume non-contingent debt & default costs
2. Contracting framework: endogenize debt contract & default costs:
3. Solution strategy & analytical results:
   concavity: default model -> complete markets model
   dealing with 'natural borrowing limits'
4. Quantitative & welfare analysis

Model Setup: Households and Firms

- Small open economy & representative consumer:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \qquad ,c_t \geq 0 \forall t \geq 0$$

- Domestic income risk:

$$y_t = z_t k_{t-1}^{\alpha} - \bar{c}$$

$z_t \in Z = \{z^1, ..., z^N\}$,transition law $\pi(z'|z)$

$\bar{c} \geq 0$: fixed expenditures (prod./cons.): non-transferable

- Risk neutral foreign investors

Model Setup: Government

- Fully committed government max'es utility of representative HH
- Seeks to smooth consumption implications of domestic income risk

Model Setup: Government

- Investment in 1-period riskless foreign bonds $F_t \geq 0$

  Gross interest rate $1 + r = 1/\beta$

  (longer maturity foreign bonds make not difference)

Model Setup: Government

- Issue non-contigent domestic bonds $D_t \geq 0$
- Default generates proportional dead weight costs $\lambda \geq 0$
- Cost accrue to borrower (b) and lender (l): $\lambda = \lambda^b + \lambda^l$
- Contracting framework:
  microfoundations for asset market structure & default costs

Model Setup: Government

- **Commitment:**
  Decision to default on bonds maturing in t taken in $t-1$

$$\Delta_{t-1} = (\delta_{t-1}^{1}, ..., \delta_{t-1}^{N}) \in [0,1]^{N},$$

  Partial default allowed for!

- Total repayment in t in state z^n

$$D_{t-1} \cdot (1 - (1 - \lambda^{b}) \cdot \delta_{t-1}(z^{n}))$$

  $\lambda^b \geq 0$: borrower's default costs

Optimal Policy Problem

- Ramsey allocation problem

$$\max_{\{F_t \geq 0, D_t \geq 0, \Delta_t \in [0,1]^N, k_t \geq 0, c_t \geq 0\}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$s.t.: c_t + \bar{c} + k_t + \frac{F_t}{1+r} = w_t + \frac{D_t}{1+R(z_t, \Delta_t)}$$

$$w_{t+1} \geq NBL(z_{t+1}) \quad \forall t \, \forall z_{t+1} \in Z$$

$$w_0 : \text{given}$$

- Beginning-of-period wealth:

$$w_t \equiv z_t k_{t-1}^{\alpha} + F_{t-1} - D_{t-1}(1 - (1 - \lambda^b)\delta_{t-1}(z_t)).$$

Foreign Lenders & Lower Bound on Default Costs

- Risk neutral international investors:

$$1 + r = \frac{1 - (1 + \lambda^l) \sum_{n=1}^{N} \delta^n \Pi(z^n | z_t)}{\frac{1}{1+R(z_t,\Delta)}}$$

- Allows estimating default costs: λ^l

- Combine data from Klingen, Weder, Zettelmeyer (2004) & Cruces and Trebesch (2011)

$$\lambda^l = 6.1\%$$

- $\lambda = \lambda^b + \lambda^l =>$ quantitative analysis: $\lambda = 10\%$ and $\lambda = 20\%$
- $\lambda \in [10\%, 20\%]$ makes huge difference relative to $\lambda = 0\%$

Outline of the Talk

1. Introduce the model & formulate Ramsey problem:
   assume non-contingent debt & default costs

2. **Contracting framework: endogenize debt contract & default costs**

3. Solution strategy & analytical results
   concavity: default model -> particular complete markets model
   dealing with 'natural borrowing limits'

4. Quantitative & welfare analysis

Optimal Government Debt Contracts

- One particular microfoundation, others may exist...

Optimal Government Debt Contracts

- Debt contract consists of an **explicit & implicit** component
  - explicit component: repayment specified in the legal text
  - implicit repayment: not formalized, commonly(!) understood
- **Default:** implicitly promised payment falls short of explicit one

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- Without frictions: explicit & implicit components perfect substitutes
- 2 frictions $=>$ non-contingent sovereign debt & default costs λ

Optimal Government Debt Contracts

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  explicit contract: costly, requires input of resources (lawyers, staff...)

  implicit contract: common understanding may be lost
  after maturity date

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- Without common understanding of implicit contract:
  - courts rule by interpreting the explicit contract
  - default exposes gov. to risk of being sued for full repayment
  - anticipation of such action following default:
    gov. needs to reach an *explicit* legal settlement
    costly => default costs.

Optimal Government Debt Contracts

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  requires paying contracting costs for sure ex-ante

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- **Implicit contracting dominates:**

  costs have to be paid only with certain probability & later

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2. Contracting framework: endogenize debt contract & default costs:
3. **Solution strategy & analytical results:**
   concavity: default model -> particular complete markets model dealing with 'natural borrowing limits'
4. Quantitative & welfare analysis

Solving the Optimal Policy Problem

- Ramsey allocation problem

$$\max_{\{F_t \geq 0, D_t \geq 0, \Delta_t \in [0,1]^N, k_t \geq 0, c_t \geq 0\}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$s.t.: c_t + \bar{c} + k_t + \frac{F_t}{1+r} = w_t + \frac{D_t}{1 + R(z_t, \Delta_t)}$$

$$w_{t+1} \geq NBL(z_{t+1}) \quad \forall t \, \forall z_{t+1} \in Z$$

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Solving the Optimal Policy Problem

- Interest rate $R(z_t, \Delta_t)$ depends on default policy: unclear if problem is concave & use of FOCs justified....
- Borrowing limits:

  very loose $=>$ non-existence of optimal policies

  too tight $=>$ exclude feasible policies
- Loosest constraints consistent w existence & non-explosive debt:

  show how to compute & uniqueness

Solving the Optimal Policy Problem

- Derive a concave equivalent formulation of the Ramsey problem
- Show how to compute 'marginally binding' NBL
  - rule out Ponzi schemes (but nothing else!)
  - policies with non-explosive debt exists above these limits.

Concave Equivalent Problem

- Equivalent optimization problem:

$$\max_{\{b_t, a_t \geq 0, k_t \geq 0, c_t \geq 0\}} E_0 \sum_{t=0}^{\infty} \beta^t u(\widetilde{c}_t)$$

$$s.t. \quad \forall t : \widetilde{c}_t = \widetilde{w}_t - \overline{c} - \widetilde{k}_t - \frac{1}{1+r} b_t - p_t \cdot a_t$$

$$\widetilde{w}_{t+1} \geq NBL(z_{t+1}) \quad \forall z_{t+1} \in Z$$

$p_t(z^n) = \pi(z^n | z_t)/(1+r)$ indep. of government policy!

$=>$ problem concave!

Concave Equivalent Problem

- Equivalent optimization problem:

$$\max_{\{b_t, a_t \geq 0, k_t \geq 0, c_t \geq 0\}} E_0 \sum_{t=0}^{\infty} \beta^t u(\tilde{c}_t)$$

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$p_t(z^n) = \pi(z^n | z_t)/(1+r)$ indep. of government policy!

$=>$ problem concave!

- Beginning-of-period wealth

$$\tilde{w}_t \equiv z_t k_{t-1}^{\alpha} + b_{t-1} + (1-\lambda) a_{t-1}(z_t)$$

No unique pricing kernel/price system not arbitrage free for $\lambda > 0$

Concave Equivalent Problem

- b has an interpretation as the net foreign asset position

$$b_t = F_t - D_t,$$

- Arrow securities capture state contingent default on D_t
- With 2 productivity states:

$$a_t = \begin{pmatrix} D_t \delta^1 \\ D_t \delta^2 \end{pmatrix}$$

- $a \geq 0$ reflects $\delta \geq 0, D_t \geq 0$: no 'creation of default costs' possible.

Concave Equivalent Problem

Proposition

A consumption path $\{c_t\}_{t=0}^{\infty}$ is feasible in the original problem if and only if $\{\widetilde{c}_t\}_{t=0}^{\infty}$, with $\widetilde{c}_t = c_t$ for all $t \geq 0$, is feasible in the equivalent Problem.

Marginally Binding NBLs

- Define 'Marginally Binding Natural Borrowing Limits':
  Loosest constraints consistent with non-explosive debt dynamics

- Implicitly defined by a fixed point problem:

$$\begin{aligned} NBL(z^n) = \arg\min \widetilde{w}(z^n) \; s.t. \\ \widetilde{w}'(z^j) \geq NBL(z^j) \text{ for } j = 1, \ldots, N, \end{aligned} \quad (1)$$

Marginally Binding NBLs

Proposition

Under some regularity assumptions, there exists - generically for all model parameterizations - a unique solution to the fixed point problem (1).

Multiplicity problematic: uniformly laxest set of borrowing limits $NBL(z^n)$ may not exist...

Marginally Binding NBLs

Proposition

Suppose the regularity condition holds. Given a productivity state $z_t = z^n$ *and a beginning-of-period wealth level* $\widetilde{w}$*:*

1. *If* $\widetilde{w} \geq NBL(z^n)$*, then there exists a policy that is consistent with non-explosive debt dynamics along all future contingencies.*
2. *If* $\widetilde{w} < NBL(z^n)$ *then there exists no policy that does not violate* **any** *finite debt limit with positive probability.*

Optimal Default Policies: Analytic Results

- $\lambda = 0$: complete markets allocation, constant c
  frequent default: $N - 1$ productivity states
  optimal default independent of net wealth position
  net wealth position constant over time

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Optimal Default Policies: Analytic Results

- $\lambda = 0$: complete markets allocation, constant c
  frequent default: $N - 1$ productivity states
  optimal default independent of net wealth position
  net wealth position constant over time
- $\lambda \geq 1$: default never optimal, only self-insurance
- $\lambda \in (0, 1)$: trade-off between default & self-insurance:

  1. sufficiently high wealth: default never optimal (discontinuity)
  2. wealth at the NBL: default optimal depending on λ

Optimal Default Policies: Analytic Results

Proposition

Suppose $\lambda > 0$ *and consumption preferences satisfy* $\lim_{c\to\infty} u''(c) = 0$. *Consider a time horizon* $T < \infty$ *and for* $j = 0, ..., T$ *%*

$$\begin{aligned}
\widetilde{c}_{t+j} &= (1-\beta)(\Pi(z_{t+j}) + \widetilde{w}_{t+j}) \\
\widetilde{k}_{t+j} &= k^*(z_{t+j}) \\
b_{t+j} &= (1+r)\left(\widetilde{w}_{t+j} - k^*(z_{t+j}) - (1-\beta)(\Pi(z_{t+j}) + \widetilde{w}_{t+j}) - \overline{c}\right) \\
a_{t+j}(z^n) &= 0 \text{ for all } n = 1, ..., N
\end{aligned}$$

For any $\epsilon > 0$ *we can find a wealth level* $\bar{w} < \infty$ *so that for all initial wealth levels* $\widetilde{w}_t < \infty$ *satisfying* $\widetilde{w}_t \geq \bar{w}$, *the Euler equation errors* e_{t+j} *implied by the policies above satify* $e_{t+j} < \epsilon$ *for all periods* $j = 0, ..., T-1$.

- Default policies discontinuously switch from $\lambda = 0 \rightarrow \lambda > 0$!

Optimal Default Policies: Wealth at the NBL

- Critical productivity index n_t^* :

$$n_t^* = \arg \max_{n \in [1,...,N]} n \quad (2)$$
$$s.t. \sum_{i=n}^{N} \pi(z^i | z_t) \geq 1 - \lambda$$

Note: n_t^* is increasing in λ

- Default will be optimal for all states z^n with $n > n^*$

Optimal Default Policies: Analytic Results

Proposition

Suppose the regularity condition holds and $\widetilde{w}_t = NBL(z_t)$, *then*

$$\widetilde{k}_t = \left(\alpha\beta\left(\frac{\sum_{n=1}^{n_t^*}\pi(z^n|z_t)z^{n_t^*} - \lambda}{1-\lambda} + \frac{\sum_{n=n_t^*+1}^{N}\pi(z^n|z_t)z^n}{1-\lambda}\right)\right)^{\frac{1}{1-\alpha}} \quad (3)$$

$$\widetilde{c}_t = 0 \text{ and } b_t = NBL(z^{n_t^*}) - z^{n_t^*}\widetilde{k}_t^{\alpha} \quad (4)$$

$$a_t(z^n) = 0 \text{ for } n \leq n_t^* \quad (5)$$

$$a_t(z^n) = \frac{NBL(z^n) - z^n\widetilde{k}_t^{\alpha} - b_t}{(1-\lambda)} > 0 \text{ for } n > n_t^* \quad (6)$$

Characterizes optimal policy for single period!
Default neg. related to productivity realization
Investment distortion increasing in λ (& larger for 'good' states)
Consumption zero...

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   concavity: default model -> particular complete markets model
   dealing with 'natural borrowing limits'
4. **Quantitative & welfare analysis**

Quantitative Analysis: Calibration

1. Optimal default policies for intermediate wealth levels?
2. Size of the welfare gains'?

Quantitative Analysis: Calibration

- Calibrate the model at annual frequency $z^h = 1.0133$, $z^l = 0.9868$
- Transition matrix for the states, given by

$$\pi = \begin{pmatrix} 0.8077 & 0.1923 \\ 0.1923 & 0.8077 \end{pmatrix}$$

- Utility function is given by

$$u(c) = \frac{(c - \bar{c})^{1-\sigma}}{1 - \sigma}$$

- $\overline{c}$: if bonds must be repaid always, max sustainable NFA equals -100% of GDP (Lane and Milesi-Ferretti (2007))
- Remaining parameters:

α	β	σ	$\bar{c}$	1+r
0.34	0.97	2	0.357	$1/\beta - 0.0005$

Quantitative Analysis: Optimal Def. Policies

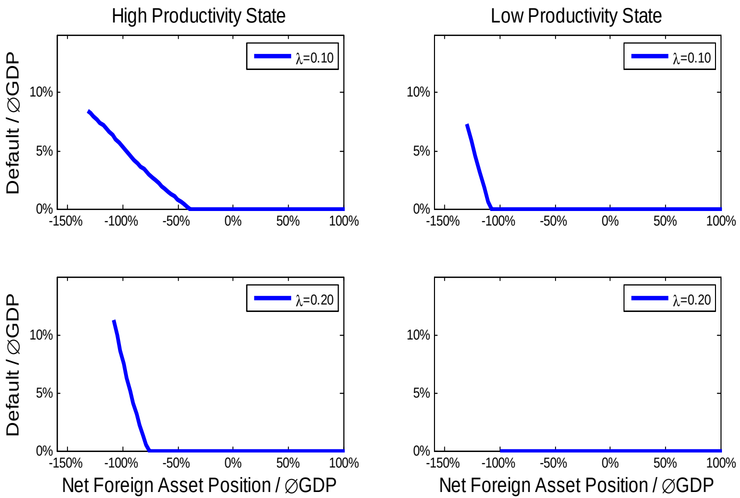


Figure: Optimal Default Policies (top row: λ=10%, bottom row: λ=20%)

Quantitative Analysis:Welfare

- Consumption equivalent welfare variation
- c_t^1 : optimal consumption if repayment of debt must always occur (presumption of Ramsey)
- c_t^2 : optimal consumption with optimal default decisions

$$E_0\left[\sum_{t=0}^{500}\beta^t\frac{\left(\left(c_t^1+\omega(c_t^1+\bar{c})\right)\right)^{1-\gamma}}{1-\gamma}\right]=E_0\left[\sum_{t=0}^{500}\beta^t\frac{(c_t^2)^{1-\gamma}}{1-\gamma}\right]$$

- increase measured with respect to $c_t^1+\bar{c}$: understatement
- non-stationary nature of domestic wealth: finite horizon of 500 yrs

Quantitative Analysis:Welfare

- Welfare increases as λ falls
- Welfare increases as initial wealth falls towards NBL of no default economy
- Upper bound on welfare: $\lambda = 10\%$, and net foreign asset at -100% of GDP:
  $$\omega = 1.9\%$$
  but quickly falls for higher net foreign asset values...

Quantitative Analysis: Economic Disasters

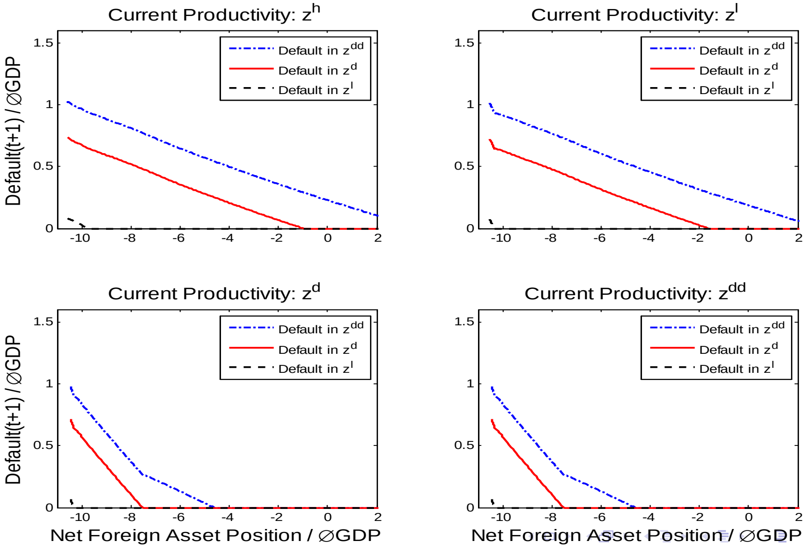
- Calibrating Economic Disasters following Barro and Jin (2011):

  shock process
  $Z = \left\{z^h, z^l, z^d, z^{dd}\right\} = \{1.0133, 0.9868, 0.9224, 0.6696\}$
  with transition matrix

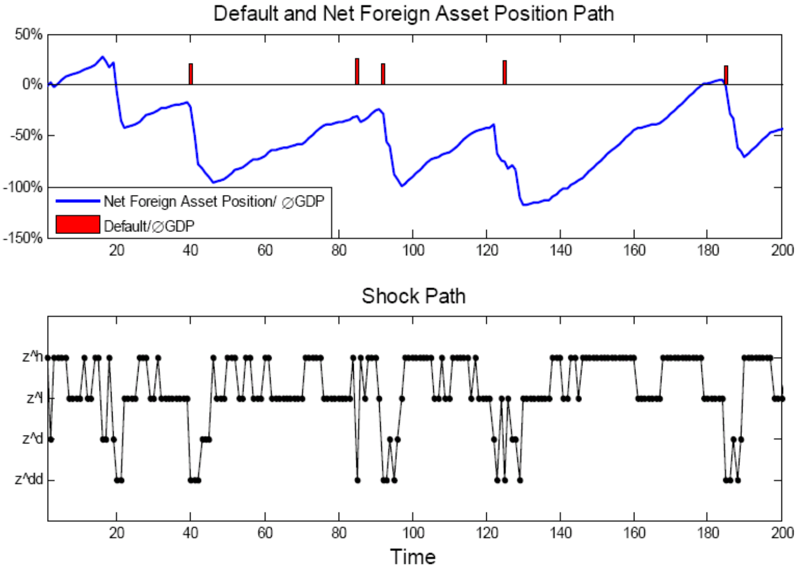
$$\pi = \begin{pmatrix} 0.7770 & 0.1850 & 0.019 & 0.019 \\ 0.1850 & 0.7770 & 0.019 & 0.019 \\ 0.1429 & 0.1429 & 0.3571 & 0.3571 \\ 0.1429 & 0.1429 & 0.3571 & 0.3571 \end{pmatrix}.$$

- Recalibrate the subsistence level of consumption to $\bar{c} = 0.198$.

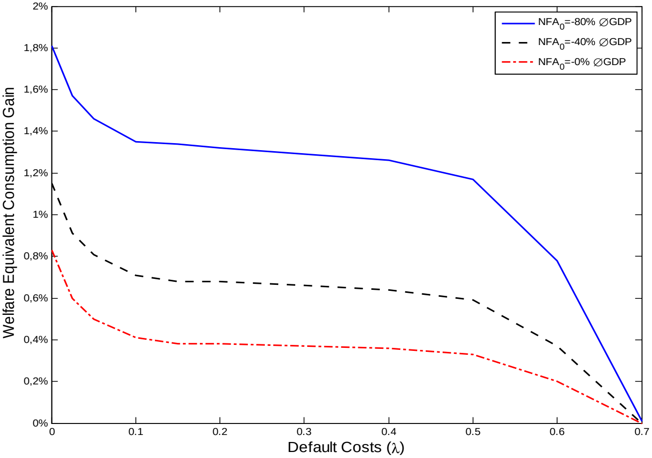
Quantitative Analysis: Economic Disasters



Quantitative Analysis: Economic Disasters



Quantitative Analysis: Economic Disasters & Welfare



Quantitative Analysis: Economic Disasters & Welfare

- Sources of welfare gains

  (1) relaxed borrowing limits

  (2) more efficient investment policies

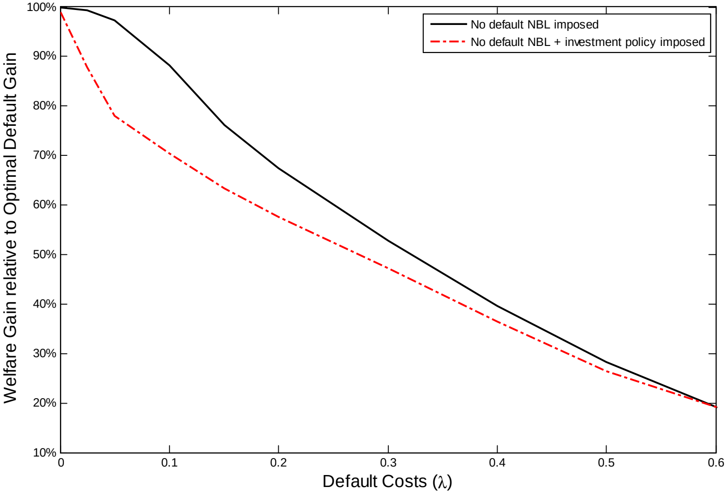
  (3) risk sharing gains from default

- Systematically decompose the welfare gains

  For small λ gains are all from (3)

  For $\lambda \approx 30\%$ gains are 50% from (3) and about 50% from (1)

Quantitative Analysis: Economic Disasters & Welfare



Long Maturities

- No difference from introducing long foreign bonds: no value for insurance
- No difference from long domestic bonds if repayment is assumed (unlike in Angeletos(2001))
- Long domestic bonds with default option:

  (partial) default *in the future* after bad event *today* $=>$ bonds fall in value

  repurchase at depreciated value & realize a capital gain
- Improvements possible: if repurchase has lower costs than default....

Conclusion

- Sovereign default is optimal under commitment if bond markets incomplete
- Relaxes borrowing limits, increases efficiency of investment, increases risk sharing
- Default optimal after bad output realizations and for low NFA
- Welfare gains large (1-2% of cons.) & not very sensitive to default costs
- Long bonds coupled with buyback potentially even more efficient

Conclusion

Commitment view on SD of normative interest...

... upon closer inspection also reasonable from a positive perspective

Commitment view

- Can rationalize high outstanding debt levels:

  default option *relaxes* the borrowing limits

- Default optimal following negative shocks to domestic income.

- Optimal default can look like a 'willingness to pay problem'

  sufficient resources for repayment available!