Population Ageing and the Macroeconomy

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Online Appendix

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A Appendix: Additional Model Details

A.1 Bequests and non-labour income

At each time t, the non-housing assets of the generations that died in the previous period must be distributed, along with the accrued interest, to living households through bequests, B_t, given by

$$B_t = (1 + r_t) \sum_{\tau=1}^{T} (1 - \psi_{\tau,t-\tau}) \tilde{\psi}_{\tau,t-\tau} s_{t-\tau} a_{\tau,t-\tau}$$

$$\tilde{B}_t = (1 + r_t) \frac{\sum_{\tau=1}^{T} (1 - \psi_{\tau,t-\tau}) \tilde{\psi}_{\tau,t-\tau} s_{t-\tau} a_{\tau,t-\tau}}{S_t}$$

Similarly, the housing wealth of the agents that died in the previous period must be distributed among remaining agents. This is aggregated analogously to savings above

$$\tilde{B}^h_t = \frac{\sum_{\tau=1}^{T} (1 - \psi_{\tau,t-\tau}) \tilde{\psi}_{\tau,t-\tau} s_{t-\tau} h_{\tau,t-\tau}}{S_t}$$

The additional housing endowment, added in each period to maintain a stable level of housing per capita, is added to the aggregate asset and housing bequests to form aggregate non-labour income, Π_t. In other words

$$\tilde{\Pi}_t = \tilde{B}_t + p^h_t \tilde{B}^h_t + p^h_t \left(\frac{S_{t+1}}{S_t} - 1 \right) \tilde{H}$$

A.2 Calibration Procedure

We set the parameters of the CES production function $\sigma = 0.7$ and $\alpha = 1/3$, and the annualised depreciation rate $\delta = 6\%$.

In order to set the parameters of the household's problem to match our moments, we consider a steady state of the model in which all households have the same demographic characteristics as the 1945 cohort. For a given calibration of this steady state, we run the dynamic simulations given the demographic transition, and back out the implied average life-cycle profiles in 1990-2015, and aggregate moments in the 1970s. We then adjust the calibration of the initial steady state until the moments that come out of the dynamic simulation match our targets from the data.

The steady state of the model gives us a stationary vector of relative population weights, $\tilde{\rho}$, with which we normalise the productivity profile such that aggregate labour productivity is 1, that is $\tilde{\rho}'\epsilon = 1$. We set hours worked at 0.3 throughout working life, hence $l_\tau = 0.3$ for $\tau = 1, \ldots, T^r - 1$, and $l_\tau = 0$ for $\tau \geq T^r$. Hence aggregate labour supply is $L = 0.3$, the value commonly used in the literature. The wage is then set as the marginal product of labour consistent with the firm's first order condition with respect to labour. Households are assumed to retire at age 65, corresponding to $T^r = 10$. They start receiving bequests at age $T^b = 7$, i.e. age 50.

We normalise the life-cycle profile of assets, a, such that aggregate wealth is consistent, using the firm's first order condition with respect to capital, with the annualised interest rate target of 3.42%, and debt, in the first periods of life, is consistent with the debt-to-GDP target of 40%. Since assets in the final period of life are non-zero, we set $\phi > 0$ to satisfy the first order condition with respect to a_T for the observed level of a_T.

Finally, we normalise housing wealth over the life-cycle such that the aggregate housing stock, $\tilde{H}$, is consistent with the housing wealth-to-GDP target of 147%. As mentioned above, we do not allow households to re-optimise their housing wealth in every period, and correspondingly, we use a step-wise function to fit the estimated life-cycle profile. Since this profile is found to be significantly above zero in both the first and last age groups, we set both $\tau = 1$ and $\tau = T$ as "move dates" in the household's problem. In order to match the observed peak in housing wealth in middle age and subsequent fall at around age 70, we allow $\tau = 5$ and $\tau = 11$, corresponding to ages 40 and 70, to also be "move dates". For simplicity, we set θ_1, θ_5, θ_{11} and θ_T to satisfy the first order condition with respect to housing with $\theta_\tau = 0$ for all other τ.

For the final step of the calibration, for a given life-cycle profile of labour and non-labour income, housing wealth and net assets, the steady state budget constraint gives consumption over the life-cycle

$$c_\tau = w l_\tau \epsilon_\tau + (1 + r) a_\tau - a_{\tau-1} - p^h (h_\tau - h_{\tau-1}) + \pi_\tau$$

Following Glover et al. (2014), we set $\beta_1 = 1$ and calibrate β_τ, $\tau > 1$, such that the Euler equations are satisfied given this stream of consumption

$$\beta_\tau = \frac{\beta_{\tau-1}}{1+r} \frac{\tilde{\psi}_{\tau-1}}{\tilde{\psi}_\tau} \frac{c_\tau}{c_{\tau-1}}$$

A.3 Life-cycle profile data

Labour income

We calibrate productivity to match *"Wage Income"* data from the SCF, which corresponds to total labour income, irrespective of hours worked. Hence, since hours worked are inelastic in the model, we are effectively subsuming all life-cycle hours and wage decisions into the productivity profile. The estimated labour income profile falls close to zero from around age 65, and in fact *median* wage income is exactly zero from the 65-70 age group. Hence we assume retirement begins at age 65, that is $T^r = 10$.

Housing and non-housing wealth

To calibrate housing wealth over the life-cycle, we take the sum of *"Primary Residence"* and *"Other Residential Real Estate"* in the SCF. The SCF includes a measure of *"Net Worth"* that aggregates all financial and non-financial assets and liabilities: to ensure that the profile of total net worth in the model corresponds to this observed net worth, we calibrate non-housing assets, a, to match the SCF *"Net Worth"* minus housing wealth as defined above. Note that, in this way, housing wealth measures only housing *assets*, and any *debt* related to housing, such as mortgages, are included in other assets, a.

Data construction

To create the life-cycle profiles for each of these variables, we put the survey respondents into 5-year age buckets corresponding to the life-cycle of households in the model, and calculate the average level of each variable for each age group, using the sampling weights provided in the SCF. This gives us an estimated life-cycle profile for each survey year from 1989-2016. We then take the average over the survey years, weighting by the number of observations in each age group in each survey year.[1] This procedure gives us an estimated life-cycle profile for each of the three variables, corresponding to the average cross-sectional age profile between 1989-2016.

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[1] Using the coefficients on the age group in fixed effects panel regressions yields similar results.

B Appendix: Additional Results

B.1 Drivers decomposition

Population structure and household decisions

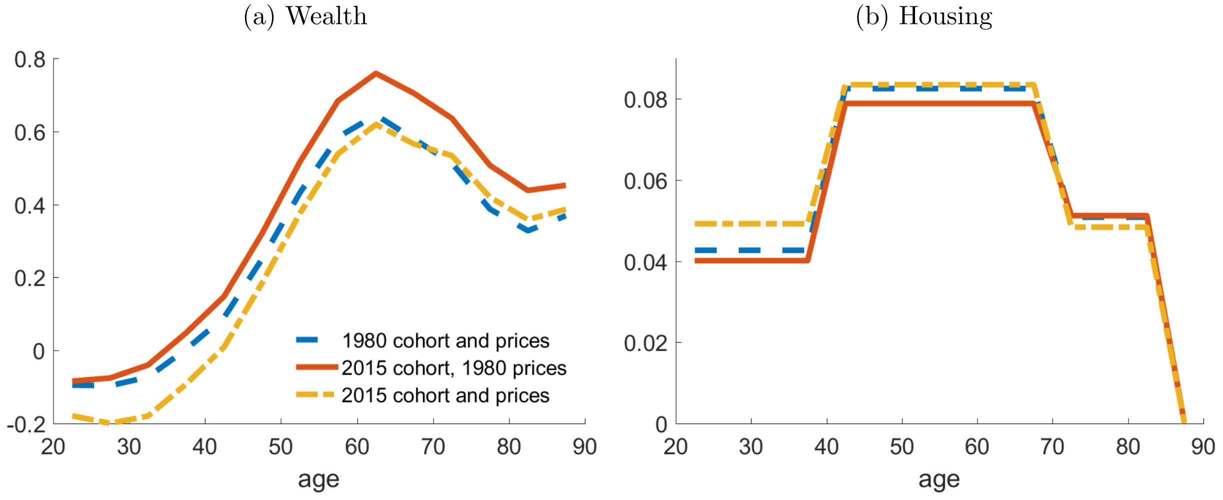
To decompose the changes in aggregate savings into two distinct drivers: changes in the age composition of the population and changes in the life-cycle savings decisions of each household, we proceed as follows. The baseline aggregate variables are calculated as weighted sums of the alive cohorts' per capita variables. These aggregate variables are therefore driven by the interaction of the changes in the weights of each cohort in the total population and the changes in the individual housing and saving decisions of the alive cohorts. To compute the impact of population weights only, we re-calculate the aggregate variables keeping fixed the alive cohorts' per capita variables at their 1950 level. On the opposite, to obtain the impact of optimisation decisions only, we re-calculate the aggregate variables keeping the weights fixed at their 1950 level. The results shown in Figure 8 in the paper are therefore *not* the outcome of a general equilibrium transition, but an ex-post decomposition. Finally, as population weights and individual household decisions multiply each other to obtain the baseline aggregate variables, it is normal that the separate effects of these two drivers do not add up to the baseline path.

Partial and general equilibrium effects

A second exercise we can carry out is to decompose the change in the life-cycle savings profile into the partial equilibrium effect of increased longevity, holding all prices constant, and the general equilibrium impact of the change in interest rates and house prices. This is shown in Figure 1, where the blue dashed lines and yellow dash-dotted lines show the general equilibrium savings and housing-wealth profiles of the 1980 and 2015 cohorts respectively, and the red solid lines show the partial equilibrium optimal savings and housing-wealth profile of the 2015 cohort if prices were held fixed at their 1980 levels.

Without any price adjustment, the 2015 cohort, which has a higher life expectancy, decides to save more and hold less housing than the 1980 cohort (comparison between the blue dashed and the red solid curves). When prices do adjust, however, the equilibrium interest rate is lower in 2015, and for the 2015 cohort saving is less attractive and borrowing in youth is more attractive, so that the desired non-housing wealth of that

Figure 1: Counterfactual wealth and housing profiles


(a) Wealth

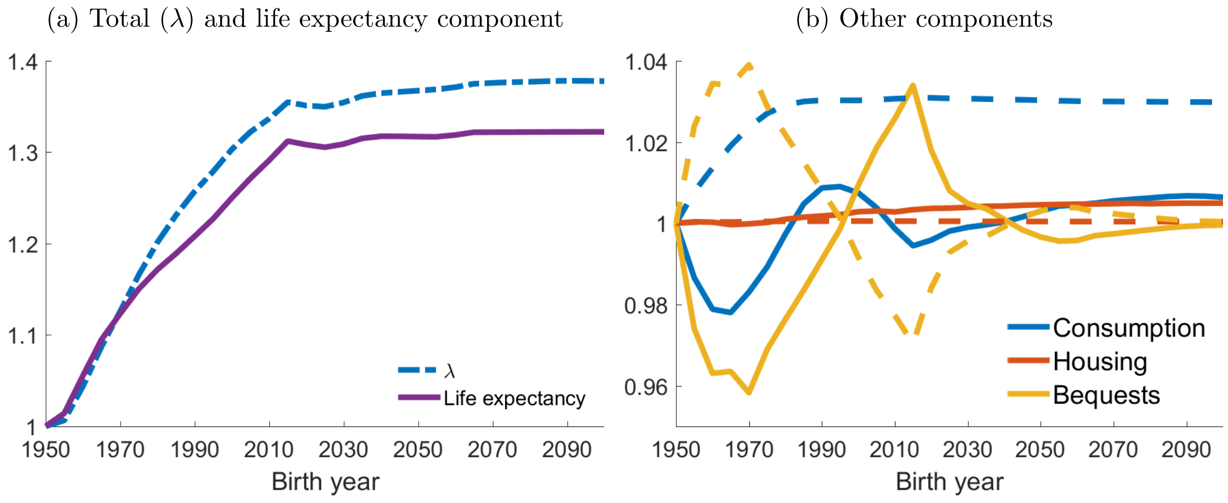
(b) Housing

cohort is lower than that of the 1980 cohort (comparing the blue dashed and the yellow dashed-dotted curves). Conversely, as house prices rise, housing wealth is higher for the 2015 cohort.

B.2 Consumption-equivalent variation and its decomposition

Following the method developed by Jones and Klenow (2016), we calculate the proportion by which the consumption of a household born in 1950 would need to be adjusted to equalise his welfare to that of a household born at another date t. The intuition is the following. Take a household born at time t, and change his living conditions (in terms of life expectancy, share of GDP used for consumption, inequality, and so on) for the ones of a household born in 1950. To bring this household's welfare back to its initial level, we need to adjust his consumption by a certain amount. This is the consumption-equivalent variation, measured as a proportion of the consumption of a household born in 1950. A consumption-equivalent variation smaller (resp. larger) than 1 means that the household born in 1950 is better (resp. worse) off than the one born at time t. Similarly to Jones and Klenow (2016), we are further able to decompose this measure into various components reflecting: (i) life expectancy; (ii) consumption; (iii) housing; (iv) bequests; (v) consumption smoothing; (vi) housing smoothing and (vii) bequest smoothing. All these components are measured relative to the reference cohort born in 1950. Components (i) to (iv) are clearly positively related to the welfare of the household born at time t, as

Figure 2: Consumption-equivalent variation and its components


(a) Total (λ) and life expectancy component

(b) Other components

Dashed lines in panel (b) show the 'smoothing' component for each term.

they directly enter the utility function. Components (v) to (vii), instead, measure the difference at time t between the average utility from, for example, consumption, $E(u(c))$, and the utility obtained from average consumption, $u(E(c))$.[2] An improved smoothing of consumption over time corresponds to a mean-preserving contraction in consumption across age groups. It keeps the utility from average consumption fixed, but increases the average utility from consumption, thus increasing the term (v) so that the relative welfare of the household born at time t improves.

Figure 2 shows the evolution of this consumption-equivalent variation and its components over time. Figure 2a shows a clear welfare increase over time. The 1950 cohort's consumption would need to be increased by 13% percent to render a household born in 1970 indifferent between being born in 1970 and in 1950. This goes up to 26% for a household born in 1990 and 34% for 2010, before stabilising towards 38%. The purple solid line in Figure 2a shows that the main driver is life expectancy. Figure 2b further shows that the improvement in consumption smoothing in our model is the second most important driver of welfare, reflecting the same mechanisms that lowered the Gini coefficient for consumption shown in Figure 12 in the paper, while total consumption and bequest have been oscillating and tend to compensate each others. Here again, the

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[2] These components correspond to the ones labeled "inequality" in Jones and Klenow (2016). Given that we have one representative agent of each age in our set-up, there is no inequality within age groups, and these components only measure variations of consumption across age groups over time, and not variations of consumption within age group. We therefore prefer to interpret them as "smoothing" rather than "inequality" components.

impact of housing and housing smoothing is very limited.

Although the effect of life expectancy appears incredibly important, there is a difficulty in including this in the welfare of different cohorts: while everything else is scale invariant, utility in death is assumed to be fixed at zero, meaning that the relative value of being alive does depend on the calibration scale, specifically if consumption, housing and bequests are above or below one. While the life expectancy component is consistently found to be a major driver of welfare over time, the precise estimation of its impact depends on the normalisation of the model. We therefore prefer to focus on its qualitative implications only. None of the other components presented in Figure 2b suffer from this conceptual problem.

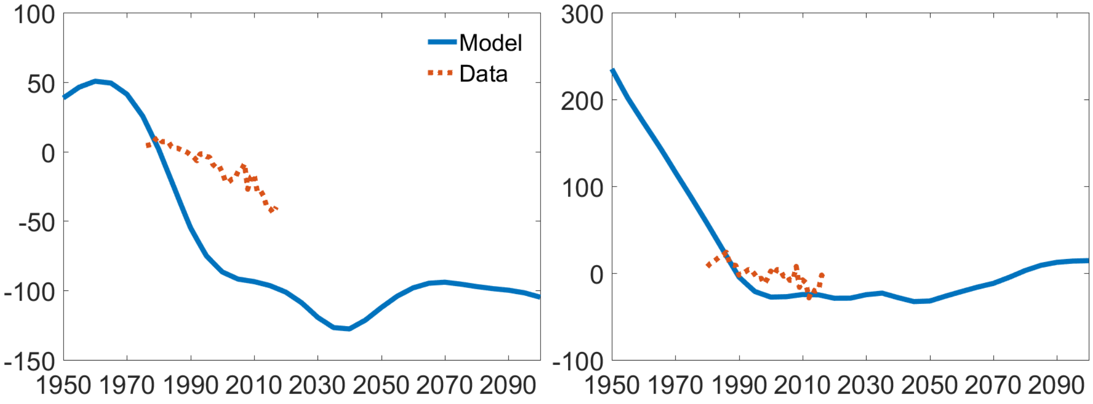
B.3 Open economy exercise

Using the open economy simulations described in the text, Figure 3 shows the path of the NFA for the US, UK, Australia and Germany. This simple exercise can capture the dynamics of NFAs, with Australia, the UK and the US having increasingly negative NFA positions both in the model and in the data, and Germany building up an increasingly positive NFA position. The model also suggests that the NFA position in the US and Germany will diverge further in the coming decades, as their demographic characteristics diverge from the aggregate of advanced economies, while for the UK and Australia it will remain stable.

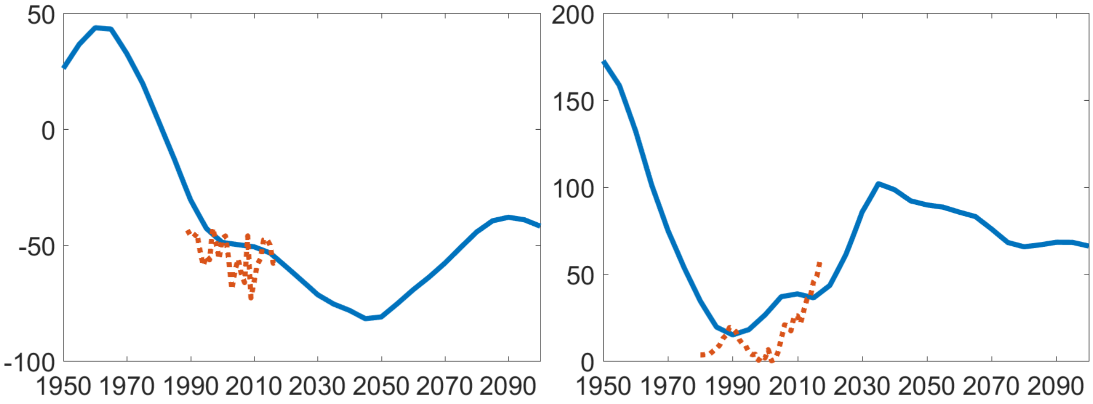
Figure 4 plots the NFA position in 2015 against the High-wealth ratio in 2015, for the model outcome and the data, across all of the 23 countries in our aggregate advanced-economies group. We see again that the model tends to predict a larger NFA position than observed in the data. Nonetheless, it does well to explain the cross-country pattern of NFA positions.

Figure 4 also includes the model predictions for NFA positions against the HWR in 2030. All countries move to the right on the HWR scale as they age. As this happens, the model predicts that some countries will move towards higher NFA positions, as they age faster than the average, while other countries will have increasingly negative NFA positions as they age more slowly than the average.

Figure 3: NFA-to-GDP (%) in the Small Open Economy Simulations


(a) United States

(b) United Kingdom


(c) Australia

(d) Germany

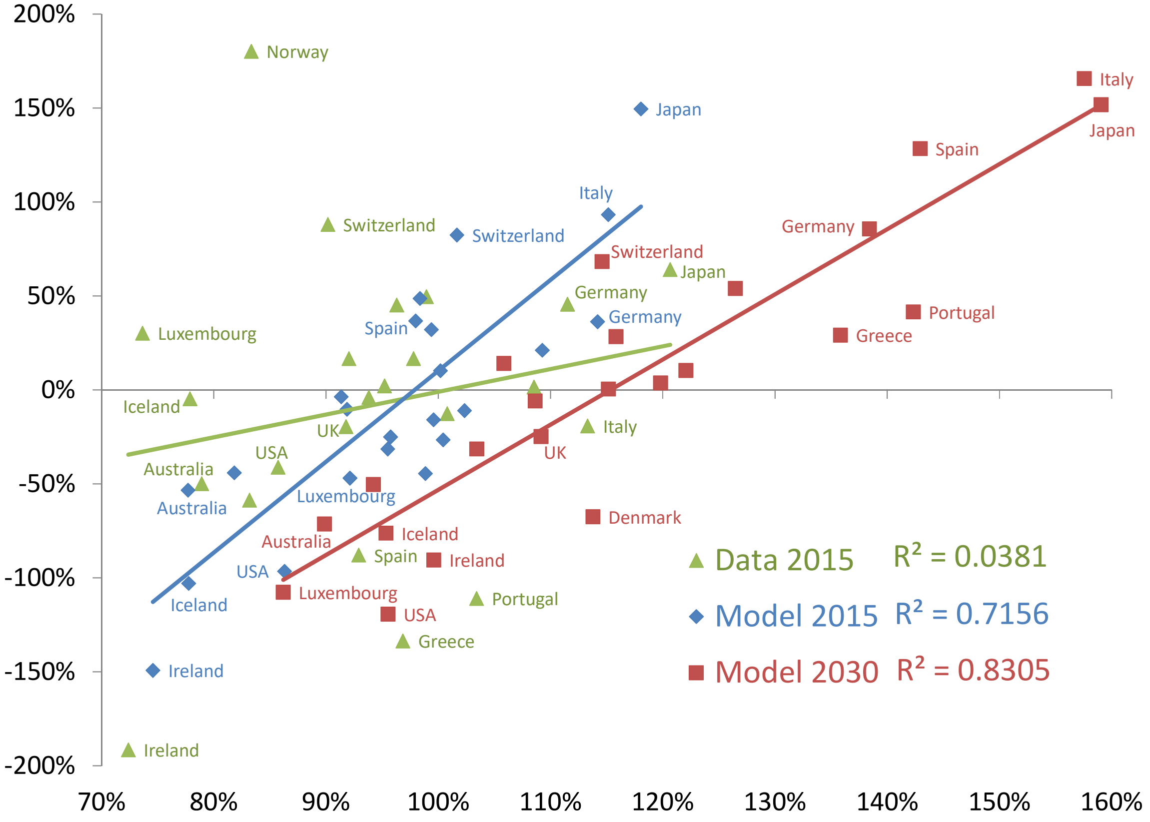
Sources: IMF IFS, authors' calculations.

C Appendix: Additional Robustness and Extensions

C.1 The role of housing

The discrete move-dates make the house price sensitive to changes in the relative size of different cohorts as they move from buying to selling housing. This means that the baby boom is important for the dynamics of house prices, as shown in Figure 5. In general equilibrium, shown in the blue line, the aggregate housing demand per capita is equal to the aggregate housing supply per capita, which is constant. This is not the case in partial equilibrium, keeping the optimal housing choice of each age cohort fixed at its 2015 level, and varying only the weight of those cohorts within the total population, shown in the red line. Clearly, this drives up the demand for housing from 1980 until

Figure 4: Demographic Changes and NFA accumulation



Note: HWR on x-axis and NFA-to-GDP on y-axis.
Sources: IMF IFS, UN population statistics, authors' calculations.

2015, while it stabilises it and drives it down from 2015 onwards.

The overall impact of housing on the model results is quantified by comparing the baseline results against the results from a model in which we exclude housing. To facilitate interpretation, we keep the parameter values obtained in the baseline case to solve the model without housing. Consequently, aggregate savings and the interest rate are higher (resp. lower) over the whole transition period, and aggregate variables without housing do not match the target set in the baseline case.

The results are shown in Figure 6. As expected, the level of the capital to output ratio increases more in the absence of housing, as households do not have any alternative for transferring wealth over time. Households also accumulate less debt, as they do not need to borrow to afford housing. Given the curvature of the production function, the impact of housing on the interest rate drop is smaller than on the level of capital-to-GDP. In terms of the marginal effect of including housing in the model, the fall in the interest rate between 1980 to 2100 is around 250bps in the model without housing, 17bps larger than the baseline. Conversely, the rise in the household debt-to-GDP ratio over the same years is 22pp lower in the model without housing. Note that the presence of housing in the model prevents the interest rate from turning negative from 2060 onwards, which is not the case any more when it is excluded.

Finally, we check the robustness of our results to the assumption that the new housing stock is added exogenously to the households' non-labour income. The alternative here is to have this new housing stock coming out of the resource constraint, which implicitly means that it is being produced from the consumption good. The results of this exercise are shown in Figure 7. The differences with the baseline are negligible, showing that this assumption is not quantitatively important.

C.2 Robustness: the retirement age

The retirement age in our model is fixed at age 65 during the whole transition. Increasing the retirement age may seem more realistic, as reforms in that direction have been implemented in most advanced economies. This is likely to offset the effects of population ageing, as households spend less time in retirement for a given life expectancy.

Solving the model with a change in the retirement age during the transition requires some

Figure 5: Aggregate housing demand, in general equilibrium and when only population weights are changing

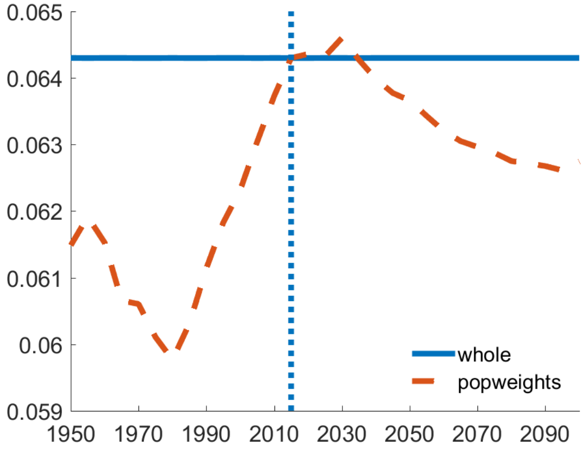
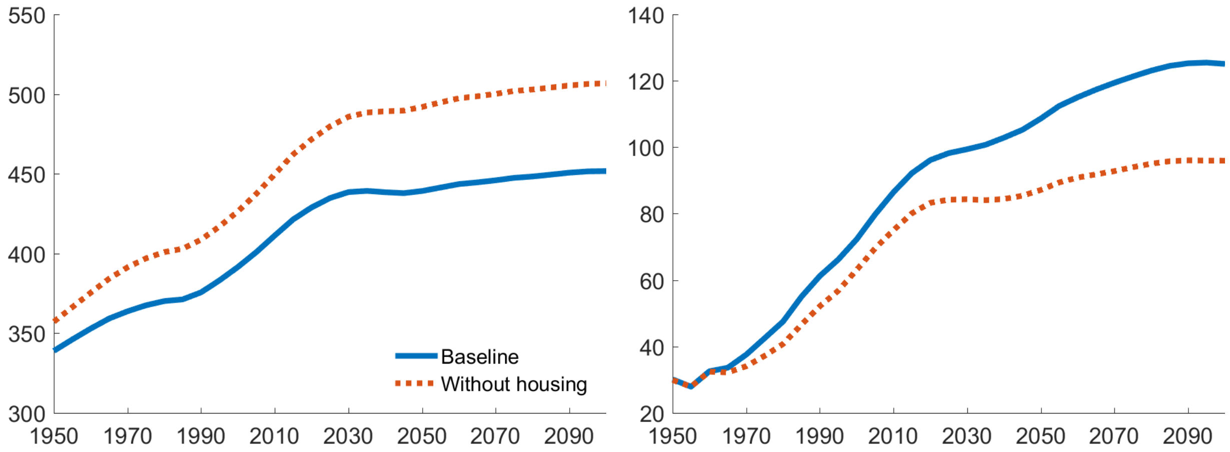


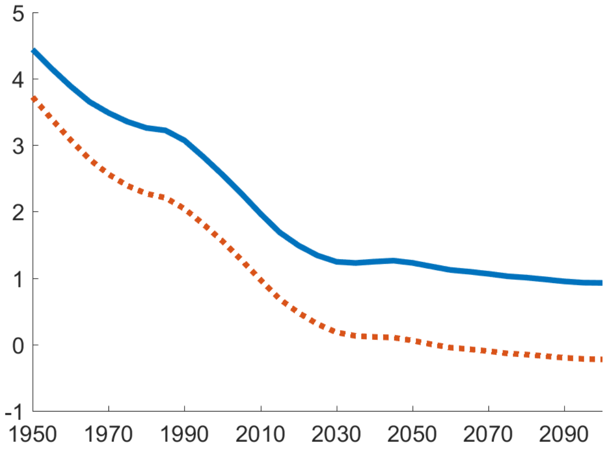
Figure 6: Simulations With and Without Housing



(a) Capital-to-GDP ratio (%)

(b) Household debt-to-GDP (%)

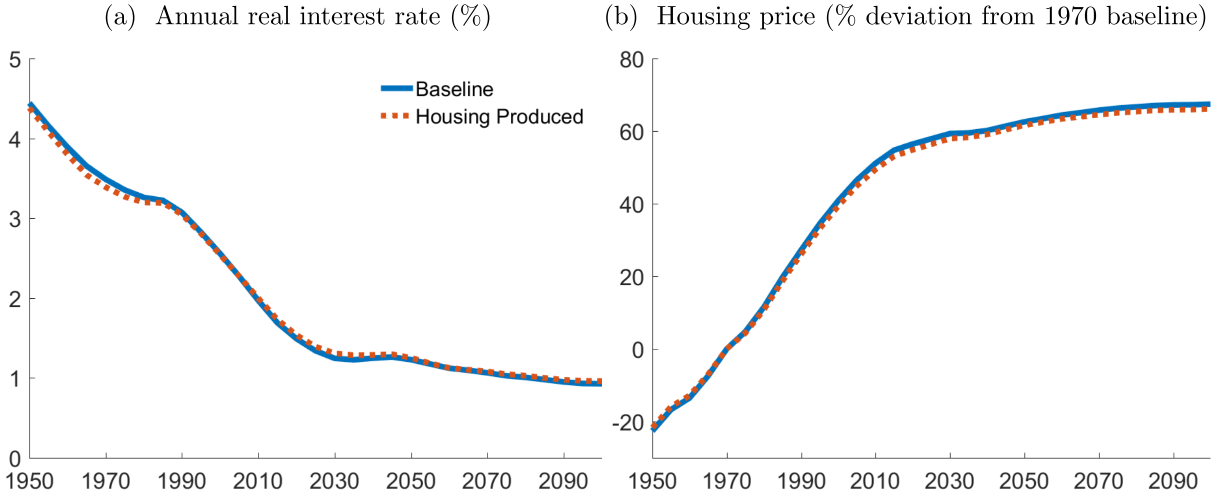
(c) Annual interest rate



additional assumptions, particularly in terms of the timing of the announcement and implementation of these changes. As a first step, however, solving the model with a higher retirement age throughout the simulation can give us an insight into the importance of the retirement age. We still need to make an additional assumption about the productivity level of older workers: as a first approximation, we assume that it is the same as the 60-64 year old cohort.

When retiring at age 70, the households save less, and the interest rate is higher. The interest rate drop between 1980 and 2050 is very similar in both cases (205bps with late retirement, against 203bps in the baseline, see Figure 8, dashed-dotted and solid lines respectively). If the retirement age were to change unexpectedly during the transition,

Figure 7: Simulations with New Housing Stock in Resource Constraint


(a) Annual real interest rate (%)

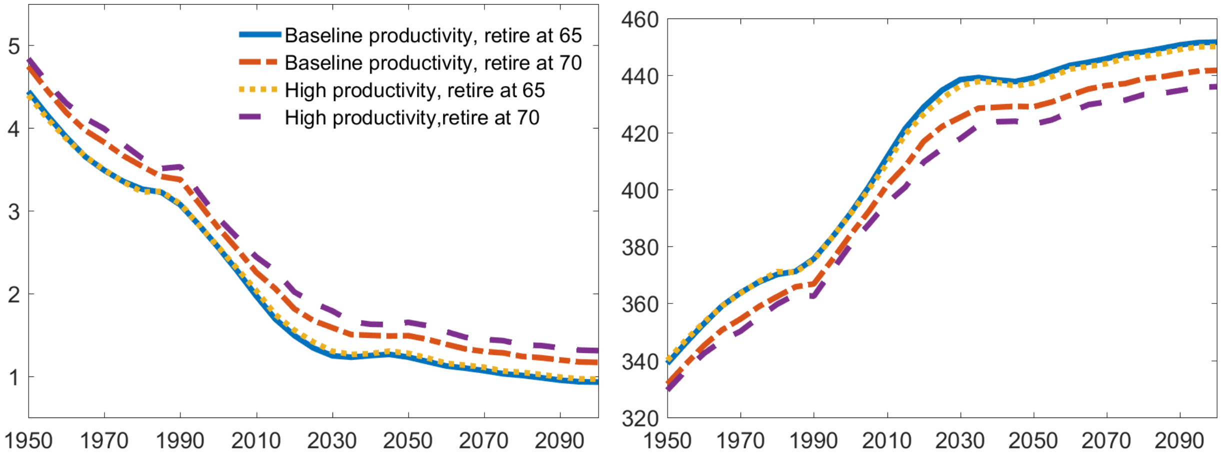
(b) Housing price (% deviation from 1970 baseline)

say in 2000, the models outcome would be identical to the baseline case until 2000. After that date, the households would start progressively adjusting their saving and housing decisions to the new retirement age, to reach a final steady state identical to the one obtained with retirement at age 70. Hence the transition would lie somewhere between these two scenarios, and at most a higher retirement age would dampen the interest rate drop by 26 bps in 2050.

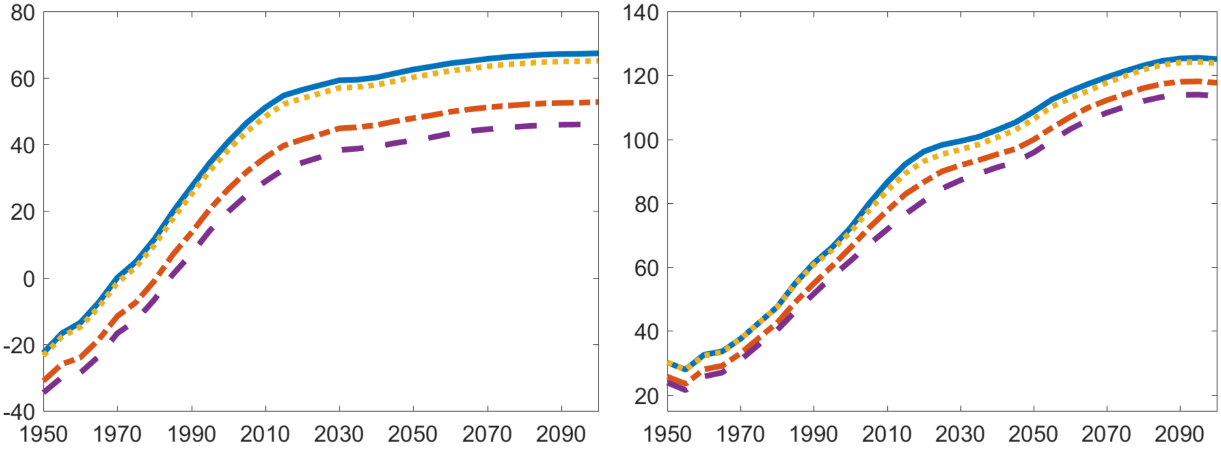
By calibrating labour productivity in our model to match labour income data, we implicitly assume that labour income changes observed in the data are entirely due to changes in labour productivity in the model. Part of the labour income decline after age 55 may however be related to a decrease in labour supply, implying that we may be underestimating the productivity of workers aged above 55. To address this, we do an additional robustness check going for the opposite extreme of assuming no productivity decline after the age 55-peak. This means that all of the decline in labor income observed in the data after age 55 is due to a decrease in labour supply, while labour productivity remains constant. We amend the calibration of the model, changing the life-cycle profile of productivity so that it stays at its highest point after age 55, and adjusting further parameters to still match the aggregate variables targets. Increasing the retirement age from 65 to 70 has a somewhat larger effect, dampening the interest rate drop between 1980 and 2050 by 37 bps (see Figure 8, dashed and dotted lines). We interpret this second robustness exercise as an upper-bound for the impact of a higher retirement age, because productivity after age 55 is set at its highest plausible level.

Figure 8: Model simulations with higher retirement age

(a) Annual real interest rate (%) (b) Capital-to-GDP ratio (%)



(c) Housing price (% deviation from 1970 baseline) (d) Household debt-to-GDP ratio (%)

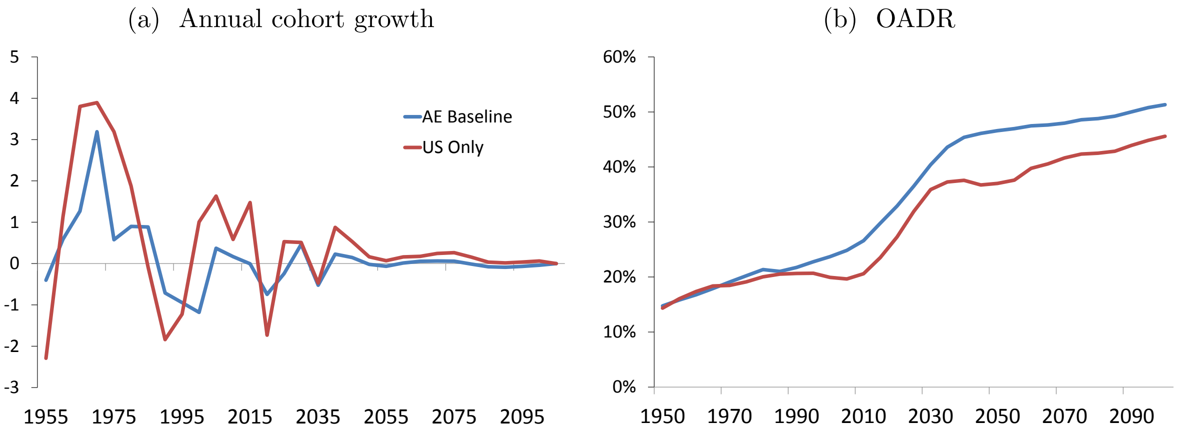


The potential effect of raising the retirement age even as high as 70 years old remains fairly modest in this model. An additional five years of labour income later in life does not offset the incentive to save in the highest productivity stage of life in order to smooth consumption. Furthermore, as life expectancy goes towards 90, five additional years of work have little effect on the overall proportion of life spent in retirement. Note that the retirement age in the UK is currently set to increase gradually from 65 to 68 by 2046, a smaller change than the one we have assumed.

C.3 Extension: The US as a closed economy

While our main results consider the aggregate evolution of advanced economies, looking at the case of the US more specifically brings useful insights. This is true not least because much of the current literature on low interest rates, and the role of demographics, has focused on the US as a closed economy. Population ageing in the US is somewhat slower than the advanced-economy average: population growth is more dynamic and life expectancy at age 60 remains below that of advanced economies. Consequently, the old age dependency ratio doubles between 1950 and 2015 in advanced economies, while it rises by only two thirds in the US (Figure 9).

Figure 9: Demographic change in the US and AEs



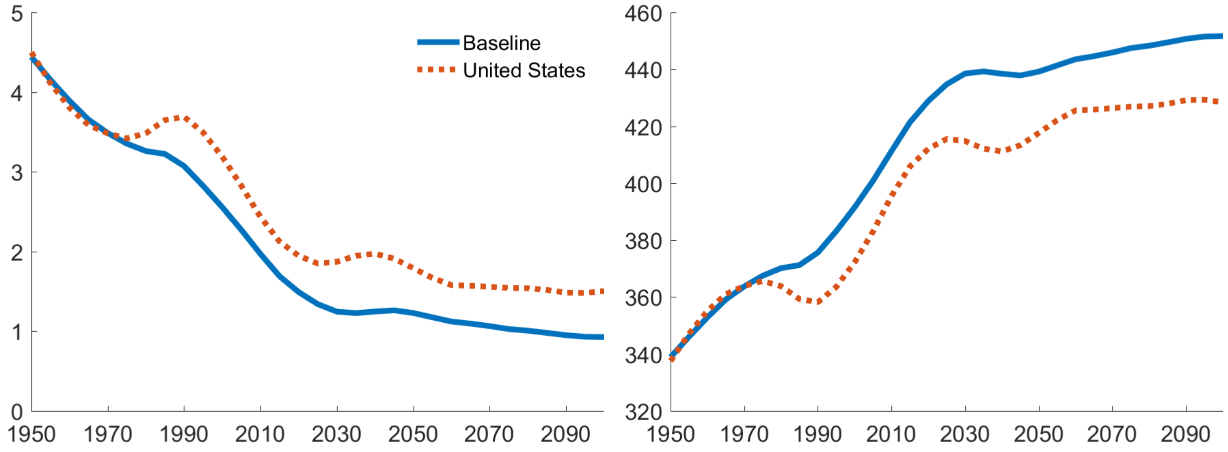
Source: UN Population Statistics

We run our model for the US as a closed economy, recalibrating the aggregate variables to match US data for the 1970s: the real interest rate, debt-to-GDP, housing wealth-

Figure 10: Simulations for the US as a Closed Economy

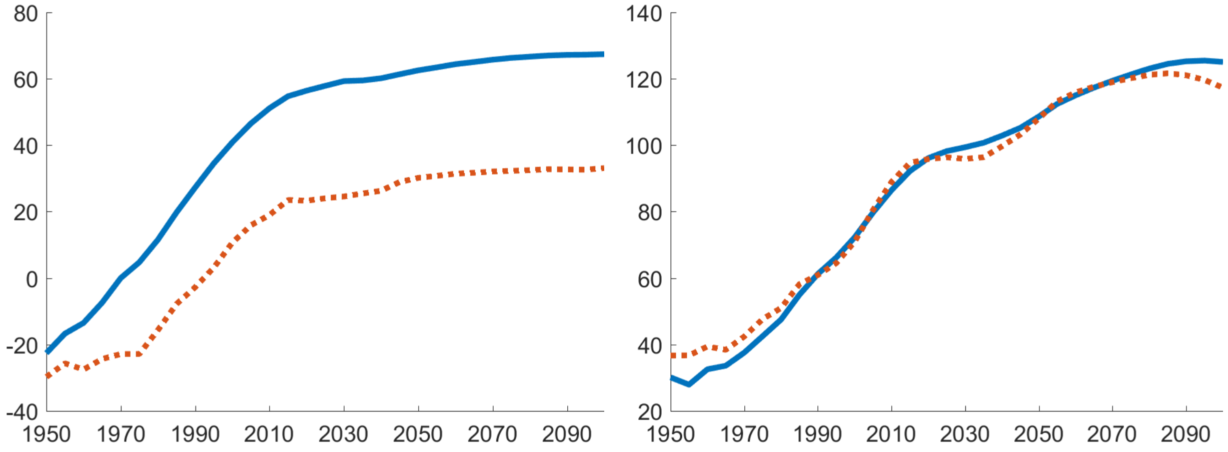
(a) Annual real interest rate (%)

(b) Capital-to-GDP ratio (%)



(c) Housing price (% deviation from 1970 baseline)

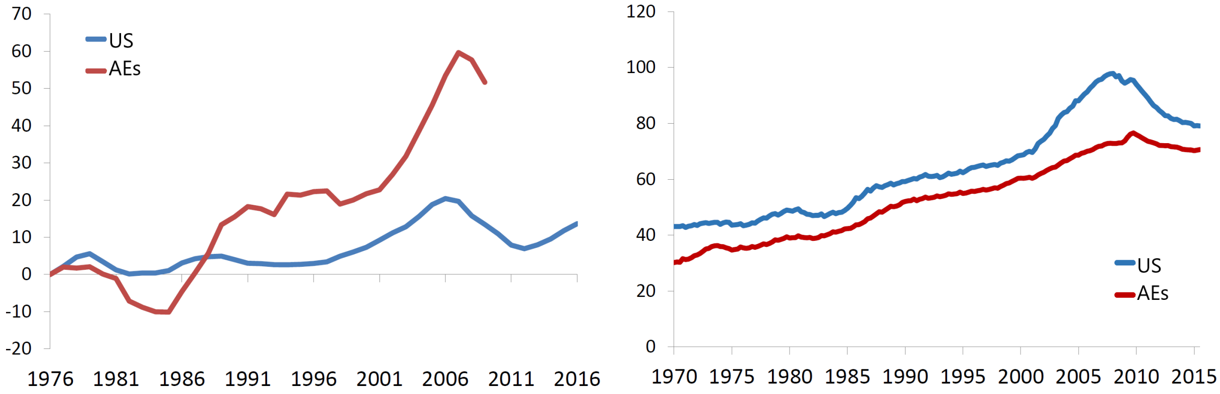
(d) Household debt-to-GDP ratio (%)



to-GDP are now set to 3.45%, 45% and 151% respectively, using the same data sources as for the baseline calibration. The life-cycle profiles for wealth and productivity were already calibrated to US data and do not change. The impact of demographic change on the interest rate is therefore smaller in the US: 136 basis points between 1980 and 2015 (see Figure 10). As the baby boom is stronger in the US, the resulting transition path of the interest rate is also less smooth. Similarly, the capital-to-GDP, household debt-to-GDP and housing price increase slower than in the advanced-economies case.

In the data, the US real interest rate starts from a higher point and decreases more between 1980 and 2015 relative to the advanced-economies interest rate, meaning that demographic changes explain a smaller part of the fall in the US interest rate. Over the

Figure 11: House prices and household debt in the US and AEs



(a) Real housing price (% deviation from 1976)

(b) Household debt-to-GDP ratio (%)

Source: BIS

same period, the increase in housing prices is however slower in the US than in advanced economies (Figure 11a), corresponding to the implications of the model. In terms of household debt-to-GDP ratio, the data for the US are more strongly influenced by the boom and bust of the 2000s, but it seems that the trend increase is equivalent in the US to advanced economies (Figure 11b).

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