DOCUMENT DE TRAVAIL N° 351

INTERPRETING THE HOURS-TECHNOLOGY TIME-VARYING RELATIONSHIP

Cristiano Cantore, Filippo Ferroni and Miguel A. León-Ledesma

November 2011



DIRECTION GÉNÉRALE DES ÉTUDES ET DES RELATIONS INTERNATIONALES

INTERPRETING THE HOURS-TECHNOLOGY TIME-VARYING RELATIONSHIP

Cristiano Cantore, Filippo Ferroni and Miguel A. León-Ledesma

November 2011

Les Documents de travail reflètent les idées personnelles de leurs auteurs et n'expriment pas nécessairement la position de la Banque de France. Ce document est disponible sur le site internet de la Banque de France « www.banque-france.fr ».

Working Papers reflect the opinions of the authors and do not necessarily express the views of the Banque de France. This document is available on the Banque de France Website "www.banque-france.fr".

Interpreting the Hours-Technology time-varying relationship*

Cristiano Cantore University of Surrey[†] Filippo Ferroni Banque de France[‡] Miguel A. León-Ledesma University of Kent[§]

^{*}We thank seminar participants at University of Surrey, at Banque de France, at Structural Change and Macroeconomic Performance workshop at Universita' di Cagliari, at the ECB, at the 17^{th} International Conference on Computing in Economics and Finance. We thank Patrick Feve for an excellent discussion of the paper. We acknowledge Vasco Carvalho, Peter McAdam, Fabio Canova and Alessio Moro for helpful conversations.

[†]Department of Economics, University of Surrey, Surrey GU2 7XH UK (e-mail:c.cantore@surrey.ac.uk)

[‡]The views expressed in this paper do not necessarily reflect those of the Banque de France.(e-mail:filippo.ferroni@banque-france.fr).

[§]School of Economics, University of Kent, Canterbury, Kent, CT27NP UK (e-mail:M.A.Leon-Ledesma@kent.ac.uk).

Résumé

Nous étudions la relation entre les heures travaillées et le progrès technique en utilisant un model structurel de cycle réel (RBC). Nous développons un modèle RBC avec une fonction de production à élasticité de substitution constante, ce qui permet de distinguer les chocs de productivité du capital des chocs de productivité du travail. Le modèle est estimé selon l'approche bayésienne. Sur l'ensemble de l'échantillon, les résultats révèlent (i) des preuves solides en faveur d'une élasticité de substitution des facteurs inferieure à un (rejetant ainsi la fonction de production Cobb-Douglas), mais aussi (ii) le rôle majeur des chocs de productivité du capital dans les fluctuations du cycle économique. Par ailleurs, et sur la base des estimations de sous-échantillons, les résultats suggèrent que la transmission des chocs technologiques aux heures travaillées a varié dans le temps, cette variation serait en effet due à une hausse de l'élasticité de substitution entre les facteurs : le travail et le capital étant de moins en moins complémentaires, le signe de la réponse des heures travaillées à un choc technologique a fini par s'inverser. On estime enfin que ce changement est le résultat d'une variation du rapport emploi qualifié/emploi non qualifié dans le total du facteur travail.

Mots clés: Heurs Travaillées et Cycle économique, Méthodes bayésiennes.

Abstract

We investigate the time varying relation between hours and technology shocks using a structural business cycle model. We propose an RBC model with a Constant Elasticity of Substitution (CES) production function that allows for capital- and labor-augmenting technology shocks. We estimate the model with Bayesian techniques. In the full sample, we find (i) evidence in favor of a less than unitary elasticity of substitution (rejecting Cobb-Douglas) and (ii) a sizable role for capital augmenting shock for business cycles fluctuations. In rolling sub-samples, we document that the transmission of technology shocks to hours worked has been varying over time. We argue that this change is due to the increase of the elasticity of factor substitution. That is, labor and capital became less complementary throughout the sample inducing a change in the sign and size of the the response of hours. We conjecture that this change may have been induced by a change in the skill composition of the labor input.

Keywords: Real Business Cycles models, Constant Elasticity of Substitution production function, Hours worked dynamics.

JEL classification: E32, E37, C53

1 Introduction

One of the most controversial issues in business cycle theory regards the impact of technology shocks on hours worked. The sign and size of the hours response to a productivity shock can have important consequences for policy analysis. And the estimated response has also been interpreted as shedding light on the ability of contrasting macro models to explain features of the business cycle. The focus of most of this literature has been on the analysis of the response of hours in full samples.¹ However, recent business cycle literature has moved the attention on the changing nature of some key data moments since the works of Kim and Nelson (1999), McConnell and Perez-Quiros (2000), and Stock and Watson (2003). Specifically concentrating on the time-varying relationship between productivity and hours worked, Gambetti (2006), Stiroh (2009), and Galí and Gambetti (2009) unveil important changes in the sign and size of these responses in the US economy since the post-war era. Technology shocks appear to have a strong negative effect before the 1980s and positive or non significant afterwards, although this increase is not monotonic. Fernald (2007) also finds that, after allowing for trend breaks in productivity, hours tend to fall when technology improves.² Hence, the time-varying structures has been considered as a possible statistical explanation for the instability of the full sample SVAR estimates. Most of this literature, however, focuses on reduced form representations that allow for limited structural interpretations in terms of deep model parameters.

In this paper, we propose a structural explanation for the time-varying nature of the reaction of hours to technology shocks. We first provide further evidence on the changes in the impulse-response of hours to technology shocks for the US economy using a standard SVAR with long-run restrictions. We then propose a parsimonious model that is potentially able to capture this observed time variation. Specifically, we propose a simple RBC model where we introduce a Constant Elasticity of Substitution (CES) production function where, as shown by Cantore, León-Ledesma, McAdam and Willman (2010), the sign of the response of hours to a technology shock depends mainly on the relative magnitude of the elasticity of capital-labor substitution and the capital intensity in production. The model contains a preference shock and two technology shocks: a labor- and a capital-augmenting shock. These shocks can be distinguished when the elasticity of capital-labor substitution differs from one (the Cobb-Douglas case). We also study the properties of our specification over the full sample. Several results stand out. First, we show that the proposed specification fits the postwar US data on productivity and hours worked reasonably well, especially when compared to a standard Cobb-Douglas specification. Second, the elasticity of substitution between capital and labor

¹There is a large literature on this issue that we do not aim to survey here. For comprehensive reviews, see Galí and Rabanal (2005) and Whelan (2009).

²Kahn and Rich (2007) and Roberts (2001) amongst other document two changes in labor productivity in US. One in early 70's and one during the mid 90's. Fernald (2007) finds two breaks in private-business labor productivity growth: 1973:2 and 1997:2. He finds also that the mean growth is similar before 1973 and after 1997. Hansen (2001), using a simple first-order autoregressive model finds a break in February 1992.

is statistically well below unity, supporting the increasing consensus in the empirical literature (see Chirinko (2008)). Third, by looking at the historical decomposition of hours worked, we find a sizable role for capital augmenting shocks in explaining business cycles fluctuations. In particular, the level of productivity is mostly explained by the labor augmenting shock, and the level of hours worked is mostly explained by the capital augmenting shock.

We then estimate the model using Bayesian techniques on rolling samples of the same length as our SVAR, and find that there is a significant sign variation of the response of hours worked to a positive technology shock. We find also that the time-varying impulse responses to a laboraugmenting shock obtained from the estimated model track satisfactorily the changes observed in the data-based SVAR, despite of the parsimonious nature of the model. Such variation is driven by a change in the magnitude of the elasticity of factor substitution which, in our model, governs the sign of the hours response. In particular, we observe an increase in the degree of factor substitution along the sample. That is, labor and capital became less complementary throughout time. We conjecture that these changes may be associated to the changing skill composition of the labor force. With heterogeneous labor, an increase in the share of skilled workers or their relative productivity can lead to an increase in the aggregate elasticity of substitution. We further explore the robustness of our claim that the time varying response of hours crucially depends on the magnitude of the elasticity of capital-labor substitution. Following Chari, Kehoe and McGrattan (2008), we study whether SVAR estimates on data simulated from our structural model would lead to impact responses similar to the ones obtained using actual data. We find little support for a significant difference between the two. Finally, it is also important to highlight that, to the best of our knowledge, this is the first attempt at directly estimating the (time-varying) elasticity of capital-labor substitution in a fully fledged DSGE model accounting for both supply and demand blocks.³

It is worth emphasizing, however, that we do not view our interpretation as exclusive of other potential sources of structural changes that may have led to time-variation in the hours-technology correlation. One explanation that has received much attention is the well known change in monetary policy at the beginning of the 80's. However, this explanation is not free from criticism. For instance, Canova and Gambetti (2009) find little support for the role of monetary policy changes in driving output and inflation dynamics and point towards the potential importance of changes in private sector behavior. Changes in the labor market can be another important source of time-variation. Along these lines, Nucci and Riggi (2009) attribute changes in the response of hours to an increase in performance-related pay schemes during the 1980s. Their model, however, can

³The literature on the estimation of CES parameters has focused almost exclusively on supply side static models as in León-Ledesma, McAdam and Willman (2010).

⁴See amongst other Clarida, Galí and Gertler (2000), Galí, Lopez-Salido and Valles (2003) and Cogley and Sargent (2005).

account for a reduction in the negative response of hours to a technology shock but not for a sign switch. In parallel to increased labor market flexibility we also observe another important change in the labor market that may have shaped aggregate hours responses. As reported by Acemoglu (2002) and Acemoglu and Autor (2011), the US labor market experienced significant changes in its skill composition. These changes can affect the elasticity of capital-labor substitution and hence the response of hours to technology shocks.⁵ These effects, however, have received little attention as potential sources of time-variation in labor market data moments. Our setup is deliberately parsimonious since the time variation of the response of hours can be seized by the change in the relative magnitude of the parameters entering the production function. For this reason, we analyze how far changes in few crucial parameters can go to explain the time-variation of hours responses. We do not go as far as claiming, however, that frictions and macroeconomic policies cannot potentially play an important role.

The paper in organized as follows. Section 2 presents some empirical evidence. Section 3 presents the model and study the response of hours with a sensible calibration exercise. Section 4 describes the estimation strategy and presents the full sample estimates. The dynamics of hours worked and productivity are reported in Section 5. Section 6 offers a theoretical discussion of potential sources of changes in capital-labor substitutability. Finally, Section 7 concludes.

2 Empirical Evidence

While there is a large literature documenting the changes in the second moments of various US times series, here we focus on response of hours worked to a technology shock. Data ranges from 1948:Q1 until 2006:Q1 and was obtained from the FRED database. The times series include output in the non-farm business sector (OUTNFB), and hours of all persons in the non-farm business sector (HOANBS). Both series are normalized by the the civilian non-institutional population of 16 years and over (CNP16OV). Labor productivity is computed as the ratio between the measure of output and hours, and we take logarithms of both series. We indicate with p_t labor productivity and with $hobs_t$ hours 6 .

To identify a technology shock we adopt the long-run restriction proposed by Blanchard and Quah (1989) where we assume that only the technology shock has a permanent effect on the level of productivity (as in Galí (1999)). We estimate the structural VAR (SVAR) model on rolling windows of fixed length, starting from the sample [1948Q1,1967Q4], and repeating the estimation moving the starting date by one year. We obtain 39 estimates of the coefficients of the reduced from VAR and

⁵During this period, we can also observe an important process of de-unionization, although this may well be the consequence of changes in skill composition of the labor force due to the introduction of skill-biased technologies as argued by Acemoglu, Aghion and Violante (2001).

⁶See the appendix for the data construction

of the identified impact matrix (one for each window) and compute the impulse response of hours to a technology shock. We considered different lag lengths for the VAR and rolling windows sizes and the results remained unchanged.⁷ We report here the results with 80 quarters and four lags in the VAR. More formally, the reduced form VAR of can be represented as

$$x_t = A_0 + A_1 x_{t-1} + \dots + A_p x_{t-p} + u_t$$

where u_t are i.i.d. zero mean normal shocks with covariance matrix Σ . We assume that $u_t = K\epsilon_t$ where $\epsilon_t = [\epsilon_t^s, \epsilon_t^d]$ is a normal i.i.d. shock with $E(\epsilon_t \epsilon_t') = I$, and where ϵ_t^s is the technology shock and ϵ_t^d a non-technology shock. It follows from the assumptions that $\Sigma = KK'$.

We consider $x_t = [\Delta p_t, hobs_t]$ in estimation.⁸ For exposition purposes it is more convenient to rewrite the system in a companion form

$$z_t = \mu + Bz_{t-1} + e_t$$

where $z_t = [x'_t, x'_{t-1}, ..., x'_{t-p+1}]'$, $e_t = [u'_t, 0, ..., 0]'$, $\mu = [A'_0, 0, ..., 0]'$, and B is the companion form matrix. The long run restriction implies that the impact matrix of cumulative effects of the shock on labor productivity has a Cholesky factor, i.e. the matrix $F = \sum_{k=0}^{\infty} S_{2,2}(B^k)$ K has a lower triangular structure where $S_{2,2}(.)$ is a selection matrix that picks the first two rows and columns of matrix B^k .

Figure 1 plots the response of hours worked to a technology shock. The response of hours worked displays significant time variations. In fact, the impact response is negative in early samples, increases up until the mid-1970s, then falls, and then increases steadily thereafter. These results are similar to those of a more parameterized set up, as in Galí and Gambetti (2009), using a VAR with time-varying coefficients and stochastic volatility, the same specification of hours, and the same identification scheme (see also the appendix B for comparisons). To ease the visual analysis, figure 2 reports the impulse responses for selected sub-samples. As it clearly stands out, the response of hours worked to an identified technology shock has changed over time. In particular, while it was negative during the 60s on impact, hours increase following a technology shock if we consider the sample including the 1990s for estimation.

In all, these results confirm the existence of important changes in the short-run technology-hours correlations in the US over the post-war period.

⁷We used rolling windows of 60, 70, 80, and 90 quarters and four lag lengths.

⁸We also considered hours in first difference, i.e. $x_t = [\Delta p_t, \Delta hobs_t]$. While we find time variations, we do not detect any sign switch. This result is due to the fact that first differencing removes the long run frequencies of hours worked. As shown in Canova, Lopez-Salido and Michelacci (2010), if secular cycles are removed from the raw series of hours worked, hours respond negatively to technology.

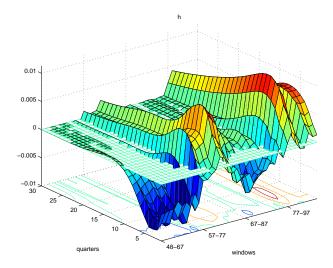


Figure 1: Response of the growth rate of productivity and of hours worked to a technology shock. The level of hours is used in estimation.

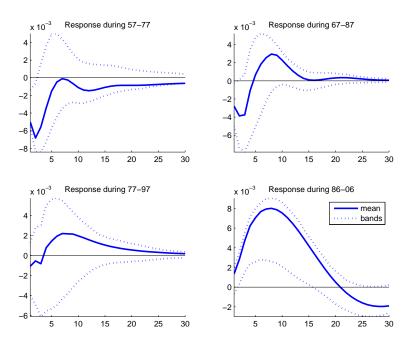


Figure 2: Response of the growth rate of productivity and of hours worked to a technology shock for selected sub-samples. The level of hours is used in estimation.

3 The Structural Model

We consider a closed economy Real Business Cycles (RBC) model. The novelty of the model is that it features a Constant Elasticity of Substitution (CES) production function, which is characterized by two sources of fluctuations, a labor- and a capital-augmenting stochastic shift to the production frontier. The model is otherwise standard, it is a single good optimizing agent framework. The

advantage of this model is that, with an elasticity of capital-labor substitution that differs from unity (the Cobb-Douglas case), even in the canonical RBC model the response of hours to a labor-augmenting technology shock can be positive or negative. Cantore et al. (2010) show analytically that the sign of the response depends on the relative magnitudes of the elasticity of substitution and the capital share.⁹

The representative household is characterized by the following preferences 10

$$U_t = \ln C_t - V_t \xi \frac{H_t^{1+\gamma}}{1+\gamma},\tag{1}$$

where C_t denotes consumption, H_t hours worked, β is the discount factor, γ is the inverse of the Frisch elasticity, ξ affects the marginal rate of substitution between consumption and leisure and determines the steady state hours and V_t is a preference shock process that has an AR(1) representation, i.e. (in log deviations from the steady state)

$$v_t = \rho_v v_{t-1} + \eta_t^v \quad \eta_t^v \sim N(0, \sigma_v). \tag{2}$$

The production is CES and presented in *normalized* form as in Cantore et al. (2010)¹¹

$$Y_t = y \left[\alpha \left(\frac{Z_t^k K_{t-1}}{k} \right)^{\frac{\sigma - 1}{\sigma}} + (1 - \alpha) \left(\frac{Z_t^h h_t}{h} \right)^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}}$$
(3)

where, as usual, output is produced by a combination of two factors, K_{t-1} the installed physical capital at time t, y and k are the steady state values of output and capital re-scaled by the labor augmenting process, and h is the steady state value for hours. α and σ are parameters controlling the capital intensity in production and the degree of substitutability between factors. As $\sigma \to 0$, factors are net complements, and the production function is Leontief. If $\sigma \to \infty$ factors are net substitutes and the production function is linear. As σ approaches 1, we have a Cobb-Douglas production function. The CES production function encompasses two types technological change, i.e. the capital augmenting, Z_t^k , and the labor augmenting technological process, Z_t^h . We assume that capital-augmenting technology has an AR(1) representation, i.e. (in log deviations from the steady state)

$$z_t^k = \rho_k z_{t-1}^k + \eta_t^k \quad \eta_t^k \sim N(0, \sigma_k),$$
 (4)

where $\rho_k < 1$ to ensure the existence of a balanced growth path. For the labor-augmenting shock we adopt a flexible specification similar to Ríos-Rull, Schorfheide, Fuentes-Albero, Kryshko and Santaeulália-Llopis

⁹The response also depends on the reaction of consumption. Cantore et al. (2010) also show that a similar change in the sign of responses can occur in a New Keynesian model, but in this case for a capital-augmenting shock.

 $^{^{10}\}mathrm{We}$ assume a log preference in consumption to guarantee a balanced growth path.

¹¹Normalization is required to compare responses when we change the elasticity of substitution. Also, it allows us to interpret directly the share parameter α as the capital income share at the point of normalization (the steady state in this case).

(2009), where it follows an AR(2) process, i.e.

$$z_t^h = \psi_{1,h}(1 - \psi_{2,h})z_{t-1}^h + \psi_{2,h}z_{t-2}^h + \eta_t^h, \tag{5}$$

with $z_t^h = \ln Z_t^H - \ln Z_0^H$ and the original autoregressive processes is rewritten in terms of partial autocorrelations $\psi_{1,h}$ and $\psi_{2,h}$. If $\psi_{1,h} = 1$, then labor-augmenting technology shocks have a permanent effect and the labor-augmenting technology process is stationary in first differences with autoregressive coefficient $-\psi_{2,h}$. If $0 < \psi_{1,h} < 1$ and $\psi_{2,h} = 0$, then the labor-augmenting technology process is persistent but stationary and follows an AR(1) process. The model is then closed by assuming that capital depreciates at rate δ and that the economy's resource constraint is given by:

$$Y_t = C_t + K_t - (1 - \delta)K_{t-1}. (6)$$

As mentioned, this model has the property that the capital intensity in production and the elasticity of factor substitution, α and σ , are the main drivers of the dynamics of output and hours worked conditional on a labor augmenting technology shock.

By means of a sensible calibration exercise, we can study the impact of a labor augmenting technology shock to hours worked for different values of the capital-labor elasticity. Without loss of generality, we assume that the labor augmenting technology process is stationary, i.e. $\psi_{2,h} = 0$ and $\psi_{1,h} = 0.8$. Moreover, we set the time discount factor, β , to 0.99, and the depreciation rate, δ , to 0.025, and the inverse of the Frisch elasticity, γ , to 1. We let the capital-labor elasticity vary between 0.1 and 1, and we fix the capital intensity in production to 0.33. Figure 13 (left panel) reports the impulse response of hours worked to a labor-augmenting technology shock for different values of σ and keeping the value of α fixed at 0.33. Approximately, when $\sigma > \alpha$ the response of hours to a labor augmenting technology shock is positive. However, hours worked decrease if $\sigma < \alpha$. The right panel of Figure 13 displays the instantaneous response of hours worked to a labor-augmenting technology shock for different values of σ and α . We let the value of capital intensity vary between 0.2 to 0.6. Thus, approximatively for values of σ larger than 0.7 and close to the Cobb-Douglas specification, the response of hours is positive regardless of values of α . The intuition behind the result is that the shock induces a substitution effect that reduces the demand for labor and a quantity effect that increases the demand for labor. Depending on the strength of these two, hours may increase or decrease.

if $\rho_{1,h} + \rho_{2,h} = 1$, then technology has a unit root and the serial correlation of its growth rates is $-\rho_{2,h}$. We can re-parameterize them in terms of partial autocorrelations $\psi_{1,h}$ and $\psi_{2,h}$ by setting:

$$\rho_{1,h} = \psi_{1,h} (1 - \psi_{2,h})
\rho_{2,h} = \psi_{2,h}$$

 $^{^{12} \}text{By assuming}$ $(\ln Z_t^H - \ln Z_0^H) = \rho_{1,h} (\ln Z_{t-1}^H - \ln Z_0^H) + \rho_{2,h} (\ln Z_{t-2}^h - \ln Z_0^h) + \eta_t^h$

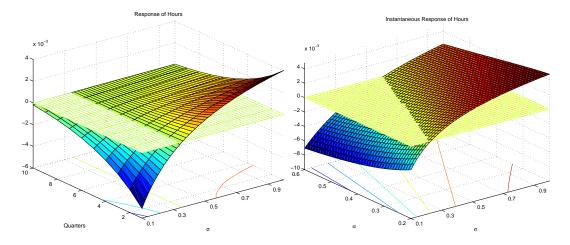


Figure 3: Impulse response of hours worked to a labor-augmenting technology shock for different values of σ and $\alpha = 0.33$ (Left panel). Instantaneous response of hours worked to a labor-augmenting technology shock for different values of σ and α (Right Panel).

Despite of its parsimonious nature, the model is a priori able to capture switches in the sign of the response of hours. And the capital-labor elasticity is the crucial parameter which might capture the time varying response observed with the SVAR analysis. Being the first attempt to estimate σ in a general equilibrium model, it is legitimate to wonder whether such parameter is identifiable. To this end, in a controlled experiment we obtain that the information contained in hours worked and productivity is sufficient to identify σ in estimation (see the appendix D).

4 Full sample estimates of a CES production technology.

We now analyze the behavior of the model when confronted with observed data on US productivity and hours worked. In particular, we are interested in verifying that the model presented fits the data reasonably and that its performance is comparable with the fit of more parsimonious yet standard specifications. Hence, we confront two specifications: an RBC model with a CES production function and an RBC model with a Cobb-Douglas production technology (i.e. $\sigma = 1$ and only the laboraugmenting technological process). We verify whether data favors a less parameterized model and thus the CES specification is redundant, or whether the latter helps characterize the data better.

Since the raw series of labor productivity displays a clear upward trend, we bridge the model to data by imposing permanent labor-augmenting technology shocks. Hence, real variables grow at the rate of the technological process and hours worked are stationary.¹³ These assumptions imply that

 $^{^{13}}$ If we assume that innovations to the labor-augmenting technology process have a permanent effect on the economy, we need to generate stationary variables in the model using the following transformations: $\frac{Y_t}{Z_t^H} \frac{K_t}{Z_t^H} \frac{C_t}{Z_t^H} \frac{W_t}{Z_t^H} H_t R_t$ where W_t is the real wage and R_t is the rental price of capital. The log-linearized equilibrium conditions are reported in Appendix ??.

 $\psi_{1,h} = 1$ and that the following measurement equations hold¹⁴

$$\Delta p_t = \Delta (y_t - h_t) + \Delta z_t^h$$
$$hobs_t = h_t$$

Table 1 reports prior and posteriors statistics for the full sample. The choice of the priors is standard. We assume inverse gamma for standard deviations, beta distributions for the autoregressive parameters, a normal distribution for the inverse of the Frisch elasticity, γ , and for the capital intensity in production, α . The prior for σ follows a gamma distribution centered around one and with a loose precision. While posterior distributions of σ are very similar using a flat prior (i.e. the posterior mean is centered around 0.15 and has a tight credible set), we prefer to use a proper priors for marginal likelihood comparisons.

A few things are worth noting. First, for many parameters, posterior distributions have different locations, spread and shape relative to the prior assumptions. This is indicative that data provide relevant information for estimation. Moreover, in most cases, the mean and median coincide ruling out asymmetric posterior distributions (not shown here). Third, the standard deviations of technology shocks are a posteriori significant implying that data favors the mechanisms induced by the CES production function.

| | Prior | | | CD | | CES 3 | | CES 2 | |
|--------------|--------|-------|------|--------|--------|--------|-------|--------|-------|
| | Distr | mean | sd | median | sd | median | sd | median | sd |
| α | Normal | 0.30 | 0.05 | 0.29 | 0.0373 | 0.33 | 0.051 | 0.35 | 0.041 |
| σ | Gamma | 1.00 | 1.00 | - | - | 0.14 | 0.031 | 0.13 | 0.023 |
| γ | Normal | 1.00 | 0.10 | 1.04 | 0.0949 | 1.00 | 0.100 | 0.98 | 0.100 |
| $ ho_v$ | Beta | 0.50 | 0.20 | 0.97 | 0.0113 | 0.95 | 0.019 | - | - |
| $ ho_k$ | Beta | 0.70 | 0.20 | - | - | 0.96 | 0.016 | 0.96 | 0.013 |
| $\psi_{2,h}$ | Beta | 0.50 | 0.20 | 0.04 | 0.0220 | 0.07 | 0.037 | 0.05 | 0.027 |
| σ_h | Igamma | 0.010 | 2.00 | 0.01 | 0.0010 | 0.010 | 0.001 | 0.010 | 0.001 |
| σ_k | Igamma | 0.010 | 2.00 | - | - | 0.019 | 0.002 | 0.018 | 0.001 |
| σ_v | Igamma | 0.010 | 2.00 | 0.01 | 0.0010 | 0.012 | 0.004 | - | _ |
| | | | | | | | | | |
| Log ML | | | 1424 | | 1430 | | 1432 | | |

Table 1: Prior, posterior statistics and marginal likelihoods across specifications. Igamma stands for the inverse gamma distribution. CES 3 and 2 refers to the number of shocks.

Concerning the parameters of interest, the posterior median of the degree of factor substitution is centered around 0.13 and the posterior distributions is quite tight in absolute terms and relative to the prior. This suggests that the data favors a more general specification for the production function. The capital share is estimated around the standard value in the RBC literature, i.e. 0.34,

¹⁴Both series are demeaned to guarantee consistency with the log-linearized variables in the model that fluctuate around a value of 0 in steady-state.

thus larger than the elasticity of substitution. This implies that, assuming no time-variation along the full sample, the point estimate of the correlation between hours worked and productivity is negative conditional on a labor augmenting technology shock. A formal comparison between the two models is reported at the bottom of Table 1 where we contrast the log of the marginal likelihood using the modified harmonic mean (see Geweke (1999)). If the two sources of technological progress and a non-unitary elasticity of substitution between inputs were not important to characterize the dynamics of output and hours, a more parsimonious model would be preferred by means of marginal likelihood. In order to favor a Cobb-Douglas production function we need a prior probability for the model with Cobb-Douglas 403 (= e^6) times larger than the one associated with a CES production function (in other words, CES beats the CD production function with posterior model probabilities of 0.9975:0.0025). Moreover, we find that regardless of the number of shocks the CES structure is preferred to the Cobb-Douglas production specification¹⁵. Given the feeble role of preference shocks in a CES setting, we expect to observe a completely different historical decomposition of the observable variables among specifications. Figure 4 reports the decomposition of hours worked in terms of structural residuals. Under the Cobb-Douglas specification, where the capital augmenting

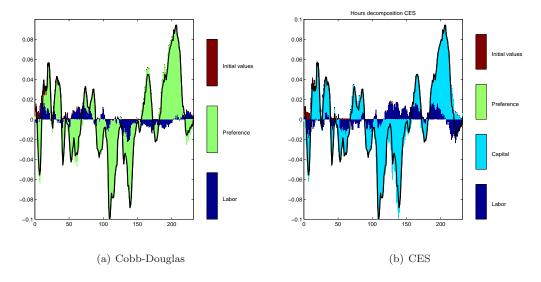


Figure 4: Historical decomposition of hours. Top panel CES specification, bottom panel Cobb-Douglas specification.

shock is absent, the preference shock plays the most important role in the historical evolution of hours worked. When we turn to the CES, the contribution of the preference shock vanishes and the capital-augmenting shock contributes significantly to the observed levels of hours worked.

¹⁵ We notice that the difference in terms log marginal likelihood is not sufficient to strictly prefer the CES specification with two shocks to the specification with three shocks. The literature adopt as a cutoff value 3, see amongst others Jeffries (1996) and Kass and Raftery (1995)).

The decomposition of productivity is similar across the two settings (not shown here), where the labor-augmenting shock represents the dominant source of the observed fluctuations of productivity. Hence, if we adopt a more general specification of the production function, we obtain that the full set of technological shocks account for the entire portion of historical fluctuations of productivity and hours experienced by the US economy within this RBC setting.

5 Time-varying dynamics

We want to investigate the dynamics of hours and technology over time through the lens of the structural model. To this end, we estimate the model on rolling windows of the same fixed length of our SVAR and we look closely at propagation mechanism of the structural shocks. Let the solution of the DSGE model be of the from,

$$y_{t+1}^{\dagger} = \Phi(\vartheta)y_t^{\dagger} + \Psi(\vartheta)\eta_{t+1},$$

where the the vector y_t^{\dagger} contains the endogenous variables of the model and η_t the structural vector of innovations with zero mean and diagonal covariance matrix Σ_{η} . Φ and Ψ are matrices which are non-linear functions of the structural parameters of the model, ϑ . Since we have a unique mapping from the structural parameters of the model to the reduced form matrix, we can back out the 'deep' parameters responsible for the changes (if any) in the transmission of shocks. Then, we look closely at the time pattern of the estimated structural parameters and try to provide intuition for such changes. Finally, we perform a 'reverse' exercise in the same spirit of Chari et al. (2008). We ask whether the estimates of SVAR on data simulated from our structural model would bring us to the same results of the SVAR on actual data. We find little support for a difference between the two.

5.1 The transmission of technology shocks

One key fact that our setup would like to explain is the time varying relationship between hours worked and technology shocks and, in particular, if the model is able to reproduce the patterns found using the SVAR model. Figure 5 plots the response of hours worked to a labor augmenting technology shock. The response of hours worked shows clear shape and sign variations along the sample. Taken literally, the very early samples are characterized by a negative response. Then, for samples that include mainly the 1970s hours react positively to technology shocks. Then, the reaction of hours turns negative and positive again in the last ten rolling windows. The resemblance with the SVAR evidence is striking. On impact, the the signs of the response of hours are correctly identified. Figure 6 plots the 68% credible sets around the instantaneous response of hours with the

¹⁶While there are variations in the level of the response, we do not detect any changes in the pattern of the response of output to a capital augmenting technology shock (not shown here). Thus, we do not report it.

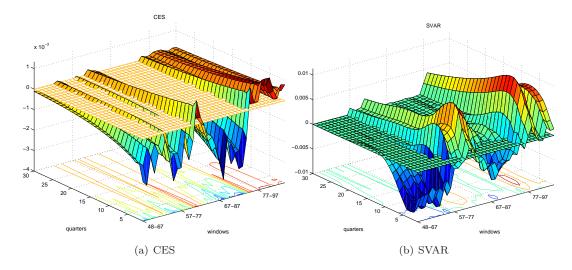


Figure 5: Impulse responses of hours to a positive labor augmenting technology shock.

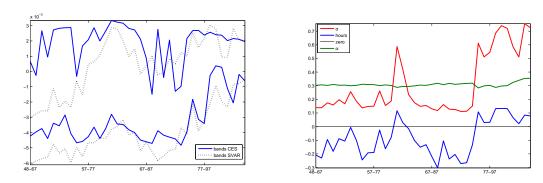


Figure 6: On the left panel credible sets of the contemporaneous impact of hours to a technology shock in the SVAR (solid line) and in the RBC with CES (dotted line). On the right panel, the median estimates of σ , α and the instantaneous impact of technology shock on hours across windows.

SVAR estimates and the RBC-CES ones. If the instantaneous response of hours were different in the two settings, we would observe windows with non overlapping bands. Looking at Figure 6, we detect no significant difference of the contemporaneous response of hours between the estimates of the SVAR and the estimates of the RBC with CES production function ¹⁷.

The question that follows is what are the driving parameters behind the change in the propagation mechanism.

Since the impulse response is computed as the marginal impact of a structural innovation to a variable, we can rule out changes in the standard deviations of the structural shocks as responsible for such variations. Even if the model is very stylized, the CES production function allows us to disentangle the scenarios where hours increase (decrease) in response to a technology innovation

¹⁷It is worth noting, however, that the lack of persistence of the structural model has to do with the lag structure of the solution of the DSGE model, which makes it difficult to replicate the hump shaped response of a four lags VAR.

because the degree of factor substitution is larger (smaller) than the capital share in production. We find that there are large variations in the posterior estimates of the degree of factor substitutability, in absolute terms and relative to the capital share parameter in production, α . Figure 6 (right panel) plots the posterior mean of the elasticity of substitution and the capital share in each of the sub-samples. Changes in the hours-technology conditional correlation on impact are associated to changes in the elasticity of capital-labor substitution.

Two things are worth mentioning.

First, the variation in the estimates of σ are significant but abrupt. This is partly due to the non parametric approach we adopt and to the uniform weighting scheme we impose on each window. One way to smooth the estimates of σ is to downsize the impact of sub-sample endpoints. As in the sample spectrum estimation (see Priestley (1982), Ch.5), we could design a bell shape distribution so that break points would have milder impact on structural estimates. However, we preferred to be agnostic and to give priority to the observables without imposing any ad hoc weighting scheme. The other approach is to parameterize the changes in σ by assuming that the capital-labor elasticity follows a slow moving exogenous process (i.e. an autoregressive process). Since first order approximations are insufficient to capture such process, higher order approximation are required. With higher order solutions the implied state space system is nor linear nor gaussian, and we need to move to particle filters to extract the likelihood. Despite important achievements have been done in this direction (see Fernndez-Villaverde and Rubio-Ramirez (2008)), the estimation of time-varying structures is still computationally burdensome and difficult to handle. Given these constraints, and for comparison with our SVAR results, we study what a computationally less intensive yet intuitively appealing structural method could tell us about the time varying relation between hours and productivity.

The second observation has to do with the interpretation of the changes in σ . In our estimates, we find periods where the sign of the response of hours switch sing, from negative to positive. These periods coincide with episodes of relatively large estimates of σ . We observe a spike in two windows that includes the 70s (i.e. the window from 1962 to 1982 and the one from 1963 to 1983) and a protracted period in the final part of the sample, i.e. the last 10 windows. The two sub-samples that includes the 70s contain very eventful years. In fact, during the 70s the US economy was hit by a sequence of negative oil price shocks. And the beginning of the 1980s is characterized by the change in the monetary policy stance. As a consequence, we suspect that the variation in σ during these windows is contaminated by the turbulence of the seventies. After 1982, however, the US economy entered in a relative quite period of time, where either 'good policy' or 'good luck' (or both) contributed to render the macroeconomic environment less volatile and more predictable. We thus believe that neither policy nor changes in the structure of the shocks are corrupting the estimated changes in the capital-labor elasticity in the latest samples. However, during the same

years, the US economy was experiencing important changes in the labor market. The literature has documented a sizable increase in the relative supply of skilled workers over time. We are prone to interpret these changes as the source of the observed changes in the estimates of the capital-labor degree of substitution. We will return to this issue in Section 6.

5.2 Is the story of change in capital-labor elasticity consistent with a SVAR?

The time-varying relationship between hours and technology identified by a SVAR with long-run restrictions is very similar to the one obtained from our RBC model with CES production function. However, Chari et al. (2008), amongst others, express concerns about the ability of SVARs with long-run restrictions to identify model shocks. This may then cast doubts about whether comparisons of model-based and SVAR-based impulse-responses constitute a reliable way to evaluate our model. To address this issue, we follow Chari et al. (2008) and simulate 50 sets of data of 100 observations from the RBC model with CES production function using the mean estimates in each window. For each simulated dataset we estimate a SVAR with 4 lags and compute the impulse response. We then compare the data-based SVAR with the SVAR with model-simulated data.

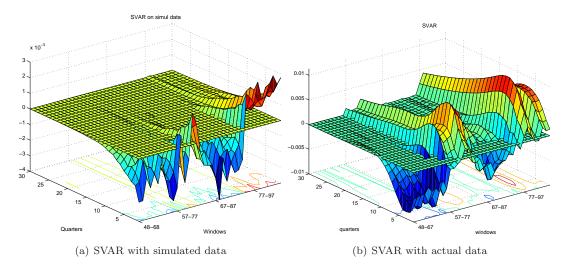


Figure 7: Impulse responses of hours to a positive labor augmenting technology shock.

Figure 7 reports the median impulse responses of hours for the SVAR (on the left panel) with simulated data, and those obtained by a SVAR with actual data. A visual inspection reveals that the instantaneous response of hours obtained with a SVAR on simulated data is similar to the one obtained with SVAR using actual data. Figure 6 plots the credible sets around the instantaneous response of hours in the SVAR on actual data (solid line) and in the SVAR with simulated data (dotted line). As in the previous case, we detect no significant difference of the contemporaneous response of hours between the estimates of the SVAR and the estimates of the RBC with CES

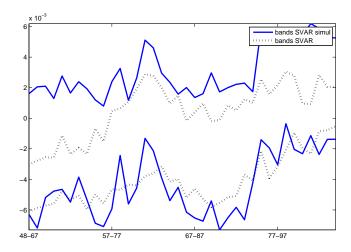


Figure 8: Credible sets of the contemporaneous impact of hours to a technology shock in the SVAR on actual data (dotted line) and in the SVAR with simulated data (solid line).

production function. Overall, we conclude that changes in the elasticity of capital-labor substitution are able to generate the observed time varying path of a SVAR with long-run restrictions.

6 Rationalizing changes in the elasticity of substitution

Our estimation results suggest that the driving factor behind the change in the response of hours is the increase in the elasticity of capital-labor substitution σ . Changes in deep parameters, such as e.g. the degree of risk aversion, are commonly used the explain the existence of instabilities in macroeconomic relationships. However, the observed change in σ deserves further attention. Here we discuss some conjectures as to what could have driven this change. We leave detailed testing strategies for future research while we keep here the focus on the change in the hours-technology correlation.

Changes in the elasticity of substitution have been associated with economic growth since La Grandville (1989). Parameter σ , nevertheless, was treated as exogenous. Hicks (1932), however, conjectured that the elasticity of substitution may be variable and a by-product of economic development. Along these lines, Miyagiwa and Papageorgiou (2007) present a multisector growth model where σ is endogenously determined and positively related to economic development. Similarly, Álvarez-Cuadrado and Van Long (2011) present a multisector model of structural change where the aggregate elasticity of substitution is endogenous as capital intensity increases in the more flexible sectors (i.e. those with higher elasticity of substitution). Since the aggregate elasticity is a weighted average of sectoral elasticities, growth and structural change can lead to changes in aggregate σ .

Although plausible explanations of long-run changes in σ , the above conjectures would naturally

lead to slow and protracted increases, rather than the more pronounced change we observe in our estimates. During the period considered, however, and especially since the mid-1980s, important changes in the US labor market occurred that could potentially be driving the increase in σ observed in the latter parts of our rolling sample. One such important change is the increased importance of skilled workers in production. The evolution of skilled to unskilled employment and wages has been widely documented in papers such as Acemoglu (2002) and Acemoglu and Autor (2011). Figure 9 reproduces the observed trends by level of skills in the US economy. It displays the share of skilled workers as a percentage of all workers using two measures. The first is the share of non-production workers in US manufacturing for the 1958-2005 period from the Annual Survey of Manufactures. The second is the share of hours of workers with college education or above, as a percentage of total hours by workers with at least high school education coming from Autor, Katz and Kearney (2008) for the whole economy and the 1963-2005 period. Although both measures differ substantially, they both show positive trends. In the case of manufacturing, however, the share falls towards the end of the sample.

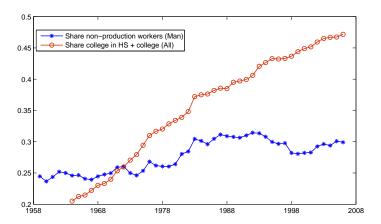


Figure 9: Shares of non-production workers (manufacturing) and college-plus hours in high-school-plus hours (aggregate economy).

The question is then whether these changes in the composition of the labor force could have affected the aggregate elasticity of substitution. In a two-factor CES production function, σ is constant. However, in the presence of heterogeneous labor (i.e. skilled and unskilled), the aggregate capital-labor elasticity of substitution is not constant and will depend, among other things, on the share of skilled labor hours in total hours input. We focus here on the case of a CES with three

¹⁸Galí and van Rens (2010) also point towards changes in the labor market to explain the decreased procyclicality of labor productivity and the increased volatility of the real wage. They, however, focus on improved matching due to increased labor market flexibility. We note that Rotemberg (2008) shows that the volatility of wages is a positive function of the elasticity of capital-labor substitution within a search and bargaining model. Also, Sargent and Wallace (1974) show that the elasticity is a key parameter to understand the cyclical behavior of productivity and wages.

factors of production (capital, skilled labor, and unskilled labor) and use the common specification of a two-level nested CES function. Here, the effects of changes in the share of skilled workers will depend on the (constant) elasticities of substitution between the three factors, and the type of nesting specified for the CES.¹⁹ Thus, we analyze the effect of changes in the proportion of skilled workers under three possible nestings.²⁰

Without loss of generality, and for simplicity, we ignore technological process terms and time subscripts and assume all variables are measured at the normalization point. We denote skilled labor as S and unskilled labor as U. The first nesting corresponds to:

$$Y = \left[\pi_X X^{\psi} + (1 - \pi_X) U^{\psi}\right]^{1/\psi} \tag{7}$$

$$X = \left[\pi_K K^{\theta} + (1 - \pi_K) S^{\theta}\right]^{1/\theta}, \tag{8}$$

where ψ and θ are the inter- and intra-class substitution parameters, π_X is the income share parameter for aggregator X at the point of normalization, and π_K is the share parameter of capital in X (also at the normalization point). The corresponding elasticities of substitution are $\sigma_{K,S} = \frac{1}{1-\theta}$ and $\sigma_{K,U} = \sigma_{S,U} = \frac{1}{1-\psi}$ with $-\infty < \theta < 1$ and $-\infty < \psi < 1$. It is worth noting that the Cobb-Douglas case occurs when ψ (θ) = 0, the Leontief case when ψ (θ) = - ∞ , and the perfect substitutes case when ψ (θ) = 1. The second nesting is:

$$Y = \left[\pi_X X^{\psi} + (1 - \pi_X) S^{\psi}\right]^{1/\psi} \tag{9}$$

$$X = \left[\pi_K K^{\theta} + (1 - \pi_K) U^{\theta} \right]^{1/\theta}, \tag{10}$$

where parameters have the same interpretation as in (7)-(8), but now $\sigma_{K,U}=\frac{1}{1-\theta}$ and $\sigma_{K,S}=\sigma_{S,U}=\frac{1}{1-\psi}$. And the third nesting is:

$$Y = \left[\pi_X X^{\psi} + (1 - \pi_X) K^{\psi}\right]^{1/\psi} \tag{11}$$

$$X = \left[\pi_S S^{\theta} + (1 - \pi_S) U^{\theta}\right]^{1/\theta}, \tag{12}$$

where we have $\sigma_{S,U} = \frac{1}{1-\theta}$ and $\sigma_{K,S} = \sigma_{K,U} = \frac{1}{1-\psi}$.

The nestings differ in terms of the assumptions imposed about the value of the elasticity of substitution across factors. While in the first nesting both K and S are equally substitutable for U but not between them, in nesting two both K and U are equally substitutable with S but not between them. Nesting (7)-(8) has been widely used in the capital-skill complementarity literature as discussed in Krusell, Ohanian, Ríos-Rull and Violante (2000). Capital-skill complementarity in this

¹⁹Papageorgiou and Saam (2008) also show that, within this kind of CES specification, the aggregate elasticity is a negative function of capital intensity. This may also help explain some of the shorter-run changes observed in our estimates.

 $^{^{20}}$ Note that, as will be apparent below, analyzing the effect of changes in the proportion of skilled workers on σ is equivalent to analyzing the effects of changes in skilled-saving relative to unskilled-saving technical change (which is skill-biased technical change if both are gross substitutes in production).

nesting simply implies that $\psi > \theta$. In nesting two, however, capital-skill complementarity implies that $\theta > \psi$ such that capital is more substitutable with U than with S. Note, however, that the third nesting does not allow for capital-skill complementarity as both skilled and unskilled workers are assumed to substitute capital the same way. In fact, it is easy to show analytically that, in this case, the aggregate elasticity of substitution between labor and capital is simply $\frac{1}{1-\psi}$ which is constant. Hence, we leave aside the third nesting as, by construction, cannot generate time-variation of σ .

In order to analyze the effect of changes in the proportion of skilled workers in the first two nestings, we define $n = \frac{U}{U+S}$ as the fraction of unskilled workers. Since total labor input is H = U+S, we can write U = nH and S = (1-n)H. Now, we use the definition of the aggregate elasticity of substitution σ :

$$\sigma = \frac{\frac{w/r}{K/H}}{\frac{\partial(w/r)}{\partial(K/H)}},\tag{13}$$

where r is the rental price of capital. Note also that, at the normalization point, $\frac{w}{r} = \frac{1 - \pi_X \pi_K}{\pi_X \pi_K} \frac{K}{H}$. Using this and expression (13), Papageorgiou and Saam (2008) show that the aggregate elasticity of substitution between H and K is a harmonic mean of the elasticities of substitution in the nested CES functions that can be expressed as:

$$\sigma = \frac{1}{(1-\theta) + (\theta - \psi)g},\tag{14}$$

$$g = \frac{\pi_K}{\frac{1 - \pi_K}{1 - \pi_X} + \pi_K}.$$
 (15)

Since θ and ψ are constants, we can analyze the effect of a change in (1-n) on σ by obtaining the derivative of g with respect to (1-n). We are then in a position to state the following lemma:

Lemma 1 The aggregate capital-labor elasticity of substitution σ is a positive function of the share of skilled workers (1-n) (and the productivity of skilled relative to unskilled workers) if:

- 1. $|\theta| > |\psi|$ for the first three-factor CES nesting (X, U);
- 2. $|\theta| < |\psi|$ for the second three-factor CES nesting (X, S).

Proof. See Appendix E. ■

Take the first nesting. This condition would imply that if capital and skills are complements (within the X aggregator), i.e. $\theta < 0$, and unskilled workers and K and U substitutes ($\psi > 0$), the degree of complementarity between K and S has to be stronger than the degree of substitutability between U and the other two factors. On the other hand, this would also be the case if all factors are substitutes ($\theta > 0$ and $\psi > 0$) but U is less substitutable for X than S and K are between each other. The same conclusions apply for the other nesting bearing in mind that, in this case, $\theta > \psi$ implies capital-skill complementarity.

The question is, of course, how likely is this to be the case? Estimates of the skilled-unskilled workers substitution parameter ψ usually range between 0.25 and 0.5.²¹ Regarding substitution between capital and skilled workers, estimates differ by study and are less abundant. Krusell et al. (2000) find and elasticity of 0.67 ($\theta \simeq -0.5$). However, given that aggregate σ is estimated to be substantially below unity (see Chirinko (2008)) and our estimates for the full sample are below 0.2, this elasticity is likely to be even lower. Hence, the conditions for a positive effect of 1-n on σ are plausible.

Based on this, we carry out a simple numerical exercise. We calibrate ψ to a value of 0.33 (corresponding to an elasticity of 1.5). Baseline values for the shares are $\pi_X = 0.6$ and $\pi_K = 0.5$, corresponding to a an aggregate capital income share of 0.27 and a skilled income share of 0.33. The initial share of skilled workers is 20% (n = 0.2). To be compatible with our low σ estimate, we then set $\theta = -3$ corresponding to a plausible elasticity of 0.25. The value of the aggregate elasticity of substitution yields 0.32. We then analyze the impact of an increase of the share of skilled workers of 0.25 (25 percentage points) similar to that observed in the data. The corresponding new value for σ is almost 0.9. This large change is thus compatible with that observed in our estimates.²²

Within reasonable bounds, hence, the effect of the change in the relative proportion of skilled workers is compatible with our conjecture and may have driven the change in the response of hours to technology shocks observed in the data. Similar conclusions could be drawn by considering changes in the skill-bias content of technical change. Indeed, these well documented changes in the US labor market can plausibly have an important effect on how shocks are transmitted into the economy.

7 Conclusions

We analyze the time variation of the response of hours worked to technology shocks observed in the US economy over the last 60 years. We first report evidence based on a SVAR model with long-run restrictions estimated on rolling samples. Consistent with previous results, the correlation between hours and the technological process conditional on technology shocks increases over the sample in a non-monotonic fashion. We then propose a structural interpretation of this time variation using a parsimonious RBC model with a Constant Elasticity of Substitution production function. Within

 $^{^{21}}$ For evidence on the elasticity of substitution between workers by skill level see, amongst many others, Katz and Murphy (1992), Autor, Katz and Krueger (1998), Ciccone and Peri (2005) and Autor et al. (2008). Most of these estimates range between 1.3 and 2.5, with consensus estimates around 1.5, corresponding to $\psi = 0.33$.

 $^{^{22}}$ Recently, Balleer and van Rens (2009) analyze the effect of skill-biased technology shocks on the labor market using a SVAR identification scheme. Their findings show that the response of the wage premium to investment-specific shocks is incompatible with capital-skill complementarity. Their preferred model would display a strong capital-skill substitutability such that $\theta > \psi > 0$. This would also be compatible with the results from Lemma 1. Nevertheless, we note that this would imply an aggregate σ much larger than 1, which clashes with a large body of evidence for the US where $\sigma \ll 1$. Also, this would imply a strongly pro-cyclical aggregate labor share. The correlation of the private sector labor share with output growth in the data, however, is about -0.4.

this setting, the sign of the response of hours crucially depends on the relative magnitudes of the elasticity of capital-labor substitution.

We estimated the model using Bayesian methods. For the whole sample, the proposed specification fits the postwar US data on productivity and hours worked reasonably well, especially when compared to a standard Cobb-Douglas specification. We then estimate the model on rolling samples of the same length as our SVAR and find that there is a significant sign variation of the response of hours worked to a positive technology shock. We find that the time-varying impulse responses to a labor-augmenting shock obtained from the estimated model track satisfactorily the changes observed in the data-based SVAR despite of its parsimonious nature. Such variation is driven by a change in the magnitude of the elasticity of factor substitution: we observe an increase in the elasticity of capital-labor substitution towards the end of the sample that leads to a change in the sign and size of the response of hours.

We conjecture that the observed increase in the aggregate elasticity of substitution driving our results may be associated to the changing skill composition of the labor force. With heterogeneous labor, an increase in the share of skilled workers or their relative productivity can lead to an increase in the aggregate elasticity of substitution. This highlights the importance of further research on the role of changes in the skill composition of the labor force and skill-biased technical change for the transmission of macroeconomic shocks.

Our analysis also brings two other important byproducts. First, as a first attempt to estimate the elasticity of factor substitution in a general equilibrium setup, our findings show a low estimated value around 0.15 over the full sample. We thus find little support for a Cobb-Douglas production function. A more general specification of the production side is preferred to better characterize the evolution of hours and productivity in the US economy. Second, capital-augmenting technology shocks are found to be the main driving force of the fluctuations of hours over the full sample.

A Data construction

In this section we explore the robustness of the empirical implications to alternative databases. For all databases the time span covers the period from 1948:Q1 until 2006:Q1 and times series are obtained from the FRED database. Labor productivity is computed as the ratio between the measure of output and hours, and we take logarithms of both series. We indicate with p_t^j labor productivity and with $hobs_t^j$ hours of database j. In parenthesis, we indicate the ID series in the FRED database of the Federal Reserve Bank of Saint Louis.

The first database (GG) follows closely the data construction in Galí and Gambetti (2009). We consider output in the non-farm business sector (OUTNFB), and hours of all persons in the non-farm business sector (HOANBS) and the civilian non-institutional population of 16 years and over (CNP16OV). We thus have

$$\Delta p_t^{GG} = \Delta \left(\ln \frac{OUTNFB_t}{CNP16OV_t} \right)$$

$$hobs_t^{GG} = \ln \frac{HOANBS_t}{CNP16OV_t}$$

An alternative database is considered following the work in Chang, Doh and Schorfheide (2007). We employ Average Weekly Hours of non-farm Business Sector (PRS85006023), total non-farm employees (PAYEMS), Civilian non institutional population of 20 years and over (CNP20OV = LNU000000025 (men) + LNU000000026(women)), real GDP (GDPC96). The database (CDS) is

$$\Delta p_t^{CDS} = \Delta \left(\ln \frac{GDPC96_t}{CNP20OV_t} \right)$$

$$hobs_t^{CDS} = \ln \frac{PRS85006023_t * PAYEMS_t}{CNP20OV_t}$$

The third data set is constructed following Ríos-Rull et al. (2009), where the series are similar to CDS but normalized by a different population structure, i.e. the civilian non-institutional population of 16 years and over (CNP16OV). The database (RRetal) is

$$\begin{split} \Delta p_t^{RR} &= \Delta \left(\ln \frac{GDPC96_t}{CNP16OV_t} \right) \\ hobs_t^{RR} &= \ln \frac{PRS85006023_t * PAYEMS_t}{CNP16OV_t} \end{split}$$

Finally, we borrow the last dataset form the work by Francis and Ramey (2009), where they propose a new measure of hours per capita and a new measure of productivity. Both series are adjusted for sectoral shifts and for changing in the composition of the age structure of the working population. The authors have kindly shared the data and are available on the authors web page (see http://weber.ucsd.edu/~vramey)

Figure 10 reports the growth rate of productivity and the evolution of hours worked across different data sets. While there are minor differences in the growth rate of productivity, the pattern

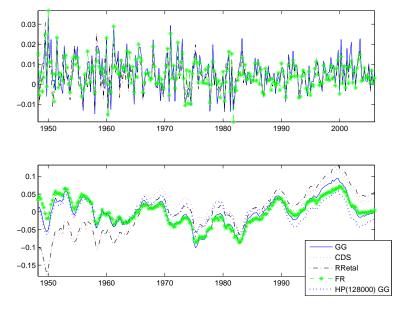


Figure 10: Growth rate of productivity and hours worked in different databases

of hour worked looks distinct across measures. In particular, the series constructed using Average Weekly Hours and total non-farm employees (as in CDS and RRelal) display a more pronounced upward trend then the one constructed in GG or in FR. This is clearly visible at the beginning and at the end of the sample. This suggests that the series display different properties at long run frequencies. To better understand the statistical features of different measures of hours worked Table 2 presents some sample moments. All measures of hours worked display very similar autoregressive

| Database | ar | sd | % of vol at BC freq | % of vol at medium term freq |
|----------|------|---------------------|---------------------|------------------------------|
| GG | 0.97 | 0.039 | 36 | 47 |
| CDS | 0.98 | 0.059 | 15 | 20 |
| RRetal | 0.99 | 0.057 | 16 | 21 |
| FR | 0.98 | 0.036 | 27 | 36 |

Table 2: AR, Standard deviations and Percentage of volatility at selected frequencies for different measures of hours worked. BC fluctuations are obtained by carving out fluctuations with a periodicity less then 32 quarters. Medium term fluctuations are obtained by carving out fluctuations with periodicity less then 48 quarters.

properties. However, there are important differences in the volatility both in terms of magnitude and in terms of location across frequencies of the spectrum. While the measures built with Average Weekly Hours and total non-farm employees are more volatile, most of their volatility is located outside Business Cycles frequencies, which is not the case for the series of hours worked constructed in GG or FR. Moreover, the measure where most of the volatility is located at typical business cycles frequencies is the measure used in GG. Hence, without a strong a priori preference for a particular

measure we tend to prefer the measure where most of the power spectrum is located between 2 and 32 quarters.

Despite of these differences, the (time varying) response of hours worked to an identified technology shock looks similar across data series. In particular, Figure 11 reports the response of hours across different settings on 38 overlapping windows of 20 years length and with the long run restriction identification scheme. While there are differences in selected sub samples, the broad picture that hours worked responded to technology negatively in early samples and positively in recent samples is consistent across different measures of hours worked.

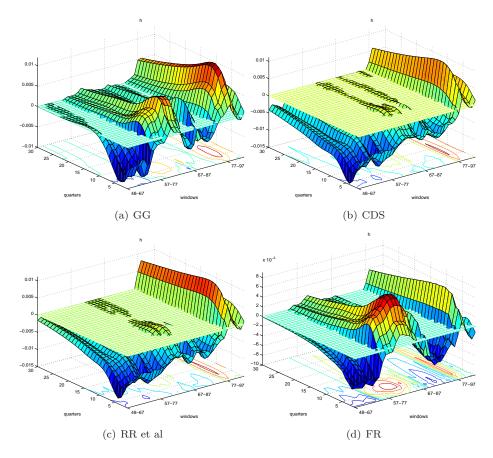


Figure 11: Impulse responses of hours to a positive labor augmenting technology shock using different database.

B Time Varying Stochastic Volatility VAR

The present section describes a VAR with time varying coefficients and with stochastic volatility. The estimation follows closely Primiceri (2005), and we borrow the specification and identification scheme form Galí and Gambetti (2009).

The reduced form Time Varying Stochastic-Volatility VAR (TVSV VAR) can be represented as

$$x_t = A_{t,0} + A_{t,1}x_{t-1} + \dots + A_{t,n}x_{t-n} + u_t$$

where $A_{t,0}$ is a time varying intercept and $A_{t,j}$ are matrices of time varying parameters. u_t are i.i.d. zero mean normal shocks with covariance matrix Σ_t . We assume that $u_t = K_t \epsilon_t$ where $\epsilon_t = [\epsilon_t^s, \epsilon_t^d]$ is a normal i.i.d. shock with $E(\epsilon_t \epsilon_t') = I$, and where ϵ_t^s is the technology shock and ϵ_t^d a non technology shock. It follows from the assumptions that $\Sigma_t = K_t K_t'$. Let α_t be the stacked vector containing the coefficients of the VAR, i.e. $\alpha_t = vec([A_{t,0}, A_{t,1}, ..., A_{t,p}])$ where vec(.) is the column stacking operator. We assume that

$$\alpha_t = \alpha_{t-1} + \eta_t^{\alpha}$$

where η_t^{α} is normally distributed with zero mean and constant variance covariance matrix Ω_{α} . We model time variation in the volatility as follows. Let $\Sigma_t = F_t D_t F_t$ where F_t is lower triangular with ones on the main diagonal and D_t is a diagonal matrix. Let ϕ_t is the stacked vector containing the non zero and non-one elements (stacked by rows) of the matrix F_t^{-1} . Recall that we are considering a 2 variables VAR, thus F_t is a 2 × 2 matrix and ϕ_t is a scalar. Let σ_t the vector containing the main diagonal of D_t . We postulate that

$$\phi_t = \phi_{t-1} + \eta_t^{\phi}$$

$$\ln \sigma_t = \ln \sigma_{t-1} + \eta_t^{\sigma}$$

where η_t^{ϕ} is normally distributed with zero mean and variance ω_{ϕ} and η_t^{σ} is normally distributed with zero mean and covariance matrix Ω_{σ} .

Priors settings and initialization follow closely Galí and Gambetti (2009) and Primiceri (2005). We use the observation form 1948:I to 1962:I to initialize the priors parametrization. The priors take the form:

$$\alpha_0 \sim N(\widehat{\alpha}_{OLS}, V(\widehat{\alpha}_{OLS}))$$

$$\phi_0 \sim N(\widehat{\phi}_{OLS}, V(\widehat{\phi}_{OLS}))$$

$$\ln \sigma_0 \sim N(\widehat{\sigma}_{OLS}, 2 * I_n)$$

$$\Omega_\alpha \sim IW(\kappa_\alpha T_0 V(\widehat{\alpha}_{OLS}), T)$$

$$\omega_\phi \sim IG(\kappa_\phi 2V(\widehat{\alpha}_{OLS}), 2)$$

$$\Omega_\sigma \sim IW(\kappa_\sigma 2I_n, 2)$$

where N is the multivariate normal distribution, IG is the inverse gamma and IW is the inverse-Wishart, $\widehat{\alpha}_{OLS}$ is the OLS estimates of the time invariant VAR coefficient in the initialization sample, and $V(\widehat{\alpha}_{OLS})$ the estimate of its covariance matrix. Similarly, $\widehat{\phi}_{OLS}$ is the OLS estimates of the

time invariant Cholesky matrix and $V(\widehat{\phi}_{OLS})$ the estimate of its covariance matrix, and $\widehat{\sigma}_{OLS}$ is the OLS estimates of the diagonal matrix D. Finally, we set $\kappa_{\alpha} = 0.005, \kappa_{\phi} = 0.1$ and $\kappa_{\sigma} = 0.005$. Figure 12 reports the impulse responses of hours worked to a technology shock starting from 1962:I

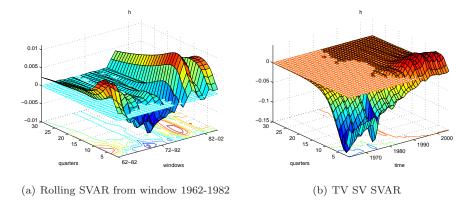


Figure 12: Impulse Responses in the rolling SVAR and in the Time Varying Stochastic Volatility SVAR

the estimate of SVAR on rolling windows and the estimates of a TVSV SVAR. The response of the TVSV SVAR are those at the beginning of each calendar year to a unit innovation in ϵ_t^s . The pattern of responses looks similar since across setups. In fact, in both cases we obtain that the response of hours to a technology shock has been increasing since 1962. While it was negative in the 60s and the 70s, it turned positive in the last part of the sample. Note that magnitudes and the degree of persistence are different since we are comparing two deeply different econometric procedures. In the TVSV SVAR the presample information is heavily affecting the negative estimates of the response of hours during the sixties. However, the broad pattern is consistent across approaches.

C RBC model with investment adjustment costs

We consider an Real Business Cycles (RBC) model with Constant Elasticity of Substitution (CES) production function and with investment adjustment costs. We model those cost so that

$$K_t = (1 - \delta)K_{t-1} + (1 - s(X_t))I_t$$

where s(1) = s'(1) = 0 and $s''(1) \neq 0$ and $X_t = I_t/I_{t-1}$; K_t is capital and I_t is investment. We assume that instantaneous utility function is given by

$$u(C_t, h_t) = \ln C_t - V_t \frac{h_t^{1+\gamma}}{1+\gamma}$$

where C_t is consumption and h_t represents hours worked. V_t is an exogenous preference process, i.e.

$$\nu_t = \ln V_t = \rho_v \nu_{t-1} + \epsilon_t^{\nu}$$

The representative consumer optimizes an infinite sum of instantaneous utility function subject to the capital accumulation equation and the period by period budget constraint, which is

$$C_t + I_t = W_t h_t + r_t K_{t-1}$$

The production side of the economy is characterized by a representative firm that minimizes the cost of factors in production subject to the production frontier is given by the CES technology, i.e.

$$Y_t = y \left[\alpha \left(\frac{Z_t^k K_{t-1}}{k} \right)^{\frac{\sigma - 1}{\sigma}} + (1 - \alpha) \left(\frac{Z_t^h h_t}{h} \right)^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}}$$
(C.1)

The CES production function encompasses two types technological change, i.e. the capital augmenting, Z_t^k , and the labor augmenting technological process, Z_t^h . We assume that capital-augmenting technology has an AR(1) representation, i.e. (in log deviations from the steady state)

$$z_t^k = \ln Z_t^k = \rho_k z_{t-1}^k + \eta_t^k \quad \eta_t^k \sim N(0, \sigma_k),$$
 (C.2)

where $\rho_k < 1$ to ensure the existence of a balanced growth path. For the labor-augmenting shock we adopt a flexible specification similar to Ríos-Rull et al. (2009), where it follows an AR(2) process, i.e.

$$z_t^h = \ln Z_t^h = \psi_{1,h} (1 - \psi_{2,h}) z_{t-1}^h + \psi_{2,h} z_{t-2}^h + \eta_t^h, \tag{C.3}$$

By the perfect competition assumption, factors are paid their marginal products and hence the system of equilibrium condition is given by

$$\begin{split} V_t h_t^{\gamma} &= \frac{W_t}{C_t} \\ 1 &= q_t \left(1 - s(X_t) - s'(X_t) X_t \right) + \beta E_t \left(q_{t+1} \frac{C_t}{C_{t+1}} s'(X_{t+1}) X_{t+1}^2 \right) \\ q_t &= E_t \beta \frac{C_t}{C_{t+1}} \left(r_{t+1} + q_{t+1} (1 - \delta) \right) \\ Y_t &= y \left[\alpha \left(\frac{Z_t^k K_{t-1}}{k} \right)^{\frac{\sigma - 1}{\sigma}} + (1 - \alpha) \left(\frac{Z_t^h h_t}{h} \right)^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}} \\ W_t &= (1 - \alpha) \left(Z_t^h \frac{y}{h} \right)^{\frac{\sigma - 1}{\sigma}} \left(\frac{Y_t}{h_t} \right)^{\frac{1}{\sigma}} \\ r_t &= \alpha \left(Z_t^k \frac{y}{k} \right)^{\frac{\sigma - 1}{\sigma}} \left(\frac{Y_t}{K_{t-1}} \right)^{\frac{1}{\sigma}} \\ K_t &= (1 - \delta) K_{t-1} + (1 - s(X_t)) I_t \\ Y_t &= C_t + I_t \\ X_t &= I_t / I_{t-1} \end{split}$$

where q_t is the Tobin q, or equivalently the ratio between the multipliers of the household constraints. Assuming $\psi_1 = 1$, the Z_t^h is not stationary and the dynamic of the model are explosive. In order to have a well defined steady state, we need to re-scale the real variables by the non stationary process. Indicating the variables in small letters as re-scaled variables, i.e. $s_t = S_t/Z_t^h$ for S = C, Y, K, W, I, X, we obtain

$$\begin{split} V_{t}h_{t}^{\gamma} &= \frac{w_{t}}{c_{t}} \\ 1 &= q_{t} \left(1 - s(x_{t} \frac{Z_{t}^{h}}{Z_{t-1}^{h}}) - s'(x_{t} \frac{Z_{t}^{h}}{Z_{t-1}^{h}}) x_{t} \frac{Z_{t}^{h}}{Z_{t-1}^{h}} \right) + \beta E_{t} \left(q_{t+1} \frac{c_{t}}{c_{t+1}} \frac{Z_{t}^{h}}{Z_{t}^{h}} s' \left(x_{t} \frac{Z_{t+1}^{h}}{Z_{t}^{h}} \right) \left(x_{t+1} \frac{Z_{t+1}^{h}}{Z_{t}^{h}} \right)^{2} \right) \\ q_{t} &= E_{t} \beta \frac{c_{t}}{c_{t+1}} \frac{Z_{t}^{h}}{Z_{t+1}^{h}} \left(r_{t+1} + q_{t+1} (1 - \delta) \right) \\ y_{t} &= y \left[\alpha \left(\frac{Z_{t}^{k} k_{t-1}}{k} \frac{Z_{t-1}^{h}}{Z_{t}^{h}} \right)^{\frac{\sigma-1}{\sigma}} + (1 - \alpha) \left(\frac{h_{t}}{h} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \\ w_{t} &= (1 - \alpha) \left(\frac{y}{h} \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{y_{t}}{h_{t}} \right)^{\frac{1}{\sigma}} \\ r_{t} &= \alpha \left(Z_{t}^{k} \frac{y}{k} \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_{t}}{K_{t-1}} \frac{Z_{t}^{h}}{Z_{t-1}^{h}} \right)^{\frac{1}{\sigma}} \\ k_{t} &= (1 - \delta) k_{t-1} \frac{Z_{t}^{h}}{Z_{t}^{h}} + (1 - s(x_{t} \frac{Z_{t}^{h}}{Z_{t-1}^{h}})) i_{t} \\ y_{t} &= c_{t} + i_{t} \\ x_{t} &= i_{t} / i_{t-1} \end{split}$$

At the non stochastic steady state we have $h^{\gamma} = \frac{w}{c}$, 1 = q, $1 = \beta (r + 1 - \delta)$, y = y, $w = (1 - \alpha) \frac{y}{h}$, $r = \alpha \frac{y}{k}$, $i = \delta k$, y = c + i, x = 1. The log linearized equilibrium conditions, around the non-stochastic steady-state, of the variables rescaled by the non stationary process are (with a small abuse of notation we indicate with small case letter the log deviation of a variable from its steady states)

$$y_t = i/y \ i_t + c/y \ c_t \tag{C.4}$$

$$k_t = (1 - \delta)(k_{t-1} - z_t^h + z_{t-1}^h) + \delta i_t \tag{C.5}$$

$$y_t = \alpha k_{t-1} + \alpha z_t^k + (1 - \alpha) h_t - \alpha (z_t^h - z_{t-1}^h)$$
(C.6)

$$q_t = c_t - c_{t+1} - z_{t+1}^h + z_t^h + \beta r r_{t+1} + \beta (1 - \delta) q_{t+1}$$
(C.7)

$$q_t = s''(1)(x_t + (z_t^h - z_{t-1}^h)) - \beta s''(1)(x_{t+1} + (z_{t+1}^h - z_t^h))$$
 (C.8)

$$w_t = v_t + \gamma h_t + c_t \tag{C.9}$$

$$w_t = 1/\sigma(y_t - h_t) \tag{C.10}$$

$$r_t = (\sigma - 1)/\sigma z_t^k + 1/\sigma (y_t - k_{t-1}) + 1/\sigma (z_t^k - z_{t-1}^k)$$
(C.11)

$$x_t = i_t - i_{t-1} (C.12)$$

C.1 A priori sensitivity analysis

By means of a sensible calibration exercise, we can study the impact of a labor augmenting technology shock on hours worked for different values of the capital-labor elasticity and of investment adjustment cost. Without loss of generality, we assume that the labor augmenting technology process is non stationary, i.e. $\psi_{1,h} = 1$. Moreover, we set the time discount factor, β , to 0.99, and the depreciation rate, δ , to 0.025, and the inverse of the Frisch elasticity, γ , to 1. We let the capital-labor elasticity vary between 0.1 and 1, and we fix the capital intensity in production to 0.33 and assume no investment adjustment cost. Figure 13 (left panel) reports the impulse response of hours worked to

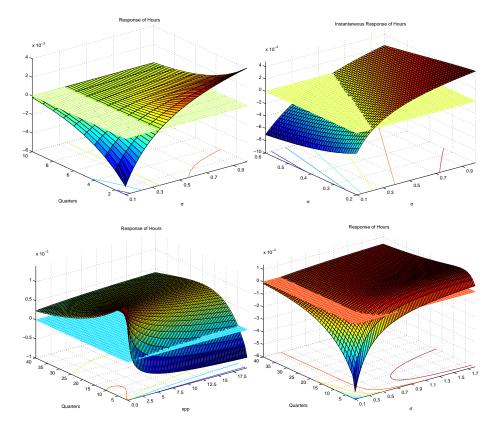


Figure 13: Impulse response of hours worked to a labor-augmenting technology shock for different values of σ and $\alpha=0.33$ (Left top panel). Instantaneous response of hours worked to a labor-augmenting technology shock for different values of σ and α (Right top Panel). Impulse response of hours worked to a labor-augmenting technology shock for different values of s'' with $\sigma \to 1$ and $\alpha=0.33$ (Left bottom panel). Instantaneous response of hours worked to a labor-augmenting technology shock for different values of σ and s''=2 and $\alpha=0.33$ (Right bottom Panel).

a labor-augmenting technology shock for different values of σ and keeping the value of α fixed at 0.33. Approximately, when $\sigma > \alpha$ the response of hours to a labor augmenting technology shock is positive. However, hours worked decrease if $\sigma < \alpha$. The right panel of Figure 13 displays the instantaneous response of hours worked to a labor-augmenting technology shock for different values

of σ and α . We let the value of capital intensity vary between 0.2 to 0.6. Thus, approximatively for values of σ larger than 0.7 and close to the Cobb-Douglas specification, the response of hours is positive regardless of values of α .

We considered also the case where the production function is Cobb-Douglas ($\sigma \to 1$) and the capital adjustment cost vary from 0 to 20. A few things are worth noticing. First, there is a switch of sign in the response of hours due to a change in investment adjustment cost. However, this change in sign occurs only on impact. Indeed, regardless of the value of the investment adjustment cost, the response of hours turns positive after few quarters. Hence, while investment adjustment cost are able to generate negative response of hours on impact, they are unable to produce a long-lasting negative response of hours worked to a technology shock ²³. Second, with positive adjustment costs the capital-labor degree of substitution is the crucial parameter which generates a long lasting positive or negative response of hours. However, the threshold is no longer uniquely determined by value of α . Third, the support of σ able to generate positive and negative response of hours has increased (0-1 without investment adjustment costs and 0-2 with investment adjustment costs).

D Identification of σ

This section verifies whether data can carry enough information to pin down σ in estimation.²⁴ Without loss of generality, we assume that the model is stationary, i.e. $0 < \psi_{1,h} < 1$ and $\psi_{2,h} = 0$. We simulate 100 observations for output and productivity assuming that $\alpha = 0.4$ and $\sigma = 0.2$ in one case (Case A) and that $\alpha = 0.4$ and $\sigma = 0.99$ in the other case (Case B). We then estimate the structural parameters of the model using Bayesian techniques. Prior elicitation is pretty standard. We assume inverse gamma for standard deviations, beta distributions for the autoregressive parameters, a normal distribution for the inverse of the Frisch elasticity, γ and for the capital intensity in production, α . All priors are centered at the true values. For the capital-labor elasticity of substitution, σ , we assume a uniform prior with 0 and 1.5 as boundaries. Posterior medians and credible sets are reported in Table 3. Data on productivity and hours worked appear informative about the parameters and shocks of interest. Typically, posterior credible sets include the parameter value used to simulate data. Even for σ , where we postulate a flat prior, the likelihood peaks very close to the true population value meaning that, if the model is correctly specified, we are able to pin down in estimation the parameters governing the CES production function.

Since the estimate of these parameters are not far off the 'true' population value, and since

 $^{^{23}}$ This result is insensitive to different calibration of the remaining structural parameters and also introducing endogenous persistence with habits in consumption. We obtain similar results for $\gamma = \{0.5, 1.0, 1.5, 2, 2.5, 3.0\}$ and for different parametrization of the exogenous processes.

 $^{^{24}}$ The estimation of σ presents some econometric challenges, especially when combined with estimates of factor-augmenting technical change. See León-Ledesma et al. (2010).

| | true | Case A | Case B |
|------------------|------------|-----------------------------|----------------------------|
| α | 0.40 | 0.39[0.31, 0.47] | 0.41 [0.35,0.48] |
| σ | 0.20/0.99 | $0.27 \ [0.18, 0.37]$ | 1.13 [0.86, 1.48] |
| γ | 1.00 | $1.01 \ [0.85, 1.18]$ | $1.00 \ [0.85, 1.15]$ |
| $ ho_v$ | 0.60 | $0.61 \ [0.43, 0.77]$ | $0.52 \ [0.42, 0.62]$ |
| $ ho_k$ | 0.60 | $0.49 \ [0.34, 0.63]$ | $0.60 \ [0.36, 0.83]$ |
| $\psi_{1,h}$ | 0.60 | $0.65 \ [0.56, 0.75]$ | $0.61 \ [0.46, 0.76]$ |
| σ_v | 0.01 | 0.0110 [0.0095, 0.0125] | $0.0093 \ [0.0064, 0.012]$ |
| σ_k | 0.01 | $0.0090 \ [0.0066, 0.0102]$ | $0.0108 \ [0.066, 0.0149]$ |
| σ_h | 0.01 | $0.0096 \ [0.0061, 0.0128]$ | $0.0094 \ [0.008, 0.0108]$ |
| | | | |
| $corr(p_t, h_t)$ | -0.15/0.15 | -0.30 [-0.06,-0.49] | 0.14 [-0.04,0.29] |

Table 3: Prior and Posterior estimates with simulated data. Median and the credible sets in parenthesis.

their relative magnitude determines the sign of the conditional and unconditional correlations, we expect to be able to track the correct sign of such correlations. In particular, the last row of Table 3 displays the unconditional correlation between hours and productivity and its estimates. On average, the signs are correctly identified for both cases. Similarly, Figure 14 reports the (true and estimates) impulse response of hours worked to a labor-augmenting technology shock. The estimated impulse response correctly captures the the sign and the persistence of the response. Regardless of the relative magnitude of σ and α , the response of productivity to a labor-augmenting technology shocks is positive and correctly estimated (not shown here). Hence, the sign of the correlation of hours and productivity conditional on an labor-augmenting technology shock crucially depends on the estimated relative magnitude of σ and α . We conclude that data on productivity and hours worked contain enough information to correctly capture conditional and unconditional moments of productivity and hours worked in our model.

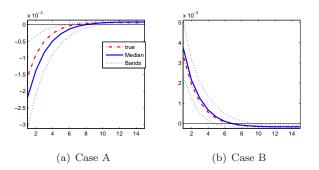


Figure 14: Impulse response (true and estimates) of hours worked to a labor-augmenting technology shock. Left panel Case A, where $0.2 = \sigma < \alpha = 0.4$ and right panel Case B, where $0.4 = \alpha < \sigma = 0.99$.

 $^{^{25}}$ The bands of the estimated correlation are obtained by simulating 50 times the model using the mean, and for each simulated data sets we compute the correlation.

E Proof of Lemma 1

We prove Lemma 1 for the first nesting corresponding to the Krusell et al. (2000) two-level CES, which we denote as (X, U) nesting. The results for the second nesting easily follow through from these. Hence, for this nesting, we first need to use the following results:

$$\frac{\partial \pi_X}{\partial X} = \frac{\psi}{X} \pi_X (1 - \pi_X),\tag{E.1}$$

$$\frac{\partial X}{\partial (1-n)} = \frac{1-\pi_K}{1-n}X,\tag{E.2}$$

$$\frac{\partial \pi_K}{\partial (1-n)} = -\theta \frac{\pi_K (1-\pi_K)}{1-n},\tag{E.3}$$

which, since, for any variables (z, q, s), $\partial z/\partial q = (\partial z/\partial s)(\partial s/\partial q)$, immediately implies

$$\frac{\partial \pi_X}{\partial (1-n)} = \psi \frac{\pi_X (1-\pi_X)(1-\pi_K)}{1-n}.$$
 (E.4)

With these results we can then calculate the partial derivative of $g = \frac{\pi_K}{\frac{1-\pi_K}{1-\pi_X} + \pi_K}$. After some tedious algebra, we can write this expression as:

$$\frac{\partial g}{\partial (1-n)} \Big|_{X,U} = \frac{-\frac{\pi_K (1-\pi_K)}{(1-n)(1-\pi_X)} (\theta + \psi)}{\left[\frac{1-\pi_K}{1-\pi_X} + \pi_K\right]^2}, \tag{E.5}$$

Since $\frac{\partial \sigma}{\partial (1-n)} = \frac{\partial \sigma}{\partial g} \frac{\partial g}{\partial (1-n)}$ we can derive, again after some algebra, the expression:

$$\left. \frac{\partial \sigma}{\partial (1-n)} \right|_{XU} = -\Pi \frac{\left(\psi^2 - \theta^2\right)}{\left[\left(1 - \theta\right) + \left(\theta - \psi\right)q\right]^2},\tag{E.6}$$

where $\Pi > 0$ is a function of share parameters:

$$\Pi = \frac{\pi_K (1 - \pi_K)}{(1 - n)(1 - \pi_X) \left[\frac{1 - \pi_K}{1 - \pi_X} + \pi_X \right]^2}$$
 (E.7)

Given that $\Pi > 0$ and that the denominator of (E.6) is positive, the effect of a change in 1 - n will be positive if $\theta^2 > \psi^2$. Hence, in the (X, U) nesting, an increase in the share of skilled workers will increase aggregate σ if $|\theta| > |\psi|$. Following the same logic, in the (X, S) nesting, the effect will be positive as long as $|\theta| < |\psi|$.

References

- Acemoglu, D.: 2002, Technical Change, Inequality and the Labor Market, *Journal of Economic Literature* **40**(1), 7–72.
- Acemoglu, D., Aghion, P. and Violante, G. L.: 2001, Deunionization, technical change and inequality, Carnegie-Rochester Conference Series on Public Policy 55(1), 229–264.
- Acemoglu, D. and Autor, D. H.: 2011, Skills, tasks and technologies: Implications for employment and earnings, in O. Ashenfelter and D. E. Card (eds), Handbook of Labor Economics Volume 4, Amsterdam: Elsevier.
- Álvarez-Cuadrado, F. and Van Long, N.: 2011, Capital-labor substitution, structural change and growth, *mimeo*, McGill University.
- Autor, D. H., Katz, L. F. and Kearney, M. S.: 2008, Trends in u.s. wage inequality: Revising the revisionists, *The Review of Economics and Statistics* **90**(2), 300–323.
- Autor, D. H., Katz, L. F. and Krueger, A. B.: 1998, Computing inequality: Have computers changed the labor market?, The Quarterly Journal of Economics 113(4), 1169–1213.
- Balleer, A. and van Rens, T.: 2009, Cyclical Skill-Biased Technological Change, *IZA Discussion Papers 4258*, Institute for the Study of Labor (IZA).
- Blanchard, O. J. and Quah, D.: 1989, The Dynamic Effects of Aggregate Demand and Supply Disturbances, *American Economic Review* **79**(4), 655–73.
- Canova, F. and Gambetti, L.: 2009, Structural changes in the US economy: Is there a role for monetary policy?, Journal of Economic Dynamics and Control 33(2), 477–490.
- Canova, F., Lopez-Salido, D. and Michelacci, C.: 2010, The Effects of Technology Shocks on Hours and Output: A Robustness Analysis, *Journal of Applied Econometrics* 25(5), 755–773.
- Cantore, C., León-Ledesma, M. A., McAdam, P. and Willman, A.: 2010, Shocking stuff: technology, hours, and factor substitution, *Working Paper Series* 1278, European Central Bank.
- Chang, Y., Doh, T. and Schorfheide, F.: 2007, Non-stationary Hours in a DSGE Model, *Journal of Money, Credit, and Banking* **69**(6), 1357–1373.
- Chari, V., Kehoe, P. J. and McGrattan, E. R.: 2008, Are structural VARs with long-run restrictions useful in developing business cycle theory?, *Journal of Monetary Economics* **55**(8), 1337–1352.
- Chirinko, R. S.: 2008, σ : The Long and Short of It, Journal of Macroeconomics 30(2), 671–686.

- Ciccone, A. and Peri, G.: 2005, Long-Run Substitutability Between More and Less Educated Workers: Evidence from U.S. States, 1950-1990, *The Review of Economics and Statistics* 87(4), 652–663.
- Clarida, R., Galí, J. and Gertler, M.: 2000, Monetary Policy Rules And Macroeconomic Stability: Evidence And Some Theory, *The Quarterly Journal of Economics* **115**(1), 147–180.
- Cogley, T. and Sargent, T. J.: 2005, Drift and Volatilities: Monetary Policies and Outcomes in the Post WWII U.S, *Review of Economic Dynamics* 8(2), 262–302.
- Fernald, J. G.: 2007, Trend breaks, long-run restrictions, and contractionary technology improvements, *Journal of Monetary Economics* **54**(8), 2467–2485.
- Fernndez-Villaverde, J. and Rubio-Ramirez, J. F.: 2008, How Structural Are Structural Parameters?, NBER Macroeconomics Annual 2007, Volume 22, NBER Chapters, National Bureau of Economic Research, Inc, pp. 83–137.
- Francis, N. and Ramey, V. A.: 2009, Measures of per capita hours and their implications for the technology-hours debate, *Journal of Money, Credit and Banking* 41(6), 1071–1097.
- Galí, J.: 1999, Technology, employment, and the business cycle: Do technology shocks explain aggregate fluctuations?, *American Economic Review* **89**(1), 249–271.
- Galí, J. and Gambetti, L.: 2009, On the Sources of Great Moderation, American Economic Journal: Macroeconomics 1(1), 26–57.
- Galí, J., Lopez-Salido, J. D. and Valles, J.: 2003, Technology shocks and monetary policy: assessing the Fed's performance, *Journal of Monetary Economics* **50**(4), 723–743.
- Galí, J. and Rabanal, P.: 2005, Technology Shocks and Aggregate Fluctuations: How Well Does the Real Business Cycle Model Fit Postwar U.S. Data?, NBER Macroeconomics Annual 2004, Volume 19, NBER Chapters, National Bureau of Economic Research, Inc, pp. 225–318.
- Galí, J. and van Rens, T.: 2010, The Vanishing Procyclicality of Labor Productivity, mimeo, CREI.
- Gambetti, L.: 2006, Time-Varying Dynamics Matter: Technology Shocks and the Response of Hours Worked. Mimeo.
- Geweke, J.: 1999, Using simulation methods for bayesian econometric models: inference, development, and communication, *Econometric Reviews* **18**(1), 1–73.
- Hansen, B. E.: 2001, The New Econometrics of Structural Change: Dating Breaks in U.S. Labor Productivity, *The Journal of Economic Perspectives* **15**(4), 117–128.

- Hicks, J. R.: 1932, The Theory of Wages, 1st Edition, London: MacMillan.
- Jeffries, H.: 1996, Theory of Probability, Oxford: Clarendon Press. Third Edition.
- Kahn, J. A. and Rich, R. W.: 2007, Tracking the new economy: Using growth theory to detect changes in trend productivity, *Journal of Monetary Economics* **54**(6), 1670–1701.
- Kass, R. E. and Raftery, A. E.: 1995, Bayes factors, *Journal of the American Statistical Association* **90**(430), pp. 773–795.
- Katz, L. and Murphy, K.: 1992, Changes in relative wages, 1963-1987: Supply and demand factors, Quarterly Journal of Economics 107, 35–78.
- Kim, C.-J. and Nelson, C. R.: 1999, Has The U.S. Economy Become More Stable? A Bayesian Approach Based On A Markov-Switching Model Of The Business Cycle, *The Review of Economics* and Statistics 81(4), 608–616.
- Krusell, P., Ohanian, L. E., Ríos-Rull, J.-V. and Violante, G. L.: 2000, Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis, *Econometrica* **68**(5), 1029–1054.
- La Grandville, Olivier de.: 1989, In Quest of the Slutzky Diamond, American Economic Review 79, 468–481.
- León-Ledesma, M. A., McAdam, P. and Willman, A.: 2010, Identifying the Elasticity of Substitution with Biased Technical Change, *American Economic Review* **100**(4), 1330–1357.
- McConnell, M. M. and Perez-Quiros, G.: 2000, Output Fluctuations in the United States: What Has Changed since the Early 1980's?, *American Economic Review* **90**(5), 1464–1476.
- Miyagiwa, K. and Papageorgiou, C.: 2007, Endogenous aggregate elasticity of substitution, *Journal of Economic Dynamics and Control* **31**(9), 2899–2919.
- Nucci, F. and Riggi, M.: 2009, The Great Moderation and Changes in the Structure of Labor Compensation, *Working Papers 124*, Sapienza University of Rome, Department of Public Economics.
- Papageorgiou, C. and Saam, M.: 2008, Two-level ces production technology in the solow and diamond growth models, *Scandinavian Journal of Economics* **110**(1), 119–143.
- Priestley, M.: 1982, Spectral analysis and time series, number v. 1-2 in Probability and mathematical statistics, Academic Press.
- Primiceri, G. E.: 2005, Time varying structural vector autoregressions and monetary policy, *Review of Economic Studies* **72**(3), 821–852.

- Ríos-Rull, J.-V., Schorfheide, F., Fuentes-Albero, C., Kryshko, M. and Santaeulália-Llopis, R.: 2009, Methods versus Substance: Measuring the Effects of Technology Shocks on Hours, NBER Working Papers 15375, National Bureau of Economic Research, Inc.
- Roberts, J. M.: 2001, Estimates of the Productivity Trend Using Time-Varying Parameter Techniques, *The B.E. Journal of Macroeconomics* **0**(1).
- Rotemberg, J. J.: 2008, Cyclical Wages in a Search-and-Bargaining Model with Large Firms, *NBER International Seminar on Macroeconomics 2006*, NBER Chapters, National Bureau of Economic Research, Inc, pp. 65–114.
- Sargent, T. J. and Wallace, N.: 1974, The Elasticity of Substitution and Cyclical Behavior of Productivity, Wages, and Labor's Share, *American Economic Review* **64**(2), 257–63.
- Stiroh, K. J.: 2009, Volatility accounting: A production perspective on increased economic stability, Journal of the European Economic Association 7(4), 671–696.
- Stock, J. H. and Watson, M. W.: 2003, Has the Business Cycle Changed and Why?, NBER Macroe-conomics Annual 2002, Volume 17, NBER Chapters, National Bureau of Economic Research, Inc, pp. 159–230.
- Whelan, K. T.: 2009, Technology shocks and hours worked: Checking for robust conclusions, *Journal of Macroeconomics* **31**(2), 231–239.

Documents de Travail

- 330. E. Challe and C. Giannitsarou, "Stock Prices and Monetary Policy Shocks: A General Equilibrium Approach,"
 June 2011
- 331. M. Lemoine, M.E. de la Serve et M. Chetouane, "Impact de la crise sur la croissance potentielle : une approche par les modèles à composantes inobservables," Juillet 2011
- 332. J. Bullard and J. Suda, "The stability of macroeconomic systems with Bayesian learners," July 2011
- 333. V. Borgy, X. Chojnicki, G. Le Garrec and C. Schwellnus, "Macroeconomic consequences of global endogenous migration: a general equilibrium analysis," July 2011
- 334. M. Kejriwal and C. Lopez, "Unit roots, level shifts and trend breaks in per capita output: a robust evaluation," July 2011
- 335. J. Ortega and G. Verdugo, "Immigration and the occupational choice of natives: a factor proportons approach," July 2011
- 336. M. Bussiere, A. Chudik and A. Mehl, "How have global shocks impacted the real effective exchange rates of individual euro area countries since the euro's creation?," July 2011
- 337. J. F. Hoarau, C. Lopez and M. Paul, "Short note on the unemployment of the "french overseas regions," July 2011
- 338. C. Lopez, C. J. Murray and D. H. Papell, "Median-unbiased estimation in DF-GLS regressions and the PPP puzzle," July 2011
- 339. S. Avouyi-Dovi and J. Idier, "The impact of unconventional monetary policy on the market for collateral: The case of the French bond market," August 2011
- 340. A. Monfort and J-P. Renne, "Default, liquidity and crises: an econometric framework," August 2011
- 341. R. Jimborean, "The Exchange Rate Pass-Through in the New EU Member States," August 2011
- 342. M.E. de la Servey and M. Lemoine, "Measuring the NAIRU: a complementary approach," September 2011
- 343. A. bonleu, G. Cette, and G. Horny, "Capital Utilisation and Retirement," September 2011
- 344. L. Arrondel, F. Savignac and K. Tracol, "Wealth effects on consumption plans: french households in the crisis," September 2011
- 345. M. Rojas-Breu, "Debt enforcement and the return on money," September 2011
- 346. F. Daveri, R. Lecat and M.L. Parisi, "Service deregulation, competition and the performance of French and Italian firms," October 2011
- 347. J. Barthélemy and M. Marx, "State-dependent probability distributions in non linear rational expectations models," October 2011
- 348. J. Idier, G. Lamé and J.-S. Mésonnier, "How useful is the marginal expected shortfall for the measurement of systemic exposure? A practical assessment," October 2011
- 349. V. Fourel and J. Idier, "Risk aversion and uncertainty in European sovereign bond markets," October 2011
- 350. V. Borgy, T. Laubach, J-S. Mésonnier and J-P. Renne, "Fiscal Sustainability, Default Risk and Euro Area Sovereign Bond Spreads," October 2011
- 351. C. Cantore, F. Ferroni and M. A. León-Ledesma, "Interpreting the Hours-Technology time-varying relationship," November 2011

Pour accéder à la liste complète des Documents de Travail publiés par la Banque de France veuillez consulter le site : http://www.banque-france.fr/fr/publications/revues/documents-de-travail/htm/accueil-documen

For a complete list of Working Papers published by the Banque de France, please visit the website: http://www.banque-france.fr/gb/publications/ner/ner_11.htm

Pour tous commentaires ou demandes sur les Documents de Travail, contacter la bibliothèque de la Direction Générale des Études et des Relations Internationales à l'adresse suivante :

For any comment or enquiries on the Working Papers, contact the library of the Directorate General Economics and International Relations at the following address:

BANQUE DE FRANCE 49- 1404 Labolog 75049 Paris Cedex 01

tél: 0033 (0)1 42 97 77 24 ou 01 42 92 63 40 ou 48 90 ou 69 81

email: 1404-ut@banque-france.fr