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## Learning Leverage Shocks and the Great Recession\*

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### L'apprentissage des chocs financiers et la Grande Récession

#### Résumé

Cet article modélise une économie dans laquelle les chocs financiers ont des effets macroconomiques de premier ordre lorsque les agents apprennent graduellement leur environnement. Lorsque les ménages réactualisent, par apprentissage adaptatif, leurs croyances relatives au processus inobservé à l'origine des chocs qui affectent le levier financier, les réponses des agrégats macroéconomiques sont significativement plus importantes que lorsque les agents sont supposés avoir des anticipations rationnelles. Le cadre de référence, calibré sur données US pour la période 1996-2008, montre que l'apprentissage adaptatif amplifie les chocs de levier dans un rapport de un à trois, par rapport aux anticipations rationnelles. Lorsque les innovations qui ont affectées pendant la période le niveau du levier financier sont introduites dans le modèle avec apprentissage, ce dernier reproduit la chute du PIB lors de la Grande Récession, contrairement au modèle avec anticipations rationnelles qui prédit une expansion pendant la même période. De plus, nous montrons qu'un levier financier procyclique renforce l'effet amplificateur de l'apprentissage et, par conséquent, qu'une politique "macro-prudentielle" imposant un levier contracyclique est stabilisante. Enfin, nous illustrons que l'apprentissage dans un modèle mal spécifié qui ignore les dépendances entre variables réelles et variables financières contribue également à amplifier les chocs financiers.

*Mots-clés*: contraintes d'endettement, garantie financière, effet de levier, apprentissage, chocs financiers, récession

Codes du Journal of Economic Literature: E32, E44, G18

### Learning Leverage Shocks and the Great Recession

#### Abstract

This paper develops a simple business-cycle model in which financial shocks have large macroe-conomic effects when private agents are gradually learning their economic environment. When agents update their beliefs about the unobserved process driving financial shocks to the leverage ratio, the responses of output and other aggregates under adaptive learning are significantly larger than under rational expectations. In our benchmark case calibrated using US data on leverage, debt-to-GDP and land value-to-GDP ratios for 1996Q1-2008Q4, learning amplifies leverage shocks by a factor of about three, relative to rational expectations. When fed with the actual leverage innovations, the learning model predicts the correct magnitude for the Great Recession, while its rational expectations counterpart predicts a counter-factual expansion. In addition, we show that procyclical leverage reinforces the impact of learning and, accordingly, that macro-prudential policies enforcing countercyclical leverage dampen the effects of leverage shocks. Finally, we illustrate how learning with a misspecified model that ignores real/financial linkages also contributes to magnify financial shocks.

Keywords: Borrowing Constraints, Collateral, Leverage, Learning, Financial Shocks, Recession Journal of Economic Literature Classification Numbers: E32, E44, G18

#### 1 Introduction

Virtually all narratives about the run-up to the US Great Recession unequivocally feature the booms in both household debt and housing prices as the main culprits. In contrast, the theoretical literature which has examined to what extent these prominent aspects matter seems somehow more cautious. In particular, the overall conclusion seems to be that movements of the loan-to-housing value are not of primary importance to explain the recent events (see e.g. Kiyotaki, Michaelides, Nikolov [22], Liu, Wang, Zha [27], Justiniano, Primiceri, Tombalotti [21] among others). In this paper, we argue that relaxing the assumption of rational expectations and acknowledging that real-world agents do not know the parameters of the stochastic process governing financial shocks leads to a very different conclusion. We develop a simple business-cycle model in which the financial sector dynamics may originate shocks that have large macroeconomic effects when private agents are gradually learning their economic environment. The key random variable is the leverage ratio, which we define by how much one can borrow out of the land market value. We show that when agents update their beliefs about the unobserved process driving shocks to the leverage ratio, the responses of output and other aggregates under adaptive learning are significantly larger than under rational expectations. More specifically, we compare two settings: (i) the model with (full information) rational expectations, in which agents know the parameters governing the AR(1) process of shocks to leverage; (ii) the model with learning, in which agents are ignorant about the "true" AR(1) parameters and update their estimates as new data arrive.

We first perform two theoretical experiments. In the simplest model, we assume that agents know the economy's steady state and, in particular, the stationary level of leverage but not its autocorrelation, which is allowed to be time-varying. More precisely, we posit that learning agents end up overestimating the persistence of leverage shocks



Figure 1: US Household Leverage Ratio 1980Q1-2010Q3. Source: Boz and Mendoza [5].

prior to 2008Q4. This assumption is motivated by the data, reported in Figure 1, on US household leverage that are provided by Boz and Mendoza [5] over the period 1980Q1-2010Q3, which can roughly be split into three phases.

The first one runs from 1980 to the early 1990s, when leverage is flat around 60%. In the second phase, leverage trends up until the last quarter of 2008, when the financial crisis is in its most severe stage. Finally, after 2008Q4 leverage seems to become flat again. We assume that leverage follows an AR(1) process and we think of the second phase, prior to 2008Q4, as a period when learning agents gradually get evidence that leverage shocks are becoming close to permanent. In Appendix A.3, we present empirical support for this assumption.

Our first findings are derived in a model that is a simple variant of Kiyotaki and Moore [23] based on Kocherlakota [24], which is linearized in percentage deviations from the known steady state. We focus on financial shocks that drive up and down the leverage ratio, which according to the data in Figure 1 are very persistent. We calibrate the model using data on leverage, debt-to-GDP and land value-to-GDP ratios for the pe-

riod 1996Q1-2008Q4 and we subject the economy to the large negative shock to leverage that was observed in 2008Q4 (see Figure 1) under the assumption that learning agents overestimate the persistence of the leverage shock, which is believed to be close to unity. We compare the responses of the linearized economy under adaptive learning, following Marcet and Sargent [29] and Evans and Honkaphoja [14], and under rational expectations. Our typical sample of results shows that learning amplifies leverage shocks by a factor of about three (see Figure 2). For example, our model predicts, when fed with the negative leverage shock of about -5% observed in 2008Q4, that output falls by about 1%, which is roughly by how much US GDP dropped at that time. In addition, aggregate consumption and the capital stock fall by about 1.2% and 2%, respectively. Under rational expectations, however, output drops only by a third of 1% while the responses of consumption and investment are divided by about four at impact. Consumption and investment go down by a significantly larger margin under learning because de-leveraging is more severe: land price and debt are much more depressed after the negative leverage shock hits when its persistence is overestimated by agents who are constantly learning their environment and, because of recent past data, temporarily pessimistic. We next show that the magnitude of the consequent recession may in part be attributed to the high level of leverage (and the correspondingly high level of the debt-to-GDP ratio) observed in 2008Q4. When the same negative leverage shock occurs in the model calibrated using 1996Q1 data, when leverage was much lower, the impact on output's response is reduced by about two thirds. In this sense, our model points at the obvious fact that financial shocks to leverage originate larger aggregate volatility in economies that are more levered.

In addition, we also ask whether procyclical leverage may act as an aggravating factor and our answer is positive. The assumption that households' leverage responds to land price is motivated by the recent evidence provided by Mian and Sufi [31] (see also the discussion in Midrigan and Philippon [32]). The counter-factual experiment with countercyclical leverage shows dampened effects of leverage shocks, with responses of aggregate variables under learning that are close to their rational expectations counterpart. One possible interpretation of this finding is that macro-prudential policies enforcing countercyclical leverage have potential stabilizing effects on the economy in the face of financial shocks, at small cost provided that non-distortionary policies are implemented (e.g. through regulation). Finally, we illustrate how learning with a misspecified model that ignores real/financial linkages also contributes to magnify financial shocks.

Our second theoretical experiment is carried out under the assumption that learning agents do not know the steady state of the economy and, in particular, that they do not know the long-run level of leverage. This is our preferred model in the sense that it is arguably a more realistic description of the difficulties that forecasting agents/econometricians face when trying to figure out the parameters governing the data generating process. In such a setting, we again feed the model with the negative leverage shock of about -5% observed in 2008Q4 and we show that the responses of the economy are again amplified under learning when agents' belief about the steady state level of leverage is overestimated (see Figure 7), which is in accordance with the data prior to 2008Q4. Summing up the results from our two model experiments, our main conclusion is that in a world where either agents know the steady state but overestimate the persistence of financial shocks or agents know the persistence parameter but overestimate the long-run level of leverage, learning amplifies the disturbances to borrowing capacity.

To take the model closer to data, we next feed the model with the actual innovations to leverage and show that the model predicts the correct magnitude for the Great Recession. More precisely, we do that in two settings that replicate both theoretical experiments. In the first one, the log-linearized model in percentage deviations from steady state is fed with the innovations obtained from the HP-detrended leverage data and it predicts that the recession under learning happens too early and is too small compared to NBER estimates (see Figure 9). However, we next assume that agents do not know the steady state of the economy and have to estimate it using constant-gain learning. When we let agents revise downward their estimate of the leverage level in accordance with the actual leverage innovations observed in 2007-2008, the learning model predicts the correct magnitude for the Great Recession (see Figure 11). In sharp contrast, the rational expectations model predicts an expansion which is at odds with the data.

To summarize, our main finding is that leverage shocks are amplified when agents gradually update their beliefs about the process driving financial shocks. We believe it is important to acknowledge that, as Figure 1 suggests, nonstationary leverage (either because it is subject to permanent shocks or because of its time-varying steady state level) may have played an important role in favoring conditions that worsened the Great Recession. Looking back in time at the data in Figure 1, there is a sense in which everybody should have foreseen that leverage could not possibly increase forever. However, figuring out when leverage would stop rising was a much harder task. Our paper stresses that when such a change comes, its macroeconomic impact when agents adaptively learn differs much from what happens under rational expectations. Moreover, on the policy side, our analysis gives an example of a macro-prudential policy that dampens the impact of financial shocks to the macroeconomy under learning by ensuring that leverage goes down when asset prices spike up.

Related Literature: Our paper connects to several strands of the literature. The macroeconomic importance of financial shocks has recently been emphasized by Jermann and Quadrini [20], among others, and our paper contributes to this literature about credit shocks by showing how learning matters. Closest to ours are the papers by Adam, Kuang and Marcet [1], who focus on interest rate changes, and by Boz and Mendoza [5], who show how changes in the leverage ratio have large macroeconomic

effects under Bayesian learning and Markov regime switching. As in Boz and Mendoza [5], we focus on leverage shocks but our setting is different. First, our model with adaptive learning is easily amenable to simulations and we solve for equilibria through usual linearization techniques. Because we assume that agents are adaptively learning through VAR estimation, it is possible to enrich the model by adding capital accumulation and endogenous production. Most importantly, our model predicts large output drops when the economy is hit by negative leverage shocks. In contrast, absent TFP shocks, output remains constant after a financial regime switch in Boz and Mendoza [5]. Our paper also relates to some of the insights in Howitt [18], Hebert, Fuster and Laibson [15, 16]. Contrary to Hebert, Fuster and Laibson [15, 16] who assume that agents use a misspecified model, in our case the overestimated persistence of shocks arises endogenously under adaptive learning when agents face the sequence of financial innovations that was observed in the run-up to the crisis. In addition, our paper stresses that changes in the beliefs about the long-run level of leverage may also go a long way explaining why shocks get amplified under adaptive learning.

In the literature, the idea that procyclical leverage has adverse consequences on the macroeconomy is forthfully developed in Geanakoplos [17] (see also Cao [8]). Although our formulation of elastic leverage is derived in an admittedly simple setup, it allows us to examine its effect in a full-fledged macroeconomic setting. Last but not least, the notion that learning is important in business-cycle models when some change in the shock process occurs has been discussed by, e.g., Bullard and Duffy [6] and Williams [36]. More recently, Eusepi and Preston [12] have shown that learning matters in a standard RBC model when the economy is hit by shocks to productivity growth (see also the related paper by Edge, Laubach, Williams [11]). Our paper adds to this literature by focusing on financial shocks under collateral constraints. As mentioned before, part of the paper's

<sup>&</sup>lt;sup>1</sup>See the discussion by Evans [13].

motivation also comes from the growing micro-evidence about the importance of house-holds' and firms' leverage for understanding consumption and investment behaviors (e.g. Mian and Sufi [31], Chaney, Sraer and Thesmar [10]).

The paper is organized as follows. Section 2 presents the model and derives its rational expectations equilibria. Section 3 relaxes the assumption that agents form rational expectations in the short run and it shows how financial shocks are amplified under learning when agents overestimate the persistence of shocks. Section 4 shows how leverage shocks get amplified under learning when agents do not know the steady state of the economy and overestimate the long-run level of leverage, while Section 5 provides evidence that such a setting predicts the correct magnitude for the Great Recession. Section 6 gathers concluding remarks and all proofs are exposed in the appendices.

## 2 The Economy with Leverage Shocks

#### 2.1 Model

The model is essentially an extension of Kocherlakota's [24] to partial capital depreciation and adaptive learning. A representative agent solves:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma} - 1}{1-\sigma} \tag{1}$$

where  $C_t \geq 0$  is consumption and  $\sigma \geq 0$  denotes relative risk aversion, subject to both the budget constraint:

$$C_t + K_{t+1} - (1 - \delta)K_t + Q_t(L_{t+1} - L_t) + (1 + R)B_t = B_{t+1} + AK_t^{\alpha}L_t^{\gamma}$$
 (2)

and the collateral constraint:

$$\tilde{\Theta}_t E_t[Q_{t+1}] L_{t+1} \ge (1+R) B_{t+1} \tag{3}$$

where  $K_{t+1}$ ,  $L_{t+1}$  and  $B_{t+1}$  are, respectively, the capital stock, the land stock and the amount of new borrowing all chosen in period t,  $Q_t$  is the land price, R is the exogenous interest rate, A is total factor productivity (TFP thereafter). In our benchmark model, leverage  $\tilde{\Theta}_t$  is subject to random shocks whereas both the interest rate and TFP are constant over time. As we focus on financial shocks, we ignore TFP disturbances and simply notice that similar results hold when the process driving technological shocks changes as well. We present first the results obtained under the collateral constraint (3), which follows Kiyotaki and Moore [23]. However, quantitatively similar results hold under the margin requirement timing stressed in Aiyagari and Gertler [3] (see Section 3.4 for robustness analysis).

Denoting  $\Lambda_t$  and  $\Phi_t$  the Lagrange multipliers of constraints (2) and (3), respectively, the borrower's first-order conditions with respect to consumption, land stock, capital stock, and loan are given, respectively, by:

$$C_t^{-\sigma} = \Lambda_t \tag{4}$$

$$\Lambda_t Q_t = \beta E_t [\Lambda_{t+1} Q_{t+1}] + \beta \gamma E_t [\Lambda_{t+1} Y_{t+1} / L_{t+1}] + \Phi_t \tilde{\Theta}_t E_t [Q_{t+1}]$$
 (5)

$$\Lambda_t = \beta E_t [\Lambda_{t+1}(\alpha Y_{t+1}/K_{t+1} + 1 - \delta)] \tag{6}$$

$$\Lambda_t = \beta(1+R)E_t[\Lambda_{t+1}] + (1+R)\Phi_t \tag{7}$$

We also incorporate into the model the feature that leverage responds to changes in the land price, which accords with the evidence documented by Mian and Sufi [31] on US micro data for the 2000s. More precisely, we posit that:

$$\tilde{\Theta}_t \equiv \Theta_t \left\{ \frac{E_t[Q_{t+1}]}{Q} \right\}^{\varepsilon} \tag{8}$$

where Q is the steady-state value of land price and the log of  $\Theta_t$  follows an AR(1) process, that is,  $\Theta_t = \overline{\Theta}^{1-\rho_\theta}\Theta_{t-1}^{\rho_\theta}\Xi_t$ . In Appendix A.1, we show how (8) can be derived in a simple setting with ex-post moral hazard and costly monitoring, similar to Aghion et al. [2]. Our goal is now to compare two cases regarding what agents know about the data generating process underlying leverage shocks:

- (i) rational expectations (with full information): agents know with certainty the "true" values of both  $\rho_{\theta}$  and  $\overline{\Theta}$ ,
- (ii) learning (with incomplete information):  $\rho_{\theta}$  and  $\overline{\Theta}$  are unknown and agents have to estimate those parameters based on available data. More precisely, two experiments are reported in Sections 3 and 4. In section 3, we follow standard practice by linearizing the model around a known steady state and we assume that learning agents do not know (and have to estimate) the persistence parameter  $\rho_{\theta}$ . In Section 4, agents are uncertain about the steady state level of leverage  $\overline{\Theta}$  instead but they know  $\rho_{\theta}$ . Before turning to that, we present the rational expectations equilibria.

## 2.2 Rational Expectations Equilibria

A rational expectations competitive equilibrium is a sequence of positive prices  $\{Q_t\}_{t=0}^{\infty}$  and positive allocations  $\{C_t, K_{t+1}, L_{t+1}, B_{t+1}\}_{t=0}^{\infty}$  such that, given the exogenous sequence  $\{\Theta_t\}_{t=0}^{\infty}$  of leverage and the exogenous interest rate  $R \geq 0$ :

- (i)  $\{C_t, K_{t+1}, L_{t+1}, B_{t+1}\}_{t=0}^{\infty}$  satisfies the first-order conditions (4)-(7), the transversality conditions,  $\lim_{t\to\infty} \beta^t \Lambda_t L_{t+1} = \lim_{t\to\infty} \beta^t \Lambda_t K_{t+1} = 0$ , and the complementarity slackness condition  $\Phi_t \left[ \tilde{\Theta}_t E_t[Q_{t+1}] L_{t+1} (1+R) B_{t+1} \right] = 0$  for all  $t \geq 0$ , where  $\tilde{\Theta}_t \equiv \Theta_t \{E_t[Q_{t+1}]/Q\}^{\varepsilon}$ , given  $\{Q_t\}_{t=0}^{\infty}$  and the initial endowments  $L_0 \geq 0, B_0 \geq 0, K_0 \geq 0$ ;
- (ii) The good and asset markets clear for all t, that is,  $C_t + K_{t+1} (1-\delta)K_t + (1+R)B_t = B_{t+1} + A_t K_t^{\alpha}$  and  $L_t = 1$ , respectively.

The above definition assumes that the interest rate is exogenous. Therefore, a natural interpretation of the model is that it represents a small, open economy. Appendix A.2 presents a closed-economy variant based on Iacoviello [19], in which borrowers and

lenders meet in a competitive credit market subject to collateral constraints and a constant debtor interest rate. Our findings reported below can be replicated in the closed-economy model when the economy is hit by negative financial and TFP shocks that occur simultaneously. As our focus is on how borrowers adaptively learn how the economy settles after financial shocks, we abstract both from TFP shocks and from further details regarding the lender's side, and we focus on the small-open-economy setting, as in Adam, Kuang and Marcet [1], Boz and Mendoza [5].

There is a unique (deterministic) stationary equilibrium such that the credit constraint (3) binds, provided that the interest factor  $1 + R \equiv 1/\mu$  is such that  $\mu \in (\beta, 1)$ , that is, if lenders are more patient than borrowers. This follows from the steady-state version of (7), that is,  $\Phi = \Lambda(\mu - \beta) > 0$ . The steady state is characterized by the following great ratios, that fully determine the linearized dynamics around the steady state. From (5) and (6), it follows that the land price-to-GDP and capital-to-GDP ratios are given by  $Q/Y = \gamma \beta/[1-\beta-\overline{\Theta}(\mu-\beta)]$  and  $K/Y = \alpha \beta/[1-\beta(1-\delta)]$ , respectively. Finally, (3) and (2) yield, respectively, the debt-to-GDP ratio  $B/Y = \mu \overline{\Theta} Q/Y$  and the consumption-to-GDP ratio  $C/Y = 1 - \delta K/Y - (1/\mu - 1)(B/Y)$ .

Appendix A.1 provides a linearized version, in percentage deviations from the steady state, of the set of equations (2)-(7) defining, together with (8) and the leverage law of motion  $\Theta_t = \overline{\Theta}^{1-\rho_\theta} \Theta_{t-1}^{\rho_\theta} \Xi_t$ , intertemporal equilibria. We assume throughout that leverage  $\Theta$  is observed while the shock  $\Xi$  remains unobserved. Eliminating  $\Phi_t$  by using (7), the linearized expectational system (in percentage deviations from steady state) can be written as:

$$X_{t} = \mathbf{A}X_{t-1} + \mathbf{B}E_{t-1}[X_{t}] + \mathbf{C}E_{t}[X_{t+1}] + \mathbf{D}\xi_{t}$$
(9)

where  $X_t \equiv (c_t \ q_t \ \lambda_t \ b_t \ k_t \ \theta_t)'$  is observed whereas  $\xi_t$  is not, and all variables in lowercase letters denote their deviations from steady-state in percentage terms (e.g.  $k_t \equiv (K_t - K)/K$ , where K is the steady-state capital stock). The derivation and the expressions of

the 6-by-6 matrices A, B, C, D as functions of parameters are given in Appendix A.1.

Anticipating our results on E-stability, we now use the fact that the linearized rational expectations equilibrium around steady state can be obtained as the unique E-stable Minimal-State-Variable solution (MSV thereafter) of the form  $X_t = \mathbf{M^{re}}X_{t-1}$ , where  $\mathbf{M^{re}}$  solves  $\mathbf{M} = [\mathbf{I_6} - \mathbf{CM}]^{-1}[\mathbf{A} + \mathbf{BM}]$  and  $\mathbf{I_6}$  is the 6-by-6 identity matrix. It is important to underline that both the autocorrelation of the leverage shock process, that is,  $\rho_{\theta}$ , and the leverage level, that is  $\overline{\Theta}$ , are known under rational expectations. In contrast, the next sections relax such an assumption and assume instead that agents have to form estimates of  $\rho_{\theta}$  and  $\overline{\Theta}$  using the available data.

## 3 Adaptive Learning when the Steady State is Known

Following Marcet and Sargent [29] and Evans and Honkapohja [14], we now relax the assumption that agents form rational expectations in the short-run. We first assume that the steady state of the economy is known, which implies that the level of leverage is common knowledge. However, the parameters governing the dynamics of the economy are not known. In particular,  $\rho_{\theta}$  is not known with certainty by agents. Because the steady state is known, we can still use the linearized dynamic system in percentage deviations from steady state, which is now:

$$X_{t} = \mathbf{A}X_{t-1} + \mathbf{B}E_{t-1}^{*}[X_{t}] + \mathbf{C}E_{t}^{*}[X_{t+1}] + \mathbf{D}\xi_{t}$$
(10)

where the operator  $E_t^*$  indicates expectations that are taken using all information available at t but that are possibly nonrational. More precisely, agents behave as econometricians by embracing the following perceived law of motion (PLM thereafter):

$$X_t = \mathbf{M}X_{t-1} + \mathbf{G}\xi_t \tag{11}$$

which agents use for forecasting. In particular, (11) yields  $E_t[X_{t+1}] = \mathbf{M}_{t-1}X_t$  and  $E_{t-1}[X_t] = \mathbf{M}_{t-2}X_{t-1}$ . The actual law of motion (ALM thereafter) results from combining (10) and (11) which gives:

$$[\mathbf{I}_6 - \mathbf{CM}_{t-1}]X_t = [\mathbf{A} + \mathbf{BM}_{t-2}]X_{t-1} + \mathbf{D}\xi_t$$
 (12)

When **M** coincides with **M**<sup>re</sup> derived in Section 2.2, then agents hold rational expectations. However, beliefs captured in **M** may differ from rational expectations and they are updated in real time using recursive learning algorithms, following Evans and Honkapohja [14]. This means that the belief matrix **M** is time-varying and its coefficients are updated using:

$$\mathbf{M}_{t} = \mathbf{M}_{t-1} + \nu_{t} \mathbf{R}_{t}^{-1} X_{t-1} (X_{t} - \mathbf{M}'_{t-1} X_{t-1})$$
(13)

$$\mathbf{R}_{t} = \mathbf{R}_{t-1} + \nu_{t}(X_{t-1}X'_{t-1} - \mathbf{R}_{t-1}) \tag{14}$$

where **R** is the estimate of the variance-covariance matrix and  $\nu_t$  is the gain sequence (which equals 1/(t+1) under ordinary least squares and  $\nu$  under constant gain, respectively OLS and CG thereafter). One difference with rational expectations that is key to our results is that agents may overestimate the autocorrelation parameter  $\rho_{\theta}$ .

The mapping from the PLM (11) into the ALM (12) is given by:

$$T(\mathbf{M}) = [\mathbf{I}_6 - \mathbf{CM}]^{-1}[\mathbf{A} + \mathbf{BM}]$$
 (15)

Adapting Proposition 10.3 from Evans and Honkapohja [14], we check that all eigenvalues of  $DT_{\mathbf{M}}(\mathbf{M})$  have real parts less than 1 when evaluated at the fixed-point solutions of the T-map (15), that is,  $\mathbf{M} = \mathbf{M^{re}}$ . Using the rules for vectorization of matrix products, we get:

$$DT_{\mathbf{M}}(\mathbf{M^{re}}) = ([\mathbf{I}_6 - \mathbf{CM^{re}}]^{-1}[\mathbf{A} + \mathbf{BM^{re}}])' \otimes [\mathbf{I}_6 - \mathbf{CM^{re}}]^{-1}\mathbf{C}$$
  
  $+ \mathbf{I}_6 \otimes [\mathbf{I}_6 - \mathbf{CM^{re}}]^{-1}\mathbf{B}$ 

All MSV solutions that we consider from now on are said to be locally E-stable when all eigenvalues of  $DT_{\mathbf{M}}(\mathbf{M^{re}})$  lie within the interior of the unit circle. In practice, we numerically compute the E-stable solutions by iterating the T-map (15), as described in Evans and Honkapohja [14, p.232].

## 3.1 Learning the Persistence of Leverage Shocks

In this section, we show that learning amplifies leverage shocks when agents' belief about the persistence parameter  $\rho_{\theta}$  is allowed to differ from rational expectations. In particular, we assume that learning agents wrongly believe that  $\rho_{\theta}$  is close to one. This is meant to capture the trend in leverage that is observed in the run-up to the 2008Q4 crisis (see Figure 1).

The model is calibrated according to Table 1, so as to deliver average values for leverage, debt-to-GDP and land value-to-GDP ratios for the period 1996Q1-2008Q4, that is  $\overline{\Theta} \approx 0.88$ ,  $B/Y \approx 0.52$  and  $QL/Y \approx 0.59$ . To calibrate those ratios, we fix the quarterly interest rate to 1% (that is,  $\mu = 0.99$ ) and  $\beta = 0.98\mu$  (consistent with the literature on heterogeneous discount rates; e.g. Krusell and Smith [25]) and then pick the land share  $\gamma$  to target the land price-to-GDP ratio. In addition, we choose  $\varepsilon = 0.5$  (consistent with the estimates of Mian and Sufi [31]).<sup>2</sup>

Table 1. Parameter Values (1996Q1-2008Q4)

$\mu$	β	δ	$\alpha$	$\gamma$	$\sigma$	$\overline{\Theta}$	ε	ν
0.99	$0.98\mu$	0.025	0.45	0.0075	1	0.88	0.5	0.04

<sup>&</sup>lt;sup>2</sup>The value we set  $\varepsilon$  to implies, for instance, that a 10% increase in land price triggers a 5% increase in leverage, which under our calibration would raise leverage from 0.88 to about 0.92.

The experiment that embodies our first results is the following. We assume that in the period preceding the financial collapse of 2008Q4, the agents in our model economy have learned that  $\rho_{\theta}$  was close to one, reflecting the leverage trend in Figure 1 that starts in the early 1990s. This means that agents' beliefs encapsulated in matrix  $\mathbf{M}$  of the PLM (11) reflect that  $\rho_{\theta} \approx 1$ . Then in 2008Q4 a large negative shock to leverage of about -5% happens (see Figure 1). The (pseudo-)impulse functions in Figure 2 report the reaction of the economy's aggregates under the assumptions that agents wrongly believe that  $\rho_{\theta} \approx 0.999$  whereas the true value is 0.984. Such a calibration is consistent with the data, as shown in Appendix A.3 where we present the real-time estimates of  $\rho_{\theta}$  under both OLS and CG, and it satisfies E-stability conditions. The blue dotted line in Figure 2 represents the RE equilibrium with  $\rho_{\theta} = 0.984$ . The solid red curve in Figure 2 occurs when agents gradually learn using (13)-(14) under the initial belief that  $\rho_{\theta} = 0.999$ , with the true value being  $\rho_{\theta} = 0.984$ . Although Figure 2 assumes CG learning with  $\nu = 0.04$ , similar results would occur under lower gains (which would imply similar effects at impact but slower recovery).<sup>3</sup>

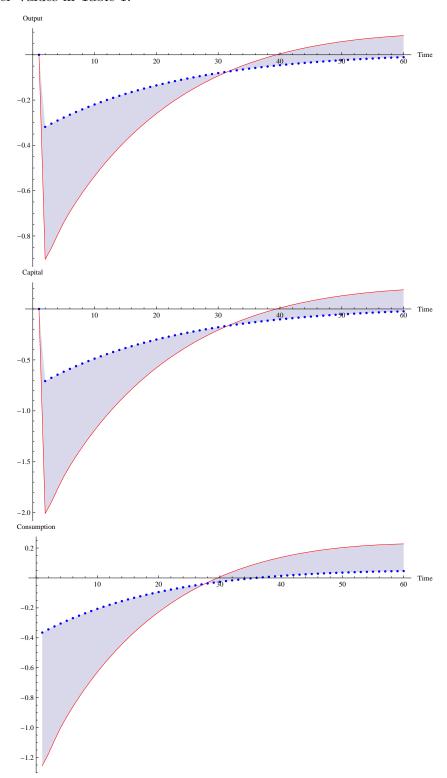
Figure 2 shows that the negative leverage shock is significantly amplified under learning. In particular, the impact on output and capital is roughly three times larger and the consumption drop is multiplied by about four compared to the rational expectations outcome. This follows from the fact that deleveraging is much more severe under learning: the fall in land price is more than five times larger and the debt decrease is multiplied by about three compared to RE.<sup>4</sup>

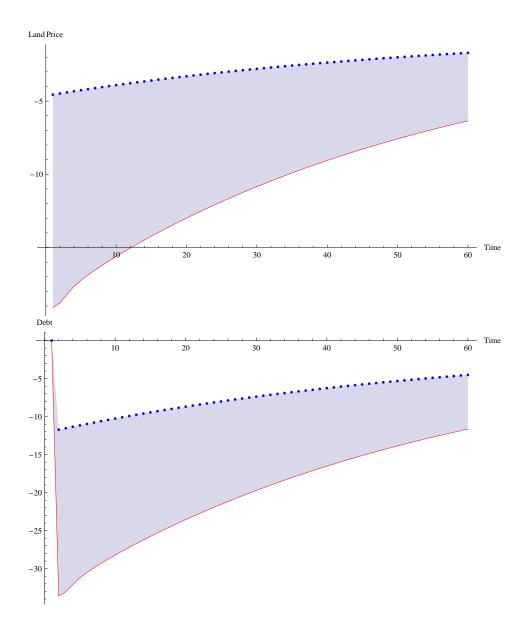
<sup>&</sup>lt;sup>3</sup>Our chosen value for the gain parameter falls within the upper range of estimates reported in Branch and Evans [4], Chakraborty and Evans [9] and it is consistent with the estimates of Malmendier and Nagel [28] for younger generations. Similar impulse response functions would be obtained from our model when fed with the Milani's [33, 34] lower gains.

<sup>&</sup>lt;sup>4</sup>In Figure 2, debt falls by much more than output. This implies that the debt-to-GDP ratio - a common definition of aggregate leverage - falls by a large amount as well.

In summary, because agents incorrectly believe that the impact of the negative leverage shock will be very persistent, they expect a much larger fall in land price and a much tighter borrowing constraint than under rational expectations, which in turn depresses consumption, investment and output. In this sense, agents are pessimistic under incorrect beliefs. Note that the magnitudes of output's and consumption's responses roughly match data, whereas investment is too volatile in our model economy without investment adjustment costs. Finally, Figure 2 shows that both capital and output overshoot their long-run levels, because initial deleveraging finances additional capital investment later on. This does not happen under rational expectations within the same time horizon.

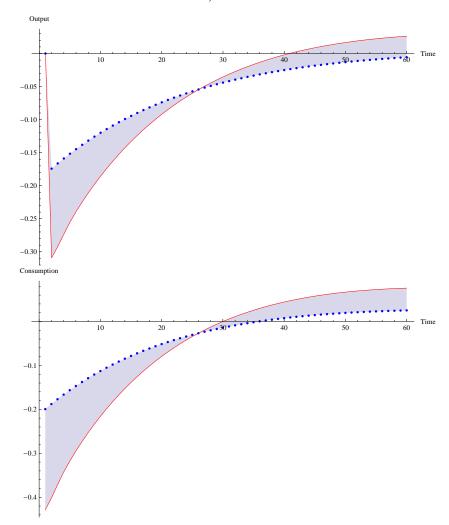
Figure 2: Responses (in Percentage Deviations from Steady State) to a -5% Leverage Shock under Learning (Red Solid Line) and Rational Expectations (Blue Dotted Line); Parameter Values in Table 1.





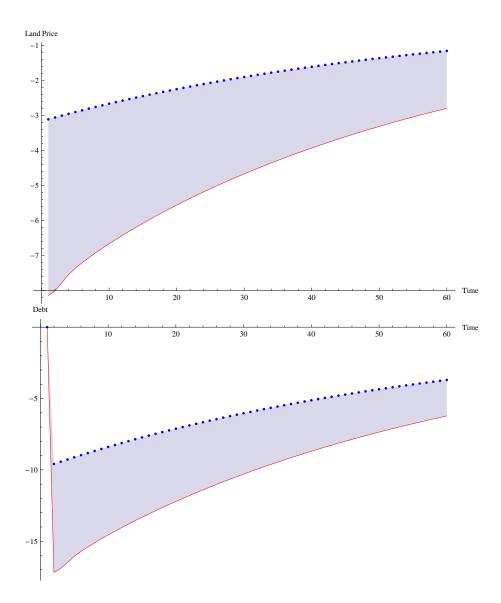
To measure how the leverage level matters for the response to a financial shock, we now calibrate the model using data from the first quarter of 1996, that is  $\overline{\Theta} \approx 0.73$  (the other values are as in Table 1), which leads to  $B/Y \approx 0.34$  and  $QL/Y \approx 0.48$ . According to most measures, this period corresponds to the starting point of the housing price "bubble". The lower level of leverage implies that both the debt-to-GDP and the land value-to-GDP are correspondingly lower than their 2008Q4 levels. Figure 3 replicates the same experiment as above, when a -5% shock to leverage hits the economy and  $\rho_{\theta}$ 

Figure 3: Responses (in Percentage Deviations from Steady State) to a -5% Leverage Shock (Learning: Red Solid Line; Rational Expectations: Blue Dotted Line) when  $\overline{\Theta} = 0.73$  (Other Parameter Values in Table 1)



goes down from 0.999 to 0.98. Direct comparison of Figures 2 and 3 reveals that higher leverage increases the effect of the shock on aggregates by more than 50% at impact under learning. In this sense, the larger the level of leverage the deeper the recession that follows after a negative financial shock.<sup>5</sup>

<sup>&</sup>lt;sup>5</sup>Output's response and capital's response are proportional so we report only the former and not the latter.



It is clear that the economy's responses to leverage shock are larger under learning because the land price forecast interact with the borrowing constraint. To stress this fact, we now report the responses of a subset of the same variables when the land price is assumed to be fixed in the borrowing constraint, that is, when (3) is replaced by:

$$\Theta_t Q L_{t+1} \ge (1+R)B_{t+1}$$
 (16)

Figure 4 reports the responses of output and consumption, which are about the same under learning and under rational expectations, in contrast to Figure 2.

#### 3.2 Macroprudential Policy

In this section, we show that countercyclical leverage dampens the impact of leverage shocks under learning. We now ask the counter-factual question: what would be the reaction of the economy to the same shock, under the same parameter values but with the leverage being now mildly countercyclical<sup>6</sup>? More precisely, we assume that  $\varepsilon = -0.5$  while the other parameters are kept unchanged and set as in Table 1. The economy's responses are reported in Figure 5. Comparing Figures 2-5 shows that countercyclical leverage dampens by a significant margin the responses to financial shocks and it brings learning dynamics closer to its rational expectations counterpart. As a consequence, a much smaller recession follows a negative leverage shock: though agents anticipate a too large deleveraging effect because they overestimate the persistence of the adverse leverage shock, the land price fall now triggers an *increase* in countercyclical leverage, which dampens the impact of the negative shock.

<sup>&</sup>lt;sup>6</sup>This feature could possibly be enforced by appropriate regulation of credit markets. Alternatively, Appendix A.1 shows how it arises if government uses procyclical taxes.

Figure 4: Responses (in Percentage Deviations from Steady State) to a -5% Leverage Shock with Fixed Land Price (Learning: Red Solid Line; Rational Expectations: Blue Dotted Line); Parameter Values in Table 1.

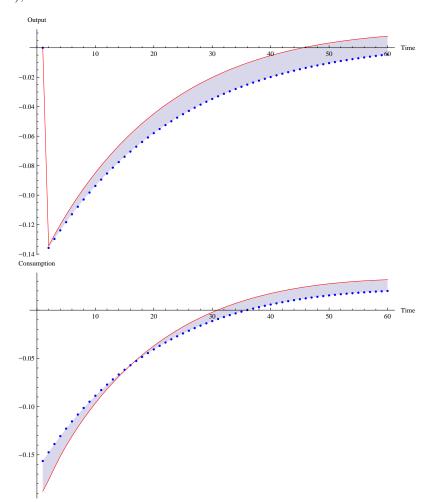
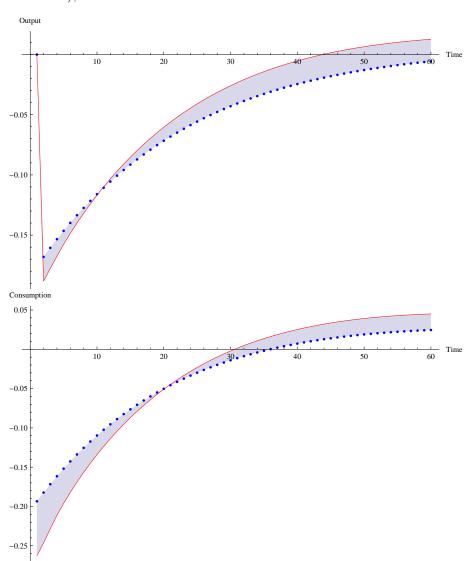
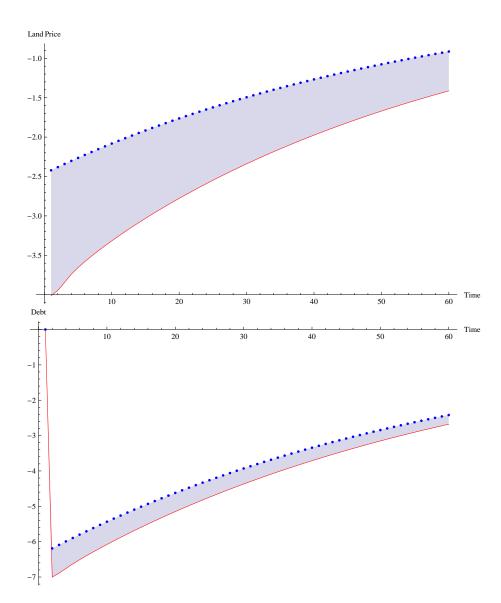


Figure 5: Responses (in Percentage Deviations from Steady State) to a -5% Leverage Shock under Countercyclical Leverage (Learning: Red Solid Line; Rational Expectations: Blue Dotted Line);  $\varepsilon = -0.5$  and other Parameter Values in Table 1.



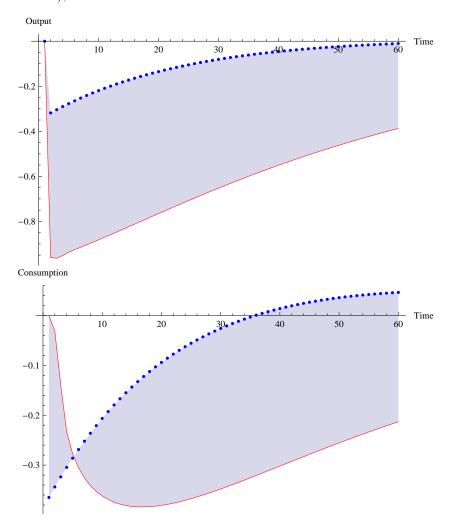


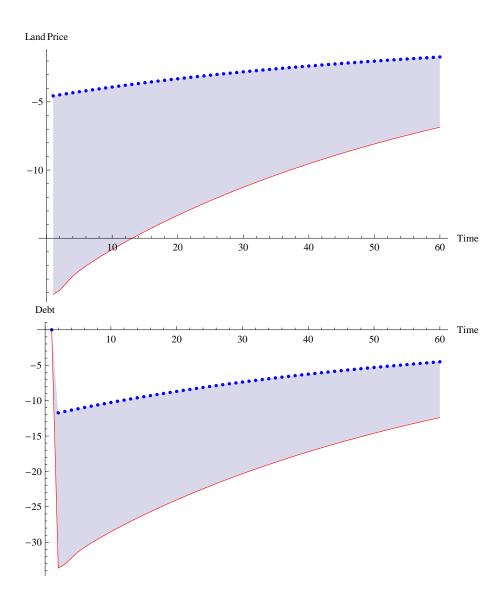
### 3.3 Learning with a Misspecified Model

In this section, we explore the idea that forecasting agents may ignore important real/financial linkages. More precisely, we assume that when forming their beliefs and when estimating matrix  $\mathbf{M}$  in (11), agents set  $\mathbf{M}(1,6) = \mathbf{M}(3,6) = \mathbf{M}(5,6) = 0$ . This means that they incorrectly believe that leverage shocks affect only financial variables (land price and debt) and not real variables (consumption and investment). Therefore, the reactions of land price and debt are not affected by this type of misspecification whereas the responses of consumption, capital and output are. A possible interpretation behind such a view could be that agents hold the belief that the effect of financial shocks are smoothed out through aggregation so that they do not matter for aggregate real variables.

We set parameter values as in Table 1 and now experiment with a case such that  $\rho_{\theta}$  is believed to equal 0.999 while it actually equals 0.98. That is, agents incorrectly believe that leverage has close to unit root. The responses are reported in Figure 6, which differs from Figure 2 in two important ways. First, not surprisingly, the reaction of consumption is now hump-shaped and exhibits more persistence. This is because agents do not take into account that leverage shocks affect consumption directly. In consequence, investment is more volatile. Second, there is no more overshooting and the recession is more persistent: the recovery occurring in Figure 2 after about 30 quarters does not show up in Figure 6. Under our formulation of model misspecification, consumption is more sluggish so that investment is more volatile when the economy is hit by a leverage shock. In that way, the impact of leverage shocks on output is amplified and more persistent under learning.

Figure 6: Responses (in Percentage Deviations from Steady State) to a -5% Leverage Shock under Model Misspecification (Learning: Red Solid Line; Rational Expectations: Blue Dotted Line); Parameter Values as in Table 1.





## 3.4 Alternative Assumptions

To assess the robustness of the findings reported in Section 3.1, we now relax two assumptions. First, we depart from logarithmic utility and we allow  $\sigma$  to take on values that are larger or smaller than one. Second, we adopt the timing assumption that is implied by the margin requirement interpretation of the borrowing constraint (Aiyagari and Gertler [3]). That is, borrowing is limited to the current market value of collateral, as opposed to tomorrow's market value. In other words, we replace (3) by  $\Theta_t Q_t L_{t+1} \geq$ 

$$(1+R)B_{t+1}.$$

In Table 2, we report the output amplification variation that obtains under learning, compared with the rational expectations equilibrium. For example, the impact of a -5% leverage shock on output's deviation (from its steady-state value, in percentage terms) is about -0.90 percentage points under learning and -0.32 percentage points under RE (see Figure 2) when parameters are set according to Table 1. Therefore, the first column of Table 2 reports that the difference is, in absolute value,  $|\Delta y| \approx 0.58$ . Similarly, the second and third columns report  $|\Delta y|$  when all parameter values are set according to Table 1, except for risk aversion  $\sigma$  which equals 0.5 and 3, respectively. Finally, the last column in Table 2 reports  $|\Delta y|$  in the margin requirement model.

Table 2. Extra Output Amplification Under Learning

Benchmark	$\sigma = 0.5$	$\sigma = 3$	Margin Model
0.58 pp	0.56 pp	0.60 pp	0.57 pp

Direct inspection of Table 2 shows that our main findings are robust both to changes in the utility function's curvature and to an alternative timing assumption. Output amplification is quantitatively similar across all different models and this turns out to be the case for the other variables as well. In addition, how the numbers change in Table 2 accords with intuition. First, under the timing assumed in (3), incorrect beliefs about the economy further amplify shocks because land price forecasts are temporarily deviating from RE. In the margin model where the borrowing limit depends on today's collateral market values, forecast errors are slightly less important during deleveraging episodes. In addition, larger risk aversion implies that consumption will fall by less and, therefore, that investment will fall by more at impact, which means that output will also fall by more.

## 4 Adaptive Learning when the Steady State is not Known

The purpose of this section is to report the outcome of our second experiment. Because the steady state is no longer known, the model has to be linearized in levels and not in percentage deviations from steady state. In Appendix A.4 we show that the dynamics equations are now:

$$X_{t} = \mathbf{A}X_{t-1} + \mathbf{B}E_{t-1}[X_{t}] + \mathbf{C}E_{t}[X_{t+1}] + \mathbf{N} + \mathbf{D}\xi_{t}$$
(17)

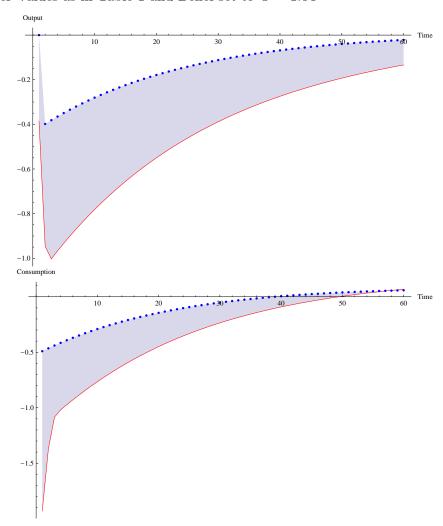
In our first experiment reported in Section 3,  $\mathbf{N}$  was a zero vector. Now equation (17) depends on a non-zero vector  $\mathbf{N}$  that will determine, together with the other matrices, the beliefs of learning agents about where the steady state is. Both the rational expectations solution and the perceived law of motion have the following VAR form:

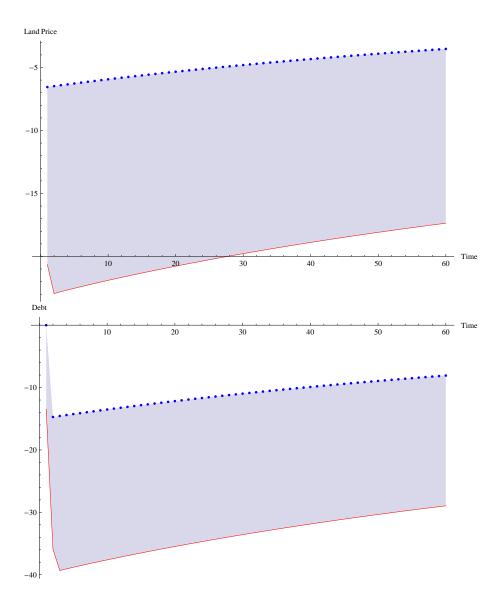
$$X_t = \mathbf{M}X_{t-1} + \mathbf{H} + \mathbf{G}\xi_t \tag{18}$$

and in Appendix A.4, we show how M, H and G are determined and how the adaptive learning algorithm has to be modified to take into account their updating.

Our goal is to report the responses of the economy to the same shock that was considered in Section 3. Our assumption is now that learning agents overestimate the leverage level  $\overline{\Theta}$  but still have a correct belief about the persistence parameter  $\rho_{\theta}$ . We assume that leverage shocks are very persistent by setting  $\rho = 0.99$ , which agrees with the OLS estimates over the sample period. In addition, we set the RE belief to be  $\overline{\Theta} = 0.88$  just as in Table 1, whereas learning agents believe that  $\overline{\Theta} = 1.04$ , which is the CG estimate obtained from the 2008Q4 data.

Figure 7: Responses (in Percentage Deviations from the RE Steady State) to a -5%Leverage Shock (Learning: Red Solid Line; Rational Expectations: Blue Dotted Line); Parameter Values as in Table 1 and Belief set to  $\overline{\Theta} = 1.04$ 





The responses of our set of variables to a -5% shock to leverage are reported in Figure 7, which features substantially larger deviations under CG learning compared to the RE benchmark. Because learning agents overestimate the level of leverage, their perception is that the negative shock is more pronounced that it actually is, which leads to more severe deleveraging. Although the responses at impact are similar in magnitude in Figures 2 and 7, the latter does not feature overshooting. Taken together, the two experiments that are described in the previous section and in this one suggest that learning amplifies negative shocks to leverage such as the one observed in 2008Q4. A natural question that

we now ask is: which experiment accords better with the actual path followed by the US output over the Great Recession?

## 5 Does Learning Help Account For The Great Recession?

The purpose of this section is to argue that learning is a plausible mechanism that helps explaining the magnitude of the Great Recession. More precisely, we now show that the responses of the model economy to observed leverage shocks is very different under learning and under rational expectations. We first derive the responses of our log-linearized model economy to the actual innovations of the HP-detrended leverage data at quarterly frequency.<sup>7</sup> This mirrors the theoretical simulations reported in Section 3. Consistent with the results reported in that section, we allow agents to estimate the autocorrelation of the AR(1) process driving leverage in a time-varying fashion under constant-gain learning. Figure 8 reports both the real-time constant-gain estimates and the OLS estimates of  $\rho_{\theta}$  obtained using the data in Figure 1, adjusted for leverage elasticity (with again  $\varepsilon = 0.5$  and  $\nu = 0.04$  as in Table 1).

The OLS estimate over the whole sample period, which turns out to be  $\rho_{\theta} \approx 0.78$ , is the benchmark in Figure 8 represented by the blue dotted line. The behavior over time of the constant-gain estimates is pictured by the red solid line and it can be decomposed into two periods: from 1980 to 2006, learning agents underestimate the autocorrelation while in the last half of the 2000's the CG estimate is higher than the OLS estimate. In particular, agents believe the AR(1) process driving leverage to be close to unit root near the end of the time period which corresponds to the recession. In view of the impulse response functions reported in the preceding sections, we should expect recessions generated by leverage shocks to be larger under learning in the last 5 years of the sam-

<sup>&</sup>lt;sup>7</sup>To produce Figure 8, we use the standard value of 1600 for the smoothing parameter.

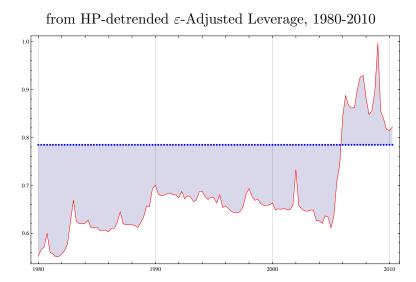


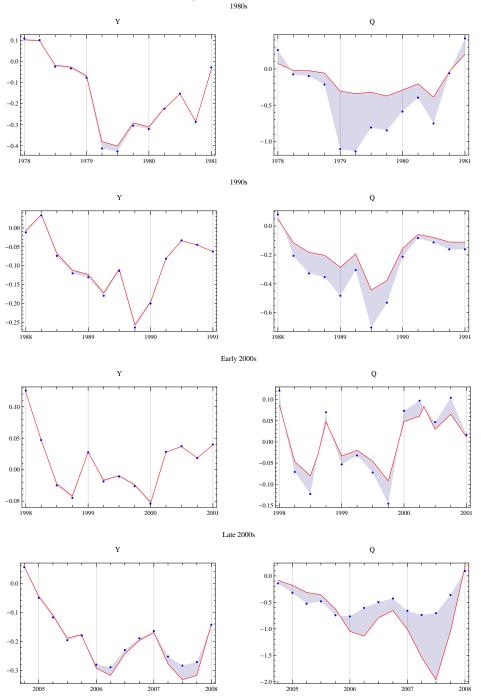
Figure 8: Estimates of  $\rho_{\theta}$  (Constant-Gain: Red Solid Line; OLS: Blue Dotted Line)

ple, as opposed to what happens at the beginning of the sample period. This is indeed confirmed in Figure 9, which reports several recessions (in percentage deviations from steady-state) that are predicted by our model when fed with the innovations drawn from the HP-detrended ( $\varepsilon$ -adjusted) leverage data over the 1980-2008 period.

The right panels in Figure 9 show that land price movements are dampened under learning, except for the late 2000's because that is a period when agents overestimate the persistence of the impact of leverage shocks (see Figure 8). On the other hand, the responses of output are quantitatively similar under rational expectations and under learning, with only a minor difference for 2006-2008. Basically, the last recession predicted by the model happens too early and is much too small compared to NBER estimates of the Great Recession. We now show that a much more realistic recession is predicted by our model if we allow agents to revise their estimate of the leverage level, in accordance with our second experiment reported in Section 4. The reason which motivates such a specification is that log-linearizing the model and feeding it with HP-detrending innovations amounts to assuming that agents know the steady-state so that they can compute the deviations from it. In view of Figure 1, this is arguably a strong

Figure 9: Model-Generated Recessions (Constant-Gain Learning: Red Solid Line;
Rational Expectations: Blue Dotted Line)

Output and Land Price Responses (Percentage Deviations from the RE Steady State)  $_{\scriptscriptstyle{1980s}}$ 



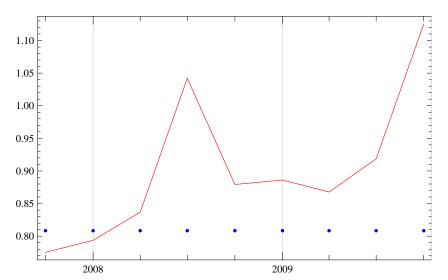


Figure 10: CG Estimate of Leverage Level over 2007Q4-2009Q4

assumption that could possibly underestimate the impact of learning. In particular, it is possible that when the Great Recession happened, agents revised downward their estimate of the leverage level, which worsened the negative impact of the leverage shock.

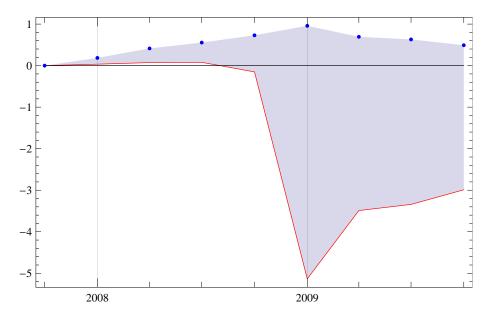
To account for this, we now turn to our second experiment. We use the model in (log)levels presented in Section 4. We make the conservative assumption that agents know the true autocorrelation  $\rho_{\theta}$  (which is estimated though OLS over the whole sample period) but not the leverage level  $\overline{\Theta}$  which they estimate under CG. All parameter values are set according to Table 1, except for  $\overline{\Theta} = 0.8$  which we set to the pre-crisis OLS estimate,  $\varepsilon = 0.25$  and  $\nu = 0.008$  which we choose so as to ensure E-stability<sup>8</sup>.

Figure 10 reports the CG estimate of the leverage level during the Great Recession period. It clearly shows how agents revised downward their estimate of  $\overline{\Theta}$  in the last quarter of 2008, from about 1.04 to 0.88. Figure 11 shows that this led to a recession under learning (red solid line). Although the trough happens one quarter too early (in 2009Q1 in the model as opposed to 2009Q2 according to NBER dating), the magni-

<sup>&</sup>lt;sup>8</sup>In addition to avoid instability of learning dynamics, reducing the leverage elasticity to land price  $\varepsilon$  and the gain parameter  $\nu$  amounts to a more conservative calibration.

Figure 11: Model-Generated Great Recession (Constant-Gain Learning: Red Solid Line; Rational Expectations: Blue Dotted Line)

Output Response Over Time (Percentage Deviations From 2007Q4)

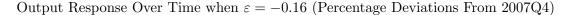


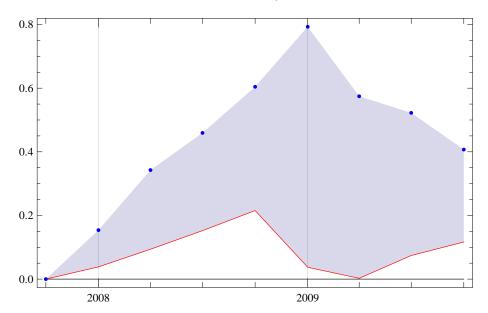
tude is correct (about -5.1% from peak to trough). This is in sharp contrast with the expansion that is predicted by the model under rational expectations (blue dotted line in Figure 11), which is driven by the positive innovations arising from OLS estimation. As a counter-factual, Figure 12 reports the output response that occurs under mildly countercyclical leverage, with  $\varepsilon = -0.16$  (implying that a 10% fall in land price increases leverage by 1.6%). Comparing Figures 11 and 12 suggests that a simple macroprudential policy may substantially attenuate leverage shocks under learning.

# 6 Conclusion

A large part of business-cycle theory relies on the assumption that agents know all parameters governing the stochastic process underlying the disturbances that hit the

Figure 12: Counter-factual Model-Generated Expansion (Constant-Gain Learning: Red Solid Line; Rational Expectations: Blue Dotted Line)





economy. This paper has shown how relaxing such an assumption in a simple model predicts that the economy's aggregates respond very differently to financial shocks when agents are gradually learning their environment, compared to rational expectations. More specifically, our theoretical experiments with a calibrated model suggest that some parameter configurations can lead to much larger amplification of the impact of shocks to leverage. This is for instance the case when learning agents overestimate either the autocorrelation parameter governing the persistence of leverage shocks or the long-run level of leverage. We have provided evidence that both cases are not inconsistent with the US data prior to the Great Recession, when borrowers probably believed that credit collateralized by real estate assets was being extended by the financial sector. In addition, the more empirically oriented counterparts of our two theoretical experiments are informative about which assumption better stands against the data. More precisely, we have shown that the linearized model in percentage deviations cannot, when fed with

the actual leverage innovations, explain the timing and magnitude of the Great Recession. Our preferred model with agents updating their estimates of the long-run level of leverage as new data arrive is more successful in that respect. In particular, it predicts the correct fall in output from peak to trough, as reported by the NBER, whereas the rational expectations model predicts a continued, counter-factual expansion in 2008 and 2009.

We believe that the main results of this paper may also be relevant for studying other settings. For example, they are suggestive about how one could try to measure to what extent unemployment variations are driven by beliefs formed by firms about either the persistence of demand shocks or the steady-state level of demand, or both. Monetary policy perhaps provides still another example in which the beliefs formed by the private sector about the persistence or about the long-run stance of monetary policy matter, as they could change the effects of policy on the economy. These are but a few examples for which extensions of the setting used in this paper could lead to fruitful research. In the same vein, another potential avenue for future research would be to model how perceptions about the process driving uncertainty shocks affect how those shocks propagate in the real economy. This requires solving higher-order approximation of nonlinear models and we believe this calls for further inquiries.

# A Appendix

## A.1 Intertemporal Equilibria around Steady State

This section derives some simple micro-foundations for the assumption of elastic leverage captured in (8) and presents the linearized version of the dynamics equations that follow Elastic leverage: the case when leverage is procyclical (that is,  $\varepsilon > 0$ ) obtains in a setting with ex-post moral hazard and costly monitoring similar to Aghion et al. [2, p.1391]. Suppose that the borrower has wealth QL and has access to investment opportunities, which can be financed by credit in the amount B. If the borrower repays next period, his income is I - (1 + R)B, where I is whatever income was generated by investing. If the borrower defaults next period, his income is now I - pQL, assuming that he loses his collateral with some probability p, which represents for example the frequency of foreclosures. Strategic default is avoided provided that  $I - (1 + R)B \ge I - pQL$ , that is, if  $pQL \ge (1 + R)B$ . The lender incurs a cost C(p)L when collecting collateral, with C'(p) > 0 and C''(p) > 0, and he chooses the optimal monitoring policy by solving:

$$\max_{p} pQL - C(p)L \tag{19}$$

which gives Q = C'(p). The higher the land price, the larger the incentives to increase effort to collect collateral. Assuming now that the cost function is  $C(p) = \phi p^{1+1/\varepsilon}/(1+1/\varepsilon)$ , with  $\varepsilon > 0$ , gives that  $p = (Q/\phi)^{\varepsilon}$ . Setting the scaling parameter  $\phi = Q^*\Theta^{-1/\varepsilon}$ , where  $Q^*$  is steady-state land value and  $\Theta$  is leverage, gives (8). Therefore, ex-post moral hazard leads to procyclical leverage.

In contrast, countercyclical leverage obtains if government implements procyclical taxes as follows. Suppose now that the lender gets  $(1-\tau)pQL - C(p)L$  when monitoring, where  $1 \ge \tau \ge 0$  is the tax rate. Under the assumption that the cost function is isoelastic, the optimal p is now  $p = ((1-\tau)Q/\phi)^{\varepsilon}$ . If the government sets time-varying taxes such that  $1-\tau = (Q/\phi)^{-\eta/\varepsilon-1}$ , for some  $\eta \ge 0$ , then it follows that  $p = (Q/\phi)^{-\eta}$  and that leverage is countercyclical. Note that this happens provided that the tax rate goes up when the land price goes up.

Linearized dynamics: we now derive the linearized version, in percentage deviations from steady-state values, of the set of equations (2)-(7) defining, together with the leverage law of motion  $\Theta_t = \overline{\Theta}^{1-\rho_\theta} \Theta_{t-1}^{\rho_\theta} \Xi_t$ , local intertemporal equilibria. In all equations

below,  $x_t$  denotes the deviation of  $X_t$  from its steady-state value in percentage terms. For example,  $k_t \equiv (K_t - K)/K$ , where K is the steady-state capital stock. Eliminating  $\Phi_t$  by using (7), one gets the following linearized equations corresponding to (2)-(7), respectively:

$$\frac{K}{Y}k_t - \frac{B}{Y}b_t = -\frac{C}{Y}c_{t-1} - \frac{(1+R)B}{Y}b_{t-1} + \left(\alpha + (1-\delta)\frac{K}{Y}\right)k_{t-1}$$
 (20)

$$b_t = (1 + \varepsilon)E_{t-1}[q_t] + \theta_{t-1}$$
(21)

$$c_t = -\lambda_t/\sigma \tag{22}$$

$$q_{t} + \lambda_{t}(1 - \mu\overline{\Theta}) = E_{t}[\lambda_{t+1}] \left(\beta(1 - \overline{\Theta}) + \gamma\beta\frac{Y}{Q}\right) + E_{t}[q_{t+1}](\beta + \overline{\Theta}(1 + \varepsilon)(\mu - \beta)) + \alpha\gamma\beta\frac{Y}{Q}E_{t}[k_{t+1}] + \theta_{t}\overline{\Theta}(\mu - \beta)$$
(23)

$$\lambda_t = E_t[\lambda_{t+1}](\beta(1-\delta) + \alpha\beta\frac{Y}{K}) + \alpha\beta(\alpha-1)\frac{Y}{K}E_t[k_{t+1}]$$
 (24)

$$\theta_t = \rho_\theta \theta_{t-1} + \xi_t \tag{25}$$

Define  $P_t \equiv (b_t \ k_t \ \theta_t)'$  and  $S_t = (c_t \ q_t \ \lambda_t)'$  the vectors of predetermined and jump variables, respectively. Then equations (20)-(25) can be decomposed into two subsystems, each pertaining to  $P_t$  and  $S_t$ . The first block is composed of (20), (21) and (25) and can be written:

$$M_0 P_t = M_1 S_{t-1} + M_2 E_{t-1} [S_t] + M_3 P_{t-1} + V \xi_t$$
(26)

where: 
$$M_0 = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{B}{Y} & \frac{K}{Y} & 0 \\ 0 & 0 & 1 \end{pmatrix}, M_1 = \begin{pmatrix} 0 & 0 & 0 \\ -\frac{C}{Y} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, M_2 = \begin{pmatrix} 0 & 1+\varepsilon & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$M_3 = \begin{pmatrix} 0 & 0 & 1 \\ -(1+R)\frac{B}{Y} & \alpha + (1-\delta)\frac{K}{Y} & 0 \\ 0 & 0 & \rho_{\theta} \end{pmatrix} \text{ and } V = (0\ 0\ 1)'.$$

The second block (22)-(24) can be written:

$$M_4S_t = M_5E_t[S_{t+1}] + M_6P_t + M_7E_t[P_{t+1}]$$
(27)

where:  

$$M_{4} = \begin{pmatrix} 0 & 1 & 1 - \mu \overline{\Theta} \\ 0 & 0 & 1 \\ \sigma & 0 & 1 \end{pmatrix}, M_{5} = \begin{pmatrix} 0 & \beta + \overline{\Theta}(1+\varepsilon)(\mu-\beta) & \beta(1-\overline{\Theta}) + \gamma\beta\frac{Y}{\overline{Q}} \\ 0 & 0 & \beta(1-\delta) + \alpha\beta\frac{Y}{\overline{K}} \\ 0 & 0 & 0 \end{pmatrix},$$

$$M_{6} = \begin{pmatrix} 0 & 0 & \overline{\Theta}(\mu-\beta) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, M_{7} = \begin{pmatrix} 0 & \alpha\gamma\beta\frac{Y}{\overline{Q}} & 0 \\ 0 & \alpha\beta(\alpha-1)\frac{Y}{\overline{K}} & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Finally, substituting the expression of  $P_t$  from (26) in (27) and piling up the resulting two blocks of equations allows one to rewrite the system as:

$$X_{t} = \mathbf{A}X_{t-1} + \mathbf{B}E_{t-1}[X_{t}] + \mathbf{C}E_{t}[X_{t+1}] + \mathbf{D}\xi_{t}$$
(28)

where  $X_t = \text{vec}(S_t P_t)$  and:

$$\mathbf{A} = \begin{pmatrix} M_4^{-1} M_6 M_0^{-1} M_1 & M_4^{-1} M_6 M_0^{-1} M_3 \\ M_0^{-1} M_1 & M_0^{-1} M_3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} M_4^{-1} M_6 M_0^{-1} M_2 & O_3 \\ M_0^{-1} M_2 & O_3 \end{pmatrix},$$

$$\mathbf{C} = \begin{pmatrix} M_4^{-1} M_5 & M_4^{-1} M_7 \\ O_3 & O_3 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} M_4^{-1} M_6 M_0^{-1} V \\ M_0^{-1} V \end{pmatrix}$$

where  $O_3$  is a 3-by-3 zeroes matrix.

### A.2 Closed-Economy Model with Constant Interest Rate

The purpose of this appendix is to show that, similar to the open-economy model developed in Section 2, the debtor interest rate is constant over time in a closed-economy version with domestic borrowers and lenders, when the preferences of the latter are appropriately chosen.

Let us now assume that lenders are domestic agents (instead of foreign countries as in Section 2), whose unique role is to provide loans to borrowers. Following Iacoviello [19], lenders derive utility from consumption and land holdings, and they get interest income

from last period's loan payments. As discussed in Pintus and Wen [35], lenders may be interpreted as financial intermediaries. The representative lender solves:

$$\max E_0 \sum_{t=0}^{\infty} \mu^t \left\{ \frac{(C_t^l)^{1-\sigma_c} - 1}{1 - \sigma_c} + \psi \frac{(L_t^l)^{1-\sigma_l} - 1}{1 - \sigma_l} \right\}$$
 (29)

with  $\sigma_c$ ,  $\sigma_l$ ,  $\psi$  all strictly greater than zero and  $\mu \in (0,1)$ , subject to the budget constraint:

$$C_t^l + Q_t(L_{t+1}^l - L_t^l) + B_{t+1} = (1 + R_t)B_t$$
(30)

where  $C_t^l$  and  $L_t^l$  denotes the lender's consumption and land holdings, respectively,  $Q_t$  is the land price,  $B_{t+1}$  is the new loan. The interest rate  $R_t$  is now endogenous and it is determined by the equality between the demand and supply of loans.

The first-order conditions obtained from (29)-(30) with respect to consumption, land, and lending are, respectively:

$$\left(C_t^l\right)^{-\sigma_c} = \chi_t \tag{31}$$

$$\chi_t Q_t = \mu E_t [\chi_{t+1} Q_{t+1}] + \mu \psi (L_{t+1}^l)^{-\sigma_l}$$
(32)

$$\chi_t = \mu E_t [\chi_{t+1} (1 + R_{t+1})] \tag{33}$$

where  $\chi_t$  is the Lagrange multiplier of constraint (30) in period t.

Assuming that lenders' utility is linear in consumption (that is,  $\sigma_c = 0$ ), one gets from (31) that in any rational expectations equilibrium  $\chi_t = 1$  for all  $t \geq 0$  so that, in view of (33), the interest factor is constant and given by  $1 + R = 1/\mu$ . As in the small-open economy model developed in Section 2, the interest rate is constant over time.

The borrower side of the model is still described by (1), (2) and (3), as in Section 2, with the addition that the total amount of land is now divided between lenders and borrowers according to:

$$L_t + L_t^l = \bar{L}.$$

where  $\bar{L}$  is the fixed supply of land. How exactly is land divided depends on both the sequence of land price and the lender's preferences, as reflected in the first-order condition (32). In addition, the representative borrower's first-order conditions are given by (4)-(7). As in Section 2, if  $\mu \in (\beta, 1)$ , then the borrower's credit constraint (3) is binding. Therefore, the main difference is that the closed-economy model allows some reallocation of land from lenders to borrowers when a shock hits the economy. Under our calibration (see Table 2), however, the effect of land reallocation is quantitatively unimportant because the land share  $\gamma$  is reasonably small. We have run simulations for the rational expectations versions of the open and closed economies and we have confirmed that the impulse-response functions of the variables involved in Section 2 are quantitatively similar under TFP shocks. In particular, the land price and debt behave in the same way in both economies.

## A.3 Time-Varying Persistence of Leverage Shocks in the Data

Figure A1 pictures both the CG and the OLS estimates for  $\rho_{\theta}$  (red solid line and blue dotted line respectively) obtained from the data sample in Figure 1, extended back to 1975, with  $\nu = 0.04$  (data not detrended).

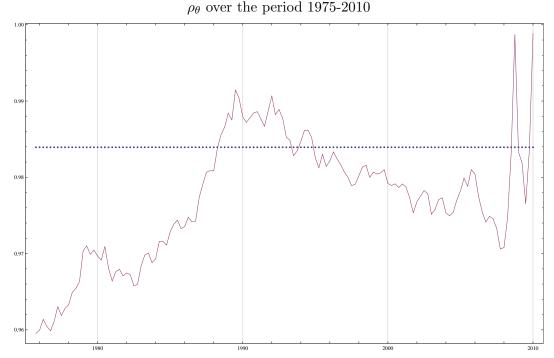
### A.4 Learning Procedure of VAR Model

#### A.4.1 Model in Levels

Define  $P_t \equiv (b_t \ k_t \ \theta_t)'$  and  $S_t = (c_t \ q_t \ \lambda_t)'$  as the vectors of predetermined and jump variables in logs, respectively (e.g.  $k_t = \log(K_t)$  where  $K_t$  is the capital stock level). Then equations (20), (21) and (25) can now be rewritten as:

$$M_0P_t = M_1S_{t-1} + M_2E_{t-1}[S_t] + M_3P_{t-1} + N_0 + V\xi_t$$

Figure A1: OLS (Blue Dotted Line) and Constant-Gain (Red Solid Line) Estimates of



while (22)-(24) can be written as:

$$M_4S_t = M_5E_t[S_{t+1}] + M_6P_t + M_7E_t[P_{t+1}] + N_1$$

with

$$N_{0} = \begin{pmatrix} b - (1+\varepsilon)q - \theta \\ \frac{K}{Y}k + \frac{(R-1)B}{Y}b + \frac{C}{Y}c - (\alpha + \frac{(1-\delta)K}{Y})k \\ (1-\rho_{\theta})\overline{\Theta} \end{pmatrix},$$

$$(1-\rho_{\theta})\overline{\Theta}$$

$$N_{1} = \begin{pmatrix} q(1-\beta-\overline{\Theta}(1+\varepsilon)(\mu-\beta)) + \lambda(1-\overline{\Theta}\mu-\beta(1-\overline{\Theta})-\beta\gamma\frac{Y}{Q}) - \alpha\beta\gamma\frac{Y}{Q}k - \overline{\Theta}(\mu-\beta)\theta \\ \lambda(1-\beta(1-\delta)-\alpha\beta\frac{Y}{K}) - \alpha(\alpha-1)\beta\frac{Y}{K}k \\ c + \frac{\lambda}{\sigma} \end{pmatrix}$$

where variables in lowercase letters denote logged steady-state levels (e.g.  $k = \log(K)$ , where K is the steady-state capital stock). Denoting  $X_t = \text{vec}(S_t P_t)$  the system can be

written as before:

$$X_{t} = \mathbf{A}X_{t-1} + \mathbf{B}E_{t-1}[X_{t}] + \mathbf{C}E_{t}[X_{t+1}] + \mathbf{N} + \mathbf{D}\xi_{t}$$
(34)

where  $\mathbf{A}$ ,  $\mathbf{B}$ ,  $\mathbf{C}$ ,  $\mathbf{D}$  are as in Appendix A.1 and:

$$\mathbf{N} = \begin{pmatrix} M_4^{-1} N_1 + M_4^{-1} M_6 M_0^{-1} N_0 \\ M_0^{-1} N_0 \end{pmatrix}$$

The rational expectations solution has a VAR form:

$$X_t = \mathbf{M}X_{t-1} + \mathbf{H} + \mathbf{G}\xi_t. \tag{35}$$

Given this form of equilibrium, the law of motion of endogenous variables can be represented using  $E_{t-1}X_t = \mathbf{M}X_{t-1} + \mathbf{H}$  and  $E_tX_{t+1} = \mathbf{M}X_t + \mathbf{H}$  as:

$$X_t = \mathbf{A}X_{t-1} + \mathbf{B}\left[\mathbf{M}X_{t-1} + \mathbf{H}\right] + \mathbf{C}\left[\mathbf{M}X_t + \mathbf{H}\right] + \mathbf{N} + \mathbf{D}\xi_t,$$
(36)

or

$$X_t = [I - \mathbf{CM}]^{-1} [\mathbf{A} + \mathbf{BM}] X_{t-1} + [I - \mathbf{CM}]^{-1} [\mathbf{BH} + \mathbf{CH} + \mathbf{N}] +$$
$$+ [I - \mathbf{CM}]^{-1} \mathbf{D} \xi_t,$$

Matrices  $\mathbf{M}$  and  $\mathbf{H}$  are given by:

$$\mathbf{M} = [I - \mathbf{C}\mathbf{M}]^{-1} [\mathbf{A} + \mathbf{B}\mathbf{M}] \tag{37}$$

$$\mathbf{H} = [I - \mathbf{CM}]^{-1} [\mathbf{BH} + \mathbf{CH} + \mathbf{N}]. \tag{38}$$

### A.4.2 VAR Estimation

To estimate the VAR we represent the model as

$$X_t = \Omega Z_t + \epsilon_t, \tag{39}$$

where  $Z_t = \begin{bmatrix} 1 & X_t \end{bmatrix}$  and  $\Omega = \begin{bmatrix} H & M \end{bmatrix}$  .

The estimator for  $\Omega$  equals

$$\hat{\Omega} = XZ'(ZZ')^{-1},\tag{40}$$

and its time T estimates,  $\hat{\Omega}_T$ , can be computed from

$$\hat{\Omega} = \left(\frac{1}{T} \sum_{t=1}^{T} X_t Z_t'\right) \left(\frac{1}{T} \sum_{t=1}^{T} Z_t Z_t'\right)^{-1}.$$
(41)

The recursive updating takes form of

$$\hat{\Omega}_{T+1} = \hat{\Omega}_T - \frac{1}{T+1} \left( \hat{\Omega}_T Z_{T+1} - X_{T+1} \right) Z'_{T+1} R_{T+1}^{-1}$$
(42)

and

$$R_{T+1} = R_T + \frac{1}{T+1} \left( Z_{T+1} Z'_{T+1} - R_T \right). \tag{43}$$

Equations (43) and (42) show how the estimates of matrix  $\Omega$  are updated as new data become available. In the above expression,  $\hat{\Omega}_T Z_{T+1} - X_{T+1}$  corresponds to a forecast error made using last period estimates.

#### A.4.3 Learning

Assume agents re-estimate the consistency with the REE model each period and use their estimates to make forecasts. These forecasts affect the behavior of the economy through equation (34).

Agents' perceived low of motion is

$$X_t = \mathbf{M}X_t + \mathbf{H} + \varepsilon_t = \Omega Z_t + \varepsilon_t. \tag{44}$$

The forecasts agents make use the estimates of this PLM over available data. Since  $X_t$  depends on agents' forecasts (so it is not available at time t regression) at time t agents

have run the regression:

$$E_t X_{t+1} = \mathbf{M}_{t-1} X_t + \mathbf{H}_{t-1} = \Omega_{t-1} Z_t$$
 (45)

$$E_{t-1}X_t = \mathbf{M}_{t-2}X_{t-1} + \mathbf{H}_{t-2} = \Omega_{t-2}Z_{t-1}$$
(46)

where now we allow agents to depart from running simply OLS regression (least-squares learning) and use constant gain,

$$R_t = R_{t-1} + \gamma_t \left( Z_t Z_t' - R_{t-1} \right)$$
  
$$\Omega_t = \Omega_{t-1} - \gamma_t \left( \Omega_{t-1} Z_t - X_t \right) Z_t' R_t^{-1}.$$

Substituting in agents' expectations, we can write the actual law of motion as

$$X_{t} = \mathbf{A}X_{t-1} + \mathbf{B}\left[\mathbf{M}_{t-2}X_{t-1} + \mathbf{H}_{t-2}\right] + \mathbf{C}\left[\mathbf{M}_{t-1}X_{t} + \mathbf{H}_{t-1}\right] + \mathbf{N} + \mathbf{D}\xi_{t}$$
(47)

or

$$X_{t} = [I - \mathbf{C}\mathbf{M}_{t-1}]^{-1} [\mathbf{A} + \mathbf{B}\mathbf{M}_{t-2}] + [I - \mathbf{C}\mathbf{M}_{t-1}]^{-1} [\mathbf{C}\mathbf{H}_{t-1} + \mathbf{B}\mathbf{H}_{t-2} + \mathbf{N}] + + [I - \mathbf{C}\mathbf{M}_{t-1}]^{-1} \mathbf{D}\xi_{t}$$
(48)

There is a mapping  $\{\mathbf{M}, \mathbf{H}\} = T(\mathbf{M}, \mathbf{H})$  from PLM to ALM,

$$T_M(\mathbf{M}, \mathbf{H}) = [I - \mathbf{C}\mathbf{M}]^{-1} [\mathbf{A} + \mathbf{B}\mathbf{M}]$$
(49)

$$T_H(\mathbf{M}, \mathbf{H}) = [I - \mathbf{C}\mathbf{M}]^{-1} [\mathbf{B}\mathbf{H} + \mathbf{C}\mathbf{H} + \mathbf{N}].$$
 (50)

Rational expectations equilibrium is a fixed-point of this mapping:

$$\mathbf{M^{re}} = [I - \mathbf{CM^{re}}]^{-1} [\mathbf{A} + \mathbf{BM^{re}}]. \tag{51}$$

Conditional on  $\mathbf{M^{re}}$  we can solve for  $\mathbf{H^{re}}$ :

$$\mathbf{H^{re}} = \left[I - \left[I - \mathbf{CM^{re}}\right]^{-1} \left(\mathbf{B} + \mathbf{C}\right)\right]^{-1} \left[I - \mathbf{CM^{re}}\right]^{-1} \mathbf{N}.$$
 (52)

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