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Abstract: This paper proposes a two-regime Bounce-Back Function augmented Self-Exciting Threshold AutoRegression (SETAR) model which allows for various shapes of recoveries from the recession regime. It relies on the bounce-back effects first analyzed in a Markov-Switching setup by Kim, Morley and Piger [2005] and recently extended by Bec, Bouabdallah and Ferrara [2011]. This approach is then applied to post-1973 quarterly growth rates of French, German, Italian, Spanish and Euro area real GDPs. Both the linear autoregression and the standard SETAR without bounce-back effect null hypotheses are strongly rejected against the Bounce-Back augmented SETAR alternative in all cases but Italy. The relevance of our proposed model is further assessed by the comparison of its short-term forecasting performances with the ones obtained from a linear autoregression and a standard SETAR. It turns out that the bounce-back models one-step ahead forecasts generally outperform the other ones, and particularly so during the last recovery period in 2009Q3-2010Q4.

Keywords: Threshold autoregression, bounce-back effects, asymmetric business cycles.
JEL classification: E32, C22.

Résumé: Dans ce papier, nous proposons une nouvelle extension d'un modèle à seuil de type SETAR (Self-Exciting Threshold AutoRegression) à deux régimes qui intègre une fonction de rebond permettant de reproduire les différentes formes de reprise en sortie d'une phase de récession. Cette approche repose sur un modèle introduit par Kim, Morley et Piger [2005] dans le cadre de changements de régimes markoviens et récemment étendu par Bec, Bouabdallah et Ferrara [2011]. Dans ce travail, nous appliquons ce modèle à seuil étendu par une fonction de rebond à des séries de taux de croissance trimestriel du PIB réel pour les quatre grandes économies de la zone euro, à savoir Allemagne, France Italie et Espagne, depuis 1973. Dans un premier temps, les tests statistiques implémentés rejettent fortement l'hypothèse nulle de linéarité, ainsi que celle de modèle à seuil standard, contre l'alternative de modèle à seuil étendu par une fonction de rebond, excepté pour l'Italie. Puis, dans un second temps, la capacité prédictive de notre modèle est évaluée par comparaison avec celle d'un modèle linéaire et d'un modèle à seuil standard. Les résultats obtenus montrent que le modèle à seuil étendu par une fonction de rebond améliore la précision des prévisions, en particulier pendant la période de reprise T3 2009 - T4 2010.

Mots-Clés: Modèles à seuil, effets de rebond, cycle des affaires asymétriques
Classification JEL: E32, C22.

Introduction

Since the early contributions by e.g. Neftci [1984], Hamilton [1989], Luukkonen and Terasvirta [1991], Anderson and Terasvirta [1991] or Beaudry and Koop [1993], the asymmetric dynamics of real output growth over the business cycle has been widely acknowledged by empirical studies. Evidence of long and soft expansion epochs followed by short and sharp recession times is generally found in nonlinear empirical work. Nevertheless, such a crude two-phase characterization of the business cycle may be too restrictive. This view is supported by more recent analysis both in a Markov-Switching (MS hereafter) framework (as in e.g. Sichel [1994] or Clements and Krolzig [1998])) or from threshold models (for instance in Tiao and Tsay [1994], Pesaran and Potter [1997], Van Dijk and Franses [1999] or Kapetanios [2003]). These studies share the feature of introducing at least one additional regime. More precisely, most of them retain a three-regime framework in which the expansion phase is decomposed in a high-growth recovery phase immediately following the trough of a cycle and a subsequent moderate-growth phase.[4] This "bounce-back" phenomenon has been put forward by Sichel [1994] for US real output data and confirmed in Kim, Morley and Piger [2005].

These authors propose an extension of the two-regime Markov-switching which allows for such bounce-back effects, without introducing a third regime. Beyond the parsimony of their proposed two-regime specification, it has also the desirable feature of allowing the bounce-back effect to depend on the duration and/or the depth of the previous recession, which is not the case of the multiple regimes models mentioned above. Recently, Bec, Bouabdallah and Ferrara [2011a] propose a generalization of the bounce-back functions used by Kim et al. [2005] which allows for more flexible shapes of recoveries as well as for simple statistical testing of specific shapes.

The main contribution of this paper is to develop a two-regime Self Exciting Threshold Auto-Regressive (SETAR) model allowing for this general bounce-back function. Actually, this threshold class of non-linear models has mainly two advantages compared to the MS class of models. First, contrary to the MS model whose maximum likelihood estimation outcome might depend heavily on the choice of the parameters values initialization, the SETAR model's estimates can be easily obtained by the non-linear least

[4] Tiao and Tsay [1994] consider a four-regime SETAR model allowing for worsening/improving recession/expansion.

squares method. Second, when modeling the output growth rate from a SETAR framework, the switching variable which governs the regime switches is the lagged output growth rate itself and hence is perfectly observable, contrary to the unobserved state variable in the MS model. Consequently, the SETAR class of models allows the regime switches to depend explicitly on the business cycle state. This probably explains the co-existence of both classes of models since more than two decades.

We then present linearity tests as well as specific recovery shapes tests against our general bounce-back augmented SETAR model alternative. When applied to French, German, Italian, Spanish and European (Euro Area) post-1973 quarterly real GDP growth rates, it turns out that the linear null hypothesis is strongly rejected in all cases but Italy. Similarly, the null of no bounce-back effect is strongly rejected in the four remaining cases. Moreover, according to our tests results, the same shape of recoveries is retained for France, Germany, Spain and the Euro Area. The relevance of our proposed model is further confirmed by comparing its short-term forecast accuracy with the ones obtained from linear or standard SETAR models.

The paper is organized as follows. Section 1 presents and discusses the bounce-back extensions of the SETAR model. Section 2 describes the data and presents the linearity test before reporting the bounce-back models estimation results. Section 3 presents the short-term forecasts evaluation exercise and Section 4 concludes.

1 A two-regime SETAR with bounce-back effects

1.1 The basic SETAR model

Let y_t denote the log of real output and Δy_t its growth rate. The basic SETAR model we will consider throughout this paper is the following:

$$\Phi(L)(\Delta y_t - \mu_t) = e_t, \tag{1}$$

with μ_t defined by:

$$\mu_t = \gamma_0(1 - s_t) + \gamma_1 s_t, \tag{2}$$

and where $\Phi(L)$ is a lag polynomial of order p with roots lying outside the unit circle and e_t i.i.d. $\mathcal{N}(0,\sigma)$. Let s_t denote the transition function which takes on the value zero or one. In our SETAR model, s_t is defined as:

$$s_t = 0 \text{ if } \Delta y_{t-1} > \kappa \text{ and } 1 \text{ otherwise.} \quad (3)$$

The model given by equations (1) to (3) allows for an asymmetric behavior across regimes. It implies that the intercept in equation (1) is γ_0 if Δy_{t-1} is larger than the threshold κ and γ_1 otherwise. Here, $s_t = 1$ is identified as the recession regime by assuming $\kappa < 0$.

1.2 Introducing bounce-back functions

As stressed in the introduction, the main drawback of the basic two-regime SETAR model presented above is that it precludes any high-growth phase following a trough before switching back to the moderate-growth phase. For this reason, a multiple regime extension of this model was considered in e.g. Tiao and Tsay [1994], Pesaran and Potter [1997] or Kapetanios [2003]. Nevertheless, this approach is not parsimonious and one could soon run out of degrees of freedom when analyzing most macroeconomic time series. This is particularly true for the topic under consideration here since, as suggested by Sichel [1994] or Kim et al. [2005], the high-growth rate phase seems to be rather short on average — with a duration shorter than two years. Yet, it is necessary that enough observations belong to each regime to get accurate estimates of the regime-dependent parameters.

Recently, Kim et al. [2005] and Bec et al. [2011a] have proposed extensions, within the two-regime class of MS models proposed by Hamilton [1989], which allow for the length and/or depth of each recession to influence the growth rate of output in the periods immediately following the recession. Kim et al. [2005] consider three kinds of bounce-back functions, which correspond respectively to "U"- or "V"- shaped recessions, or "Depth" non-linear bounce-back models. Bec et al. [2011a] develop a more general bounce-back frame, hereafter denoted BBF, which includes the "U", "V" and "D" bounce-back functions as special cases.

The BBF-augmented SETAR model is defined by replacing equation (2) in the SE-

TAR model by the following equation:

$$\begin{aligned}\mu_t &= \gamma_0(1-s_t)+\gamma_1 s_t \\ &+\lambda_1 s_t \sum_{j=\ell+1}^{\ell+m} s_{t-j}+\lambda_2(1-s_t)\sum_{j=\ell+1}^{\ell+m} s_{t-j}+\lambda_3\sum_{j=\ell+1}^{\ell+m}\Delta y_{t-j-1}s_{t-j},\end{aligned} \tag{4}$$

where e_t and s_t are defined as in equations (1) and (3) and ℓ and m are non-negative integers. The model defined here by equations (1), (4) and (3) will be denoted BBF(p,m,ℓ) hereafter.

Let us first isolate the first term of the bounce-back function: $\lambda_1 s_t \sum_{j=\ell+1}^{\ell+m} s_{t-j}$, by assuming $\lambda_2=\lambda_3=0$. So as to simplify further the interpretation, let ℓ be fixed to zero, as in Kim et al. [2005]. A positive value of parameter λ_1 will contribute to enhance the growth rate of y_t, compared to model (1), as soon as one period after the dynamics of Δy_t enters the recession regime and stays therein for at least two consecutive periods. For instance, starting from a long expansion epoch, i.e. if $s_{t-j}=0$ for $j=1,2\ldots h$ with h large enough, let us consider a four-quarter recession, i.e. $s_t+j=1$ for $j=0,1,2,3$. Then, neglecting the autoregressive terms in Δy_t, the extra growth imputable to this bounce-back effect is 0 at time t, λ_1 at time $t+1$, $2\lambda_1$ at time $t+2$, $3\lambda_1$ at time $t+3$ before going back to zero at time $t+4$, when the recession is over. Hence a bounce-back effect requires that $\lambda_1>0$. As proposed in Bec et al. [2011a], this period of extra growth may be delayed by a positive value of ℓ. Finally, its duration may vary according to the value of parameter m.

The BBU function is quite close to that case, since it is obtained from equation (4) by setting the following restrictions:

$$H_0^U : \lambda_1=\lambda_2=\lambda \;\text{ and }\; \lambda_3=0. \tag{5}$$

Hence, in the BBU case, the bounce-back term is $\lambda\sum_{j=\ell+1}^{\ell+m} s_{t-j}$. Contrary to the first term of the bounce-back function in equation (4) commented above, this BBU term can last longer than the recession if the bounce-back effect duration, measured here by the parameter m, or delay, governed by ℓ, is long enough.

The BBV function is also a special case of equation (4) in that it corresponds to the second term of the bounce-back function, $\lambda_2(1-s_t)\sum_{j=\ell+1}^{\ell+m} s_{t-j}$, and hence it is obtained from the restrictions:

$$H_0^V : \lambda_1=\lambda_3=0. \tag{6}$$

The specificity of the BBV-like bounce-back effect is that it activates only after the recession is over, when the system switches back to the expansion regime. Here again, this term will enhance the growth rate for positive values of λ_2.

The third term of the bounce-back function ($\lambda_3 \sum_{j=\ell+1}^{\ell+m} \Delta y_{t-j} s_{t-j}$) corresponds to the BBD bounce-back effect and hence to the joint restrictions below:

$$H_0^D : \lambda_1 = \lambda_2 = 0. \tag{7}$$

For this last effect to affect positively the output growth rate, the value of λ_3 must be negative since in the recession regime, the Δy_{t-j}'s are negative. This is a very simple way to introduce the idea first advocated by Friedman in 1964 that "a large contraction in output tends to be followed on by a large business expansion; a mild contraction, by a mild expansion" (see Friedman [1993]), i.e. that the vigour of the recovery is positively related to the depth, or magnitude of the contraction.

1.3 Estimation and testing

First, p is chosen as the smallest integer value for which the estimated residuals of the non-linear model are not serially correlated. Then, for this value of p, the triple (m, ℓ, κ) estimate is obtained from a triple-grid search as the one maximizing the likelihood of the BBF model. Concretely, the grid retained for the duration parameter is $m \in \{2, ..., 8\}$ while the one for the bounce-back delay parameter is $\ell \in \{0, ..., 4\}$. The grid interval, denoted K, for the threshold parameter κ is chosen so as to leave at least 10% of the observations in the recession regime. As noted earlier, we further constrain this grid interval to include non-positive values only because negative values of the output growth rate are considered as signals of a recession. Then, for these maximum likelihood estimates $(\hat{m}, \hat{\ell}, \hat{\kappa})$, the $\hat{\gamma}_i$'s, $\hat{\Phi}_i$'s, $i = 0, 1$, and $\hat{\lambda}_j$, $j = 1, 2, 3$, are obtained by non-linear least squares.

Before investigating further the estimated BBF-augmented SETAR model, we first test the null of linearity, using the $SupLR = \sup_{\kappa \in K} LR(\kappa)$ statistics corresponding to the hypothesis $\gamma_i = \gamma$, $\Phi_i = \Phi$, $\forall i = 0, 1$, and $\lambda_j = 0$, $\forall j = 1, 2, 3$ in equation (4). Even though the distribution of this test depends on nuisance parameters under the null of linearity, its asymptotic distribution derives from Hansen [1996]. The corresponding critical values cannot in general be tabulated since this distribution depends on un-known moment functionals. Therefore, we use a residual bootstrap method calculated by simulation to compute the corresponding p-values[5].

For the countries for which the linearity hypothesis is rejected, we then proceed to the tests of the specific recovery shapes described earlier. Actually, it is worth noticing that the tests of all the null hypotheses H_0^U, H_0^V and H_0^D above are nuisance-parameter free. Hence, they can be tested using a standard Likelihood Ratio — or Lagrange Multiplier, or Wald — test statistics which in turn is asymptotically Chi-squared distributed with two degrees of freedom for H_0^U, H_0^V and H_0^D. It is also possible to use a standard LR statistics to test the following null hypothesis of no bounce-back effect:

$$H_0^N : \lambda_1 = \lambda_2 = \lambda_3 = 0, \tag{8}$$

which amounts to test the null of the SETAR model given by equations (1)-(2) against the BBF model defined by equations (1)-(4). Again, the distribution of this test is nuisance-parameter free under the null and hence the corresponding LR statistics is asymptotically distributed as a $\chi^2(3)$.

2 Estimation results

2.1 The data

For France, Germany, Italy, Spain and the Euro Area, the data used for the empirical investigation are seasonally adjusted quarterly real GDP from the OECD (Main Economic Indicators) database. Since most of these countries display significant slowdowns in trend productivity growth during the early 1970s, we follow Kim et al. [2005] in considering data for a sample period beginning in 1973Q1. The last available observation for our international sample is 2010Q4. The GDP growth rate data, denoted Δy_t, are then computed as the first difference of the logarithm of the original series multiplied by 100 — see Figure 1 in Appendix.

2.2 Linearity tests

Since the linearity tests are performed from SupLR tests, they first involve the estimation of $(\hat{m}, \hat{\ell}, \hat{\kappa})$ in equation (4). As already mentioned, these parameters estimates are chosen

[5] A detailed description of the method can be found in Hansen [1996] or Hansen and Seo [2002].

from a triple-grid search so as to maximize the log-likelihood of the BBF(p, m, ℓ) model. Consequently, they also correspond to the maximum value of the SupLR statistics since they do not affect the log-likelihood of the linear model. The p-values of these statistics are then obtained by simulation, using a residuals bootstrap method. For given initial conditions, 10,000 random draws are made from the residual vectors under the linear null. From these bootstrap residuals, one can create a simulated sample of series using the linear autoregression, and for each sample, calculate the corresponding SupLR statistics. The bootstrap p-value then obtains as the percentage of simulated statistics which exceed the actual statistics. Finally, for all countries, the autoregressive lag order p is chosen so as to eliminate serial correlation in the BBF model, which leads to retain two lags for France and Italy, four lags for Germany, three lags for Spain and one lag for the US[6]. The results of the linearity tests, as well as the corresponding p, $\hat{m}$, $\hat{\ell}$ and $\hat{\kappa}$ are reported in Table 1. From the p-values reported in the last column of this Table, it appears that the

Table 1: Linearity tests results

Country	p	$\hat{m}$	$\hat{\ell}$	$\hat{\kappa}$	SupLR	p-value
FR	2	4	2	-0.059	16.94	0.005
GE	4	7	2	-0.557	19.58	0.000
IT	2	4	0	-0.287	9.63	0.245
SP	3	5	2	-0.160	21.57	0.000
EA	1	5	2	-0.132	10.80	0.030

null of linearity is strongly rejected for France, Germany, Spain and the Euro Area. By contrast, the linear AR model does not imply a significant log-likelihood loss compared to the BBF model for the Italian GDP growth rate: the corresponding p-value is 24%. Nevertheless, if the true DGP of this series is a non-linear constrained version of the more general model (4), the linearity test could gain power if it was computed from a restricted non-linear alternative. For this reason, in the Italian case, we also considered the BBU, BBV, BBD and standard SETAR without bounce-back alternatives instead of the BBF: all these non-linear alternatives failed to improve the linearity test p-value. Hence, the countries retained for the subsequent analysis are France, Germany and

[6]Even though the BBF model residuals were found not serially correlated with zero lag in the German case, it turns out that the fourth lag provides significant information regarding the non-linear dynamics of the GDP growth rate and hence, it is kept for the subsequent analysis.

Spain, together with the Euro Area. It is worth noticing that in these four nonlinear cases, the estimated delay for the bounce-back to become active after the regime switch is two quarters: $\hat{\ell} = 2$. Then, the estimated duration of the bounce-back effect lies between four quarters for France and seven quarters for Germany.

2.3 Tests for the presence and shape of the bounce-back effect

Table 2 reports the log-likelihood of the BBF model and the LR test statistics corresponding to the restrictions H_0^U, H_0^V, H_0^D and H_0^N presented above: n_p denotes the number of parameters while BBF_c denotes a constrained version of the BBF model which does not correspond to one of the four null hypotheses already tested.

First, it is worth emphasizing that our results provide strong support in favour of the presence of a bounce-back effect following a recession in all the countries considered. Actually, the LR tests of H_0^N, i.e. the standard SETAR model without bounce-back effect, against the BBF alternative, do not reject the null at the 1% level in the four cases. Then, it can be seen that the specific BBU, BBV and BBD functions are also strongly rejected. By contrast, after inspection of the general BBF model estimation results, it appeared that the null $H_0^C : \lambda_2 = \lambda_3 = 0$ was likely not to be rejected in most cases. This is confirmed by the LR-test statistics for this hypothesis reported in the bottom panel of Table 2: the null H_0^C is never rejected at the conventional level. Consequently, this constrained version of the BBF model, hereafter denoted BBF_c, is retained in the following analysis. This constrained model corresponds to the following definition for μ_t in the SETAR model given by equation (1):

$$\mu_t = \gamma_0(1 - s_t) + \gamma_1 s_t + \lambda_1 s_t \sum_{j=\ell+1}^{\ell+m} s_{t-j}, \tag{9}$$

As noticed in section 1.2, this specific form of the bounce-back function implies that it is active when $s_t = 1$ only and becomes inactive as soon as the recession time is over. Hence, it is likely to play a role in the close neighbourhood of a trough. Moreover, and contrary to the empirical evidence found in Kim et al. [2005], Morley and Piger [2009] and Bec et al. [2011a] from US data, the depth of the recession does not seem to affect the strength of the recovery in Europe.

The nonlinear least squares estimates of the selected bounce-back SETAR models are reported in Table 3. For France, Spain and the Euro Area, the estimation sample

Table 2: Testing for the presence and shape of the bounce-back effect

	FR	GE	SP	EA
		H_1: BBF		
n_p	7	9	8	6
Log-Lik	-99.84	-189.46	-138.84	-108.59
		H_0^N: SETAR		
n_p	4	6	5	3
Log-Lik	-106.98	-198.04	-144.73	-114.15
LR stat *(p-val)*	14.28 *(0.00)*	17.16 *(0.00)*	11.78 *(0.01)*	11.12 *(0.01)*
		H_0^U: BBU		
n_p	5	7	6	4
Log-Lik	-104.74	-196.27	-144.06	-112.04
LR stat *(p-val)*	9.80 *(0.01)*	13.62 *(0.00)*	10.44 *(0.00)*	6.90 *(0.03)*
		H_0^V: BBV		
n_p	5	7	6	4
Log-Lik	-106.55	-197.82	-144.69	-113.61
LR stat *(p-val)*	13.42 *(0.00)*	16.72 *(0.00)*	11.70 *(0.00)*	10.04 *(0.01)*
		H_0^D: BBD		
n_p	5	7	6	4
Log-Lik	-105.04	-195.70	-144.63	-113.41
LR stat *(p-val)*	10.40 *(0.01)*	12.48 *(0.00)*	11.58 *(0.00)*	9.64 *(0.01)*
		H_0^C: BBF$_c$		
n_p	5	7	7	–
Log-Lik	-102.32	-191.16	-141.40	-110.16
LR stat *(p-val)*	4.96 *(0.08)*	3.39 *(0.18)*	5.12 *(0.08)*	3.14 *(0.21)*

BBF$_c$ stands for $H_0^C : \lambda_2 = \lambda_3 = 0$.

is 1973Q1-2010Q4, but due to the large values of the lag order, $\hat{m}$ and $\hat{\ell}$ in the German case, the largest sample we could use is 1973Q4-2010Q4. It is worth noticing that all the bounce-back parameters have the expected sign ($\lambda_1 > 0$) and are significantly different from zero at the 5%-level. The data together with the estimated thresholds are reported in Figure 1 in the Appendix.

Table 3: Bounce-back SETAR estimates

	FR	GE	SP	EA
	BBF$_c$(2,4,2)	BBF$_c$(4,7,2)	BBF$_c$(3,5,2)	BBF$_c$(1,5,2)
λ_1	**0.57** (3.08)	**0.51** (3.68)	**0.35** (2.56)	**0.60** (2.83)
γ_0	**0.53** (5.32)	**0.51** (4.95)	**0.61** (4.12)	**0.50** (5.41)
γ_1	0.06 (0.34)	-0.69 (-2.12)	-0.03 (-0.13)	0.37 (1.99)
ϕ_1	**0.26** (3.08)	-0.03 (-0.27)	-0.11 (-1.21)	**0.55** (7.15)
ϕ_2	**0.34** (4.35)	0.08 (0.93)	**0.45** (6.62)	–
ϕ_3	–	0.02 (0.29)	**0.31** (3.88)	–
ϕ_4	–	**0.18** (2.17)	–	–
σ	0.48	0.89	0.62	0.51
n_0	136	132	134	137
n_1	16	17	18	15
Q(4) [p-val]	[0.65]	[0.99]	[0.06]	[0.13]

t-statistics in parenthesis. Q(4) is the Ljung-Box statistics. Bold figures denote the 5% level. n_0 (resp. n_1): number of observations in expansion (resp. recession) regime.

3 BBF model short-run forecast accuracy

In this section, the one-step ahead forecasts are calculated from a pseudo-real time analysis using recursive regressions. Actually, given that our final observation date, T_f, is 2010Q4, we begin the forecast performance evaluation from T_0=2000Q1. Then, for all $t \in \{T_0, ..., T_f - 1\}$, we estimate the model from the initial observation, T_i=1973Q1[7], until t, and use this estimate to compute the one-step-ahead forecasts of the real GDP growth rate, denoted $\Delta\hat{y}_{t+1|t}$. So as to assess the added value of the nonlinear features of the model, these forecasts are compared with those from a benchmark linear autoregression, i.e. imposing a constant value for μ_t in equation (1). The added value of the bounce-

[7]Except for Germany where it is 1973Q4 due to the values of p, $\hat{m}$ and $\hat{\ell}$.

back term is also assessed by comparing these forecasts to a standard SETAR model, i.e. setting all the λ_i's to zero, $i = 1, 2, 3$, in equation (4). In a first step, we focus on point forecasts accuracy as measured by the Root Mean Squared Error (RMSE) criteria. In a second step, the possible gains stemming from the asymmetrical nature of the BBF model is further explored by computing forecasts distribution from bootstrap resampling techniques.

3.1 One-step ahead forecasts RMSE's

Let us begin with an evaluation of the short-run forecast accuracy of the BBF model based on the RMSE criteria. Here, both the general unconstrained BBF and the constrained BBF_c models are still considered. Finally, particular attention is paid to the last recession driven by the subprime crisis. Therefore, in addition to 2000Q1-2010Q4, the forecast assessment for each model is also carried out for period 2008Q2-2010Q4, distinguishing the crisis period 2008Q2-2009Q2 and the recovery period 2009Q3-2010Q4. All these results are gathered in Table 4 below. When looking at the 2000Q1-2010Q4 forecasting sample (first column of this Table), it can be seen that the BBF_c model outperforms its BBF unconstrained version for France, Spain and the Euro area. Even though the unconstrained BBF model is preferred in Germany, its RMSE values are very close to the ones obtained from the BBF_c version. When comparing the results across countries, the best forecast accuracy is obtained for Spanish and French data, and to a lesser extend for the European data. Probably due to a larger volatility, the German GDP growth rate seems more difficult to forecast. Let us now turn to the relative forecast accuracy of the four models over the last crisis, hence focusing on the one-step-ahead forecast errors obtained for the period 2008Q2-2010Q4. Looking at the second column of Table 4, it turns out that the relative accuracy of the BBF_c specifications is further improved over the last crisis compared to the longer baseline forecasting sample. This evidence confirms the relevance of the bounce-back augmented model. Looking closer at the country-specific results, it appears quite expectedly that the forecast accuracy deteriorates between the last decade and this crisis episode. Exploring further the forecast performances of these models by splitting the crisis episode into the contraction (2008Q2-2009Q2) and recovery (2009Q3-2010Q4) phases, the results are more contrasted. Actually, during the contraction sub-period, all models give the less

Table 4: 1-step ahead forecasts (relative RMSE criterion)

Model	2000Q1-2010Q4	2008Q2-2010Q4	2008Q2-2009Q2	2009Q3-2010Q4
		France		
AR(2)*	0.481	0.746	1.050	0.317
SETAR(2)	1.03	0.98	0.96	1.19
BBF(2,4,2)	0.96	0.87	0.86	1.46
BBF_c(2,4,2)	**0.94**	**0.82**	**0.86**	**0.47**
		Germany		
AR(4)*	0.970	1.710	2.330	0.906
SETAR(4)	0.99	0.98	0.98	0.96
BBF(4,7,2)	**0.90**	**0.86**	0.88	**0.75**
BBF_c(4,7,2)	0.91	0.88	**0.87**	0.90
		Spain		
AR(3)*	0.467	0.878	1.240	0.368
SETAR(3)	0.96	0.96	0.89	1.47
BBF(3,5,2)	0.95	0.95	0.85	1.58
BBF_c(3,5,2)	**0.85**	**0.83**	**0.81**	**0.93**
		Euro Area		
AR(1)*	0.542	0.992	1.420	**0.344**
SETAR(1)	1.00	1.01	1.01	1.00
BBF(1,5,2)	0.99	1.00	0.92	1.73
BBF_c(1,5,2)	**0.96**	**0.95**	**0.91**	1.39

*: All RMSE, but the ones of the AR models, are given relative to the AR model RMSE.

accurate one-step-ahead forecasts. Nevertheless, the BBF_c models clearly outperform both the linear and SETAR models in terms of RMSE, the gains ranging from 9% in the Euro area to 19% in Spain. Remark that the linearity test was less favorable for the Euro area as regards its $p-$value than for other retained cases, see Table 1. This confirms Clements, Frances, Smith and Van Dijk [2003] findings that a high degree of non-linearity (as measured by the $p-$value of the linearity test) is required before non-linear models outperform the linear in terms of forecasting. For the French and German cases, the best forecasting results are obtained during the recovery phase, their RMSE relative to the linear one falling respectively to 47% and 75%. The reverse is true for the Euro area, and to a lesser extend for Spain, where the BBF models relative forecasting performance deteriorates compared to the contraction phase as well as to all other sub-periods.

3.2 Forecasts bootstrapped distribution

After having compared point forecasts, we now turn to the distributions of forecasts so as to assess whether there is gain to use BBF models beyond RMSE measures. In this subsection, we propose to compute distribution for the predictor stemming from the BBF model given by equations (1)-(4), by using bootstrap resampling techniques. In this respect, we implement two bootstrap methods, namely with and without parameters uncertainty[8] For both methods, we generate a bootstrapped vector of length B, $(\Delta\hat{y}^{(b)}_{t+1|t})_{b=1,...,B}$, for all $t \in \{T_0, ..., T_f - 1\}$, that will be used to assess the empirical distribution of the predictor $(\Delta\hat{y}_{t+1|t})$. For example, a confidence interval at the $1-\alpha$ level can be computed by taking the empirical $\alpha/2$ and $1-\alpha/2$ quantiles of the bootstrap vector $(\Delta\hat{y}^{(b)}_{t+1|t})_{b=1,...,B}$. In this study, we adopt $B = 1000$ replications, arguing that this number is sufficient to achieve stability of the results[9]. Note also that for sake of simplicity, and in opposition to the previous forecasting experience, we only consider the constrained version of the models, that is the BBF$_c$ models. We briefly present below both bootstrap methods.

In a first approach, bootstrapped distributions may be constructed by using parameters estimates as if they were the true parameters values, i.e. without taking the parameters variability into account. This method is quite simple and not time-consuming, as it only requires to bootstrap the residuals and to add the bootstrapped error to the one-step-ahead predictor estimated by the conditional expectation. Let $F_{\hat{e}}$ denote the empirical cumulative density function (cdf) of the residuals $\hat{e}_t$ computed from equations (1)-(9). The bootstrapped 1-step-ahead forecast, denoted $\Delta\hat{y}^{(b)}_{t+1|t}$, for $b = 1, \ldots, B$, is given by

$$\Delta\hat{y}^{(b)}_{t+1|t} = \hat{\mu}_{t+1|t} + \sum_{i=1}^{p} \hat{\phi}_i(\Delta y_{t+1-i} - \hat{\mu}_{t+1-i}) + e^{(b)}_{t+1}, \tag{10}$$

where $e^{(b)}_{t+1}$ is randomly drawn from $F_{\hat{e}}$ with replacement, $\hat{\mu}_t$ is the estimated conditional

[8]For the latter, we extend the method proposed by Pascual, Romo and Ruiz (2004) in the linear case and by Li (2011) in the non-linear SETAR case.

[9]We checked randomly the robustness of our results with $B = 5000$.

mean of the constrained BBF_c given by

$$\hat{\mu}_t = \hat{\gamma}_0(1 - s_t) + \hat{\gamma}_1 s_t + \hat{\lambda}_1 s_t \sum_{j=\ell+1}^{\ell+m} s_{t-j}, \tag{11}$$

and where $\hat{\mu}_{t+1|t}$ is the BBF_c conditional mean forecast defined by

$$\hat{\mu}_{t+1|t} = \hat{\gamma}_0(1 - s_{t+1}) + \hat{\gamma}_1 s_{t+1} + \hat{\lambda}_1 s_{t+1} \sum_{j=\ell+1}^{\ell+m} s_{t+1-j}. \tag{12}$$

Thus, denoting $\Delta\hat{y}^{\alpha/2}_{t+1|t}$ and $\Delta\hat{y}^{\alpha/2}_{t+1|t}$ respectively the empirical $\alpha/2$ and $1-\alpha/2$ quantiles of the cdf of $(\Delta\hat{y}^{(b)}_{t+1|t})_{b=1,\ldots,B}$, the $(1-\alpha)$-level bootstrapped confidence intervals are given by $CI_{(1-\alpha)} = \left[\Delta\hat{y}^{\alpha/2}_{t+1|t}, \Delta\hat{y}^{\alpha/2}_{t+1|t}\right]$. As in the previous section related to the 1-step-ahead point forecasts, we implement a recursive forecasting scheme to get density distributions for the predictors from 2000Q1 to 2010Q4. As a benchmark, we are also compute a confidence interval for the 1-step-ahead forecast $\Delta\hat{y}_{t+1|t}$ stemming from linear AR(p) models. The theoretical interval for this one-step ahead predictor is given by $\left[\Delta\hat{y}_{t+1|t} \pm t_{1-\alpha/2}\hat{\sigma}_e\right]$ where $t_{1-\alpha/2}$ is the quantile of the residuals distribution (supposed to be Gaussian) at the confidence level $1-\alpha$ and where $\hat{\sigma}^2_e$ is the estimated residuals variance. This interval is again computed assuming that the parameters are known, *i.e.* ignoring the parameters uncertainty. The 90% confidence intervals constructed as described above for the BBF_c and the AR models are reported together with the observed Δy_t in Appendix, Figure 4.[10] It can be seen that the $CI_{90\%}$ obtained from the BBF_c models are narrower than the ones from the linear AR in three cases out of four, namely France, Germany and the Euro Area. We also note that during the subprime crisis, observed GDP growth rates are out of the $CI_{90\%}$ bounds for all models, pointing out the unexpected amplitude of the movements that cannot be caught by auto-projective models. However, it is clear that BBF models enable to replicate the bounce-back effects that occur at the end of the recession in the second part of the year 2009, particularly for France and the Euro Area, thus confirming the relevance of our proposed model. By contrast, the linear AR model seems too rigid, especially during the recovery phase.

Notwithstanding its simplicity, the first approach could yield misleading results by neglecting the parameters uncertainty. For this reason, we check the robustness of our

[10]Since the results between 2000Q41 and 2004Q4 are similar to those obtained between 2005Q1 and, say, 2007Q2, the graphs only plot the results from 2005Q1 so as to get a better visual focus on the subprime crisis period.

conclusions by adapting to our model (eqs (1) to (4)) the bootstrap method recently proposed by Li [2011] for SETAR processes. This second approach allows to incorporate the variability due to parameters estimation into forecast distributions without assuming any specific distribution for the innovation process. The latter is only assumed to be *i.i.d.*. Compared to the first bootstrap approach described above, this one requires the following two preliminary steps :

1) B bootstrap replicates $\{\Delta y_t^{(b)}\}_{t=1}^T$ of trajectories $\{\Delta y_t\}_{t=1}^T$ are generated as $\Delta y_t^{(b)} = \Delta y_t$ for $t = 1, \ldots, \max(p, l+m)$, and

$$\Delta y_t^{(b)} = \hat{\mu}_t + \sum_{i=1}^{p} \hat{\phi}_i (\Delta y_{t-i}^{(b)} - \hat{\mu}_{t-i}) + e_t^{(b)}, \text{for } T \geq t > \max(p, l+m),$$

   with $e_t^{(b)}$ and $\hat{\mu}_t$ defined as above.

2) Model (1)-(9) is then re-estimated using the B bootstrapped series $\{\Delta y_t^{(b)}\}_{t=1}^T$ and the estimated threshold[11] $\hat{\kappa}$ in order to get $(\hat{\phi}_1^{(b)}, \ldots, \hat{\phi}_p^{(b)}, \hat{\gamma}_0^{(b)}, \hat{\gamma}_1^{(b)}, \hat{\lambda}_1^{(b)})$, for $b = 1, \ldots, B$.

Finally, for a given b, the 1-step-ahead forecast denoted $\Delta \hat{y}_{t+1|t}^{(b)}$, is given by

$$\Delta \hat{y}_{t+1|t}^{(b)} = \hat{\mu}_{t+1|t}^{(b)} + \sum_{i=1}^{p} \hat{\phi}_i^{(b)} (\Delta y_{t+1-i}^{(b)} - \hat{\mu}_{t+1-i}^{(b)}) + e_{t+1}^{(b)}, \tag{13}$$

where $\Delta \hat{y}_t^{(b)} = \Delta y_t$, for $t = T, T-1, \ldots, T-p+1$, and:

$$\hat{\mu}_{t+1|t}^{(b)} = \hat{\gamma}_0^{(b)} (1 - s_{t+1}) + \hat{\gamma}_1^{(b)} s_{t+1} + \hat{\lambda}_1^{(b)} s_{t+1} \sum_{j=\ell+1}^{\ell+m} s_{t+1-j}. \tag{14}$$

Note that we compute the bootstrap forecasts conditional on the last observations of the observed series. The bootstrap confidence intervals that integrate parameter uncertainty are again obtained from the empirical $\alpha/2$ and $1-\alpha/2$ quantiles of the cdf of $(\Delta \hat{y}_{t+1|t}^{(b)})_{b=1,\ldots,B}$. Obviously, if parameters variance is small, accounting for parameters uncertainty should not affect significantly the bootstrapped confidence intervals. Moreover, if the distribution of the innovation process $(e_t)_t$ is known, then theoretical and bootstrapped confidence interval are equivalent. Figure 4 in the Appendix plots

[11] Note that both methods ignore the sampling variability of the estimated threshold $\hat{\kappa}$, based on its super-consistency (Chan [1993]) and on Li (2011) simulation exercises with this regard in the case of SETAR processes.

together the bootstrapped $CI_{90\%}$ obtained with and without parameters uncertainty for the BBF models. Before the beginning of the last recession, the two bootstrap methods produce remarkably similar results, revealing a strong stability in parameters estimates. Nevertheless, it can be seen that around the turning point, mid-2009, taking parameters uncertainty into account widens the $CI_{90\%}$ as expected — see France and the Euro Area (resp. panels (a) and (d) in Figure 4), and to a lesser extent Germany. Actually, this result originates in the large variance characterizing this period. Actually, there is a sudden increase in the variance located in 2009 when realized growth rates were very negative. Basically, the variance was multiplied by two for France and Spain and by three for the euro area, reflecting the uncertainty in parameter estimates due to those strong negative evolutions. After this shock, the variances go down to the pre-recession level, suggesting that this sudden rise in uncertainty was only short-lived. Nevertheless, for Spain, taking parameters uncertainty into account drastically modifies the $CI_{90\%}$ during the first quarters of the recovery: it hardly contains the observed value from 2009Q3 on.

From this bootstrapped distributions, the empirical skewness and kurtosis are straightforward to compute. It is noteworthy that these results convey very useful information related to the shape of the distribution, especially by comparison with a linear model with a Gaussian distribution without any asymmetry and with small tail risks. These statistics are reported in Table 5 for the pre- and post-peak of the last recession subsamples. In the German, Spanish and European cases, the slightly negative skewness values

Table 5: Bootstrapped empirical skewness and excess kurtosis (with parameters uncertainty)

Sample	France	Germany	Spain	Euro Area
		Skewness		
2000Q1-2008Q1	0.10	-0.22	-0.04	-0.14
2008Q2-2010Q4*	-0.07	-0.42	-0.13	-0.19
		Excess Kurtosis		
2000Q1-2008Q1	0.65	0.78	1.70	1.88
2008Q2-2010Q4*	0.66	1.22	2.14	2.01

associated to positive excess kurtosis point to a left-skewed, heavy-tailed distribution of

$\Delta\hat{y}_{t+1|t}$, i.e. the forecast is more likely to be far below the point-forecast (its mean) than it is to be far above. These features amplify after the last recession: for instance the left skewness increases by 90% in Germany. The kurtosis also increases after the beginning of the subprime crisis, indicating even more mass in the tails than a Gaussian distribution with the same variance. This can be interpreted as higher tail risks around the central projections, in line with the deterioration of economic conditions during the economic recession. This is not the case in France, where the excess kurtosis was almost constant. Note also that France is the only country where positive skewness is found in the distribution of $\Delta\hat{y}_{t+1|t}$ before the crisis, then the skewness becomes negative, pointing out that risks are since then tilted to the downside.

4 Conclusion

In this paper, we propose to augment the standard Self-Exciting Threshold Autoregression by a Bounce-Back function which allows for more general and more flexible shape of recessions, particularly in the recovery phase. When applied to post-1973 quarterly growth rate of real GDPs, we find evidence for a bounce-back effect in France, Germany, Spain and the Euro area. Furthermore, the forecast accuracy analysis based on these BBF-SETAR estimates clearly supports the relevance of this model for the one-step-ahead forecasts, where the accuracy gains generally lie between 10% and 20% compared to the linear autoregression forecasts. Moreover, bootstrap simulations experiments reveal an improvement in the forecasts confidence intervals which are found to be narrower for the bounce-back model than for the linear autoregression, without any noticeable deterioration of the coverage rates.

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Appendix

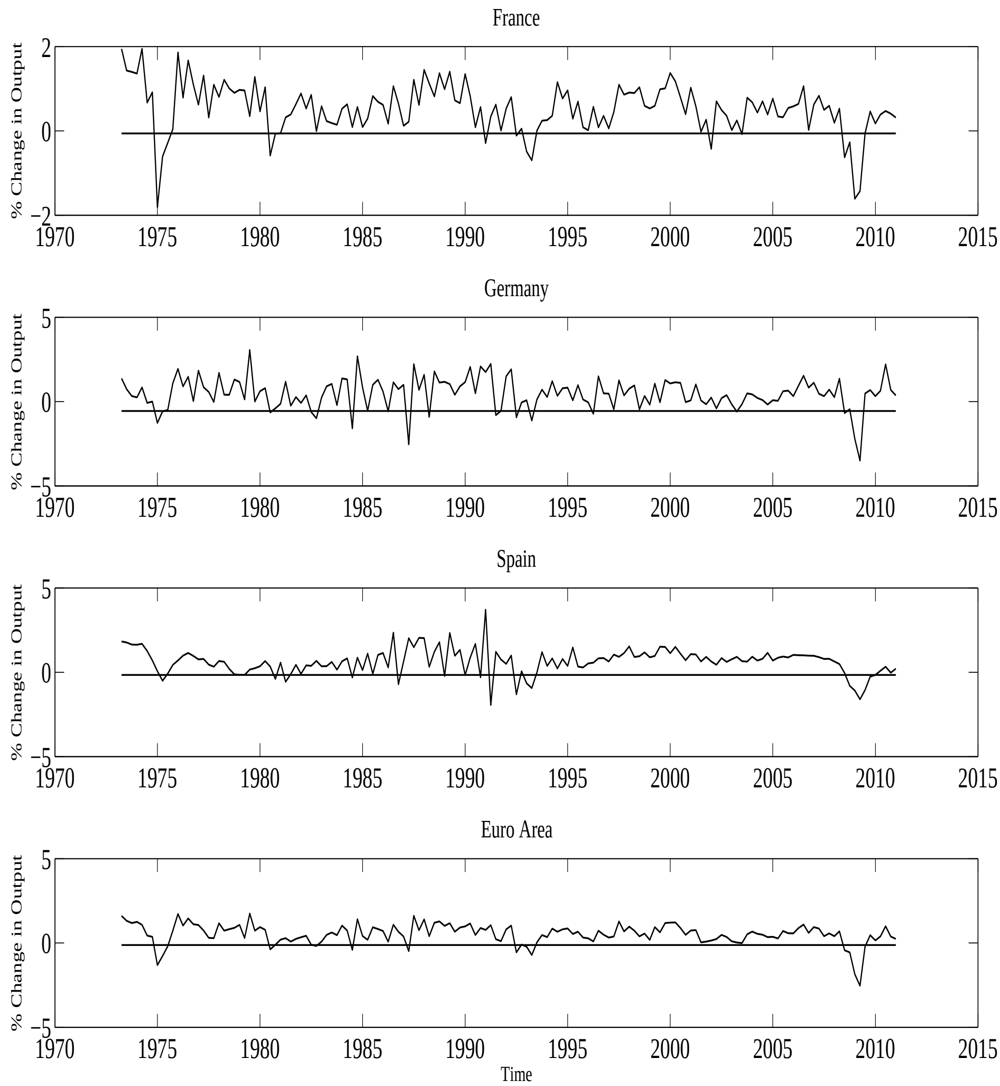
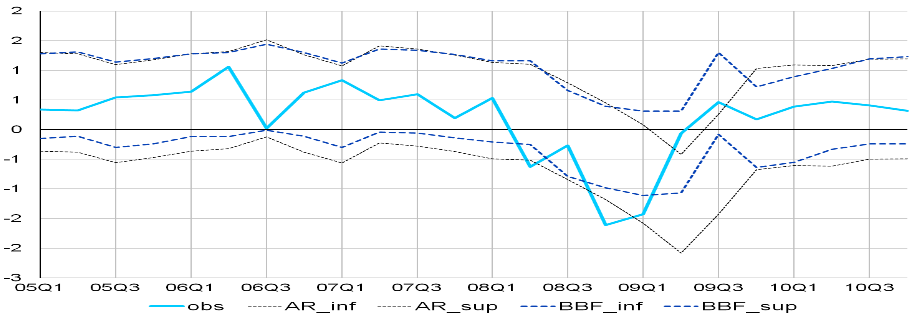
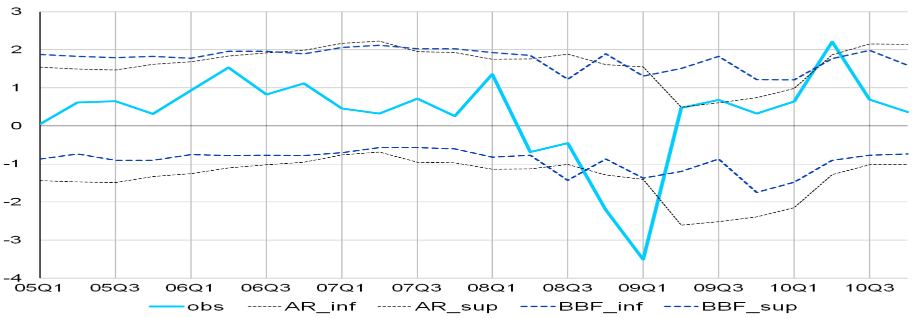


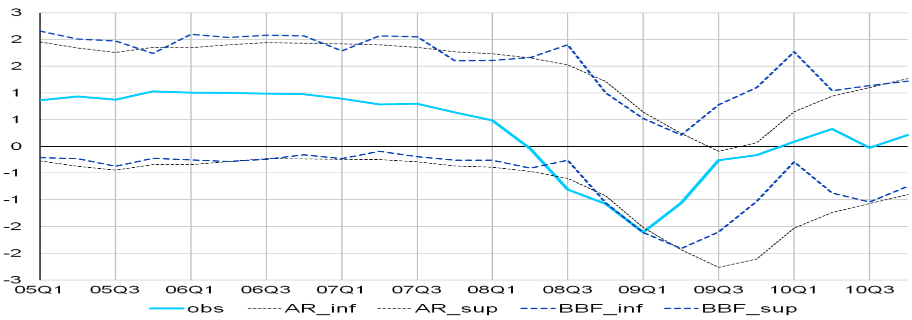
Figure 1: Data and estimated thresholds



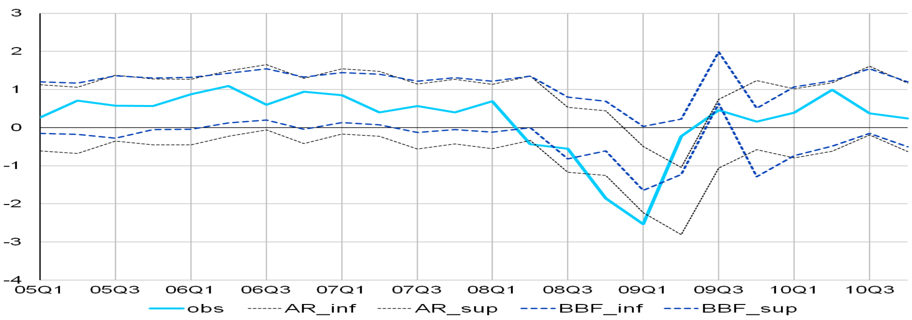
(a) France



(b) Germany

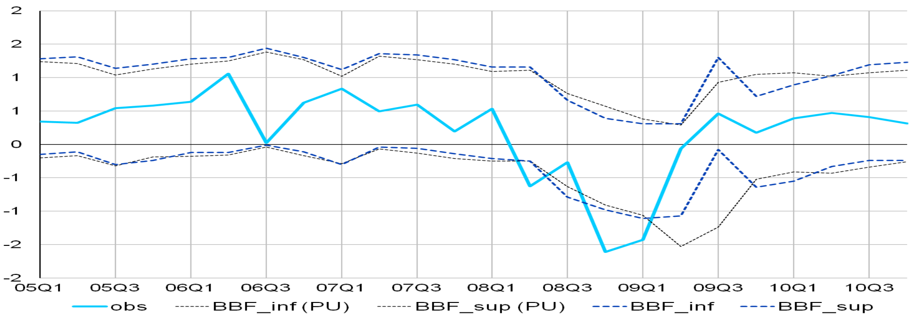


(c) Spain

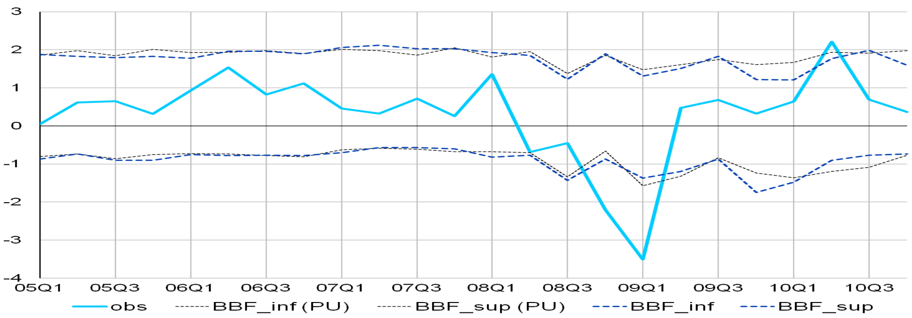


(d) Euro Area

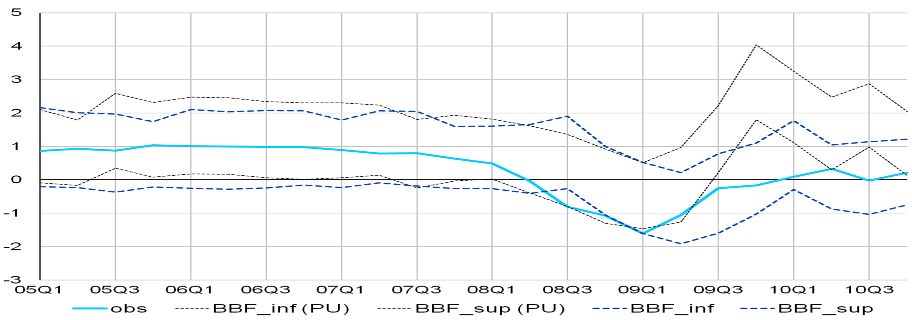
Figure 2: 90% Confidence Intervals for AR and BBF$_c$-SETAR models (without parameters uncertainty)



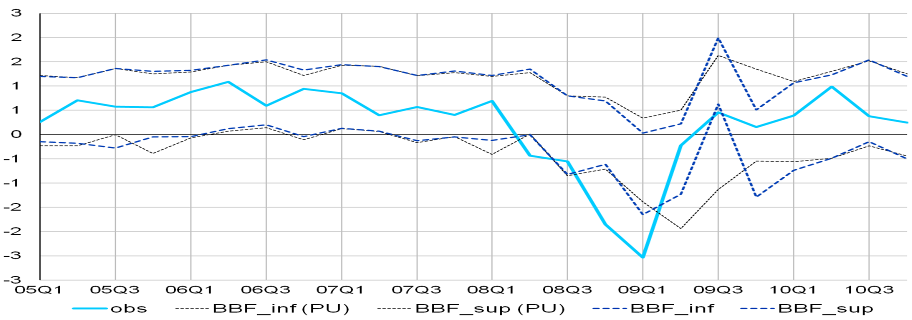
(a) France



(b) Germany



(c) Spain



(d) Euro Area

Figure 3: 90% Confidence Intervals for BBF$_c$-SETAR models with and without parameters uncertainty

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