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Debt enforcement and the return on money*

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Résumé

L'utilisation comme moyen de paiement d'actifs à rendement faible, comme la monnaie fiduciaire, alors qu'il existe des actifs à rendement plus élevé sans risque demeure une énigme en théorie monétaire. Dès lors qu'une réponse satisfaisante à cette question n'a pas été formulée, les conclusions tirées des modèles monétaires qui supposent, de manière arbitraire, une limite à la liquidité des actifs alternatifs pour assurer un prix positif de la monnaie à l'équilibre s'avèrent difficiles à évaluer. Cet article présente un cadre dans lequel la monnaie a un prix positif à l'équilibre malgré l'existence d'un actif à rendement plus élevé et l'absence de contraintes légales et de coûts de transaction associés à l'utilisation de cet actif. L'argument proposé est que l'utilisation de la monnaie est associée avec des frictions sous-jacentes aux contrats de dette. Dans une économie dans laquelle les débiteurs peuvent échapper à leurs obligations contractuelles - la capacité des agents à s'engager à rembourser des dettes est limitée -, le rendement effectif des actifs est déterminé par les incitations au remboursement volontaire des dettes. Il est montré que l'inflation ou, plus généralement, le taux de dépréciation d'un actif dans lequel les dettes sont libellées peut opérer comme un dispositif d'engagement. Par conséquent, la monnaie est utilisée à l'équilibre et le taux optimal d'inflation est positif.

Mots-clés: Monnaie, Inflation, Pouvoir exécutoire des dettes, Banque

Codes JEL: E41, E50, E51

Abstract

The rate-of-return-dominance puzzle asks why low-return assets, like fiat money, are used in actual economies given that risk-free higher-return assets are available. As long as this question remains unresolved, most conclusions from monetary models which arbitrarily restrict the marketability properties of alternative assets to make money valuable are difficult to assess. In this paper, I provide a framework in which fiat money has value in equilibrium, even though a higher-return asset is available and there are neither restrictions nor transaction costs in using it. I suggest that the use of money is associated with frictions underlying debt contracts. In an environment where full enforcement is not feasible, the actual rate of return on assets is determined by incentives eliciting voluntary debt repayment. I show that the inflation rate or, more generally, the depreciation rate of an asset in which debts are denominated may function as a commitment device. As a result, money is used in equilibrium and the optimal inflation rate is positive.

Keywords: Money, Inflation, Debt Enforcement, Banking

JEL Classification: E41, E50, E51

1 Introduction

At least since Hicks (1935) raised this issue as crucial in the theory of money, monetary theorists have been concerned with the rate-of-return dominance puzzle, which asks why fiat money, an intrinsically useless object, is held given that alternative assets which exhibit higher return are available. For many of them, this question remains greatly unanswered (Wallace (1990, 1998); Hellwig (1993)). In order to circumvent this difficulty, most monetary models designed to study monetary policy issues exclude the existence of higher-return assets which compete with money or limit their marketability properties by means of *ad hoc* assumptions. However, the conclusions stemming from these models are difficult to assess as long as the features that make money valuable are not completely understood.

In this paper, I provide a framework in which fiat money has value in equilibrium, even though a higher-return asset is available and there are neither restrictions nor transaction costs which prevent agents from using this asset as a medium of exchange. I suggest that the use of money is associated with frictions underlying debt contracts. I consider the issuance of credit backed by deposits, in an environment where full enforcement is not feasible. This entails that the actual rate of return on assets is in part determined by incentives eliciting voluntary debt repayment. I show that the inflation rate or, more generally, the depreciation rate of an asset in which debts are denominated may function as a commitment device.

The mechanism can be described as follows. Consider that the institutions which provide credit, that for simplicity I call banks, are only able to enforce debts by agents who carry out transactions in the market. Furthermore, they are only able to do so temporarily. In this case, the asset in which debts are denominated affects the outside option for defaulters: The punishment on defaulters may be stronger if the asset in which they take out loans loses value faster. The reason is that defaulters would choose not to participate in the market for some time in order to avoid enforcement by banks, until banks' enforcement power vanishes. Doing this would entail a lower benefit if the rate of return were lower, since the asset would be less valuable at the moment to use it to purchase goods. Thus, borrowing a low-return asset is a better commitment device than borrowing a high-return asset. In equilibrium, less binding borrowing constraints translate into higher deposit rates for loans denominated in the low-return asset which more than compensate agents for the depreciation of this asset and provide a motive for holding it.

In order to put in place the mechanism described, I develop a Lagos and Wright (2005) model where agents can make deposits and take out loans, as in Berentsen, Camera and Waller (2007). I consider two assets, money and a real asset. Each period, agents get preference shocks after banks have closed which determine whether they desire to purchase goods in the market in the current period. Agents who do not purchase goods hold the borrowed assets when the settlement stage arrives. Banks must establish conditions for

voluntary repayment by these agents: Since they do not desire to purchase goods, they could refrain from trading in the market to avoid enforcement. I refer to the debt-enforcement technology described as limited enforcement, since it enables banks to force repayment only by agents who take part in the market and, further, banks can force repayment by these agents only for some time. I show that under limited enforcement a monetary equilibrium exists even when the inflation rate is higher than the depreciation rate of the real asset. To highlight the role of the debt-enforcement technology in the existence of the monetary equilibrium, I also consider the perfect-enforcement case in which banks can fully enforce all debts. In this set-up, money has never value if the inflation rate is higher than the depreciation rate of the real asset.

The main argument to explain why money is not driven out from circulation stressed in the literature on the rate-of-return dominance puzzle is the presence of features in actual economies which favor money over the competing assets. As a notable example, the legal-restrictions theory highlights the existence of legal restrictions which limit the use of bonds as media of exchange or forbid financial intermediaries to transform large-denomination bonds into small-denomination claims that could play a monetary role (e.g., Wallace (1983)). The existence of frictions that inhibit arbitrage opportunities can also make money valuable. Aiyagari, Wallace and Wright (1996) build a search-model *à la* Trejos and Wright (1995) with money and nominal bonds. Owing to search frictions which make the redemption of bonds difficult to materialize, money is used in equilibrium. Contrary to these articles, this paper provides an example for money not being driven out from circulation even when there are no legal restrictions and agents can perfectly exploit arbitrage opportunities between the competing assets. Zhu and Wallace (2007) construct a model to show that if the bargaining protocol between a buyer and a seller is such that the allocation is selected according to the money holdings of the buyer, then money coexist with interest-bearing bonds. Lagos (2011) shows that if fiat money is heterogeneous in some physical property (e.g., serial numbers) then there is a continuum of equilibria in which money coexists with risk-free bonds. In Lagos (2010), bonds can coexist with equity in absence of legal restrictions because the return on equity is not only higher on average but also stochastic. Compared to this literature, the framework developed in this paper considers two competing assets which exhibit potentially different rates of return but are otherwise identical. Thus, it allows to show that the relatively low rate of return of an asset can itself be the source of the use of this asset in equilibrium and, thereby, provides insight on the specific link between the rate of return of an asset and how its liquidity is determined.

This paper is very much related to the literature on endogenous debt constraints developed by Kehoe and Levine (1993), Alvarez and Jermann (2000) and Hellwig and Lorenzoni (2009) among others. These papers study how much debt can be sustained in equilibrium

when full commitment is not feasible but defaulters can be punished by being excluded from all financial operations, as in Alvarez and Jermann, or from future credit, as in Hellwig and Lorenzoni. In those economies, agents can issue state-contingent securities whose rates of return are endogenously determined.¹ By contrast, I consider two distinguishable outside assets (i.e., assets which do not cancel out within the private sector) with exogenous rates of return, which allows me to explicitly address the rate of return dominance between the assets. A strand of this literature states a benefit from inflation owing to its effect on incentives to default. Aiyagari and Williamson (2000) and Berentsen, Camera and Waller show that inflation may be optimal because it makes the outside option for defaulters less attractive. Indeed, defaulters are more exposed to inflation than non-defaulters because they are excluded from the banking system. However, the punishment on defaulters in those economies crucially depends on the assumption that money is the only asset available to conduct transactions: The mechanism at stake would not occur if defaulters could resort to an alternative asset.² In the set-up described in this paper, a positive rate of inflation is optimal despite the existence of a higher-return asset available to defaulters.

In the next section the model is presented. In section 3, the symmetric equilibrium is characterized. Section 4 is devoted to the study of the perfect-enforcement case as a benchmark. Section 5 presents the limited-enforcement set-up. Finally, section 6 concludes.

2 Environment

Time is discrete and continues forever. There is a continuum of infinitely-lived agents of unit mass and two types of perfectly divisible and non-storable goods: a market-good and a home-made good. Agents can only consume the home-made good produced by themselves and the market good produced by other agents. They discount across periods with factor $\beta \in (0, 1)$. As in Lagos and Wright (2005), each period is divided into two subperiods.

In each period, two competitive markets open sequentially. Before the first market opens, agents get an idiosyncratic preference shock by which they cannot produce the market good (with probability $(1 - s)$) or they can produce the market good and get no utility from consumption (with probability s). I call consumers the agents who get the first type of shock and sellers those who get the second type.

After the first shock, consumers get a second preference shock: They learn that they only get utility from consuming the home-made good (with probability $(1 - b - s) / (1 - s)$), and so they are home-consumers, or that they only get utility from consuming the market good

¹Hellwig and Lorenzoni also consider an alternative economy with unbacked public debt.

²Diaz and Perera-Tallo (2011) consider an economy with money and private bonds, but only money can be used for transaction purposes.

(with probability $b/(1-s)$), and so they are buyers. Preferences on the type of good that an agent likes which are determined by the second shock apply to the whole period (first and second subperiods).

In the first market, buyers get utility $u(q)$ when they consume a quantity q of the market good, with $u'(q) > 0$, $u''(q) < 0$, $u'(0) = +\infty$ and $u'(\infty) = 0$. For simplicity, I assume that home-consumers get utility q when they consume a quantity q of the home-made good. For sellers and home-consumers, producing a quantity q represents a disutility equal to $c(q) = q$.

In the second market, all agents can produce both types of goods. Consuming a quantity x of the market good (home-made good) gives utility $U(x) = \ln(x)$ to buyers and sellers (home-consumers).³ Disutility from producing x is equal to h , where one hour of work yields one unit of good.

There are two storable and perfectly divisible assets in the economy: a real asset and an intrinsically useless object called fiat money. Both assets can potentially be used as media of exchange with neither restrictions nor specific transaction costs. In the second market, one unit of the real asset can be transformed into one unit of labor and one unit of labor can be transformed into one unit of the real asset. The real asset depreciates at a rate γ_a across periods, with $\gamma_a > \beta$. The quantity of money at the beginning of period t is denoted as M . The money supply grows at the gross rate $\gamma_m = M_{+1}/M$ where the subscript $+1$ indicates the following period and $\gamma_m \geq \gamma_a$. Agents receive monetary lump-sum transfers from the central bank equal to $T = (\gamma_m - 1)M_{-1}$ after the second market in period t .

Agents can deposit and borrow the assets (money as well as the real asset) by resorting to banks. Banking activities take place after the first preference shock and before the second one. Banks are competitive and face an exogenous level of reserve requirements r ; i.e., they must keep a ratio r of deposits to loans in a particular asset. As in Berentsen, Camera and Waller (2007), loans are issued as bilateral contracts between an agent and a bank by which the bank gives an amount of money to the agent and the agent must pay it back during the second market together with the interest on it. Deposits are taken by banks and paid back during the second subperiod with the corresponding interest. The timing of events is depicted in Figure 1.

The key assumption in this environment is that debt enforcement is limited. Banks possess an enforcement technology by which they can force repayment by those agents who enter

³The assumption on a logarithmic utility function in the second subperiod is sufficient although not necessary for the results that follow. What is necessary is that $U(x^*) - x^* - U(0)$, where x^* is determined by $U'(x^*) = 1$, is sufficiently high. The assumed preferences by which some agents (those who have spent the loan to purchase goods) will be subject to enforcement while some agents (those who have not used the loan and carry no good) will not be subject to enforcement could be rationalized as the existence of collateral only available to the former group of agents. To keep the model as simple as possible, no collateral is modeled here.

the second market, in which debt settlement takes place. However, they cannot force repayment by agents who do not trade in the second market. In addition, in a particular period t banks' enforcement power only allows them to ensure the repayment of loans contracted upon at the beginning of t (i.e., banks cannot force agents to repay loans issued before t).⁴ Banks keep track of financial histories in order to punish defaulters by excluding them from the banking system for the rest of their lifetime; i.e., after defaulting, agents are prevented from borrowing and depositing. Defaulters are also excluded from the monetary transfers. The enforcement technology and the record keeping on borrowers entail a banking cost κ per unit of money loaned. As a benchmark, I will consider the perfect-enforcement set-up; i.e., the case in which banks are able to fully force all agents to repay their debts.

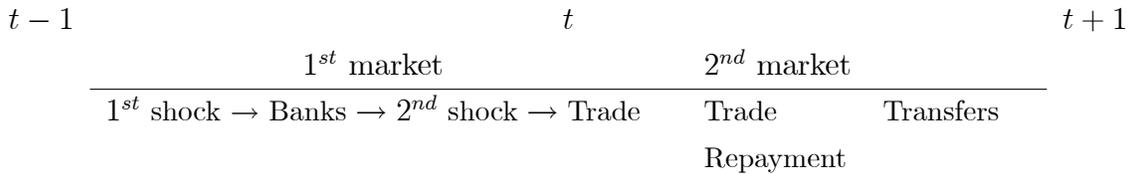


Figure 1: Timing of events

In order to motivate a role for a medium of exchange, traders are assumed to be anonymous so that sellers require compensation at the same time as they produce. This assumption rules out bilateral credit; however, it does not conflict with the existence of lending in this model because this only requires that agents are identified by banks.

3 Symmetric equilibrium

I will focus on symmetric and stationary equilibria. Imposing stationarity implies that end-of-period real money holdings are constant:

$$\phi/\phi_{+1} = M_{+1}/M = \gamma_m \tag{1}$$

where ϕ is the price of money in real terms. Let $V(a, z)$ denote the value function of an agent who holds an amount a of the real asset and an amount z of real money holdings at the beginning of a period. $W(a, z, \ell_a, \ell_z)$ is the expected value from entering the second market with a units of the real asset, z units of real money balances, an amount ℓ_a of real-asset loans and a real amount ℓ_z of money loans ($\ell_a, \ell_z < 0$ denote deposits). In what follows, I use the index $j = a, z$ to refer to a generic asset, real money balances or the real asset.

⁴This assumption is made for simplicity. The model could be extended to endow banks with a longer-lasting enforcement technology.

3.1 The second market

The program for an agent in the second market is to solve:

$$W(a, z, \ell_a, \ell_z) = \max_{x, h, a_{+1}, z_{+1}} U(x) - h + \beta V_{+1}(a_{+1}, z_{+1}) \quad (2)$$

$$\text{s.t. } h = x + \gamma_a a_{+1} + z_{+1} \phi / \phi_{+1} - a - z + (1 + \mathbf{1}_a) \ell_a + (1 + \mathbf{1}_z) \ell_z - T$$

Denote the interest rate on loans and deposits as i_j^ℓ and i_j^d respectively. In the budget constraint, $\mathbf{1}_j = i_j^d$ if $\ell_j < 0$ and $\mathbf{1}_j = i_j^\ell$ if $\ell_j > 0$.

Insert the budget constraint into (2) to replace h and use (1), to get the first-order conditions on x , a_{+1} and z_{+1} :

$$U'(x) = 1$$

$$\partial V_{+1} / \partial a_{+1} = \gamma_a / \beta, \quad \partial V_{+1} / \partial z_{+1} = \gamma_m / \beta \quad (3)$$

The envelope conditions on a , z , ℓ_a and ℓ_z are:

$$W_j = 1 \quad (4)$$

$$W_{\ell_j} = -(1 + \mathbf{1}_j)$$

3.2 The first market

3.2.1 Sellers

In the first market, sellers decide how much to produce in exchange for money and the real asset, q_a^s and q_z^s , and how much to deposit, ℓ_a^s and ℓ_z^s . Their program is:

$$\max_{q_a^s, q_z^s, \ell_a^s, \ell_z^s} -q^s + W(a_{-1} + \ell_a^s + p_a q_a^s, z_{-1} + \ell_z^s + p_z q_z^s, \ell_a^s, \ell_z^s)$$

$$\text{s.t. } -\ell_j^s \leq j_{-1}$$

where p_a (p_z) is the price of first-market goods in terms of the real asset (real money balances), a_{-1} (z_{-1}) is the amount of the real asset (real money balances) brought from the previous period and $q^s = q_a^s + q_z^s$. The constraints mean that the seller's deposits are limited by his holdings of the real asset and real money balances.

The first-order conditions on q_a^s and q_z^s yield:

$$p_j = 1 \quad (5)$$

The first-order condition on ℓ_a^s and ℓ_z^s is:

$$W_j + W_{\ell_j} - \lambda_j^s = 0$$

where λ_j^s is the Lagrange multiplier associated with the deposit constraint for asset j . Using (4), it implies

$$i_j^d = \lambda_j^s \quad (6)$$

3.2.2 Consumers

Consumers must choose the consumption quantities q_z and q_a to be purchased with money and the real asset and the amount of loans in each asset, ℓ_z and ℓ_a , before the second shock; i.e., before learning if they have a preference for the market good or for the home-made good.⁵ Their program is:

$$\begin{aligned} \max_{q_a, q_z, \ell_a, \ell_z} \quad & \frac{b}{1-s} [u(q) + W(a_{-1} + \ell_a - p_a q_a, z_{-1} + \ell_z - p_z q_z, \ell_a, \ell_z)] \\ & + \frac{1-b-s}{1-s} W(a_{-1} + \ell_a, z_{-1} + \ell_z, \ell_a, \ell_z) \\ \text{s.t.} \quad & p_j q_j \leq j_{-1} + \ell_j, \quad \ell_j \leq \bar{\ell}_j, \quad q = q_a + q_z \end{aligned}$$

where $\bar{\ell}_j$ is the borrowing limit for loans in asset j .

The first-order condition on q_z and q_a is:

$$u'(q) - W_j p_j - \lambda_j^b p_j = 0$$

where λ_j^b is the multiplier associated to the cash constraint for asset j . Using (4) and (5), this condition becomes:

$$u'(q) = 1 + \lambda_j^b \quad (7)$$

The first-order condition on ℓ_z and ℓ_a can be written as:

$$u'(q) = 1 + (1-s)(\lambda_j^\ell + i_j^\ell) / b \quad (8)$$

where λ_j^ℓ is the multiplier associated to the borrowing constraint for loans in asset j .

3.2.3 Banks

Banks must hold a proportion r in the form of deposits for each unit of money loaned. They solve the following problem per borrower:

$$\begin{aligned} \max_{\ell_j} \quad & \sum \ell_j (i_j^\ell - r i_j^d - \kappa) \\ \text{s.t.} \quad & \ell_j \leq \bar{\ell}_j, \quad b [u(q) - (1 + i_j^\ell) \ell_j] - (1 - b - s) i_j^\ell \ell_j \geq \Gamma_j \end{aligned}$$

where the borrowing limit ℓ_j will be endogenized later. The first constraint is the borrowing constraint. The second constraint is the participation constraint of the borrower: Each bank has to offer a pay-off to the borrower that is at least the same as the pay-off he may get while resorting to another bank, Γ_j .

⁵Given assumptions on preferences, any consumption quantity entails zero net utility by home consumers in the first subperiod.

Using (5), the first-order condition on ℓ_j is:

$$i_j^\ell - r i_j^d - \kappa - \lambda_j^c + \lambda_j^\Gamma [bu'(q) - b - (1-s)i_j^\ell] = 0$$

where λ_j^c and λ_j^Γ are the multipliers associated to the borrowing constraint and the participation constraint, respectively.

3.3 Marginal value of money

The expected utility for an agent who starts a period with portfolio (a, z) is:

$$\begin{aligned} V(a, z) = & b[u(q) + W(a + \ell_a - q_a, z + \ell_z - q_z, \ell_a, \ell_z)] \\ & + s[-q^s + W(a + \ell_a^s + q_a^s, z + \ell_z^s + q_z^s, \ell_a^s, \ell_z^s)] \\ & + (1 - b - s)W(a + \ell_a, z + \ell_z, \ell_a, \ell_z) \end{aligned}$$

Using (4), (6) and (7), the marginal value of asset j is:

$$\partial V / \partial j = bu'(q) + s i_j^d + 1 - b$$

Using (3), this condition becomes:

$$\begin{aligned} \gamma_a / \beta - 1 &= b[u'(q) - 1] + s i_a^d \\ \gamma_m / \beta - 1 &= b[u'(q) - 1] + s i_z^d \end{aligned} \tag{9}$$

The right-hand side of this equation represents the marginal cost of acquiring an additional unit of asset j while the left-hand side represents its marginal benefit given by the increase in consumption with probability b and in interests earned with probability s .

3.4 Market clearing

In a symmetric equilibrium, the market-clearing conditions for the first market and the credit market are:

$$\begin{aligned} s q_j^s &= b q_j \\ -s \ell_j^s &= r(1-s)\ell_j \end{aligned} \tag{10}$$

Total output in the second market is $H = bh^b + sh^s + (1-b-s)h^c$ where h^b , h^s and h^c are the amounts of hours worked in the second subperiod of each period by the buyer, the seller and the home-consumer and satisfy

$$\begin{aligned} h_s &= x + \ell_a^s(1 + i_{d,a}) + \ell_z^s(1 + i_{d,z}) - q^s + \gamma_a a_{+1} + \gamma_m z_{+1} - T \\ h_b &= x + \ell_a(1 + i_{\ell,a}) + \ell_z(1 + i_{\ell,z}) + \gamma_a a_{+1} + \gamma_m z_{+1} - T \\ h_c &= h_b - a_{-1} - z_{-1} - \ell_a - \ell_z \end{aligned} \tag{11}$$

Using (11) and (10)

$$H = x + (1 - s)(\ell_a + \ell_z)\kappa + (\gamma_a - 1)a_{-1} \quad (12)$$

The output produced in the second market is used for agents' consumption, to afford banks' operating cost and to adjust real-asset holdings. I use (12) to state the expected lifetime utility \mathcal{W} of the representative agent as

$$\mathcal{W}(1 - \beta) = bu(q) - sq^s + U(x) - x - (1 - s)(\ell_a + \ell_z)\kappa - (\gamma_a - 1)a_{-1} \quad (13)$$

4 Perfect enforcement

Next, I assess the existence of the monetary equilibrium according to the prevailing debt enforcement technology. First, I consider the perfect-enforcement case as a benchmark in order to analyze the unconstrained-credit equilibrium; i.e. an equilibrium in which agents may borrow as much as they desire since banks have the power to fully enforce all debts. In this equilibrium, $\lambda_j^\ell = 0$ ($\bar{\ell}_a = \bar{\ell}_z = \infty$). Hence, from (8):

$$u'(q) = 1 + (1 - s)i_j^\ell/b \quad (14)$$

Banks are competitive so banks' profits are zero; this implies:

$$ri_j^d = i_j^\ell - \kappa \quad (15)$$

if $\ell_j > 0$.

Definition 1 *An equilibrium with perfect enforcement is a vector of consumption quantities $\{q, q_a, q_z\}$ and interest rates $\{i_a^\ell, i_z^\ell, i_a^d, i_z^d\}$ that satisfy $q_a + q_z = q$, (9), (14) and (15) for $j = a, z$. An equilibrium is monetary if $q_z > 0$ and non-monetary otherwise.*

Proposition 1 *Assume enforcement is perfect. A non-monetary equilibrium with credit exists iff $\kappa/(1 - r) \geq \gamma_a/\beta - 1 \geq (1 - s)\kappa$. A monetary equilibrium with credit exists iff $\kappa/(1 - r) \geq \gamma_m/\beta - 1 \geq (1 - s)\kappa$ and $\gamma_m \leq \gamma_a$. No monetary equilibrium exists for $\gamma_m > \gamma_a$.*

Proposition 1 replicates the standard result in monetary theory: If agents can choose between fiat money and a higher-return asset to conduct transactions and there are neither restrictions nor transaction costs which limit arbitrage between them, then fiat money is driven out in equilibrium.

5 Limited enforcement

When enforcement is limited, banks are able to force agents who voluntarily enter the second market and trade to repay their debts. However, they cannot enforce debts by agents who do not trade in the second market. Given assumptions on preferences, buyers always choose to enter the market because they desire the market good. Therefore, banks are only concerned with home-consumers who do not desire the market good and could refrain from trading in order to avoid being forced to repay. Banks set a borrowing constraint to prevent default by these agents. They choose $\bar{\ell}_j$ such that the expected lifetime utility for a home-consumer who does not default equals the expected lifetime utility for a home-consumer who defaults:

$$U(x) - h_c + \beta V_{+1}(a, z) = U(x) - \bar{h}_c + \beta \hat{V}_{+1}(a, z)$$

where \bar{h}_c is the amount of hours worked by the defaulter in the period in which he defaults and \hat{V}_{+1} corresponds to the expected lifetime value for a defaulter. The borrowing constraint can be written as

$$\begin{aligned} -h_c + \frac{\beta}{1-\beta} [bu(q) - sq^s - bh_b - sh_s - ch_c] \\ = -\bar{h}_c + \frac{\beta}{1-\beta} [bu(\hat{q}) - sq^s - b\hat{h}_b - s\hat{h}_s - c\hat{h}_c] \end{aligned} \quad (16)$$

where \hat{q} is the quantity consumed by a defaulter when he is a buyer and \hat{h}_b , \hat{h}_s and \hat{h}_c are the amounts of hours worked by the defaulter each time he turns out to be buyer, seller or home-consumer, respectively. It is straightforward to show that defaulters only use the real asset because they cannot use the banking system and inflation is assumed to be equal or higher than the rate at which the real asset depreciates.⁶ The marginal value of the real asset for a defaulter determines \hat{q} :

$$\gamma_a/\beta - 1 = b[u'(\hat{q}) - 1] \quad (17)$$

In a symmetric equilibrium, real money holdings by a non-defaulter are $z_{-1} = \phi M_{t-1}$ (in a non-monetary equilibrium $z_{-1} = \phi = 0$). Since $\gamma_m > \beta$, the deposit constraint and cash constraint bind, so that real money holdings by a non-defaulter satisfy:

$$\begin{aligned} z_{-1} &= -\ell_z^s \\ p_z q_z &= z_{-1} + \ell_z \end{aligned} \quad (18)$$

Using (5), (10) and (18) the real amount of an individual money-loan is:

$$\ell_z = \frac{sq_z}{s + r(1 - s)} \quad (19)$$

⁶When $\gamma_m = \gamma_a$, defaulters are actually indifferent between both assets.

Similarly, real-asset loans and real-asset holdings satisfy

$$q_a = a_{-1} + \ell_a \quad (20)$$

$$\ell_a = \frac{s q_a}{r(1-s) + s} \quad (21)$$

Lemma 1 *Assume enforcement is imperfect. The borrowing constraint set by banks satisfies*

$$\begin{aligned} & -\bar{\ell}_z i_{\ell,z} - \bar{\ell}_a i_{\ell,a} + \frac{\beta}{1-\beta} [bu(q) - bq - (1-s)(\bar{\ell}_z + \bar{\ell}_a)\kappa - (\gamma_a - 1)a_{-1}] \\ & \geq -\gamma_a \left(\hat{q} - \frac{z_{-1} + \bar{\ell}_z}{\gamma_m} - \frac{a_{-1} + \bar{\ell}_a}{\gamma_a} \right) + \frac{\beta}{1-\beta} [bu(\hat{q}) - (\gamma_a - 1 + b)\hat{q}] \end{aligned} \quad (22)$$

The left-hand side of the borrowing constraint in Lemma 1 represents the pay-off to an agent who does not default. In period t , this agent has to work to pay the interest on his loan. From $t+1$ on, his expected utility is determined by the net utility of consuming q each time he turns out to be a buyer ($bu(q) - bq$) minus the expected cost of having access to the banking system $((1-s)(\bar{\ell}_z + \bar{\ell}_a)\kappa)$ and the cost due to the depreciation of the real asset if $a_{-1} > 0$. The right-hand side represents the pay-off to a defaulter. The gain of defaulting is given by a lower disutility of working in the period of default, since the defaulter does not need to work to pay the interest on the loan taken at the beginning of the period. In addition, the defaulter can use the unspent loan to purchase goods in $t+1$, which allows him to save working effort in t ($\bar{\ell}_z/\gamma_m + \bar{\ell}_a/\gamma_a$). The cost of defaulting consists of being excluded from the banking system from $t+1$ on. The inability to access banks may imply a lower utility from consumption since \hat{q} may be lower than q ($bu(\hat{q}) - (\gamma_a - 1 + b)\hat{q}$).

Definition 2 *A constrained-credit equilibrium with limited enforcement is a vector of consumption quantities $\{q, q_a, q_z, \hat{q}\}$, interest rates $\{i_z^\ell, i_z^d, i_a^\ell, i_a^d\}$, loans $\{\ell_z, \ell_a\}$, asset holdings $\{a_{-1}, z_{-1}\}$ and borrowing limits $\{\bar{\ell}_z, \bar{\ell}_a\}$ that satisfy $\bar{\ell}_z = \ell_z$, $\bar{\ell}_a = \ell_a$, $q = q_z + q_a$, (9), (15), (17), (18), (19), (20), (21) and (22). An equilibrium is monetary if $q_z > 0$ and non-monetary otherwise.*

Lemma 2 *Denote $\bar{\gamma}$ the value of $\gamma_a = \gamma_m$ such that $i_a^d = i_z^d = 0$ when enforcement is imperfect. $\bar{\gamma}$ satisfies $\bar{\gamma} = 1 + (1 - \beta s) s \kappa / [s + r(1 - s)]$.*

According to Lemma 2, $\bar{\gamma} > 1$ if $\kappa > 0$; i.e., the inflation rate must be positive to support an equilibrium with credit. To see why, consider the case in which $\kappa = 0$. The gain for an agent who defaults consists of the working hours saved in the current period, $(1 + i_{\ell,z})\bar{\ell}_z = \bar{\ell}_z$ when $\kappa = 0$ and $i_a^d = i_z^d = 0$. The cost of defaulting is given by the inability of using the banking system, which entails an increase in working time per period equal to $(\bar{\gamma} - \beta)\bar{\ell}_z + T$ at $\bar{\gamma} = \gamma_m = \gamma_a$. The discounted lifetime sum of this cost is

$\sum_{t=0}^{\infty} \beta^t [(\bar{\gamma} - \beta) \bar{\ell}_z + T] = [(\bar{\gamma} - \beta) \bar{\ell}_z + T] / (1 - \beta)$. Hence, $\bar{\gamma} = 1$ is the value of $\bar{\gamma}$ for which the agent is indifferent between defaulting or not when $\kappa = 0$. As a consequence, when $\kappa > 0$, $\bar{\gamma}$ must be higher than one; otherwise the gain for a defaulter would be higher than in the case with $\kappa = 0$ but the cost would be the same, which cannot occur in equilibrium.

Denote r^* the value of r that solves $r^* = s\beta/\bar{\gamma}(r^*)$. Notice that $r^* \in (0, 1)$.

Proposition 2 *Consider enforcement is limited. If $r \leq r^*$, there is $\gamma_m^* > \gamma_a = \bar{\gamma}$ such that a monetary equilibrium exists. Welfare at $\gamma_m = \gamma_m^*$ is higher than welfare at $\gamma_m = \bar{\gamma}$.*

Proposition 2 states that, if enforcement is limited and r is sufficiently low, a monetary equilibrium exists, even though agents could freely dispose of money and switch to a higher-return asset at no cost. The explanation for this result resides on the link between the rate of return of an asset and the borrowing constraint on loans denominated in that asset. Since defaulters are obliged to skip one period to spend the defaulted loan, the rate of return of the asset in which the loan is denominated affects incentives to default. The lower the rate of return of an asset across periods is, the less valuable a defaulted loan denominated in that asset is when it can be used to purchase goods, and the smaller the incentives to default are. Consequently, the borrowing constraint on loans denominated in the low-return asset is less binding than the borrowing constraint on loans denominated in the high-return asset.

The key feature of a constrained-credit equilibrium is that borrowing interest rates are set below their market-clearing level to prevent default.⁷ Therefore, a decrease in the rate of return of money which reduces incentives to default allows interest rates to be closer to market-clearing levels. Higher borrowing interest rates reflect in higher deposit rates to satisfy the zero-profit condition by banks. As a result, agents are compensated for the higher marginal cost of holding money (instead of holding the real asset) across periods: If they were to deposit their asset holdings, money deposits would be more profitable than real-asset deposits. The higher demand for money entails a higher price of money, which allows buyers to attain higher consumption. In the monetary equilibrium at $\gamma_m^* > \bar{\gamma}$, q and hence expected lifetime utility are higher compared to a monetary equilibrium with $\gamma_m = \bar{\gamma}$ or a real-asset equilibrium with $\gamma_a = \bar{\gamma}$.

The condition $r \leq s\beta/\bar{\gamma}(r^*)$ in Proposition 2 is necessary for a monetary equilibrium to exist when $\gamma_m > \bar{\gamma}$. At the denominator of the right-hand side of this condition, $\bar{\gamma}$ reflects the negative effect of an increase in γ_m on a defaulter caused by the depreciation of his money holdings, whereas r in the left-hand side determines the extent to which the increase in deposit interest rates is translated into an increase in borrowing interest rates.

⁷See articles by Alvarez and Jermann; Hellwig and Lorenzoni; Kehoe and Levine and Berentsen, Camera and Waller already cited.

Thus, this condition states that, when γ_m increases, the negative effect on the defaulter must be sufficiently high compared to the increased punishment on the non-defaulter owing to a higher i_z^ℓ . This ensures that a higher γ_m effectively reduces incentives to default and entails a higher price of money.⁸

6 Conclusion

I have presented a model in which money is used in equilibrium even though a real asset which displays a higher rate of return across periods is available and neither restrictions nor transaction costs are associated to its use as a substitute for money. Thus, in this framework the rate-of-return dominance has arisen as an equilibrium outcome. In doing this, this paper has suggested a novel connection between endogenously determined borrowing constraints in limited-commitment environments and the persistence of low-return assets that we observe in actual economies. Indeed, the rationale proposed for the use of low-return assets like money as a commitment device for borrowers can help explain why bank loans are mainly claims to outside money, or why inflationary currencies are not driven out from the banking system even in countries in which no restrictions to operate in foreign currencies exist. The results presented suggest that the link between debt enforcement and the assets which are effectively used is worth being further explored. In particular, this line of research can be fruitful to understand the consequences of removing currency restrictions or legal restrictions on the use of assets which could play the same role as fiat money. The study of debt enforcement as a variable subject to policy decisions should be considered and is left for future research.

⁸Other frameworks like the one in Berentsen, Camera and Waller do not require this condition for inflation to be welfare improving because agents cannot choose between different assets; i.e., there is no real asset which functions as an outside option. As a result, a higher inflation rate punishes the defaulter relatively more than it does in this model, and an increase in inflation can support a stronger increase in borrowing interest rates.

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Appendix

Proof of Proposition 1. For a non-monetary equilibrium with credit to exist, it must be that $i_a^d \geq 0$ and $i_a^\ell \geq i_a^d$. Using (9), (14) and (15), these conditions imply $\kappa/(1-r) \geq \gamma_a/\beta - 1 \geq (1-s)\kappa$. Analogously, for a monetary equilibrium with credit to exist, it must be that $\kappa/(1-r) \geq \gamma_m/\beta - 1 \geq (1-s)\kappa$. If $\gamma_a < \gamma_m < \beta[1 + (1-s)\kappa]$, first-order conditions on money and the real asset are $\gamma_m/\beta - 1 \geq bu'(q) - b$ and $\gamma_a/\beta - 1 \geq bu'(q) - b$, hence agents maximize their utility by using only the real asset. If $\gamma_a < \beta[1 + (1-s)\kappa] < \gamma_m$, agents maximize their utility by using only the real asset since $\frac{(\gamma_m/\beta - 1)r + s\kappa}{r(1-s) + s}(1-s) > \gamma_a/\beta - 1 = bu'(q) - b$. If $\beta[1 + (1-s)\kappa] \leq \gamma_a < \gamma_m$, first-order conditions on money and the real asset can be written as $\gamma_m/\beta - 1 \geq b\{1 + s/[r(1-s)]\}[u'(q) - 1] - s\kappa/r$ and $\gamma_a/\beta - 1 \geq b\{1 + s/[r(1-s)]\}[u'(q) - 1] - s\kappa/r$ so money is driven out in equilibrium.

Proof of Lemma 1. Immediate from inserting the value of $h_b, h_s, h_c, \bar{h}_c, \hat{h}_b, \hat{h}_s$ and \hat{h}_c into (16) and setting $\bar{\ell}_z = \ell_z$ and $\bar{\ell}_a = \ell_a$, where $\bar{h}_c = x + \gamma_a(\hat{q} - z/\gamma_m - \ell_z/\gamma_m)$ and $\hat{h}_c = x + (\gamma_a - 1)\hat{q}$, $\hat{h}_b = \hat{h}_c + \hat{q}$ and $\hat{h}_s = \hat{h}_c - q^s$.

Proof of Lemma 2. If $i_a^d = i_z^d = 0$, from (9) and (17) $q = \hat{q}$ and from (15) $i_a^\ell = i_z^\ell = \kappa$ if $\ell_a, \ell_z > 0$. Then $1 + (1 - \beta s)s\kappa/[s + r(1 - s)]$ solves (22) with equality.

Proof of Proposition 2. Consider $z_{-1} + \ell_z = q$ and $a_{-1} = 0$ to conjecture a monetary equilibrium. Set (22) to equality and differentiate it with respect to γ_m using (9), (15), (17) and (19). Evaluating at $\bar{\gamma}$ yields

$$\partial q / \partial \gamma_m \Big|_{\gamma_a = \gamma_m = \bar{\gamma}} = \frac{r/\beta - [s + r(1-s)]/\bar{\gamma}}{rbu''(q)} \quad (23)$$

which is positive if $r \leq r^*$.

Aside from the potential deviation by an agent who holds a representative portfolio and defaults in t , the following deviation must be ruled out. An agent could have incentive in $t - 1$ to plan to default in t if he turns out to be home consumer in t and, consequently, to bring to t a portfolio which takes into account the probability of defaulting on his debt in t . Next, I show that agents do not have incentive in $t - 1$ to plan to default in t .

Consider an agent in $t - 1$ who plans to default in t if he turns out to be home-consumer. Denote as $\{\check{a}_{-1}, \check{z}_{-1}\}$ the portfolio taken from $t - 1$ to t by this agent. This portfolio is determined by:

$$\begin{aligned} \gamma_m/\beta &= bu'(\check{q}) + s(1 + i_z^d) + (1 - b - s)\gamma_a/\gamma_m & (" \geq " \text{ if } \check{z}_{-1} = 0) \\ \gamma_a/\beta &= bu'(\check{q}) + 1 - b & (" \geq " \text{ if } \check{a}_{-1} = 0) \end{aligned} \quad (24)$$

where \check{q} is the consumption quantity by this agent if he is a buyer in t . In (24), with probability $(1 - b - s)$ the agent defaults, in which case the value of an extra unit of the

real asset or money is given by its value in $t + 1$. At $\gamma_m = \gamma_a = \bar{\gamma}$, this agent is indifferent between taking a portfolio of money only, real asset only or both. If $\gamma_m > \bar{\gamma} = \gamma_a$, the agent could be better off by taking only money or only the real asset. Denote \check{q}_a (\check{q}_z) the consumption quantity in t if the agent takes only real asset (money) to t . If $\check{q}_a > \check{q}_z$ (i.e., $\gamma_m/\beta - s(1 + i_z^d) - (1 - b - s)\gamma_a/\gamma_m > \gamma_a/\beta - 1 + b$), the agent takes the real asset and if $\check{q}_a < \check{q}_z$ the agent takes money (if $\check{q}_a = \check{q}_z$ the agent is indifferent).

From (24), $\partial\check{q}_a/\partial\gamma_m = 0$ and

$$\partial\check{q}_z/\partial\gamma_m = \left[1/\beta - s \frac{\partial i_z^d}{\partial\gamma_m} + \frac{(1 - b - s)\gamma_a}{(\gamma_m)^2} \right] \frac{1}{bu''(\check{q}_z)}$$

Using (9) and (23), $\partial\check{q}_z/\partial\gamma_m$ evaluated at $\gamma_m = \bar{\gamma}$ is

$$\partial\check{q}_z/\partial\gamma_m = \left[1/\beta - \frac{b + s/r}{\bar{\gamma}} \right] \frac{1}{bu''(\check{q}_z)} \quad (25)$$

which is positive if $r \leq r^*$. Therefore, the agent who plans in $t - 1$ to default in t takes $\check{z}_{-1} > 0$ and $\check{a}_{-1} = 0$. Hence, $\partial\check{q}_z/\partial\gamma_m = \partial\check{q}/\partial\gamma_m$. For $\gamma_m > \gamma_a = \bar{\gamma}$, $\check{z}_{-1} < z_{-1}$ and $\check{q} < q$.

Consider that this agent has been buyer in the first period in $t - 1$ (the same will hold for a seller or a home-consumer). In the second subperiod in $t - 1$, the expected lifetime utility for this agent is

$$\begin{aligned} & U(x) - x - \ell_z i_z^\ell - \gamma_m \check{z} + T + \beta \left[bu(\check{q}) - bq + U(x) - \check{H} \right] \\ & + \frac{\beta^2(b + s)}{1 - \beta} [bu(q) - bq + U(x) - x - (1 - s)\ell_z \kappa] \\ & + \frac{\beta^2(1 - b - s)}{1 - \beta} [bu(\hat{q}) + U(x) - x - (\gamma_a - 1 + b)\hat{q}] \end{aligned} \quad (26)$$

where \check{H} is the expected number of hours worked by this agent in t equal to

$$\check{H} = x + b\ell_z i_z^\ell + s(z - \check{z} - \check{z}i_z^d) + (1 - b - s)\gamma_a \left(\hat{q} - \frac{\check{z} + \ell_z}{\gamma_m} \right) \quad (27)$$

\check{H} reflects that with probability $(b + s)$ the agent does not default in t and with probability $(1 - b - s)$ the agent defaults and produce the real asset in order to be able to consume \hat{q} as a buyer in $t + 1$.

The expected lifetime utility for an agent who does not plan to default and brings real money balances z to t is

$$U(x) - x - \ell_z i_z^\ell - z + \frac{\beta}{1 - \beta} [bu(q) - bq + U(x) - x - (1 - s)\ell_z \kappa] \quad (28)$$

At $\gamma_m = \gamma_a = \bar{\gamma}$, it is straightforward to show that (26) and (28) are equal. To assess how utility by the agent who plans in $t - 1$ to default in t changes when γ_m increases, insert (27)

into (26), differentiate it with respect to γ_m , and evaluate it at $\gamma_m = \gamma_a = \bar{\gamma}$ using (9), (18), (19), (23), (24), (25) and Lemma 2 to get:

$$-\frac{\partial \ell_z}{\partial \gamma_m} \kappa - \frac{\partial i_z^l}{\partial \gamma_m} \ell_z - \frac{\partial z}{\partial \gamma_m} + \frac{\beta}{1-\beta} \left\{ [bu'(q) - b] \frac{\partial q}{\partial \gamma_m} - (1-s) \frac{\partial \ell_z}{\partial \gamma_m} \kappa \right\}$$

which is equal to (28) differentiated with respect to γ_m . Therefore, agents have no incentive in $t-1$ to plan defaulting in t .

Differentiate expected utility defined in (13) using (10) with respect to γ_m to get $\partial \mathcal{W} / \partial q = \{bu'(q) - b - (1-s) s\kappa / [s + r(1-s)]\} \partial q / \partial \gamma_m$ which is equal to $\{\gamma_m / \beta - 1 - (1-s) s\kappa / [s + r(1-s)]\} \partial q / \partial \gamma_m$ at $\bar{\gamma} = \gamma_m$. Since $\gamma_m / \beta - 1 - (1-s) \kappa \geq 0$ for an equilibrium with credit to exist, expected utility is an increasing function of q . Given that $\partial q / \partial \gamma_m > 0$ for $r \leq r^*$, a monetary equilibrium exists for some $\gamma_m = \gamma_m^* > \bar{\gamma}$: Agents do not have incentive to switch to the real asset since they attain higher utility by using money.

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