DOCUMENT
DE TRAVAIL
N° 507

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September 2014

BANQUE DE FRANCE
EUROSYSTÈME

DIRECTION GÉNÉRALE DES ÉTUDES ET DES RELATIONS INTERNATIONALES

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*I would like to thank my supervisor F. Langot for motivating my research by asking the right questions, as well as N. Dromel, A. Sahin and A. Chéron for their comments and suggestions; I am also grateful to seminar participants in the University of Le Mans, France (January 2014) and in the "Ecole Thematique des Evaluations des Politiques Publiques" in Aussois, France (March 2013). All errors are mine. The views expressed in this paper solely reflect those of the author and not necessarily those of the Banque de France.

Résumé

Dans ce papier j'étudie la volatilité des variables du marché du travail générée dans les modèles d'appariement et la corrélation de ces variables avec la productivité du travail. D'un côté Shimer (2005) écrit que « Non seulement il y a peu d'amplification, mais il y a aussi très peu de propagation d'un choc de productivité du travail dans le modèle [d'appariement] ». De l'autre, à partir de Galì (1999) la littérature empirique semble indiquer qu'un choc de productivité puisse avoir comme effet de réduire les heures travaillées à l'impact ; cette évidence remet en cause le mécanisme de transmission décrit par Shimer (2005) en accord avec le fonctionnement des modèles de cycle réels. Je montre qu'un modèle dans la tradition des Nouveaux Keynésiens avec rigidités nominales peut reproduire simultanément les moments concernant la volatilité relative et la corrélation des variables du marché du travail par rapport à la productivité. Le modèle présente deux caractéristiques : le facteur travail diminue après un choc technologique positif, et la stratégie de calibration des coûts d'ouverture d'un poste vacant diffère par rapport à celle-ci suivie par Shimer (2005), ce qui est cohérent avec les modèles de cycle réels traditionnels. Je montre aussi qu'en choisissant des préférences de type traditionnel (séparables par rapport à la consommation et au loisir) la critique de Shimer est d'autant plus valable, à cause des effets richesse qui agissent sur l'offre de travail.

Mots-clés: fluctuations du marché du travail, choc technologique, rigidités des prix

Codes JEL: E24, E32, J60

Abstract

In this paper I shed light on the issues of the (low) volatilities of labor market variables implied by the search and matching model and the (high) values of the correlations between these variables and labor productivity. On the one hand, Shimer (2005) claims that "Not only there is little amplification, but there is also no propagation of the labor productivity shock in the [search and matching] model." On the other, starting from Galì (1999) empirical evidence about the reaction of employment to a neutral positive technological shock seems to indicate a recessionary effect in the short term, thus casting doubts about the whole transmission mechanism as described by Shimer (2005) in line with a RBC framework. I claim that a New Keynesian model with nominal rigidities is able to replicate the set of moments of both volatilities and correlations; the model presents two distinctive features: employment decreases after a positive technological shock and the calibration strategy in choosing the vacancy posting cost is different with respect of Shimer (2005) and in line with the RBC tradition. I show also that the use of the traditional separable preferences in consumption and leisure worsens the Shimer's critique, via the consequences of wealth effects on labor supply.

Keywords: labor market fluctuations, technology shock, price rigidities

JEL codes: E24, E32, J60

Non-technical summary

The search and matching framework has become the workhorse model to analyze the phenomenon of unemployment, yet Shimer (2005) in a widely cited paper casts doubts about the ability of the search and matching model to replicate the relative volatilities of labor market variables with respect to a labor productivity shock: his calibrated version of the model generates unemployment and vacancies which are ten times less volatile than in the data. Shimer (2005) points out another drawback, which has received less attention: the model lacks also a "propagation mechanism" (in addition to the "amplification mechanism" just described), since the shock is transferred almost one-to-one from productivity to the labor market variables: the values of the correlations of labor market variables with the labor productivity are in fact much higher (in absolute value) than in the data.

The "puzzle" raised by Shimer (2005) about the relative volatilites of labor market variables in the search and matching model has received much attention in the literature: on the one hand, authors like Hagedorn, Manovski (2008) have pointed that the calibration strategy is very important, mostly for what it concerns the values of the bargaining power of the workers and the value of the "outside options" of unemployed workers in the wage bargaining process; on the other hand, authors like Hall, Milgrom (2008) have claimed that it is necessary to go beyond the Nash bargaining scheme, in order to obtain wages which are more rigid: if the adjustment following a shock cannot be done through prices (wages), then quantities would react more, enhancing the volatility of such variables as unemployment and vacancies.

It is important at this point to remember that Shimer (2005) and his followers work within a real business cycle framework to explain the transmission mechanism of a technological shock; after an increase in productivity firms can produce more, so they post more vacancies, but this creates a tension on the labor market: tightness increases, so do wages, and the initial additional profitability is squeezed, so that in fact firms do not have incentives to post vacancies any more.

The main point of my work is to analyze the impact that a different transmission mechanism of a labor productivity shock can have on both the volatility and the correlation puzzles: starting from Galì (1999) in fact there has been a (sometimes harsh) debate about the reaction of hours worked after a positive technological shock, but it seems that a consensus is built around the fact that hours worked decrease on impact, but then increase and finally go back to their initial level[1].

In a context of "rigid demand" (if for example firms can change price only with a certain probability, i.e. there are some price rigidities "à la Calvo"), after a positive technological shock the firm wants to produce the same quantity of the good (since the demand it faces is fixed), but it is now more productive: it can therefore use "less" labor, so that on impact vacancies decrease and unemployment rises; as long as firms can change their prices, we go back to the functioning of a "real" model: firms can decrease their prices and produce more, they therefore want to hire more workers, so that vacancies increase and unemployment decreases. Even if hours worked decrease only in the "short run", this additional transmission mechanism has the effect of both enhancing the overall volatility and reducing the unconditional correlation of labor market variables with labor productivity.

I therefore claim that if one takes into account this difference between the "short" and the "medium" run, a simple New Keynesian general equilibrium model is able to replicate both set of moments (volatilities and correlations).

The main features of the model are two: on the one hand I follow a different discipline with respect to Shimer (2005) for what it concerns the calibration of the vacancy posting costs, since the model is a general equilibrium one, so that the critique of Shimer about the relative volatilites of variables is

[1]See for example Fève, Guay (2009)

already substantially mitigated[2]. On the other hand, the model includes prices rigidities à la Calvo and a monetary authority which sets the nominal interest rate according to a Taylor rule: the overall reaction of employment following a technological shock depends on the calibration of the Taylor rule (in particular it depends on how much the monetary authority is "accommodating" with respect to output); in my model the Taylor rule is calibrated according to Clarida, Gali, Gertler (2000) and I do have a contractionary effect on employment in the short run.

[2]It is important to remark that my calibration strategy implies (and does not postulate) a value for leisure (i.e. the value of the "outside options" of unemployed workers) which is even lower than that one calibrated by Shimer (2005), thus at the opposite of the strategy adopted by Hagedorn, Manovski (2008).

1 Introduction

Models in the New Keynesian framework have been extended to introduce labor market frictions à la Mortensen-Pissarides, in order to be able to speak about "unemployment" in a non trivial manner and to consider the interactions between inflation and unemployment.

In this paper I want to deal with two well known issues which concern the variable "labor", as it is considered in macroeconomic models: the first one (chronologically speaking) has been pointed out by Galì (1999) in his seminal paper about the reaction of "employment"[3] to a positive (neutral) technological shock; the second issue has been raised by Shimer (2005) in his equally seminal paper in which he questions the ability of the labor market model à la Mortensen-Pissarides to replicate the relative volatilities and correlations of labor market variables with respect to labor productivity.

My main point is that if one takes into consideration the empirical evidence in line with Galì (1999), the "puzzle(s)" raised by Shimer (2005) must be reinterpreted.

The key point lies in the transmission mechanism which is at the heart of many models in the New Keynesian framework: if we accept the idea that there exists some form of nominal rigidities in price setting, under some restrictions about how much monetary policy is "accommodating" with respect to technology shocks[4], a neutral technology shock has a "contractionary" effect in the short run on labor input.

This short term adjustment is in line with the Keynesian interpretation that "demand determines supply": if demand is fixed in the short run (since it depends on prices, which cannot be adjusted immediately by all firms), after a positive productivity shock firms will be willing to produce the same quantity, for which they need less labor.

Nevertheless, price rigidities are transitory: if the price adjustment process is not too slow (i.e. nominal rigidities are not too important), and the technological shock is enough persistent, the New Keynesian model can potentially also account for the "classical" dynamics we find in Real Business Cycle models. This means that once prices adjust, the effect of a productivity shock becomes "expansionary", i.e. it boosts the creation of vacancies and then employment.

According to this reasoning, labor market tightness in a New Keynesian model can be characterized by a non-monotonic adjustment after a transitory shock: a decrease in the very short run, followed by an increase and then the adjustment in the long run to go back to the steady state.

Thus, the elasticity of labor market tightness to a (permanent) shock[5] doesn't provide sufficient information about the labor market variables volatilities, because the adjustment is not monotonic: in my model labor market tightness decreases on impact and then overshoots its steady state value, before converging back to it.

Moreover, this different transmission mechanism breaks the tight link between labor market variables and labor productivity, which is reflected in the high (absolute) values of the correlations of these variables with productivity as in Shimer (2005), thus addressing also the "propagation mechanism" issue.

In this paper therefore I develop and calibrate a simple version of the standard New Keynesian model with search and matching frictions, in order to assess quantitatively the importance of the

[3]Employment (or labor input) in his empirical model corresponds to total hours (baseline specification) or to the employed civilian labor force (alternative specification)

[4]The reaction to technology shocks can be direct , as in Galì (1999), if we consider a money rule, or indirect if for example we consider a Taylor rule in which the authority sets the interest rate looking at the inflation and output gap

[5]Shimer (2005) approximates the quantitative implications of a transitory technological shock via the elasticities of labor market variables at steady state. See also Mortensen, Nagypàl (2007) for a discussion about the robustness of this approximation of the short run dynamics of the real matching model after a transitory shock

different transmission mechanisms.

My quantitative results are linked to three important aspects of the calibration strategy.

First of all and mostly important, I follow the RBC literature in choosing the calibration strategy procedure, as opposed to that one chosen by Shimer (2005): in particular this means that I derive the value of the cost of posting a vacancy from steady state restrictions, once I set a target value for the total costs of recruiting in terms of output; consequently, I derive endogenously the value of the "outside options" or the value of leisure. Shimer (2005) instead sets a value for leisure (connecting it with unemployment benefits) and derives the value of the vacancy posting cost. This could seem a detail, but instead it has important consequences: I show in fact that if I had to follow Shimer's procedure, I would have a much bigger value for the recruiting costs than what it is normally accepted in the RBC literature[6]. The calibration of the parameter which represents the vacancy posting cost has obviously a very important impact quantitatively on the overall functioning of the model: as we will see, it enters in the expression of the elasticity of labor market tightness with respect to labor productivity which is proposed by Shimer (2005) as informative of the functioning of the model.

Secondly, for what it regards the dynamics of employment, I consider that hirings are "immediately productive", as in Blanchard and Galì (2010). The logic behind this choice is that I want to allow employment to react immediately after a shock, as it seems to happen according to the empirical evidence in line with Galì (1999).

When new hired workers are immediately productive, a reformulation of the search and matching model in discrete time is necessary. In this setting in fact we can imagine that in each period there are two sub-periods: the market for input (which is only labor) and then production; at the beginning of the first sub-period, some matches are destroyed, but the separated agents can immediately look again for a job; the firm then decides how many workers to hire and in the second sub-period it produces using the "new hired workers". The pool of job seekers in the first sub-period is therefore given by the unemployed workers coming from the previous period plus the ones ($s * N$) whose matches have been destroyed: those workers can therefore look for a job, find one and become productive before the end of the period.

This sequence of events implies that those who look for a job constitute a larger set than normally considered to estimate the job finding rate, since some of them are "employed" at the end of the previous period and at the beginning of the following period, but in the meantime they have been "job seekers".

For what it regards the separation rate, the same gap with the usual measure also exists: during the period some matches are destroyed but new ones are formed before the end of the period, leading to an underestimation of the separation rate when only transitions from employment to unemployment are considered. Intuitively, if this second underestimation of the separation rate is less important, the corresponding job finding rate will be smaller than that one measured by Shimer (2005) for the same level of unemployment rate[7].

[6] As we will see in detail, in my calibration exercise I set the ratio of recruiting costs in terms of output to $\frac{\omega V}{Y} = 0.8\%$ while following the procedure of Shimer (2005) would imply a ratio of around 4%; the range for this ratio accepted in the RBC literature goes from 0.5% to 1%.

[7] In my calibration I set the quarterly separation rate to $s = 0.1$ and the employment rate to $N = 0.9455$, in line with Shimer (2005); these values, together with the "immediate hiring" hypothesis, imply a job finding probability of $p = 0.6344$, which corresponds to a monthly probability of $x_m = 0.285$, thus in line with Hall (2005), computed in order to verify $x_m + (1 - x_m)x_m + (1 - x_m)^2 x_m = 0.6344$.

In the Appendix I check that the model calibrated on a monthly basis, where $s = 0.034$ and $p = 0.45$ as in Shimer (2005), does give the same type of quantitiative results; anyway it has to be highlighted that because of instability problems, the calibration of the Calvo parameter cannot be the exact equivalent of the one of the quarterly model.

The adoption of the "instantaneous hirings" dynamics is another step in the direction of enhancing the reaction of labor market quantities to shocks, but as we will see in details, the overall quantitative effects are not revolutionary, mostly if compared to the impact of the calibration of the vacancy posting costs.

Finally, it is important to highlight that in calibrating the parameters of the Taylor rule, I consider that the monetary authority reacts to both inflation and output and I follow Clarida, Galì, Gertler (1999) in choosing the values identified for the Volcker-Greenspan period: this calibration allows to obtain a recessionary effect of a positive technological shock on employment.

To sum up, the steps which bring me to my model, by progressively distancing from Shimer (2005), can be outlined as follows: first of all I embed the search and matching model in a general equilibrium framework (with almost no nominal rigidities) and I show that the different calibration procedure with respect to Shimer (2005) allows to substantially reduce the "puzzle" about the relative volatilities of labor market variables; secondly, I analyze the different transmission mechanism of a labor productivity shock which arises in a context in which there is some degree of nominal rigidities (expressed as nominal price rigidities à la Calvo): in this case I claim that I can address both puzzles, i.e. the issues about the volatilities and the correlations of labor market variables with labor productivity.

Considering the related literature of New Keynesian models with search and matching frictions, some papers have addressed explicitly the relation between nominal frictions and the volatilities of labor market variables.

Andrés, Doménech and Ferri (2006) check the importance of four potential mechanisms present in a DSGE New Keynesian model to increase the volatility of labor market variables and they assess that among price rigidity, inter-temporal substitution effect, endogenous match destruction, capital accumulation and distortionary taxes, it is the presence of price rigidity which is the most important feature, but they do not discuss the transmission mechanism and they do not therefore look at the values of correlations of labor market variables with productivity.

Barnichon (2010) identifies a switch in the sign of the correlations of labor variables and labor productivity around 1984 and, among other possible mechanisms, he proposes a change in the structural parameters of the monetary policy rule which influence the reaction of unemployment to a positive technological shock.

Thomas (2011) develop a New Keynesian model with firms which not only decide how many vacancies to post to hire workers and produce, but also set their prices, since they are monopolistically competitive[8]; as the author himself acknowledges[9], even if his paper is not directly addressed to answer to the unemployment volatility puzzle, it proposes an amplification mechanism which enhances the volatility of labor market variables in a New Keynesian framework; interestingly his model calibration implies a contractionary effect of a positive technological shock on both employment and hours per worker.

Balleer (2012) strongly questions the existence of the puzzle about unemployment volatility as set by Shimer (2005): she shows that looking at unconditional moments can be misleading since, according to her VAR evidence, the volatility of unemployment conditional to a technological shock in the data is in line with the one implied by the model. According to the author, another and more important "puzzle" arises: after a technological shock unemployment increases and job finding rate falls, thus questioning all the transmission mechanism à la Shimer (which is the same as in a standard RBC

[8]This feature contrasts to the standard modelling device according to which there are two sectors, one in which the representative firm produces using labor and sells its output in a perfectly competitive market, and one in which retailers just "repackage" the differentiated good and set its price, as we find for example in Trigari (2006)

[9]Thomas (2011), p. 1133 footnote 3

Table 1: **Volatilities of labor market variables, Shimer (2005)**

Summary statistics, quarterly US data, 1951-2003

	U	V	θ	p	$productivity$
St dev	0.19	0.202	0.382	0.118	0.02
Correlation matrix	U	V	θ	p	$productivity$
U	1	-0.894	-0.971	-0.949	-0.408
V		1	0.975	0.897	0.364
θ			1	0.948	0.396
p				1	0.396
$productivity$					1

(U stands for unemployment, V for vacancies, θ for labor market tightness, p is job finding rate and $productivity$ is the average labor productivity -the seasonally adjusted real average output per person in the non-farm business sector-constructed by BLS; variables are expressed in logs as deviations from HP trend with smoothing parameter 10^5)

Summary statistics: relative volatilities

	U	V	θ	p	$productivity$
$\frac{\sigma(X)}{\sigma(productivity)}$	9.5	10.1	19.1	5.9	1

model); her point is thus that it is necessary to consider the presence of a demand shock (but then her preference shocks are not suitable to replicate the overall correlations of labor market variables with productivity).

Finally I am aware of the potential effects of the introduction of search and matching frictions on the Taylor principle, as stressed by Kurozumi, Van Zandweghe (2010), so I take into considerations their findings about the calibration of the Taylor rule for monetary policy.

The remainder of the paper is organized as follows: Section 2 illustrates the two puzzles raised by Shimer (2005) about the volatilities and the correlations of labor market variables with respect to productivity; Section 3 presents the model and the results; Section 4 illustrates graphically the main mechanisms; finally Section 5 contains the conclusions and Section 6 (the appendix) presents additional details of the model and some checks.

2 The Shimer (2005) puzzle(s) in the data

2.1 The values of relative volatilities

The criticism addressed by Shimer (2005) to the search and matching model about its ability to reproduce the main labor market variables volatilities is well known; I report in Table 1 the empirical moments identified by Shimer (2005) as the targets to reproduce.

The Shimer (2005) puzzle consists in the fact that the labor market variables obtained by his theoretical model are ten times less volatile than in the data.

Some authors, as Balleer (2012), have pointed out that Shimer (2005) considers unconditional moments, while the comparison between the data and the simulated moments should be done conditionally on the type of shocks. In the proceedings I will adopt the same approach as Shimer (2005), so I look at the unconditional moments in the data.

Table 2: **Model volatilities conditional to labor productivity shocks, Shimer (2005)**

Simulated moments, smoothing parameter= 10^5

	U	V	θ	p	*productivity*
St dev	0.009	0.027	0.035	0.010	0.02
Correlation matrix	U	V	θ	p	*productivity*
U	1	-0.927	-0.958	-0.958	-0.958
V		1	0.996	0.996	0.995
θ			1	1	0.999
p				1	0.999
productivity					1

Summary statistics: relative volatilities

	U	V	θ	p	*productivity*
$\frac{\sigma(X)}{\sigma(productivity)}$	0.45	1.35	1.75	0.5	1

2.2 The values of correlations

There is another failure in the search and matching model with only productivity shocks, in addition to the lack of amplification of the shock: as Shimer (2005) stresses, there is almost no propagation of the shock.

The correlations of labor market variables with productivity implied by the model are in fact too high: considering the whole data sample from 1951 to 2003, Shimer (2005) reports empirical (unconditional) correlations within the 0.35-0.4 range[10], while the values implied by the model are above 0.95 (in absolute value); Shimer (2005) moreover highlights that the empirical values can hide a switch in sign around the mid-eighties[11].

Considering the period 1951-2011, I find values roughly in line with those reported by Shimer (2005) and Michaillat (2012) for the correlations between vacancies, unemployment and tightness with labor productivity; when I look at the two sub-periods 1951-1984 and 1985-2011, I find evidence of a decrease in their absolute values, but no switch in sign[12], as we can see in Table 3.

Considering the large literature that has documented the presence of a break in many macroeconomic series after the arrival of Volcker at the Federal Reserve Bank, I decide to consider as my empirical counterpart the period post-1984.

3 The Galì (1999) evidence

The question about the reaction of hours to a technological shock raised by Galì (1999) has been the object of a vast discussion in macroeconomics.

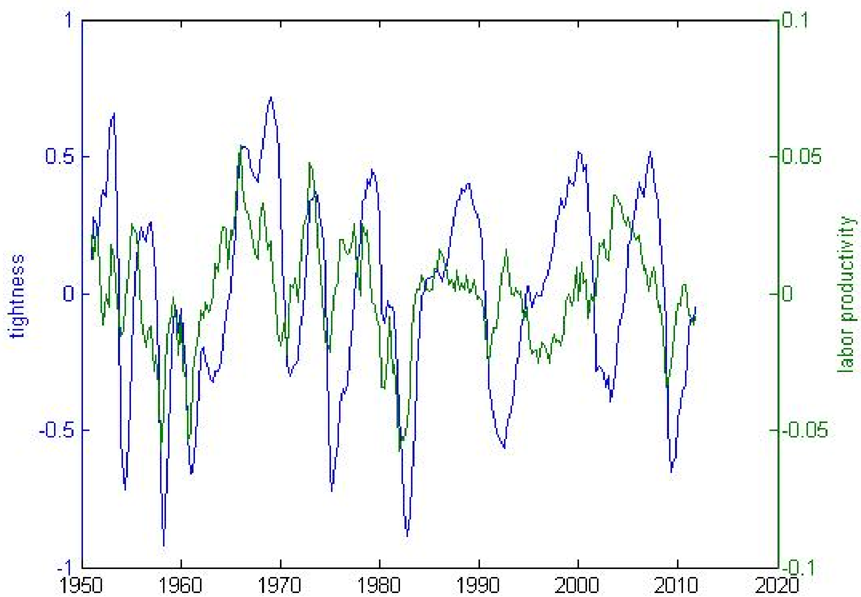
Without attempting to give here an exhaustive discussion, I just want to summarize the conclusions I retain to be of interest for this paper; in doing this, I rely heavily on the clear exposition one can

[10]Michaillat (2012) reports slightly higher values, in the range 0.5-0.65, for the period 1964-2009

[11]Shimer (2005), p. 33 : "From 1951 to 1985, the contemporaneous correlation between detrended labor productivity and and the v-u ratio was 0.57 and the peak correlation was 0.74. From 1986 to 2003, however, the contemporaneous and peak correlations are negative, -0.37 and -0.43, respectively.[...]"

[12]If I consider the correlation between labor market tightness and productivity for the same sub-period analyzed by Shimer (2005), i.e. 1986-2003, however, I find that it is indeed negative

Figure 1: **Productivity and labor market tightness 1951-2011**



Labor productivity is, as in Shimer (2005), the seasonally adjusted real average output per person in the non-farm business sector as constructed by the LBS;
labor market tightness is the ratio of vacancies and unemployment expressed in levels;
for vacancies I use the composite Help Wanted Index proposed by Barnichon (2010b),
the series of unemployment is taken from BLS.

Table 3: **Summary statistics U.S quarterly data**

	Michaillat (2012)	**my calculations**		
	1964-2009	**1951-2011**	**1951-1984**	**1985-2011**
σ_Y	0.03	0.0333	0.0343	0.0309
σ_A	0.02	0.0202	0.0227	0.0159
$\frac{\sigma(U)}{\sigma(A)}$	8.5	9.5325	9.3295	10.3446
$\frac{\sigma(V)}{\sigma(A)}$	9.2	9.3541	8.8242	10.5140
$\frac{\sigma(\theta)}{\sigma(A)}$	17.1	18.3171	17.2945	20.1422
$\frac{\sigma(w)}{\sigma(A)}$	1.05	1.1043	1.2558	1.0086
$\frac{\sigma(Y)}{\sigma(A)}$	1.5	1.6525	1.5115	1.9422
$corr(U,A)$	-0.561	-0.4140	-0.4856	-0.1175
$corr(V,A)$	0.524	0.40	0.5249	0.0739
$corr(\theta,A)$	0.559	0.4188	0.5169	0.0984
$corr(w,Y)$	0.502	0.3519	0.4213	0.3327
$corr(Y,A)$	0.891	0.7071	0.7650	0.5510
$corr(U,V)$	-0.889	-0.8827	-0.9046	-0.8589
$corr(U,\pi)$	-	-0.2663	-0.3967	-0.1953

(Y stands for production,A is labor productivity, U stands for unemployment, V for vacancies, θ for labor market tightness, w is real wage, π is inflation)

All series are provided by the BLS: labor productivity (A) is real average output per person in the non-farm business sector,U is the unemployment level from CPS, for vacancies (V) I use the composite Help Wanted Index proposed by Barnichon (2010b), real wage (w) is the real hourly average earnings of production and non supervisory employees (available from 1964), production is non-farm business sector output, the rate of inflation is calculated from the CPI for all urban consumers. All variables are detrended with an HP filter with smoothing parameter of $10^5$and reported in log as deviation from the trend, as in Shimer (2005) and Michaillat (2012).

find in Fève, Guay (2009).

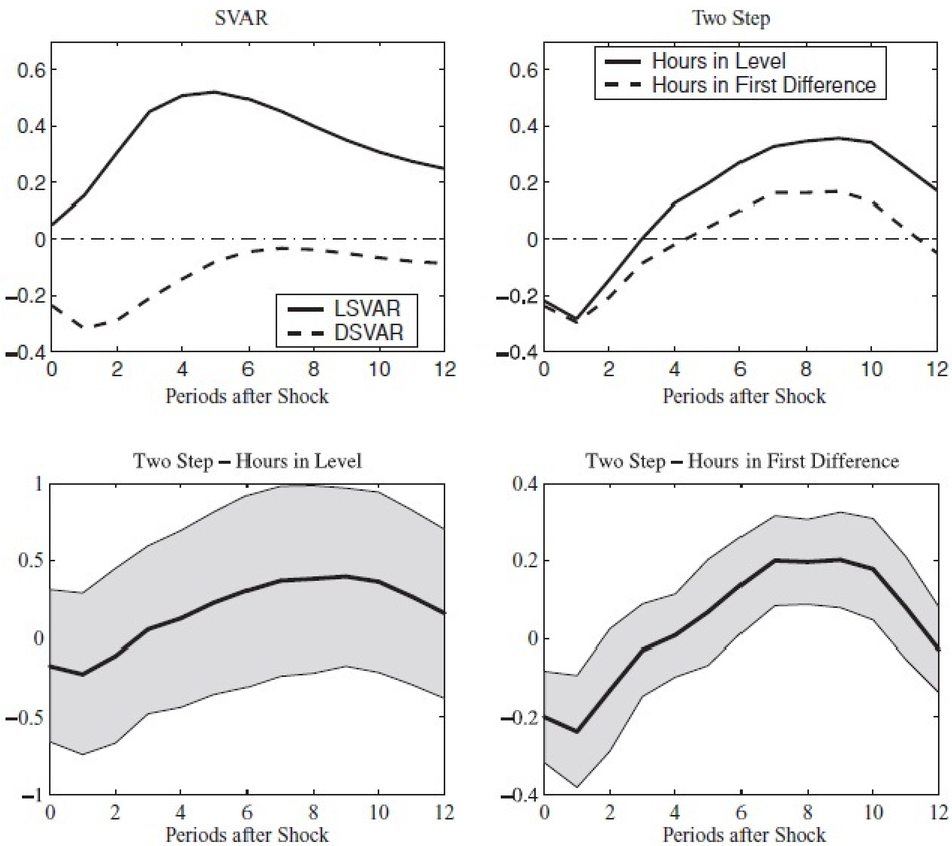
As the authors say, Basu, Fernald and Kimball (2006) and Francis, Ramey (2005) confirm the findings of Galì (1999): hours decrease after a positive technological shock in the U.S.[13] Numbers of other papers have also confirmed these findings, for example Balleer (2012) or Barnichon (2012). Nevertheless the paper by Christiano, Eichenbaum and Vigfusson (2004) questions these results: the authors find that, using a SVAR where hours enter in levels, the response of this variable after a technological shock is positive and hump-shaped.

The issue then focused on the stationarity of the series of hours, and thus on the choice of introducing them in levels or in first difference, but the presence of a unit root in the series cannot be stated or denied easily.

The paper by Fève, Guay (2009) tries to give an answer with an alternative methodology: they propose a VAR with a two-step estimation; first the technological shocks are identified without using the hours worked as a variable, and then hours are regressed on the technological shocks for different lags. The results of the authors are interesting since they embed the previous findings of both Galì (1999) (and followers) and Christiano, Eichenbaum and Vigfusson (2004): according to the authors in fact the two-step methodology indicates that hours decrease after a positive technological shock in the short run and increase (with an hump-shape) afterwards. According to them, the results are not sensitive to the choice of introducing hours (in the second step) in levels or in first differences, even if the IRFs are more precisely estimated when hours enter in first differences. We report in Figure 2 the main findings of the authors. We use this evidence in the following of the paper to claim that the reaction of hours after a positive technological shock is negative, at least in the short run.

[13] The former paper uses a direct measure of "purified technology" while the latter, as Galì (1999), uses a Structural VAR where hours enter in first difference.

Figure 2: **IRFs of hours to a technological improvement, Fève, Guay (2009)**



From Fève, Guay (2009) p. 1003

4 The model

I set up a New Keynesian model with search and matching frictions in the labor market, in the tradition of models such as Trigari (2006), Andrés, Doménech, Ferri (2006), Blanchard, Galì (2010), Kurozumi, Van Zandweghe (2010)[14].

As in the last cited paper, I abstract from the intensive labor margin, i.e. I consider an indivisible labor framework, such that if the agent is employed, she can supply h (fixed) hours[15]; this brings me to an environment which is close to Shimer (2005) set-up. I remain in a perfect risk sharing framework and I look at the decentralized equilibrium (the only public authority here is the monetary authority).

My approach in describing the economy is in line with the "Keynesian" interpretation: in the short run prices are rigid, so that quantities adjust after a shock; since hours worked per worker are fixed, it is the extensive margin which has to be able to react. I therefore adopt a different description for the dynamics of labor market flows with respect to the canonical search and matching model: I consider in fact that newly hired workers become immediately productive, so that I write the law of motion of employment as $N_t = (1-s)N_{t-1} + \Upsilon S_t^{1-\psi} V_t^{\psi}$, where $S_t = 1-(1-s)N_{t-1}$ indicates the agents who are actually looking for a job[16].

This convention changes the definition of employment: as in KZ (2010), I can say that N_t is not anymore a state variable, while N_{t-1} remains a predetermined state variable.

For what it regards preferences, my baseline specification considers non-separable preferences in consumption and leisure, so that I do not have wealth effects on labor supply, thus remaining comparable with Shimer (2005)[17]. I then show that the wealth effects on labor supply implied by the traditional set of separable preferences are quantitatively important and they can worsen Shimer's critique in a RBC framework.

In my model I keep the decisions about prices separated from those about posting vacancies, i.e. I assume that there are three sectors: wholesale firms produce output using only labor (I abstract from the presence of physical capital, again to analyze a framework close to Shimer (2005)); these firms need to post vacancies in order to hire workers, since the labor market is frictional, but they can sell their output in a perfectly competitive one; the second sector is composed by retail firms, who are monopolistically competitive: these firms buy the output of wholesale firms, differentiate it and sell to a perfectly competitive final producer who has a CES aggregation function and who sells the final production to households[18].

For what it regards the nominal frictions, I suppose that the retail firms cannot change freely the price of their output, since they face nominal frictions à la Calvo: they are able to reset their price optimally each period with a certain probability $(1-\alpha)$; alternatively they will leave it unchanged. Finally, I will close the model by supposing a monetary authority which adopts a Taylor rule to set the nominal interest rate.

I proceed now to describe my model and calibration strategy. As I said in the Introduction, the hypothesis about the behavior of monetary authority with respect to deviations of output from its steady state are of fundamental importance, since they change the behavior of the agents in the model

[14]From now on KZ (2010)

[15]In Subsection 4.9.6 I generalize to the case in which hours per worker can vary

[16]Other papers who adopt this convention are for example Blanchard, Galì (2010), KZ (2010). Note that therefore unemployed and workers searching for a job do not coincide any more

[17]Shimer (2005) considers risk-neutral agents with linear utility in consumption

[18]As we know, it would have been the same to suppose that households have CES preferences for the differentiated goods produced by retail firms.

and so the answers of variables to the shocks[19].

I will therefore analyze the reaction of variables conditional to the specifications of monetary policy. Many authors have emphasized the different characteristics of the "Great Moderation" period and claimed that the conduct of monetary policy can have changed around mid 1980s: I therefore choose to consider the post-1984 period as reference and I show that in this case the predictions of my model are consistent with the data.

4.1 Labor market flows

I suppose that the law of motion of employment is given by the equation

$$N_t = (1-s)N_{t-1} + \Upsilon S_t^{1-\psi} V_t^{\psi} \tag{1}$$

with $S_t = 1-(1-s)N_{t-1}$, $\Upsilon > 0$, $0 < s < 1$ and $0 < \psi < 1$. Accordingly, the definition of job finding rate and labor market tightness take into account the actual searching agents, and not the unemployed ones, who are defined as all those who are not employed ($U_t = 1 - N_t$); if I call $M_t = \Upsilon S_t^{1-\psi} V_t^{\psi}$, then $p_t = \frac{M_t}{S_t}$ and $\theta_t = \frac{V_t}{S_t}$ are respectively the job finding rate and the labor market tightness; finally the job filling rate is given by $\Phi_t = \frac{M_t}{V_t}$.

4.2 Households

The representative household plays an employment lottery at the beginning of each period: with a probability N_t she can work h (fixed) hours[20], if not she doesn't work at all; the household can save by buying bonds so that I can write her problem in terms of value function as

$$W(N_{t-1}) = max\,\{N_t U(C_t^e, \Gamma^e) + (1-N_t)U(C_t^u, \Gamma^u) + \beta E_t W(N_t)\}$$

$$s.t. \begin{cases} N_t C_t^e + (1-N_t)C_t^u + \frac{B_{t+1}}{P_t}\frac{1}{(1+r_t^n)} & = w_t N_t h + \frac{B_t}{P_t} + \frac{D_t}{P_t} \\ N_{t+1} & = (1-s)N_t + p_{t+1}S_{t+1} \\ p_t & = \frac{M_t}{S_t} \\ S_t & = 1-(1-s)N_{t-1} \end{cases}$$

where C_t^e and C_t^u represent respectively the consumption of the employed and the unemployed, B_{t+1} is the amount of bonds expressed in currency units, r_t^n is the nominal interest rate, w_t is the real wage and D_t the nominal profits eventually rebated by firms; P_t is the price of the consumption good. As I said, the state variable now is N_{t-1}, so it appears in the value function with the right timing[21].

4.2.1 Preferences

With respect to the utility function, I retain the specification of non-separable preferences à la Greenwood, Hercowitz and Huffmann, henceforth GHH (1988); I consider that these preferences have qualitative implications which are more realistic than the traditional separable preferences.

In GHH (1988) the utility function is the following

$$U(c, 1-h) = \varphi[c + \nu(1-h)]$$

[19]As pointed out by Dotsey (1999): "[...] the central bank's systematic behavior can alter the correlations between variables in the model."

[20]See Subsection 4.9.6 for the case with variable hours

[21]See the Appendix for more details about the derivation of a unique budget constraint

where φ, ν are strictly increasing and concave functions, c is consumption, h is hours of work, with $h \in \{0, 1\}$.

The functional form is then the standard one

$$U(C_t^z, L_t^z) = log(C_t^z + \Gamma_t^z) \tag{2}$$

for $z = e, u$, with $\gamma, \varepsilon > 0$, where $\Gamma_t^e = \gamma \frac{(1-h_t)^{1-\varepsilon}}{1-\varepsilon}$ and Γ_t^u=Γ^u.

In my case, since I do not have variable hours, I set $\Gamma_t^e = \Gamma^e = \gamma \frac{(1-h)^{1-\varepsilon}}{1-\varepsilon}$ and Γ_t^u=Γ^u, with h fixed at the steady state value[22].

I suppose, as in CL (2004), that at the steady state the unemployed worker values leisure more than the employed one, i.e. $\Gamma^u - \Gamma^e > 0$.

The FOCs for the households problem imply that the marginal utility of consumption is the same for both agents, so I have that

$$\lambda_t = \frac{1}{C_t^e + \Gamma_t^e} = \frac{1}{C_t^u + \Gamma^u} \tag{3}$$

which implies that $C_t^e = C_t^u + \Gamma^u - \Gamma^e$ so that $C_t^e > C_t^u$, under the assumption $\Gamma^u - \Gamma^e > 0$.

If I consider the utility gap, I obtain

$$U(C_t^e, \Gamma^e) - U(C_t^u, \Gamma^u) = log(C_t^e + \Gamma^e) - log(C_t^u + \Gamma^u) = 0 \tag{4}$$

i.e. the unemployed are as well off as the employed.

When preferences are separable and take the standard form

$$U(C_t^z, L_t^z) = log(C_t^z) + \Gamma_t^z \tag{5}$$

for $z = e, u$, with $\gamma, \varepsilon > 0$, where $\Gamma_t^e = \gamma \frac{(1-h_t)^{1-\varepsilon}}{1-\varepsilon}$ and Γ_t^u=Γ^u, I obtain that $C_t^e = C_t^u$ and therefore $U(C_t^u, \Gamma^u) > U(C_t^e, \Gamma^e)$, i.e. employed agents are worse off than the unemployed.

To sum up, the non-separable preferences à la GHH with $\gamma > 0$ imply that consumption and leisure are substitutes so that the employed workers consumer more than the unemployed, and their utility level is the same as that one of the unemployed; on the contrary, the traditional separable preferences imply that unemployed workers are better off than the employed, which is at odd with common sense and with all the well-being literature.

When preferences are separable, we know that labor supply depends also on consumption, so that wealth effects have an impact on labor supply choices: in Section 4.9.4 I show that with separable preferences the marginal utility of wealth λ_t enters in the wage equation, and that it has important quantitative consequences.

4.3 Intermediate firms

The intermediate representative firm produces its output using only labor with a linear production function $Y_t = A_t N_t h$, where A_t is the stochastic technological process. This firm faces a frictional labor market: in order to hire workers, it must post vacancies; it sells its output to retail firms in a perfectly competitive environment.

[22] I use this convention in order to easily allow for variable hours in the extension of the model that I develop in Subsection 4.9.6

The problem of the firm in terms of value function is therefore
$V(N_{t-1}) = max_{V_t}\left\{x_t A_t N_t h - w_t N_t h - \omega V_t + \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}V(N_t)\right]\right\}$
$s.t. \begin{cases} N_t &= (1-s)N_{t-1} + \Phi_t V_t \\ \Phi_t &= \frac{M_t}{V_t} \end{cases}$
where the problem is expressed in real terms: x_t is the relative price of the intermediate output i.e. $x_t = \frac{P_t^{intermed}}{P_t}$; ω is the real cost (in terms of consumption good) of posting a vacancy (i.e. in nominal terms I would have ωP_t).

The first order condition, combined with the expression of the marginal value of one additional worker for the firm, gives the vacancy opening condition

$$\frac{\omega}{\Phi_t} = (x_t A_t h - w_t h) + \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{\omega}{\Phi_{t+1}}(1-s)\right] \tag{6}$$

As usual, the firm posts vacancies till the cost it incurs (given by ω during the time the vacancy is open) is equal to the benefit, given by the net production obtained and the "saved" cost of recruiting if the job remains filled.

4.4 Wage equation

I define the total surplus of a match as $Surplus_t = \frac{\partial V(N_{t-1})}{\partial N_{t-1}} + \frac{1}{\lambda_t}\frac{\partial W(N_{t-1})}{\partial N_{t-1}}$, where the marginal value of a worker for the household is expressed in terms of consumption goods.

I assume a Nash-bargaining framework, so that the sharing rule is obtained by solving the Nash product maximization problem
$max_{w_t}\left(\frac{1}{\lambda_t}\frac{\partial W(N_{t-1})}{\partial N_{t-1}}\right)^{1-\xi}\left(\frac{\partial V(N_{t-1})}{\partial N_{t-1}}\right)^{\xi}$
where the parameter ξ represents the bargaining power of the firm.

I derive the sharing rule and I finally obtain the wage equation by using the expressions for the marginal value of employment for the firm and the worker[23]

$$w_t h = (1-\xi)\left[x_t A_t h + \beta(1-s)\omega E_t\frac{\lambda_{t+1}}{\lambda_t}\theta_{t+1}\right] + \xi\left[(\Gamma^u - \Gamma^e)\right] \tag{7}$$

Let's consider the first square bracket in the RHS of the wage equation, which represents the contribution of the worker to production from the firm's point of view: the first element is the real marginal product of N_t, while the second is the saving coming from the fact that the firm doesn't need to post a vacancy in the future, if the match is not destroyed.

The real marginal product of a worker is given by her productivity A_t multiplied by the relative price of the homogeneous intermediate good x_t; this term, as we will see in detail, is also the marginal cost of the retailer: we know that in a New Keynesian model the real marginal cost decreases after a positive technological shock (i.e. when A_t increases), so that the overall effect on the term $x_t A_t h$ is not unambiguous and it depends on the relative strength of the two effects; as we will see, this is the crucial element which brings non-monotonicity in the dynamics of adjustment.

The fact that labor market tightness enters with its expected value depends on the instantaneous hiring hypothesis: if the match is destroyed in the following period, the firm has to hire another worker, so it would be needed to post vacancy with the cost ω, while with the "traditional" dynamics

[23] See the Appendix for details.

of employment, in order to hire a worker in the following period the firm should post a vacancy today (since the hired worker is productive only after one period), and this is the reason why with the traditional dynamics I have the value of tightness at time "t" and not "t+1".

The important point here is that the dynamics of vacancies and unemployment are not any more unambiguous as in the real model of Shimer (2005): due to the presence of nominal frictions, according to the behavior of the monetary authority, I could have a recessionary or an expansionary effect of a positive productivity shock on employment (as I said, it depends on the fact that monetary policy is enough accommodating with respect to output)[24]: in the short run I can have the "Keynesian" reaction that vacancies and so employment decrease, while as prices adjust (so the effect of the decrease of the marginal cost vanishes) the effect of productivity prevails, so that vacancies increase.

In addition, as emphasized by KZ (2010), the presence of the expected value of labor market tightness can give rise to a "vacancy channel" of monetary policy, which is potentially a source of indeterminacy: when for example the nominal interest rate is raised, I have the traditional "demand channel" which leads to a decrease in inflation rate because future demand is expected to raise, while current one decreases; here in addition, since firms expect a lower future supply (lower current vacancy posting affects current and so future employment), they expect future vacancies posting to increase, which leads to higher real marginal cost and therefore higher inflation.

Let's consider now the second element of the RHS of the equation: it represents the outside options of the worker, i.e. the threatening point should she leave the bargaining.

With non-separable preferences the term inside the bracket is not changing along the cycle: I find here the equivalent of what Shimer (2005) and Hagedorn and Manovskii (2008)[25] call "z", i.e. the value of leisure.

For what it regards the calibration of the value of leisure, we know that Shimer (2005) calibrates $z = 0.4$: considering that he normalizes productivity to the unity and that his mean labor income has a value of 0.993, when interpreted as unemployment benefit the choice of the value of z implies a replacement rate of 40%; HM (2008) stress that with a higher value for z (together with a low value for the bargaining power of the worker), the standard search and matching model can replicate the relative volatilities of labor market variables[26].

If one wants to address the Shimer puzzle about volatilities without using the HM (2008) calibration approach, it is necessary to be careful about the calibration of the value of leisure.

In my model this value comes from steady state conditions, since I follow the calibration strategy as in Andolfatto (1996): I target a value for the costs of posting vacancies in terms of output and therefore I derive from steady state conditions the value of non-market activity, as we will see in details in Subsection 4.8; in my calibration exercise the value of leisure corresponds to a replacement rate of 30%, therefore even lower than that one in Shimer (2005).

4.5 Retailers

There is a continuum of monopolistically competitive retailer firms $i \in [0, 1]$ which buy the intermediate firm good and differentiate it at no cost; they sell their output to the final producer, which buys the

[24]This feature of the monetary policy implies that aggregate demand will be less or more sticky. Once the retailers fix their price, demand is given and they have to satisfy it; if there is a positive technological shock which increases the productivity of labor, the retailer which cannot change its price will see its mark-up to increase, which means that the relative price of intermediate firms output decreases, so that those firms can be induced to decrease employment, since each worker is now more productive.

[25]From now on HM (2008)

[26]HM (2008) normalize the value of productivity to the unit as Shimer (2005) and set $z = 0.955$ and $\xi = 0.052$

differentiated goods and aggregate them according to a CES production function and finally sells to the households its output.

The problem of the retailer i can be therefore written in the following way[27]:

$max_{P_t(i)} E_t \sum_{j=0}^{\infty} \alpha^j v_{t+j} \left[P_t(i)Y_{t+j}(i) - P_{t+j}x_{t+j}Y_{t+j}(i)\right]$

$s.t. Y_{t+j}(i) = \left(\frac{P_t(i)}{P_{t+j}}\right)^{-\eta} C_{t+j}$

where α is the probability that the retailer cannot reset optimally its price, $P_t x_t$ is therefore the nominal marginal cost and $v_{t+j} = \beta^j \frac{\lambda_{t+j}\frac{1}{P_{t+j}}}{\lambda_t \frac{1}{P_t}}$.

The first order condition for this problem gives the pricing setting rule

$P_t(i) = \frac{E_t \sum_{j=0}^{\infty} \theta^j \beta^j \lambda_{t+j} C_{t+j} P_{t+j}^{\eta} \frac{\eta}{\eta-1} x_{t+j}}{E_t \sum_{j=0}^{\infty} \theta^j \beta^j \lambda_{t+j} C_{t+j} P_{t+j}^{\eta-1}}$

where $\frac{\eta}{\eta-1}$ is the price mark-up.

As it is standard in sticky price literature, I can derive the expression for the evolution of the price index: $P_t^{1-\eta} = \left[(1-\theta)P_t(i)^{(1-\eta)} + \theta P_{t-1}^{(1-\eta)}\right]$.

The market clearing condition implies the total production of retailers to be equal to total demand for their goods expressed by the final producer: $y_t \equiv \int_0^1 Y_t(i)di = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\eta} C_t di = \Delta_t C_t$, where $\Delta_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\eta} di$ is the price distortion.

As it is standard in the sticky price literature, I express the price distortion as

$\Delta_t = P_t^{\eta}\left[\left(\int_0^1 P_t(i)^{-\eta}di\right)^{-\frac{1}{\eta}}\right]^{-\eta} = P_t^{\eta}\left(P_t^*\right)^{-\eta} = \left(\frac{P_t^*}{P_t}\right)^{-\eta}$; I have therefore that $P_t^* \equiv \left(\int_0^1 P_t(i)^{-\eta}di\right)^{-\frac{1}{\eta}}$

so that I can have an expression for the law of motion of the price distortion

$P_t^* = \left[(1-\alpha)P_t(i)^{-\eta} + \alpha\left(P_{t-1}^*\right)^{-\eta}\right]^{\frac{-1}{\eta}}$. We know that in steady state $\Delta = 1$ and that up to a first order linear approximation around a zero steady state inflation the log-linearized price dispersion is always null, so that I will ignore it in my solution.

4.6 Aggregate relations

The aggregate relations which hold in the economy tell us that production of the intermediate firms is given by $Y_t = A_t N_t h$ and the economy wide resource constraint is $Y_t = y_t + \omega V_t = \Delta_t C_t + \omega V_t$.

4.7 Technology and monetary policy

The stochastic technological process is specified as a standard AR (1) process: $A_t = A_{t-1}^{\varrho_A} A^{(1-\varrho_A)} e^{\varepsilon_t^A}$, where $0 < \varrho_A < 1$ and $\varepsilon_t^A \sim N(0, \sigma_{\varepsilon_A})$.

I suppose the existence of a monetary policy authority which adopts a Taylor rule; with this respect, I follow KZ (2010) who study indeterminacy issues in a model similar to mine, where nonetheless the only source of aggregate uncertainty is given by sunspot shocks to inflation expectations: the main conclusion of the authors is that a Taylor rule which is strictly inflation forecast targeting gives almost always rise to indeterminacy; their study justifies the widespread approach in the literature[28] to adopt a Taylor rule of the type proposed by Clarida, Galì, Gertler (2000), which includes a smoothing element and the reaction of monetary policy to both (contemporaneous) inflation and output[29].

[27] See the Appendix for a detailed derivation

[28] For example Trigari (2006), Andrés, Doménech, Ferri (2006)

[29] Notice however that KZ (2010) use a Taylor rule which includes directly employment and not output

My specification of the Taylor rule is then the following:
$\frac{R_t^n}{R^n} = (\frac{R_{t-1}^n}{R^n})^{\varrho_m} \left(\frac{\pi_t}{\pi}\right)^{(1-\varrho_m)\gamma_\pi} \left(\frac{Y_t}{Y}\right)^{(1-\rho_m)\gamma_y} e^{\varepsilon_t^m}$ where $\varepsilon_t^m \sim N(0, \sigma_{\varepsilon_m})$

4.8 Calibration and solution

The period of reference is considered to be a quarter, as it is standard in the DSGE literature[30].

In order to be comparable with the labor market literature, I choose to target the high value for employment adopted by Shimer (2005) of $N = 0.9455$ which, together with the choice of the widely accepted value for the separation rate of $s = 0.1$ at a quarterly level, allows me to pin down the job finding rate[31].

The choice of these values implies that the labor market tightness and the job finding rate in steady state are respectively $\theta = 0.7049$, $p = 0.6344$.

For the worker's bargaining power, Shimer (2005) sets it to $1 - \xi = 0.72$, but in the literature the more commonly adopted values stay in the range $0.4 - 0.6$,[32]so I set $1 - \xi = 0.6$.

For what it regards vacancy costs in terms of output, the RBC literature has adopted values in the range 0.5%-1%. To set this value, I follow CL (2004) who consider the evidence in Abowd, Kramarz (1997): according to the statistics of this paper, total hiring costs account for 1.03% of total labor costs[33]; this value, in terms of my model, can be expressed as $\frac{\frac{\omega V}{q}}{whN} = 0.0103$, where total wage costs can be recovered from the intermediate firm program as $whN = xAhN - \omega V$[34].

Considering the value I have for the job filling rate of $\Phi = 0.9$, I end up with a value for $\frac{\omega V}{Y}$ between 0.7% and 0.8%: I therefore choose a value of $\frac{\omega V}{Y} = 0.8\%$, which belongs to the range of possible values that one finds in the RBC literature.

Another widely cited paper in the literature to calibrate the cost of posting a vacancy is that one by Silva, Toledo (2009): the authors provide evidence of the fact that total hiring costs account for around 14% of the quarterly compensation of an employee; in terms of my model, this calibration would imply a ratio of total vacancy costs in terms of output of $\frac{\omega V}{Y} = 1.2\%$[35], so that I consider my calibration choice not in contrast to this alternative view.

The steady state value of the consumption ratio will be determined by steady state restrictions.[36]

The calibrated value for vacancy costs implies therefore a consumption ratio of 0.2405 for the benchmark case of non-separable preferences; the "value of leisure" (in Shimer's terms) would be 0.25,

[30] In the Appendix I check that a model calibrated on a monthly basis does not give different results.

[31] I remember that here, due to the different dynamics of employment, the steady state value of employment is given by $N = \frac{p}{s+(1-s)p}$

[32] Consider that Shimer (2005) imposes the Hosios condition, setting the same value of 0.72 for the elasticity of the matching function with respect to unemployment while, according to Petrongolo, Pissarides (2001), the range of admissible values for this parameter is 0.3-0.7. Finally, consider that the value proposed by Hagedorn, Manovskii (2008) for the worker's bargaining weight is 0.052.

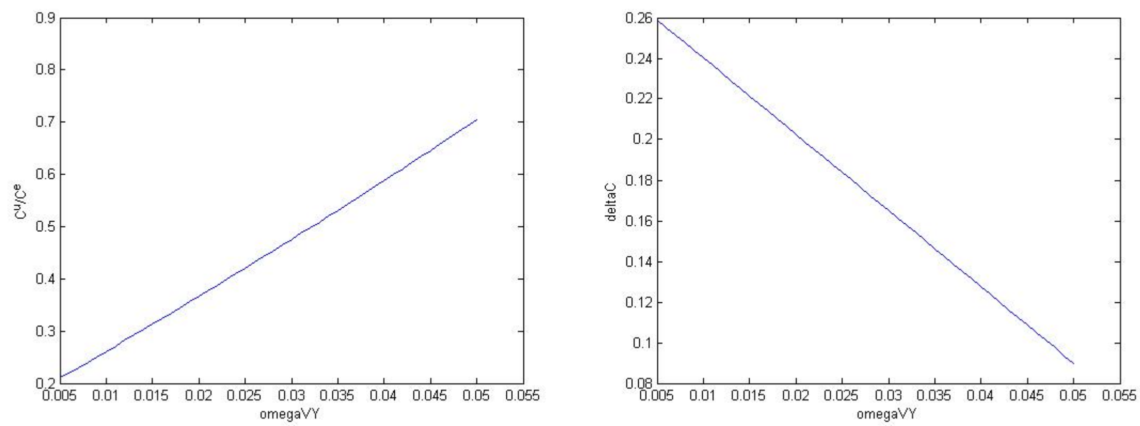
[33] From Abowd, Kramarz (1997), Table 1 p. 22, I find that $\frac{HiringCostsperHire*TotalHiring}{AverLaborcost*TotalEmployment} = 0.0103$

[34] One can alternatively think of the total wage costs as approximately 80% of total output, since there is no physical capital, and find the same result as in the main text

[35] The average cost of recruiting is 14% of the compensation of an employee, which in terms of my model can be written as $\frac{\omega}{\Phi} = 0.14wh$; using the job creation condition, this implies that $\omega = \frac{xAh}{[(1/0.14)-\beta(1-s)+1]}\Phi$, which gives a value for ω of 0.0345 and therefore a ratio of $\frac{\omega V}{Y} = 0.0115$

[36] From the firm's foc for vacancies I have $w = x \cdot A + (\frac{\omega}{\Phi})(1/h)\left[(\beta(1-s)) - 1\right]$and from the wage equation I have that $C^e - C^u = (\frac{1}{\xi}) * \left\{w \cdot h - (1-\xi)\left[x \cdot A \cdot h + \beta(1-s)(\frac{\omega}{\Phi}))p\right]\right\}$: once I calibrate the parameters of the labor market and the vacancies costs, I derive the value for the consumption gap.

Figure 3: **Consumption ratio (left) and value of leisure '$C^e - C^u$' (right)**



as we can see in Figure 3, which means a replacement rate in terms of hourly wage of $\frac{(C^e - C^u)}{w} = 0.3$ (while in Shimer (2005) this ratio has a value of 0.4).

As I stressed in the Introduction, my calibration procedure is in line with the RBC tradition starting from Andolfatto (1996), while Shimer (2005) sets the value for the outside options and derives from the steady state restrictions the value for the parameter ω.

If I had to follow Shimer's procedure, setting a value for non-market activity as 40% of the wage [37], I would have obtained a value for the parameter of the vacancy posting cost of $\omega = 0.13$ and therefore a ratio of $\frac{\omega V}{Y} = 4.46\%$, which is above any value usually considered in the literature.

As expected, a model calibrated with such a high cost of posting vacancies delivers much less volatility of labor market variables with respect to a model where the recruiting costs are lower, as we will see in Section 4.9.1.

The value for the elasticity of substitution implies a steady state mark-up of 20%; for what it concerns nominal frictions, the Calvo parameter, about which there is also a considerable uncertainty and which has a fundamental impact on the reaction of employment to a technology shock, is set to a value of $\alpha = 0.5$, which implies an average price spell of almost 6 months[38].

The parameters on the Taylor rule are standard: in line with Clarida, Galì, Gertler (2000) for the Volcker-Greenspan period, in my baseline calibration I suppose that monetary policy reacts to output and inflation, and there is a component of interest rate smoothing.

The calibrated values I use for quarterly data in the benchmark case are reported in the Table 4.

In order to solve the model, I calibrate the model and log-linearize it around a zero steady state

[37] Shimer (2005) normalizes labor productivity to the unity and he sets the value of leisure to $z = 0.4$; since the mean wage is $w = 0.993$, these values imply the following ratios: $\frac{z}{w} = 0.4028$ and $\frac{w}{y} = 0.993$. If I want to follow Shimer's calibration procedure in my model, I set the value of leisure, i.e. $(C^e - C^u)$ so that I keep the same ratios of Shimer (2005), i.e. $(C^e - C^u)/w = 0.4$ and $\frac{w}{xAh} = 0.993$, which imply a value of leisure of $(C^e - C^u) = 0.11$ and a wage of $w = 0.28$

[38] In the empirical literature I find estimates for the duration of the price spell ranging from 1.5 quarters according to Bils, Klenow (2004) to 3 or 4 quarters, according to Nakamura, Steinsson (2008); the calibration strategies adopted in the DSGE literature reflect this heterogeneity, so that one can find calibrated values for the price spells going from 4.5 months (Thomas (2011)) to almost 20 months (Trigari (2009)), passing through a value between 9 and 12 months chosen by Blanchard, Galì (2010).

Table 4: **Baseline calibration for nominal model**

h	ε	ϱ_A	σ_{ε_A}	$\omega\frac{V}{Y}$	$\frac{C^u}{C^e}$	β	η	γ_π	γ_Y	α	ϱ_m	N	Φ	ξ	ψ	s
1/3	4	0.9	0.009	0.8%	0.24	0.985	6	1.5	0.5	0.5	0.8	0.9455	0.9	0.4	0.4	0.1

inflation to obtain IRFs and theoretical moments, using the software Dynare[39].

4.9 Results

The reasoning behind my quantitative results is the following: once I depart from the calibration procedure of Shimer (2005), I can obtain a model which already partially solves the issue about the relative volatilities of labor market variables in a framework where there are almost no nominal rigidities.

Considering this model, I then analyze the importance of the transmission mechanism when there is some degree of price stickiness, i.e. I analyze the quantitative importance of the different reaction of employment in the short and in the long run.

4.9.1 A first step in solving the issue about relative volatilities: the value of the vacancy posting costs

First of all I present the quantitative properties of my model considering the two different calibration procedures (à la Andolfatto (1996) and à la Shimer (2005)), and therefore the two different values for the vacancy posting costs.

For exposition concerns, I consider a framework in which nominal rigidities are almost nonexistent, in order to be exactly in the framework of Shimer (2005) for what it regards the transmission mechanism of a positive labor productivity shock: this means that I set my Calvo parameter to a value of $\alpha = 0.01$.

The results of this first step are presented in Table 5.

We clearly see in the last column that when I calibrate my general equilibrium model following the procedure of Shimer (2005) for what it regards the value of the outside options (and therefore the vacancy posting cost), I end up with the problems about the relative volatilities and the correlations of labor market variables with labor productivity. In this sense, I am in the same line of HM (2008) who stress the importance of the calibration of some parameters (even if my calibration strategy is far from theirs).

4.9.2 The steady state elasticities

In order to be more directly comparable with Shimer (2005), since in the case in which nominal rigidities are absent the transmission mechanism is as in the real business cycle framework, I can compute the steady state elasticities.

Let's remember first Shimer (2005) set up: as we know, the equilibrium in the labor market can be summarized by two equations, i.e. the job creation condition (the "labor demand", eq. 8) and the wage equation (the "labor supply", eq. 9):

[39] Stéphane Adjemian, Houtan Bastani, Michel Juillard, Frédéric Karamé, Ferhat Mihoubi, George Perendia, Johannes Pfeifer, Marco Ratto and Sébastien Villemot (2011), "Dynare: Reference Manual, Version 4," Dynare Working Papers, 1, CEPREMAP

Table 5: **The importance of the vacancy posting costs in a "real" model** ($\alpha = 0.01$)

	U.S. data			$\frac{\omega V}{Y} = 0.8\%$	$\frac{\omega V}{Y} = 4.46\%$
	1951-2011	**1951-1984**	**1985-2011**	**Tech shock**	**Tech shock**
σ_Y	0.0333	0.0343	0.0309	0.0251	0.018
σ_A	0.0202	0.0227	0.0159	0.0161	0.0161
$\frac{\sigma(U)}{\sigma(A)}$	9.5325	9.3295	10.3446	9.8012	2.0124
$\frac{\sigma(V)}{\sigma(A)}$	9.3541	8.8242	10.5140	7.6025	1.5528
$\frac{\sigma(\theta)}{\sigma(A)}$	18.3171	17.2945	20.1422	10.0435	2.0621
$\frac{\sigma(w)}{\sigma(A)}$	1.1043	1.2558	1.0086	0.9068	1.0435
$\frac{\sigma(Y)}{\sigma(A)}$	1.6525	1.5115	1.9422	1.5590	1.118
$\rho(U,A)$	-0.4140	-0.4856	-0.1175	-0.9794	-0.9767
$\rho(V,A)$	0.40	0.5249	0.0739	0.9745	0.9786
$\rho(\theta,A)$	0.4188	0.5169	0.0984	0.9999	0.9995
$\rho(w,Y)$	0.3519	0.4213	0.3327	0.9981	1
$\rho(Y,A)$	0.7071	0.7650	0.5510	0.9973	0.9998
$\rho(U,V)$	-0.8827	-0.9046	-0.8589	-0.9092	-0.9116
$\rho(U,\pi)$	-0.2663	-0.3967	-0.1953	0.6219	0.5878

(Y stands for production,A for labor productivity, U for unemployment -i.e. $U = 1 - N$, V stands for vacancies, θ for labor market tightness, w for real wage, π for inflation)

$$\frac{\omega}{\Phi(\theta)} = \frac{A - w}{r + s} \tag{8}$$

$$w = \xi z + (1 - \xi)(A + \omega\theta) \tag{9}$$

where ω is the cost of opening a vacancy, A is the marginal product of a filled job, $\Phi(\theta) = \Upsilon\theta^{\psi-1}$ is the job filling rate, r is the real interest rate, s is the fixed separation rate, ξ is the bargaining power of the firm and z represents the outside options (unemployment benefit or home production) or the value of leisure[40]. If I combine these two equations, I obtain

$$\frac{(r + s)}{\Phi(\theta)} + (1 - \xi)\theta = \xi\frac{A - z}{\omega} \tag{10}$$

and by total differentiation I can obtain an expression for the elasticity of tightness with respect to productivity

$$\epsilon_{\theta,A} = \frac{d\theta}{dA}\frac{A}{\theta} = \frac{\xi}{\omega\left[(1 - \xi) - (r + s)\frac{\partial\Phi(\theta)}{\partial\theta}\frac{1}{(\Phi(\theta))^2}\right]}\frac{A}{\theta} \tag{11}$$

This expression enables me to say what happens to labor market tightness when productivity increases (permanently), all the other variables remaining constant: with the benchmark calibration of Shimer (2005)[41], this elasticity has a value of 1.0118: if the productivity of a filled job increases,

[40] In terms of Shimer (2005) notation, I actually have $\frac{c}{q(\theta)} = \frac{p-z}{r+s}$ and $w = (1 - \beta)z + \beta(p + c\theta)$

[41] For quarterly values he sets $A = 1, z = 0.4, \xi = 0.28, \omega = 0.213, r = 0.012, s = 0.1, \theta = 1, \Upsilon = 1.355, \psi = 0.28$

the incentive of a firm to increase vacancies increases, but also the wage, so that the overall effect is that quantities barely move.

Since I want to perform the same exercise than Shimer (2005), let's derive the expressions for elasticity in my model and consider the quantitative differences which come from the calibration strategy.

If I combine my labor demand and labor supply (equations 6 and 7) at the steady state I obtain equation 12, which is the equivalent to equation 10 in Shimer (2005). If I differentiate it, I get the expression for elasticity which is given by equation 13

$$\frac{1-\beta(1-s)}{\Phi(\theta)} + (1-\xi)(1-s)\beta\theta = \frac{\xi\left[xAh - (\Gamma^u - \Gamma^e)\right]}{\omega} \tag{12}$$

$$\epsilon_{\theta,A} = \frac{d\theta}{dA}\frac{A}{\theta} = \frac{\xi x h}{\omega\left[(1-\xi)\beta(1-s) - [1-\beta(1-s)]\frac{\partial\Phi(\theta)}{\partial\theta}\frac{1}{(\Phi(\theta))^2}\right]}\frac{A}{\theta} \tag{13}$$

With my benchmark calibration [42], I get $\epsilon_{\theta,A} = 10.2744$.

The value I get is therefore ten times bigger than the corresponding one in Shimer (2005); the reason behind this result lies in two aspects: first of all I set the bargaining power of the firm to a higher value with respect to Shimer (2005)[43], but mostly important, the cost of posting a vacancy is obtained in my case by a different calibration procedure, which gives a much lower value than in Shimer (2005).

4.9.3 What about the effect of the "instantaneous hirings" hypothesis?

The quantitative impact of the adoption of the instantaneous hirings hypothesis is not changing the results for vacancies and tightness (the impact on the variable "unemployment" is more important, since its definition is different), once I consider the calibration of the vacancy posting costs. I therefore report the results for completeness in the Appendix in Table 9.

4.9.4 What about separable preferences?

In order to stress the importance of the choice of the non-separable preferences, I proceed to check quantitatively the relevance of the wealth effects on the movements of labor supply when I adopt the traditional set of separable preferences.

I report for convenience the functional form of the utility function in this case:

$$U(C_t^z, L_t^z) = log(C_t^z) + \Gamma^z \tag{14}$$

for $z = e, u$, where $\Gamma^e = \gamma\frac{(1-h)^{1-\varepsilon}}{1-\varepsilon}$ and $\gamma, \varepsilon > 0$, where the superscripts e and u refer respectively to the employed and the unemployed agent.

The FOCs for the households problem imply that the marginal utility of consumption is the same for both agents, therefore in the benchmark case I have $\lambda_t = \frac{1}{C_t^e} = \frac{1}{C_t^u}$ and so $C_t^e = C_t^u$: the consumption of the two agents is equalized, which implies that the utility gap between the employed and the unemployed is negative

[42] I already discussed the calibration and steady state restrictions, but I report here the values for convenience: $A = 1, \xi = 0.4, \psi = 0.4, \omega = 0.024, \beta = 0.985, s = 0.1, \theta = 0.7, \Upsilon = 0.7296, x = 0.83, h = 0.33$

[43] $\xi = 0.4$ instead of $\xi = 0.28$

Table 6: **Comparison between separable and non-separable preferences**

Theoretical moments, smoothing parameter= 10^5

	U.S. data 1985-2011	Non separable pref Low nom rigidit Tech shock	Separable pref Low nom rigidit Tech shock
σ_Y	0.0309	0.0251	0.0168
σ_A	0.0159	0.0161	0.0161
$\frac{\sigma(U)}{\sigma(A)}$	10.3446	9.8137	0.7329
$\frac{\sigma(V)}{\sigma(A)}$	10.5140	7.6149	0.5714
$\frac{\sigma(\theta)}{\sigma(A)}$	20.1422	10.0559	0.7516
$\frac{\sigma(w)}{\sigma(A)}$	1.0086	0.9068	1.0062
$\frac{\sigma(Y)}{\sigma(A)}$	1.9422	1.5590	1.0435
$\rho(U, A)$	-0.1175	-0.9794	-0.9810
$\rho(V, A)$	0.0739	0.9745	0.9712
$\rho(\theta, A)$	0.0984	0.9999	0.9997
$\rho(w, Y)$	0.3327	0.9981	1
$\rho(Y, A)$	0.5510	0.9973	1
$\rho(U, V)$	-0.8589	-0.9092	-0.9076
$\rho(U, \pi)$	-0.1953	0.6219	0.6164

(Y stands for production,A for labor productivity, U for unemployment -i.e. $U = 1 - N$, V stands for vacancies, θ for labor market tightness, w for real wage, π for inflation)

$$U(C_t^e, \Gamma^e) - U(C_t^u, \Gamma^u) = log(C_t^e) + \gamma\frac{(1-h)^{1-\varepsilon}}{1-\varepsilon} - log(C_t^u) - \Gamma^u = \Gamma^e - \Gamma^u < 0 \qquad (15)$$

since it is assumed that near the steady state the unemployed value leisure more than the employed.

If I substitute in the model these preferences and I derive the wage equation, we see the differences with respect to the behavior of the outside options of the workers.

Let's consider the expression for the wage equation:

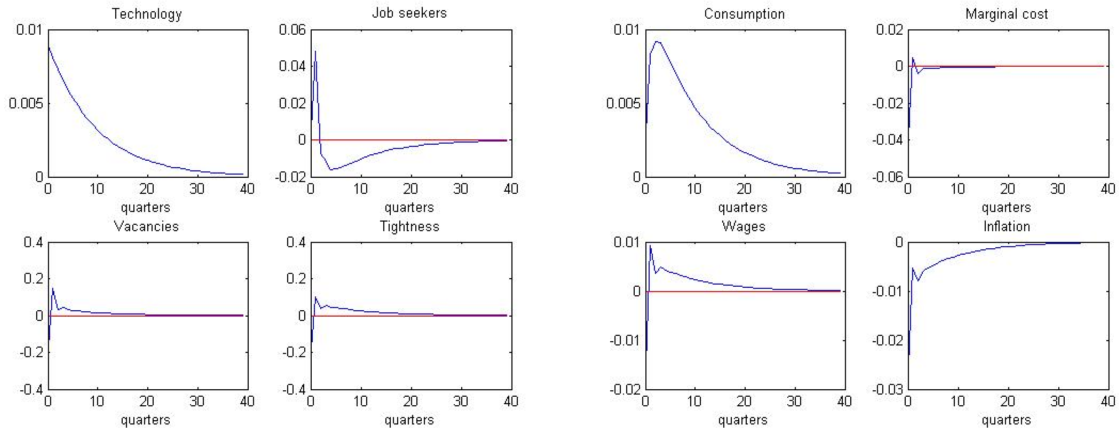
$$w_t h = (1-\xi)\left[x_t A_t h + \beta(1-s)\omega E_t \frac{\lambda_{t+1}}{\lambda_t}\theta_{t+1}\right] + \xi\left[\frac{\Gamma^u - \Gamma^e}{\lambda_t}\right] \qquad (16)$$

As we can see, the element in the second square bracket in the RHS of the equation implies that wealth effects have an impact on wages: when λ_t decreases (i.e. when consumption increases), there is a upward pressure on the real wage.

In the version of the model with low nominal rigidities, therefore, the mechanism described by Shimer (2005) is worsened: since wages increase, firms profits are squeezed so that the effect of the increase in productivity is dampened even more. The quantitative impact in this case is very relevant, as we can see if we look at the last column of Table 6[44].

[44]I solve and simulate a model which is calibrated as the benchmark one: the steady state values are the same, in particular for the "value of leisure" I have that $\frac{\Gamma^u - \Gamma^e}{\lambda}$ in the model with separable preferences is exactly equivalent to $(\Gamma^u - \Gamma^e)$in the benchmark model

Figure 4: **IRFs after a positive technological shock when $\alpha = 0.5$**



4.9.5 The analysis of the transmission mechanism in a New Keynesian model with no wealth effects (non-separable preferences)

Once I highlighted the importance of my calibration strategy for taking a first step in the direction of a model which reproduces the right volatilities of labor market variables, I consider the main point about the transmission mechanism of a positive productivity shock.

The idea is that the discussion about the Shimer puzzles should be conditional on the transmission mechanisms of the economy: the effect of a "supply" shock (positive technology shock) on total hours (here employment) in a New Keynesian model with sticky prices can be ambiguous. In my setting the Galì (1999) effect, i.e. the fact that employment decreases after a positive productivity shock, is linked to the formulation of the Taylor rule: I expect that, for a given level of nominal rigidities, with a non-accommodating monetary policy with respect to output, employment can decrease after a positive productivity shock, therefore the calibration of the parameter γ_Y is of fundamental importance.

In my benchmark specification (when $\alpha = 0.5$), a positive technology shock has a recessionary effect on employment: as we can see in the IRFs in Figure 4, in the very short run job seekers increase, vacancies and employment decrease; then as soon as the effect of the technological shock prevails on that one of the marginal cost, vacancies and so employment and tightness increase. This different transmission mechanism implies a lower correlation of labor market variables with productivity, as we can see in the bottom part of Table 7.

If I consider the "real" model (when $\alpha = 0.01$), as I stressed I already obtain an important quantitative reaction of labor market variables to a productivity shock, but I have also very high (absolute) values for the correlations between productivity and labor market variables; moreover, the model with only a technological shock is not able to reproduce the Phillips curve[45].

We can finally notice that this Keynesian propagation mechanism makes the wages less pro-cyclical than usually obtained in RBC models.

[45] It would be necessary to introduce a demand shock, as already emphasized by Chéron, Langot (2000)

Table 7: **Model with non-separable preferences**

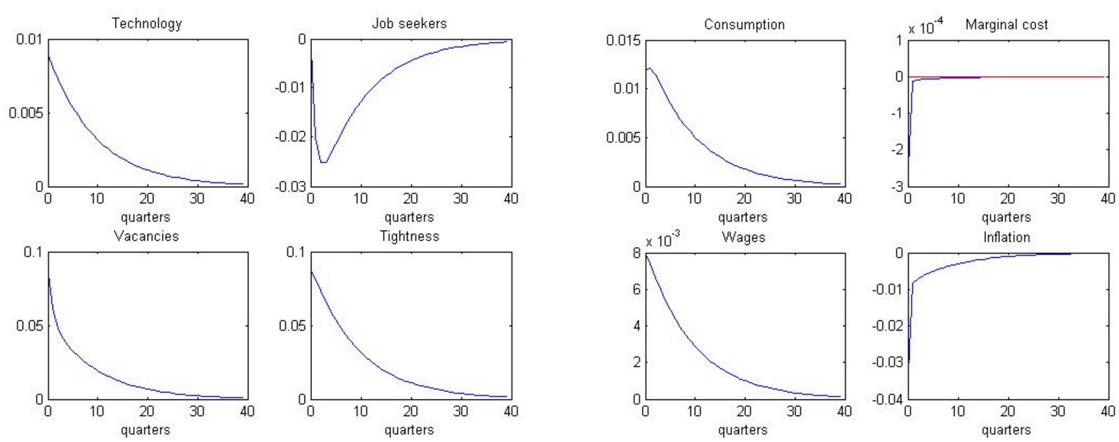
Benchmark calibration

Theoretical moments, smoothing parameter= 10^5

	U.S. data 1985-2011	$\alpha = 0.5$ **Tech shocks**	$\alpha = 0.01$ **Tech shock**
σ_Y	0.0309	0.0199	0.0251
σ_A	0.0159	0.0161	0.0161
$\frac{\sigma(U)}{\sigma(A)}$	10.3446	11.4907	9.8012
$\frac{\sigma(V)}{\sigma(A)}$	10.5140	16.6398	7.6025
$\frac{\sigma(\theta)}{\sigma(A)}$	20.1422	15.9752	10.0435
$\frac{\sigma(w)}{\sigma(A)}$	1.0086	1.3913	0.9068
$\frac{\sigma(Y)}{\sigma(A)}$	1.9422	1.2360	1.5590
$\rho(U, A)$	-0.1175	-0.0599	-0.9794
$\rho(V, A)$	0.0739	0.0435	0.9745
$\rho(\theta, A)$	0.0984	0.0522	0.9999
$\rho(w, Y)$	0.3327	0.5643	0.9981
$\rho(Y, A)$	0.5510	0.8445	0.9973
$\rho(U, V)$	-0.8589	-0.8454	-0.9092
$\rho(U, \pi)$	-0.1953	-0.5325	0.6219

(Y stands for production,A for labor productivity, U for unemployment -i.e. $U = 1 - N$, V stands for vacancies, θ for labor market tightness, w for real wage, π for inflation)

Figure 5: **IRFs after a positive technological shock when** $\alpha = 0.01$



4.9.6 What about variable hours?

Till now, I considered that the hours per worker are fixed and equal to h, which is calibrated to the steady state value of $1/3$.

One can wonder if my results continue to be valid once the hours per worker are also a variable of choice: in this case, in fact, there are two margins of adjustment of the labor input, the number of employees (N_t) and the hours worked by each worker (h_t), so that if the adjustment on the second margin is more important than that one on the first margin, I could again incur in the critique of Shimer (2005). I therefore extended the previous model in order to have variable hours.

There are at least two possible ways of modelling: one consists in considering the "efficient bargaining model", in which hours are bargained at the same time in which the wage is chosen by the worker and the firm; the second one consists in considering the "right-to-manage" model, in which the firm chooses unilaterally the hours worked, after the wage has been set through the bargaining procedure.

I choose the efficient bargaining setup, in order to have results which are directly comparable to what has been used in the literature[46]; in this case, the only difference with respect to my benchmark model consists in the fact that in the Nash product maximization problem there are now two variables of choice, w_t and h_t.

The Nash product problem is therefore
$max_{w_t,h_t} \left(\frac{1}{\lambda_t}\frac{\partial W(N_{t-1})}{\partial N_{t-1}}\right)^{1-\xi}\left(\frac{\partial V(N_{t-1})}{\partial N_{t-1}}\right)^{\xi}$
from which I obtain the wage and the hours equations

$$w_t h_t = (1-\xi)\left[x_t A_t h_t + \beta(1-s)\omega E_t \frac{\lambda_{t+1}}{\lambda_t}\theta_{t+1}\right] + \xi\left[(\Gamma^u - \Gamma^e_t)\right] \tag{17}$$

$$\gamma(1-h_t)^{-\varepsilon} = A_t x_t \tag{18}$$

where I remind that $\Gamma^e_t = \gamma\frac{(1-h_t)^{1-\varepsilon}}{1-\varepsilon}$.

The calibration strategy and therefore the steady state of the model are exactly equal to those of the benchmark model.

I therefore perform the same exercise as in the previous Section 4.9.5: I check the IRFs and the moments implied by the model with productivity shocks.

As we can see in Figure 6, when hours per worker are also a variable of choice (left panel), employment decreases less than in the benchmark case, so that I expect the volatilities of the job market variables to be lower than in the benchmark case.

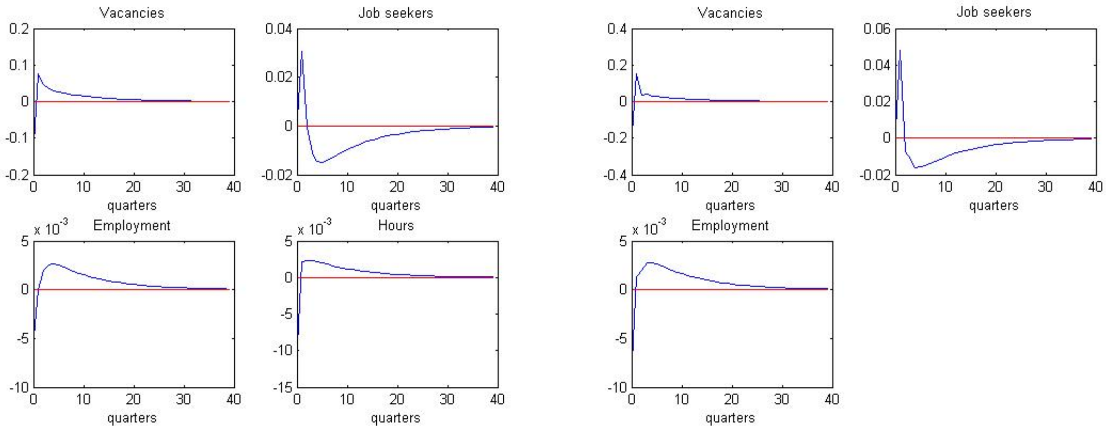
The relative importance of the intensive (hours worked per worked) and extensive (number of employees) margins in accounting for the total change of labor input across the business cycle has been widely studied: the recent paper by Ohanian, Raffo (2012) confirms a result known since King, Rebelo (1999), i.e. the fact that the extensive margin accounts for more than two thirds of the total labor input variations over the business cycle. This consideration has led to the use of models which do not explicitly modelize the intensive margin.

Before analyzing the quantitative properties of the model with variable hours in Table 8, I precise that I relied on the database created by Ohanian, Raffo (2012) for the series on hours worked per worker and employment, which anyway are taken from the Bureau of Labor Statistics website.

As one expects, since the reaction of the labor input is given by both the extensive and the extensive margin, the volatilities of the employment rate (and so of the unemployment) and of the vacancy rate

[46] As in CL (2004) for example

Figure 6: **Selected IRFs with variable hours (left) and fixed hours (right)**



are lower than in the benchmark case, but anyway if we look for example at unemployment, it is far from the order of magnitude of the Shimer's critique.

The model anyway presents an important drawback: if one compares the relative volatilites of the extensive and the intensive margins, one realizes that the model implies an intensive margin which is more volatile than the extensive one, something which is at odd with what we see in the data[47].

[47]This is a problem shared with other models, as for example in Thomas (2008)

Table 8: **Model with variable hours**

Theoretical moments, smoothing parameter= 10^5

	U.S. data 1985-2011	**Variable hours Tech shock**	$h = 1/3$ **Tech shocks**
σ_Y	0.0309	0.0258	0.0199
σ_A	0.0159	0.0161	0.0161
$\frac{\sigma(U)}{\sigma(A)}$	10.3446	8.6584	11.4907
$\frac{\sigma(V)}{\sigma(A)}$	10.5140	10.6894	16.6398
$\frac{\sigma(\theta)}{\sigma(A)}$	20.1422	10.9317	15.9752
$\frac{\sigma(N)}{\sigma(A)}$	0.6149	0.4969	0.6646
$\frac{\sigma(h)}{\sigma(A)}$	0.1845	0.7453	-
$\frac{\sigma(w)}{\sigma(A)}$	1.0086	0.9627	1.3913
$\frac{\sigma(Y)}{\sigma(A)}$	1.9422	1.6025	1.2360
$\rho(U, A)$	-0.1175	-0.1571	-0.0599
$\rho(V, A)$	0.0739	0.1134	0.0435
$\rho(\theta, A)$	0.0984	0.1389	0.0522
$\rho(w, Y)$	0.3327	0.8513	0.5643
$\rho(Y, A)$	0.5510	0.6483	0.8445
$\rho(U, V)$	-0.8589	-0.8306	-0.8454
$\rho(U, \pi)$	-0.1953	-0.3012	-0.5325

(Y stands for production,A for labor productivity, U for unemployment -i.e. $U = 1 - N$, V stands for vacancies, θ for labor market tightness, N for employment, $h$for hours per worker, w for real wage, π for inflation.

All series except hours per worker are provided by the BLS: labor productivity (A) is real average output per person in the non-farm business sector,U is the unemployment level from CPS, for vacancies (V) I use the composite Help Wanted Index proposed by Barnichon (2010b), real wage (w) is the real hourly average earnings of production and non supervisory employees (available from 1964), production is non-farm business sector output, the rate of inflation is calculated from the CPI for all urban consumers. Employment (N) and hours per worker (h) are taken from the dataset of Ohanian, Raffo (2012); employment is total employment from the BLS and the hours per worker are the results of their estimation. All variables are detrended with an HP filter with smoothing parameter of $10^5$and reported in log as deviation from the trend)

5 The main mechanism: a simple graphical illustration

The Shimer puzzle about the ability of the MP model to generate a reaction of labor market variables after a positive labor productivity shock can be easily illustrated using the textbook treatment of the equilibrium conditions as in Pissarides (2000); this exercise is useful because it allows me to see how embedding the model in a general equilibrium framework with nominal rigidities can potentially change the implications.

In this section I want to provide an illustration of the mechanisms behind my quantitative results: I will therefore look at the IRFs of unemployment and labor market tightness and I will show the differences between the two cases with and without nominal rigidities using the tool of a phase diagram. I will also consider, for the sake of clarity, an illustration of the results in terms of comparative statics.

Let's consider what happens in my models with and without nominal rigidities when preferences are non-separable, i.e. without wealth effects.

My job creation condition and wage equation are given by

$$\frac{\omega}{\Phi(\theta)}\left[1-\beta(1-s)\right] = xAh - wh \tag{19}$$

and

$$wh = (1-\xi)\left[xAh + \beta(1-s)\omega\theta\right] + \xi(\Gamma^u - \Gamma^e) \tag{20}$$

Looking at the equilibrium conditions, we see that two additional elements can intervene after a (transitory) productivity shock with respect to a partial equilibrium framework with no nominal rigidities: the real interest rate (since $\beta = \frac{1}{1+r}$) and the mark-up.

If the real interest rate decreases, the firm has an incentive to post vacancies, so we would have a rotation in the labor creation condition towards the right, such that both wage and tightness increase; at the same time the wage equation shifts towards the left, since the firm discounts more the value of employment and it is ready to pay a higher wage.

If the mark-up decreases, there is a downward pressure on wages coming from both the labor demand (which shifts towards the left) and the wage equation (which shifts towards the right), with different amplitudes.

In a demand-constrained setting, when firms are more productive, they see their marginal cost to decrease (i.e. the mark-up increases), since they cannot cut prices and the demand is fixed.

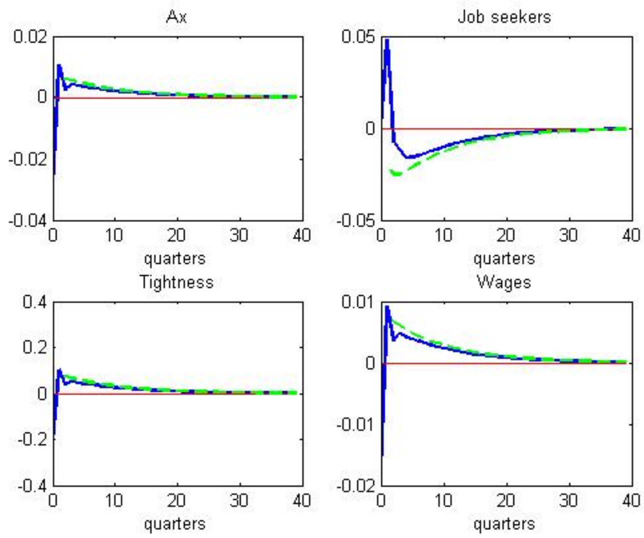
The overall effect on the term xAh depends on the relative strength of the two opposed forces moving x and A: in my framework on impact the effect of marginal cost prevails, and then after few periods that one of technology wins, as we can see in the blue line IRFs in Figure 7. This overall negative shock on the marginal product of employment pushes the intermediate firm to decrease vacancies and therefore employment.

If I want to represent these movements in a "comparative statics" graph (see Figure 8), I would have to represent two movements: a first one which corresponds to a "recessionary" shock (red curves) and a second one which corresponds to the positive technological shock in Shimer (2005), i.e. the classical one that characterizes a real model (blue curves).

I can show the mechanisms underlying my results in terms of the phase diagram in the space (S, θ), where I remember that S_t indicates the pool of job seekers[48] and the variable on the y-axis is the labor market tightness.

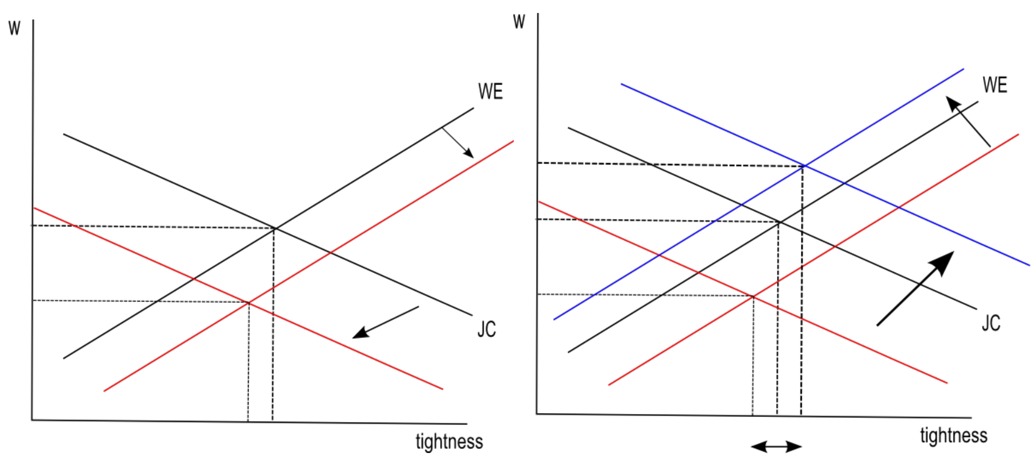
[48] $S_t = (1 - N_{t-1}) + sN_{t-1}$

Figure 7: **Selected IRFs with and without nominal rigidities**



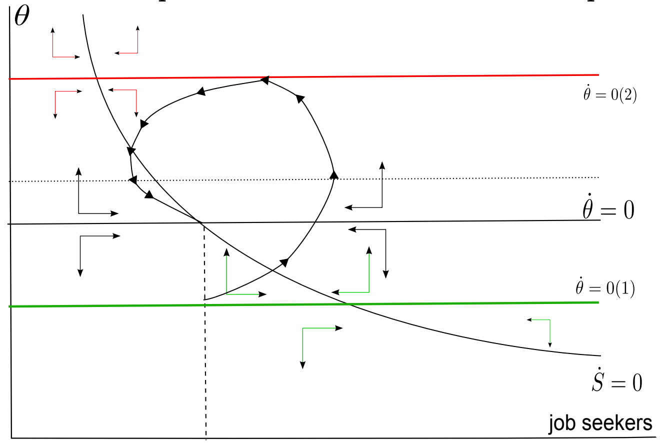
In green the IRFs when there are almost no nominal rigidities
and in blue the IRFs of the benchmark model with $\alpha = 0.5$

Figure 8: **The Shimer puzzle in a NK framework: short and medium run effects**



After a positive productivity shock which has a contractionary effect, at impact both the labor demand (JC) and the labor supply (WE) shift: the black curves represent the initial situation and the red ones the reaction at impact; then the blue curves represent the long term mouvements, as long as price can adjust, as in a "classical" model

Figure 9: **The Shimer puzzle in a NK framework: the phase diagram**



First of all let's combine the job creation condition and the wage equation in order to obtain the following equation:

$$\frac{\omega}{\Phi(\theta_t)} = \xi\left(x_t A_t h - (\Gamma^u - \Gamma^e)\right) + \beta(1-s)\omega\left[E_t\frac{\lambda_{t+1}}{\lambda_t}\frac{1}{\Phi(\theta_{t+1})} - (1-\xi)E_t\frac{\lambda_{t+1}}{\lambda_t}\theta_{t+1}\right] \tag{21}$$

The dynamic system I want to represent, following Pissarides (2000) and Miao (2014), is composed by two equations: equation 1 which gives the dynamics of employment (that I re-write in terms of job seekers) and equation 21.

$$\begin{cases} S_{t+1} &= s + (1-s)[1-\Phi(\theta_t)\theta_t]S_t \\ \frac{\omega}{\Phi(\theta_t)} &= \xi\left(x_t A_t h - (\Gamma^u - \Gamma^e)\right) + \beta(1-s)\omega\left[E_t\frac{\lambda_{t+1}}{\lambda_t}\frac{1}{\Phi(\theta_{t+1})} - (1-\xi)E_t\frac{\lambda_{t+1}}{\lambda_t}\theta_{t+1}\right] \end{cases}$$

The graphical representation in Figure 9 is qualitatively equivalent to the standard one which can be found in the textbook of Pissarides (2000) of a transitory technological shock.

I report in Figure 10 the "empirical" phase diagram I obtain if I plot my simulated values for the pool of job seekers and labor market tightness obtained after a positive technological shock.

We can notice that at the beginning labor market tightness jumps down, then the adjustment begins but the second sub-shock brings the system to a point where both tightness and job seekers are higher than in the steady state; at this point the dynamics is the same as in a "real" model where a positive technological shock has an expansionary effect on employment.

To stress this statement I report in Figures 11 and 12 the "theoretical" and the "empirical" phase diagrams for my model when I set the Calvo parameter such that nominal rigidities are almost nonexistent.

In this case we see that we have the traditional dynamics of adjustment: tightness jumps up after a positive productivity shock (since the marginal cost barely moves); then the dynamics (which is theoretically approximated under the hypothesis that the production per worker doesn't change any more) of adjustment follows till the return to the steady state, as we can see also in the IRFs in Figure 7 (green dashed lines).

Figure 10: **The Shimer puzzle in a NK framework: the "empirical" phase diagram**

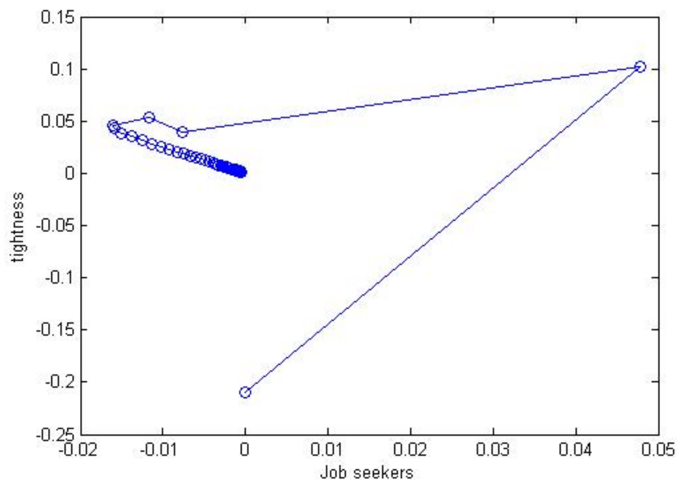


Figure 11: **The Shimer puzzle in a NK framework: the phase diagram with no nominal rigidities**

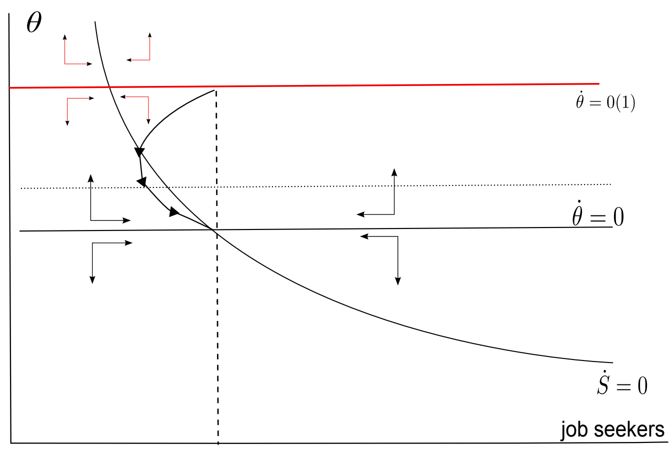
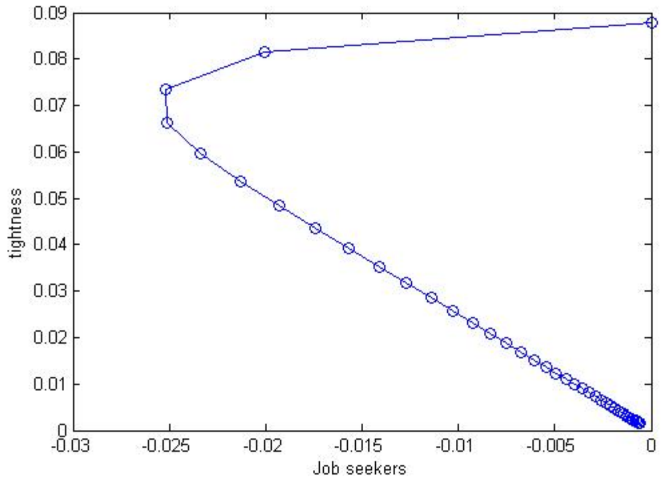


Figure 12: **The Shimer puzzle in a NK framework: the "empirical" phase diagram with no nominal rigidities**



6 Concluding remarks

In this work I addressed the issue of the volatilities of labor market variables implied by the search and matching model. I claimed that this issue cannot be separated by the consideration of the value of the correlations between labor market variables and labor productivity: in order to reconcile these aspects, I develop a version of the New Keynesian model with nominal rigidities and a frictional labor market with non-separable preferences.

My model allows me to obtain, under a certain specification for the monetary policy rule and calibration strategy, a negative reaction of employment after a positive productivity shock, in the spirit of Galì (1999).

The quantitative performance of the model seems to indicate that the combined presence of nominal and labor market rigidities allows to explain both the volatilities of labor market variables and their correlations with productivity of the Great Moderation period conditional on a technology shock.

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8 Appendix

8.1 The model in detail

I provide a detailed derivation of my nominal model.

Households

The original problem of the households can be written as

$$W(\Omega_t^H) = max_{C_t^e, C_t^u, B_{t+1}^e, B_{t+1}^u, Ins_t}\{N_t\left[U(C_t^e,\Gamma^e) + \beta E_t W(\Omega_{t+1}^{H,e})\right] + (1-N_t)\left[U(C_t^u,\Gamma^u) + \beta E_t W(\Omega_{t+1}^{H,u})\right]\}$$

$$s.t\begin{cases} \frac{B_t}{P_t} + \frac{D_t}{P_t} + w_t h - C_t^e - \tau_t Ins_t - \frac{B_{t+1}^e}{P_t}(1+r_t^n)^{-1} & = 0 \\ \frac{B_t}{P_t} + \frac{D_t}{P_t} + Ins_t - C_t^u - \tau_t Ins_t - \frac{B_{t+1}^u}{P_t}(1+r_t^n)^{-1} & = 0 \end{cases}$$

where I suppose that there exists an insurance company which sells a contract denoted by Ins_t; the zero profit condition for this insurance firm is given by $\Pi_t^{ins} = \tau_t Ins_t - (1-N_t)Ins_t = 0$ and I assume also that employed and unemployed agents start with the same amount of bonds B_t.

I write the FOCs:

$\frac{\partial L}{\partial C_t^e} = 0 \rightarrow \lambda_t^e = U_1(C_t^e,\Gamma^e)$

$\frac{\partial L}{\partial C_t^u} = 0 \rightarrow \lambda_t^u = U_1(C_t^u,\Gamma^u)$

$\frac{\partial L}{\partial Ins_t} = 0 \rightarrow (1-N_t)\lambda_t^u(1-\tau_t) = N_t\lambda_t^e\tau_t \rightarrow \lambda_t^u = \lambda_t^e = \lambda_t$

$\frac{\partial L}{\partial B_{t+1}^e} = 0 \rightarrow \lambda_t^e[P_t(1+r_t^n)]^{-1} = \beta E_t\frac{\partial W(\Omega_{t+1}^{H,e})}{\partial B_{t+1}^e}$

$\frac{\partial L}{\partial B_{t+1}^u} = 0 \rightarrow \lambda_t^u[P_t(1+r_t^n)]^{-1} = \beta E_t\frac{\partial W(\Omega_{t+1}^{H,u})}{\partial B_{t+1}^u}$

therefore $E_t\frac{\partial W(\Omega_{t+1}^{H,e})}{\partial B_{t+1}^e} = E_t\frac{\partial W(\Omega_{t+1}^{H,u})}{\partial B_{t+1}^u} \rightarrow B_{t+1}^e = B_{t+1}^u = B_{t+1}$because of the continuity and concavity of the value function.

Therefore I can write the problem of the households with only one budget constraint for both types of agents.

I then report the value function for the households and derive the FOCs:

$$W(\Omega_t^H) = max_{C_t^e, C_t^u, B_{t+1}}\{N_t\left[U(C_t^e,\Gamma^e)\right] + (1-N_t)\left[U(C_t^u,\Gamma^u)\right] + \beta E_t W(\Omega_{t+1}^H)\}$$

$$s.t.\begin{cases} N_t C_t^e + (1-N_t)C_t^u + \frac{B_{t+1}}{P_t}\frac{1}{(1+r_t^n)} & = w_t N_t h + \frac{B_t}{P_t} + \frac{D_t}{P_t} \\ N_{t+1} & = (1-s)N_t + p_{t+1}S_{t+1} \\ p_t & = \frac{M_t}{S_t} \\ S_t & = 1-(1-s)N_{t-1} \end{cases}$$

where $\Omega_t^H = \{N_{t-1}, B_t\}$

The FOCs are:

$\frac{\partial L}{\partial C_t^e} = 0 \rightarrow \lambda_t = U_1(C_t^e,\Gamma^e)$

$\frac{\partial L}{\partial C_t^u} = 0 \rightarrow \lambda_t = U_1(C_t^u,\Gamma^u)$

where obviously $\lambda_t$is the Lagrangian multiplier of the budget constraint, so it expresses the marginal value of real wealth (so the marginal value not of one unit of currency, as it would be if the budget constraint had been written in nominal terms, but the marginal value of the wealth necessary to have one unit more of the consumption good, i.e. the marginal value of $P_t$units of nominal income).

$\frac{\partial L}{\partial B_{t+1}} = 0 \rightarrow \frac{\lambda_t}{P_t(1+r_t^n)} = \beta E_t\frac{\partial W(\Omega_{t+1}^H)}{\partial B_{t+1}}$

I apply the envelope theorem to find

$\lambda_t = \beta E_t \lambda_{t+1} \frac{(1+r_t^n)}{\frac{P_{t+1}}{P_t}} \rightarrow \lambda_t = \beta E_t \lambda_{t+1}(1+r_t)$

where therefore r_t indicates the real net interest rate and the gross rate of inflation is given by $\pi_{t+1} = \frac{P_{t+1}}{P_t}$: the Fisher parity condition $(1+r_t) = \frac{(1+r_t^n)}{\pi_{t+1}}$ holds.

Then, the first order conditions for the households problem as reported in the text are

$$\begin{cases} \lambda_t = & \frac{1}{C_t^e + \Gamma^e} \\ \lambda_t = & \frac{1}{C_t^u + \Gamma^u} \\ \lambda_t = & \beta E_t \lambda_{t+1}(1+r_t) \end{cases}$$

which imply that $C_t^e = C_t^u + \Gamma^u - \Gamma^e$; since I assume $\Gamma^u - \Gamma^e > 0$, I have that the consumption of the employed is higher than that one of the unemployed.

Firms

The hypothesis that markets are complete brings me to derive the expression of the discount factor that is used by firms to evaluate their future flow of profits. I assume that intermediate firms are owned by households, so that their budget constraint be written , more precisely, as

$N_t C_t^e + (1-N_t) C_t^u + \frac{Q_t}{P_t} B_{t+1} + \frac{Q_t^s}{P_t} \tilde{s}_{t+1} = w_t N_t h + \frac{B_t}{P_t} + \frac{(Q_t^s + D_t)}{P_t} \tilde{s}_t$

where $Q_t = \frac{1}{(1+r_t^n)}$ and I explicitly consider that at the beginning of the period the household holds a number $\tilde{s}_t$ of the firm's share , whose price is given by Q_t^s, and that she decides how many shares to hold for the following period.

The problem of the household can be written, for the sake of completeness, as

$$max_{C_t^e, C_t^u, B_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t \left[N_t U(C_t^e) + (1-N_t) U(C_t^u, \Gamma^u) \right]$$

$$s.t. \begin{cases} N_t C_t^e + (1-N_t) C_t^u + \frac{Q_t}{P_t} B_{t+1} + \frac{Q_t^s}{P_t} \tilde{s}_{t+1} & = w_t N_t + \frac{B_t}{P_t} + \frac{(Q_t^s + D_t)}{P_t} \tilde{s}_t \\ N_{t+1} & = (1-s) N_t + p_{t+1} S_{t+1} \\ p_t & = \frac{M_t}{S_t} \\ S_t & = 1 - (1-s) N_{t-1} \end{cases}$$

In addition to the previous FOCs, I have also that one for $\tilde{s}_{t+1}$, which gives

$\beta E_t \lambda_{t+1} \frac{(Q_{t+1}^s + D_{t+1})}{P_{t+1}} = \lambda_t \frac{Q_t^s}{P_t}$

If I iterate forward the previous equation for one period I get

$\lambda_{t+1} \frac{Q_{t+1}^s}{P_{t+1}} = \beta E_{t+1} \lambda_{t+2} \frac{(Q_{t+2}^s + D_{t+2})}{P_{t+2}} \rightarrow \lambda_t \frac{Q_t^s}{P_t} = \beta E_t \left[\beta E_{t+1} \lambda_{t+2} \frac{(Q_{t+2}^s + D_{t+2})}{P_{t+2}} + \lambda_{t+1} \frac{D_{t+1}}{P_{t+1}} \right]$

so if I continue, with the addition of the transversality condition, according to which I eliminate the possibility of bubbles, i.e. $lim_{j \rightarrow \infty} E_t \beta^j \lambda_{t+j} \frac{Q_{t+j}^s}{P_{t+j}} = 0$ I can write

$\lambda_t \frac{Q_t^s}{P_t} = E_t \sum_{j=1}^{\infty} \beta^j \lambda_{t+j} \frac{D_{t+j}}{P_{t+j}}$,

which is the pricing condition of the share of the firm: the price (in real terms) of a share is given by the present value of the (real) dividends it promises, where the discount factor, in terms of marginal value of real consumption, is given by $\beta^j \frac{\lambda_{t+j}}{\lambda_t}$.

A firm which wants to maximize its real value, then, has to solve the problem

$max \frac{(Q_t^s + D_t)}{P_t} \tilde{s}_t = max(E_t \sum_{j=1}^{\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} \frac{D_{t+j}}{P_{t+j}} + \frac{D_t}{P_t}) \tilde{s}_t$

Normalizing the number of shares as $\tilde{s}_t = \tilde{s}_{t+1} = 1$, I find that the problem can be written as

$max E_t \sum_{j=0}^{\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} \frac{D_{t+j}}{P_{t+j}}$, i.e. the firm maximizes the value of the real dividends using as discount factor the term $Z_{t+j} \equiv \beta^j \frac{\lambda_{t+j}}{\lambda_t}$, which means that at the end the real dividends are evaluated according to the marginal utility of consumption they provide to the households.

Wage equation

The sharing rule which derives form the maximization of the Nash product is

$(1-\xi)(1-p_t)\frac{\partial V(N_{t-1})}{\partial N_{t-1}} = \xi \frac{1}{\lambda_t}\frac{\partial W(N_{t-1})}{\partial N_{t-1}}$.

The expressions of the marginal values of employment for the firm and the household are given by $\frac{\partial V(N_{t-1})}{\partial N_{t-1}} = (1-s)\frac{\omega}{\Phi_t}$ and

$\frac{\partial W(N_{t-1})}{\partial N_{t-1}} = \left[U(C_t^e,\Gamma^e) - U(C_t^u,\Gamma^u) + \lambda_t[w_t h - C_t^e + C_t^u] + \beta E_t \frac{\partial W(N_t)}{\partial N_t}\right](1-s)(1-p_t)$;

from the maximization of the Nash product I get the FOC

$(1-\xi)(1-p_t)\frac{\partial V(N_{t-1})}{\partial N_{t-1}} = \xi \frac{1}{\lambda_t}\frac{\partial W(N_{t-1})}{\partial N_{t-1}}$.

Since $\frac{\partial V(N_{t-1})}{\partial N_{t-1}} = (1-s)\frac{\omega}{\Phi_t}$, this implies that $\frac{\partial W(N_t)}{\partial N_t} = \frac{(1-\xi)}{\xi}\lambda_{t+1}(1-s)(1-p_{t+1})\frac{\omega}{\Phi_{t+1}}$.

Final producer and the retailers

The problem of the final producer is

$max P_t Y_t^F - \int_0^1 Y_t(i)P_t(i)di$

$s.t. \left\{ Y_t^F = \quad \left\{\int_0^1 Y_t(i)^{\frac{\eta-1}{\eta}} di\right\}^{\frac{\eta}{\eta-1}}\right.$

where Y_t^F is the production of the final producer, and since it sells to households I also know that $Y_t^F = C_t$; $Y_t(i)$ is the production of retailer i whose price is $P_t(i)$ desired by the final producer and η is the elasticity of substitution.

The solution of this problem gives the demand for good i which is faced by the retailer: $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\eta} C_t$.

By substituting back the demand function I can derive the expression of the price index: $P_t = \left\{\int_0^1 P_t(i)^{1-\eta}di\right\}^{\frac{1}{1-\eta}}$, whose evolution is given, following Calvo (1983), by $P_t^{1-\eta} = \left[(1-\alpha)P_t(i)^{(1-\eta)} + \alpha P_{t-1}^{(1-\eta)}\right]$.

I write here explicitly the equations needed to derive the Phillips curve, as it is standard in sticky price literature; I saw that the FOC for price setting decision is given by

$P_t(i) = \frac{E_t \sum_{j=0}^{\infty} \theta^j \beta^j \lambda_{t+j} \frac{1}{P_{t+j}} C_{t+j} P_{t+j}^{(\eta+1)} \frac{\eta}{\eta-1} x_{t+j}}{E_t \sum_{j=0}^{\infty} \theta^j \beta^j \lambda_{t+j} \frac{1}{P_{t+j}} C_{t+j} P_{t+j}^{(\eta-1)}}$

I can also re-write the FOC in the following way

$E_t \sum_{j=0}^{\infty} \theta^j \beta^j \lambda_{t+j} \frac{1}{P_{t+j}} C_{t+j} P_{t+j}^{(\eta+1)} \left[\left(\frac{P_t(i)}{P_t}\frac{1}{P_{t+j}/P_t}\right) - \frac{\eta}{\eta-1}x_{t+j}\right] = 0$

so that if I define $\frac{P_t(i)}{P_t} = p_t^*(i)$ and $X_{t,j} = \begin{cases} \frac{1}{\pi_{t+j}\cdots\pi_{t+1}} & j \geq 1 \\ 1 & j = 0 \end{cases}$, so that $X_{t,j} = X_{t+1,j-1}\frac{1}{\pi_{t+1}}$, I can rewrite the previous expression as

$E_t \sum_{j=0}^{\infty} \theta^j \beta^j \lambda_{t+j} C_{t+j} X_{t,j}^{(-\eta)} \left[p_t^*(i)X_{t,j} - \frac{\eta}{\eta-1}x_{t+j}\right] = 0$ which gives

$p_t^*(i) = \frac{E_t \sum_{j=0}^{\infty} \theta^j \beta^j \lambda_{t+j} C_{t+j} X_{t,j}^{(-\eta)} \frac{\eta}{\eta-1} x_{t+j}}{E_t \sum_{j=0}^{\infty} \theta^j \beta^j \lambda_{t+j} C_{t+j} X_{t,j}^{(1-\eta)}} = \frac{R_t}{F_t}$

At this point I need to find the expressions for R_t and F_t: simply by taking out from the sum the first element, and using the definition of $X_{t,j}$, I can re-write the expression of R_t as follows:

$R_t = \lambda_t C_t x_t \frac{\eta}{\eta-1} + \beta\theta E_t \pi_{t+1}^{\eta} E_{t+1} \sum_{j=0}^{\infty} \theta^j \beta^j \lambda_{t+1+j} C_{t+1+j} X_{t+1,j}^{(-\eta)} \frac{\eta}{\eta-1} x_{t+1+j}$

which gives the following relation

$R_t = \lambda_t C_t x_t \frac{\eta}{\eta-1} + \beta\theta E_t \pi_{t+1}^{\eta} R_{t+1}$

Similarly I obtain for F_t the following expression

$F_t = \lambda_t C_t + \beta\theta E_t \pi_{t+1}^{\eta-1} F_{t+1}$

I notice that here, since I do not suppose any efficient subsidy to the labor input, I find the term which represents the distortion coming from monopolistic competition: $\frac{\eta}{\eta-1}$.

If I consider the dynamics for the aggregate price index, $P_t^{1-\eta} = \left[(1-\theta)P_t(i)^{(1-\eta)} + \theta P_{t-1}^{(1-\eta)}\right]$, I can re-write this expression as following

$1 = (1-\theta)p_t^*(i)^{(1-\eta)} + \theta\pi_t^{(\eta-1)}$ so that finally $p_t^*(i) = \left\{\frac{1}{(1-\theta)}\left[1-\theta\pi_t^{(\eta-1)}\right]\right\}^{\frac{1}{1-\eta}}$.

The equations of the model

1. $N_{t+1} = (1-s)N_t + \Upsilon S_{t+1}^{1-\psi} V_{t+1}^{\psi}$
2. $S_t = 1-(1-s)N_{t-1}$
3. $\lambda_t = \beta E_t \lambda_{t+1}(1+r_t)$
4. $r_t = \frac{r_t^n}{E_t \pi_{t+1}}$
5. $\lambda_t = \frac{1}{C_t^u + \Gamma^u}$
6. $C_t = C_t^e N_t + C_t^u(1-N_t)$
7. $C_t^e = C_t^u + \Gamma^u - \Gamma^e$
8. $\frac{\omega}{\Phi_t} = (x_t A_t h - w_t h) + \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{\omega}{\Phi_{t+1}}(1-s)\right] \rightarrow x_t = \frac{w_t}{A_t} + \frac{\frac{\omega}{\Phi_t} - \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{\omega}{\Phi_{t+1}}(1-s)\right]}{A_t h}$
9. $Y_t = A_t N_t h$
10. $Y_t = \Delta_t C_t + \omega V_t$
11. $w_t h = (1-\xi)\left[x_t A_t h + \beta(1-s)\omega E_t \frac{\lambda_{t+1}}{\lambda_t}\theta_{t+1}\right] + \xi\left[\Gamma^u - \Gamma^e\right]$
12. $p_t^*(i) = \frac{E_t \sum_{j=0}^{\infty} \theta^j \beta^j \lambda_{t+j} C_{t+j} X_{t,j}^{(-\eta)} \frac{\eta}{\eta-1} x_{t+j}}{E_t \sum_{j=0}^{\infty} \theta^j \beta^j \lambda_{t+j} C_{t+j} X_{t,j}^{(1-\eta)}} = \frac{R_t}{F_t}$
13. $R_t = \lambda_t C_t x_t \frac{\eta}{\eta-1} + \beta\theta E_t \pi_{t+1}^{\eta} R_{t+1}$
14. $F_t = \lambda_t C_t + \beta\theta E_t \pi_{t+1}^{\eta-1} F_{t+1}$
15. $p_t^*(i) = \left\{\frac{1}{(1-\theta)}\left[1-\theta\pi_t^{(\eta-1)}\right]\right\}^{\frac{1}{1-\eta}}$
16. $\frac{R_t^n}{R^n} = (\frac{R_{t-1}^n}{R^n})^{\varrho_m} \left(\frac{\pi_t}{\pi}\right)^{(1-\varrho_m)\gamma_\pi} \left(\frac{Y_t}{Y}\right)^{(1-\rho_m)\gamma_y} e^{\varepsilon_t^m}$
17. $A_t = A_{t-1}^{\varrho_A} A^{(1-\varrho_A)} e^{\varepsilon_t^A}$

Log-linearization

1. $\hat{N}_t = (1-s)\hat{N}_{t-1} + s\left[\psi\hat{V}_t + (1-\psi)\hat{S}_t\right]$

2. $S\hat{S}_t + (1-s)\hat{N}_{t-1} = 0$

3. $\hat{\theta}_t = \hat{V}_t - \hat{S}_t$

4. $\hat{p}_t = \psi\hat{\theta}_t$

5. $\hat{\Phi}_t = (\psi - 1)\hat{\theta}_t$

6. $\hat{\lambda}_t = E_t\hat{\lambda}_{t+1} + \hat{r}_t^n - E_t\pi_{t+1}$

7. $\left[\frac{1}{C^u+\Gamma^u}\right]\hat{\lambda}_t + \frac{c^u}{(C^u+\Gamma^u)^2}\hat{C}_t^u = 0$

8. $\frac{C}{N}\hat{C}_t^e + (C^e - C^u)\hat{N}_t - C^e\hat{C}_t^e - \frac{1-N}{N}C^u\hat{C}_t^u = 0$

9. $\hat{C}_t^e = \frac{C^u}{C^e}\hat{C}_t^u$

10. $\hat{Y}_t = \hat{A}_t + \hat{N}_t$

11. $\hat{Y}_t = \frac{C\Delta}{Y}(\hat{C}_t + \hat{\Delta}_t) + \frac{\omega V}{Y}\hat{V}_t$ with $\Delta = 1$and $\hat{\Delta}_t = 0 \forall t$

12. $\frac{\omega}{\Phi}\beta(1-s)(E_t\hat{\Phi}_{t+1} - \frac{1}{\beta(1-s)}\hat{\Phi}_t + \hat{\lambda}_t - E_t\hat{\lambda}_{t+1}) = xAh(\hat{x}_t + \hat{A}_t) - wh\hat{w}_t$

13. $wh\hat{w}_t = (1-\xi)\left[xAh(\hat{x}_t + \hat{A}_t) + \beta(1-s)\omega(E_t\hat{\theta}_{t+1} + E_t\hat{\lambda}_{t+1} - \hat{\lambda}_t)\right] + \xi\left[(C^e\hat{C}_t^e - C^u\hat{C}_t^u)\right]$

14. $\hat{\pi}_t = \frac{(1-\beta\theta)(1-\theta)}{\theta}\hat{x}_t + \beta E_t\hat{\pi}_{t+1}$

15. $\hat{A}_t = \varrho^A\hat{A}_{t-1} + \hat{\varepsilon}_t^A$

16. $\hat{r}_t^n = \varrho^m\hat{r}_{t-1}^n + (1-\varrho^m)\left[\gamma_y\hat{Y}_t + \gamma_\pi\hat{\pi}_t\right] + \hat{\varepsilon}_t^m$

8.2 The quantitative effects of the "instantaneous hirings" hypothesis

I report in Table 9 the quantitative results for two models which differ for the dynamics of labor market flows but present the same ratio of vacancy posting costs in terms of output of $\frac{\omega V}{Y} = 0.8\%$.

The different dynamics implies that the same values for the separation rate and the job finding rate are consistent with different values of the employment rate: in my benchmark specification $N = 0.9455$ while with the traditional dynamics the implied rate of employment is $N = 0.8638$.

8.3 A monthly calibration exercise

In order to check that the choice of a calibration strategy based on a quarterly basis (as it is standard in the RBC literature) does not affect the quantitative results, I calibrate the model using as reference period one month: in this case I use for the separation and the job finding rate the same values proposed by Shimer (2005), i.e. $s = 0.034$ and $p = 0.45$. In Table 10 I report the values of the calibrated parameters and in Table 11 the quantitative results.

The moments of the monthly calibrated model are computed as it is standard: from the monthly series I obtain the quarterly average and then I apply the HP filter with a smoothing parameter of $10^5$to the log of the series.

Table 9: **The impact of the "instantaneous hirings" hypothesis**

	U.S. data			Instantaneous hirings	Traditional dynamics
	1951-2011	**1951-1984**	**1985-2011**	**Tech shock**	**Tech shock**
σ_Y	0.0333	0.0343	0.0309	0.0251	0.0222
σ_A	0.0202	0.0227	0.0159	0.0161	0.0161
$\frac{\sigma(U)}{\sigma(A)}$	9.5325	9.3295	10.3446	9.8012	2.7329
$\frac{\sigma(V)}{\sigma(A)}$	9.3541	8.8242	10.5140	7.6025	6.0807
$\frac{\sigma(\theta)}{\sigma(A)}$	18.3171	17.2945	20.1422	10.0435	8.2298
$\frac{\sigma(w)}{\sigma(A)}$	1.1043	1.2558	1.0086	0.9068	0.9068
$\frac{\sigma(Y)}{\sigma(A)}$	1.6525	1.5115	1.9422	1.5590	1.3789
$\rho(U, A)$	-0.4140	-0.4856	-0.1175	-0.9794	-0.8156
$\rho(V, A)$	0.40	0.5249	0.0739	0.9745	0.9843
$\rho(\theta, A)$	0.4188	0.5169	0.0984	0.9999	0.9981
$\rho(w, Y)$	0.3519	0.4213	0.3327	0.9981	0.9879
$\rho(Y, A)$	0.7071	0.7650	0.5510	0.9973	0.9835
$\rho(U, V)$	-0.8827	-0.9046	-0.8589	-0.9092	-0.7008
$\rho(U, \pi)$	-0.2663	-0.3967	-0.1953	0.6219	0.3774

Table 10: **Monthly calibration parameters**

h	ε	ϱ_A	σ_{ε_A}	$\omega\frac{V}{Y}$	$\frac{C^u}{C^e}$	β	η	γ_π	γ_Y	α	ϱ_m	N	Φ	ξ	ψ	s
1/3	4	$0.9^{1/3}$	0.0056	0.8%	0.24	0.995	6	1.5	0.5/3	0.73	$0.8^{1/3}$	0.9601	0.64	0.4	0.4	0.034

As I highlighted in the Introduction, the model based on a monthly base it is not exactly equivalent to the benchmark one: with a Calvo parameter equal to 0.83 (so that the average price spell would have been of almost 6 months, as in the benchmark case) the model is unstable, so I report the results obtained for the highest possible value of the Calvo parameter compatible with a unique and stable equilibrium; this value ($\alpha = 0.73$) implies an average duration of the price spell of 3.5 months. As we can see, the quantitative eeffects are not changed by the different calibration strategy for what it regards the period of reference.

Table 11: **Comparison between quarterly (benchmark) and monthly calibration**

	U.S. data 1985-2011	**Benchmark model Tech shocks**	**Monthly calibration Tech shock**
σ_Y	0.0309	0.0199	0.0167
σ_A	0.0159	0.0161	0.0159
$\frac{\sigma(U)}{\sigma(A)}$	10.3446	11.4907	6.3260
$\frac{\sigma(V)}{\sigma(A)}$	10.5140	16.6398	14.8034
$\frac{\sigma(\theta)}{\sigma(A)}$	20.1422	15.9752	15.7109
$\frac{\sigma(w)}{\sigma(A)}$	1.0086	1.3913	4.1665
$\frac{\sigma(Y)}{\sigma(A)}$	1.9422	1.2360	1.0503
$\rho(U, A)$	-0.1175	-0.0599	-0.0439
$\rho(V, A)$	0.0739	0.0435	0.1353
$\rho(\theta, A)$	0.0984	0.0522	0.1107
$\rho(w, Y)$	0.3327	0.5643	0.2275
$\rho(Y, A)$	0.5510	0.8445	0.9676
$\rho(U, V)$	-0.8589	-0.8454	-0.6777
$\rho(U, \pi)$	-0.1953	-0.5325	-0.6932

(Y stands for production,A for labor productivity, U for unemployment -i.e. $U = 1 - N$, V stands for vacancies, θ for labor market tightness, w for real wage, π for inflation)

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