DOCUMENT

DE TRAVAIL

N° 387

PRIVATE UNCERTAINTY AND MULTIPLICITY

Gaetano Gaballo

July 2012

BANQUE DE FRANCE

EUROSYSTÈME

DIRECTION GÉNÉRALE DES ÉTUDES ET DES RELATIONS INTERNATIONALES

DIRECTION GÉNÉRALE DES ÉTUDES ET DES RELATIONS INTERNATIONALES

PRIVATE UNCERTAINTY AND MULTIPLICITY

Gaetano Gaballo

July 2012

Les Documents de travail reflètent les idées personnelles de leurs auteurs et n'expriment pas nécessairement la position de la Banque de France. Ce document est disponible sur le site internet de la Banque de France « www.banque-france.fr ».

Working Papers reflect the opinions of the authors and do not necessarily express the views of the Banque de France. This document is available on the Banque de France Website "www.banque-france.fr".

Private Uncertainty and Multiplicity

Gaetano Gaballo[*][†]

[*]Banque de France, Monetary Policy Research Division [DGEI-DEMFI-POMONE), 31 rue Croix des Petits Champs 41-1391, 75049 Paris Cedex 01, France. Comments welcome at : *gaetano.gaballo@banque-france.fr*.

[†]I would like to thank George Evans, Christian Hellwig, Boyan Jovanovic, David K. Levine, Ramon Marimon and Michael Woodford for their invaluable comments at different stages of this project. It goes without saying that any errors are mine own.

Résumé: Ce document présente les conditions dans lesquelles une petite incertitude privé sur un état endogène global de l'économie peut invalider l'unicité de l'équilibre. Le principal résultat est présenté dans un modèle macroéconomique entièrement micro fondé où les agents apprennent à partir des prix d'équilibre. Les résultats s'appliquent à une large classe de problèmes statiques d'extraction de signal où la corrélation fondamentale et les externalités stratégiques contribuent conjointement à la multiplicité d'équilibres. Les cas où une seule de ces deux elements est suffisante pour une multiplicité sont également isolés et discutés.

Classification JEL: D82, D83, E3.

Mots-clés: information dispersée, coordination des anticipations, croyances du second ordre.

Abstract: This paper provides the conditions under which small enough private uncertainty on an aggregate endogenous state of the economy can invalidate uniqueness of the equilibrium. The main result is presented in a fully microfounded macroeconomic model where agents learn from equilibrium prices. The findings apply to a broad class of static signal extraction problems where both fundamental correlation and pay-off externalities jointly contribute to a multiplicity of equilibria. The cases where only one of these two determinants is sufficient for a multiplicity are also isolated and discussed.

JEL Classification: D82, D83, E3.

Keywords: dispersed information, coordination of expectations, second-order beliefs.

1 Introduction

This paper shows that a small enough degree of private uncertainty can generate a multiplicity of equilibria in macro-models that have a unique equilibrium under perfect knowledge. This finding contrasts with a classical result of the global games literature (Hans and van Damme, 1993) maintaining that multiplicity can be solved by perturbing the model away from perfect information. In an influential paper Morris and Shin (2000) put forward this view in macroeconomics. They propose 'rethinking' multiplicity as the artifact of two extreme assumptions: "First, [a] economic fundamentals are assumed to be common knowledge; and second, [b] economic agents are assumed to be certain about each other's behavior in equilibrium"[1]. Arbitrarily small private uncertainty on the fundamentals would lead therefore to uniqueness. Nevertheless, later works have showed that when agents have access to public information generated by market transactions, private uncertainty on fundamentals is generally not enough to pin down the number of equilibria[2]. That is, when signals also reveal information about others' beliefs then agents' ability to coordinate on multiple equilibria improves. However, it is not clear from the present literature whether marginal endogenous uncertainty can only restore a pre-existing multiplicity or have the potential to generate truly new equilibria.

This paper fills the gap by looking at the possibility that *private* - rather than public - endogenous signals can confuse agents and be a source of multiplicity. It identifies the conditions under which an endogenous information structure allowing for infinitesimal departures from the assumptions [a] and [b] can indeed invalidate the uniqueness of the equilibrium. In particular, it shows that a *small enough* private uncertainty on an endogenous aggregate state of the economy can generate three rational expectation equilibria (REE) in models where if this uncertainty is null or large enough then a unique equilibrium exists. This result reverses Morris and Shin's argument on the effect of a marginal relaxation of perfect information. At the same time it still maintains that less information prevents multiplicity since uniqueness is restored as uncertainty increases. The paper also isolates a particular case where a multiplicity arises even with perfect knowledge of fundamentals, that

[1] Morris and Shin (2000), pag. 140, square brackets added.

[2] Angeletos and Werning (2006), and Hellwig, Mukherji and Tsyvinski (2006). See the discussion below.

is, when [a] holds but [b] does not. This case further emphasizes the crucial role of uncertainty about others' beliefs in sustaining a multiplicity of equilibria.

To provide microfoundations to the informational frictions of interest I present a macro-model with money-in-the-utility-function. The economy is segmented in a continuum of islands, each inhabited by a representative consumer and a representative final producer. The final producer hires island-specific inputs - labor and capital - at local prices to produce an homogenous good traded across islands at a global price. The consumer supplies island-specific labor and one unit of endowment that is sold at a global price to intermediate producers of the island-specific capital. The informativeness of local prices is blurred by three sources of fundamental randomness. An idiosyncratic productivity shock hits the production of the island-specific capital; and the money supply on each island is determined by an aggregate and an island-specific stochastic component.

Only once the shocks hit and input markets simultaneously clear, final production is implemented and the consumption good is traded across islands. Therefore the final producers do not observe the price of the consumption good when they hire inputs. They instead observe the prices of local transactions. From the island-specific wage they are able to infer the island-specific supply of money which is a *private exogenous signal* of the aggregate monetary shock. From the price of the island-specific capital they learn a noisy signal about the price of the endowment blurred by an idiosyncratic productivity shock. This second piece of information is a *private endogenous signal* about both the aggregate monetary shock and the aggregated price expectations of all final producers. An equilibrium requires that producers' expectations and the movements of the local prices they observe be mutually consistent.

The main proposition of the paper demonstrates that small enough private uncertainty about the price of the endowment creates room for a multiplicity of equilibria. In particular, if the variance of the nominal idiosyncratic shocks is above a certain threshold, then for any small enough variance of the productivity shocks, three rational expectation equilibria exist. With either zero or a large enough variance of the productivity shocks the economy has instead a unique equilibrium.

The microfoundation of the information structure clarifies the conditions under which private uncertainty matters; namely, when it concerns an endogenous aggregate state that responds in opposite ways to an aggregate shock in the two extreme scenarios: no information and perfect foresight. The price of the endowment has this feature in the model. With full information - i.e. at the limit of zero variance of the real shocks - all global prices move together as neutrality of money holds. With no information - i.e. at the limit of infinite variance of the real shocks - producers hire less inputs as they underestimate a positive increase in the consumption price, and this in turn depresses the price for the endowment.

Therefore, for a high enough degree of private uncertainty the price for the endowment is negatively correlated with the aggregate monetary shock. This means the weigh put on the local price for capital, which gives an optimal forecast of the aggregate shock, must be negative. As private uncertainty decreases this weigh further decreases (increases in size) as the signal becomes more informative. Nevertheless, at the limit of no private uncertainty the correlation between the aggregate monetary shock and the price for the endowment is positive. Hence, by continuity when the signal is sufficiently informative an optimal positive weight must exist too. In particular, for a small enough degree of private uncertainty three equilibria coexist: two in which the endowment price co-moves with the fundamental shock and one in which it goes in the opposite direction. One equilibrium of either kind vanishes with perfect information and only one equilibrium in which the price for the endowment moves at the same rate of the money supply remains.

In the process of proving the main result, the paper presents a fairly general analysis of static signal extraction problems that goes beyond the specific restrictions of the model. The formal analysis proceeds in two sequential steps. First, I uncover the conditions under which multiple equilibria can arise when only a private signal about an endogenous aggregate variable is available. I distinguish between two effects: a fundamental effect arising from the correlation between the signal and the unobserved stochastic fundamental, and a pay-off externality effect determined by the correlation of the signal with the average price expectation. The fundamental effect is actually sufficient to generate a multiplicity - that is, multiple equilibria arise even in the case agents forecast a purely exogenous realization. In addition, pay-off substitutability (complementarity below the unity) merely reinforces (dampens) the fundamental effect. In a second step, I study the original problem where both an endogenous and an exogenous signal are present. In this case exogenous information shrinks the areas where a multiplicity arises.

Finally, in the last section I discuss an ad-hoc variation of the model where even if final producers are perfectly informed about fundamentals a partially-correlated signal on an endogenous state can trigger three determinate equilibria. In this case, a pure pay-off externality alone can sustain a multiplicity of equilibria and two determinate equilibria emerge - beyond a fundamental one - where prices move with non-fundamental noise. This occurs with a degree of pay-off complementarity that exceeds unity.

This paper relates to the debate on the robustness of multiplicity of equilibria in the classic currency attack model. Morris and Shin (1998) first noticed that small private uncertainty on fundamentals leads to a unique equilibrium. In fact, this possibility relies on the exogenous nature of the information structure; when markets transactions generate public information then the original multiplicity is restored (Angeletos and Werning (2006), and Hellwig, Mukherji and Tsyvinski (2006)). The present finding completes the picture showing how private - rather than public - endogenous signals can even generate a multiplicity in the context of models that exhibit equilibrium uniqueness - rather than a multiplicity - with perfect knowledge.

This work also relates to a large body of literature concerning dispersed information in macro-models dating back to Lucas (1973). Notable applications of that approach are Angeletos and La'O (2008), Hellwig and Venkateswaran (2009), Lorenzoni (2009) among others. In all of these works a unique equilibrium exists whose welfare properties are challenged by the interaction of public and private signals as in Morris and Shin (2002). The same objective is shared by Amador and Weill (2010) that also noticed that three REE are possible with private uncertainty lying in between two strictly positive boundaries. Similar findings for more ad-hoc information structures are in Ganguli and Yang (2009), Benhabib, Wang and Wen (2012) and Desgranges and Rochon (2012). In contrast to this paper, in those models a multiplicity vanishes with small enough private uncertainty. That is, they do not meet the aforementioned conditions for which a multiplicity results from a *marginal* perturbation of perfect information. However, the characterization of their case is provided as an additional result of the general analysis in this study.

2 A microfounded macro-model

This section presents a dynamic macro-model encompassing the reduced form of the seminal Cobweb model (Muth, 1961). The model's main objective is to microfound in the most transparent way the whole class of signal extraction problems to which the results apply. In this economy final producers are the only type of agents imperfectly informed about a single aggregate shock. They learn from market interactions two private noisy signals: one about the aggregate shock and one about an aggregate endogenous state. This constitutes the backbone of the information structure that will studied in the next section.

Preferences and Technology

Consider an endowment economy composed of a continuum of islands with unit mass. Each island $i \in I \equiv [0,1]$ is inhabited by a representative consumer and a representative producer. The representative consumer maximizes the following utility function

$$U_i\left(C_{i,t}, M_{i,t}, L_{i,t}\right) \equiv \mathbf{E}_{i,0}\left[\sum_{t=0}^{\infty} \delta^t \left(\frac{C_{i,t}^{1-\psi}}{1-\psi} + \log\frac{M_{i,t-1}}{P_t} - \frac{\left(L_{i,t}^s\right)^{1+\gamma}}{1+\gamma}\right)\right], \quad (1)$$

subject to a budget constraint for each period

$$\frac{R_t}{P_t} Z_{i,t}^s + \frac{W_{i,t}}{P_t} L_{i,t}^s + \frac{M_{i,t-1}}{P_t} = C_{i,t} + \frac{M_{i,t}}{P_t} + \frac{T_{i,t}}{P_t}, \quad (2)$$

where ψ and γ are positive constants, R is the return on an unspecific type of capital $Z_i^s = 1$ that expires in one period and whose unitary endowment is renewed each time in each island, W_i is a island-specific wage, L_i^s is supply of island-specific working hours, C_i is the consumption of the final good whose price is P, and M_i is the money demand on island i. $T_{i,t}$ is a redistributive nominal transfer such that $\int T_{i,t}\, di = 0$.[3]

The endowment is acquired in an inter-island market to be transformed in island-specific capital K_i. The transformation is operated by competitive firms maximizing profits

$$R_{i,t} K_{i,t}^s - R_t Z_{i,t}, \quad (3)$$

[3]The only scope of the transfer is ensuring that in equilibrium the bagdet constraints holds for each i.

using the following linear technology

$$K_{i,t}^{s} \equiv e^{-\hat{\eta}_i} Z_{i,t}, \tag{4}$$

where $e^{-\hat{\eta}_i}$ is a productivity factor specific to the production of the island-specific capital K_i^s which is produced using Z_i units of the endowment acquired in a inter-islands market at a price R.

Finally island-specific capital and labour are used by the representative producer in island i to produce an homogeneous consumption good that is consumed across islands. Competitive firms maximizes profits

$$P_t Y_{i,t} - W_{i,t} L_{i,t} - R_{i,t} K_{i,t}, \tag{5}$$

under the constraint of a Cobb-Douglas technology with constant return to scale

$$Y_{i,t}\left(K_{i,t}, L_{i,t}\right) \equiv K_{i,t}^{1-\alpha} L_{i,t}^{\alpha}, \tag{6}$$

with $\alpha \in (0,1)$, where K_i and Y_i denote respectively the demand of capital and the produced quantity of the consumption good, generated in island i. Notice that production is island-specific, that is, each representative producer hires labor and capital from his own island only. Input markets are segmented and there is one different price for each input on each island.

Shocks

Following Amador and Weill (2010) I suppose that at the initial time $t = 0$ the economy is hit by aggregate and island-specific disturbances. The productivity of the intermediate sector is affected by the stochastic noise

$$\hat{\eta}_i \sim N\left(0, \hat{\sigma}\right), \tag{7}$$

where $\hat{\eta}_i$ is an island-specific realization distributed independently across the islands. A second source of randomness concerns the supply of money available in each island M_i^s. It is determined by

$$M_i^s = \frac{1-\delta}{\delta} e^{\varepsilon + \hat{\phi}_i}, \tag{8}$$

where $\left(1-\delta\right)/\delta$ is a scaling factor included for notational convenience,

$$\varepsilon \sim N\left(0, 1\right), \tag{9}$$

is an aggregate permanent shock drawn from a white noise distribution, and

$$\hat{\phi}_i \sim N(0, \sigma_\phi), \tag{10}$$

is a monetary transmission disturbance independently distributed across the islands.

It is worth anticipating here that potential deviations of global prices from the equilibrium steady state will be a function of the aggregate shock only. This implies that observing a global price is informationally equivalent to observing the aggregate monetary shock.

Equilibrium and Actions

The timing of actions in the economy is as follows. First shocks occur, all input markets open and clear simultaneously, then production of the consumption good is implemented and the final market operates. All agents in the economy, have the same unbiased prior on the distributions of the shocks and acquire information through the equilibrium prices they deal with. Final producers, in contrast to the other types in the economy, do not trade on any global market. Therefore they will be uncertain about the aggregate monetary shock and the consumption price at the time of planning production[4].

This economy has a unique deterministic equilibrium. Nevertheless, I will demonstrate that in the stochastic version of the model the information structure just outlined can make multiplicity arise. As usual in the literature on noisy rational expectations from Grossman (1975) and Hellwig (1980) onward, I restrict attention to symmetric equilibria with a linear (log) representation that in the case of this model it is possible with no approximation.

Definition 1 *Given the stochastic realizations $(\varepsilon, \{\hat{\phi}_i, \hat{\eta}_i\}_I)$, a symmetric log-linear rational expectation equilibrium at the initial time is characterized by a distribution of local prices $\{R_i, W_i\}_I$, global prices (P, R) and relative individual and aggregate quantities such that:*

- *(optimality) agents optimize their actions conditional to the prices and quantities they observe;*

[4] In other words, the consumption price does not reveal simultaneously to the production choice. Lack of simultaneity is what makes informational frictions matter. For a discussion of the issue see Hellwig and Venkateswaran (2011).

- *(market clearing) intra-island markets demand and supply match,* $L_i = L_i^s$ *and* $K_i = K_i^s$*, the money market clears,* $\int M_i^d di = M^s$*, the endowment market clears* $\int Z_i di = \int Z_i^s di = 1$*, and the good market clears,* $\int Y_i di = \int C_i di$*;*

- *(log-representation) and log-deviations of individual actions from their equilibrium steady state are linear functions of the shocks.*

The first condition requires that agents' actions are optimal with respect to the information that agents infer from the equilibrium prices they observe. In other words, actions and the informational content of prices must be mutually consistent. The requirement of a log-linear equilibrium is quite natural for this model. It ensures tractability of the aggregate relations and allows to write down the first-order conditions in log-linear terms as deviations from a unique steady state[5] (the issue is fully treated in appendix A.2.2). In what follows I derive the log-linear first-order conditions and discuss the implied information structure.

The final producer i forms an expectation about the price of the final good

$$E_t^i(p_t) \equiv \mathbf{E}[p_t|\omega_{i,t}], \tag{11}$$

conditional on an information set $\omega_{i,t}$ available at time t. The information set held by producers embodies all the information generated by their market transactions in the input markets. The producer acquires the quantities of inputs that satisfy

$$\begin{aligned}
w_{i,t} &= E_t^i(p_t) + (1-\alpha)k_{i,t} + (\alpha-1)\, l_{i,t}, && \text{(12a)}\\
r_{i,t} &= E_t^i(p_t) - \alpha k_{i,t} + \alpha l_{i,t}, && \text{(12b)}\\
l_{i,t} &= \alpha^{-1} y_{i,t} - \alpha^{-1}(1-\alpha)k_{i,t}, && \text{(12c)}
\end{aligned}$$

where $x_i \equiv \ln X_i - \ln \bar{X}$, that is small cases denote deviations in log-terms from the equilibrium steady state. Hence, the information set of final producers

$$\omega_{i,t} = \{r_{i,t}, w_{i,t}\}, \tag{13}$$

[5]In fact, the equilibrium steady state linearly will depend on some constant covariance-variance terms arising from the aggregation of actions across agents. Nevertheless, in equilibrium, deterministic drifts do not affect the final producers' signal extraction problem on deviations from the equilibrium steady state.

consists of the prices arising from the transactions they carry out: local wages and the island-specific price for capital. Final producers' price expectations and prices in the input markets are formed simultaneously, hence, the accuracy of the final producers' forecasts depends on the informativeness of local equilibrium prices.

Since future fundamentals are constant, and final producers' uncertainty is solved after the first period (once the producers observe the realized price) the consumer's expectations over the future course of the economy is a unique deterministic equilibrium where aggregate and island-specific prices and quantities will be constant. As in Amador and Weill (2010), the intertemporal first order condition for money in island i, namely

$$\frac{\Lambda_{i,t}}{P_t} = \delta \mathbf{E}\left[\frac{\Lambda_{t+1,i}}{P_{t+1}}\right] + \delta \frac{1}{M_{i,t}},$$

collapses to the one-period equilibrium relation (details in appendix A.1.)

$$\lambda_i - p = -\varepsilon - \hat{\phi}_i, \tag{14}$$

where I already substituted $m_i = m_i^s$ with m_i^s given by (8). From here onward I will omit time indices as the following relations are all simultaneous and I will focus on the initial period only. The description of the consumers' problem is finally completed by

$$\begin{aligned} -\psi c_i &= \lambda_i, & (15a)\\ \gamma l_i^s &= w_i + \lambda_i - p, & (15b)\end{aligned}$$

that are the first-order conditions relative to consumption and labor. Notice that both consumers and intermediate producers can infer the aggregate monetary shock (and so the consumption price) because they observe the equilibrium price of the endowment that is traded on a global market[6].

In contrast final producers only observe the price of the island-specific capital, that is a noisy signal of the price of the endowment. In fact, the competitive supply curve of the island-specific capital is given by

$$\begin{aligned} r_i &= r + \hat{\eta}_i, & (16a)\\ k_i^s &= z_i - \hat{\eta}_i. & (16b)\end{aligned}$$

[6]However, the supply of labor can be expressead without any reference to the price substituting (14) in (15b) .

In other words, any quantity of the island-specific capital is supplied at a price equal to (or more precisely, at the minimum price not smaller than) the cost of the unspecific capital acquired in the inter-islands market augmented for an i.i.d. productivity disturbance. Therefore, the real disturbances prevent the local price for capital to fully reveal the price for the endowment.

The signal extraction problem

Let us look at how the consumption price moves with the average final producers' expectation. Working the equations out (see Appendix A.2.3) one can write down the consumption price in equilibrium as a linear combination of the aggregate exogenous shock and the aggregate expectation, that is

$$p = \varepsilon + \beta \left(E\left(p\right) - \varepsilon \right), \tag{17}$$

with

$$\beta \equiv -\frac{\alpha\psi}{1+\gamma-\alpha},$$

measuring the impact of the aggregate expectation

$$E\left(p\right) \equiv \int E^{i}\left(p\right) \ di, \tag{18}$$

on the consumption price. The two extreme scenarios of perfect information and no information obtain respectively as $E(p) = p = \varepsilon$ and $E(p) = 0$ with $p = (1-\beta)\varepsilon$. In the latter the consumption price overreacts to the aggregate shock to clear a suboptimal production. In the former instead, since the neutrality of money holds, the price increases at the same rate as the monetary base. Finally notice that $\beta < 0$, that is, the model replicates the same reduced form of the celebrated Cobweb model (Muth, 1961) for which the impact of the aggregate expectation on the actual price is strictly negative.

Let's come now to the information set available to final producers. Notice that in equilibrium the information set (13) can be rewritten as

$$\omega_{i,t} = \{r + \hat{\eta}_i, \ \varepsilon + \hat{\phi}_i\}, \tag{19}$$

composed by private noisy signals of respectively the price of the endowment and the aggregate monetary shock obtained respectively from (16a) and (15b)

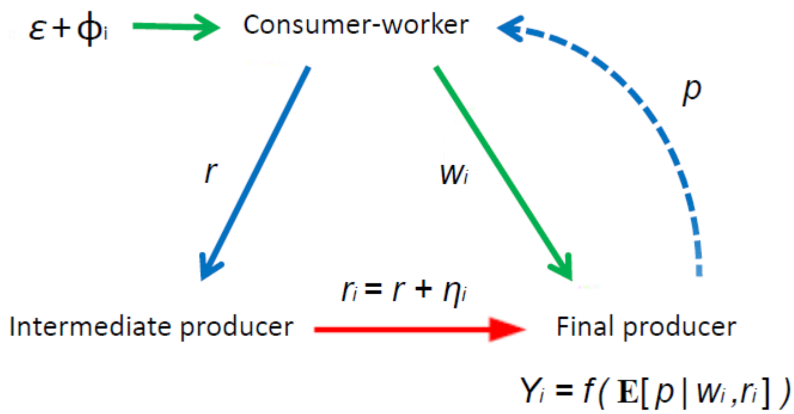

Figure 1: Informational flows in the economy.

after substituting for (14)[7]. The latter is a purely exogenous information on the aggregate monetary shock blurred by truly private uncertainty, whereas the former is an endogenous piece of information in the form of a private signal about an aggregate endogenous variable. The exogenous signal is directly informative about the aggregate unobserved shock. The endogenous signal instead embodies information about both the aggregate shock and producers' expectations that cannot be untangled.

Figure 1 summarizes the flows of information in the economy. The final producer aquires information about the money supply in his/her island through the local labor market, that is observing the local equilibrium wage. He/she also observes the price for the island-specific capital that is a noisy observation of the price for the endowement. The latter is traded on a global market which thus enables both sides - consumers and intermediate producers - to infer the aggregate money supply shock. These markets operate simultaneously (solid lines). Once the input markets have cleared, the final production is implemented in each island at the best of the information gathered. Finally the global market for consumption opens and the price p revelas itself (dashed line).

[7]The final producer infers the money supply since he/she observes the inslad-specific wage and knows the equilibrium quantity traded on the intra-island labor market.

In analogy with the consumption price, one can express the price of the endowment as

$$r = \varepsilon + \varkappa \left(E\left(p\right) - \varepsilon \right), \tag{20}$$

with

$$\varkappa \equiv \frac{1+\gamma}{1+\gamma-\alpha},$$

that is a linear function of the aggregate shock ε and the aggregate price expectation (details in appendix). Notice that $\varkappa > 1$, that is, the price of the endowment exhibits opposite reactions in the extreme cases of perfect information ($r = \varepsilon$) or no information ($r = (1 - \varkappa)\,\varepsilon$). In other words, the price of the endowment reacts more than one-to-one to an aggregate expected departure from the perfect information outcome. The underlying economic intuition is simple. When the aggregate monetary shock is perfectly observed, all prices must increase at the same rate whereas quantities do not move. Now suppose instead final producers do not expect a positive shock to occur. Labor supply shrinks in front of an unchanged labor demand as producers do not expect variations in the consumption price. As a consequence, in equilibrium the final producers hire less labour and the wage increases less fast than the money supply. This determines a fall of the productivity of the island-specific capital and indirectly a reduction of the price of the endowment.

Note that the case of no information is plausible only at the limit of infinite volatility of island-specific shocks. In this case final producers cannot infer from the shrinking of the labor supply and the decrease of the price of the capital that in fact an aggregate shock has occurred and hence the consumption price will move. With intermediate degree of private uncertainty instead an equilibrium requires that final producers' expectations and observed price movements are mutually consistent. Whenever the signal extraction problem yields more than one solution, multiple equilibria arises. The following proposition states a multiplicity result for the model presented.

Proposition 2 (Multiplicity) *If*

$$\sigma_\phi > \frac{1+\gamma}{\alpha} - (1-\psi),$$

then there exists a finite threshold $\hat{\sigma}_*$ such that for any $\hat{\sigma} \in (0, \hat{\sigma}_*)$ the economy has three determinate REE. A unique equilibrium obtains otherwise for a small enough $\hat{\sigma}$.

This proposition follows as a direct application of proposition 7 as proved in the following section. In particular, that proposition requires two more conditions, namely $\beta < 1$ and $\varkappa > 1$, which are satisfied by the model at hand for any feasible calibration. In this sense, the model isolates the whole class of signal extraction problems to which the results apply.

The proposition states that, provided the precision of the exogenous information is below a certain threshold, *small enough* private uncertainty on the price of the endowment generates a multiplicity of equilibria. In other words, a marginal perturbation away from perfect knowledge invalidates the uniqueness of the equilibrium. Moreover the *more precise* is endogenous information conveyed by the price of the island-specific capital, the more likely is that the economy exhibits a multiplicity of equilibria, whereas the uniqueness is restored with large private uncertainty. As we will see, the endogenous nature of a private signal makes possible the coexistence of one equilibrium where the signal has a negative fundamental correlation (r decreases when ε hits) and it is negatively weighted, and two equilibria where instead the signal has a positive fundamental correlation (r increases when ε hits) and it is positively weighted. In particular, the possibility of a multiplicity is reinforced by the precision of the endogenous information as the signal becomes more informative, and so more largely weighted.

Finally, multiplicity is more likely as the exogenous information is *less precise*, specifically, when the degree of private uncertainty on exogenous information σ_ϕ is above a certain threshold defined as a surprisingly simple combination of the CES parameters of the utility function and technology process. Notice that such a combination is an index of the convexity of the problem. In particular, multiplicity is more likely as the model approaches linearity, that is, as the endogenous variables are more reactive to shocks.

3 Analysis

This section analyzes the signal extraction problem of final producers embodied in the model above. However, the exposition aims to be self-contained so that no reference to the specific model is strictly needed. The great benefit of having microfoundations beyond the informational structure of interest come at the stage of the economic interpretation of the results. These will be outlined in dedicated paragraphs.

To illuminate the role of each component of the analysis I will proceed in two subsequent steps. First, I will consider the presence of a single private signal of an endogenous state. Second, I will build on this result extending the analysis to the case where both an endogenous signal and an exogenous signal are available.

3.1 Private uncertainty about an endogenous state

A Single Endogenous Signal

Suppose a continuum of agents $i \in (0,1)$ needs to forecast a price

$$p = \varepsilon + \beta \left(E \left(p \right) - \varepsilon \right), \tag{21}$$

reacting linearly to an exogenous normally distributed disturbance $\varepsilon \sim N(0,1)$ and the aggregate expectation $E(p) \equiv \int E^i(p) \;\; di$ where $E_t^i(p_t) \equiv \mathbf{E}[p_t|\omega_i]$ is an individual expectation conditional to the set of signals held by agent i.

The parameter β measures the nature and impact of the payoff externalities. For $\beta = 0$ the price process is completely exogenous. In this case an incentive to use signals of the aggregate supply shock can only concern its fundamental content. Examples of signal extraction problems of this kind are found in Amador and Weill (2010), Desgranges and Rochon (2011) and Ganguli and Yang (2009). For $\beta \neq 0$ instead, the price moves with the average expectation, so that also pay-off externalities are involved as in Morris and Shin (2002) and subsequent literature. With this in mind, I consider all values $\beta < 1$ in order to provide results that are directly applicable to a larger class of economies. Cases of extreme degrees of expectational complementarity providing for $\beta > 1$ are finally discussed in the last section of this paper.

Suppose the case each agent holds a single private endogenous signal written as

$$\omega_i \equiv \left\{ E\left(p\right) + \zeta^{-1} \varepsilon + \eta_i \right\}, \tag{22}$$

where $\eta_i \sim N\left(0, \zeta^{-2}\sigma\right)$ is an i.i.d. normal disturbance. This signal accounts for a generic private observation of an aggregate variable that moves with the aggregate expectation $E(p)$ and the fundamental noise ε. Two parameters of crucial importance are ζ and σ. The latter represents the variance of the private noise whereas the former is the covariance of the fundamental component with the aggregate shock ε when both are expressed in terms of the variance of the fundamental component ζ^{-2}. The limit values $\sigma \to \infty$ and $\sigma \to 0$ entail the extreme situations where informational heterogeneity vanishes and agents have respectively no information and perfect information on the fundamental realization. All the intermediate cases consist of dispersed information.

Back to the model. The signal (22) corresponds to the endogenous signal $r_i = r + \hat{\eta}_i$ in the microfounded model. To see this rewrite r_i using (20) and rescale by $\varkappa$. The equivalence obtains defining $\eta_i \equiv \varkappa^{-1}\hat{\eta}_i$ so that $\sigma = (1-\varkappa)^{-2}\hat{\sigma}$, and $\zeta \equiv \varkappa/(1-\varkappa)$. In particular notice that $\varkappa > 1$ implies $\zeta < -1$. For $\zeta < -1$ the signal (22) represents a private observation of an aggregate state that reacts in opposite directions to ε in the case of no information ($E(p) = 0$) and full information ($E(p) = \varepsilon$). In fact, for the model above it is $\varkappa/(1-\varkappa) = -(1+\gamma)/\alpha < -1$.

The Actual Law of Motion

A private signal about an aggregate endogenous variable provides information on the unknown fundamental but also on second-order agents' beliefs. The two pieces of information cannot be identified separately because when agents use heterogeneous information - that in this case means they put weight on the signal itself - then the problem of forecasting the forecasts of others is in play. As a result, the signal generates non trivial feedback informational effects. Once agents collectively put weight on the signal then the aggregate expectation reacts to it, and so the precision of the signal itself is affected by the use of the signal.

To impose order in the analysis let us fix a liner forecasting strategy. Notice that, when the random variable to be forecasted is normally distributed - and here it is the case - a linear forecasting strategy is the optimal one as it correctly identifies the first and second moment of the objective conditional distribution. Agent i's forecast is written as

$$E^i(p) = b_i\left(E(p) + \zeta^{-1}\varepsilon + \eta_i\right), \tag{23}$$

where b_i is a constant coefficient to be determined that weights the expectational signal. In other words, agent type i expects a displacement of the actual price from the deterministic equilibrium that is proportional to the signal as defined in (22). If all agents use the rule above then by definition (18) the aggregate expectation is

$$E\left(p\right)=\frac{\mathbf{b}}{1-\mathbf{b}}\zeta^{-1}\varepsilon, \tag{24}$$

where

$$\mathbf{b}\equiv\int b_i\ di, \tag{25}$$

is the average weight across agents. Therefore an individual expectation can be rewritten as

$$E^i\left(p\right)=b_i\left(\frac{1}{1-\mathbf{b}}\zeta^{-1}\varepsilon+\eta_i\right), \tag{26}$$

where the signal is now expressed as a function of exogenous shocks depending on the average weight. In fact, the collective strategy of weighing the expectational signal according to (23) has a non-linear effect on the variance of the fundamental component of the signal as it is shown by (26). This happens because the aggregate shock does not vanish in the aggregation feeding back into the aggregate expectation and in turn into the signal, coming full circle. Nevertheless, the variance of the private component is never affected. Hence, the informativeness of the signal of the fundamental innovation - as well as the overall variance of the signal itself - changes non linearly with the average weight.

Plugging (25) in (21), we finally obtain the actual law of motion of the price

$$p=\varepsilon+\beta\left(\frac{\mathbf{b}}{1-\mathbf{b}}\zeta^{-1}\varepsilon-\varepsilon\right), \tag{27}$$

as functions of the average weight and the aggregate shock only. Importantly the correlation between the signal and the price can take either sign depending on the extent of $\mathbf{b}$. Therefore, different combinations of variances and correlation are in principle possible depending on the average weight given to the signal. This very feature creates room for the emergence of a multiplicity of equilibria.

It is worth noticing here that the law of motion for the price has been obtained without guessing any a-priori form, but just using definitions and temporary equilibrium conditions. This means that these relations are still valid for *disequilibrium* beliefs, that is they entail the price course given an arbitrary profile of weights restricting agents' expectations.

The Set of Equilibria

The course of the economy is entirely determined by (25)-(26)-(27) for a given profile of individual weights. A rational expectation equilibrium (REE) is obtain when agents' beliefs are consistent with the actual conditional distribution of price fluctuations according to (11). In other words, the forecast error of each agent has to be orthogonal to the available information. This implies a restriction on the profile of individual weights $\{b_i\}_I$. The orthogonality restriction

$$\mathbf{E}\left[\left(\frac{1}{1-\mathbf{b}}\zeta^{-1}\varepsilon+\eta_i\right)\left((1-\beta)\,\varepsilon+\frac{\beta\mathbf{b}}{1-\mathbf{b}}\zeta^{-1}\varepsilon-b_i\left(\frac{1}{1-\mathbf{b}}\zeta^{-1}\varepsilon+\eta_i\right)\right)\right]=0,$$

entails what I will call the *best individual weigh function*

$$b_i(\mathbf{b})=\underbrace{\frac{\zeta\,(1-\mathbf{b})}{1+\sigma\,(1-\mathbf{b})^2}}_{\textit{fundamental effect}}+\underbrace{\frac{\beta\,(\mathbf{b}-\zeta\,(1-\mathbf{b}))}{1+\sigma\,(1-\mathbf{b})^2}}_{\textit{pay-off externality effect}} \tag{28}$$

provided $\mathbf{b}\neq 1$, that is the optimal weight that each agent must put on his own expectational signal as a function of the average weight $\mathbf{b}$. It is instructive to distinguish between two components determining the best individual weight. The first and second term on the right hand side reflect the informativeness of the signal about respectively the fundamental shock and the average expectational mistake. The latter interacts with the former when the price moves with the aggregate expectation (in the case $\beta\neq 0$) and so generates pay-off externalities in the signal extraction problem. Notice that both this effects are stronger as the precision of the signal increases, that is as σ decreases.

An equilibrium requires that (28) holds for each agent so that an optimal value obtains imposing $b_i=\mathbf{b}$. In particular an equilibrium value of $\mathbf{b}$ determines an aggregate expectation and in turn an equilibrium in the economy as the shocks unfold. The set of the REE of the economy is therefore characterized by

$$\mathbf{b}^3-2\mathbf{b}^2+\left(1+\sigma^{-1}(1-\beta)(1+\zeta)\right)\mathbf{b}-\sigma^{-1}(1-\beta)\,\zeta=0 \tag{29}$$

that is, the the locus of the fix points of (28). The following proposition states analytical conditions for a multiplicity of REE.

Proposition 3 *Given a $\zeta < -1$ and a $\beta < 1$, there always exists a threshold σ_* that is monotonically decreasing in both ζ and β such that for any $\sigma \in (0, \sigma_*)$ equation (29) has three real solutions, whereas a unique one otherwise.*

Proof. Postponed in appendix. ■

The result just obtained does not consider the effects of exogenous signals, nevertheless, it embodies all the key determinants beyond the multiplicity result in the previous section. In what follows I will spell out the core insights originated by the presence of endogenous private uncertainty. The next section will show instead how the joint presence of exogenous uncertainty changes the picture.

The fundamental effect

The core mechanism beyond multiplicity can be discussed looking at the particular case $\beta = 0$ when a pure fundamental effect is at work and pay-off externalities do not matter. Figure 2 plots the individual weight function $b_i(\mathbf{b})$ in for $\beta = 0$, $\zeta = -4$ (that is $\varkappa = 4/3$) and different values of σ namely 0.5 (dashed line), 1 (solid line) and 5 (dotted line).

The best individual weight function is plotted in figure 2. It is a cubic function taking the value 0 at $\mathbf{b} = 1$ and in the limits of $\mathbf{b} \to \pm\infty$. It is positive with $\mathbf{b} > 1$ and negative otherwise. The first derivative is zero at $\mathbf{b} \to \pm\infty$ and $-\zeta$ at $\mathbf{b} = 1$. In particular notice as $\sigma \to \infty$ then the curve approaches the x-axis, whereas as $\sigma \to 0$ the curve approaches the line $\zeta - \zeta\mathbf{b}$. For σ high enough the curve is sufficiently close to the x-axis so that one equilibrium only arises in the negative quadrant. Ceteris paribus, as σ decreases the signal becomes more precise, so the optimal weight increases in absolute size, that is, the curve is further away from the x-axis and closer to the line $\zeta - \zeta\mathbf{b}$ before approaching zero as $\mathbf{b} \to \pm\infty$. The latter intersects the bisector in the positive quadrant only if $-\zeta > 1$ that is therefore a necessary condition to obtain multiple fix points for a small enough variance σ. This proves the following remark.

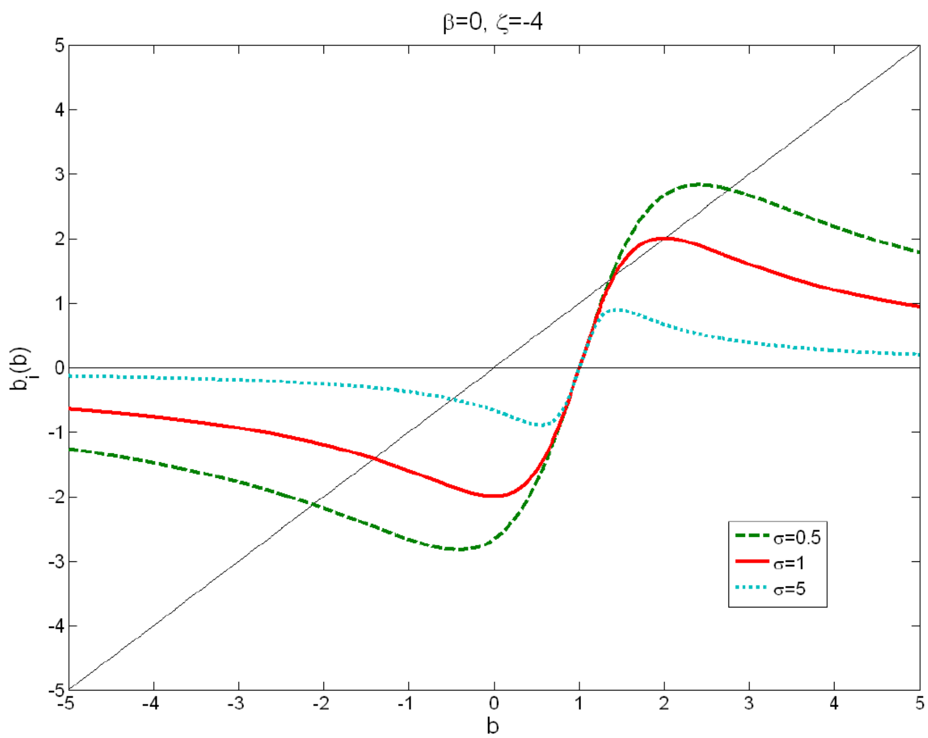

Figure 2: Plot of the best individual weight for different values of σ.

Remark 4 *The fundamental effect alone, that is when pay-off externalities are null ($\beta = 0$), can be sufficient to sustain a multiplicity of equilibria provided $\zeta < -1$.*

Figure 2 gives also insights on the out-of-equilibrium properties of the multiple equilibria generated by the signal extraction problem. In particular, with a small enough degree of private uncertainty, the rational expectation equilibrium in the middle is not strongly rationalizable in the sense of Guesnerie (1992, 2005). In fact, if agents expect that the average weight on the endogenous signal is in a neighborhood of that equilibrium than their best individual weight must be further away from the equilibrium (notice $\lim_{\sigma \to 0} b_i' (\mathbf{b}) = -\zeta > 1$). But since this is common knowledge, a second-order rational belief on the average weight must equally lie further away from the equilibrium, etc. in other words, this equilibrium cannot be obtained as a singleton from a rationalizability process. Importantly notice that this equilibrium is the one surviving under perfect knowledge. This would suggest that small degrees of private uncertainty generate two new equilibria that can work instead as absorbing points of a rationalizability dynamic. In other words, rationalizability and multiplicity are two deeply interconnected phenomena. This conjecture surely deserves a closer assessment that will be the object of a future paper.[8] I prefer to keep the present work focused on the issue of the existence of multiple equilibria. At the same time it is worth alerting the reader of the possibility that the closest equilibrium to the one under perfect knowledge could not be the one selected by a process of iterated deletion of never best replies.

Interaction with the pay-off effect

Consider now how changes in β can affect the number of equilibria. The existence of a unique fix point in the region $\mathbf{b} < 1$ is not affected because the curve has the same qualitative behavior for $\beta < 1$. To assess the existence of a multiplicity one has to check how the best individual weight function moves

[8] Notice that $b_i' (\mathbf{b})$ is always negative at the low and high equilibria. Nevertheless to prove that such equilibria are strongly rationalizable rational expectation equilibria one has to show that $|b_i' (\mathbf{b})| < 1$ at the equilibria. Unfortunately we cannot write down this condition in a closed form. Moreover, the investigation of this property in the full model requires a cumbersome multidimensional analysis that can take a whole paper to be spelled out.

with β for values $\mathbf{b} > 1$. Since $\zeta < -1$ the quantity $\mathbf{b} - \zeta (1 - \mathbf{b})$ is lower than one for $\mathbf{b} > 1$ and it is linearly decreasing in $\mathbf{b}$. Hence, for a positive β this effect is pro-multiplicity (increases $b_i(\mathbf{b})$) only for moderate increase of $\mathbf{b}$ beyond one. Nevertheless, for the case $\beta \to 1$, where this impact is maximal, the two multiple equilibria collapses on the single unfeasible limit point $b_i(\mathbf{b}) = \mathbf{b} = 1$. Therefore, given the monotonicity of the pay-off effect, no strictly positive value of β below the unity can sustain a multiplicity unless the fundamental effect is not already sufficient to generate it. On the other hand, for negative values of β, provided $\mathbf{b}$ is large enough the effect becomes pro-multiplicity, and more importantly, can be arbitrarily large for a β low enough. That is, a multiplicity can arise for a β low enough even in the case the fundamental effect is not sufficient alone. This feature is illustrated in figure 3 whose discussion is postponed to next subsection. The following remark finally summarizes the contribution of the pay-off effect.

Remark 5 *Pay-off substitutability ($\beta < 0$) promotes multiplicity whereas sub-unitary ($\beta \in (0,1)$) pay-off complementarity does not.*

Back to the model. It is instructive to interpret some of the conditions found in terms of the microfounded macro-model of the previous section. As already noticed the condition $\zeta < -1$ corresponds to $\varkappa > 1$ that implies the price of the endowment exhibits opposite reactions to a nominal aggregate shock in the scenarios of perfect foresight and no information. This condition is met for any feasible calibration in the model. In particular substituting (24) in (20) (holding at the limit case of $\sigma_\phi \to \infty$) we have

$$r = \frac{1 - \varkappa}{1 - \mathbf{b}} \varepsilon,$$

that is, with $\mathbf{b} > 1$ ($\mathbf{b} < 1$) the price of the endowment increases (decreases) with the aggregate shock and the private signal is weighted accordingly. Moreover as $\mathbf{b}$ displaces away from 1 the volatility of r decreases. Hence, for decreasing values of σ there exist two equilibria, one of either kind, entailing increasingly small variance of r. These two vanish with perfect knowledge and only one equilibrium in which r (and p) increases at the rate ε remains. Therefore the three equilibria entail dramatically different responses to a single aggregate shock.

3.2 The effect of exogenous information

This section extends the previous analysis to the original case presented in the microfounded model where agents deal with both endogenous and exogenous imperfect information. The analysis confirms the intuition that more precise exogenous information reduces the likelihood of a multiplicity of equilibria. Nevertheless, still the smaller the degree of uncertainty on the aggregate state the more likely a multiplicity arises.

An Endogenous and an Exogenous Signal

Here I question the robustness of the previous proposition to the introduction of exogenous signals. In formal terms, I am interested in assessing the existence of a multiplicity of REE when the price moves according to (21) and agents observe

$$\omega_i = \left\{E\left(p\right) + \zeta^{-1}\varepsilon + \eta_i,\ \zeta^{-1}\varepsilon + \phi_i\right\} \tag{30}$$

where $\phi_i \sim N\left(0, \zeta^{-2}\sigma_\phi\right)$. The strategy of the analysis mimics the one already discussed, so I will proceed more quickly through the same steps: I fix a linear forecasting rule, I recover the law of motion for the price, and finally I characterize the conditions for the existence of a multiplicity of REE.

Back to the model. This section analyses the original case entailed by (19). The endogenous signal is expressed as before, whereas the exogenous one $\varepsilon + \hat{\phi}_i$ is rescaled by ζ^{-1} so that $\phi_i \equiv \zeta^{-1}\hat{\phi}_i$.

The Actual Law of Motion

In this case, agents have two possibly correlated pieces of information. Their forecasting strategy is written as

$$E^i\left(p\right) = a_i\left(\zeta^{-1}\varepsilon + \phi_i\right) + b_i\left(E\left(p\right) + \zeta^{-1}\varepsilon + \eta_i\right), \tag{31}$$

where a_i and b_i are constants weighting respectively the exogenous and the endogenous signal. Since all agents use the rule above then by definition of aggregate expectation it is

$$E\left(p\right) = \frac{\mathbf{a} + \mathbf{b}}{1 - \mathbf{b}}\zeta^{-1}\varepsilon, \tag{32}$$

where

$$\mathbf{a} \equiv \int a_i \; di \quad \text{and} \quad \mathbf{b} \equiv \int b_i \; di \tag{33}$$

is the average weight across agents. An individual expectation can be rewritten as

$$E^i(p) = a_i \left(\zeta^{-1}\varepsilon + \phi_i\right) + b_i \left(\frac{\mathbf{a}+1}{1-\mathbf{b}}\zeta^{-1}\varepsilon + \eta_i\right) \tag{34}$$

and the actual law of motion of the market price (17) is given by

$$p = (1-\beta)\,\varepsilon + \beta\frac{\mathbf{a}+\mathbf{b}}{1-\mathbf{b}}\zeta^{-1}\varepsilon, \tag{35}$$

as functions of weights and exogenous shocks only. As before, the law of motion for the price has been recovered without using any a-priori guess on the form of the aggregate law.

The Set of Equilibria

Rational expectations imply restrictions on both the profile of $\{a_i\}_I$ and $\{b_i\}_I$. Spelling out the orthogonality conditions we can express the locus of REE as the profile $\{a_i = \mathbf{a}, b_i = \mathbf{b}\}_{i \in I}$ that is a solution to the following equation

$$\begin{aligned}(1-\beta)(\mathbf{a}+1)(1-\mathbf{b})\,\zeta + \beta(\mathbf{a}+\mathbf{b})(\mathbf{a}+1) + \\ -\,\mathbf{a}(\mathbf{a}+1)(1-\mathbf{b}) - \mathbf{b}\left((\mathbf{a}+1)^2 + (1-\mathbf{b})^2\sigma\right) = 0,\end{aligned}$$

with

$$\mathbf{a} = \frac{(1-\beta)\left(\zeta(1-\mathbf{b}) - \mathbf{b}\right)}{\mathbf{b} - \beta + (1+\sigma_\phi)(1-\mathbf{b})}.$$

The relation above entails an equation of the fifth degree in $\mathbf{b}$. Nevertheless one can divide the both sides by $-(1-\mathbf{b})^2(1-\beta+(1-\mathbf{b})\,\sigma_\phi)^{-2}$, ruling out the unfeasible solution $\mathbf{b} = 1$ and reducing the problem to the study of the following cubic fixed-point equation

$$\Phi_1\mathbf{b}^3 + \Phi_2\mathbf{b}^2 + \Phi_3\mathbf{b} + \Phi_4 = 0, \tag{36}$$

with

$$
\begin{aligned}
\Phi_1 &\equiv \sigma\sigma_\phi^2, \\
\Phi_2 &\equiv -\left(1+\sigma_\phi-\beta\right)2\sigma\sigma_\phi, \\
\Phi_3 &\equiv \left(1-\beta\right)\left(1+\zeta\right)\sigma_\phi\left(\left(1-\beta\right)\left(1+\zeta\right)+\sigma_\phi\right)+\left(1-\beta+\sigma_\phi\right)^2\sigma, \\
\Phi_4 &\equiv -\left(\left(1-\beta\right)\left(1+\zeta\right)+\sigma_\phi\right)\left(1-\beta\right)\zeta\sigma_\phi.
\end{aligned}
$$

So, as before, an eventual multiplicity would concern the existence of not more than three equilibria. Moreover, as expected, the fix equation (36) is equivalent to the one previously studied in the limit case of $\sigma_\phi \to \infty$ in which the exogenous signal is not informative on the aggregate shock. Therefore, we can state the following as a corollary of the proposition 3.

Corollary 6 *At the limit $\sigma_\phi \to \infty$ the parameter region where a multiplicity of equilibria arises corresponds to the one characterized in proposition 3.*

At the two limit cases $\sigma_\phi \to 0$ and $(\sigma, \sigma_\phi) \to (0,0)$ it is easy to check that the system has a unique equilibrium respectively at ($\mathbf{a} =\zeta, \mathbf{b} =0$) and ($\mathbf{a} =0, \mathbf{b} =\zeta/\left(1+\zeta\right)$), where both solutions entail a unitary weight on the aggregate shock. The proposition below establishes all the intermediate cases.

Proposition 7 *Given a $\zeta < -1$ and a $\beta < 1$, if*

$$
\sigma_\phi > -\left(1+\zeta\right)\left(1-\beta\right), \tag{37}
$$

then there exists a threshold σ_* that is monotonically decreasing in ζ and β such that for any $\sigma \in (0, \sigma_*)$ equation (36) has three real and distinct solutions. A unique real solutions obtains otherwise for a small enough σ.

Proof. Postponed in appendix. ■

The proposition above corresponds to proposition 2 back to the model. It confirms the intuition that the introduction of exogenous information makes a multiplicity of equilibria less likely. Still the potential of an endogenous signal to generate multiple equilibria increases with its precision. That is, the lower is σ the larger is the area were a multiplicity arises, or equivalently, a multiplicity of equilibria is *more likely with more endogenous information.*

Figure 3 shows a numerical exploration of the parametric space $\zeta < -1$ and $\beta < 1$ for some calibration of σ and σ_ϕ. Each box illustrates one different

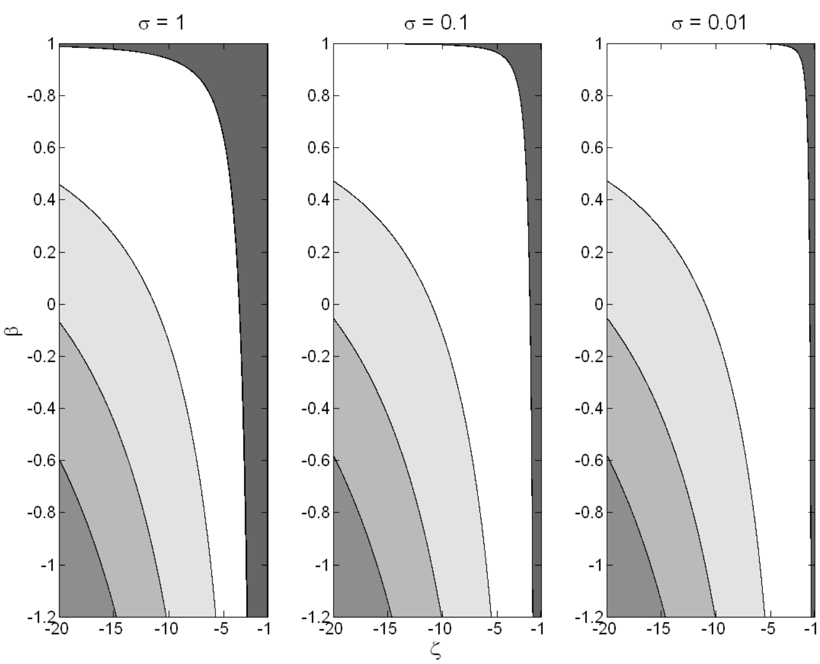

Figure 3: Multiplicity regions in the space $\zeta < -1$ and $\beta < 1$ for different calibrations of σ and σ_ϕ.

case among $\sigma = (1, 0.1, 0.01)$ whereas in all are considered the values $\sigma_\phi = (10, 20, 30, \infty)$. The white area is the one where a multiplicity arises for any of these σ_ϕ-values. With darker grey is denoted the area in which a multiplicity arises for increasing values of σ_ϕ, with the exception of the darkest one where a multiplicity never arises. The border line between the white and the darkest region represents the locus of calibrations for which a strictly smaller σ is necessary to obtain a multiplicity. All the other borders instead denote the lower bounds of the multiplicity area for the different σ_ϕ values. For all the calibrations such curves are very close to their limit (37) as σ approaches zero .

3.3 Additional results

The framework developed above can be easily extended to investigate a full range of static signal extraction problems beyond the specific case provided by the model. Now, suppose agents need to forecast a price (21), holding two private signals

$$\omega_i = \{x + \hat{\eta}_i,\ \varepsilon + \hat{\phi}_i\},$$

where now x represent a generic aggregate variable and all remaining shocks are defined as before. In particular consider x has the following linear law of motion

$$x = f(x)\,\varepsilon + \varkappa_x \left(E(p) - \varepsilon\right), \tag{38}$$

with $f(x)$ being the value of x in the scenario of perfect information and $\varkappa_x$ a scalar measuring the impact of an average expectation of displacement from the perfect information outcome. In particular, when neutrality of money holds then $f(x) = 1$ if x is a price, whereas $f(x) = 0$ if x is an aggregate quantity. The signal (38) can be written as (30) substituting ζ with the more general $\zeta_x \equiv \varkappa_x / \left(f(x) - \varkappa_x\right)$. Notice that if x is an aggregate quantity it is always $\zeta_x = -1$.

Once the problem is set up in this way one can go through the same steps as before to obtain the same fixed-point equation (36). We can explore therefore the possibility of a multiplicity of equilibria for values $\zeta_x \geq -1$. The following proposition states the results.

Proposition 8 *Given a $\zeta_x \geq -1$, a $\beta < 1$ and a finite σ_ϕ, then there exists a compact region contained in $\zeta_x > 8$ such that three determinate REE exist* *if*

$$\sigma_\phi > \frac{8\left(1+\zeta_x\right)}{\left(\zeta_x - 8\right)}\left(1-\beta\right), \tag{39}$$

for any σ lying in between two strictly positive boundaries

$$0 < m\left(\zeta_x, \beta, \sigma_\phi\right) < \sigma < M\left(\zeta_x, \beta, \sigma_\phi\right), \tag{40}$$

where $\lim_{\sigma_\phi \to \infty} m\left(\zeta_x, \beta, \sigma_\phi\right) = 3\left(1+\zeta_x\right)\left(1-\beta\right)$ *and* $\lim_{\sigma_\phi \to \infty} M\left(\zeta_x, \beta, \sigma_\phi\right) = 27\zeta_x\left(1-\beta\right)/8$*.*

Proof. Postponed in appendix. ■

This region is the one to which equilibria of the kind found in Amador and Weill (2010) belong to. Such equilibria arise for a degree of uncertainty on the endogenous state being in between two strictly positive boundaries. Differently from the equilibria characterized in proposition 7 these equilibria *disappear* for a small enough degree of private uncertainty. Moreover given σ the region in the parametric space $(\zeta_x, \beta, \sigma_\phi)$ that satisfies (39)-(40) is wider as σ increases, that is, a multiplicity arises as more likely with higher private uncertainty.

Figure 4 shows a numerical exploration of the parametric space $\zeta > 8$ and $\beta > 1$ for some calibration of σ and σ_ϕ. Each box illustrates one different case among $\sigma = (1, 27, 50)$ whereas in all are considered the values $\sigma_\phi = (10, \infty)$. As for figure 3, the white area is the one where a multiplicity always arises. The darker grey denotes the area in which a multiplicity arises for higher values of σ_ϕ, with the exception of the darkest one that is reserved for the area where a multiplicity never arises.

Merging the results from proposition 7 and 8 it is possible to derive a general claim on the necessary conditions for private endogenous signals to sustain a multiplicity of equilibria. It is stated as follows.

Remark 9 *Suppose neutrality of money holds and agents forecast a price as in (21) with $\beta < 1$, holding a single noisy signal of an arbitrary endogenous variable (38) and possibly some imperfect exogenous information, then:*

- *if x is a global price with $\varkappa_x > 1$ (that is $\zeta_x < -1$) then three REE can arise more likely provided a small enough degree of private uncertainty;*

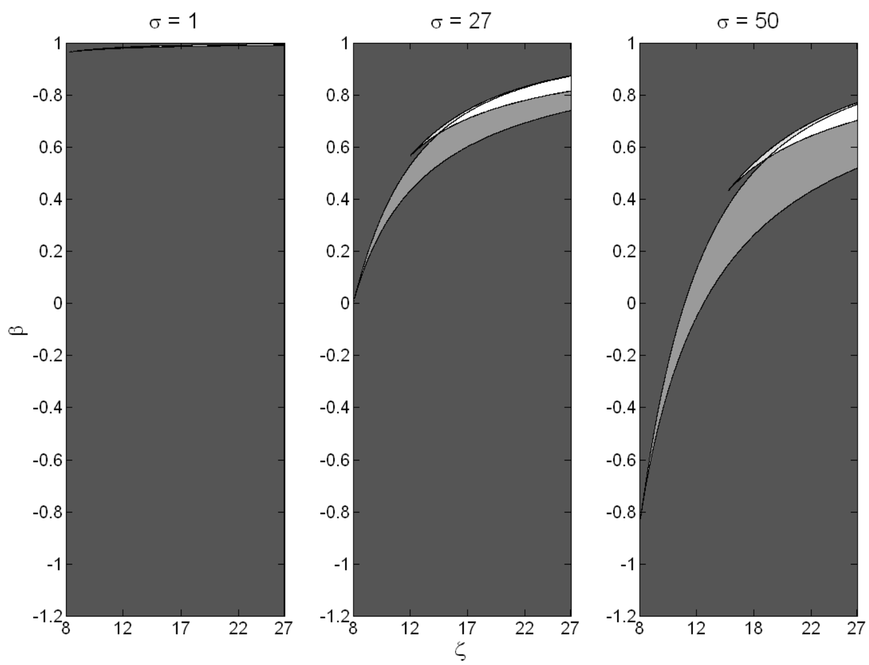

Figure 4: Multiplciity regions in the space $\zeta > 8$ and $\beta < 1$ for different calibrations of σ and σ_ϕ.

- *if x is a global price with $\varkappa_x \in (8/9, 1)$ (that is $\zeta_x > 8$) then three REE can arise more likely provided private uncertainty is between two strictly positive boundaries;*

- *if x is an aggregate quantity (that is $\zeta_x = -1$) a unique equilibrium always exists.*

This proposition sheds light on a property of neutrality of money that relates to the informational content of prices and quantities. It establishes that private uncertainty on the state of aggregate quantities does not qualitatively alter the set of REE. On the contrary, private uncertainty on a global price can originate a multiplicity of equilibria depending on its reaction to the average expectation of displacement from the perfect foresight outcome.

4 Multiplicity with perfect knowledge of fundamentals

In the cases investigated above, perfect knowledge of the aggregate shock leaves no room for a multiplicity of REE. In this section I will extend the model to isolate an extreme case in which endogenous signals can generate a multiplicity of equilibria even with common knowledge of the aggregate monetary shock ε. This result is due to strong pay-off externalities that arise with $\beta \in (1, \infty)$, a region that concludes the possible range of cases for the class of static signal extraction problems studied in this paper.

Extension of the Model

In this section I will introduce a policy parameter affecting the supply of money with the only aim of studying a particular case of signal extraction problem. The supply of money is modified as

$$M_i^s = \frac{1-\delta}{\delta} e^{\varepsilon + \hat{\phi}_i} \left(\frac{P}{\bar{P}} \right)^{-\varphi},$$

where now the quantity of money is conditional to the price emerging at the initial time with elasticity measured by φ. Ceteris paribus, for a negative (positive) φ the monetary authority increases (shrinks) the money supply injected in the initial period if the current price increases above its equilibrium steady state value $\bar{P}$. Such a modification changes the first order condition for money in

$$\lambda_i - p = \varphi p - \varepsilon - \hat{\phi}_i, \tag{41}$$

expressed as before as deviations from the unique deterministic equilibrium in log terms. With this specification, the actual law of motion of the consumption price is given by (see Appendix A.2.)

$$p = \frac{1}{1+\varphi}\varepsilon + \beta\left(E\left(p\right) - \frac{1}{1+\varphi}\varepsilon\right), \tag{42}$$

where, crucially, the expectational feedback

$$\beta = \frac{-\alpha\psi}{\left(1+\gamma-\alpha\right)\left(1+\varphi\right)+\alpha\psi\varphi},$$

can now be positive and specifically greater than one in the case

$$-1 < \varphi < -\frac{1+\gamma-\alpha}{1+\gamma-\alpha+\alpha\psi}, \tag{43}$$

that is when money supply increases with the current price. With $\beta > 1$ the consumption price itself reacts in opposite directions to an aggregate nominal shock in the two cases of no information and perfect foresight. Moreover, the overall price volatility is also enhanced by the policy choice. Notice however that whenever producers are able to perfectly forecast the consumption price, there is no role for policy because real quantities remain unchanged irrespective of nominal changes.

Timing and Information

To illuminate the main point I will assume some extreme informational assumptions. Suppose that now producers cannot simultaneously condition their expectations to the prices arising in the market; information is sticky so they fix a conditional demand schedule before the input markets open. Assume instead that the central bank announces with full transparency the aggregate realization of the monetary shock ε and also releases some qualitative expectation surveys on price expectations of the kind often published or commented by monetary authorities. In particular, I model the latter as an endogenous signal of producers' forecasts written as $s_i \equiv \{E\left(p\right) + \xi + \eta_i\}$, where $\xi \sim N\left(0,1\right)$ is an independently-drawn noise that is common to private signals across agents, whereas $\eta_i \sim N(0,\sigma)$ is an idiosyncratic individual-specific component. The common shock represents a statistical measurement error in the survey and the individual-specific one could be the result of genuine private interpretation. The information set can be now written as

$$\omega_i = \{\varepsilon,\ E(p) + \xi + \eta_i\}.$$

Two features matter here: first, both observational shocks are mutually independent and identically distributed in time and across agents; second, ξ represents a non-fundamental component. The aggregate component of the monetary shock ε is actually all the fundamental information agents need. In other words, there is no uncertainty on fundamentals in the model. Nevertheless, I will demonstrate that expectational complementarities can be so strong that non-fundamental equilibria are self-fulffilled by the collective use of correlated private endogenous signals. In this sense we can label *bad* policies the ones (the range of φ) for which a multiplicity arises.

The actual law of motion

As before, let us consider a linear forecasting rule,

$$E^i(\pi) = b_i \left(E(\pi) + \xi + \eta_i\right), \tag{44}$$

where $\pi \equiv p - (1+\varphi)^{-1}\varepsilon$ labels the distance of the actual price from $(1+\varphi)^{-1}\varepsilon$, the fundamental value that is now known. Again b_i denotes the coefficient weighting the expectational signal. Given that all agents follow the same strategy, by definition, one can write the average expectation as

$$E(\pi) = \frac{\mathbf{b}}{1-\mathbf{b}}\xi, \tag{45}$$

with $\mathbf{b} \neq 1$, where again $\mathbf{b}$ is the average weight across the population. We can then rewrite the forecasting rule of the agents

$$E^i(\pi) = b_i \left(\frac{1}{1-\mathbf{b}}\xi + \eta_i\right), \tag{46}$$

and the actual law of motion for the consumption price

$$\pi = \beta\frac{\mathbf{b}}{1-\mathbf{b}}\xi,$$

as non-linear functions of the exogenous shocks and the parameters only. Exactly as in the first case discussed in this paper, the equilibrium of the economy is entirely determined by a profile of b_i.

The set of equilibria

The equilibrium is now the same as stated by definition 1 with due correspondences. It is easy to check that the *fundamental equilibrium* - that is the one in which everybody puts a zero weight on the expectational signal - is always a REE. That is intuitive because agents already have all the fundamental information they need. Nevertheless equilibria different from the fundamental one - for which it is optimal for producers to put weight on the expectational signal - cannot be a-priori excluded. To obtain a closed form for the optimal b_i for a given $\mathbf{b}$ one needs to spell out the orthogonality conditions to pin down the set of optimal $\{b_i\}_I$. Imposing zero covariance between the expectational signal and the forecast error, one obtains that the best individual weight function

$$b_i(\mathbf{b}) = \frac{\beta \mathbf{b}}{1 + \sigma (1 - \mathbf{b})^2}, \tag{47}$$

in response to an average weight across the population. Notice that (47) corresponds to (28) with $\zeta = 0$. An equilibrium obtains when (47) holds for every i, so that every agent puts the same weight $b_i = \mathbf{b}$ on the expectational signal. The following proposition states a simple analytical result.

Proposition 10 *The fundamental equilibrium entailed by $b_i = \mathbf{b} = 0$ for each $i \in I$ is always an equilibrium of the economy. Two distinct non-fundamental equilibria exist for $b_i = \mathbf{b}_\pm$ for each $i \in I$ taking values*

$$\mathbf{b}_\pm = 1 \pm \sqrt{(\beta - 1)/\sigma},$$

if and only if $\beta > 1$.

This case is one in which a multiplicity of equilibria is sustained by a pure pay-off externality effect arising for strong values of expectational complementarity $\beta > 1$. Even if expectational signals are not informative on fundamentals, private signals provide information on the average second-order belief of others when the price overreacts to the average forecasting mistake. Non-fundamental equilibria necessarily arise with partially correlated expectational signals and disappear in the limit cases of perfect correlation or independence. Hence, as before the multiplicity of equilibria are strictly linked to arbitrarily small amount of private uncertainty due to the idiosyncratic observational noises. In particular, non-fundamental fluctuations are sustained by a signal extraction problem on an additional imperfect information being a common non-fundamental shock. This implies that non-fundamental equilibria exhibit necessarily aggregate non-fundamental volatility driven by the correlated component in private expectational signals.

The key mechanism beyond this result is similar to the one underlying an exogenous sunspot equilibrium. To see this notice that for $\beta > 1$ it is possible to build up private sunspot equilibria when agents observe truly exogenous signals for an ad-hoc value of cross-sectional correlation. Suppose to replace the private endogenous signal s_i with an exogenous white noise signals $\varsigma_i = \{\xi + \eta_i\}$ where the shocks have the same characteristics as before. For the specific value of private uncertainty $\sigma = \beta - 1$ - that requires necessarily $\beta > 1$ - we obtain a continuum of *indeterminate* equilibria exhibiting non-fundamental aggregate volatility driven by ξ.[9] In our original case, instead the non-linearity introduced by endogenous signals prevents indeterminacy and generates only two non-zero *determined* equilibria values $\mathbf{b}$ for which non-fundamental volatility shows up. In other words, the non-linearity of the optimal individual weight (28) is key to determinacy.

5 Conclusions

This paper has laid out the general conditions under which private uncertainty on a certain endogenous state of the economy determine a multiplicity of equilibria in models that have a unique equilibrium under perfect knowledge or absence of endogenous signals. This occurs when agents are privately uncertain about a global price that exhibits opposite reactions to an aggregate shock in the scenarios of no information and perfect foresight. The results hold even in absence of pay-off complementarities, that is, when the unobserved variable to be forecasted is purely exogenous. Nevertheless, the paper also discussed a case in which a multiplicity arises with strong pay-off externalities when the fundamentals are perfectly known.

Some important theoretical questions are left in the background. The existence of a multiplicity of determinate equilibria raises the issue of agents'

[9] This mechanism is the same at work in Benhabib, Wang and Wen (2012). Nevertheless in their paper the sunspot equilibrium is determinate because the signal includes private fundamental disturbances.

coordination. Different approaches inquiring the out-of-equilibrium dynamics of agents' beliefs have been implemented to answer this question. In particular, a selection on a unique equilibrium could occur conditionally to a given learning scheme. Some preliminary work in this direction has been done in Gaballo (2011). Another important issue concerns the extent to which endogenous signals can originate a multiplicity in dynamic settings. The model I presented here is essentially static in nature, as the realization of the shock is not informative about the future course of the economy. When this is not the case agents accumulate additional correlated information through time. As showed by Angeletos, Hellwig and Pavan (2007) the dynamic interaction between exogenous and endogenous information can still sustain a multiplicity of equilibria in the context of coordination games encompassing the currency attack model. How this can survive in a microfounded macro-model with a unique equilibrium under complete information is a question that hopefully the analysis in this paper can help to address in a near future.

Appendix

A.1 Demand for money

The intertemporal first-order condition for money in island i is

$$\frac{\Lambda_{i,0}}{P_0} = \delta \mathbf{E}_0 \left[\frac{\Lambda_{1,i}}{P_1} \right] + \frac{\delta}{M_i^s},$$

after substituting for the equilibrium relation $M_{i,0} = M_i^s$. Given that it is common knowledge that uncertainty on the aggregate monetary shock realization will vanish after one period, then the relation collapses to the equilibrium restriction valid for any i at every $t + \tau > t$

$$\mathbf{E}_{t+\tau-1} \left[\frac{\Lambda_{i,t+\tau}}{P_{t+\tau}} \right] = \frac{\delta}{1-\delta} \frac{\delta}{M_i^s},$$

where $\mathbf{E}_t \left[M_{i,t+\tau} \right] = M_{i,t+\tau} = M_i^s$. Therefore, at $t = 0$ one can rewrite the first order condition for money demand as

$$\frac{\Lambda_{i,0}}{P_0} = \frac{\delta}{1-\delta} \frac{\delta}{M_i^s} + \frac{\delta}{M_i^s} = \frac{\delta}{1-\delta} \frac{1}{M_i^s},$$

that is equivalent to (14) after a logaritmic transformation.[10] Notice that both the assumption of money in the utility function and the assumption of permanent shock hitting at the initial time are introduced to yield a multiplicative demand for money as shown above. This allows for the expression of the whole system in log-terms without any approximation (see below).

[10] Explosive solutions can be ruled out using an argument from Obstfeld and Rogoff (1983) (see Amador and Weill, 2010, appendix A.1).

A.2 Aggregate relations and the stochastic steady state

A.2.1 First-order conditions and the deterministic steady state

The whole list of first-order conditions of the model are

$$
\begin{aligned}
W_i &= \alpha E^i(P) L_i^{\alpha-1} K_i^{1-\alpha} && \text{(48a)}\\
R_i &= (1-\alpha) E^i(P) L_i^{\alpha} K_i^{-\alpha} && \text{(48b)}\\
Y_i &= L_i^{\alpha} K_i^{1-\alpha} && \text{(48c)}\\
L_i^{\gamma} &= W_i \Lambda_i P^{-1} && \text{(48d)}\\
C_i^{-\psi} &= \Lambda_i && \text{(48e)}\\
\Lambda_i P^{-1} &= e^{-\varepsilon-\hat{\phi}_i} && \text{(48f)}\\
R &= e^{-\hat{\eta}_i} R_i && \text{(48g)}\\
K_i &= e^{-\hat{\eta}_i} Z_i && \text{(48h)}
\end{aligned}
$$

where the first three refer to the problem of final producers, the last two to the problem of intermediate producers and the rest to the consumer's problem.

The unique deterministic price and aggregate production obtain respectively as $P^* = \alpha^{\frac{-\alpha\psi}{1+\gamma-\alpha+\alpha\psi}}$ and $Y^* = \alpha^{\frac{1}{1+\gamma-\alpha+\alpha\psi}}$ after solving the system for $\varepsilon = 0$, $\eta_i = 0$ and constant actions across agents.

A.2.2. Uniqueness of the temporary equilibrium

The requirement that the equilibrium has a log-linear representation requires that any variable in the model has the form

$$X_i = \bar{X} e^{\mathbf{x}_{i,1}\varepsilon + \mathbf{x}_{i.2}\hat{\phi}_i + \mathbf{x}_{i,3}\hat{\eta}_i}$$

that is any deviation of $X_i/\bar{X}$ from the stochastic steady state $\bar{X}$ is a linear combination of the shocks hitting the economy. In particular, the aggregate values will depend on the aggregate shock ε only. Let us prove here that given a (possibly non optimal) symmetric forecasting rule for final producers

$$E^i(P) = \bar{P} e^{\mathbf{e}_{i,1}\varepsilon + \mathbf{e}_{i,2}\hat{\phi}_i + \mathbf{e}_{i,3}\hat{\eta}_i} \tag{49}$$

then a unique log-linear (temporary) equilibrium exists: that is, for each variable in the model there exists a unique steady state and a unique log-linear deviation implied by fixing (49).

Production side. The aggregation of the demand for endowment

$$Z = \bar{Z}e^{\mathbf{z}_1\varepsilon} = \int Z_i di = \int \bar{Z}_i e^{\mathbf{z}_{i,1}\varepsilon + \mathbf{z}_{i,2}\hat{\phi}_i + \mathbf{z}_{i,3}\hat{\eta}_i} di = \bar{Z}_i e^{\mathbf{z}_{i,1}\varepsilon + \frac{\mathbf{z}_{i,2}^2\sigma_\phi + \mathbf{z}_{i,3}^2\hat{\sigma}_i}{2}}$$

which satisfies the market clearing condition $Z = 1$ for any ε for

$$\bar{Z}_i = e^{-\frac{\mathbf{z}_2^2\sigma_\phi + \mathbf{z}_3^2\hat{\sigma}_i}{2}}$$

and $\mathbf{z}_1 = 0$. Using (48h) we obtain

$$K = \int K_i di = \int \bar{K}_i e^{\mathbf{k}_{i,1}\varepsilon + \mathbf{k}_{i,2}\hat{\phi}_i + \mathbf{k}_{i,3}\hat{\eta}_i} di = \int \bar{Z}_i e^{\mathbf{z}_1\varepsilon + \mathbf{z}_2\hat{\phi}_i + (\mathbf{z}_3 - 1)\hat{\eta}_i} di = e^{\frac{\hat{\sigma}_\eta}{2}},$$

where therefore $\mathbf{k}_{i,1} = \mathbf{z}_{i,1} = 0$, $\mathbf{k}_{i,2} = \mathbf{z}_{i,2}$ and $\mathbf{k}_{i,3} = \mathbf{z}_{i,3} - 1$ and $\bar{Z}_i = \bar{K}_i$. According to (48g)

$$R = \bar{R}_i e^{\mathbf{r}_{i,1}\varepsilon + \mathbf{r}_{i,2}\hat{\phi}_i + (\mathbf{r}_{i,3-1})\hat{\eta}_i}$$

where

$$R = \bar{R}e^{\mathbf{r}_1\varepsilon} = \int R_i di = \bar{R}_i e^{\mathbf{r}_{i,1}\varepsilon + \frac{\mathbf{r}_{i,2}^2\sigma_\phi + \left(\mathbf{r}_{i,3} - 1\right)^2\hat{\sigma}_i}{2}} \tag{50}$$

so that $\bar{R} = \bar{R}_i$,$\mathbf{r}_1 = \mathbf{r}_{i,1}$,$\mathbf{r}_{i,2} = 0$ and $\mathbf{r}_{i,3} = 1$ entail the unique solution. Plugging (48f) in (48d) and the resulting in (48a) we get

$$L_i = \alpha^{\frac{1}{1-\alpha+\gamma}} e^{\frac{-(\varepsilon+\hat{\phi}_i)}{1-\alpha+\gamma}} E^i\left(P\right)^{\frac{1}{1-\alpha+\gamma}} K_i^{\frac{1-\alpha}{1-\alpha+\gamma}} \tag{51a}$$

and substituting the expression above in (48b) we have

$$R_i = (1-\alpha)\,\alpha^{\frac{\alpha}{1-\alpha+\gamma}} e^{\frac{-\alpha(\varepsilon+\hat{\phi}_i)}{1-\alpha+\gamma}} E^i\left(P\right)^{\frac{1+\gamma}{1-\alpha+\gamma}} K_i^{\frac{-\alpha\gamma}{1-\alpha+\gamma}}$$

where $R_i = \bar{R}e^{\mathbf{r}_1\varepsilon + \hat{\eta}_i}$. So the following restrictions on log-deviations

$$\begin{aligned}
\mathbf{r}_1 &= \frac{(1+\gamma)\,\mathbf{e}_1 - \alpha}{1-\alpha+\gamma}, \\
0 &= \frac{1+\gamma}{1-\alpha+\gamma}\mathbf{e}_2 - \frac{\alpha\gamma}{1-\alpha+\gamma}\mathbf{k}_2 - \frac{\alpha}{1-\alpha+\gamma}, \\
1 &= \frac{1+\gamma}{1-\alpha+\gamma}\mathbf{e}_3 - \frac{\alpha\gamma}{1-\alpha+\gamma}\mathbf{k}_3
\end{aligned}$$

and on the steady state

$$\bar{R}_i = (1-\alpha)\,\alpha^{\frac{\alpha}{1-\alpha+\gamma}}\,\bar{Z}_i^{\frac{-\alpha\gamma}{1-\alpha+\gamma}}\,\bar{P}^{\frac{1+\gamma}{1-\alpha+\gamma}}$$

are implied. Therefore for *given* $\bar{P}, \mathbf{e}_1$,$\mathbf{e}_2$ and $\mathbf{e}_3$ there exist unique steady state values of R, R_i, K_i, Z_i and unique relative deviations defined by the relations above. Once K_i is defined then also deviations from L_i are uniquely defined with a steady state

$$\bar{L}_i = \alpha^{\frac{1}{1-\alpha+\gamma}}\,\bar{P}^{\frac{1}{1-\alpha+\gamma}}\,\bar{Z}_i^{\frac{1-\alpha}{1-\alpha+\gamma}}.$$

Analogously we can find the unique implied steady state and deviation of W_i and Y_i working respectively on (48d) (after plugging (48f) in) and (48c). In particular

$$Y_i = \alpha^{\frac{\alpha}{1-\alpha+\gamma}}\,e^{\frac{-\alpha\left(\varepsilon+\hat{\phi}_i\right)}{1-\alpha+\gamma}}\,E^i\,(P)^{\frac{\alpha}{1-\alpha+\gamma}}\,K_i^{\frac{(1-\alpha)(1+\gamma)}{1-\alpha+\gamma}}$$

which implies

$$\begin{aligned}
\mathbf{y}_1 &= \frac{\alpha\,(\mathbf{e}_1{-}1)}{1-\alpha+\gamma}, \\
\mathbf{y}_2 &= \frac{\alpha\mathbf{e}_2 + (1-\alpha)\,(1+\gamma)\,\mathbf{k}_2{-}\alpha}{1-\alpha+\gamma}, \\
\mathbf{y}_3 &= \frac{\alpha\mathbf{e}_3 + (1-\alpha)\,(1+\gamma)\,(\mathbf{k}_3{+}1)}{1-\alpha+\gamma},
\end{aligned}$$

and steady state $\bar{Y}_i = \alpha^{\frac{\alpha}{1-\alpha+\gamma}}\,\bar{P}^{\frac{\alpha}{1-\alpha+\gamma}}$.

Demand side. From (48e) we have

$$C_i = \bar{P}^{-\frac{1}{\psi}}\,e^{-\frac{1}{\psi}\left((\mathbf{p}_1-1)\varepsilon-\hat{\phi}_i\right)}$$

using (48f) after substituting for (48e), which gives the restrictions $\mathbf{c}_1 = -\frac{1}{\psi}\,(\mathbf{p}_1-1)$, $\mathbf{c}_2 = -1$, $\mathbf{c}_3 = 0$ and steady state $\bar{C}_i = \bar{P}^{-\frac{1}{\psi}}$. The clearing condition for the good market is therefore

$$\int Y_i di = \bar{Y} e^{\frac{\alpha(\mathbf{e}_1-1)}{1-\alpha+\gamma}\varepsilon} = \bar{P}^{-\frac{1}{\psi}}\,e^{-\frac{1}{\psi}(\mathbf{p}_1-1)\varepsilon+\frac{\hat{\sigma}_\phi}{2\psi^2}}$$

with steady state$\bar{Y} = \alpha^{\frac{\alpha}{1-\alpha+\gamma}}\,\bar{P}^{\frac{\alpha}{1-\alpha+\gamma}}\,e^{\left(\mathbf{y}_{i,2}^2\sigma_\phi+\mathbf{y}_{i,3}^2\sigma_\eta\right)/2}$. Finally we have

$$\bar{P}^{-\frac{1}{\psi}}\,e^{\frac{\sigma_\phi}{2\psi^2}} = \alpha^{\frac{\alpha}{1-\alpha+\gamma}}\,\bar{P}^{\frac{\alpha}{1-\alpha+\gamma}}\,e^{\frac{\mathbf{y}_{i,2}^2\sigma_\phi+\mathbf{y}_{i,3}^2\sigma_\eta}{2}}$$

that is

$$\bar{P} = \alpha^{-\frac{\alpha\psi}{1-\alpha+\gamma+\alpha\psi}} e^{-\frac{\left(\psi \mathbf{y}_{i,2}^2 - \psi^{-1}\right)\sigma_\phi + \psi \mathbf{y}_{i,3}^2 \sigma_\eta}{2}}$$

and hence a one-to-one relation between $\mathbf{p}_1$ and $\mathbf{e}_1$. Notice that for σ_ϕ and σ_η going to zero the stochastic steady state price $\bar{P}$ is equal to the deterministic one P^*.

A.2.3. The system of log-linear deviations from the steady state

Therefore the aggregate economy in its more general specification (including $\varphi = 0$ as a special case) can be represented in log linear terms by the following system of first order conditions

$$\begin{bmatrix} w \\ r \\ l \\ \lambda \\ y \\ p \end{bmatrix} = \begin{bmatrix} 0 & 0 & \alpha - 1 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha^{-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1+\varphi \\ 0 & 0 & 0 & -\frac{1}{\psi} & 0 & 0 \\ 1 & 0 & -\gamma & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} w \\ r \\ l \\ \lambda \\ y \\ p \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & -1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} E(p) \\ \varepsilon \end{bmatrix}$$

where $x_i = \ln X_i - \ln \bar{X}_i$ with $\bar{X}_i$ being the individual action at its equilibrium steady state and letters without the index i representing aggregate values. In particular $\bar{X}_i$ is the individual action at the specific realization $\varepsilon = 0$ and $\eta_i = \phi_i = 0$. The system is solved as

$$\begin{bmatrix} w \\ r \\ l \\ \lambda \\ y \\ p \end{bmatrix} = \begin{bmatrix} \frac{\gamma(1+\varphi)+\alpha\psi\varphi}{(1+\gamma-\alpha)(1+\varphi)+\alpha\psi\varphi} & \frac{1-\alpha}{(1+\gamma-\alpha)(1+\varphi)+\alpha\psi\varphi} \\ \frac{(1+\gamma)(1+\varphi)+\alpha\psi\varphi}{(1+\gamma-\alpha)(1+\varphi)+\alpha\psi\varphi} & \frac{-\alpha}{(1+\gamma-\alpha)(1+\varphi)+\alpha\psi\varphi} \\ \frac{\varphi+1}{(1+\gamma-\alpha)(1+\varphi)+\alpha\psi\varphi} & \frac{-1}{(1+\gamma-\alpha)(1+\varphi)+\alpha\psi\varphi} \\ \frac{-\alpha\psi(\varphi+1)}{(1+\gamma-\alpha)(1+\varphi)+\alpha\psi\varphi} & \frac{\alpha\psi}{(1+\gamma-\alpha)(1+\varphi)+\alpha\psi\varphi} \\ \frac{\alpha(\varphi+1)}{(1+\gamma-\alpha)(1+\varphi)+\alpha\psi\varphi} & \frac{-\alpha}{(1+\gamma-\alpha)(1+\varphi)+\alpha\psi\varphi} \\ \frac{-\alpha\psi}{(1+\gamma-\alpha)(1+\varphi)+\alpha\psi\varphi} & \frac{\gamma-\alpha+\alpha\psi+1}{(1+\gamma-\alpha)(1+\varphi)+\alpha\psi\varphi} \end{bmatrix} \begin{bmatrix} E(p) \\ \varepsilon \end{bmatrix},$$

giving the log relations used in the text.

A.3 Proof. of propositions 3, 7 and 8

A.3.1. Preliminaries

The fixed-point equations (29) and (36) have the same structure. Here I will prove a lemma that will be useful in the following proofs.

Lemma 11 *The equation*

$$x^3 - 2\vartheta x^2 + \left(\vartheta^2 - \mu\right) x - \kappa = 0, \tag{52}$$

with $\vartheta > 0$ *and* κ, μ *real scalars, has three real roots if and only if*

$$\mu > -\vartheta^2/3, \tag{53}$$

and

$$\kappa \in [k_-, k_+], \tag{54}$$

where

$$k_- = -\left(\frac{2}{9}\mu + \frac{2}{27}\vartheta^2\right)\sqrt{\vartheta^2 + 3\mu} - \frac{2}{3}\mu\vartheta + \frac{2}{27}\vartheta^3, \tag{55}$$

$$k_+ = \left(\frac{2}{9}\mu + \frac{2}{27}\vartheta^2\right)\sqrt{\vartheta^2 + 3\mu} - \frac{2}{3}\mu\vartheta + \frac{2}{27}\vartheta^3. \tag{56}$$

Proof. Consider the equation rewritten as

$$\underbrace{x\left(\vartheta - x\right)^2}_{\equiv y(x)} = \underbrace{\kappa + \mu x}_{\equiv z(b)}, \tag{57}$$

where $y(x)$ and $z(x)$ are two real continuous and differentiable functions defined as respectively the left-hand term and the right-hand term of the equation above. The latter is a line with intercept κ and slope μ, whereas the former is a cubic passing through the origin with roots at $(0,0)$ and $(\vartheta, 0)$, and with local maximum and minimum respectively at $\left(\vartheta/3, 4\vartheta^3/27\right)$ and $(\vartheta, 0)$.

Consider two points $\{\hat{x}_\pm(\mu), y(\hat{x}_\pm(\mu))\}$ such that the slope of the curve is equal to a given constant μ. These are

$$\hat{x}_\pm(\mu) = \frac{2\vartheta \pm \sqrt{\vartheta^2 + 3\mu}}{3},$$

$$y(\hat{x}_\pm(\mu)) = \left(\frac{2\vartheta \pm \sqrt{\vartheta^2 + 3\mu}}{3}\right)\left(\vartheta - \frac{2\vartheta \pm \sqrt{\vartheta^2 + 3\mu}}{3}\right)^2,$$

where $\hat{x}_{\pm}(\mu)$ solves $y'(\hat{x}_{\pm}(\mu)) = \mu$.

From the theorem of the mean value we know that if there exist at least two distinct values x_1 and x_2 with $x_1 < x_2$ such that $y(x_1) = z(x_1)$ and $y(x_2) = z(x_2)$ (that is multiple intersections exist) then it also exists an intermediate value $x_3 \in [x_1, x_2]$ such that $y'(x_3) = \mu$. Therefore if the latter does not exist then the former condition is violated. Hence a second restriction for $z(x)$ having three intersections with $y(x)$ provides for (53). In this region we have to assess whether or not $\kappa \in [k_-, k_+]$ where

$$
\begin{aligned}
k_+ &= y(\hat{x}_-(\mu)) - \mu\hat{x}_-(\mu), \\
k_- &= y(\hat{x}_+(\mu)) - \mu\hat{x}_+(\mu),
\end{aligned}
$$

are the intercepts of the two lines having slope μ and being tangents at a point of $y(x)$. This is a necessary and intersection with $y(x)$. ■

Remark 12 *Notice that within the parameter region $\mu > -\vartheta^2/3$ with $\vartheta > 0$ it is true that:*

i *k_+ is always positive with a minimum at 0 and $\partial k_+/\partial\mu > 0$ for $\mu > \vartheta^2$;*

ii *k_- is negative for $\mu > 0$ and has a maximum at $8\vartheta^3/27$ corresponding to the lower parameter bound $\mu = -\vartheta^2/3$;*

iii *k_- is a decreasing and concave in μ, that is*

$$
\frac{\partial k_-}{\partial \mu} = -\frac{1}{3}\left(\sqrt{\vartheta^2 + 3\mu} + 2\vartheta\right) < 0 \quad \textit{and} \quad \frac{\partial k_-}{\partial \mu^2} = -\frac{1}{2\sqrt{\vartheta^2 + 3\mu}} < 0,
$$

where in particular $\partial k_-/\partial\mu > -\vartheta$ for $\mu < 0$.

A.3.2. Proposition 3

Proof. The fixed-point equation (29) corresponds to (52) with

$$
\begin{aligned}
\vartheta &= 1, \\
\kappa &= \zeta\frac{(1-\beta)}{\sigma}, \\
\mu &= -(1+\zeta)\frac{(1-\beta)}{\sigma}.
\end{aligned}
$$

To check the existence of multiple solutions we are going to investigate when (54) holds with $\zeta < -1$ and $\beta < 1$, that is, in the case $\mu > 0$, $\kappa < 0$. We need to prove that given a couple $\left(\bar{\zeta}, \bar{\beta}\right)$ there exists a small enough σ such that a (54) is satisfied. Firstly, let us write down the derivative of k_- and κ with respect to σ given respectively by

$$\frac{\partial k_-}{\partial \mu}\frac{\partial \mu}{\partial \sigma} = \frac{\partial k_-}{\partial \mu}(1+\zeta)\frac{(1-\beta)}{\sigma^2} \quad \text{and} \quad \frac{\partial \kappa}{\partial \sigma} = -\zeta\frac{(1-\beta)}{\sigma^2}, \tag{58a}$$

so that we have

$$\frac{\partial k_-}{\partial \sigma} > \frac{\partial \kappa}{\partial \sigma} \quad \text{whenever} \quad \frac{\partial k_-}{\partial \mu}(1+\zeta) > -\zeta, \tag{59}$$

where notice $\partial k_- \backslash \partial \sigma$ and $\partial \kappa \backslash \partial \sigma$ are both positive since $\partial k_- \backslash \partial \mu$ is always negative (remark 12.iii). Now consider a point $\left(\bar{\zeta}, \bar{\sigma}, \bar{\beta}\right)$ such that

$$\bar{\kappa}\left(\bar{\zeta}, \bar{\sigma}, \bar{\beta}\right) \leq \bar{k}_-\left(\bar{\zeta}, \bar{\sigma}, \bar{\beta}\right) \leq 0 < \bar{k}_+\left(\bar{\zeta}, \bar{\sigma}, \bar{\beta}\right),$$

that is (54) is violated. Given that

$$\lim_{\sigma \to 0} \frac{\partial k_-}{\partial \mu} = -\infty \leq -\frac{\bar{\zeta}}{\bar{\zeta}+1} \tag{60}$$

it is always true, then (59) holds and so k_- always decreases monotonically faster than κ as σ approaches its lower bound. Since k_- is negative whereas k_+ remains always positive (remark 12.i and 12.ii), there must exist a σ_* small enough such that (54) holds for all $\sigma < \sigma^*$. Finally notice that such threshold σ_* must increase with decreasing $\bar{\beta}$ (and $\bar{\zeta}$) because ceteris paribus it increases μ (and at the same time relaxes the constraint (60)). For a proof that a multiplicity does not obtain for $\sigma \to 0$ in the case of $\zeta \geq -1$ see A.3.4. ■

A.3.3. Proposition 7

Proof. The fixed-point equation (36) corresponds to (52) with

$$\begin{aligned}
\vartheta &= \frac{1-\beta+\sigma_\phi}{\sigma_\phi}, \\
\kappa &= \zeta\frac{1-\beta}{\sigma}\left(\frac{(1-\beta)(1+\zeta)+\sigma_\phi}{\sigma_\phi}\right), \\
\mu &= -(1+\zeta)\frac{1-\beta}{\sigma}\left(\frac{(1-\beta)(1+\zeta)+\sigma_\phi}{\sigma_\phi}\right)
\end{aligned}$$

and it coincides with (29) in the limit of $\sigma_\phi \to \infty$. To check the existence of multiple solutions for $\zeta < -1$ and $\beta < 1$ we need to assess how the new parameter σ_ϕ changes existence conditions uncovered before. Observe that

$$\frac{\partial k_\pm}{\partial \mu}\frac{\partial \mu}{\partial \sigma} = \frac{\partial k_\pm}{\partial \mu}\left(1+\zeta\right)\frac{\left(1-\beta\right)}{\sigma^2}\left(\frac{\left(1-\beta\right)\left(1+\zeta\right)+\sigma_\phi}{\sigma_\phi}\right) \tag{61a}$$

and

$$\frac{\partial \kappa}{\partial \sigma} = -\zeta\frac{\left(1-\beta\right)}{\sigma^2}\left(\frac{\left(1-\beta\right)\left(1+\zeta\right)+\sigma_\phi}{\sigma_\phi}\right),$$

so that

$$\frac{\partial k_-}{\partial \sigma} > \frac{\partial \kappa}{\partial \sigma} \text{ if } \frac{\partial k_-}{\partial \mu}\left(\zeta+1\right) > -\zeta \text{ provided } \sigma_\phi > -\left(1-\beta\right)\left(1+\zeta\right), \tag{62a}$$

$$\frac{\partial k_-}{\partial \sigma} > \frac{\partial \kappa}{\partial \sigma} \text{ if } \frac{\partial k_-}{\partial \mu}\left(\zeta+1\right) < -\zeta \text{ provided } \sigma_\phi < -\left(1-\beta\right)\left(1+\zeta\right). \tag{62b}$$

The argument put forward for the proof of proposition 3 can be exactly replicated for a given $\left(\bar{\beta},\bar{\zeta},\bar{\sigma}_\phi\right)$ such that $\bar{\sigma}_\phi > -\left(1-\bar{\beta}\right)\left(1+\bar{\zeta}\right)$. The latter is therefore a sufficient condition for a multiplicity.

Let us focus therefore on the case $\bar{\sigma}_\phi < -\left(1-\bar{\beta}\right)\left(1+\bar{\zeta}\right)$ for which $\mu < 0$ and $\kappa > 0$. Since $\lim_{\sigma\to\infty}\mu = 0$ and $\lim_{\sigma\to\infty}\partial k_-/\partial\mu = -\vartheta$ we have

$$\left|\lim_{\sigma\to\infty}\frac{\partial k_-}{\partial \mu}\right|_{\bar{\sigma}_\phi < -\left(1-\bar{\beta}\right)\left(1+\bar{\zeta}\right)} < \left|\lim_{\sigma\to\infty}\frac{\partial k_-}{\partial \mu}\right|_{\bar{\sigma}_\phi = -\left(1-\bar{\beta}\right)\left(1+\bar{\zeta}\right)} = -\frac{\bar{\zeta}}{1+\bar{\zeta}},$$

since for decreasing σ_ϕ ϑ increases. The latter is a necessary condition for the existence of a multiplicity. Nevertheless, since $\partial k_-/\partial\mu^2$ is negative (remark 12.iii) then it is also true that for lower values of σ,

$$\begin{aligned} -\frac{2}{3}\vartheta = \left|\lim_{\mu\to-\vartheta^2/3}\frac{\partial k_-}{\partial \mu}\right|_{\bar{\sigma}_\phi < -\left(1-\bar{\beta}\right)\left(1+\bar{\zeta}\right)} > \\ > \left|\frac{\partial k_-}{\partial \mu}\right|_{\bar{\sigma}_\phi < -\left(1-\bar{\beta}\right)\left(1+\bar{\zeta}\right)} > \left|\lim_{\sigma\to\infty}\frac{\partial k_-}{\partial \mu}\right|_{\bar{\sigma}_\phi < -\left(1-\bar{\beta}\right)\left(1+\bar{\zeta}\right)}, \end{aligned}$$

as μ decreases until it reaches its lower bound $-\vartheta^2/3$. Therefore when

$$-\frac{2}{3}\vartheta \le -\frac{\bar{\zeta}}{1+\bar{\zeta}},$$

that is when

$$\sigma_\phi \leq -\frac{2}{1-2\bar{\zeta}}\left(1-\bar{\beta}\right)\left(1+\bar{\zeta}\right) < -\left(1-\bar{\beta}\right)\left(1+\bar{\zeta}\right), \tag{63}$$

(62b) is never satisfied so that $0 > \partial\kappa/\partial\sigma > \partial k_-/\partial\sigma$, κ increases always strictly slower than k_- and so a multiplicity does not arise. In other words, (63) is a sufficient condition for non existence of a multiplicity. This demonstrates that for a σ_ϕ small enough a multiplicity does not exists irrespective of σ.

For intermediate values

$$-\frac{2}{2-2\zeta}\left(1-\beta\right)\left(1+\zeta\right) < \sigma_\phi < -\left(1-\beta\right)\left(1+\zeta\right),$$

instead a multiplicity would arise as before as σ decreases since (62b) would hold from a certain point onward. Nevertheless, notice that for small enough σ now (53) becomes binding so that the following restriction applies

$$-\frac{\left(1-\beta+\sigma_\phi\right)^2}{3\sigma_\phi^2} < -\frac{\left(1-\beta\right)\left(1+\zeta\right)\left(\left(1-\beta\right)\left(1+\zeta\right)+\sigma_\phi\right)}{\sigma\sigma_\phi}, \tag{64}$$

that entails a lower bound to σ

$$\sigma > m\left(\zeta,\beta,\sigma_\phi\right) \equiv \frac{3\sigma_\phi\left(1-\beta\right)\left(1+\zeta\right)\left(\left(1-\beta\right)\left(1+\zeta\right)+\sigma_\phi\right)}{\left(1-\beta+\sigma_\phi\right)^2}, \tag{65}$$

that constitutes a third necessary condition for existence of a multiplicity. In particular, notice that

$$\frac{\partial m\left(\zeta,\beta,\sigma_\phi\right)}{\partial\sigma_\phi} = \frac{3\left(\beta-1\right)^2\left(\zeta+1\right)\left(\left(1+\zeta\right)\left(1-\beta\right)+\left(1-\zeta\right)\sigma_\phi\right)}{\left(1-\beta+\sigma_\phi\right)^3} < 0,$$

since

$$-\frac{1}{1-\zeta}\left(1+\zeta\right)\left(1-\beta\right) < -\frac{2}{1-2\zeta}\left(1-\beta\right)\left(1+\zeta\right) < \sigma_\phi,$$

that is as σ_ϕ increases m lowers. So, for a multiplicity to arise, the smaller is σ the closer σ_ϕ has to be to $-\left(1-\bar{\beta}\right)\left(1+\bar{\zeta}\right)$. This demonstrates that $\sigma_\phi > -\left(1-\beta\right)\left(1+\zeta\right)$ is the condition for a multiplicity to arise for a σ small enough, that is for *any* σ under a certain threshold. For a proof that a multiplicity does not obtain for $\sigma \to 0$ in the case of $\zeta \geq -1$ see A.3.4. ■

A.3.4. Proposition 8

Proof. Here we consider the fixed-point equation (36) for $\zeta \geq -1$ (here I will write simply ζ for ζ_x). First of all notice that (54) cannot be satisfied for $\zeta \in [-1, 0]$ that is for $\mu < 0$ and $\kappa < 0$ because $y(x)$ is non-monotone only in the first quadrant. Hence, a multiplicity may eventually arise for $\zeta > 0$ for which $\mu < 0$ and $\kappa > 0$. In this case the restriction (53) is now binding and it implies (65) as lower bound to σ which reduces to

$$\lim_{\sigma_\phi \to \infty} m(\zeta, \beta, \sigma_\phi) = 3(1+\zeta)(1-\beta), \tag{66}$$

in the limit $\sigma_\phi \to \infty$. Notice that for $\sigma \to \infty$ we have

$$\lim_{\sigma \to \infty} k_\pm \left(\bar{\zeta}, \sigma, \bar{\beta}, \bar{\sigma}_\phi\right) = 4\vartheta^3/27, \tag{67}$$

whereas

$$\lim_{\sigma \to \infty} \frac{\partial k_-}{\partial \mu} = -\theta < -1 < -\frac{\zeta}{\zeta+1}, \tag{68}$$

for whatever $\zeta > 0$. According to (62a) the latter implies that $0 > \partial\kappa/\partial\sigma > \partial k_-/\partial\sigma$ so that, for decreasing σ, k_- increases initially faster than κ. Therefore at least locally there does not exist any multiplicity region in the limit $\sigma \to \infty$.

Nevertheless, (68) can be eventually reverted for smaller σ. Suppose now we start from a point $\left(\bar{\zeta}, \bar{\sigma}, \bar{\beta}, \bar{\sigma}_\phi\right)$ such that $\mu\left(\bar{\zeta}, \bar{\sigma}, \bar{\beta}, \bar{\sigma}_\phi\right) = -\vartheta\left(\bar{\zeta}, \bar{\sigma}, \bar{\beta}, \bar{\sigma}_\phi\right)^2/3$. At this point, $\lim_{\mu \to -\vartheta^2/3} k_\pm = 8\vartheta^3/27$. By continuity of the conditions (62a) we can conclude that if and only if

$$\kappa \geq 8\,\vartheta^3/27, \tag{69}$$

for some σ, then there exists a compact region of the parameter space such that (54) is satisfied whereas such a region does not exist otherwise. Disequality (69) corresponds to

$$\frac{(1-\beta)\zeta\left((1-\beta)(1+\zeta)+\sigma_\phi\right)}{\sigma\sigma_\phi} > \frac{8}{27}\left(\frac{1-\beta+\sigma_\phi}{\sigma_\phi}\right)^3,$$

that is

$$\sigma < M(\zeta, \beta, \sigma_\phi) \equiv \frac{27\sigma_\phi^2(1-\beta)\zeta\left((1-\beta)(1+\zeta)+\sigma_\phi\right)}{8(1-\beta+\sigma_\phi)^3}, \tag{70}$$

that provides a higher bound to σ with

$$\lim_{\sigma_\phi \to \infty} M(\zeta, \beta, \sigma_\phi) = \frac{27}{8}(1-\beta)\zeta,$$

in the limit of $\sigma_\phi \to \infty$.

Finally from intersection of (65) and (70), one obtains a necessary condition for a multiplicity of multiple real solutions as

$$\sigma \in (m(\zeta, \beta, \sigma_\phi), M(\zeta, \beta, \sigma_\phi)),$$

that is a non empty interval if and only if

$$\sigma_\phi > \frac{8(1+\zeta)}{(\zeta-8)}(1-\beta),$$

with $\zeta > 8$. ■

References

[1] Manuel Amador, Pierre-Olivier Weill. Learning from Prices: Public Communication and Welfare. Journal of Political Economy, 118:866-907, 2010.

[2] Jess Benhabib, Pengfei Wang and Yi Wen. Sentiments and Aggregate Demand Fluctuations, NYU mimeo, 2012.

[3] George-Marios Angeletos, Christian Hellwig and Alessandro Pavan. Dynamic global games of regime change: learning, multiplicity, and the timing of attacks. Econometrica, 75:711-756, 2007.

[4] George-Marios Angeletos and Jennifer La'O. Dispersed information over the business cycle: Optimal fiscal and monetary policy. Working paper, MIT, 2008.

[5] George-Marios Angeletos and Ivan Werning. Crises and prices: Information aggregation, multiplicity and volatility. American Economic Review, 96:1721-1737, December 2006.

[6] Hans Carlsson and van Damme Eric E. "Global Games and Equilibrium Selection". Econometrica, 61:989-1018, 1993.

[7] Gabriel Desgranges and Celine Rochon. Conformism and public news. Economic Theory, forthcoming.

[8] Jayant V. Ganguli and Lyian Yang. Complementarities, multiplicity, and supply information. Journal of the European Economic Association, 7:90-115, 2009.

[9] Christian Hellwig, Arijit Mukherji, and Aleh Tsyvinski. Self-fulfilling currency crises: The role of interest rates. American Economic Review, 96:1769-1787, December 2006.

[10] Christian Hellwig and Venky Venkateswaran. Setting the right prices for the wrong reasons. Journal of Monetary Economics, 56:S57-S77, 2009.

[11] Christian Hellwig and Venky Venkateswaran. Hayek Vs Keynes: Incomplete Information and Market Prices. Toulouse School of Economics and Penn State University, mimeo, 2011.

[12] Martin Hellwig. On the aggregation of information in competitive markets. Journal of Economic Theory, 22:477-498, 1980.

[13] Gaetano Gaballo. Good Luck or Good Policy? An Expectational Theory of Macro-Volatility Switches, EUI Max Weber Working Papers 19/2011.

[14] Sanford J. Grossman. On the efficiency of competitive stock markets where trades have diverse information. Journal of Finance, 31:573-585, 1976.

[15] Roger Guesnerie. An Eductive Exploration of Eductive Justifications of the Rational-Expectations Hypothesis, American Economic Review 82:1254-1278, 1992.

[16] Roger Guesnerie. Assessing Rational Expectations 2, MIT press, 2005.

[17] Guido Lorenzoni. Optimal monetary policy with uncertain fundamentals and dispersed information. Review of Economic Studies, 2009. Forthcoming.

[18] Robert E. Jr. Lucas. Expectations and the neutrality of money. Journal of Economic Theory, 4:103-124, 1972.

[19] Stephen Morris and Hyun Song Shin. Rethinking Multiple Equilibria in Macroeconomic Modeling. American Economic Review, 88:587-597, 1998.

[20] Stephen Morris and Hyun Song Shin. Unique equilibrium in a model of self-fulfilling currency attacks. NBER Macroeconomics Annual 15:139-182, 2000.

[21] Stephen Morris and Hyun Song Shin. The social value of public information. American Economic Review, 92:1521-1534, 2002.

[22] John F. Muth. Rational expectations and the theory of price movements. Econometrica, 29:315-335, 1961.

[23] Maurice Obstfeld and Kenneth Rogoff. Speculative hyperinflations in maximizing models: Can we rule them out? Journal of Political Economy, 91:675-87, 1983.

Documents de Travail

380. M. Boutillier and J. C. Bricongne, "Disintermediation or financial diversification? The case of developed countries," April 2012

381. Y. Ivanenko and B. Munier, "Price as a choice under nonstochastic randomness in finance," May 2012

382. L. Agnello and R. M. Sousa, "How does Fiscal Consolidation Impact on Income Inequality?," May 2012

383. L. Ferrara, M. Marcellino, M. Mogliani, "Macroeconomic forecasting during the Great Recession: The return of non-linearity?," May 2012

384. M. Bessec and O. Bouabdallah, "Forecasting GDP over the business cycle in a multi-frequency and data-rich environment," June 2012

385. G. Dufrénot and K. Triki, "Public debt ratio and its determinants in France since 1890 – Does econometrics supports the historical evidence? ," July 2012

386. G. Dufrénot and K. Triki, "Why have governments succeeded in reducing French public debt historically and can these successes inspired us for the future? An historical perspective since 1890," July 2012

387. G ; Gaballo, "Private Uncertainty and Multiplicity," July 2012

Pour accéder à la liste complète des Documents de Travail publiés par la Banque de France veuillez consulter le site : www.banque-france.fr

For a complete list of Working Papers published by the Banque de France, please visit the website: www.banque-france.fr

Pour tous commentaires ou demandes sur les Documents de Travail, contacter la bibliothèque de la Direction Générale des Études et des Relations Internationales à l'adresse suivante :

For any comment or enquiries on the Working Papers, contact the library of the Directorate General Economics and International Relations at the following address :

BANQUE DE FRANCE
49- 1404 Labolog
75049 Paris Cedex 01
tél : 0033 (0)1 42 97 77 24 ou 01 42 92 63 40 ou 48 90 ou 69 81
email : 1404-ut@banque-france.fr