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## PRECAUTIONARY SAVING AND AGGREGATE DEMAND

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**ABSTRACT:** We formulate and estimate a tractable macroeconomic model with time-varying precautionary savings. We argue that the latter affect aggregate fluctuations via two main channels: a stabilizing aggregate supply effect working through the supply of capital; and a destabilizing aggregate demand effect generated by a feedback loop between unemployment risk and consumption demand. Using the estimated model to measure the contribution of precautionary savings to the propagation of recent recessions, we find strong aggregate demand effects during the Great Recession and the 1990–1991 recession. In contrast, the supply effect at least offset the demand effect during the 2001 recession.

**KEYWORDS:** Incomplete markets, DSGE model, Bayesian estimation, Great Recession.

**JEL CLASS.:** E32

**RÉSUMÉ:** Nous formulons et estimons un modèle macroéconomique analytiquement simple où l'épargne de précaution varie au cours du cycle. Nous montrons que cette dernière affecte les fluctuations agrégées au travers de deux principaux canaux : un effet d'offre agrégée stabilisant, qui transite par l'offre de capital; un effet de demande agrégée destabilisant, qui tire son origine d'une boucle de rétroaction néfaste entre le risque de chômage et la demande de biens de consommation. En utilisant le modèle estimé pour mesurer la contribution de l'épargne de précaution aux récessions récentes, nous trouvons un puissant effet de demande agrégée au cours de la Grande Récession et pendant la récession de 1990–1991. Par contraste, l'effet d'offre agrégée semble avoir neutralisé les effets de demande agrégée au cours de la récession de 2001.

**MOTS-CLÉS:** Marchés incomplets, modèles DSGE, estimation bayésienne, Grande Récession.

**CLASSIFICATION JEL:** E32

## NON TECHNICAL SUMMARY

How do fluctuations in households' precautionary wealth contribute to the propagation of aggregate shocks? In this paper, we attempt to answer this question by formulating and estimating a tractable structural macroeconomic model in which the precautionary motive for holding wealth is operative and its quantitative impact on the business cycle can be precisely evaluated. The framework that we propose is designed to be sufficiently rich to incorporate the main theoretical channels through which precautionary wealth and its variations over time may affect aggregate outcomes, but also sufficiently simple to accommodate the variety of state variables and shocks that are necessary for a likelihood-based estimation of the underlying parameters.

Our framework combines three basic frictions: (i) nominal rigidities (in goods prices and wages), (ii) matching frictions in the labor market, and (iii) imperfect insurance against idiosyncratic unemployment risk coupled with borrowing constraints. All three frictions are known, even in isolation, to capture some important features of the business cycle. In this paper, however, we argue that it is their interactions that matter for the aggregate impact of time-varying precautionary savings, because such interactions give rise to a powerful feedback loop between unemployment risk and aggregate consumption demand. More specifically, following aggregate shocks that lower demand, job creation is discouraged, unemployment persistently rises, and hence so does unemployment risk. Imperfectly-insured households rationally respond to this rise in idiosyncratic risk by increasing precautionary wealth, thereby cutting consumption and degrading demand even more; this in turn magnifies the initial labor market contraction, further raises unemployment risk, and so on. In other words, it is the endogenous response of households' precautionary wealth (due to imperfect insurance) in an equilibrium where aggregate demand matters (due to nominal rigidities) and when idiosyncratic risk is endogenous (due to labor-market frictions) that explains the potentially large effect of aggregate shocks on outcomes.

This "aggregate demand" effect of time-varying precautionary savings is, however, usually not the only one at work in economies with imperfect insurance. As is well understood at least since the work of [Krusell and Smith \(1998\)](#), time-varying precautionary savings also have an "aggregate supply" effect. More specifically, imperfectly insured households are reluctant to dissave in bad times (when idiosyncratic risk is high) and somewhat unwilling to save in good times (when idiosyncratic risk is low), so precautionary wealth tends to smooth fluctuations in investment, capital and ultimately output. In the absence of the aggregate demand effect described above (e.g., under full nominal flexibility), this aggregate supply effect of precautionary wealth necessarily dominates the demand effect, causing output to be *less*, not more, volatile under imperfect insurance than under perfect insurance. The presence of (and competition between) the two effects implies

that determining which effect actually dominates, and hence whether time-varying precautionary savings ultimately makes the economy more or less responsive to aggregate shocks, essentially becomes an empirical question. However, designing a quantitative model that can incorporate the main general-equilibrium effects of time-varying precautionary savings and can at the same time be taken to the data remains a challenge. Our modeling strategy aims to answer this challenge by constructing a framework that integrates the three frictions discussed above, whilst remaining tractable and thereby amenable to estimation using likelihood methods.

Our framework is “tractable” in the sense that its equilibrium dynamics admit a finite-dimensional state-space representation. This representation is achieved by designing the model so that the wealth of a household will only depend on a finite history of idiosyncratic unemployment shocks; this will in turn imply that the support of the cross-sectional distribution of wealth has finite support. This is in contrast to the usual imperfect-insurance models, which typically generate a cross-sectional distribution of wealth with continuous support, due to the dependence of a household’s wealth on its full history of idiosyncratic shocks (see, for example, [Aiyagari, 1994](#); [Krusell and Smith, 1998](#)). The key to the reduction of the dimensionality of the state space lies in our modeling of the household sector. First, we divide households into two classes, “firm owners” and “workers”. Firm owners are patient households who not only work but also hold both the capital stock and firms’ equities, while enjoying perfect insurance against idiosyncratic shocks; their cross-sectional distribution of wealth is thus degenerated by construction. Workers, in contrast, are impatient, only enjoy labor income, and are imperfectly insured against idiosyncratic unemployment risk. We then make two specific assumptions about the financial frictions suffered by the workers. First, we assume that employed workers are perfectly insured within (large) “families”, while unemployed workers are taken charge of by the public unemployment insurance scheme (which only provides imperfect insurance). This can be seen as a minimal departure from the representative-agent assumption in that perfect insurance takes place among employed workers but no longer extends to unemployed workers. This minimal departure is, however, sufficient to capture the essence of the precautionary motive, because the imperfect insurance that workers have against unemployment motivates the accumulation of precautionary wealth, when they are still employed. The second assumption we make about financial frictions is that workers face a borrowing limit that is strictly tighter than the natural limit. This can be shown to imply that a worker who remains unemployed eventually faces the borrowing limit in finite time.<sup>1</sup> As we show, taken

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<sup>1</sup>An agent facing the natural limit never borrows more than what he or she would be able to repay in the worst individual history, here a history with permanent unemployment. If the borrowing limit is tighter than the natural limit, then a sufficiently long unemployment spell necessarily causes the borrowing limit to bind, which directly limits the number of wealth states that one must keep track of (see [Challe and Ragot, 2014](#) for further discussion of this point).

together these two assumptions imply that the cross-sectional distribution of wealth among workers has finite support (without being degenerated), while the Euler equation of a typical worker has exactly the same form as with a continuous-support distribution.

One important feature of our empirical approach is the incorporation, at the estimation stage, of cross-sectional information available at the business-cycle frequency. In particular, the model generates a rich cross-sectional distribution of consumption, as a consequence of heterogeneity in wealth and income. The empirical counterpart of this cross-sectional distribution is the consumption shares of the households by income quantiles, which can be measured at the business-cycle frequency (from the U.S. Consumption Expenditure Survey). We treat these consumption shares as time series in our likelihood-based estimation, next to the usual time series for macroeconomic aggregates (consumption, investment), monetary variables (inflation and the nominal interest rate) and labor market transitions rates (i.e., the job-finding and job-loss rates). We also make use of unconditional moments of cross-sectional data (for example, the average fall in consumption experienced by individuals transiting into unemployment) to discipline the steady state of the model. Our framework and empirical approach are thus a first step towards incorporating cross-sectional information into the estimation of Dynamic Stochastic General Equilibrium models. At present, these models typically feature perfect insurance, implying that cross-sectional information is irrelevant. Our framework naturally overcomes this limitation.

Once the joint posterior distribution of the structural parameters of the model has been recovered, we ask: has precautionary wealth mattered in the propagation of the recent US recessions, including the Great Recession? In these instances, has the aggregate demand effect of time-varying precautionary savings dominated the aggregate supply effect, making the precautionary motive inherently destabilizing? To answer these questions we extract the structural shocks that affected the U.S. economy during these periods and then feed them into a counterfactual perfect-insurance model, wherein the precautionary motive for holding wealth is turned off by construction. For the Great Recession, we find evidence of a powerful feedback loop between idiosyncratic unemployment risk and consumption demand, so that the aggregate demand effect of time-varying precautionary saving largely dominates the aggregate supply effect: not only does aggregate consumption fall a lot more with than without the precautionary motive, but idiosyncratic unemployment risk also rises much more, that is, low aggregate demand feeds back to labor market conditions. We find qualitatively similar, though quantitatively less important, amplifying effects of time-varying precautionary savings during the 1990–1991 recession. In contrast, we find no evidence of strong aggregate demand effects of precautionary savings during the 2001 recession – if anything the supply effects may have slightly dominated, that is, we observe marginally less aggregate volatility

without than with the precautionary motive). The general conclusion that comes out of this analysis is that both the aggregate demand and supply effects of time-varying precautionary savings are likely to matter quantitatively. When the demand effect is ignored, the potentially destabilizing role of the precautionary motive is entirely missed. When, on the contrary, the supply effect is ignored, this destabilizing role is likely to be overstated.



## 1. INTRODUCTION

How do fluctuations in households' precautionary wealth contribute to the propagation of aggregate shocks? In this paper, we attempt to answer this question by formulating and estimating a tractable structural macroeconomic model in which the precautionary motive for holding wealth is operative and its quantitative impact on the business cycle can be precisely evaluated. The framework that we propose is designed to be sufficiently rich to incorporate the main theoretical channels through which precautionary wealth and its variations over time may affect aggregate outcomes, but also sufficiently simple to accommodate the variety of state variables and shocks that are necessary for a likelihood-based estimation of the underlying parameters.

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Our analysis relates to several strands of the business cycle literature. Sticky-price models emphasize the role of aggregate demand as a key driver of the business cycle (see, e.g., [Christiano, Eichenbaum and Evans, 2005](#); [Galí, 2010](#); [Smets and Wouters, 2007](#); [Woodford, 2003](#)). These models have recently been extended to incorporate labor market frictions and involuntary unemployment – see [Blanchard and Galí \(2010\)](#); [Gertler, Sala and Trigari \(2008\)](#); [Heer and Maussner \(2010\)](#); [Leduc and Liu \(2014\)](#); [Trigari \(2009\)](#); [Walsh \(2005\)](#), and [Galí \(2010\)](#) for a survey. We relax the perfect-insurance assumption from this framework, so that households' savings depends on the idiosyncratic unemployment risk that they face.

[Krusell, Mukoyama and Sahin \(2010\)](#), [Nakajima \(2012\)](#) and more recently [Kehoe, Midrigan and Pastorino \(2014\)](#) analyze imperfect-insurance, heterogenous-agent models with search frictions wherein the idiosyncratic unemployment risk faced by households is endogenized through firms' job creation policy. These models assume flexible prices, implying that only the aggregate supply effect of time-varying precautionary saving is operative (while in our model the supply effect competes with the demand effect).

Other papers combine nominal frictions with imperfect insurance, but like [Krusell and Smith \(1998\)](#) treat labor-market flows as exogenous constraints on labor supply; this, by construction rules out any feedback from aggregate demand to unemployment risk, which is the key amplification mechanism in our model. This class of models includes [Guerrieri and Lorenzoni \(2011\)](#), which studies the impact of a tightening of the borrowing constraint, as well as [Oh and Reis \(2012\)](#) and [McKay and Reis \(2013\)](#), which study the impact of fiscal and transfer policies.

Two papers consider the same frictions in goods, labor and asset markets as we do: [Gornemann, Kuester and Nakajima \(2012\)](#), and [Ravn and Sterk \(2013\)](#). There are important differences between these papers and ours, both in terms of focus and in terms of method. [Gornemann et al. \(2012\)](#) is concerned with the redistributive impact of monetary policy shocks. The authors thus construct an imperfect-insurance model with large-dimensional cross-sectional heterogeneity and show that an increase in the policy rate raises income and wealth inequalities, consistent with the empirical findings of [Coibion, Gorodnichenko, Kueng and Silvia \(2012\)](#). [Ravn and Sterk \(2013\)](#) studies how an exogenous shock to the job separation rate can explain the depth and length of the Great Recession. The latter paper illustrates the feedback loop between unemployment risk and aggregate

demand but it has no capital and hence the aggregate supply effect of precautionary savings is shut down. In contrast to both contributions, we allow a variety of structural shocks (including, but not only, the monetary policy shock or the separation shock) to affect aggregate dynamics.

Our interest in the aggregate demand effect of time-varying precautionary savings is shared by several recent theoretical contributions, most notably [Rendhal \(2014\)](#), [Beaudry, Galizia and Portier \(2014\)](#) and [Heathcote and Perri \(2014\)](#). [Rendhal \(2014\)](#) shows how a zero lower bound problem coupled with labor market frictions and rigid nominal wages can cause the economy to fall into a liquidity trap. [Beaudry et al. \(2014\)](#) shows that when the economy has excess capital then precautionary savings against unemployment risk and may cause a demand shortage. Their approach is closely related to [Heathcote and Perri \(2014\)](#), which shows that the feedback loop between aggregate demand and unemployment risk may lead to multiple equilibria. Like [Beaudry et al. \(2014\)](#) and [Heathcote and Perri \(2014\)](#), our paper focuses on the interactions between households' wealth and unemployment risk, although the specific mechanism by which this occurs in our model is different from theirs and is embedded into the standard sticky-price framework. Moreover, in contrast to all three papers, we use our structural model not only for theoretical investigation, but also to extract the strength of the unemployment risk-aggregate demand feedback loop from the data.

The rest of the paper is organized as follows. Section 2 describes the model, from agents' behavior to the definition of the recursive equilibrium. Section 3 shows how our assumptions lead a collapse of the dimension of the state space, while preserving the precautionary motive. Section 4 takes the model to the data and evaluates its empirical performance. Section 5 discusses our counterfactual experiment and investigates the amplifying role of the precautionary motive during the last U.S. recessions. Section 6 offers some concluding remarks.

## 2. THE MODEL

**2.1. Model Overview.** The model introduces imperfect insurance against (time-varying) idiosyncratic unemployment risk into a quantitative "New Keynesian" model with labor market frictions. There are two household types: "workers" and "firm owners". All households participate in a market for one-period nominal bonds, supply labor when employed, and transit between employment and unemployment. However, only firm owners own the capital stock as well as all firms. Idiosyncratic unemployment risk is time-varying and cannot be perfectly insured by workers, who also face a borrowing constraint (as in, e.g., [Krusell and Smith, 1998](#)). Such financial frictions will

motivate workers' accumulation of precautionary wealth. Hence, to the extent that the idiosyncratic unemployment risk is time-varying, so will be the amount of precautionary wealth in the economy.

The production side has four types of firms, in the spirit of, e.g., [Trigari \(2009\)](#) or [Heer and Maussner \(2010\)](#). Labor intermediaries hire labor from the households in a market with matching frictions (modeled as in [Mortensen and Pissarides, 1994](#)) and transform it into labor services. Competitive wholesale good firms buy labor and capital services to produce wholesale goods that are then used as inputs by intermediate goods firms. Every intermediate good firm is the monopolistic supplier of the differentiated good it produces, but faces [Calvo \(1983\)](#)-type nominal frictions when setting nominal prices (as in, e.g., [Christiano et al., 2005](#); [Smets and Wouters, 2007](#)). Finally, intermediate good firms sell their goods to a competitive final good sector, which aggregates them into a single final good used for consumption, investment, as well as utilization and vacancy posting costs. The final good serves as the numeraire, in which all prices are expressed. A Central Bank determines the nominal interest rate via a Taylor-like rule.

The timing of events within a period is as follows (see [Figure 1](#)). A period is divided into three stages: the "labor market transitions", "production", and "consumption-saving" stages. In the first stage, after the innovations to the exogenous aggregate state have been revealed, some existing employment relationships are destroyed, then hiring decisions are made and new relationships are formed.<sup>3</sup> In the production stage, production takes place and total income is shared between the agents in the following forms: net wages (for employed workers and firm owners), unemployment benefits (for unemployed workers and firm owners), and capital income and monopolistic rents (for firm owners, whether employed or unemployed). Finally, households' consumption and savings are determined in the last stage.

We present the model recursively and use primes to denote next period's values. We call the aggregate state  $X$ , a vector containing all the relevant aggregate state variables in the model. We assume that all agents know the current value of  $X$  as well as its law of motion  $X' = \Gamma(X, \epsilon')$ , where  $\epsilon'$  is the innovation to the exogenous aggregate state (which is assumed to be first-order Markovian). Henceforth, all expectation operators are conditional on  $X$ . For expositional clarity, we summarize the content of  $X$  only in [Section 2.6](#) below, after the presentation of the model has been completed. We first present the behavior of the households ([Section 2.2](#)), then that of the firms ([Section 2.3](#)), and finally turn to the market clearing conditions ([Section 2.5](#)) and the definition of the equilibrium ([Section 2.6](#)). In the recursive presentation of the model, all variables either belong

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<sup>3</sup>Our timing assumption (the standard one for quarterly models) allows a separated worker to be re-matched immediately with positive probability, in which case the worker does not experience unemployment.

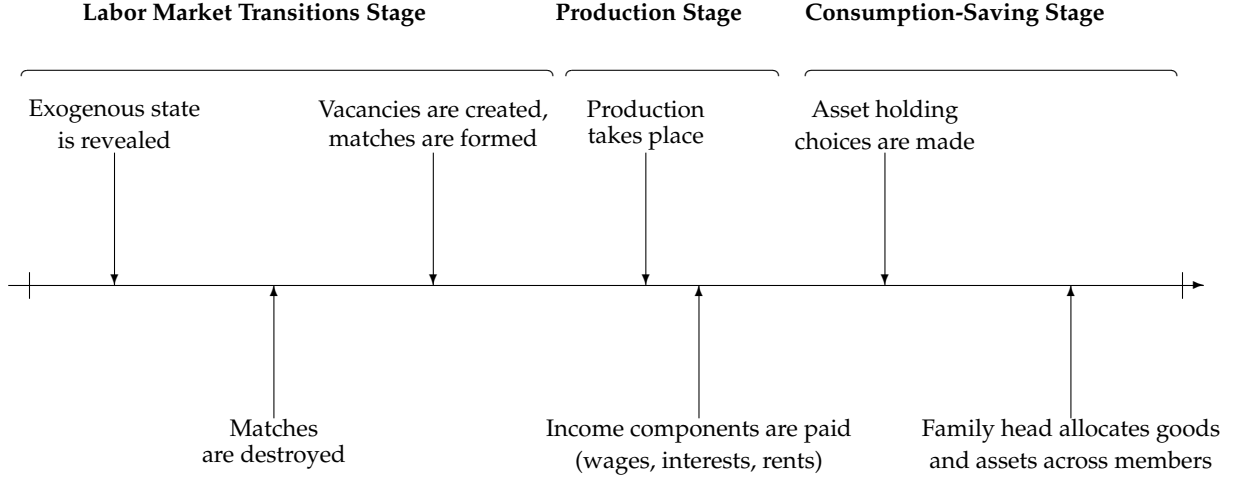


FIGURE 1. Model timeline within a period

to  $X$  or are a function of  $X$ . To save on notation when presenting the model we will only make this explicit for the value and policy functions. The rest of the relationships will be clarified when describing the equilibrium.<sup>4</sup>

**2.2. Households.** There is a unit mass of households, each of whom is endowed with one unit of labor that is supplied inelastically during the production stage, if the household is employed at the end of the labor market transitions stage. All households are subject to idiosyncratic changes in their employment status: a share  $f$  (resp.  $s$ )  $\in [0, 1]$  of the households who are unemployed (resp. employed) at the beginning of the labor market transitions stage will be employed (resp. unemployed) at the end of that stage. We refer to  $f$  and  $s$  as the “job-finding” and “job-loss” rates.

There are two types of households: there is a measure  $\Omega \in [0, 1]$  of “workers” (indexed by  $W$  henceforth) and a measure  $1 - \Omega$  of “firm owners” (indexed by  $F$ ). Workers and firm owners have subjective discount factor  $\beta^W$  and  $\beta^F$ , respectively, and firm owners are more patient than workers:  $0 < \beta^W < \beta^F < 1$ . All households have the same period utility function  $u(c - hc)$ , where  $c$  is consumption,  $c$  is the level of consumption habits,  $h \in (0, 1)$  a constant habit parameter, and where  $u(\cdot)$  satisfies  $u'(\cdot) > 0$ ,  $u''(\cdot) < 0$ . Habits are external and defined as follows. We let  $c^F$  be the common consumption habit of firm owners in the current period and it is assumed to be equal to the average consumption of firm owners in the previous period. Regarding workers, we let  $c^W(\aleph)$  denote the habit level of workers in the current period having been continuously unemployed for  $\aleph \in \{0, 1, \dots\}$  periods. It is assumed to be equal to the average consumption of workers having experienced the same number of consecutive periods of unemployment ( $= \aleph$ )

<sup>4</sup>In this section, we only describe the households and firms problems, as well as aggregation and the market-clearing conditions. See the technical Appendix for further details on first-order conditions and how to obtain the aggregation conditions.

in the previous period. For example,  $\mathbf{c}^W(0)$  is the habit level of currently employed workers and it is equal to the last period average consumption of employed workers. Similarly  $\mathbf{c}^W(1)$  is the consumption habit of an unemployed workers who was employed in the previous period, and it is equal to the average consumption of those workers who had lost their job in the previous period, and so on. This implies that all workers with the same  $\aleph$  share the same habit level while two workers with different  $\aleph$ s in general have different habit levels.

2.2.1. *Workers.* The only assets that workers can trade are one-period nominal bonds. We let  $\tilde{\mu}(a, \aleph)$  denote the cross-sectional distribution of workers over individual bond wealth  $a$  and length of unemployment spell  $\aleph$  at the beginning of the labor market transitions stage. That is,  $\tilde{\mu}(a, \aleph)$  is the share of workers with nominal bonds holdings less than or equal to  $a$  and having experienced exactly  $\aleph \geq 0$  consecutive periods of unemployment at that point in time. We refer to this distribution as the “cross-sectional wealth distribution” for short. This distribution satisfies  $\sum_{\aleph \geq 0} \int_a d\tilde{\mu}(a, \aleph) = 1$ . We let  $\mu(a, \aleph, X)$  characterize the same distribution at the end of labor market transitions stage. Finally, we let  $\tilde{\mathbf{n}}^W \equiv 1 - \sum_{\aleph \geq 1} \int_a d\tilde{\mu}(a, \aleph)$  and  $\mathbf{n}^W = f(1 - \tilde{\mathbf{n}}^W) + (1 - s)\tilde{\mathbf{n}}^W$  denote the workers’ employment rates at the beginning and the end of labor market transitions stage, respectively. Obviously we have  $\mathbf{n}^W = \tilde{\mathbf{n}}^{W'}$ , i.e., employment at the end of the current-period labor market transition stage is the same as employment at the beginning of same stage in the next-period.

Employed workers earn the net labor income  $(1 - \tau)w$ , where  $w$  is the real wage received by workers and  $\tau$  the social contribution rate. Unemployed workers earn the unemployment benefit  $b^u e^z$ , where  $b^u$  is a constant and where  $e^z$  is the economy’s stochastic trend (to be specified later). The unemployment insurance scheme is balanced in every period, i.e.

$$\tau w \mathbf{n}^W = b^u e^z (1 - \mathbf{n}^W). \quad (1)$$

We now turn to the two basic assumptions which, as we will show below, will keep the cross-sectional distribution of wealth  $\tilde{\mu}(\cdot)$  endogenously finite. The first assumption ensuring tractability concerns the extent of risk sharing.

**Assumption 1.** *Every employed worker (for whom  $\aleph = 0$ ) belongs to a “family” whose head pools resources and allocate consumption goods and nominal bonds across family members so as to maximize the family’s intertemporal utility. When an employed worker falls into unemployment, the worker leaves the family with his or her assets and is taken charged of by the unemployment insurance scheme. When an unemployed worker finds a job, the worker is randomly reallocated to one of the families and, again, pools resources with its members.*



There is a measure  $\Omega$  of such families, each with their own employment level or “size”. We let  $\tilde{n}^W$  and  $n^W$  denote, respectively, the size of a given family before and after the labor market transitions stage.<sup>5</sup> As will become clear in Section 3, Assumption 1 implies that at the behavior of an employed worker is characterized by the same Euler equation as in a standard imperfect-insurance model (e.g., Krusell and Smith, 1998), except that there is a single type of employed worker – instead of infinitely many types. The second assumption ensuring tractability is about how much debt a worker can take.

**Assumption 2.** *Workers’ bonds holdings cannot fall below the borrowing limit  $\underline{a}e^z$ , where  $\underline{a}$  is a negative constant, and this borrowing limit is strictly tighter than the natural limit.<sup>6</sup>*

Intuitively, under Assumption 2, a worker who remains continuously unemployed will eventually face a binding borrowing limit in finite time – rather than decumulate wealth indefinitely.

The key choice that workers make is how much to consume and save, and this takes place at the beginning of the consumption-saving stage, hence after labor market transitions have taken place. We denote by  $V^u(a^u, \aleph, X)$  the intertemporal utility of a worker who is unemployed after the labor market transition stage and has nominal bonds  $a^u$  and unemployment duration  $\aleph \geq 1$  when the aggregate state is  $X$ . Similarly, we denote by  $V^e(n^W, A^e, X)$  the value function for a family of employed workers of size  $n^W$  and total wealth  $A^e$  when the aggregate state is  $X$ . Since all workers face the same labor market transition rates, the law of motion for  $n^W$  is  $n^{W'} = f'(1 - n^W) + (1 - s')n^W$  and we have that  $n^W = \tilde{n}^{W'}$ .

*Unemployed workers.* The intertemporal utility for an unemployed worker is:

$$V^u(a^u, \aleph, X) = \max_{a^{u'}, c^u} \left\{ u(c^u - hc^W(\aleph)) + \beta^W \mathbb{E}_X \left[ (1 - f')V^u(a^{u'}, \aleph + 1, X') + f' \frac{V^e(n^{W'}, A^{e'}, X')}{n^{W'}} \right] \right\}, \quad (2)$$

where  $c^u$  is the unemployed worker’s consumption and  $\mathbb{E}_X$  denotes the expectations operator. Notice that because  $V^e(n^W, A^e, X)$  is the value function for the whole family, it must be divided by the size of the family,  $n^W$ , to find its per-member analogue. Moreover, because a worker is atomistic relative to the size of a family, the bonds holding choice of an unemployed worker,  $a^{u'}$ , does not affect the intertemporal utility of the family that he may join (formally,  $\partial A^{e'} / \partial a^{u'} = 0$ ).

<sup>5</sup>In other words,  $\tilde{n}^W$  and  $n^W$  are the family level counterparts of the economy-wide employment rates  $\tilde{\mathbf{n}}^W$  and  $\mathbf{n}^W$ .

<sup>6</sup>Loosely speaking, the natural debt limit is the level of debt that a worker would be able to repay for certain, i.e., even if this worker stayed unemployed forever (see, e.g., Aiyagari, 1994). In our model, provided that aggregate shocks have sufficiently small magnitude, then a sufficient condition for the debt limit to be tighter than the natural limit is that  $\underline{a} > -b^u \beta^F / (1 - \beta^F)$ .

The worker solves (2), taking as given the law of motion for the aggregate state variables and subject to the following budget and borrowing constraints:

$$a^{u'} + c^u = b^u e^z + (1 + r)a^u \text{ and } a^{u'} \geq \underline{a}e^z, \quad (3)$$

where

$$1 + r = (1 + \mathbf{R}_{-1})e^{\varphi_c} / (1 + \pi). \quad (4)$$

In the latter expression,  $\mathbf{R}_{-1}$  is last-period nominal policy rate,  $\pi$  is realized inflation, and  $\varphi_c$  ( $\in X$ ) a “risk premium” shock that drives a wedge between the gross nominal policy rate and the actual gross nominal return on bonds held by households (see, e.g., [Smets and Wouters, 2007](#)). The solution to the problem of an unemployed worker is made of the optimal policy functions for nominal bonds and consumption, i.e.,  $a^{u'} = g_{a^u}(a^u, \aleph, X)$  and  $c^u = g_{c^u}(a^u, \aleph, X)$ .

*Employed workers.* As mentioned above the value function for a family of employed workers features as a family-level state variable its wealth at the beginning of the consumption-saving stage,  $A^e$ . The choice variable of the family head is  $a^{e'}$ , i.e., the per-family member wealth at the end of the consumption-saving stage. The transition from  $a^{e'}$  to  $A^e$  must take into account the wealth flows into and out of the family that will take place during the labor market transitions stage of the next period. During that stage,  $s'n^W$  family members will loose their job, taking out a total amount of wealth of  $s'n^W a^{e'}$ . At the same time, some currently unemployed workers will join the family, bringing into the family a total wealth inflow denoted  $B'$ . It follows that

$$A^e = (1 - s')n^W a^{e'} + B', \quad (5)$$

where  $B$  is given by  $B = f \sum_{\aleph \geq 1} \int_a a d\tilde{\mu}(a, \aleph)$ . The latter expression means that the current-period wealth inflow aggregates the wealth of job finders, who are randomly drawn from the workers who are unemployed at the beginning of the labor market transition stage. By the law of large numbers the distribution of workers experiencing this transition is the same across families, so  $B$  is the same for all families.

The intertemporal utility of a family of employed workers is:

$$V^e(n^W, A^e, X) = \max_{a^{e'}, c^e} \{n^W u(c^e - hc^W(0)) + \beta^W \mathbb{E}_X [V^e(n^{W'}, A^{e'}, X') + s'n^W V^u(a^{e'}, 1, X')]\}, \quad (6)$$

where  $c^e$  is the per family member consumption. Note that the family head values the potential utility loss that current members may experience in the next period if they fall into unemployment. This is precisely what motivates the accumulation of wealth for precautionary purpose. There will be  $s'n^W$  such members, hence the corresponding weight before  $V^u(a^{e'}, 1, X')$ .

The family head solves (6), taking as given the laws of motions for the aggregate ( $X$ ) and family-level ( $(n^W, A^e)$ ) state variables, and subject to the following budget and borrowing constraints:

$$n^W(a^{e'} + c^e) = (1 - \tau)wn^W + (1 + r)A^e \text{ and } a^{e'} \geq \underline{a}e^z. \quad (7)$$

The solution to this problem is summarized by the policy functions  $a^{e'} = g_{a^e}(n^W, A^e, X)$  and  $c^e = g_{c^e}(n^W, A^e, X)$ .

2.2.2. *Firm owners.* Besides earning either wages or unemployment benefits and participating in the bond market like the workers, firm owners rent out capital and own all the firms (and thus receive all profits). In contrast to the workers, we assume that there is perfect insurance among the firm owners. More specifically, we assume that every firm owner belongs to a family, whose head pools resources and allocate consumption goods and assets so as to maximize the intertemporal utility of all the family members. There is a measure  $1 - \Omega$  of such families, and each family is now of measure one: it includes both employed and unemployed firm owners at every point in time (i.e., firm owners never leave their family). It follows that all firm owners within a family consume and save the same, regardless of their employment status. The fraction of employed members within every family of firm owners before and after the labor market transitions stage are denoted by  $\tilde{n}^F$  and  $n^F$ , respectively. Just as with the workers we have  $n^{F'} = f'(1 - n^F) + (1 - s')n^F$  and  $n^F = \tilde{n}^{F'}$ . As before, these are family-level variables. The corresponding aggregate variables are denoted  $\tilde{\mathbf{n}}^{F'}$  and  $\mathbf{n}^F$ .

The intertemporal utility of a family of firm owners at the beginning of the consumption-saving stage is given by:

$$V^F(n^F, k, a^F, i, X) = \max_{a^{F'}, c^F, i', v, k'} \{u(c^F - hc^F) + \beta^F \mathbb{E}_X[V^F(n^{F'}, k', a^{F'}, i', X')]\},$$

where  $k$  and  $a^F$  are the family's capital and bonds wealth,  $i$  its investment in the previous period (which enters  $V^F(\cdot)$  due to the presence of investment adjustment costs),  $c^F$  is consumption, and  $v \in [0, 1]$  is the capital utilization rate. Firm owners face the budget constraint:

$$c^F + i' + a^{F'} = w^F n^F + [r_k v - \eta(v)]k + (1 + r)a^F + Y, \quad (8)$$

where  $w^F$  is the real wage earned by firm owners,  $r_k$  the real rental rate of capital services and  $Y$  denotes profits from intermediate goods firms and labor intermediaries, both rebated to their owners as dividends (measured in units of the final good).<sup>7</sup> The function  $\eta(v)k$  is the real cost that

<sup>7</sup>Ownership of the firms here takes the form of a fully diversified portfolio of private (hence untraded) equity. We could equivalently allow the flows of rents to be traded among firm owners.

the utilization rate entails in units of the final good, where  $\eta(v)$  is such that  $\eta(1) = 0$ ,  $\partial\eta(v)/\partial v > 0$  and  $\partial^2\eta(v)/\partial^2v > 0$ . Finally, the capital stock evolves as:

$$k' = (1 - \delta)k + e^{\varphi_i}(1 - S(i'/i))i', \quad (9)$$

where  $\delta \in [0, 1]$  is the depreciation rate,  $\varphi_i$  is an investment-specific shock and  $S(\cdot)$  an investment adjustment cost function satisfying  $\partial^2S(\cdot)/\partial(i'/i)^2 > 0$  and  $S(g_i) = \partial S(\cdot)/\partial(i'/i)|_{i'/i=g_i} = 0$ , where  $g_i$  is the steady-state value of  $i'/i$ .

The family head maximizes intertemporal utility subject to (8)–(9) and taking as given the laws of motions for  $X$  and  $n^F$ . We focus on a symmetric equilibrium. Hence, all families of firm owners are identical, i.e. the cross-sectional distribution of families over the family-level state vector  $(n^F, k, a^F, i)$  is degenerate. We may thus write the optimal policy functions as functions of  $X$  only, i.e.,  $a^{F'} = g_{a^F}(X)$ ,  $c^F = g_{c^F}(X)$ ,  $i' = g_i(X)$ ,  $v = g_v(X)$ , and  $k' = g_k(X)$ . Importantly, the homogeneity of firm owners implies that a single pricing kernel serves to price all future profits paid by the firms in the economy. This pricing kernel is given by:

$$M^{F'} = \beta^F \frac{u'(c^{F'} - hc^F)}{u'(c^F - hc^F)}, \quad (10)$$

where we have used the fact that  $c^F = g_{c^F}(X) = \mathbf{c}^{F'} \in X'$  (i.e., current consumption determines next period's habit).

**2.3. Firms.** There are four types of firms in the economy, each forming a continuum of unit mass.

**2.3.1. Final goods firms.** The final good  $y$  is produced by a continuum of identical and competitive firms that combine intermediate differentiated goods according to the production function:

$$y = \left( \int_0^1 y_\zeta^{(\theta-1)/\theta} d\zeta \right)^{\theta/(\theta-1)}, \quad (11)$$

where  $y_\zeta$  is the input of intermediate good  $\zeta$  and  $\theta > 1$  is the cross-partial elasticity of substitution between any two intermediate goods.

Let  $p_\zeta$  denote the price of intermediate good  $\zeta$  in terms of the final good. This price is taken as given by the final-good firms. The program of the representative final-good producer is thus:

$$\max_{y, y_\zeta} y - \int_0^1 p_\zeta y_\zeta d\zeta, \quad (12)$$

subject to (11). From the optimal choices of final-good firms, one can deduce the demand function for the intermediate-good firms,  $\zeta \in [0, 1]$ :

$$y_\zeta(p_\zeta) = p_\zeta^{-\theta} y. \quad (13)$$

The zero-profit condition for final-good producers implies that:

$$\left( \int_0^1 p_\zeta^{1-\theta} d\zeta \right)^{1/(1-\theta)} = 1. \quad (14)$$

2.3.2. *Intermediate goods firms.* The intermediate-good sector is imperfectly competitive. Intermediate-good firm  $\zeta \in [0, 1]$  is the monopolistic supplier of the good it produces, which it does by means of a linear production function with a fixed cost:

$$y_\zeta = x_\zeta - \kappa_y e^z, \quad (15)$$

where  $x_\zeta$  is the quantity of wholesale goods used in production and  $\kappa_y e^z$  is the fixed cost measured in units of the wholesale good. The term  $e^z$  is included to ensure the existence of a BGP. Firm  $\zeta$ 's current profit, measured in units of the final good, is given by:

$$\Xi = (p_\zeta - p_m)y_\zeta - p_m \kappa_y e^z,$$

where  $p_m$  is the price of wholesale goods in term of the final good (which is taken as given by intermediate-good firms).

Firm  $\zeta$  chooses  $p_\zeta$  to maximize the present discounted value of future profits, taking as given the demand curve (13). Following Calvo (1983), we assume that in every period every intermediate-good firm can be in one of the following two idiosyncratic states: either the firm can freely reoptimize its price, or it cannot and simply rescales the existing price according to the indexation rule:

$$p_\zeta = \frac{(1 + \bar{\pi})^{1-\gamma_p} (1 + \pi_{-1})^{\gamma_p}}{1 + \pi} p_{\zeta,-1}. \quad (16)$$

where  $\gamma_p \in (0, 1)$  measures the degree of indexation to the most recently available final good inflation measure,  $\pi_{-1}$  is final good inflation in the previous period,  $\bar{\pi}$  is the steady-state inflation rate, and  $p_{\zeta,-1}$  is last period's relative price of intermediate good  $\zeta$ .<sup>8</sup> The ex ante probability of each firm being able to reoptimize the price in the next period is constant and equal to  $1 - \alpha \in [0, 1]$ , irrespective of the time elapsed since the period in which the price of the firm was last revised.

It follows from this price adjustment mechanism that the behavior of a firm can be described by two Bellman equations, corresponding to the two idiosyncratic states in which the firm can be. The value of a firm that is allowed to reset its price is given by  $V^R(X)$  and only depends on the aggregate state. The value of a firm not allowed to reset its selling price and with last period's price  $p_{\zeta,-1}$  is denoted as  $V^N(p_{\zeta,-1}, X)$ . The corresponding Bellman equations are:

$$V^R(X) = \max_{p_\zeta} \{ \Xi + \alpha \mathbb{E}_X [M^{F'} V^N(p_\zeta, X')] + (1 - \alpha) \mathbb{E}_X [M^{F'} V^R(X')] \} \text{ and}$$

<sup>8</sup>This indexation rule is simply the standard rule  $P_\zeta = (1 + \bar{\pi})^{1-\gamma_p} (1 + \pi_{-1})^{\gamma_p} P_{\zeta,-1}$ , written in terms of final good units (where the  $P_\zeta$ s are nominal prices).

$$V^N(p_{\zeta-1}, X) = \Xi + \alpha \mathbb{E}_X[M^{F'} V^N(p_{\zeta}, X')] + (1 - \alpha) \mathbb{E}_X[M^{F'} V^R(X')],$$

taking as given the demand curve (13) and where  $p_{\zeta}$  is given by (16) (i.e., is not the result of an optimization by the firm). We let  $p^* = g_{p^*}(X)$  denote the corresponding policy function.

We focus on a symmetric equilibrium wherein the solution to intermediate goods firms' problem is the optimal reset price common to all price resetting firms. After straightforward algebraic manipulations, the first-order conditions associated with the determination of the optimal reset price is  $p^* = \frac{K}{F}$ , where  $K$  and  $F$  are defined recursively as follows

$$K = \mu e^{\varphi_p} p_m y + \alpha \mathbb{E}_X \left[ M^{F'} \left( \frac{1 + \pi'}{(1 + \bar{\pi})^{1-\gamma_p} (1 + \pi)^{\gamma_p}} \right)^{\theta} K' \right] \text{ and}$$

$$F = y + \alpha \mathbb{E}_X \left[ M^{F'} \left( \frac{1 + \pi'}{(1 + \bar{\pi})^{1-\gamma_p} (1 + \pi)^{\gamma_p}} \right)^{\theta-1} F' \right],$$

where  $\mu = \theta/(1 - \theta)$  and where we allow for exogenous variations in the markup through the shock  $\varphi_p \in X$ . This optimal reset price, together with the Calvo price setting mechanism, the zero profit condition (14), and the indexation rule (16), imply the following law of motion for inflation:

$$\pi = \frac{\alpha((1 + \bar{\pi})^{1-\gamma_p} (1 + \pi_{-1})^{\gamma_p})}{(1 - (1 - \alpha)(p^*)^{1-\theta})^{1/(1-\theta)}} - 1. \quad (17)$$

The price-setting mechanism generates a cross-sectional distribution of prices, since the selling price of a firm not reoptimising its price depends on the time that has elapsed since the last time the price was reset. However, the price dispersion index  $\Lambda \equiv \int_0^1 p_{\zeta}^{-\theta} d\zeta$  suffices to capture the relevant properties of the distribution, and it evolves according to the law of motion:

$$\Lambda = (1 - \alpha)(p^*)^{-\theta} + \alpha \left( \frac{(1 + \bar{\pi})^{1-\gamma_p} (1 + \pi_{-1})^{\gamma_p}}{1 + \pi} \right)^{-\theta} \Lambda_{-1}, \quad (18)$$

where  $\Lambda_{-1}$  is the value of the index in the previous period.

**2.3.3. Wholesale goods firms.** The wholesale good is produced by a continuum of identical and competitive firms. The representative wholesale-good firm produces with the technology  $y_m = \check{k}^{\phi} (e^z \check{n})^{1-\phi}$ ,  $\phi \in (0, 1)$ , where  $\check{n}$  and  $\check{k}$  denote labor and capital services. It solves the following program:

$$\max_{\check{n}, \check{k}} \{ p_m \check{k}^{\phi} (e^z \check{n})^{1-\phi} - Q \check{n} - r_k \check{k} \}, \quad (19)$$

where  $Q$  is the real unit price of labor services. The solution to (19) gives the optimal demands for factor services  $\check{n} = g_{\check{n}}(X)$  and  $\check{k} = g_{\check{k}}(X)$ .

2.3.4. *Labor intermediaries and labor market flows.* Labor services are sold to wholesale goods firms by labor intermediaries, which hire labor from households in a market with search frictions. Our timing convention about job destruction and creation is as in [Walsh \(2005\)](#), [Galí \(2010\)](#) and many others. More specifically, at the beginning of the labor market transition stage, a fraction  $\rho = \rho(\varphi_s)$  of existing employment relationships are destroyed, where  $\varphi_s$  is a job destruction shock. The workers who loose their job on that occasion enter the unemployment pool, where they join the workers who where already unemployed at the end of the previous period.

After job destruction has taken place, labor intermediaries post vacancies, at the unit cost  $\kappa_v e^z$  in terms of the final goods. As before, the term  $e^z$  is included to ensure a BGP. One employed worker provides one unit of labor services but a firm owner provides  $\psi > 1$  units of labor services. We call this parameter the “skill premium”.<sup>9</sup> The the wage paid by a labor intermediary to a firm owner is  $w^F = \psi w$ . The values to the labor intermediary of a match with a worker and a firm owner are, respectively:

$$J^e = Q - w + \mathbb{E}_X[(1 - \rho')M^{F'}J^{e'}] \text{ and } J^F = \psi(Q - w) + \mathbb{E}_X[(1 - \rho')M^{F'}J^{F'}]. \quad (20)$$

We note that  $J^F = \psi J^e$ , i.e., the value of a match with a firm owner is proportional to that with a worker (with a coefficient of proportionality equal to the skill premium). We assume that, when posting a vacancy, labor intermediaries cannot target a particular skill type. Labor intermediaries thus adjust vacancies until the expected payoff on a posted vacancy is equal to its cost, i.e.,

$$\lambda[\Omega J^e + (1 - \Omega)J^F] = \kappa_v e^z, \quad (21)$$

where  $\lambda$  is the economy-wide vacancy-filling rate.

Let  $\tilde{\mathbf{n}} = \Omega \tilde{\mathbf{n}}^W + (1 - \Omega)\tilde{\mathbf{n}}^F$  denote the economy-wide employment rate before the labor market transitions stage and  $\mathbf{n} = \Omega \mathbf{n}^W + (1 - \Omega)\mathbf{n}^F$  the same rate after the labor market transitions stage. These two definitions imply that  $\tilde{\mathbf{n}}' = \mathbf{n}$ . The unemployment pool is made of workers who are unemployed at the beginning of the labor market transitions stage (in number  $1 - \tilde{\mathbf{n}}$ ) as well as workers who where employed at the beginning of that stage but lose their job after the job destruction shock (in number  $\rho \tilde{\mathbf{n}}$ ). The matching technology produces  $m$  employment relationships using as inputs the unemployment pool and the aggregate number of vacancies  $v$ . This technology has the form:

$$m = \bar{m}(1 - (1 - \rho)\tilde{\mathbf{n}})^\chi v^{1-\chi}, \quad (22)$$

<sup>9</sup>The extent of consumption dispersion across U.S. households cannot be entirely accounted for by wealth: dispersion in labor income is needed in addition to dispersion in asset income. This is adequately captured by a skill premium (see, e.g., [Challe and Ragot, 2014](#)).

where  $\bar{m}$  is a scaling parameter and  $\chi \in (0, 1)$  the elasticity of match production with respect to the size of the unemployment pool. Accordingly, the economy-wide job-finding and vacancy-filling rates are given by, respectively:

$$f = \frac{m}{(1 - (1 - \rho)\bar{\mathbf{n}})} \text{ and } \lambda = \frac{m}{v}. \quad (23)$$

Note that under our timing convention the workers that are separated from their firm at the beginning of the period can be re-matched within the period (in which case they effectively do not change employment status). It follows that the period-to-period job-loss rate  $s$  is given by:

$$s = \rho(1 - f). \quad (24)$$

As usual, there are two equivalent ways of viewing labor market flows. From the point of view of the households, employment dynamics is determined by the flows of job losers and job finders, i.e.,  $\mathbf{n} = f(1 - \bar{\mathbf{n}}) + (1 - s)\bar{\mathbf{n}}$ . From the point of view of the labor intermediaries, it follows from the natural process of job destruction and the intensity of vacancy posting, i.e.,  $\mathbf{n} = (1 - \rho)\bar{\mathbf{n}} + \lambda v$ .

2.3.5. *Wages.* The presence of labor market frictions implies that there may exist a full bargaining set over which a labor intermediary and an employee (whether worker or firm owner) find it mutually profitable to be matched. However, the theory does not pin down the specific way in which the match surplus is shared among the parties (see [Hall, 2005](#)). We assume that both  $w$  and  $w^F = \psi w$  belong to the relevant bargaining sets, in the sense that when either a worker or a firm owner is matched with a labor intermediary then both parties extract a positive surplus from the match.<sup>10</sup> Moreover, we assume that there are some rigidities in nominal wage adjustment, and only use the implied expression for the real wage. It is given by:

$$w = \left( \frac{\mathbf{w}_{-1}}{1 + \pi} \right)^{\gamma_w} \left( \bar{w} e^{z + \varphi_w} \left( \frac{\mathbf{n}}{\bar{n}} \right)^{\psi_n} \right)^{1 - \gamma_w}. \quad (25)$$

Here  $\mathbf{w}_{-1}$  denotes last period's real wage rate,  $\bar{w}$  is a scale factor,  $\gamma_w$  is the degree of indexation to past wages, and  $\psi_n$  is the sensitivity of wages to the business cycle, here measured as the ratio of aggregate employment  $\mathbf{n}$  to its steady-state value  $\bar{n}$ . The wage equation is also perturbed by a wage shock  $\varphi_w$  and is appropriately scaled by the technology shock  $e^z$ , to ensure a BGP.

2.4. **Central Bank.** The Central Bank is assumed to set the nominal interest rate  $R$  according to the following rule (see, e.g., [Christiano, Motto and Rostagno, 2014](#)):

$$\log \left( \frac{1 + R}{1 + \bar{R}} \right) = \rho_R \log \left( \frac{1 + \mathbf{R}_{-1}}{1 + \bar{R}} \right) + (1 - \rho_R) \left[ a_\pi \log \left( \frac{1 + \pi}{1 + \bar{\pi}} \right) + a_y \log \left( \frac{1 + g}{1 + \bar{g}} \right) \right] + \varphi_R \quad (26)$$

<sup>10</sup>We verify ex post that this condition hold over our sample, once the full model has been estimated (see the technical Appendix for details).



where  $\bar{R}$  is the steady-state nominal interest rate,  $\rho_R \in (0, 1)$  an interest rate smoothing parameter,  $(a_\pi, a_y)$  the reaction coefficients to inflation and output growth,  $g = y/y_{-1} - 1$  the growth rate of final output, where  $y_{-1}$  is last-period final output, and  $\varphi_R$  a monetary policy shock.

## 2.5. Market clearing.

2.5.1. *Labor services.* Recall from Section 2.2 that all households face the same labor market transition rates  $(f, s)$ . Hence, in the steady state the employment rates for workers and firm owners are the same. Assuming that employment is symmetric at the beginning of the date-0 labor market transition stage, by the law of large numbers they remain symmetric at every point in time, i.e.,

$$\tilde{n}^W = \tilde{n}^F = \tilde{\mathbf{n}}^W = \tilde{\mathbf{n}}^F \equiv \tilde{\mathbf{n}} \quad \text{and} \quad n^W = n^F = \mathbf{n}^W = \mathbf{n}^F \equiv \mathbf{n}. \quad (27)$$

Because a matched firm owner provides  $\psi$  times more units of labor services than a worker, the total supply of labor services is  $\Omega \mathbf{n}^W + (1 - \Omega)\psi \mathbf{n}^F = (\Omega + (1 - \Omega)\psi)\mathbf{n}$ . It follows that clearing of the market for labor services requires:

$$(\Omega + (1 - \Omega)\psi)\mathbf{n} = \check{n}. \quad (28)$$

2.5.2. *Asset markets.* Recall that firm owners are symmetric and in measure  $1 - \Omega$ . Since each of them supplies  $vk$  units of capital services, the total supply of capital services is  $(1 - \Omega)vk$ . The market-clearing condition for capital services is thus:

$$(1 - \Omega)vk = \check{k}. \quad (29)$$

All the households may participate in the market for nominal bonds, which are in zero net supply. In symmetric equilibrium, at the end of the consumption and saving stage, every firm owners hold the same bond wealth  $a^{F'}$  and every employed worker hold the same bond wealth  $a^{e'}$ , while the bond wealth of unemployed workers varies depending on values of  $a^u$  and  $\aleph$ , i.e.,  $a^{u'} = g_{a^u}(a^u, \aleph, X)$ . Clearing of the market for bonds thus requires:

$$(1 - \Omega)a^{F'} + \Omega \mathbf{n}a^{e'} + \Omega \sum_{\aleph \geq 1} \int_a a^{u'} d\mu(a, \aleph, X) = 0. \quad (30)$$

2.5.3. *Goods markets.* The aggregate demand for final goods is made of total investment (by firm owners), the consumption of all households, as well as capital utilization costs (directly paid by firm owners) and vacancy costs (paid by the labor intermediaries). Because of symmetry, only the consumption of unemployed workers only varies across values of  $a^u$  and  $\aleph$ , i.e.,  $c^u = g_{c^u}(a^u, \aleph, X)$ . Clearing of the market for final goods requires that demand equal supply, i.e.,

$$(1 - \Omega)(c^F + i' + \eta(v)k) + \Omega \mathbf{n}c^e + \Omega \sum_{\aleph \geq 1} \int_a c^u d\mu(a, \aleph, X) + \kappa_v e^z v = y. \quad (31)$$

The intermediate-good sector demands one unit of wholesale goods for any unit of intermediate goods. Hence the market-clearing condition for the wholesale-good sector is:

$$\int_0^1 x_\zeta d\zeta = y_m = \check{k}^\phi (e^z \check{n})^{1-\phi}. \quad (32)$$

The total demand for intermediate goods by the final-good sector is  $\int_0^1 y_\zeta(X, p_\zeta) d\zeta = \Lambda y$ , where  $\Lambda$  evolves as shown in equation (18). The total supply of wholesale goods is equal to  $\int_0^1 x_\zeta d\zeta - \kappa_y e^z$ . Hence, clearing of the market for intermediate goods requires, using (32):

$$\Lambda y = \check{k}^\phi (e^z \check{n})^{1-\phi} - \kappa_y e^z. \quad (33)$$

**2.6. Aggregate state and equilibrium.** We are now in a position to summarize the content of the aggregate state.<sup>11</sup> Again, because we are focusing on a symmetric equilibrium (i.e., one where family-level variables are symmetric across firms), the aggregate state is given by:

$$X = \{\tilde{\mu}(\cdot), k, a^F, i, \mathbf{c}^F, \mathbf{c}^W(\aleph)_{\aleph \in \mathbb{N}}, a^e, \mathbf{R}_{-1}, \mathbf{\Lambda}_{-1}, \boldsymbol{\pi}_{-1}, \mathbf{y}_{-1}, \mathbf{w}_{-1}, \Phi\}, \quad (34)$$

where  $\Phi \equiv \{z, \varphi_i, \varphi_c, \varphi_s, \varphi_R, \varphi_w, \varphi_p\}$  is the vector of exogenous state variables. Note that knowledge of the cross-sectional distribution of wealth  $\tilde{\mu}(\cdot)$  allows one to compute employment at the beginning of labor market transitions stage, i.e.  $\check{\mathbf{n}} = \check{\mathbf{n}}^W = \int_a d\tilde{\mu}(a, 0)$ .

**Definition 1.** *A symmetric recursive equilibrium is a set of value and policy functions, a set of prices, and labor market flows such that:*

- (1) *Workers.* Given  $r(X), w(X), \tau(X), b^u e^z, \mathbf{c}^W(\aleph)_{\aleph \in \mathbb{N}}, f(X)$  and  $s(X)$ , the value function  $V^u(a^u, \aleph, X)$ ,  $V^e(n^W, a^e, X)$  and policy functions  $g_{a^u}(a^u, \aleph, X)$ ,  $g_{c^u}(a^u, \aleph, X)$ ,  $g_{a^e}(X)$ , and  $g_{c^e}(X)$  solve the workers' problems;
- (2) *Firm owners.* Given  $r(X), r_k(X), w^F(X), \mathbf{c}^F, Y(X), f(X)$  and  $s(X)$  the value function  $V^F(n^F, k, a^F, i, X)$  and policy functions  $g_{a^F}(X)$ ,  $g_{c^F}(X)$ ,  $g_i(X)$ ,  $g_v(X)$ , and  $g_k(X)$  solve the problem of a firm owner;
- (3) *Final goods firms.* Given  $p_\zeta, \zeta \in [0, 1]$ , the demands for wholesale goods  $y_\zeta(p_\zeta, X)$  is optimal from the point of view of final goods firms;
- (4) *Intermediate goods firms.* Given  $p_m(X), y_\zeta(p_\zeta, X)$ , and  $M^F(X, X')$ , the value functions  $V^R(X)$  and  $V^N(p_{\zeta-1}, X)$  and the reset price  $p^*(X)$  solve the problem of intermediate goods firms;
- (5) *Wholesale goods firms.* Given  $p_m(X), Q(X)$  and  $r_k(X)$ , the demand for labor and capital services  $\check{n}(X)$  and  $\check{k}(X)$  solve the problem of wholesale good firms;

<sup>11</sup>See the technical Appendix for a description of the equilibrium conditions. There, we show how one can obtain both a recursive and a sequential representations of the equilibrium. We use the second representation to solve the model.

- (6) *Labour intermediaries.* Given  $Q(X)$ ,  $w(X)$ , and  $M^F(X, X')$ , the value to the labor intermediaries of their matches with the households  $J^e(X)$  and  $J^F(X)$  are given by (20), the free entry condition (21) determines the vacancy-filling rate  $\lambda(X)$ , and  $m(X)$ ,  $f(X)$ ,  $v(X)$ , and  $s(X)$  are determined according to (22), (23), and (24);
- (7) *Profits.* The profit function  $Y(X)$  results from the optimal decision of the intermediate goods firms and the labor intermediaries.
- (8) *Social contribution rate, real interest rate, SDF, wages, and nominal interest rate.* Given  $y(X)$ ,  $\pi(X)$ , and  $b^u e^z$ , the social contribution rate  $\tau(X)$  is so that (1) holds; the real return on nominal bond holdings  $r(X)$  follows (4); the stochastic discount factor  $M^F(X, X')$  is given by (10), firm owners' wage  $w^F(X)$  is equal to  $\psi w(X)$ , where  $w(X)$  is given by (25); the nominal interest rate  $R(X)$  is given by (26); .
- (9) *Market clearing.* The market clearing conditions (28) to (33) hold.
- (10) *Laws of motion.* Given  $p^*(X)$ , inflation  $\pi(X)$  and price dispersion  $\Lambda(X)$  evolve according (17) and (18), respectively. Given  $f(X)$ ,  $s(X)$ ,  $g_{a^u}(a^u, \aleph, X)$ , and  $g_{a^e}(X)$  the law of motion of  $\tilde{\mu}(\cdot)$  is consistent with policy rules (only labor moves from  $\tilde{\mu}(\cdot)$  to  $\mu(\cdot)$ ):

$$\mu(a, 0, X) = f(X) \sum_{\aleph \geq 1} \tilde{\mu}(a, \aleph) + (1 - s(X))\tilde{\mu}(a, 0) \text{ and } \mu(a, 1, X) = s(X)\tilde{\mu}(a, 0),$$

and for  $\aleph \geq 2$  we have  $\mu(a, \aleph, X) = (1 - f(X))\tilde{\mu}(a, \aleph - 1)$ . As before, note that knowledge of  $\mu(\cdot)$  allows one to compute the end-of-labor market transitions stage employment rate  $\mathbf{n} = \mathbf{n}^W = \int_a d\mu(a, 0, X)$ . The law of motion of  $\mu(\cdot)$  is consistent with policy rules (only assets moves from  $\mu(\cdot)$  to  $\tilde{\mu}'(\cdot)$ ):

$$\tilde{\mu}'(a', 0, X) = \int_a \mathbf{1}_{a'=g_{a^e}(X)} d\mu(a, 0, X) \text{ and } \tilde{\mu}'(a', \aleph, X) = \int_a \mathbf{1}_{a'=g_{a^u}(a, \aleph, X)} d\mu(a, \aleph, X).$$

- (11) *Habits.* Given  $g_{c^F}(X)$  and  $g_{c^u}(a^u, \aleph, X)$ , tomorrow's habit level of a particular household type is equal to the average consumption of this type today, i.e.,

$$\mathbf{c}^{F'} = g_{c^F}(X) \text{ and } \mathbf{c}^{W'}(\aleph) = \int_a g_{c^u}(a, \aleph, X) d\mu(a, \aleph).$$

### 3. EQUILIBRIUM DYNAMICS

We now show how Assumption 1 and 2 allow us to simplify the cross-sectional distribution of wealth,  $\tilde{\mu}(\cdot)$ , and thereby the computation of the equilibrium. We first show that the support of  $\tilde{\mu}(\cdot)$  is finite, which will in turn imply that the aggregate state  $X$  is finite. Then, we construct an equilibrium in which this distribution has exactly two distinct states (one for employed workers, one for unemployed workers), and derive a set of sufficient conditions for this equilibrium to exist. In our empirical analysis in Section 4, we use these sufficient conditions to verify that our conjecture holds empirically in our sample (with high probability, given the uncertainty surrounding the

parameters). Finally, we show that time-varying precautionary savings are maintained under a linear approximation to employed workers' first-order condition.

**3.1. A cross-sectional distribution of wealth with finite support.** Let us begin by showing that the support of the cross-sectional distribution of wealth is finite.

**Proposition 1.** *If Assumption 1 and 2 hold, the cross-sectional distribution of wealth  $\tilde{\mu}(a, \mathbb{R})$  has a finite number of wealth states (i.e. the support of  $a$  is finite).*

This property follows directly and generically from our assumptions. Assumption 1 ensures that a worker who loses its job during the labor market transitions stage starts the consumption-saving stage with nominal bond holdings  $a^u = a^e$ . Since unemployment insurance is imperfect, this newly unemployed worker will decumulate assets during the unemployment spell in order to partly insulate consumption from the corresponding income fall. Despite this gradual decumulation, our framework is tractable because Assumption 2 implies that unemployed workers eventually hit a binding borrowing constraint in finite time; hence the number of wealth levels that one must keep track of is itself finite.

While the proposition is helpful as a general characterization of the support of the cross-sectional distribution of wealth, it does not tell us how many point the support actually has. The latter is determined by the number periods of consecutive unemployment it takes before a worker reaches the borrowing limit defined in Assumption 2. Hence, it depends on, first, the amount of bond holdings at the start of the unemployment spell, and second, on the speed at which asset decumulation takes place. Both are functions of the deep parameters of the model, and one can always set them to generate equilibria with as many wealth states as one wishes without losing tractability. However, the data imposes some discipline on these figures: the amount of wealth that workers hold (hence the initial bond wealth of a worker falling into unemployment) must be consistent with the broad features of the empirical cross-sectional distribution of wealth, and the job transition rates (which are a key determinant of both initial bond holdings and the pace of asset decumulation), must be consistent with their empirical counterparts. We will show in Section 2 that the data favors a specification where (liquid) wealth is fully liquidated after one period of unemployment, so that the cross-sectional distribution of wealth among workers has exactly two states.

**3.2. The two-wealth state equilibrium: construction and existence conditions.** We proceed by construction. First, we derive the cross-sectional distributions of wealth and consumption under the conjecture that, by the end of consumption-saving stage, the debt limit is (i) never binding for employed workers, and (ii) always binding for unemployed workers. This will imply that workers' cross-sectional wealth distribution has exactly two states. Second, we derive a set of

sufficient conditions for the conjectured equilibrium to hold. Finally, discuss how this drastically limits the dimensionality of the aggregate state.

3.2.1. *Construction.* In the equilibrium under consideration, by the end of consumption-saving stage all unemployed workers face a binding borrowing limit, so that  $a^{uu} = \underline{a}e^z$ , but none of the employed workers do, i.e.,  $a^{ee} > \underline{a}e^z$ . This implies that the support of  $a$  in  $\tilde{\mu}(a, \aleph)$  has exactly two distinct points.

Since all employed workers hold the same quantity of nominal bonds at the end of the consumption-saving stage,  $a^{ee}$ , while the wealth of a typical family of employed workers at the beginning of the consumption-saving stage is  $A^e$ , then the consumption of every employed worker (“ $e$  worker” thereafter) is given by:

$$c^e = (1 - \tau)w + (1 + r)A^e/n - a^{ee}.$$

As already mentioned, in the conjectured equilibrium the nominal bond holdings of any unemployed workers at the end of the consumption-saving stage is  $a^{uu} = \underline{a}e^z$ . Since the consumption level of any household depends on both beginning-of-period and end-of-period wealth, there can be only two types of unemployed workers. First, those who were employed at the beginning of the labor market transitions stage (“ $eu$  workers” thereafter) receive income  $b^ue^z + (1 + r)a^e$  during the production stage and thus consume:<sup>12</sup>

$$c^{eu} = b^ue^z + (1 + r)a^e - a^{uu}.$$

Second, those who were unemployed at the beginning of the labor market transitions stage (“ $uu$  workers” thereafter) receive income  $b^uu + (1 + r)a^u$  during the production stage and thus consume:

$$c^{uu} = b^uu + (1 + r)a^u - a^{uu}.$$

While  $e$  workers are a fraction  $\mathbf{n}$  of the workers by the end of the labor market transitions stage,  $eu$  workers and  $uu$  workers are in proportion  $\mathbf{n}^{eu} = s\tilde{\mathbf{n}}$  and  $\mathbf{n}^{uu} = 1 - \mathbf{n} - \mathbf{n}^{eu} = (1 - f)(1 - \tilde{\mathbf{n}})$ , respectively, by the end of the same stage.

To summarize, under our conjecture, there are only three worker types, i.e.,  $e$ ,  $eu$ , and  $uu$  workers. After the labor market transitions stage, these types are in measures  $\mathbf{n}$ ,  $\mathbf{n}^{eu}$ , and  $\mathbf{n}^{uu}$ , and during the consumption-saving stage they consume  $c^e$ ,  $c^{eu}$  and  $c^{uu}$ , respectively. There are only two possible levels of bond holdings among workers at the end of that stage, namely  $a^{ee} > \underline{a}e^z$  (for  $e$  workers) and  $a^{uu} = \underline{a}e^z$  (for  $eu$  and  $uu$  workers). Finally, because there are only three workers’

<sup>12</sup>Recall that workers who fall into unemployment leave the family with wealth equal to the per-member wealth of the end of the previous period,  $a^e$ . In contrast, those who stay in the family pool income with the other members, so that the relevant per-member wealth is  $A^e/n$ .

types, there are only three relevant habit levels to keep track of:  $\mathbf{c}^W(0)$  (for  $e$  workers),  $\mathbf{c}^W(1)$  (for  $eu$  workers) and  $\mathbf{c}^W(2)$  (for  $uu$  workers, i.e., all workers for whom  $\aleph \geq 2$ ). Since habits levels are determined by the average consumption of the relevant group in the previous period, we have  $\mathbf{c}^{W'}(0) = c^e$ ,  $\mathbf{c}^{W'}(1) = c^{eu}$  and  $\mathbf{c}^{W'}(2) = c^{uu}$ .

3.2.2. *Existence conditions.* In order to construct the two-wealth state equilibrium, we have assumed that, by the end of consumption-saving stage, all unemployed workers face a binding borrowing limit, while none of the employed workers do. We now derive a set of sufficient conditions for this to be the case in every period. A first set of conditions insures that the debt limit is binding for both  $eu$  and  $uu$  workers in every period. Formally, their optimal asset holding condition must be corner, i.e.,

$$\mathbb{E}_X[M^{ju'}(1+r')] < 1, \quad j = e, u, \quad (35)$$

where  $M^{ju'}$  is the SDF of  $ju$  workers,  $j = e, u$ , which is given by:

$$M^{ju'} = \beta^W(1-f') \frac{u'(c^{juu} - h\mathbf{c}^{W'}(2))}{u'(c^{ju} - h\mathbf{c}^W(\mathbf{s}))}$$

for  $j = e, u$ , with  $\mathbf{s} = 1$  if  $j = e$  and  $\mathbf{s} = 2$  if  $j = u$ .

With probability  $1 - f'$  an unemployed worker will stay unemployed, and thus stay (or becomes) a  $uu$  worker, with marginal utility  $u'(c^{juu} - h\mathbf{c}^W(2))$ ; or with complementary probability  $f'$  this worker will find a job and become an  $e$  worker. In the latter case, the worker will join a family and pool his remaining assets with the rest of the family. However, because this worker is atomistic relative to the size of the family we have  $\partial A^e / \partial a^u = 0$ . Condition (35) holds if and only if  $a^{ju} = \underline{a}e^z$  for both  $eu$  and  $uu$  workers.

The third sufficient condition for the two-wealth state equilibrium to hold is that  $e$  workers never face a binding borrowing limit. Formally, their optimal asset-holding condition must be interior, i.e.,

$$\mathbb{E}_X[M^{e'}(1+r')] = 1, \quad (36)$$

where  $M^{e'}$  is the stochastic discount factor of  $e$  workers, which is given by:

$$M^{e'} = \beta^W \frac{(1-s')u'(c^{e'} - h\mathbf{c}^{W'}(0)) + s'u'(c^{eu'} - h\mathbf{c}^{W'}(1))}{u'(c^e - h\mathbf{c}^W(0))}. \quad (37)$$

The stochastic discount factor is the ratio of next-period to current marginal utility. The marginal utility of a currently employed worker (the denominator) is  $u'(c^e - h\mathbf{c}^W(0))$ . Next-period marginal utility (the numerator) must be broken into two idiosyncratic states, because the worker will either stay employed in the next period (with probability  $1 - s'$ ), in which case he will enjoy marginal utility  $u'(c^{e'} - h\mathbf{c}^{W'}(0))$ , or fall into unemployment (with probability  $s'$ ) and enjoy marginal utility  $u'(c^{eu'} - h\mathbf{c}^{W'}(1))$ . Condition (36) holds if and only if  $a^{e'} \geq \underline{a}e^z$ . In the empirical evaluation of

the model we will make sure that equations (35)–(36) hold in every period with high probability, given the posterior distribution of estimated parameters and the realizations of aggregate shocks over our sample.

In the two-wealth state equilibrium, the aggregate state  $X$  in (34) becomes small-dimensional: the cross-sectional distribution of wealth  $\tilde{\mu}(\cdot)$  has only two points in its support ( $a^{e'}$  for the employed,  $a^{u'}$  for the unemployed) and there are only three relevant habit levels for workers ( $c^W(0)$ ,  $c^W(1)$  and  $c^W(2)$ ).<sup>13</sup>

**3.3. Time-varying precautionary savings.** We are now in a position to isolate the key determinants of the precautionary motive and its variations over time in our economy. A worker's incentive to save is summarized by the behavior of its intertemporal marginal rate of substitution, i.e., the SDF defined in equation (38). To the clarity of the argument, we first abstract from consumption habits. When  $h = 0$  equation (38) becomes:

$$M^{e'} = \beta^W \frac{(1 - s')u'(c^{e'}) + s'u'(c^{eu'})}{u'(c^e)}. \quad (38)$$

There is uninsured idiosyncratic unemployment risk whenever  $s' > 0$  (i.e., the job-loss rate is positive) and  $c^{eu'} < c^{e'}$  (falling into unemployment generates a consumption loss). In this case, an increase in the job-loss rate  $s'$  raises future marginal utility, i.e., raises the incentive to save. This is the precautionary motive for holding wealth. Importantly, the job loss rate  $s'$  affects precautionary wealth even if we consider a first order approximation to workers' Euler equation. To see this, observe that the log-deviation of the SDF from its steady-state value is given by:

$$\hat{M}^{e'} \simeq \underbrace{-\sigma \left( \frac{c^{e'} - c^e}{c^e} \right)}_{\text{perfect-insurance SDF}} + \underbrace{\sigma \left( \mathbb{E} \left[ \frac{c^{e'} - c^{eu'}}{c^e} \right] \right)}_{\text{impact of imperfect insurance}} s'.$$

The second term in the equation correspond to the usual (log-linear) SDF under perfect insurance (i.e. if all workers consumed the same). The third term in is a correction to this SDF coming from the fact that workers are imperfectly insured. More specifically, the term  $\mathbb{E}[(c^{e'} - c^{eu'})/c^e]$  is the mean (i.e., steady-state) consumption growth differential between a worker who stays employed from the current to the next period and a worker who falls into unemployment in the next period, where  $s'$  is the probability of the latter event occurring. The last term is positive whenever insurance against unemployment risk is imperfect (so that  $\mathbb{E}[(c^{e'} - c^{eu'})/c^e] > 0$ ) and there is a possibility of falling into unemployment (i.e.,  $s' > 0$ ). Again, an increase in the job-loss rate translates into a greater (log-linear) SDF, hence a greater desire to save. That the precautionary is

<sup>13</sup>See the technical Appendix for further details.

preserved in the first-order approximation of workers' Euler equation implies that it will remain operative when we solve and estimate the linear state-space representation of the full model.

#### 4. ESTIMATING THE IMPERFECT-INSURANCE MODEL

In this section, we take our model to the data. We first describe the functional forms, the structural shocks, and the Bayesian empirical strategy we use. We emphasize our ability to incorporate cross-sectional data in our likelihood-based estimation, next to the traditional aggregate macroeconomic, monetary and labor-market transitions data. We will split the parameter set into two subsets. The first subset will be chosen to match some unconditional moments related to steady state of the model. It is important to note that we also make use of cross-sectional data to discipline this subset of parameters because we will match some cross-sectional unconditional moments. We call this first subset the calibrated parameters. The second subset will be estimated and called accordingly. For this second set, we describe the prior choice together with the posterior estimates. We then report a set of impulse-response functions to monetary policy and technology shocks implied by the posterior estimates.

**4.1. Functional forms and structural shocks.** Before proceeding, we must specify the functional forms adopted for the utilization cost function,  $\eta(\cdot)$ , and the investment adjustment cost function,  $S(\cdot)$ . In particular, we assume:

$$\eta(v) = \frac{\bar{r}_k}{\tilde{\nu}_v} [e^{\tilde{\nu}_v(v-1)} - 1] \quad \text{and} \quad S\left(\frac{i'}{i}\right) = \frac{\nu_i}{2} \left(\frac{i'}{i} - \bar{g}_i\right)^2, \quad \text{with } \tilde{\nu}_v, \nu_i > 0.$$

Here  $\bar{r}_k$  and  $r_k$ ,  $\bar{g}_i$  are the steady-state values of the rental rate of capital and the growth factor of investment  $i'/i$ , while  $\tilde{\nu}_v$  and  $\nu_i$  are the curvatures of the utilization and investment adjustment cost functions. These functional forms ensure that in a steady state both costs vanish. As in [Smets and Wouters \(2007\)](#), we define  $\nu_v \equiv \tilde{\nu}_v / (1 + \tilde{\nu}_v)$  and estimate  $\nu_v$  rather than  $\tilde{\nu}_v$ . This allows us to eliminate numerical problems at the estimation stage. Moreover, we impose the following functional form for the exogenous job destruction rate:

$$\rho(\varphi_s) = \frac{1}{1 + e^{\bar{\rho} - \varphi_s}},$$

where  $\bar{\rho}$  is a constant that pins down the steady-state value of  $\rho$ . With this functional form, we ensure that  $\rho$  varies only in the compact set  $[0, 1]$ . Finally, the utility function  $u(\cdot)$  takes the form

$$u(c) = \lim_{\tilde{\sigma} \rightarrow \sigma} \frac{c^{1-\tilde{\sigma}} - 1}{1 - \tilde{\sigma}}, \quad \text{with } \sigma > 0.$$

In order to simplify the presentation of the empirical analysis we are going to index variables by a time subscript, where  $t$  will denote current variables and  $t - 1$  lagged ones. The considered



structural shocks are of two classes. First,  $\varphi_{h,t}$ , for  $h \in \{i, c, s, R, w, p\}$ , are stationary and are assumed to follow AR(1) processes of the form:

$$\varphi_{h,t} = \rho_h \varphi_{h,t-1} + \sigma_h \epsilon_{h,t}, \text{ with } \epsilon_{h,t} \sim \mathcal{N}iid(0, 1).$$

Second, the technology shock  $z_t$  follows a random walk with drift  $\mu_z > 0$  and, hence, it is not stationary:

$$z_t = \mu_z + z_{t-1} + \sigma_z \epsilon_{z,t}, \text{ with } \epsilon_{z,t} \sim \mathcal{N}iid(0, 1).$$

**4.2. Empirical strategy and data.** Before taking the model to the data, we first need to induce stationarity by normalizing the first-order conditions of the model by  $e^{z_t}$ . Then, we linearize the resulting system in the neighborhood of the normalized steady state.<sup>14</sup> Let  $\hat{X}_t$  denote the vector collecting the deviation from steady state of the normalized state variables and let  $\epsilon_t$  denote the vector collecting the innovations to the structural shocks. The law of motion of  $\hat{X}_t$  is of the form:

$$\hat{X}_t = \mathbf{F}(\boldsymbol{\vartheta}) \hat{X}_{t-1} + \mathbf{G}(\boldsymbol{\vartheta}) \epsilon_t, \quad (39)$$

where

$$\boldsymbol{\vartheta} = (\Omega, \sigma, h, \beta^F, \beta^W, \delta, \theta, \phi, \kappa_v, \kappa_y, \bar{\rho}, \chi, \bar{m}, v_i, v_v, \bar{\eta}, \alpha, \gamma_p, \psi, \bar{w}, \psi_n, \gamma_w, \\ b_u, \underline{a}, \bar{\pi}, \rho, a_\pi, a_y, \mu_z, \rho_x \text{ for } x \in \{c, i, w, s, p, R\}, \sigma_x \text{ for } x \in \{c, i, w, s, p, R, z\})$$

is the vector of model's parameters. The matrices  $\mathbf{F}(\boldsymbol{\vartheta})$  and  $\mathbf{G}(\boldsymbol{\vartheta})$  are functions of the model's parameters.

The vector of variables used as observable in estimation,  $O_t$ , consists of the growth rates of consumption and total investment,  $\Delta \log(c_t)$  and  $\Delta \log(\tilde{i}_t)$  respectively, inflation  $\pi_t$ , the nominal interest rate  $R_t$ , nominal wage inflation  $\Delta \log(W_t)$ , the job-loss rate  $s_t$ , and the job-finding rate  $f_t$ . Total investment  $\tilde{i}_t$  is defined as the sum of investment  $i_t$  and utilization and vacancy-posting costs, so that  $\tilde{i}_t = i_t + \eta(v_t)k_{t-1} + \kappa_v v_t$ . Nominal wage inflation,  $\Delta \log(W_t)$ , is defined as real wage inflation plus inflation, i.e.  $\Delta \log(W_t) = \Delta \log(w_t) + \pi_t$ .

The vector  $O_t$  also contains the share of consumption of the 60 percent poorest in total consumption  $c_{60,t}^*/c_t^*$ , i.e. cross-sectional data. While in our theoretical setup it is the case that  $c_t^*$  and total consumption  $c_t$  coincide exactly, a statistical discrepancy between their data counterpart from CEX and BEA has been documented (see [Heathcote, Perri and Violante, 2010](#)). In an attempt to capture this discrepancy, we append a white noise measurement error,  $u_t$ , to the consumption share of the 60 percent poorest households. Formally:

$$O_t \equiv (\Delta \log(c_t), \Delta \log(\tilde{i}_t), \pi_t, R_t, \Delta \log(W_t), s_t, f_t, c_{60,t}^*/c_t^* + u_t)'$$

<sup>14</sup>See the technical Appendix for a description of the normalized steady state.

hence, the measurement equation is:

$$O_t = \mathbf{M}(\boldsymbol{\theta}) + \mathbf{H}(\boldsymbol{\theta})\hat{X}_{t-1} + \mathbf{J}(\boldsymbol{\theta})\varepsilon_t + \mathbf{e}_8 u_t, \quad (40)$$

where  $\mathbf{e}_8$  is the eighth column of the identity matrix of dimension 8, and we use  $\mathbf{M}(\boldsymbol{\theta})$  to denote the vector of means of observed variables.

Our sample runs from 1982Q1 to 2007Q4.<sup>15</sup> Consumption is defined as the sum of personal consumption expenditures on nondurable goods and services, as well as government consumption expenditures and gross investment. The resulting series is deflated by the implicit GDP deflator. Investment is defined as the sum of gross private domestic investment and personal consumption expenditures on durable goods. The resulting series is also deflated by the implicit GDP deflator. These two series are converted to per-capita terms by dividing them by the civilian population, age 16 and over. Inflation is calculated using the GDP deflator and the nominal interest rate is defined as the Effective Federal Funds Rate. Finally, we measure nominal wages as the average weekly earnings of production and nonsupervisory employees, from the Current Employment Statistics survey.

For the labor-market transition probabilities  $f$  and  $s$ , we proceed as follows. First, we compute monthly transition rates using Current Population Survey (CPS) data on unemployment and short-run unemployment, using the approach of [Shimer \(2005, 2012\)](#). Using these series, we construct transition matrices across employment statuses for every month in the sample and then multiply those matrices over the three consecutive months of each quarter to obtain quarterly transition rates.

To construct the consumption share of the 60 percent poorest, we first aggregate nondurable items in the Consumer Expenditure Survey (CEX) to compute individual nondurables consumption (using the same categories as [Heathcote et al. \(2010\)](#)), and then sort consumption by income levels to obtain  $c_{60,t}^*/c_t^*$ .

We follow the Bayesian approach to estimate the model's parameters. Based on the state-space representation for the dynamic system represented by (39) and (40), we (i) evaluate the likelihood of the observed variables at any value of  $\boldsymbol{\theta}$  using the Kalman filter and (ii) form the posterior distribution by combining the likelihood function with a joint density characterizing some prior beliefs.

Given the specification of the model, the posterior distribution cannot be recovered analytically but we may numerically draw from it, using a Monte-Carlo Markov Chain (MCMC) sampling

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<sup>15</sup>The data used for estimation come from the Bureau of Economic Analysis (BEA), the Federal Reserve Bank of St. Louis' FRED II database, and the Bureau of Labor Statistics (BLS). See the technical Appendix for further details on the data.

approach. More specifically, we rely on the Metropolis-Hastings algorithm to obtain a random draw of size 1,000,000 from the posterior distribution of the parameters.

**4.3. Calibrated parameters.** As mentioned, the vector of parameters  $\vartheta$  is split into two subsets  $\vartheta_1$  and  $\vartheta_2$ . The first one,

$$\vartheta_1 = (\delta, \theta, \chi, \bar{m}, \Omega, \underline{a}, \bar{\pi}, \mu_z, \beta^F, \beta^W, b^u, \phi, \kappa_v, \bar{\rho}, \kappa_y, \psi, \bar{w}),$$

contains parameters that are not estimated using the just described bayesian estimation. These parameters are either outright calibrated or tied to some restriction implied by the fact that we force the steady-state to match some unconditional moments. The remaining parameters, contained in  $\vartheta_2$ , are estimated.

The depreciation rate  $\delta = 0.015$  implies a 6 percent annual depreciation of physical capital. We choose  $\theta$  such that the markup is 20 percent. The elasticity of the matching function with respect to vacancies is set 0.5, so that  $\chi = 0.5$ . This value is standard. Finally, the parameters  $\bar{m}$  is normalised to 1. Finally, we set the share of workers in the model to  $\Omega = 0.6$  and restrict borrowing by setting  $\underline{a} = 0$ .

The remaining parameters in  $\vartheta_1$  are chosen as follow. The steady-state inflation rate  $\bar{\pi}$  is set to match the average value of inflation over the sample. The growth rate of technical progress  $\mu_z$  is set to match the average growth of output over the sample. The subjective discount factor of firm owners  $\beta^F = 0.9985$  is set so that, given both  $\mu_z$  and  $\bar{\pi}$ , the steady-state nominal real interest rate matches its average empirical counterpart. The subjective discount factor of workers  $\beta^W = 0.9835$  is set so that the average nondurables consumption loss when falling into unemployment is 21 percent, as documented by [Chodorow-Reich and Karabarbounis \(2014\)](#). The parameter  $b^u$  is set so as to impose a replacement rate of 50 percent. Note that the values of  $\beta^W$  and  $b^u$  determine workers' incentive to hold precautionary wealth and hence the shape the cross-sectional distribution of wealth. Our parameters imply that workers hold less than 1 percent of total wealth, as is consistent with the empirical cross-sectional distribution of liquid wealth in the Survey of Consumer Finances (SCF).<sup>16</sup>

We pin down  $\phi$  so that the labor share in income is 64 percent. We set  $\kappa_v$  so that the share of vacancy costs in output is 1 percent. The parameter  $\bar{\rho}$  is pinned down by imposing that the steady-state value of  $s$  coincides with its empirical average value. The parameter  $\kappa_y$  is set so that steady-state monopolistic profits are zero. We set the skill premium parameter  $\psi$  so as to match

<sup>16</sup>Liquid wealth is the relevant wealth concept to think about households' ability to smooth nondurables consumption in the face of idiosyncratic income shocks occurring at the business-cycle frequency (see, [Challe and Ragot, 2014](#); [Kaplan and Violante, 2014](#); [Kaplan, Violante and Weidner, 2014](#)). In the 2007 wave of the SCF, the bottom 60 percent of the households in terms of liquid wealth held 0.31 percent of total liquid wealth (or about 700\$ on average).

the average share of income of the 60 percent poorest in total income, as backed out from CEX data. This last choice underscores a clear advantage of our setup, in that it allows us to make explicit contact with cross-sectional data at the calibration stage. Given the preceding restrictions, we select  $\bar{w}$  to match the average value of  $f$  in the data.

It is important to note that because most of the steady-state restrictions also involve the parameters in  $\boldsymbol{\vartheta}_2$ , the parameters  $\boldsymbol{\vartheta}_1$  will be functions of the posterior draws. More precisely, for each draw of  $\boldsymbol{\vartheta}_2$ , we readjust the parameters in  $\boldsymbol{\vartheta}_1$  to meet all the steady-state restrictions listed above.

**4.4. Estimated parameters.** The remaining parameters contained in  $\boldsymbol{\vartheta}_2$  are estimated. They are listed in table 1, together with information on their prior and posterior distributions. In addition to the prior, for each parameter, the table reports the posterior mean and standard deviation together with the bounds of the 90 percent Highest Posterior Density interval (HPD, labeled “low” and “high”).<sup>17</sup>

We now comment on the estimation results. The posterior means of  $\sigma$  and  $h$  are 0.40 and 0.79, respectively. This implies that the coefficient of relative risk aversion along the balanced growth path  $-cu''(c)/u'(c) \approx (1 + \mu_z)\sigma/(1 + \mu_z - h)$  is equal to 1.87, which is well within the range of available estimates. The posterior mean of transformed curvature of the utilization cost  $v_u$  is 0.48, hardly different from the prior mean. This implies that the actual degree of curvature of the utilization cost function  $\tilde{v}_u = v_u/(1 - v_u)$  is about 0.92. The posterior mean for  $v_i$  is close to 2.9. The degree of price stickiness  $\alpha$  has a posterior mean equal to 0.79, implying an average price duration of almost 4.8 quarters. The posterior means of the degrees of price and wage indexation are equal to 0.3 and 0.9, respectively. The value for  $\gamma_p$  is in the range of previous estimates obtained in the literature. The value for  $\gamma_w$  is quite high, suggesting a relatively strong degree of nominal wage stickiness. In turn, this is partly reinforced by a posterior mean degree of responsiveness of wages to employment of about 0.6, slightly higher than its prior mean. The posterior mean of the degree of interest rate smoothing is also higher than its prior mean, with  $\rho$  close to 0.77. The responsiveness of the nominal interest rate has a posterior mean of 2.02, higher than its prior mean. This is indicative of a strong reaction to inflation on the part of the monetary policy authority, consistent with the historical record over the post-Volcker era. The posterior mean of the responsiveness to economic activity is relatively higher than the prior mean. Finally, the parameters controlling the structural shocks are within the results obtained in the literature. Here, it is important to bear in mind that our estimate for the standard error of the markup shock  $\sigma_p$  is not directly comparable to results discussed in the literature. The reason is that in general the markup shock is rescaled by the slope of the New Keynesian Phillips curve, resulting in a relatively low standard error of markup

<sup>17</sup>Further discussion of our choice of prior distributions can be found in the technical Appendix.

TABLE 1. Prior and Posterior Distribution of Structural Parameters

Parameter	Prior shape	Prior Mean	Prior S.D.	Post. Mean	Post. S.D.	Low	High
$\sigma$	Gamma	1.00	0.50	0.40	0.07	0.28	0.52
$h$	Beta	0.50	0.10	0.79	0.05	0.72	0.87
$v_i$	Gamma	1.00	0.50	2.93	0.91	1.47	4.44
$v_u$	Beta	0.50	0.10	0.48	0.10	0.31	0.64
$\alpha$	Beta	0.50	0.10	0.79	0.04	0.72	0.86
$\gamma_p$	Beta	0.50	0.10	0.34	0.09	0.19	0.49
$\gamma_w$	Beta	0.50	0.10	0.88	0.03	0.83	0.92
$\psi_n$	Beta	0.50	0.10	0.61	0.09	0.47	0.76
$\rho$	Beta	0.50	0.10	0.77	0.03	0.72	0.82
$a_\pi$	Gamma	1.50	0.15	2.02	0.15	1.76	2.26
$a_y$	Gamma	0.13	0.05	0.24	0.10	0.09	0.38
$\rho_p$	Beta	0.50	0.10	0.74	0.08	0.62	0.87
$\rho_c$	Beta	0.50	0.10	0.74	0.05	0.65	0.82
$\rho_w$	Beta	0.50	0.10	0.70	0.07	0.59	0.81
$\rho_i$	Beta	0.50	0.10	0.47	0.08	0.34	0.59
$\rho_s$	Beta	0.50	0.10	0.85	0.03	0.81	0.90
$\sigma_c$	Inverted Gamma	1.00	2.00	0.52	0.18	0.25	0.80
$\sigma_w$	Inverted Gamma	1.00	2.00	0.37	0.03	0.32	0.42
$\sigma_i$	Inverted Gamma	1.00	2.00	4.57	1.43	2.29	6.95
$\sigma_p$	Inverted Gamma	1.00	2.00	2.52	1.11	1.12	3.96
$\sigma_z$	Inverted Gamma	1.00	2.00	1.18	0.09	1.03	1.32
$\sigma_R$	Inverted Gamma	1.00	2.00	0.18	0.02	0.15	0.21
$\sigma_s$	Inverted Gamma	1.00	2.00	9.67	0.75	8.44	10.89
$\sigma_u$	Inverted Gamma	1.00	2.00	1.46	0.11	1.28	1.65

**Note:** Low and High stand for the lower and upper boundaries of the 90 percent HPD interval, respectively.

shocks. In the present paper,  $\sigma_p$  is not rescaled. If it were, our estimate would broadly fall in the ballpark of available estimates.

**4.5. Verification of the existence conditions.** In the Section 3.2.2, we presented a set of sufficient conditions for the two-wealth state equilibrium to exist. Here, we want to check whether the conditions are met given our posterior estimates. Recall that for the sufficient conditions to be met, it must be the case that equations (35)-(36) hold. If equation (35) holds, we have, equivalently, that:

$$\Psi_t^{eu} \equiv u'(c_t^{eu} - hc_{t-1}^{eu}) - \beta^W \mathbb{E}_t\{(1 - f_{t+1})u'(c_{t+1}^{uu} - hc_t^{uu})(1 + r_{t+1})\} > 0$$

and

$$\Psi_t^{uu} \equiv u'(c_t^{uu} - hc_{t-1}^{uu}) - \beta^W \mathbb{E}_t\{(1 - f_{t+1})u'(c_{t+1}^{uu} - hc_t^{uu})(1 + r_{t+1})\} > 0.$$

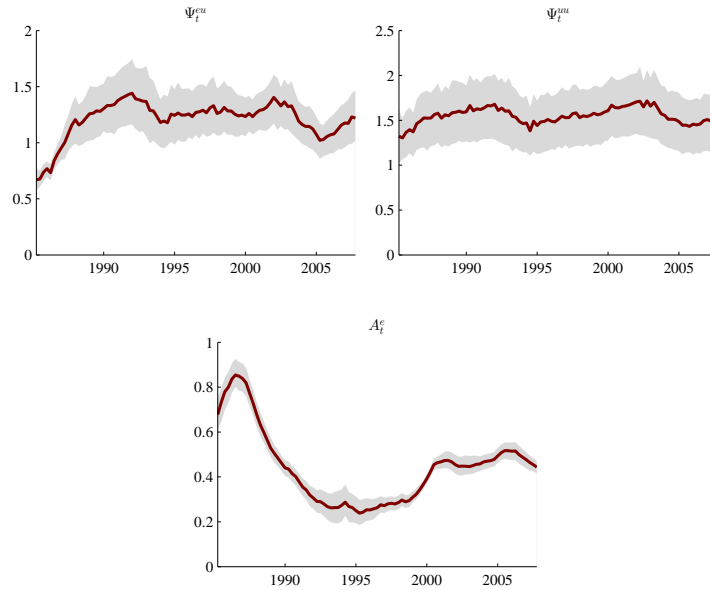


FIGURE 2. Existence Conditions

**Note:** The thick red line is the posterior mean path, the grey area is the 90 percent HPD interval.

If condition (36) holds, we have, equivalently, that  $A_t^e > 0$ .

From left to right and top to bottom, the panels in Figure 2 report the posterior mean (thick red line) of  $\Psi_t^{eu}$ ,  $\Psi_t^{uu}$  and  $A_t^e$  (each appropriately normalized), respectively, over the estimation sample, as implied by the smoothed values of the state variables. In each panel, we also report the associated 90 percent HPD interval (the grey area delineated by the thin, black dashed lines). Figure 2 makes clear the posterior probability that the sufficient conditions are indeed satisfied is close to one.

**4.6. Empirical performance.** In this section, we show that our baseline imperfect-insurance model empirically outperforms the perfect-insurance benchmark, so that taking into account time-varying precautionary savings improves the fit to the data. The perfect-insurance benchmark is one that is structurally identical to our baseline model, except that workers also enjoy perfect insurance (not only firm owners). That is, workers never leave their family regardless of their employment status, so all workers enjoy equal consumption and hold identical end-of-period bond wealth. Because workers are impatient relative to firm owners, they then borrow up to the borrowing limit in every period, as in, e.g., [Kiyotaki and Moore \(1997\)](#) or [Iacoviello \(2005\)](#). This model does not completely eliminate household heterogeneity (only heterogeneity among workers), so that the cross-sectional distribution of consumption is not degenerated (workers and firm owners consume different amounts) and we can compare the two models using the observables described above,

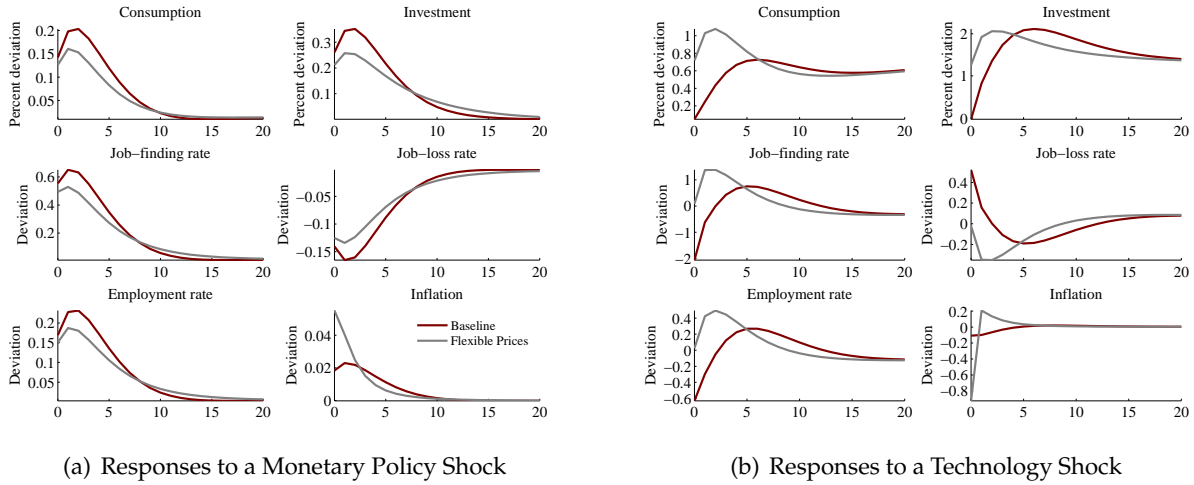


FIGURE 3. Impulse Response Functions

**Note:** Impulse response functions of Consumption, Investment (both in log) as well as the job-finding, job-loss, employment and inflation rates after a monetary shock (panel a) and a technology shock (panel b). The size of the technology shock is set to the respective standard deviation obtained at the estimation stage. That of the monetary shock is set to induce a 25-basis-point drop in the annualised nominal interest rate. The black solid line is the baseline, the grey line is the flex-price version. For Consumption and Investment, the IRFs are reported in percent deviation from the pre-shock levels; for the other variables, the IRF correspond to the deviation from the pre-shock level.

i.e. including the consumption shares. However, in the perfect-insurance model workers no longer hold any precautionary wealth in excess of the borrowing limit (by construction), so the aggregate demand and supply effects of time-varying precautionary savings are absent.<sup>18</sup>

We compare the fit of the two specifications by comparing the marginal likelihoods. To that end, let  $\mathcal{M}_{II}$  and  $\mathcal{M}_{PI}$  denote the our imperfect-insurance model and its perfect-insurance counterpart, respectively. Also, let  $p(O_{1:T}|\mathcal{M}_j)$  denote the log marginal likelihood of model  $\mathcal{M}_j$ , for  $j \in \{II, PI\}$ , where  $O_{1:T}$  denotes the sample observations of the data vector  $O_t$ . We obtain  $p(O_{1:T}|\mathcal{M}_{II}) = -797.9$  and  $p(O_{1:T}|\mathcal{M}_{PI}) = -807.4$ . Note that the imperfect-insurance model still outperforms the perfect-insurance model when we exclude the consumption share of the 60 percent poorest from the set of observables:  $p(O_{1:T}^*|\mathcal{M}_{II}) = -630.0$  and  $p(O_{1:T}^*|\mathcal{M}_{PI}) = -643.5$ , where  $O_{1:T}^*$  denotes the history of observables when the consumption share of the 60 percent poorest is excluded.

**4.7. Impulse response functions to monetary policy and technology shocks.** Figures 3(a) and 3(b) report the Impulse response functions (IRFs) of consumption, investment (both in log) as well as the job-finding, job-loss, employment and inflation rates to a 25 basis points monetary policy shock and a one-standard-deviation shock to the growth rate of technology, respectively. For consumption and investment, the IRFs after a technology shock are simply obtained by cumulating

<sup>18</sup>See the technical Appendix for a complete formal description of the perfect-insurance benchmark.

their growth rates. We have computed the IRFs by setting the model's parameters to their posterior mean.<sup>19</sup> The reported dynamics for consumption and investment correspond to percent deviations from their levels prior to the shock, while for the other variables it corresponds to the deviations from the pre-shock levels.

To get a sense of how nominal frictions affect the economy's response to these shocks, the figure also reports the IRFs for the case where intermediate-good prices are almost fully flexible (i.e., with  $\alpha = 0.05$  and  $\gamma_p = 0$ ). Unsurprisingly, price flexibility attenuates the response of real variables to monetary policy shocks, but magnifies that of inflation. In response to a technology shock all variables respond less in the sticky-price model than in the flexible-price model, a reflection of the stronger short-run demand constraints that prevail in the sticky-price economy—see, e.g., [Galí and Rabanal \(2005\)](#).

## 5. PRECAUTIONARY SAVINGS DURING POST-VOLCKER RECESSIONS

**5.1. Measuring the contribution of time-varying precautionary savings.** We now use our estimated imperfect-insurance model to measure the contribution of time-varying precautionary savings in the propagation of the recent U.S. recessions (the 1990-1991 recession, the 2001 recession, and the Great Recession). From a theoretical point of view, our model embodies the two aggregate effects of time-varying precautionary savings discussed in the introduction: the aggregate demand effect is operative because the three basic frictions that we have assumed generate a mutually reinforcing feedback between idiosyncratic unemployment risk and aggregate consumption demand; but the aggregate supply effect is also operative because our model has capital and thus the traditional smoothing effect of imperfect-insurance models (e.g., [Krusell and Smith, 1998](#)). In the presence of both effects the question naturally arises as to which effect dominates, i.e., whether time-varying precautionary savings ultimately amplify or dampen recessions.

Answering this question requires running counterfactual experiments wherein the precautionary motive is shut down. If a counterfactual experiment shows more volatility, this means that the stabilizing aggregate supply effect has dominated. Otherwise, it is the aggregate demand effect that has dominated. We run these experiments as follows. First, using the posterior mean of the parameter distribution computed in Section 4, we run the Kalman smoother to extract the sequences of structural shocks experienced by the U.S. economy during recession episodes. Second, we feed these shocks into the perfect-insurance counterpart of our model. The perfect-insurance model is similar to that described in Section 4.6, but with all the deep parameters maintained at

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<sup>19</sup>The IRFs to the other five structural shocks as well as a variance-decomposition exercise can be found in the technical Appendix.





FIGURE 4. The Great Recession

**Note:** The red lines correspond to the actual paths of consumption, investment, the job-finding rate, the job-loss rate, and the employment rate. The dashed, grey lines correspond to the counterfactual sample paths. Consumption and investment are reported in proportional deviation from their level at the beginning of the recession. All the other variables are expressed in level deviation from their values at the beginning of the recession. The grey area indicates the recession dates.

the posterior mean of the estimated imperfect-insurance model. Third, we compare the time-series generated by the perfect-insurance model (grey dashed line of Figures 4 and 5) with the historical time series (red lines) for consumption, investment as well as the job-finding, job-loss and employment rates (the first two variables are expressed as log-deviations from their value at the NBER peak, while the last four are in level-deviations from their value at the peak). Because the the precautionary motive is shut down in the perfect-insurance benchmark, this comparison gives us a measurement of the propagating role of time-varying precautionary savings in the last three recessions. Figures 4 analyzes the Great Recession, while Figure 5 considers the 1990 and the 2001 recessions.

**5.2. The Great Recession.** Figure 4 compares the actual and counterfactual paths of the above-mentioned variables during the Great Recession. The amplification generated by the precautionary motive is striking. The fall in consumption from peak to trough would have been three times

smaller without the precautionary motive than it actually was in the data (it is -3.5% in the data but only -1.2% in our counterfactual experiment). This fall in consumption reflects the rational response of imperfectly-insured workers to the huge increase in idiosyncratic labor market risk that they have faced, as is illustrated by the sharp rise in the job-loss rate over the period (about two and a half standard deviations of the job-loss rate in the pre-recession sample). Moreover, there is evidence of a strong feedback from aggregate demand to idiosyncratic labor market risk. This can be inferred from the differential responses of the job-finding and job-loss rates between the data and perfect-insurance counterfactual. Just to fix ideas, on the eve of the Great Recession the quarter-to-quarter job-finding and job-loss rates were 76% and 4%, respectively, while during the Great Recession the former crashed down to 50% and the latter went up to 6%. Since the model allows for within-period labor market transitions, the job-loss rate  $s = \rho(1 - f)$  combines an exogenous separation component  $\rho$  and an endogenous job-finding component  $f$  that responds to aggregate demand. During the Great Recession, all of the increase in  $s$  is explained by the latter component.<sup>20</sup> Our counterfactual analysis indicates that without this feedback the rise in the job-loss rate would have been half as large as it actually was. But since as explained above  $s$  in turn drives time-varying precautionary savings and thereby consumption demand, this closes the feedback loop. Combined with the greater fall in the job-finding rate, this manifested itself as a large drop in the employment rate. Hence, as far as consumption and labor-market risk are concerned, the aggregate demand effect of the precautionary motive has dominated the supply effect during the Great Recession.

Our counterfactual analysis also indicates that the fall in investment was not significantly affected by the precautionary motive. But this is to be expected, because the precautionary motive has two contradicting effects on investment. On the one hand, larger precautionary wealth in a recession takes down the real interest rate. This drop is transmitted to the market for capital claims by firm owners (who participate in both asset markets) and ultimately stimulates investment. This is precisely the aggregate supply effect of the precautionary motive, which tends to smooth the fall in investment relative to the perfect-insurance case. On the other hand, because of the aggregate demand effect, consumption and output are depressed, which tends to discourage investment. The overall impact of these two forces on investment is a priori ambiguous, and in the present case they roughly offset each other. Similarly, inflation is little affected by time-varying precautionary savings. This reflects the fact that the New Keynesian Phillips curve has flattened in the post-Volcker

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<sup>20</sup>More specifically, having computed quarterly values for  $f$  and  $s$  on the basis of the monthly rates (Section 4), we then calculated the value taken by the separation shock  $\rho = s/(1 - f)$  as a residual. It turns out that  $\rho$  has slightly fallen (from 15% to 12%) during the Great Recession, so the rise in  $s$  entirely comes from the fall in  $f$ .

period, i.e., there are significant nominal price rigidities (see, e.g., [Coibion and Gorodnichenko, 2013](#); [Sbordone, 2007](#)). Hence, large variations in aggregate demand –including those due to the precautionary motive– are associated with limited movements in inflation.

**5.3. The 1990-1991 and 2001 recessions.** We now turn to the other two recessions in our sample. [Figure 5](#) compares the actual and counterfactual paths of consumption, investment, the job-finding rate, the job-loss rate, and the employment rate over the 1990-1991 and 2001 recessions. The impact of the precautionary motive on the propagation of the 1990-1991 recession shares several features of the Great Recession, except, of course, for the size of the effect. More specifically, over the duration of the recession, and also over the three quarters after the recession ended, consumption stagnated. Our counterfactual experiment indicates that it would have kept growing at a moderate pace all over this time had the precautionary motive not been active. The feedback from stagnating consumption to depressed labor market conditions is also apparent from the comparison between the paths of actual labor market transition rates and those that would have prevailed without the precautionary motive; in particular, the job-loss rate would have roughly stayed at pre-recession level during most of the year 1991. In other words, just as in the Great Recession, the aggregate demand effect of the precautionary motive dominated the supply effect during the 1990-1991 recession.

The 2001 recession was short and mild, even when compared to the 1990-1991 recession. For that reason, the precautionary motive may have been weak, as is reflected by the small difference between the actual paths and those implied by the counterfactual perfect-insurance model of all the variables of interest. If anything the perfect-insurance model generates more volatility than the imperfect-insurance model, as is revealed by the dynamics of the labor market. This suggests that the precautionary motive has tended to stabilize the economy, i.e., the aggregate supply effect of the precautionary motive has dominated the demand effect during this period.

## 6. CONCLUDING REMARKS

This paper has investigated the impact of time variations in aggregate precautionary wealth for the propagation of business cycle shocks, with special focus on its role during the recent U.S. recessions. The key difference between the theoretical framework that we have used to structure the data and existing sticky price/wage models is the assumption that insurance against labor market risk is imperfect. This assumption has two main implications. First, it generates a precautionary motive for holding wealth, and thus the aggregate demand and supply effects of time-varying precautionary savings on which our paper has focused. Second, the model produces cross-sectional dispersion (in wealth, income and consumption) and thereby heterogenous responses to shocks

## PRECAUTIONARY SAVING AND AGGREGATE DEMAND

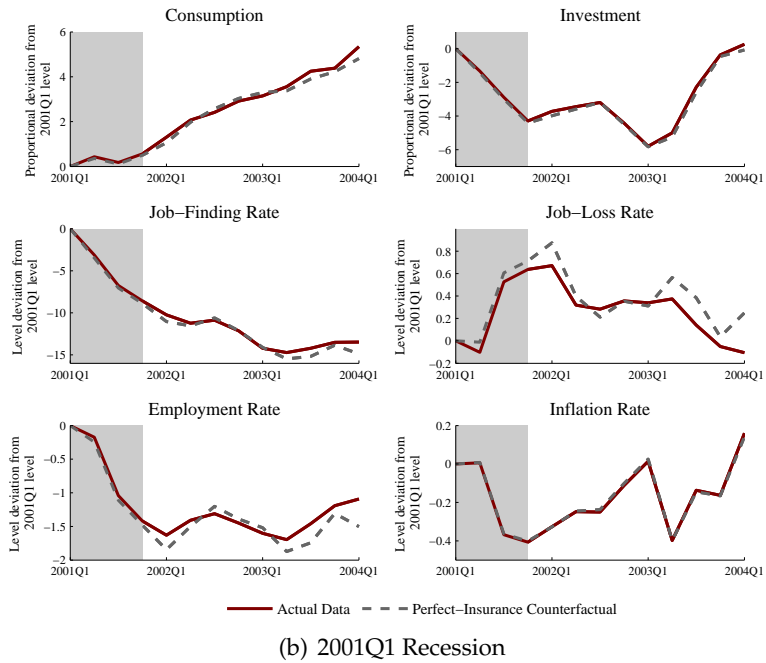
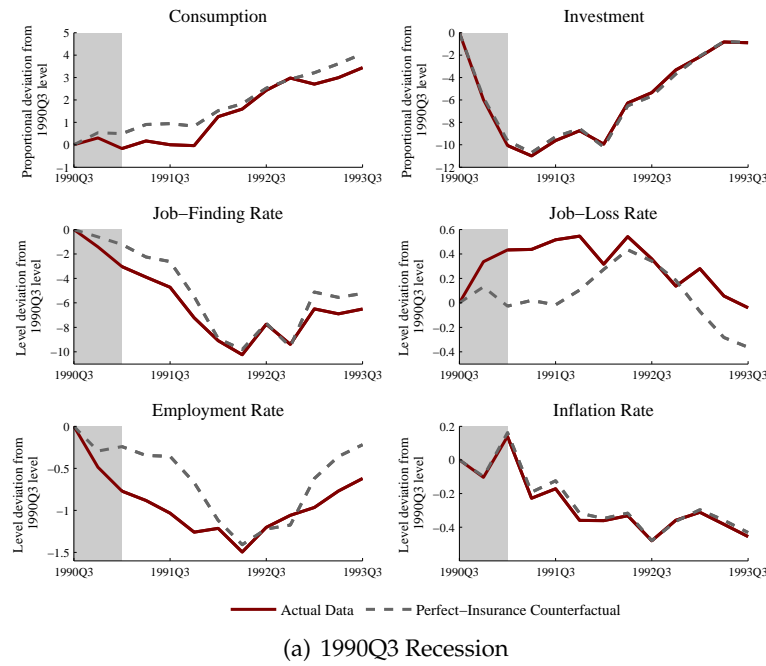


FIGURE 5. The 1990Q3 and 2001Q1 Recessions

**Note:** The red lines correspond to the actual paths of consumption, investment, the job-finding rate, the job-loss rate, and the employment rate. The dashed, grey lines correspond to the counterfactual sample paths. Consumption and investment are reported in proportional deviation from their level at the beginning of the recession. All the other variables are expressed in level deviation from their values at the beginning of the recession. The grey area indicates the recession dates.

across households. In this paper, we have essentially used the cross-sectional heterogeneity generated by the model as additional theoretical restrictions that can be taken to the data in order to further discipline the estimation of its deep parameters (as when the cross-sectional distribution of consumption is incorporated into the measurement equation). There are, however, many other dimensions in which household heterogeneity matters, and thus where the approach that we have taken can usefully be applied. One area that immediately comes to mind is the impact of transfer-based fiscal policy and its interactions with other macro policies. [Kaplan and Violante \(2014\)](#) and [McKay and Reis \(2013\)](#) have taken the first steps in that direction, but in models that abstract from the unemployment risk-aggregate demand feedback that is likely to be activated following fiscal shocks. Note also that it is straightforward to introduce other assets into our framework, including public debt. Cyclical variations in the public debt cannot a priori be neglected when studying fiscal policy in heterogeneous-agent environments, because the very reason why heterogeneity matters – imperfect insurance and borrowing constraints – makes the economy non-Ricardian, causing the induced changes in public debt to have first-order effects on the equilibrium. We leave these themes for future research.

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