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THE SWAP SPREAD

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Modelling the swap spread

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Abstract

Swaps are one of the major innovations of the 80s but there are little empirical studies on interest rates swaps (IRS), especially on US and European markets. To understand how swap pricing works, we estimate IRS valuation models for the US, German and French swap markets. On one hand, we derive swap rate from the market value of the swap contract formula. On the other hand, questioning the role of default credit risk in valuing the swap contract, we show that the swap rate can be expressed as a function of corporate bond rate and default risk indicators; the empirical analysis indicates some elements of validity for both approaches.

Key words: Swap market; Interest rate swaps; Swap valuation.

JEL classification: C12, C32.

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1 Introduction

Swaps are one of the major innovations of the 1980's. Taking into account the fact that swaps do not exist before 1980, the current size of the swaps market is quite impressive. Measuring the recent development of this market by the amount of outstanding of interest rate swaps (IRS) indicates that the notional principal amount of outstanding of IRS is evaluated at 22.2 trillions of dollars at the end of 1997 (BIS, 1999). At the end of 1992, this amount reached 3.9 trillions only. Note that the total amount of outstanding of notional principal of currency swap is estimated to 3 trillions at the end of 1997. Furthermore the European IRS market is the largest in terms of notional principal outstanding. However, at our knowledge, few empirical studies have been devoted to this zone. More generally, there is little empirical research on IRS, and the largest part of empirical academic papers has been devoted to the US market.

Nevertheless, many theoretical studies deal with IRS. Whittaker (1987) evaluates the impact of the credit risk on the valuation of IRS. In a more general framework, which could be applied to any off-balance sheet contracts, Hull (1989) proposes a way to compute the value of contracts when there is a credit risk, using different formulations of the default risk. Hull gives an application on currency swaps but it may be adapted to IRS.

In addition, Duffie and Huang (1996) extend the valuation model for defaultable claims proposed by Duffie and Singleton (1994) to the case in which the two counterparties have an asymmetric risk. Duffie and Huang provide an empirical simulation based on their theoretical model (in the continuous-time framework). Moreover, Duffie and Singleton (1997) propose an econometric model of the term structure of swap rates. Assuming that the counterparties have symmetric probabilities of default, they show that a swap is priced by the present value of its cash flows discounted by risk- and liquidity-adjusted short rate process.

These examples show that some theoretical models on IRS are available. Sun, Sunderasan and Wang (1993) distinguish three main research areas in the swap pricing theory:

- the first approach is based on the economic rationale for the existence and the evaluation of IRS markets. In this case, one evaluates the impact of interest rate and credit risk on the structure and valuation of swaps (see also Bicksler and Chen, 1986, Smith et al., 1986, 1988, or Turnbull, 1987);

- in the second one, IRS can be valued by finding the level of fixed payments that will have the same present value as the floating payments (see Smith et al., 1986, 1988, Brown et al., 1994). This approach is a direct valuation procedure for IRS. For instance Hull (1989), Sundaresan (1991), and Cooper and Mello (1991) used this procedure to price currency or interest rate swaps subject to market or default risk;

- the last approach explores the regulatory framework for swaps and capital adequacy for swap markets makers (see Whittaker, 1987, Hull, 1989, Duffie and Huang, 1996, or Duffie and Singleton, 1997).

In empirical studies the three theoretical approaches are often combined. Most of the empirical work deals with the US market (see Litzenberger, 1992, Sun et al., 1993, Brown et al., 1994, Minton, 1997). However Cossin and Pirotte (1997) examine for a European market (Swiss Franc) both interest swaps and currency swaps using actual transactions data.

More precisely, Litzenberger (1992) attempts to find a characterization consistent with standardized swap contract and which allows to obtain an appropriate theory of swap pricing. Litzenberger approach is empirical because he analyzes the market participants behavior with the observed data but he does not provide a formulation (econometric estimation) of the underlying model. Sun et al. (1993) try to evaluate the impact of the credit reputation of swap dealers on interest swaps and bid/offer spreads. Analyzing the relationship between the quoted swaps rates and the estimated par bond yields in the interbank market, Sun et al. conclude that: (i) there is a positive spread between AAA (rating) swap offer rates and Treasury yields at all maturities. This result does not depend on the shape of Treasury yield curve; (ii) the spread between swap rates and Treasury yield increases generally with maturity, but the intensity of this movement is correlated with the slope of the yield curve; (iii) the bid/offer spread of swaps dealers is sensitive to the credit reputation.

Note that Sun et al. paper is one of the most detailed study concerning IRS, with an important work on the data collecting. This problem is pointed out here because the data unavailability partly explains the lack of empirical work on IRS.

Brown et al. (1994) provide an empirical analysis of the swap spread. In the first step, they derive the swap spread model from a pure expectations approach, using forward rates. However the approximations used seem quite imprecise. Brown et al. then introduce the cost of financial intermediation in the swap spread valuation model. Lastly, they analyze the role of the default risk or the impact of the supply of corporate debt in the swap spread equation. In the similar way, Chen and Selender (1995) show the importance of the exogenous variables depending on the swaps maturity. For instance, Eurodollars rates play a major role in explaining spreads for short-term swaps, whereby spreads for 5 years are significantly correlated to the level of Treasury yield and the slope of the yield curve. For the long-term spread, the credit quality of counterparties could be introduced in the swap spreads equation.

Cossin and Pirotte (1997) specifically analyze the swap spread for a European market. Their goal is to study the credit risk spreads on transaction data. They compare the transaction data (both for IRS and currency swaps) to the quoted rates. For IRS, Cossin and Pirotte mainly explain the difference between transaction and quoted rates by a credit risk indicator, and the amount issued. This paper is interesting but unfortunately the sample size is quite small. Moreover the data comes from a medium size bank and are, perhaps, not representative of the behavior of the large market participants.

Minton (1997) tests two models drawn from Smith et al. (1988). In the first one, the swap is a function of a portfolio of consecutive 3-month Eurodollar futures contracts. In the second model, the swap is priced as a portfolio of bonds. She deduces relationships between long-term interest rate swaps (namely, 10-year swaps) and some determinants (corporate bond yields, the slope of yield curve, the Treasury bond yield, default risk indicators). The default riskis measured with two variables: the corporate quality spread (with bilateral default risk) and the aggregate default spread (with unilateral default risk). Minton's article is one of the most complete empirical paper on IRS valuations on the US market. Note however that her investigation requires a huge collecting data work.

To sum up, it is clear that the empirical research on modelling swap rate is rather poor. We can note two common characteristics of the empirical papers briefly presented above: first, these papers are broadly based on the main standard swap pricing models; second, the estimation of these models requires a tedious collecting data work, complicated by the fact that a part of information is confidential. The relative lack of empirical research can be explained by this problem.

Our objective is to model the IRS rate on the US, German, and French markets using standard pricing models. It is worth noting that these models cannot be used to forecast swap rates. Indeed most of the determinants (mainly, the Treasury bond rate and the corporate bond rate) are partly endogenous. However such models are helpful to understand how swap pricing works: which are the main relevant approaches? What risk premium indicators can help to explain the swap rate? The paper proceeds as follows. Section 2 details the theoretical determinants of the swap spread. Section 3 describes the data and presents some summary statistics. The relation between the swap and a portfolio of short-term futures contracts is considered in section 4. Similarly the relation between the swap and the corporate bond is examined in section 5. Section 6 presents our main conclusions.

2 Theoretical determinants of the swap rate

Several factors theoretically affect the swap rate. In this section we outline these factors, insisting on the econometric implications of the relationships obtained.

2.1 Basic setup

We consider a "plain vanilla" interest rate swap, as shown in Figure 1. Counterparty X agrees to receive a fixed rate, whereas counterparty Y agrees to receive a floating rate. Transactions are generally arranged by swap dealers, that absorb the counterparties' credit risks and receive the bid-offer spread.

[Insert Figure 1]

In interest rate swap, only net cash flows are exchanged, but not principal amounts. Therefore the credit default only concerns interest flows, not the principal. We now consider the valuation of swap contracts in an arbitrage-free setting, in the line of Cox, Ingersoll, and Ross (1980) and Sundaresan (1991). The basic setup is characterized by the following assumptions:

A1. The counterparties have the same degree of creditworthiness (see Cooper and Mello, 1991, for an analysis of the default risk of swap);

A2. There are no transactions and information costs, taxes or other frictions; capital markets are perfect and competitive;

A3. Reset dates coincide with payment dates of the floating leg.

The market value of the swap contract equals the difference between the present-value of the expected cash flows generated by the floating-rate and the fixed-rate side of the transaction. The market value of a N-period agreement is:

$$V_t = E_t \sum_{i=1}^{N} \rho_{t+i} Fv_{t+i} - \sum_{i=1}^{N} \rho_{t+i} S_{t+i}.$$

The first sum corresponds to the floating leg of the swap. We assume a floating payment at each date, with $Fv_{t+i} = \frac{n}{360} f_{t+i}$ with f_{t+i} the floating rate between $t+i-1$ and $t+i$; for US and German swaps, the floating rates are the euro-dollar and the euro-mark rates respectively; for French swaps, the floating rate is the PIBOR rate; the floating rate is denoted P_t in the following; n is the number of days between two payments dates ($n = 90$ days typically). ρ_{t+i} is the discount factor between t and $t+i$, that is $\rho_{t+i} = 1/\left(1 + z_t^{(i)}\right)^i$, where $z_t^{(i)}$ is the zero-coupon discount rate between t and $t+i$.

The second sum corresponds to the fixed leg of the swap. So S_{t+i} is equal to the swap rate, which can be expressed as $S_{t+i} \equiv S_t^{(N)} = Y_t^{(N)} + SS_t^{(N)}$ where $Y_t^{(N)}$ is the Treasury bond yield maturing in $t+N$ and $SS_t^{(N)}$ defines the swap spread. The swap rate is constant over the life of the swap contract.

The market value V_t can therefore be rewritten as:

$$V_t = E_t \sum_{i=1}^{N} \rho_{t+i} P_{t+i} - \left(Y_t^{(N)} + SS_t^{(N)}\right) \sum_{i=1}^{N} \rho_{t+i}.$$

2.2 The swap as a portfolio of short-term forward contracts

Under the assumption that there are no arbitrage opportunities, the market value of the swap contract should be zero. So the swap fixed rate can be expressed as:

$$Y_t^{(N)} + SS_t^{(N)} = \frac{E_t \sum_{i=1}^{N} \rho_{t+i} P_{t+i}}{\sum_{i=1}^{N} \rho_{t+i}} = E_N\left(P_t\right).$$

The swap fixed rate equals the weighted average of expected future floating rates, where $E_N\left(X_t\right) = E_t \sum_{i=1}^{N} \rho_{t+i} X_{t+i} / \sum_{i=1}^{N} \rho_{t+i}$ denotes the weighted average of expected future path of X over N periods.

The swap spread should then be equal to the difference between the weighted average of expected floating rates and the Treasury bond yield:

$$SS_t^{(N)} = E_N\left(P_t\right) - Y_t^{(N)}. \tag{1}$$

However the future path of short-term rates is unknown. Smith et al. (1988) show that the cash flow of a par swap can be replicated by the cash flows of a portfolio of consecutive 3-month forward contracts on the short-term rate. The swap spread can then be expressed as:

$$SS_t^{(N)} = \frac{E_t \sum_{i=1}^{N} \rho_{t+i} f_t^{(1,i-1)}}{\sum_{i=1}^{N} \rho_{t+i}} - Y_t^{(N)} = \Sigma_{N,t} - Y_t^{(N)} \tag{2}$$

where $f_t^{(1,i-1)}$ denotes the 3-month forward rate on euro-currency or PIBOR at time t for $t+i-1$. $\Sigma_{N,t}$ is the weighted average of 3-month forward rates over N periods.

From an empirical point of view, equation (2) allows to take account of some statistical properties of interest rates. Indeed many papers (since Campbell and Shiller,

1987) stressed that interest rates may behave as non-stationary processes. In such a case, relationships between variables in level cannot be estimated using standard (OLS) techniques. On the contrary, if swap rates and Treasury bond rates are cointegrated, equation (2) involves only stationary variables and standard techniques can be implement. In section 3, we will check some stationarity properties of interest rates.

2.3 The swap as a function of corporate bond

Many authors questioned the role of default risk in valuating the swap contracts (Bicksler and Chen, 1986, Cooper and Mello, 1991, Sun et al., 1993, Duffie and Huang, 1996). As in Brown et al. (1994), the link between default risk and swap spread can be introduced as follows. Firms X and Y can issue short-term one-period debt at $P_t + CS_t^{(1)}$, where $CS_t^{(1)}$ denotes credit spread, at each period between t and $t+N-1$, or a long-term N-period debt at a rate $B_t^{(N)} = Y_t^{(N)} + BS_t^{(N)}$, where $BS_t^{(N)}$ denotes corporate bond spread. Note that the corporate spread is based on the interbank short-term rate, whereas the corporate bond spread is based on the Treasury bond rate. Both spreads are therefore based on different reference agents for low risk.

If firms X and Y are assumed to face the same costs of funds, in the absence of arbitrage opportunity the expected cost of floating-rate funds over N periods equals the fixed-rate cost of funds, such that:

$$E_N\left(P_t + CS_t\right) = Y_t^{(N)} + BS_t^{(N)}. \tag{3}$$

Combining (3) with (1) yields

$$SS_t^{(N)} = BS_t^{(N)} - E_N\left(CS_t^{(N)}\right). \tag{4}$$

Under arbitrage-free assumption, the swap spread should equal the difference between the default risk premia across short-term and long-term debt markets.

Similarly, Smith et al. (1988) show that a swap can be replicated by a portfolio of bonds with the same par value and the same maturity. The net cash flows of a fixed-rate payer in a swap can be replicated by a combination of a long position in a variable short-term bond that sells at par on reset dates and a simultaneous short position in a bond of equal par value that makes fixed-rate interest payments on the same reset dates. The swap rate should then be equal to the coupon rate on the fixed-rate bond issued at par. It should be noted nevertheless that the probability of default in a swap is lower than that in a corporate bond, because the former's probability of default is equal to the joint probability of the firm being in bankrupt and the swap having negative value to the firm (Hull, 1989). Duffie and Huang (1996) show that default premia on fixed- and floating-rate bonds differ by a significant amount, indicating that this differential may be an important determinant of swap rates.

A measure of default risk may be the spread between high-rated and low-rated corporate bond rates (cf. Minton, 1997, on US data). However, for Germany and France, such data are not available for a long sample. An aggregate measure of default risk is then proxied by the corporate bond spread or by the Treasury term spread. Fama and French (1989) suggest that the Treasury term spread may capture business condition risk resulting from monetary policy shocks. The swap rate can therefore be expressed as:

$$S_t^{(N)} = f\left(B_t^{(N)}, default\ risk_t^{(N)}\right) = f\left(B_t^{(N)}, BS_t^{(N)}, Y_t^{(N)} - Y_t^{(1)}\right). \tag{5}$$

As previously, we want to take account of the possible non-stationarity of interest rates. In order to render both sides of equation (5) stationary, we model the swap spread as a function of the corporate bond spread and the Treasury term spread, such that:

$$SS_t^{(N)} = g\left(BS_t^{(N)}, Y_t^{(N)} - Y_t^{(1)}\right). \tag{6}$$

2.4 Financing cost

In Figure 1, the swap dealer is able to negotiate simultaneous swaps with counterparties X and Y. So there is no direct interest rate exposure to hedge, and the bid/offer spread to the swap spread should reflect only compensation for bearing credit risk, but not the cost for hedging. On the contrary, if swaps do not "match", the unbalanced position of the swap dealer has to be hedged. If the swap dealer has to pay the fixed rate to counterparty X, it is exposed to a decrease in the Treasury bond yield or in the swap spread. If the swap dealer has to receive the fixed rate from counterparty Y, it is exposed to an increase in the Treasury bond yield or in the swap spread. In each case, the swap dealer is exposed to the fixed rate on the swap, while the hedge will only cover the Treasury bond yield component of that rate. So the swap dealer will remain exposed to changes in the swap spread.

The hedge can be achieved by purchasing (or short-selling) Treasury bonds in the cash market. This position will be financed (or invested) with a series of overnight positions in the "repo" market (see Brown et al., 1994). So if the swap dealer has to pay the fixed rate, a higher repo rate will increase the cost of the hedge and then tend to widen its bid/offer spread. If the swap dealer has to receive the fixed rate, a higher repo rate will increase the return on the investment on the Treasury bond market and then tend to narrow its bid/offer spread, in a competitive market. The final effect of the repo rate on the swap spread is not clear: the sign of this effect depends on the proportion of swap dealers being in the offer side and in the bid side.

3 Data description and statistical diagnostics

3.1 The swap market

We first provide some evidence on the recent development of the IRS market (BIS, 1999). Remind that at the end of 1997, the notional principal amount of outstanding of IRS was evaluated at 22.2 trillions of dollars. At the end of 1992, this amount reached only 3.9 trillions. Between 1992 and 1997, the average rate of growth of the outstanding of this notional principal is 42%. The breakdown of the notional principal outstanding of IRS by classes of maturity (0 to 2 years, 2 to 5 years, 5 to 10 years, over 10 years) enables to identify the specific behavior of the different categories of IRS. First the outstanding corresponding to 0-2 year swaps represents more than 40% of the total amount of outstanding over the period covered. Second in opposite side, the lowest outstanding is associated with the highest maturity (over 10 years). For instance, at the end of 1997, the total amount of outstanding of this kind of swaps equals 1 trillion (4.6% of the total amount of outstanding). More generally, the weight of IRS with maturity less than 5 years (so called the short- and medium-term IRS) is very high, more than 70% between 1992 and 1997. However, the average rate of growth of the short- and medium-term swaps is weaker than that of the long-term one (over 5-year maturity). For the former it is equal to 40.2% (versus 48.4% for the long-term swaps). The 10-year swaps analysis could be justified by the trend of the long-term swaps.

It is difficult to understand the changes observed on the swaps markets by examining the total amount of outstanding only. The above breakdown by maturity shows that the behavior of the different components of IRS could be quite distinct. But we cannot explain the dramatic growth of the outstanding at this stage. Broadly we are not able to indicate whether this growth is due to changes in the interbank counterparty or to changes in all end-users behavior. First we split the amount of outstanding into two parts: all end-users (including brokers) and the interbank market (using here the BIS terminology). For this broad breakdown it appears that the outstanding of IRS concerning the interbank activity rises slightly more sharply than that of all end-users swaps. The average rate of growth of the interbank swaps is equal to 45% versus 39% for all end-users. At the end of 1997, the amount of outstanding of interbank swaps is 12 trillions of dollars, that of all end-users swaps equals 10.3 trillions.

Both interbank and all end-users play a major role on the swap markets. The second breakdown concerns all end-users counterparties. Financial institutions, corporates and governments are included in this group. The heterogeneity of this group (mixture of public and private sectors) is confirmed in terms of the changes of the different amounts of outstanding. As expected, the amounts of outstanding of these three counterparties increase dramatically over the 1990's (The average annual rate of growth is around 25% for corporates, 31% for governments and 47% for financial institutions.) However among the end-users, the weight of the financial institutions increases very sharply (around 70% at the end of the period). In opposite, the weight of the corporates declines strongly (around 24% in 1996 versus 34% at the end of 1992). This brief comparison shows that it is quite difficult to draw precise conclusions from the aggregate indicators.

To complete the descriptive analysis of the notional principal of IRS, we could examine the breakdown of the outstanding by location of the end-users, namely between North America, Europe and Asia. Over the period covered, Europe appears as the most important IRS market, preceding the American market. These markets represent about 50% and 40% of the all end-users amount of outstanding respectively.

To sum up this descriptive presentation, we can note that IRS market grew dramatically, during the 1990's. However some differences exist between the counterparties, the locations of the market and the maturities of swaps:

- among all end-users, Europe is preceding North America. Note that these two zones are the most important swap markets. From this point of view, it seems necessary to carefully analyze the European IRS market;
- the Asian IRS market is quite small as compared to those of Europe and North America;
- by maturity, the short- and medium-term (maturities less than 5 years) amounts of outstanding of IRS are heavily larger than the long-term one but the former

appears less dynamic (in terms of rate of growth);

- the IRS market is characterized by a very intensive interbank activity. Nevertheless all end-users contribute significantly to the swap market expansion in the 1990's;
- among all end-users group, financial institutions are the most important counterparties. Corporates appear at the second rank. However due to the unavailability of data, most of the studies have been devoted to corporates swaps.

To assess the liquidity of the swap market as compared to the euro-currency market, we lastly consider the bid/offer spread. Indeed, this spread is often used to measure the market liquidity.

Table 1 provides summary statistics for the bid/offer spreads on the euro-currency market (for maturities overnight, 7 day, 1 month, 3 months, 6 months and 1 year) and on the swap market (for maturities 2 years, 3 years, 4 years, 5 years, 7 years and 10 years). For the three countries, the average as well as the median bid/offer spreads are larger on the euro-currency market than on the swap market. If we concentrate on the median spreads, that are less sensitive to extreme values, we obtain a median spread of 12.5 bp for all maturities of the euro-currency markets; on the swap markets the median spread is between 3 bp and 5 bp. In fact, the main difference the three countries concerns the volatility of the spread on the euro-currency markets. Indeed the standard deviation of shortest-term spreads (overnight rates) are larger in Europe than in the US. During the French currency turmoils between mid-1992 and mid-1993, the bid-offer spreads clearly widen on the euro-franc market, with a maximum of 500 bp for the overnight rate and 75 bp for the 1-year rate. However, even during this troubled period, the bid/offer spread on the swap market did not exceed 15 bp.

3.2 Data description

To test the models presented in the previous section for the US, German and French swap markets, we use two data sets. The first one consists in usual data, such as euro-currency and interbank interest rates (with maturities of one day, seven days, one, three, six and twelve months), swap rates (with maturities of two, three, four, five, seven and ten years), the 10-year Treasury bond rate and the 10-year AAA-rated corporate bond rate. The series mainly come from Datastream. Our second data set, with the same covering period and frequency than the first one, contains zero-coupon yield curves for the swap market. The data and the methodology used for these interpolations are described in the Appendix. The data covers the period from January 3, 1992 to December 25, 1998 at a weekly frequency (Friday closing quotes).

Estimating the weighted average of 3-month forward rates is rather challenging. The cash flows of 3-month forward contracts on the reference short-term rates (euro-dollar, euro-mark or PIBOR rates) can be approximated by the 3-month forward rates obtained from the zero-coupon curve. Therefore we need both zero-coupon rates (to evaluate discount factors) and forward rates. In a first step, we compute zero-coupon swap rates, using the approach presented in the Appendix. Theoretically we should interpolate a complete zero-coupon curve to evaluate the whole sequence of 3-month forward rates (that is about 40 forward rates for $N = 10$ years). This approach would necessitate the use of an algorithm for generating the complete underlying zero-coupon yield curve. We use a simplified approach, that avoids such an interpolation. Indeed using 3-month, 6-month, 1-year reference short-term rates and 2-year to 10-year swap rates, it is easy to evaluate zero-coupon rates for the same maturities (see the Appendix). We are then able to estimate the corresponding forward rates, according to:

$$f_t^{(m,n)} = \left(\frac{\left(1 + z_t^{(n+m)}\right)^{n+m}}{\left(1 + z_t^{(n)}\right)^{n}} \right)^{1/m}$$

where $f_t^{(m,n)}$ is the forward rate at time t for a contract maturing at date $t+n$ for $t+n+m$, with m and n expressed in months. Therefore with the data at hand, we are able to compute $f_t^{(3,3)}$, $f_t^{(6,6)}$, $f_t^{(12,12j)}$, for $j = 1, ..., 9$. The weighted sequence of forward rates was then evaluated as:

$$\frac{\sum_{i=1}^{M} \rho_{t+n_i} f_t^{(m_i,n_i)}}{\sum_{i=1}^{M} \rho_{t+n_i}} \qquad \text{with} \qquad \rho_{t+n_i} = \frac{1}{\left(1 + z_t^{(n_i)}\right)^{n_i}}$$

and $M = 11$, $m = \{3, 6, 12, ..., 12\}$ and $n = \{3, 6, 12, 24, ..., 108\}$. This estimated sequence of future forward rates is very close to the true one, and it avoids to interpolate zero-coupon curves.

In section 2, we have shown that the swap rate can be expressed, under the arbitrage-free assumption, as the weighted average of short-term forward rates, that is $S_t^{(N)} = \Sigma_{N,t}$. In most countries, such data are not available, on a historical basis, for forward contracts maturing in the far future (typically beyond one year).[1] We have then used interpolated zero-coupon yield curves based on euro-currency and PIBOR rates (for maturities below one year) and on swap rates (for maturities beyond one year). We obtain implicit forward rates that can be used to compute $\Sigma_{N,t}$. But in this case, it can be shown that both terms of the previous expression should be identically equal.[2]

Table 2 provides some summary statistics on the spread between the swap rate $S_t^{(N)}$ and the weighted average of short-term implicit forward rates $\Sigma_{N,t}$, for $N = 2, ..., 10$ years. The spread is very low for the three countries, whatever the maturity. The median spread is lower than 1 bp for the 10-year spread and between 2 and 3 bp for the 2-year spread. The maximum reaches 7 bp for the French 2-year spread. This difference mainly comes from the approximation detailed in section 3.2.

It is worth noting that these weighted averages of short-term implicit forward rates cannot be directly used to estimate the swap rates, since they are identically related. But as suggested by Brown et al. (1994), 3-month euro-currency and PIBOR futures contracts can be used for such a purpose. As already said however, such contracts are not available for maturities longer than one year over a long sample, at least for Germany and France. We therefore consider in the following the weighted average of 3-month future rates over one year, denoted $\Sigma_{1,t}$.

[1] Using Eurodollar futures contracts with maturity up to 4 years, Minton (1997) studies the relation between OTC swap rates and swap rates derived from Eurodollar futures prices. Sun et al. (1993) deal with the relation between LIBOR par bond yields and swap rates. In both cases, the series are highly correlated.

[2] If discount factors and forward rates are extracted from the same euro-currency and PIBOR-swap curve, the expression $\Sigma_{N,t}$ can be rewriten as $\left(1 - \rho_{t+N}\right) / \sum_{i=1}^{N} \rho_{t+i}$.

3.3 Non-stationarity of interest rates

We now check some statistical properties of the four different long-term interest rates used in this paper (the swap rate, the theoretical swap rate, the corporate bond rate and the Treasury bond rate). We first consider long-run properties, that is non-stationarity and cointegration.

From a theoretical point of view, interest rates have to be stationary (since nominal rates are bounded by zero). But many papers give empirical evidence in favor of the non-stationarity of interest rates (Hall, Anderson, and Granger, 1992, for short-term rates, Campbell and Shiller, 1987, and Shea, 1992, for long-term rates). In this section we only present standard tests for stationarity (namely, augmented Dickey and Fuller and KPSS tests), but we do not consider more recent and more sophisticated tests. Indeed our study is based on some results concerning the non-stationarity of interest rates, but it does not focus on this particular point.

Table 3 Panel A reports the ADF test and the KPSS test. The first one is based on the null hypothesis of non-stationarity, whereas the second one is based on the null hypothesis of stationarity. Both tests allow to conclude that our different long-term rates can be considered as non-stationary in the three countries. This result only means that over the sample under study, interest rates behave as non-stationary series. Therefore dealing with rates in level may be misleading, since standard techniques cannot be used. But we do not want to model the dynamics of interest rates (in first difference), because pricing models are based on an arbitrage free hypothesis, and therefore it says nothing about the dynamics of series. On the contrary we are interested in understanding links between spreads, such as swap spreads, corporate bond spreads and term spreads.

3.4 Statistical properties of spreads

Table 3 Panel B reports stationarity tests concerning spreads. As far as swap spreads and corporate bond spreads are concerned, results are rather ambiguous: for the US, we reject the stationarity of the swap spread, whereas we conclude in favor of a stationary bond spread. We obtain the reverse result for German data. Last, for France, both spreads appear to be non-stationary. Note that the US swap spread can be considered as stationary over the 1992-97 subperiod. Fig. 2 display the swap spread and the corporate bond spread for the three countries. For US and French data, the swap spread and the bond spread are in line over the whole sample. For Germany, the two spreads display similar patterns from 1996 only: the bond spread decreased significantly between 1992 and 1994, whereas the swap spread remained stable. All in all, the non-stationary component of these series does not appear so predominant. We therefore present in Table 4 summary statistics on these spreads. First German and French swap spreads have similar summary statistics: the median spread is 27 bp and 23 bp respectively, whereas it is as high as 41 bp for US data. The standard deviation is basically the same. However, as shown in Fig. 1, swap spreads display rather different patterns in the three countries. The US swap spread remained very stable at about 40 bp until the beginning of 1998; and then it dramatically increased up to 90 bp. The development of the German swap spread is rather similar, at about 20-30 bp between 1992 and 1997 and with an increase up to 60 bp in 1998; in France however we note a first decrease between 1992 and 1994 from 50 bp to 10 bp; as in the US and Germany, the French swap spread lastly increases in 1998 up to

40 bp.

The bond spread is almost always positive. In few cases for German and French data, the bond spread is negative, that is not consistent with a positive default risk premium. This can be explained by differences in the collecting data procedures. We note that the median spread is slightly higher in the US than in Europe. However we use the same rating (AAA) for corporate bonds in the three countries. The median US bond spread is 37 bp, whereas it is only 26 bp on German and French data. Fig. 1 displays the relation between the swap spread and the corporate bond spread.

Considering now the theoretical swap spread and the term spread, we obtain more clear-cut results. First both spreads are clearly non-stationary in all countries (Table 3 Panel B). Moreover, Fig. 2 displays the theoretical swap spread and (minus) the term spread. Both series present very similar patterns. In particular they do not look like stationary processes.

4 The estimates results

4.1 The IRS as a function of short-term forward contracts

We first analyze the relation established in section 2.2 between the swap rate, the sequence of future short-term rate and the Treasury bond rate (equation (1)). Under arbitrage-free assumption, the swap spread is expressed as the difference between the theoretical swap rate and the long-term Treasury bond rate (the so-called theoretical spread).

We first consider the system $\left(S_t^{(10)}, \Sigma_{1,t}, Y_t^{(10)}\right)$. The three series of this system are non-stationary. The stationarity of the spreads obtained from these rates is not well established. So we want to test for the cointegration rank and the form of the cointegration vector.

We estimated the joint dynamics of the system using Johansen's (1988) maximum likelihood methodology. Tests for the deterministic components of the system lead us to allow a constant term in the cointegration relationships. The optimal lag length, selected by the HQ (Hannan and Quinn, 1979) information criterion, is $p = 1$ for the US and $p = 2$ for European countries, whatever the cointegration rank. Lastly, based on the usual maximal eigenvalue λ_{max} and trace statistics λ_{trace}, proposed by Johansen (1988), at the significance level of 5%, we choose one cointegration vector ($r = 1$) (Table 5 Panel A).

If we now consider the cointegration vector, we note that the coefficients of the three interest rates nearly sum to 0. This indicates that the cointegration relationship could involve spreads only. The likelihood ratio test statistic for the hypothesis that the cointegration vector is $(1, -\alpha, \alpha - 1, \beta)$ is distributed as a χ^2 with 1 degree of freedom. The test of this hypothesis is not rejected for the US at the usual significance level; for European countries however the null hypothesis is not rejected at a 2% and a 3% significance level respectively. Moreover in the case of Germany, we notice that the parameter of the theoretical swap spread ($\Sigma_{1,t} - Y_t^{(10)}$) has the wrong sign.

We now turn to the univariate estimate of relation (2) with the following form

$$SS_t^{(N)} = \alpha_0 + \alpha_1 \left(\Sigma_{1,t} - Y_t^{(N)}\right) + \varepsilon_{1,t}. \tag{7}$$

Table 6 presents the estimates of this regression. First the information content of the theoretical swap spread about the swap spread depends on the country. In the US, the slope parameter is significantly positive but rather low and the adjusted R^2 is 0.18 only. In Germany the slope parameter is negative and the information content is almost null. Last in France on the contrary the information content appears to be quite high with an adjusted R^2 equal to 0.63. We next consider the effect of introducing the overnight repo rate in the RHS of the previous relations. This allows to take into account the hedging cost of the swap dealer. The repo rate is measured as the Friday overnight interbank rate. As in Brown et al. (1994), we obtain a significantly negative parameter for the repo rate. It is higher on US data than on German and French data. In this case, the slope parameter is higher than in the previous regression: it is positive for the three countries. Adding the Treasury term spread in the previous relation does not help to explain the swap spread. Indeed the term spread parameter is never significant, and the theoretical spread parameter remains unchanged.

It is worth noting that a small slope parameter (significantly less than one) does not imply the rejection of the theoretical model. Indeed the theoretical model of section 2 predicts a slope parameter equal to one for a sequence of short forward rates over 10 years. Here we use a sequence of forward rates over one year only. Therefore, nothing can be said on the true value of this slope parameter.

4.2 The IRS as a function of corporate bonds

In equation (4), we expressed the swap spread as a function of the bond spread and the weighted average of the sequence of future credit spreads over euro-currency and PIBOR rates. These credit spreads are however determined on non-public markets (e.g. bank loans) or quite illiquid over-the-counter markets (e.g. floating-rate notes). This prevents to directly test relation (4) including the credit spread. Due to the lack of relevant data, the credit risk is often measured by an aggregate measure, the corporate bond spread or Treasury term spread.

Table 7 presents cointegration analysis in the system $\left(S_t^{(10)}, B_t^{(10)}, Y_t^{(10)}\right)$. As previously we are able to identify one cointegration vector at a 5% significance level (Panel A). The null hypothesis of the 0-sum of parameters in this cointegration vector is not rejected for the US, but should be rejected in the European countries; in Germany we note that the parameter of the corporate bond spread has the wrong sign as compared with the theory. The US swap spread appears to overreact to the corporate bond spread, since the slope is 1.44. For French data, the slope is 0.84 but we have to reject the null hypothesis of 0-sum (Panel B).

The theoretical model can now be tested with the regression between the swap spread and the corporate bond spread:

$$SS_t^{(N)} = \beta_0 + \beta_1 BS_t^{(N)} + \varepsilon_{2,t}. \tag{8}$$

If the swap spread is a function of the corporate bond spread and if the bond spread and the credit spread are not correlated, the slope estimate in (8) should equal unity. Some other proxies of the credit risk may be introduced in relation (8). Fama and French (1989) suggest that the Treasury term spread may capture business condition risk resulting from monetary policy shocks. Similarly Longstaff and Schwartz (1995) obtain that the level of interest rates are negatively correlated to the corporate credit spreads. In this case however, we should emphasize the high correlation between the bond rate and the Treasury long-term bond rate, that may introduce multi-colinearity in such a regression.

Table 8 reports estimates of these relationships. As far as the swap spread is concerned, we find a positive effect of the corporate bond spread. It is close to 0 for Germany, but for the US and France it is rather high. Moreover, when the repo rate is added, in order to take account of financing costs, the parameter of the corporate bond rate increases slightly, particularly for German data, in which case it reaches 0.30. The repo rate has a negative effect on the swap spread for Germany. But it is surprisingly positive for the US and France.

Lastly when the Treasury term spread is introduced in the regression as a proxy of the credit risk, the slope parameter of the corporate bond spread increases a little as compared to the previous regression in Germany and France. The sign of the term spread is ambiguous however: it is negative in the US, but positive in European countries. When the repo rate is added, the term spread has a negative effect on the swap spread in the three countries, as suggested by Fama and French (1989). The effect of the repo rate is still negative, except in the US.

5 Conclusion

We considered in this paper two standard theoretical swap pricing models: the first one expresses the swap as a sequence of short-term forward contracts; in the second one, the swap is interpreted as a portfolio of corporate bonds. The empirical analysis indicates some elements of validity for both approaches.

The estimated coefficients of the model based on short-term forward rates are quite difficult to interpret, since the main determinant –the theoretical spread– has to be replaced by a sequence of forward rates over a shorter period of time. This model seems to be relevant for French swaps and, to a lesser extent, for US swaps. For the second model, as expected the swap rate reacts to corporate bond rate changes. The slope parameter is close to one for US data. In Germany the introduction of the repo rate to account for financial costs increases the information content of the corporate bond spread about the swap spread.

If we take into account the relative lack of the raw information, the results are cheerful. The extension of this work to the Euro zone could be considered.

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Appendix: Method for interpolating the swap yield curves

Swap yield curves are constructed using two kinds of data: the short end of the yield curve is based on euro-currency and interbank rates (PIBOR), for maturities 1 day, 1 week, 1, 3, 6 and 12 months; the short end of the yield curve is based on swap rates, for maturities 2, 3, 4, 5, 7 and 10 years.

Before interpolating the zero-coupon yield curve, two problems have to be solved: First, swap data begins with the two-year rate, so it is necessary to use interbank rates for the short end of the curve; Some maturities (6, 8 and 9 years) are not collected in our database, so we use a linear interpolation.

First interbank rates $interb$ (expressed as exact/360) have to be converted in actuarial rates as exact/exact r_m:

$$r_{m,i} = \left(1 + interb_i \frac{t_{m,i}}{360}\right)^{b/t_{m,i}} - 1$$

where b is the base for actuarial rates ($b = 365$ or 366).

We then obtain zero-coupon actuarial rates for maturities 1, 3, 6 ,12 months ($r_{m,1}$, $r_{m,2}$, $r_{m,3}$, $r_{m,4}$). Swap zero-coupon rates are iteratively obtained as:

i) the one-year zero-coupon rate is equal to the one-year interbank rate, $r_1 = r_{m,4}$. In the following, one notes $C_1 = r_{m,4}$;

ii) the two-year zero-coupon rate (r_2) can be obtained from the two following strategies: one purchases a two-year swap with price 1, with an annual coupon C_2; one borrows $C_2/(1+r_1)$ at rate r_1 for one year, in order to obtain a null intermediate flow after one year (one receives C_2 from the swap, one repays C_2 from the borrowing). If the initial investment is set equal to the expected cash flows, one obtains:

$$1 - \frac{C_2}{1+r_1} = \frac{1+C_2}{(1+r_2)^2} \qquad \Leftrightarrow \qquad r_2 = \left[\frac{1+C_2}{1-\frac{C_2}{1+r_1}}\right]^{1/2} - 1$$

iii) using the same procedure for following maturities, one obtains a recursive formula for the zero-coupon rate maturing in $t+n$ as a function of the $(n-1)$ preceding zero-coupon:

$$r_n = \left[\frac{1+C_n}{1 - C_n \sum_{i=1}^{n-1} \frac{1}{(1+r_i)^i}}\right]^{1/n} - 1 \qquad n = 2, ..., N.$$

The discount factor, defined by $\rho_n = 1/(1+r_n)^n$, can be directly obtained as:

$$\rho_n = \frac{1 - C_n \sum_{i=1}^{n-1} \rho_i}{1+C_n} \qquad n = 2, ..., N$$

with $\rho_1 = 1/(1+r_1)$. Noting that $C_n \sum_{i=1}^{n-1} \rho_i + (1+C_n)\rho_n = 1$, $n = 2, ..., N$, this problem can be directly solved as:

$$\begin{pmatrix} 1+C_1 & 0 & 0 & \cdots & 0 \\ C_2 & 1+C_2 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ C_N & C_N & C_N & \cdots & 1+C_N \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_N \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$$

or $AP = e$, where P is the vector containing the N discount factors. The discount factors can then be defined as $P = A^{-1}e$. Lastly the vector of zero-coupon rates is obtained by $r_n = (1+\rho_n)^{1/n}$, $n = 1, ..., N$.

Table 1a: Summary statistics on bid-offer spreads: interbank market rates

92-98	overnight	7 days	1 month	3 months	6 months	1 year
US						
mean	13.7	12.6	12.7	13.0	12.9	13.3
std	4.0	3.4	3.9	4.2	4.2	4.8
min	6.2	3.1	3.1	3.1	3.1	3.1
median	12.5	12.5	12.5	12.5	12.5	12.5
max	25.0	31.3	31.3	31.3	37.5	50.0
Germany						
mean	17.1	13.4	12.9	12.8	12.9	13.4
std	10.2	4.6	4.7	4.6	4.8	5.4
min	6.3	3.1	3.1	3.1	3.1	3.1
median	12.5	12.5	12.5	12.5	12.5	12.5
max	87.5	31.2	43.8	28.1	34.4	34.4
France						
mean	19.2	18.3	16.6	15.8	15.7	15.7
std	39.3	24.4	16.5	13.7	11.8	8.4
min	3.1	6.2	6.3	6.2	6.3	6.3
median	12.5	12.5	12.5	12.5	12.5	12.5
max	500.0	300.0	200.0	200.0	100.0	75.0

Table 1b: Summary statistics on bid-offer spreads: swap rates

1992-98	2 years	3 years	4 years	5 years	7 years	10 years
US						
mean	3.4	3.4	3.5	3.5	3.5	3.5
std	0.6	0.6	0.6	0.6	0.6	0.6
min	1.0	2.0	2.0	1.0	2.0	2.0
median	3.0	3.0	3.0	3.0	3.0	3.0
max	5.0	5.0	5.0	5.0	6.0	6.0
Germany						
mean	4.0	4.1	4.5	4.5	4.6	4.6
std	0.4	0.5	0.6	0.6	0.7	0.7
min	2.0	2.0	3.0	3.0	3.0	3.0
median	4.0	4.0	5.0	5.0	5.0	5.0
max	6.0	6.0	6.0	6.0	10.0	10.0
France						
mean	4.8	4.8	4.8	4.7	4.8	4.8
std	1.6	1.6	1.5	1.5	1.7	1.7
min	3.0	3.0	4.0	4.0	4.0	3.0
median	4.0	4.0	4.0	4.0	4.0	4.0
max	15.0	15.0	15.0	15.0	15.0	15.0

Table 2: Summary statistics on the spread between the swap rate, $S_t^{(i)}$, and the weighted average of short-term implicit forward rates, $\Sigma_{i,t}$, $i = 2, ..., 10$

1992-98	2 years	3 years	4 years	5 years	7 years	10 years
US						
mean	2.6	2.0	1.6	1.4	1.1	0.9
std	0.7	0.5	0.4	0.3	0.2	0.2
min	1.3	1.1	0.9	0.8	0.7	0.6
median	2.7	2.0	1.6	1.4	1.1	0.9
max	4.7	3.4	2.7	2.2	1.8	1.5
Germany						
mean	2.4	1.8	1.5	1.3	1.0	0.8
std	1.4	0.8	0.6	0.5	0.4	0.3
min	0.7	0.6	0.3	0.2	0.1	0.0
median	2.0	1.6	1.4	1.2	1.0	0.8
max	6.4	4.2	3.1	2.6	1.9	1.5
France						
mean	2.9	2.2	1.7	1.4	1.1	1.0
std	1.6	1.0	0.8	0.7	0.5	0.4
min	0.8	0.4	0.4	0.2	0.2	0.1
median	2.6	1.9	1.6	1.4	1.1	0.9
max	7.0	4.8	3.6	3.0	2.3	1.9
max	7.1	4.9	3.9	3.2	2.4	1.9

Table 3: Stationarity tests for long-term interest rates and spreads

Panel A: Interest rates			
	ADF	*KPSS*	
US			
$S_t^{(10)}$	-2.302	0.644	^{b}
$Y_t^{(10)}$	-1.494	0.780	^{a}
$B_t^{(10)}$	-2.149	0.808	^{a}
$\Sigma_{1,t}$	-1.683	1.370	^{a}
Germany			
$S_t^{(10)}$	-0.335	2.065	^{a}
$Y_t^{(10)}$	-0.536	2.073	^{a}
$B_t^{(10)}$	-0.844	2.182	^{a}
$\Sigma_{1,t}$	-2.408	2.372	^{a}
France			
$S_t^{(10)}$	-1.120	2.116	^{a}
$Y_t^{(10)}$	-0.750	2.029	^{a}
$B_t^{(10)}$	-0.850	2.125	^{a}
$\Sigma_{1,t}$	-1.670	2.424	^{a}

Panel B: Interest rate spreads				
	ADF		*KPSS*	
US				
$SS_t^{(10)}$	-0.330		0.894	^{a}
$BS_t^{(10)}$	-2.247		0.265	
$\Sigma_{1,t} - Y_t^{(10)}$	-0.659		2.203	^{a}
$Y_t^{(10)} - Y_t^{(1)}$	-0.640		2.005	^{a}
Germany				
$SS_t^{(10)}$	-3.006	^{b}	0.382	^{c}
$BS_t^{(10)}$	-2.342		1.491	^{a}
$\Sigma_{1,t} - Y_t^{(10)}$	-2.030		1.649	^{a}
$Y_t^{(10)} - Y_t^{(1)}$	-1.681		1.591	^{a}
France				
$SS_t^{(10)}$	-2.113		1.292	^{a}
$BS_t^{(10)}$	-2.258		2.144	^{a}
$\Sigma_{1,t} - Y_t^{(10)}$	-1.662		2.107	^{a}
$Y_t^{(10)} - Y_t^{(1)}$	-1.741		1.908	^{a}

The ADF statistics are based on the following regression: $\Delta x_t = a + bx_{t-1} + \sum_{i=1}^{l} c_i \Delta x_{t-i} + u_t$, where x_t is the interest rate, u_t is the error term. The order of the autoregressive process, l, is selected in order to whiten the residuals. The critical values are from Fuller (1976). The KPSS statistics are computed with 12 lags. The critical values are from Kwiatkowski et al. (1992). ^{a} and ^{b} indicate that the statistics are significant at a 1% and 5% significance level respectively.

Table 4: Summary statistics on variables used in the estimations

	$SS_t^{(10)}$	$\Sigma_{1,t} - Y_t^{(10)}$	$BS_t^{(10)}$	$Y_t^{(10)} - Y_t^{(1)}$
US				
mean	43.1	-110.8	38.1	191.2
std	11.2	105.8	8.4	111.1
min	21.9	-311.2	19.2	-14.7
median	41.0	-73.1	37.0	163.1
max	94.1	61.9	77.6	390.1
Germany				
mean	27.5	-110.5	33.6	145.3
std	10.5	139.8	23.4	121.6
min	-0.4	-314.9	-11.2	-129.6
median	26.6	-144.5	26.3	178.5
max	62.9	205.7	119.4	320.7
France				
mean	26.1	-83.8	28.1	65.1
std	11.7	123.2	14.8	180.5
min	2.6	-253.1	-24.0	-453.5
median	22.5	-111.2	26.0	130.2
max	62.0	213.3	85.0	280.7

Table 5: Tests for the cointegration rank in the system $\left(S_t^{(10)}, Y_t^{(10)}, \Sigma_{1,t}\right)$

Panel A: Test for cointegration rank					
country $k-r$	λ	λ_{max}		λ_{trace}	
US ($p=1$)					
3	0.069	26.01	[b]	36.87	[b]
2	0.022	7.94		10.86	
1	0.008	2.92		2.92	
Germany ($p=2$)					
3	0.076	28.37	[a]	42.74	[a]
2	0.036	13.30		14.37	
1	0.003	1.07		1.07	
France ($p=2$)					
3	0.067	25.14	[b]	34.94	[b]
2	0.023	8.54		9.77	
1	0.003	1.23		1.23	

Panel B: Cointegration vector					
country	$S_t^{(10)}$	$Y_t^{(10)}$	$\Sigma_{1,t}$	intercept	0-sum hypothesis ($\chi^2(1)$)
US					
unconstrained	1	-0.913	-0.041	-0.767	
constrained	1	-0.949	-0.051	-0.488	2.18 (0.14)
Germany					
unconstrained	1	-0.965	0.012	-0.548	
constrained	1	-1.028	0.028	-0.226	5.68 (0.02)
France					
unconstrained	1	-0.887	-0.088	-0.510	
constrained	1	-0.927	-0.073	-0.327	4.78 (0.03)

Note: The estimation is performed with a constant term in the cointegration relations and $p = 1$ for the US and $p = 2$ for European countries. k is the number of variables, r is the cointegration rank. λ contains the eigenvalues of the system. λ_{max} and λ_{trace} are the cointegration test statistics proposed by Johansen (1988). The critical values for λ_{max} and λ_{trace} are from Osterwald-Lenum (1992, Table 2*). [a] and [b] indicate that the statistics are significant at a 1% and 5% significance level respectively. The constrained cointegration vector is $(1, -\alpha, \alpha - 1, \beta)$ and the 0-sum hypothesis test statistic is distributed as a χ^2 with one degree of freedom.

Table 6: The swap as a function of short-term forward contracts

	independent variables				adj. R^2	see
	intercept	$\Sigma_{1,t} - Y_t^{(10)}$	$Y_t^{(10)} - Y_t^{(1)}$	repo rate		
US						
	0.4813	0.0448			0.1777	0.1015
	(0.0077)	(0.0050)				
	0.7248	0.0846		-0.0434	0.2197	0.0989
	(0.0543)	(0.0100)		(0.0096)		
	0.4825	0.0434	-0.0014		0.1755	0.1016
	(0.0148)	(0.0166)	(0.0158)			
	0.7221	0.0927	0.0075	-0.0439	0.2181	0.0990
	(0.0546)	(0.0194)	(0.0155)	(0.0097)		
Germany						
	0.2650	-0.0889			0.0116	0.1041
	(0.0069)	(0.0039)				
	0.5147	0.0434		-0.0368	0.1477	0.0967
	(0.0332)	(0.0077)		(0.0048)		
	0.2743	-0.0245	-0.0183		0.0105	0.1042
	(0.0138)	(0.0203)	(0.0234)			
	0.5171	0.0382	-0.0059	-0.0368	0.1455	0.0968
	(0.0343)	(0.0206)	(0.0218)	(0.0048)		
France						
	0.3247	0.0751			0.6291	0.0712
	(0.0045)	(0.0030)				
	0.4381	0.1045		-0.0151	0.6542	0.0688
	(0.0221)	(0.0063)		(0.0029)		
	0.3321	0.0948	0.0140		0.6318	0.0710
	(0.0059)	(0.0107)	(0.0073)			
	0.4380	0.1047	0.0001	-0.0151	0.6532	0.0689
	(0.0227)	(0.0106)	(0.0076)	(0.0031)		

Table 7: Tests for the cointegration rank in the system $\left(S_t^{(10)}, Y_t^{(10)}, B_t^{(10)}\right)$

Panel A: Test for cointegration rank					
country $k-r$	λ	λ_{max}		λ_{trace}	
US ($p=2$)					
3	0.080	30.16	^{a}	44.21	^{a}
2	0.033	12.11		14.05	
1	0.005	1.95		1.95	
Germany ($p=2$)					
3	0.062	23.16	^{b}	40.76	^{b}
2	0.035	12.83		17.60	
1	0.013	4.77		4.77	
France ($p=2$)					
3	0.142	55.21	^{a}	68.48	^{a}
2	0.024	8.80		13.27	
1	0.123	4.47		4.47	

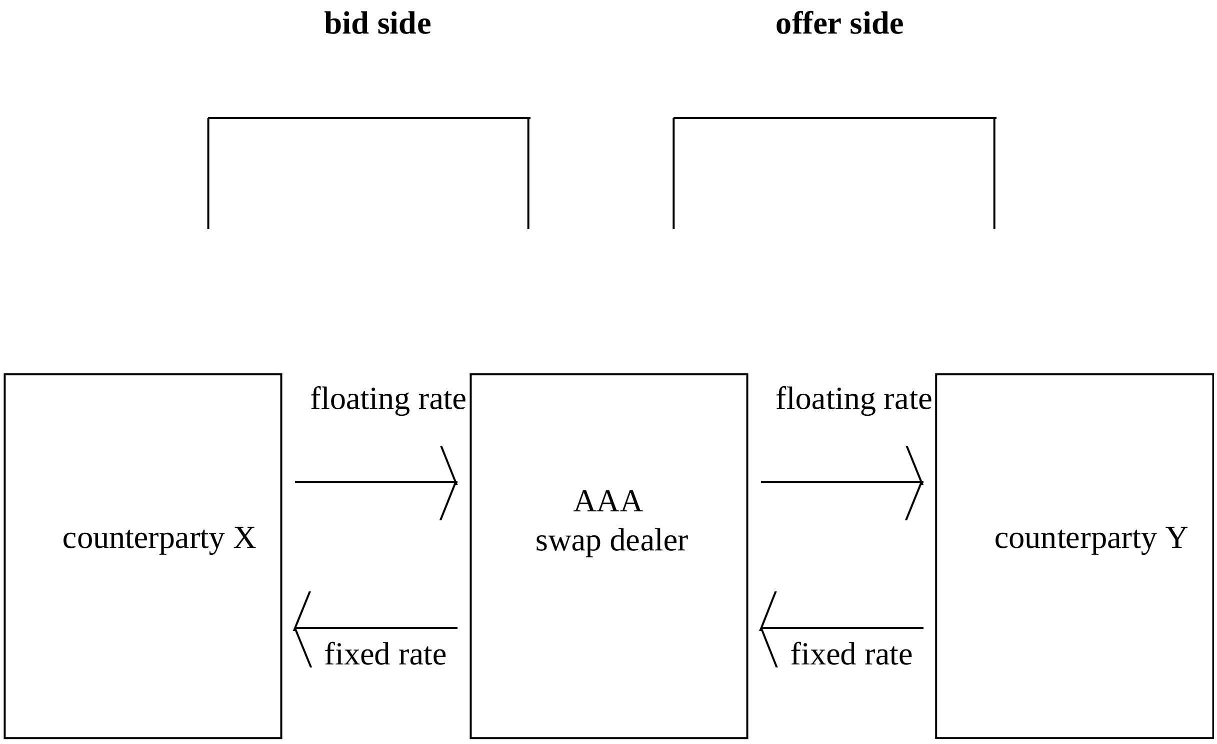
Panel B: Cointegration vector					
country	$S_t^{(10)}$	$Y_t^{(10)}$	$B_t^{(10)}$	intercept	0-sum hypothesis ($\chi^2(1)$)
US					
unconstrained	1	0.749	-1.803	0.600	
constrained	1	0.442	-1.442	0.121	3.31 (0.07)
Germany					
unconstrained	1	-0.732	-0.206	-0.628	
constrained	1	-1.061	0.061	-0.327	5.15 (0.02)
France					
unconstrained	1	-0.032	-0.940	-0.212	
constrained	1	-0.160	-0.840	-0.053	12.20 (0.00)

Note: The estimation is performed with a constant term in the cointegration relations and $p=2$. k is the number of variables, r is the cointegration rank. λ contains the eigenvalues of the system. λ_{max} and λ_{trace} are the cointegration test statistics proposed by Johansen (1988). The critical values for λ_{max} and λ_{trace} are from Osterwald-Lenum (1992, Table 2*). ^{a} and ^{b} indicate that the statistics are significant at a 1% and 5% significance level respectively. The constrained cointegration vector is $(1, -\alpha, \alpha-1, \beta)$ and the 0-sum hypothesis test statistic is distributed as a χ^2 with one degree of freedom.

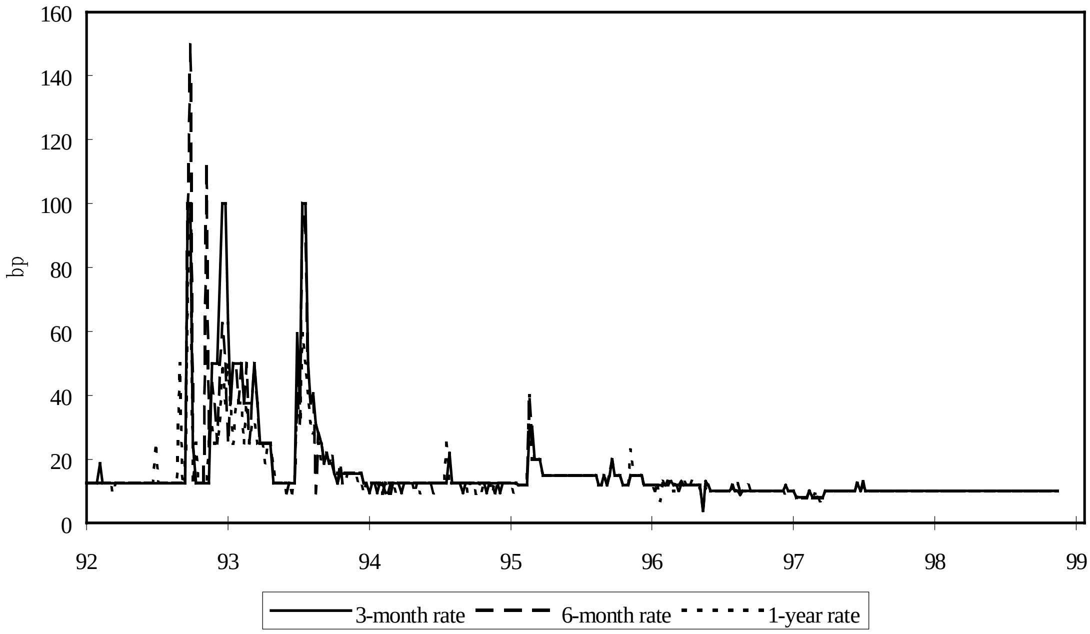
Table 8: The swap as a function of the corporate bond

		independent variables			adj. R^2	see
	intercept	$B_t^{(10)}$	$Y_t^{(10)} - Y_t^{(1)}$	repo rate		
US						
	0.0591	0.9754			0.5315	0.0766
	(0.0188)	(0.0480)				
	-0.1367	1.0376		0.0374	0.6662	0.0646
	(0.0226)	(0.0408)		(0.0031)		
	0.1418	0.9211	-0.0324		0.6327	0.0678
	(0.0185)	(0.0429)	(0.0032)			
	-0.1132	1.0272	-0.0034	0.0345	0.6656	0.0647
	(0.0458)	(0.0445)	(0.0057)	(0.0057)		
Germany	0.2634	0.0338			0.0030	0.1045
	(0.0096)	(0.0233)				
	0.3650	0.3007		-0.0367	0.2687	0.0895
	(0.0120)	(0.0306)		(0.0032)		
	0.1795	0.1504	0.0307		0.0597	0.1015
	(0.0198)	(0.0333)	(0.0064)			
	0.4672	0.2742	-0.0269	-0.0472	0.2908	0.0882
	(0.0315)	(0.0310)	(0.0077)	(0.0043)		
France	0.1041	0.6424			0.6471	0.0695
	(0.0071)	(0.0248)				
	0.0933	0.5863		0.0042	0.6506	0.0691
	(0.0087)	(0.0360)		(0.0019)		
	0.1630	0.4547	-0.0197		0.6843	0.0657
	(0.0112)	(0.0369)	(0.0030)			
	0.2485	0.4747	-0.0367	-0.0135	0.7018	0.0639
	(0.0211)	(0.0360)	(0.0046)	(0.0029)		

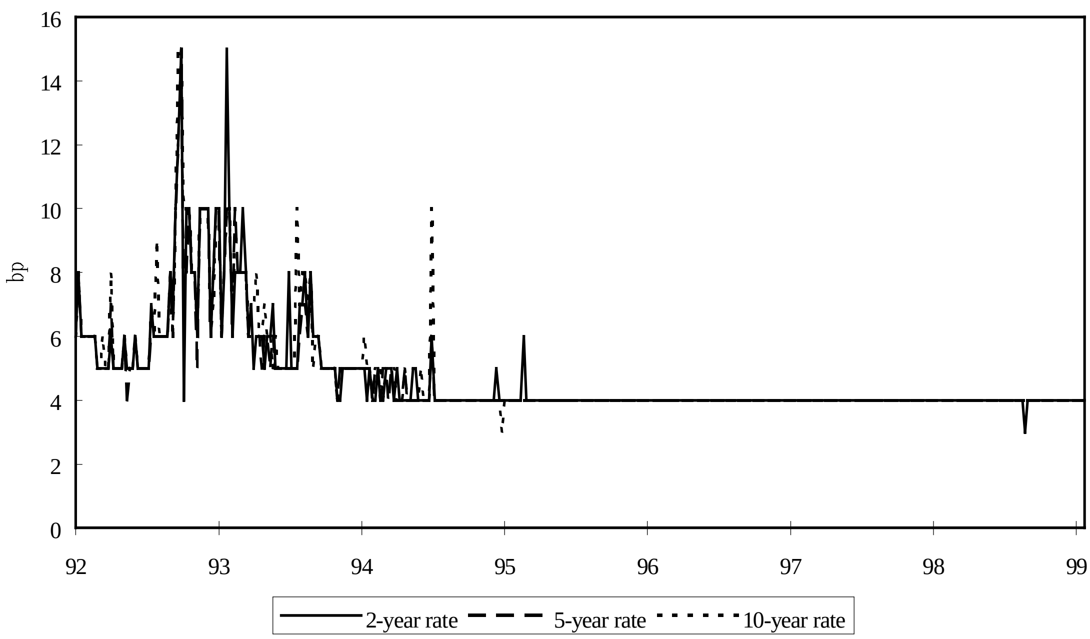
Figure 1: An intermediated interest rate swap



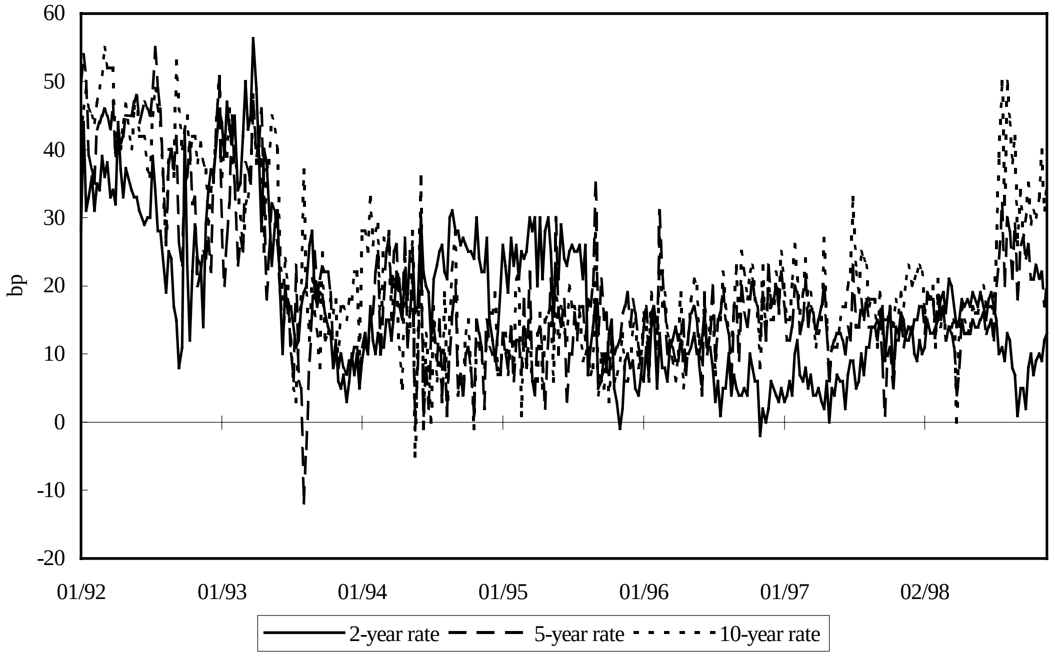
Graph 1: Bid/offer spreads on the interbank market



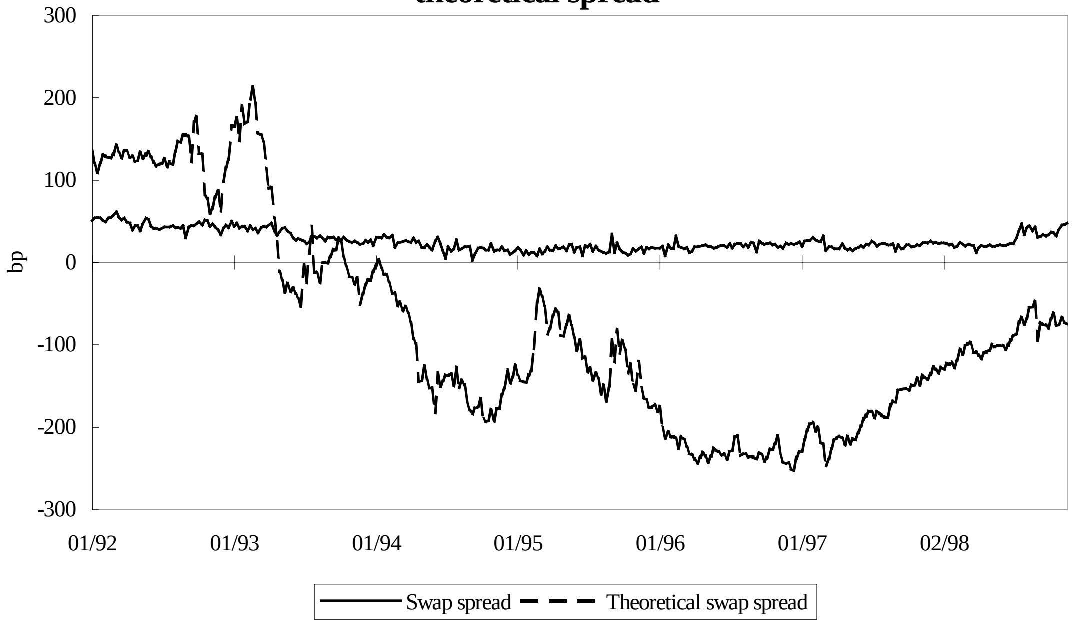
Graph 2: Bid/offer spreads on the swap market

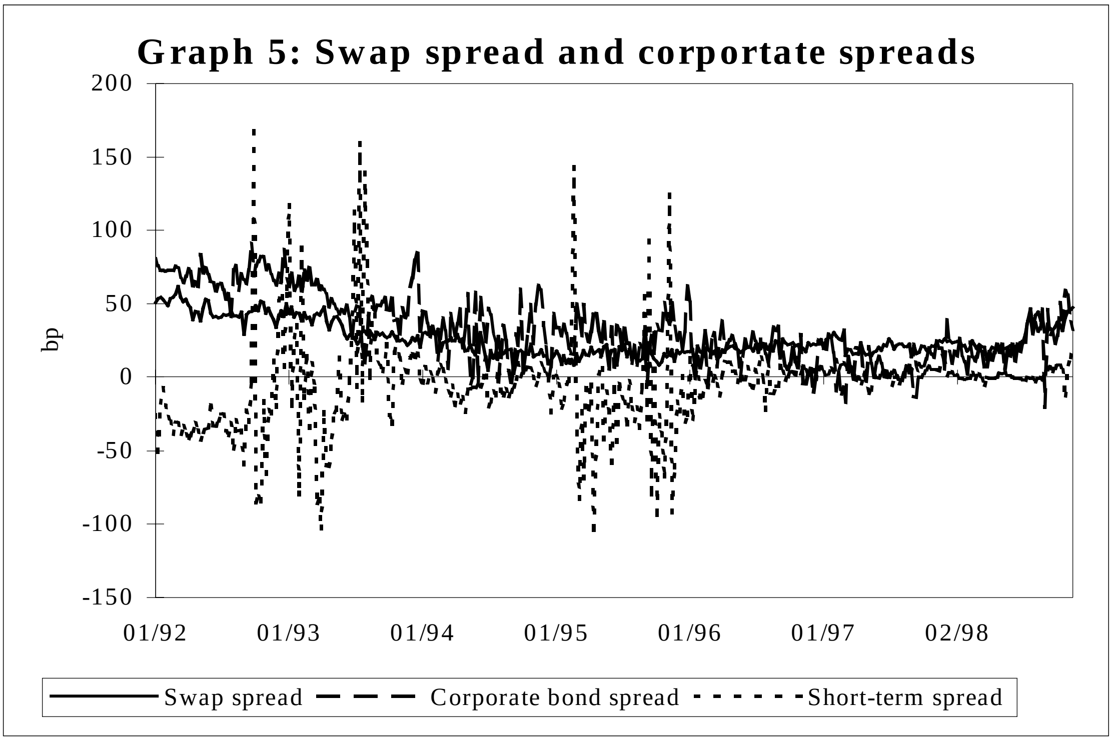


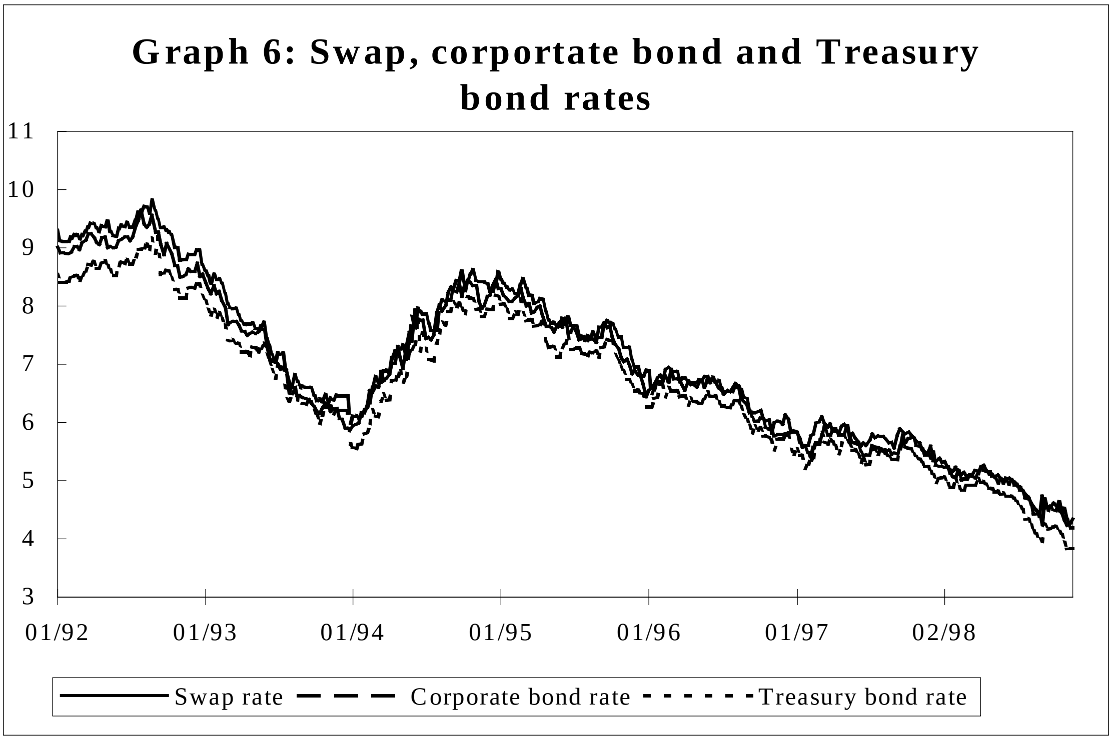
Graph 3: Zero-coupon swap spreads



Graph 4: Swap spread and 1-year theoretical spread




Graph 5: Swap spread and corportate spreads
200
150
100
50
0
-50
-100
-150
bp
01/92
01/93
01/94
01/95
01/96
01/97
02/98
Swap spread
Corporate bond spread
Short-term spread


Graph 6: Swap, corportate bond and Treasury bond rates
11
10
9
8
7
6
5
4
3
01/92
01/93
01/94
01/95
01/96
01/97
02/98
Swap rate
Corporate bond rate
Treasury bond rate

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