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Central bank reputation in a forward-looking model

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Résumé:
Ce papier examine si des préoccupations de réputation peuvent amener la banque centrale à mettre en œuvre la politique monétaire optimale temporellement incohérente dans un modèle néo-keynésien standard. La nature prospective de ce modèle est à cet égard intéressante à double titre: d'une part, elle accentue l'incohérence temporelle de la politique monétaire optimale en ajoutant un biais de stabilisation à l'éventuel biais d'inflation; d'autre part, elle permet de modéliser la réputation de la banque centrale de manière plus satisfaisante en expliquant la coordination des agents privés sur la durée de punition. Nos résultats suggèrent que les biais d'inflation et de stabilisation peuvent être surmontés pour toutes les calibrations utilisées dans la littérature. Ces résultats permettent d'endogénéiser la perspective atemporelle de Woodford et tendent à mettre en doute le bien-fondé de récentes propositions de délégation de politique monétaire.

Mots-clefs: biais de stabilisation, biais d'inflation, discrétion, engagement, perspective atemporelle, réputation.

Abstract:
This paper examines whether reputation concerns can induce the central bank to implement the time-inconsistent optimal monetary policy in a standard New Keynesian model. The forward-looking nature of this model is in this respect interesting on two accounts: first, it worsens the time-inconsistency problem of optimal monetary policy by adding a stabilization bias to the possible inflation bias; second, it enables us to model more satisfactorily the reputation of the central bank by accounting for the coordination of the private agents on the punishment length. Our results suggest that the inflation bias and the stabilization bias can be overcome for the calibrations used in the literature. These results enable us to endogenize Woodford's timeless perspective and weaken the case for monetary policy delegation.

Keywords: commitment, discretion, inflation bias, reputation, stabilization bias, timeless perspective.

JEL codes: E52, E58, E61.

Résumé non technique:

La politique monétaire optimale est connue pour être "temporellement incohérente" (*time-inconsistent*) sous certaines hypothèses depuis l'article pionnier de Kydland et Prescott (1977). La plus célèbre conséquence de ce problème d'incohérence temporelle est l'existence d'un biais d'inflation, mis en évidence par Barro et Gordon (1983a), lorsque la banque centrale cherche à stabiliser la production au-dessus de son niveau potentiel. Barro et Gordon (1983b) montrent cependant que des considérations de réputation peuvent inciter la banque centrale à mettre en œuvre la politique monétaire optimale sous une hypothèse simple de "mécanisme de punition" (*grim-trigger mechanism*). Une solution alternative pour surmonter ce biais d'inflation, proposée par Rogoff (1985), consiste à déléguer la politique monétaire à un banquier central conservateur. Quelle que soit la solution retenue, le biais d'inflation ne semble plus poser problème dans l'environnement actuel d'inflation faible.

Le problème d'incohérence temporelle de la politique monétaire optimale a cependant suscité un nouvel intérêt ces dernières années avec le développement des modèles néo-keynésiens. Comme l'ont montré Clarida, Galí et Gertler (1999) et Woodford (1999), la nature "prospective" (*forward-looking*) de ces modèles donne en effet naissance à un autre biais, dit de stabilisation, en rendant la politique monétaire optimale temporellement incohérente même lorsque la banque centrale ne cherche pas à stabiliser la production au-dessus de son niveau potentiel. Ce biais de stabilisation provient du fait que la politique monétaire optimale à une date donnée consiste à influencer les anticipations des agents privés concernant la politique monétaire future de façon à faciliter la stabilisation du taux d'inflation et de l'écart de production à la date courante – or cette politique monétaire future anticipée, qui permet la mise en œuvre de la politique monétaire optimale à la date courante, ne coïncide pas avec la politique monétaire qui sera optimale aux dates futures (même en l'absence de nouveaux développements).

Un certain nombre de projets de délégation de la politique monétaire sont déjà apparus dans la littérature académique pour remédier à l'apparition de ce nouveau biais. Il a ainsi été proposé d'introduire dans la fonction de perte assignée à la banque centrale un objectif de stabilisation du niveau des prix, un objectif de stabilisation de la croissance de la masse monétaire, un objectif de stabilisation de la croissance de la production nominale ou encore un objectif de stabilisation de la variation d'écart de production. Mais à notre connaissance il n'existe encore à présent aucune étude portant sur la réputation de la banque centrale dans les modèles néo-keynésiens. Une telle étude serait pourtant la bienvenue, puisque la délégation de la politique monétaire pourrait bien être inutile si le seul souci de sa réputation suffisait à ce que la banque centrale mette en œuvre la politique monétaire optimale.

Notre papier cherche à combler cette lacune en étudiant la réputation de la banque centrale dans un modèle prospectif, plus précisément dans un modèle néo-keynésien standard, choisi pour sa popularité et sa simplicité. Nous définissons la réputation de la banque centrale comme sa capacité à influencer les anticipations des agents privés, capacité qui dépend de la politique monétaire passée sous une hypothèse simple de mécanisme de punition (*i.e.* la crédibilité se gagne en joignant le geste à la parole). Nous montrons notamment que le biais de stabilisation réduit le bien-être d'une façon non négligeable (puisqu'il équivaut à une augmentation permanente du taux d'inflation allant de 0,26 à 1,23 points de pourcentage), mais qu'il peut être surmonté par des considérations de réputation pour toutes les calibrations considérées dans la littérature. Ce résultat nous amène en particulier à mettre en doute le bien-fondé des récentes propositions de délégation de politique monétaire évoquées ci-dessus.

Non-technical summary:

Optimal monetary policy is known to be time-inconsistent under certain assumptions since the seminal work of Kydland and Prescott (1977). The best known consequence of this time-inconsistency problem is Barro and Gordon's (1983a) inflation bias which arises when the central bank seeks to stabilize output above its potential level. Barro and Gordon (1983b) show however that reputation considerations can then make the optimal monetary policy sustainable under a simple grim-trigger mechanism assumption. An alternative way to overcome this inflation bias, proposed by Rogoff (1985), is to delegate monetary policy to a conservative central banker. Whatever the solution implemented, the inflation bias is arguably no longer a relevant issue in the current low inflation environment.

The time-inconsistency problem of optimal monetary policy has however aroused renewed interest in recent years with the development of New Keynesian models. As shown by Clarida, Galí and Gertler (1999) and Woodford (1999), the forward-looking nature of these models gives indeed rise to a new bias, called the stabilization bias, by making optimal monetary policy time-inconsistent even when the central bank does not seek to stabilize output above its potential level. More precisely, in these models the optimal current monetary policy requires to raise some expectations about the future monetary policy (in order to facilitate the stabilization of the economy in the present) which the central bank will however have no incentive to validate subsequently − even in the absence of new developments in the meantime.

The literature has already come up with a number of monetary policy delegation schemes as remedies for this stabilization bias. It has thus been proposed to introduce into the loss function assigned to the central bank a price level stabilization objective, a money growth stabilization objective, a nominal income growth stabilization objective or an output gap change stabilization objective. But to our knowledge there is so far no study on central bank reputation in New Keynesian models. Such a study would however be welcome, since monetary policy delegation may well be useless if reputation concerns alone can induce the central bank to implement the time-inconsistent optimal monetary policy.

This paper aims at filling this gap in the literature by considering the issue of central bank reputation in a forward-looking model, namely a standard New Keynesian model chosen for its popularity and analytical tractability. We define the reputation of the central bank as its ability to influence the private agents' expectations, which depends on its monetary policy record under a simple grim-trigger mechanism assumption (*i.e.* credibility is gained by matching deeds with words). We notably show that the stabilization bias reduces social welfare in a non-negligible way (as much as a permanent increase in the inflation rate of 0,26 to 1,23 percentage points), but that it can be overcome by reputation considerations for all the calibrations considered in the literature. This result weakens the case for monetary policy delegation.

1 Introduction

Optimal monetary policy is known to be time-inconsistent under certain assumptions since the seminal work of Kydland and Prescott (1977). Under these assumptions the central bank is therefore doomed to implement the suboptimal discretionary equilibrium if it cannot credibly commit to implementing the optimal monetary policy when the private agents have rational expectations.

The best known consequence of this time-inconsistency problem is Barro and Gordon's (1983a) inflation bias which arises when the central bank seeks to stabilize output above its potential level. Barro and Gordon (1983b) show however that reputation considerations can then make the optimal monetary policy sustainable under a simple grim-trigger mechanism assumption. An alternative way to overcome this inflation bias, proposed by Rogoff (1985), is to delegate monetary policy to a conservative central banker. Now whether because they are conservative or concerned for their reputation, nowadays central bankers do not seem in practice to aim at stabilizing output above its potential level, as observed by Blinder (1997), so that the inflation bias is arguably no longer a relevant issue in the current low inflation environment.

The time-inconsistency problem of optimal monetary policy has however aroused renewed interest in recent years with the development of New Keynesian models. As shown by Clarida, Galí and Gertler (1999) and Woodford (1999), the forward-looking nature of these models gives indeed rise to a new bias, called the stabilization bias[1], by making optimal monetary policy time-inconsistent even when the central bank does not seek to stabilize output above its potential level. More precisely, in these models the optimal current monetary policy requires to raise some expectations about the future monetary policy (in order to facilitate the stabilization of the economy in the present) which the central bank will however have no incentive to validate subsequently − even in the absence of new developments in the meantime.

The literature has already come up with a number of monetary policy delegation schemes as remedies for this stabilization bias: Vestin (2000) proposes to introduce a price level stabilization objective, Söderström (2001) a money growth stabilization objective, Jensen (2002b) a nominal income growth stabilization objective, Walsh (2003b) an output gap change stabilization objective and Svensson and Woodford (2005) a state-contingent linear inflation contract, into the loss function assigned to the central bank. But to our knowledge there is so far no study on central bank reputation in New Keynesian models. Such a study would however be welcome, since monetary policy delegation may well be useless if reputation concerns alone can induce the central bank to implement the time-inconsistent optimal monetary policy[2].

[1] The stabilization bias is presented by Dennis (2003) in a non-technical way and by Walsh (2003a, chapter 11) and Woodford (2003a, chapter 7) in a more technical but very pedagogical way.

[2] As acknowledged by Woodford (2003b, p. 885), monetary policy delegation can even be counterproductive for a similar reason: "Of course, the assignment to the central bank of an objective different from the true social loss

This paper aims at filling this gap in the literature by considering the issue of central bank reputation in a forward-looking model, namely a standard New Keynesian model chosen for its popularity and analytical tractability. We define the reputation of the central bank as its ability to influence the private agents' expectations, which depends on its monetary policy record under a simple grim-trigger mechanism assumption (*i.e.* credibility is gained by matching deeds with words). More precisely, we generalize Currie and Levine's (1993, chapter 5) framework − itself an extension of Barro and Gordon's (1983b) framework to a dynamic stochastic model − to a finite punishment length. For simplicity, we assume that monetary policy is fully transparent so that the private agents understand the central bank's incentive to deviate from the optimal monetary policy[3] and can immediately detect such a deviation[4].

The consideration of grim-trigger mechanisms is particularly interesting in our standard New Keynesian model for two reasons. First, numerical calibrations of this model enable us to conclude unambiguously about the sustainability of the optimal monetary policy for a given punishment length. By contrast, "qualitative models" usually lead to inconclusive results since the optimal monetary policy (and more generally any time-inconsistent monetary policy superior to the discretionary monetary policy) is necessarily non-sustainable when the discount factor is close enough to zero and sustainable when the discount factor is close enough to one and the punishment is long enough, as implied by the folk theorem. Second, the forward-looking nature of this model is shown to facilitate greatly the coordination of the atomistic private sector on the punishment length − except in the particular case of serially uncorrelated cost-push shocks. By contrast, this coordination is usually left unexplained in non-forward-looking models.

The remaining of the article is organized as follows. Section 1 gives an overview of our framework and methodology. Section 2 focuses on central bank reputation as a means to overcome the inflation bias. Though apparently no longer (if ever[5]) a cause for concern, the inflation bias is

function, in the expectation that it will pursue that objective with discretion, is not the only possible approach to the achievement of a desirable pattern of responses to disturbances. One defect of the "optimal delegation" approach considered here is that it presumes that the stationary Markov equilibrium associated with a particular distorted objective will be realized. Yet there may well be other possible rational expectations equilibria consistent with discretionary optimization by the central bank, "reputational" equilibria in which the bank may do a *better* job of minimizing the objective it has been assigned, but as a consequence bring about a pattern of responses that is *less* desirable from the point of view of the true social objective." (Woodford's emphasis.)

[3] The publication by the central bank of an explicit loss function with a numerical relative weight for the output gap stabilization objective, along the lines set by Svensson (2003), would undoubtedly help the private agents to understand the central bank's incentive to deviate from the optimal monetary policy.

[4] This assumption is more easily justified in a rule-based policy-making framework along the lines set by Kydland and Prescott (1977, p. 487): "In a democratic society, it is probably preferable that selected rules be simple and easily understood, so it is obvious when a policymaker deviates from the policy", and by Woodford (1999, p. 292): "A simple feedback rule would make it easy to describe the central bank's likely future conduct with considerable precision, and verification by the private sector of whether such a rule is actually being followed should be straightforward as well." In the real world, the monetary policy frameworks closest to transparent rule-based policy-making are currently those of the Reserve Bank of New Zealand and the Bank of Canada, which publish macroeconomic projections conditional on a future nominal interest rate path derived from a pre-determined monetary policy reaction function.

[5] This question is notably addressed by Ireland (1999), who argues that the inflation bias can explain the behaviour

worth considering precisely as a way to test the relevance of our analysis by checking whether reputation considerations can indeed overcome this bias in our framework. Section 3 focuses on the main issue at stake, namely central bank reputation as a means to overcome the stabilization bias. We then shortly conclude and provide a technical appendix.

2 Central bank reputation in a standard New Keynesian model

This section gives an overview of our framework and methodology.

2.1 A standard New Keynesian model

We consider a standard New Keynesian model with structural inflation inertia developed by Woodford (2003a)[6], whose reduced form (log-linearized around the steady state) is isomorphic to, and includes as a particular case, that of the canonical New Keynesian model without structural inflation inertia used notably by Clarida, Galí and Gertler (1999), Walsh (2003a) and Woodford (2003a). For our purpose, this reduced form can be limited to a Phillips curve and a social loss function.

The Phillips curve, derived from the firms' profits maximization programme, is written:

$$z_t = \beta \widetilde{E}_t \{z_{t+1}\} + \kappa x_t + \delta \eta_t \text{ with } z_t \equiv \pi_t - \gamma \pi_{t-1},$$

where π_t denotes the inflation rate and x_t the output gap at date t, while $\widetilde{E}_t \{.\}$ stands for the private agents' expectation operator conditionally on the information available at date t, which includes the past and present variables and shocks. Parameters β, γ, δ and κ are such that $0 < \beta < 1$, $0 \leq \gamma \leq 1$, $\delta \in \{0, 1\}$ and $\kappa > 0$. This Phillips curve is forward-looking (both in terms of the inflation rate π and in terms of the quasi-differenced inflation rate z) because of the underlying Calvo-type price-setting assumption, as firms know that the price they choose today will remain effective for more than one period on average. When $\gamma > 0$ it is also backward-looking (in terms of the inflation rate π) because of the underlying assumption that non-optimally reset prices are indexed to past inflation, with parameter γ measuring the degree of indexation. The exogenous cost-push disturbance η, which captures any factor altering the relationship between real marginal costs and the output gap, is assumed to follow an autoregressive process of order one: $\eta_t = \rho \eta_{t-1} + \varepsilon_t$ with $0 \leq \rho < 1$, where ε is a white noise of variance $V_\varepsilon > 0$. We assume that

of inflation in the United States if one allows for time variation in the natural rate of unemployment.

[6] Walsh (2005) uses a very close model, whose reduced form is identical to that of our model except for the specification of the cost-push shock stochastic process.

the distribution of ε is continuous, symmetric and bounded[7], that is to say that the set of possible values for this shock is the real interval $[-\overline{\varepsilon}, \overline{\varepsilon}]$ where $\overline{\varepsilon} > 0$.

The social loss function at date t, derived by Woodford (2003a, chapter 6) as negatively related to the second-order approximation of the representative household's utility function taken in the neighbourhood of the steady state, is written:

$$L_t = E_t \left\{ \sum_{k=0}^{+\infty} \beta^k \left[(z_{t+k})^2 + \lambda (x_{t+k} - x^*)^2 \right] \right\},$$

where $E_t \{.\}$ stands for the rational expectation operator conditionally on the information available at date t, which includes the past and present variables and shocks, while parameters λ and x^* are such that $\lambda > 0$ and $x^* \geq 0$. The presence of a nominal variable (namely the quasi-differenced inflation rate) in this loss function comes from the fact that the absence of price-setting synchronization entails relative price distortions which lead to an inefficient sectoral allocation of labour, even when the output gap is equal to zero. The case $x^* = 0$ can be justified by the existence of structural policies offsetting first-order distortions.

In what follows, we proceed as if the central bank controlled directly variables z and x to minimize the social loss function subject to the Phillips curve[8]. The central bank may minimize the loss function under discretion (*i.e.* at each date) or under commitment (*i.e.* once and for all). We note $\left(z_t^{c,T}, x_t^{c,T}, L_t^{c,T}\right)$ for $t \geq T$ the value of (z_t, x_t, L_t) when the central bank optimizes once and for all at a given date T, $\left(z_t^d, x_t^d, L_t^d\right)$ for $t \in \mathbb{Z}$ the value of (z_t, x_t, L_t) when the central bank re-optimizes at each date and $\left(z_t^{tp}, x_t^{tp}, L_t^{tp}\right)$ for $t \in \mathbb{Z}$ the value of (z_t, x_t, L_t) when the central bank optimizes at each date from Woodford's (1999) timeless perspective, that is to say equivalently the value $\lim_{T \longrightarrow -\infty} \left(z_t^{c,T}, x_t^{c,T}, L_t^{c,T}\right)$, where "$c, T$", "$d$" and "$tp$" stand respectively for "commitment at date T", "discretion" and "timeless perspective". We also note $\left(L^c, L^d, L^{tp}\right)$ the unconditional mean of $\left(L_t^{c,t}, L_t^d, L_t^{tp}\right)$ for any date $t \in \mathbb{Z}$. Finally, when $\gamma \neq 1$ we measure the gain from commitment by Jensen's (2002b) "inflation equivalent", which corresponds to the value of the permanent increase in the inflation rate (expressed in percentage points) leading to an increase in the unconditional mean of the loss function equal to $L^d - L^c$ or $L^d - L^{tp}$, that is to say in our context:

$$\Pi^c \equiv 100 \frac{\sqrt{(1-\beta)\left(L^d - L^c\right)}}{1-\gamma} \text{ and (if } L^{tp} < L^d\text{) } \Pi^{tp} \equiv 100 \frac{\sqrt{(1-\beta)\left(L^d - L^{tp}\right)}}{1-\gamma}.$$

Whatever $t \in \mathbb{Z}$, $L_t^{c,t}$ is by construction the minimal value which the loss function L_t can take (and in particular $L_t^{c,t} < L_t^d$). This value may however not be achievable when a commitment

[7] The finite distribution assumption, necessary for reputation concerns to overcome the stabilization bias in all situations (as clear from subsection 3.2), is actually unavoidable in our log-linearized framework.

[8] Interest rate rules implementing the discretionary equilibrium or the precommitment equilibrium can be easily designed from the missing IS equation to justify this procedure.

technology is lacking and when the private agents have rational expectations because the precommitment equilibrium is time-inconsistent, so that the central bank would choose to (re-)commit at each date if the private agents did wrongly trust each commitment. The next subsection examines how this time-inconsistency problem can be overcome when the central bank may credibly re-commit at the prior cost of a temporary loss of reputation. Note finally that the precommitment equilibrium proves time-inconsistent when $x^* \neq 0$ because of an inflation bias *à la* Barro and Gordon (1983a) and when $\delta = 1$ because of a stabilization bias *à la* Clarida, Galí and Gertler (1999) and Woodford (1999). We choose to consider these two biases alternatively since if it exists the inflation bias is likely to overshadow the stabilization bias. Thus section 2 focuses on the inflation bias by assuming $\delta = 0$ and $x^* \neq 0$, while section 3 focuses on the stabilization bias by assuming $\delta = 1$ and $x^* = 0$.

2.2 Central bank reputation

In this subsection the private agents are assumed to behave according to a grim-trigger mechanism such that once lost, the central bank's reputation takes D periods to be restored, where $D \in \mathbb{N}^*$ is an exogenous parameter. More precisely, at each date $t \geq T_0$ the private agents form their expectation $\widetilde{E}_t \{z_{t+1}\}$ in accordance with the T_0-commitment equilibrium if and only if this equilibrium has been implemented from date T_0 to date t included. Let T_0' denote the first date when the central bank deviates from this equilibrium (if it does). By recurrence on $j \in \mathbb{N}$, we assume that if T_j' exists then: i) if $D \geq 2$ then at each date $t \in \{T_j', ..., T_j' + D - 2\}$ the private agents expect the central bank to act in a discretionary way from date $t + 1$ to date $T_j' + D - 1$ included; ii) at each date $t \in \{T_j', ..., T_j' + D - 1\}$ the private agents expect the central bank to implement the T_{j+1}-commitment equilibrium at date $T_{j+1} \equiv T_j' + D$; and iii) at each date $t \geq T_{j+1}$ the private agents form their expectation $\widetilde{E}_t \{z_{t+1}\}$ in accordance with the T_{j+1}-commitment equilibrium if and only if this equilibrium has been implemented from date T_{j+1} to date t included. Finally, T_{j+1}' is then defined as the first date when the central bank deviates from this equilibrium (if it does).

Four points are worth noting at this stage. First, we need to resort to a recurrence on $j \in \mathbb{N}$ to define this grim-trigger mechanism because in our dynamic model with a finite punishment length, the situation prevailing immediately after a punishment interval (*i.e.* at date T_j for a given $j \in \mathbb{N}^*$) differs from the situation which would have prevailed at the same date had the central bank not deviated from the T_{j-1}-commitment equilibrium. More precisely, we assume that at each date T_j (if this date exists) for $j \in \mathbb{N}^*$ the meter is reset to zero in the sense that the central bank can start to implement the T_j-commitment equilibrium, but cannot revert to the implementation of the T_0-commitment equilibrium[9]. Second, the punishment equilibrium implemented from date T_j'

[9] Alternative assumptions to examine the sustainability of the timeless perspective equilibrium (rather than that

to date $T'_j + D - 1$ included (if these dates exist) for $j \in \mathbb{N}$ will take into account the private agents' expectations of the central bank's re-commitment at date T_{j+1} and will therefore not correspond to the (permanent) discretionary equilibrium displayed in subsections 2.1 and 3.1. Third, the private agents' "cynical" expectations during the punishment intervals (as opposed to their "trustful" expectations outside these intervals) are rational, so that the central bank has no incentive to surprise the private agents during these intervals. Fourth, because the private agents are assumed not to be unionized, each atomistic private agent is "expectations-taker" so that this grim-trigger mechanism is not strategically chosen but should rather be considered as a postulate about the private sector's behaviour, which explains why D is exogenous[10] and why there is no possibility of renegotiation during the punishment intervals between the private agents on the one hand and the central bank on the other hand.

The timing of the model at each date t can be presented as follows: 1) shock ε_t occurs, which is observed by the central bank and the private agents; 2) if $\exists j \in \mathbb{N}$ such that $t = T_j$ or such that the T_j-commitment equilibrium has been implemented from date T_j to date $t-1$ included, then the central bank decides whether to implement the T_j-commitment equilibrium or to deviate from this equilibrium, and this decision is observed by the private agents; 3) the private agents form their expectation $\widetilde{E}_t \{z_{t+1}\}$; 4) the central bank chooses z_t and x_t. As clear from this timing, we assume for simplicity that the central bank and the private agents have an information set which goes beyond the perfect knowledge of the structure of the model and the value of its parameters. In particular, the central bank observes the realization of the current shock before deciding which equilibrium to implement[11]. Moreover, the private agents can adjust their expectation of the future quasi-differenced inflation rate to the central bank's decision[12], which implies that the central bank may surprise the private agents only by disappointing their previous expectations (and not by disappointing their current expectations), so that the cost to renege may be biased upwards. Finally, the fact that z_t and x_t are chosen after the private agents form their expectation is a

of the precommitment equilibrium) could be formulated in exactly the same way except that the expressions "the T_j-commitment equilibrium" for $j \in \mathbb{N}$ would be replaced by "the timeless perspective equilibrium". In this framework, the necessary and sufficient condition for the inflation bias to be overcome is found to be independent of parameter D and to be satisfied for calibration 1, while the necessary and sufficient condition for the stabilization bias to be overcome proves not analytically determinable except in the particular case $\rho = 0$.

[10] Even in **appendix F** where the private agents manage to coordinate on an endogenized value of D, this value of D is not strategically chosen by the private agents.

[11] Were steps 1 and 2 inverted, the condition for the stabilization bias to be overcome would no longer be analytically determinable when $0 < \rho \leq \frac{1}{2}$.

[12] This assumption may be better understood in a framework with an explicit IS equation. Indeed, the corresponding timing would then be the following one: 1) shock ε_t occurs, which is observed by the central bank and the private agents; 2) the central bank sets the nominal interest rate, which is observed by the private agents; 3) the private agents form their expectations about the future situation, set their prices, produce and consume. Jensen (2002a) also assumes that the private agents observe the central bank's action before forming their expectation of future inflation. As argued by Walsh (2000, p. 249): "With most major central banks using a short-term interest rate to implement monetary policy, policy changes are immediately and widely noted in the press. Expectations about future inflation can respond immediately to any change in policy, affecting both current and future equilibria. This response has the potential to discipline an opportunistic central bank".

natural consequence of the Calvo-type price-setting assumption, which implies that the current quasi-differenced inflation rate depends on the private agents' expectation of the future quasi-differenced inflation rate and not the other way round.

Now let us set T_0 equal to 0 for simplicity, consider a given $n \in \mathbb{N}$ and assume that the 0-commitment equilibrium has been implemented from date 0 to date $n-1$ included if $n \geq 1$. At date n, the central bank either implements this equilibrium (option A) or deviates from this equilibrium (option B). Let $\left(z_{n+k}^{\phi}, x_{n+k}^{\phi}\right)$ for $k \in \mathbb{N}$ denote the value of (z_{n+k}, x_{n+k}) with option $\phi \in \{A, B\}$, so that the value taken by the loss function at date n with option $\phi \in \{A, B\}$ is

$$L_n^{\phi} = E_n \left\{ \sum_{k=0}^{+\infty} \beta^k \left[\left(z_{n+k}^{\phi} \right)^2 + \lambda \left(x_{n+k}^{\phi} - x^* \right)^2 \right] \right\}. \quad (1)$$

Let finally $\left(z_t^{d,T}, x_t^{d,T}\right)$ for $t < T$ denote the value of (z_t, x_t) when from date t to date $T-1$ included the private agents expect the central bank to act in a discretionary way from date t to date $T-1$ included and to implement the T-commitment equilibrium at date T. We then have in particular:

$$\left(z_n^A, x_n^A\right) = \left(z_n^{c,0}, x_n^{c,0}\right) \text{ and}$$
$$\left(z_{n+k}^B, x_{n+k}^B\right) = \left(z_{n+k}^{d,n+D}, x_{n+k}^{d,n+D}\right) \text{ for } 0 \leq k \leq D-1.$$

In the absence of any commitment technology, the central bank seeks to minimize L_t at each date $t \geq 0$. At date n in particular, it chooses the option $\phi \in \{A, B\}$ which minimizes L_n. A necessary and sufficient condition for the central bank never to deviate from the 0-commitment equilibrium is that it chooses not to deviate in the most tempting situation. Given that L_n^A and L_n^B depend only on η_{-1} and ε_i for $0 \leq i \leq n$[13], this necessary and sufficient condition can be written in the following way:

$$S \equiv \underset{\substack{n,\, \eta_{-1} \text{ and } \varepsilon_i \\ \text{for } 0 \leq i \leq n}}{Sup} \left(L_n^A - L_n^B\right) \leq 0 \text{ and, if it exists, } M \equiv \underset{\substack{n,\, \eta_{-1} \text{ and } \varepsilon_i \\ \text{for } 0 \leq i \leq n}}{Max} \left(L_n^A - L_n^B\right) < 0.$$

Now let us define $\widehat{L}_n^A$ as the value taken by L_n^A when z_{n+k}^A and x_{n+k}^A are arbitrarily replaced by $z_{n+k}^{c,0}$ and $x_{n+k}^{c,0}$ respectively for $k \geq 1$ in equation (1), and $\widehat{L}_n^B$ as the value taken by L_n^B when z_{n+k}^B

[13] Indeed, given the grim-trigger mechanism considered, at each date $t \geq n$ the central bank implements either a T-commitment equilibrium (where $T \geq 0$) or a punishment equilibrium. As shown by **appendices B** and **C**, the corresponding quasi-differenced inflation rates and output gaps depend only on η_{-1} and ε_i for $0 \leq i \leq t$. As a consequence, $\left(z_{n+k}^A, x_{n+k}^A\right)_{k \geq 0}$ and $\left(z_{n+k}^B, x_{n+k}^B\right)_{k \geq 0}$ depend only on η_{-1} and ε_i for $0 \leq i \leq n+k$, so that L_n^A and L_n^B depend only on η_{-1} and ε_i for $0 \leq i \leq n$.

and x^B_{n+k} are arbitrarily replaced by $z^{c,n+D}_{n+k}$ and $x^{c,n+D}_{n+k}$ respectively for $k \geq D$ in equation (1). Let us further define

$$\widehat{S} \equiv \underset{\substack{n,\, \eta_{-1} \text{ and } \varepsilon_i \\ \text{for } 0 \leq i \leq n}}{Sup} \left(\widehat{L}^A_n - \widehat{L}^B_n\right) \text{ and, if it exists, } \widehat{M} \equiv \underset{\substack{n,\, \eta_{-1} \text{ and } \varepsilon_i \\ \text{for } 0 \leq i \leq n}}{Max} \left(\widehat{L}^A_n - \widehat{L}^B_n\right).$$

Appendix D shows the following equivalence:

$$(S \leq 0 \text{ and, if it exists, } M < 0) \Longleftrightarrow \left(\widehat{S} \leq 0 \text{ and, if it exists, } \widehat{M} < 0\right), \tag{2}$$

so that $\left(\widehat{S} \leq 0 \text{ and, if it exists, } \widehat{M} < 0\right)$ is a necessary and sufficient condition for the central bank never to deviate from the 0-commitment equilibrium, that is to say a necessary and sufficient condition for the precommitment equilibrium to be a reputational equilibrium. This result: i) greatly facilitates our analysis, since $\widehat{S}$ and $\widehat{M}$ are much easier to compute than S and M; ii) can easily be generalized to other dynamic stochastic models (whether forward-looking or not), as clear from **appendix D**; and iii) improves on the existing literature about central bank reputation in dynamic stochastic models, like Currie and Levine (1993, chapter 5) for instance, since this literature typically avoids handling $\left(z^B_{n+k}, x^B_{n+k}\right)_{k \geq D}$ by considering an infinite punishment length.

Appendix E shows that $\frac{\Delta \widehat{S}}{\Delta D} \leq 0$ when $\delta = 0$ (as in section 2) or $x^* = 0$ (as in section 3). As a consequence, the longer the punishment length D, the more deterred the central bank from deviating from the precommitment equilibrium and thus the more likely the precommitment equilibrium to be a reputational equilibrium. This conventional result ensures the unicity (but not the existence) of $\underline{D} \in \mathbb{N}^*$ such that the precommitment equilibrium is a reputational equilibrium if and only if $D \geq \underline{D}$, that is to say such that $\left(\widehat{S} \leq 0 \text{ and, if it exists, } \widehat{M} < 0\right) \Longleftrightarrow (D \geq \underline{D})$. Now calibrating parameter D is a challenging task, which we wisely choose to circumvent. We agree with Rogoff (1987, p. 151) that "it is not intuitively appealing to have a long or infinite punishment interval". We argue moreover that the most plausible values for D are of the same order as monetary policy committees' terms of office, which are typically a few years long. As a rule of thumb, we therefore decide that the precommitment equilibrium qualifies as a reputational equilibrium if and only if $\underline{D}$ exists and is of the order of a few years.

3 Central bank reputation and the inflation bias

This section focuses on the inflation bias, that is to say on the case $\delta = 0$ and $x^* \neq 0$.

3.1 The inflation bias

In this subsection the private agents are assumed to have rational expectations, so that they form the same expectations as the central bank: $\widetilde{E}_t\{.\} = E_t\{.\}$ at all dates $t \in \mathbb{Z}$.

We first determine the discretionary equilibrium, that is to say the equilibrium obtained when at each date $t \in \mathbb{Z}$ the central bank chooses z_t and x_t so as to minimize L_t subject to the Phillips curve taken at date t. As shown in **appendix A**, the central bank then chooses the same trade-off between a quasi-differenced inflation rate higher than 0 and an output gap lower than x^* at each date t:

$$z_t^d = \frac{\kappa\lambda x^*}{\kappa^2 + \lambda(1-\beta)} \text{ and } x_t^d = \frac{\lambda(1-\beta)x^*}{\kappa^2 + \lambda(1-\beta)}.$$

We then determine the precommitment equilibrium or more precisely (without any loss in generality) the 0-commitment equilibrium, that is to say the equilibrium obtained when at date 0 the central bank chooses z_t and x_t for all $t \geq 0$ so as to minimize the loss function L_0 subject to the Phillips curve taken at all dates $t \geq 0$. As shown in **appendix B**, we obtain the following results for $t \geq 0$:

$$z_t^{c,0} = \frac{\lambda(1-\omega)\omega^t x^*}{\kappa} \text{ and } x_t^{c,0} = \omega^{t+1}x^*,$$

where $\omega \in]0,1[$ is a parameter depending on β, κ, λ given in **appendix B**.

Unlike the discretionary equilibrium, the precommitment equilibrium makes therefore the quasi-differenced inflation rate and the output gap vary over time, and more precisely decrease exponentially towards zero. This comes from the fact that the commitment technology enables the central bank to trade off not only between a quasi-differenced inflation rate higher than 0 and an output gap lower than x^* at date t, but also between the situation at date t and the future situations. By lowering the private agents' expectations $\widetilde{E}_t\{z_{t+1}\} = E_t\{z_{t+1}\}$ in the Phillips curve taken at date t, the central bank indeed improves the trade-off between a value of z_t higher than 0 and a value of x_t lower than x^*. Of course, the precommitment equilibrium proves time-inconsistent since at date $t+1$ the central bank faces the same optimization problem as at date t and has therefore no incentive to choose a quasi-differenced inflation rate different from the quasi-differenced inflation rate chosen at date t. This inflation bias implies that the quasi-differenced inflation rate is higher in the discretionary equilibrium than in the precommitment equilibrium:

$$z_t^d - z_t^{c,0} \geq \frac{\beta\lambda^2(1-\omega)^2 x^*}{\kappa[\kappa^2 + \lambda(1-\beta)]} > 0 \text{ for } t \geq 0.$$

The social loss function is easily shown to take the following values:

$$L_t^d = \frac{\kappa^2\lambda\left(\kappa^2+\lambda\right)x^{*2}}{(1-\beta)\left[\kappa^2+\lambda(1-\beta)\right]^2} = L^d \text{ for } t\in\mathbb{Z},$$

$$L_0^{c,0} = \frac{\lambda(1-\omega)^2\left[\kappa^2+\lambda(1-\beta\omega)^2\right]x^{*2}}{(1-\beta)\kappa^2(1-\beta\omega)(1-\beta\omega^2)} = L^c,$$

$$L_t^{c,0} = \left[\frac{1}{1-\beta} - \frac{\omega^{t+1}(2-\omega^t)}{1-\beta\omega}\right]\lambda x^{*2} \text{ for } t\geq 0,$$

$$L_t^{tp} = \frac{\lambda x^{*2}}{1-\beta} = L^{tp} \text{ for } t\in\mathbb{Z}.$$

Note finally that $L_t^{c,0}$ is a strictly increasing function of $t\in\mathbb{N}$, so that $L^c < L_t^{c,0} < L^{tp}$ for all $t>0$. We naturally also have $L^c < L^d$ and we find moreover that $L^{tp} < L^d$ if and only if $\kappa^2 > 2\kappa^2(1-\beta)+\lambda(1-\beta)^2$, which implies that $L^{tp} > L^d$ is obtained only for very unlikely values of parameters β, κ and λ.

3.2 Central bank reputation

In this subsection the private agents are assumed to behave according to the grim-trigger mechanism described in subsection 1.2. As shown in **appendix C**, we have for $0\leq k\leq D-1$:

$$\begin{aligned} z_{n+k}^{d,n+D} &= \left[\kappa^2 - \beta\lambda(1-\omega)^2\left(\frac{\beta\lambda}{\kappa^2+\lambda}\right)^{D-k}\right]\frac{\lambda x^*}{\kappa\left[\kappa^2+\lambda(1-\beta)\right]}, \\ x_{n+k}^{d,n+D} &= \left[(1-\beta)+\beta(1-\omega)^2\left(\frac{\beta\lambda}{\kappa^2+\lambda}\right)^{D-k}\right]\frac{\lambda x^*}{\kappa^2+\lambda(1-\beta)}. \end{aligned} \tag{3}$$

This result shows in particular the existence of a bijective relationship, for a given value of $(\beta,\kappa,\lambda,x^*)$ and at each date $n+k\in\{n,...,n+D-1\}$ of the punishment interval, between the quasi-differenced inflation rate $z_{n+k} = z_{n+k}^{d,n+D}$ or the output gap $x_{n+k} = x_{n+k}^{d,n+D}$ on the one hand and the punishment length D on the other hand. The existence of this bijective relationship, which is a consequence of the forward-looking nature of the model and the fact that $E_n\left\{z_{n+D}^B\right\} = E_n\left\{z_{n+D}^{c,n+D}\right\} \neq 0$ (since $x^*\neq 0$), would greatly facilitate the coordination of the private agents on a given punishment length, should they know the value of all the model's parameters except D. Indeed, they could then coordinate on a given value of D as soon as date $n+1$, as shown by **appendix F**, because the observation of z_n and x_n would reveal to each private agent some information about the beliefs of other private agents about the punishment length. By contrast, in non-forward-looking models the observation of aggregate variables by each private agent during the punishment interval cannot reveal anything about the beliefs of other private agents about the punishment length, so that the coordination problem is more serious.

Appendix G shows that the precommitment equilibrium is a reputational equilibrium if and only if

$$\frac{\kappa^2}{(1-\beta)\lambda}+\frac{2\beta^{D+2}\lambda^2(1-\omega)^2}{\left[\kappa^2+\lambda(1-\beta)\right]^2}\left[1-\left(\frac{\lambda}{\kappa^2+\lambda}\right)^D\right]\leq\frac{\left(1-\beta^D\right)\kappa^2\left[\kappa^2+\lambda(1-\beta)^2\right]}{(1-\beta)\left[\kappa^2+\lambda(1-\beta)\right]^2}+$$
$$\frac{\beta^D(1-\omega)^2\left[\kappa^2+\lambda(1-\beta\omega)^2\right]}{(1-\beta)\lambda(1-\beta\omega)(1-\beta\omega^2)}+\frac{\beta^2\left(\kappa^2+\lambda\right)\lambda(1-\omega)^4}{\left[\kappa^2+\lambda(1-\beta)\right]^2}\left(\frac{\beta\lambda}{\kappa^2+\lambda}\right)^{2D}\left[\frac{1-\frac{\left(\kappa^2+\lambda\right)^{2D}}{\beta^D\lambda^{2D}}}{1-\frac{\left(\kappa^2+\lambda\right)^2}{\beta\lambda^2}}\right]. \quad (4)$$

Two points are worth noting about this proposition. First, whether the precommitment equilibrium is a reputational equilibrium does not depend on the value of x^*. This result comes from the fact that the values taken by the loss function under commitment and under discretion are both proportional to x^{*2}. Second, as shown in **appendix G**, deviating from the precommitment equilibrium becomes more tempting with time for the central bank, so that $\widehat{S}$ is attained only asymptotically (*i.e.* for $n \to +\infty$, which implies that $\widehat{M}$ does not exist). This result implies that the precommitment equilibrium is a reputational equilibrium if and only if the timeless perspective equilibrium is a reputational equilibrium.

The consideration of one calibration of (β, κ, λ) found in the literature (detailed in **appendix J**) leads to the numerical results reported in **table 1**, where the notation NA stands for "non-available".

Table 1: numerical results for calibration 1.

No.	ω	Π^c	Π^{tp}	$\underline{D}$
1a	$0,64$	$9,44$	$9,43$	3
1b	$0,64$	$18,88$	$18,86$	3

Three points are worth noting about these results. First, Π^c and Π^{tp} are very close to each other, so that the timeless perspective equilibrium hardly reduces welfare compared to the precommitment equilibrium. Second, Π^c and Π^{tp} are as high as 9,5% or even 19%, that is to say that the welfare gain from commitment is very large. Third, $\underline{D}$ exists and is very small. More precisely, the precommitment equilibrium proves a reputational equilibrium provided that the private agents "punish" the central bank during at least three quarters only. This result suggests that today's apparent absence of inflation bias can be explained by reputation considerations in our standard New Keynesian model.

4 Central bank reputation and the stabilization bias

This section focuses on the stabilization bias, that is to say on the case $\delta = 1$ and $x^* = 0$.

4.1 The stabilization bias

In this subsection the private agents are assumed to have rational expectations, so that they form the same expectations as the central bank: $\widetilde{E}_t\{.\} = E_t\{.\}$ at all dates $t \in \mathbb{Z}$.

We first determine the discretionary equilibrium, that is to say the equilibrium obtained when at each date $t \in \mathbb{Z}$ the central bank chooses z_t and x_t so as to minimize L_t subject to the Phillips curve taken at date t. As shown in **appendix A**, the central bank then faces a trade-off between a positive quasi-differenced inflation rate and a negative output gap following a positive cost-push shock, and makes the current variables z_t and x_t depend only on the current disturbance η_t:

$$z_t^d = \frac{\lambda}{\kappa^2 + \lambda(1-\beta\rho)}\eta_t \text{ and } x_t^d = \frac{-\kappa}{\kappa^2 + \lambda(1-\beta\rho)}\eta_t.$$

We then determine the precommitment equilibrium or more precisely (without any loss in generality) the 0-commitment equilibrium, that is to say the equilibrium obtained when at date 0 the central bank chooses the state-contingent values of z_t and x_t for all $t \geq 0$ so as to minimize the loss function L_0 subject to the Phillips curve taken at all dates $t \geq 0$. To that aim, we follow the undetermined coefficients method and specify the variables prior to optimization as the following linear combinations of shocks[14]: $z_t = \sum_{j=0}^{+\infty} a_{j,t}\varepsilon_{t-j}$ and $x_t = \sum_{j=0}^{+\infty} b_{j,t}\varepsilon_{t-j}$ for $t \geq 0$, with $(a_{j,t}, b_{j,t}) \in \mathbb{R}^2$ for $(j,t) \in \mathbb{N}^2$. As shown in **appendix B**, we obtain the following results[15] for $t \geq 0$:

$$z_t^{c,0} = \frac{\omega\left[(1-\rho)\eta_t - (1-\omega)\xi_t\right]}{(1-\beta\rho\omega)(\omega-\rho)} \text{ and } x_t^{c,0} = \frac{\kappa\omega(\rho\eta_t - \omega\xi_t)}{\lambda(1-\beta\rho\omega)(\omega-\rho)},$$

where $\xi_t \equiv \sum_{j=0}^{t} \omega^j \varepsilon_{t-j} + \omega^t \rho\eta_{-1}$. Thus written, these results hold only in the case $\omega \neq \rho$, but they can easily be extended by continuity to the case $\omega = \rho$.

Unlike the discretionary equilibrium, the precommitment equilibrium makes therefore the current variables z_t and x_t depend not only on the current disturbance η_t, but also on a linear combination of present and past shocks ξ_t which differs from η_t when $\omega \neq \rho$. This result is best understood in the particular case $\gamma = \rho = 0$. In that case indeed, the discretionary equilibrium makes the current variables π_t and x_t depend only on the current shock ε_t, while the precommitment equilibrium makes the current variables π_t and x_t depend not only on the current shock ε_t, but also on past shocks ε_{t-j} for $j \geq 1$. This inertia or "history-dependence" of the precommitment equilibrium

[14] Note that we allow the opportunist central bank to make the quasi-differenced inflation rate and the output gap depend on the shocks occurred before the commitment date by considering (possibly time-variant) linear combinations of the entire history of shocks.

[15] These results imply that the response of the inflation rate to a positive cost-push shock is hump-shaped if and only if $\gamma + \rho + \omega > 2$ (which requires in particular $\gamma > 0$ and $\rho > 0$), while the response of the output gap to a negative cost-push shock is hump-shaped if and only if $\rho + \omega > 1$ (which requires in particular $\rho > 0$). This makes our New Keynesian model less vulnerable to the lack-of-empirical-validity criticism which has been addressed to the canonical New Keynesian model (corresponding to the particular case $\gamma = \rho = 0$) for its inability to match the hump-shaped responses of variables to shocks observed in the data.

comes from the fact that the commitment technology enables the central bank to spread the burden of the adjustment to shocks over time: following a positive cost-push shock ε_t, the central bank can trade off not only between a higher inflation rate and a lower output gap at date t, but also between the situation at date t and the future situations. As shown by Clarida, Galí and Gertler (1999), this trade-off between the present and the future improves the trade-off between output and inflation in the present, as an expected future inflation term $\widetilde{E}_t\left\{\pi_{t+1}\right\} = E_t\left\{\pi_{t+1}\right\}$ in-between $-\frac{\varepsilon_t}{\beta}$ and 0 offsets part of the effect of the cost-push shock ε_t in the Phillips curve taken at date t. Of course, the precommitment equilibrium proves time-inconsistent since at date $t+1$ the central bank has no incentive to go on reacting to the bygone shock ε_t in this purely forward-looking framework, and this time-inconsistency gives rise to the so-called stabilization bias.

The social loss function is easily shown to take the following values:

$$L_t^d = \frac{\lambda\left(\kappa^2+\lambda\right)\left[\left(1-\beta\right)\eta_t^2+\beta V_\varepsilon\right]}{\left(1-\beta\right)\left(1-\beta\rho^2\right)\left[\kappa^2+\lambda\left(1-\beta\rho\right)\right]^2} \text{ for } t \in \mathbb{Z},$$

$$L^d = \frac{\lambda\left(\kappa^2+\lambda\right)V_\varepsilon}{\left(1-\beta\right)\left(1-\rho^2\right)\left[\kappa^2+\lambda\left(1-\beta\rho\right)\right]^2},$$

$$L_0^{c,0} = \frac{\omega\left[\left(1-\beta\right)\eta_0^2+\beta V_\varepsilon\right]}{\left(1-\beta\right)\left(1-\beta\rho^2\right)\left(1-\beta\rho\omega\right)^2},$$

$$L^c = \frac{\omega V_\varepsilon}{\left(1-\beta\right)\left(1-\rho^2\right)\left(1-\beta\rho\omega\right)^2},$$

$$L^{tp} = \frac{\left[\kappa^2\left(1+\rho\omega\right)+2\lambda\left(1-\rho\right)\left(1-\omega\right)\right]\omega^2 V_\varepsilon}{\left(1-\beta\right)\lambda\left(1-\rho^2\right)\left(1-\omega^2\right)\left(1-\rho\omega\right)\left(1-\beta\rho\omega\right)^2}.$$

Finally, the unconditional mean of $L_t^{c,0}$ is easily shown to be a strictly increasing function of $t \in \mathbb{N}$, so that we have $L^c < E\left\{L_t^{c,0}\right\} < L^{tp}$ for all $t > 0$, where $E\{.\}$ represents the unconditional mean operator. We naturally also have $L^c < L^d$ and we find moreover that $L^{tp} < L^d$ is obtained for all calibrations reported in **appendix J**[16].

4.2 Central bank reputation

In this subsection the private agents are assumed to behave according to the grim-trigger mechanism described in subsection 1.2. As shown in **appendix C**, we have for $0 \leq k \leq D-1$:

$$z_{n+k}^{d,n+D} = \left[1-\frac{\beta\omega\left(1-\omega\right)}{1-\beta\rho\omega}\left(\frac{\beta\lambda\rho}{\kappa^2+\lambda}\right)^{D-k}\right]\frac{\lambda\eta_{n+k}}{\kappa^2+\lambda\left(1-\beta\rho\right)},$$

$$x_{n+k}^{d,n+D} = -\left[1-\frac{\beta\omega\left(1-\omega\right)}{1-\beta\rho\omega}\left(\frac{\beta\lambda\rho}{\kappa^2+\lambda}\right)^{D-k}\right]\frac{\kappa\eta_{n+k}}{\kappa^2+\lambda\left(1-\beta\rho\right)}.$$

[16] We also find that $L^{tp} > L^d$ is theoretically possible, at least for some unlikely values of the parameters, in accordance with Blake's (2001) results.

Like in subsection 2.2, this result shows in particular the existence of a bijective relationship, for a given value of $(\beta, \kappa, \lambda, \rho)$ such that $\rho \neq 0$ and at each date $n+k \in \{n, ..., n+D-1\}$ of the punishment interval provided that $\eta_{n+k} \neq 0$, between the quasi-differenced inflation rate $z_{n+k} = z_{n+k}^{d,n+D}$ or the output gap $x_{n+k} = x_{n+k}^{d,n+D}$ on the one hand and the punishment length D on the other hand. The existence of this bijective relationship, which is a consequence of the forward-looking nature of the model and the fact that $E_n\left\{z_{n+D}^B\right\} = E_n\left\{z_{n+D}^{c,n+D}\right\} \neq 0$ (when $\rho \neq 0$)[17], would greatly facilitate the coordination of the private agents on a given punishment length, as shown by **appendix F**, should they initially ignore the value of D.

Appendix H shows that a necessary and sufficient condition when $0 \leq \rho \leq \frac{1}{2}$ and a sufficient condition when $\frac{1}{2} < \rho < 1$ for the precommitment equilibrium to be a reputational equilibrium is

$$\frac{\omega\bar{\varepsilon}^2}{\beta(1-\rho)(1-\omega)V_\varepsilon} + \frac{\left(1-\beta^{D-1}\right)(1-\rho)}{(1-\beta)(1-\beta\rho^2)} \leq \frac{\lambda\left(\kappa^2+\lambda\right)(1-\beta\rho\omega)^2}{(1+\rho)\left[\kappa^2+\lambda(1-\beta\rho)\right]^2\omega}\left[\left(\frac{1-\beta^{D-1}}{1-\beta} - \rho^2\frac{1-\beta^{D-1}\rho^{2D-2}}{1-\beta\rho^2}\right) - \frac{2\beta\omega(1-\omega)}{1-\beta\rho\omega}\left(\frac{\beta\lambda\rho}{\kappa^2+\lambda}\right)^{D-1}\left(\frac{1-\frac{\left(\kappa^2+\lambda\right)^{D-1}}{\lambda^{D-1}\rho^{D-1}}}{1-\frac{\kappa^2+\lambda}{\lambda\rho}} - \rho^2\frac{1-\frac{\left(\kappa^2+\lambda\right)^{D-1}\rho^{D-1}}{\lambda^{D-1}}}{1-\frac{\left(\kappa^2+\lambda\right)\rho}{\lambda}}\right) + \frac{\beta^2\omega^2(1-\omega)^2}{(1-\beta\rho\omega)^2}\left(\frac{\beta\lambda\rho}{\kappa^2+\lambda}\right)^{2D-2}\left(\frac{1-\frac{\left(\kappa^2+\lambda\right)^{2D-2}}{\beta^{D-1}\lambda^{2D-2}\rho^{2D-2}}}{1-\frac{\left(\kappa^2+\lambda\right)^2}{\beta\lambda^2\rho^2}} - \rho^2\frac{1-\frac{\left(\kappa^2+\lambda\right)^{2D-2}}{\beta^{D-1}\lambda^{2D-2}}}{1-\frac{\left(\kappa^2+\lambda\right)^2}{\beta\lambda^2}}\right)\right]. \quad (5)$$

Two points are worth noting about this proposition. First, thus written this inequality is defined only for $\rho > 0$, but it can easily be extended by continuity to the case $\rho = 0$. Second, as shown in **appendix H**, if $0 \leq \rho \leq \frac{1}{2}$ then $\widehat{S}$ is attained only asymptotically (*i.e.* for $n \to +\infty$, which implies that $\widehat{M}$ does not exist) and more precisely in the case of an infinite sequence of shocks equal to $\bar{\varepsilon}$. Like in subsection 2.2, this result implies that the precommitment equilibrium is a reputational equilibrium if and only if the timeless perspective equilibrium is a reputational equilibrium.

Now let us define F as the function

$$\begin{aligned}]0;1[\times\mathbb{R}^*\times\mathbb{R}^*\times[0;1[\times\mathbb{N}^* &\longrightarrow \mathbb{R} \\ (\beta,\kappa,\lambda,\rho,D) &\longmapsto F(\beta,\kappa,\lambda,\rho,D) \end{aligned}$$

[17] The condition $\rho \neq 0$ is necessary and sufficient for $E_n\left\{z_{n+D}^B\right\} = E_n\left\{z_{n+D}^{c,n+D}\right\}$ to differ from zero in our canonical New Keynesian model with $(x^*, \delta) = (0, 1)$ and under our grim-trigger mechanism assumption. But $E_n\left\{z_{n+D}^B\right\} = E_n\left\{z_{n+D}^{c,n+D}\right\} \neq 0$ could also be obtained for $\rho = 0$ in the same model under an alternative grim-trigger mechanism assumption (for instance if the equilibrium expected to be implemented immediately after the punishment interval is the timeless perspective equilibrium instead of the precommitment equilibrium) or under the same grim-trigger mechanism assumption in an alternative model (for instance the model with the same Phillips curve and social loss function as ours except that the inflation rate π would replace the quasi-differenced inflation rate z in the social loss function).

such that inequality (5) is equivalent to

$$\frac{\overline{\varepsilon}^2}{V_\varepsilon} \leq F(\beta,\kappa,\lambda,\rho,D).$$

In the particular case $\rho = 0$, we have

$$F(\beta,\kappa,\lambda,0,D) = \frac{\beta^2\left(1-\beta^{D-1}\right)\lambda(1-\omega)^2}{(1-\beta)(\kappa^2+\lambda)\omega}.$$

Appendix I determines function F's variations and limits when $\rho = 0$, reported in **table 2**.

Table 2: function F's variations and limits when $\rho = 0$.

Par.	Left limit	Variation	Right limit
β	$\lim_{\beta\longrightarrow 0^+} F(\beta,\kappa,\lambda,0,D) = 0$	$\frac{\partial F}{\partial \beta} > 0$	$\lim_{\beta\longrightarrow 1^-} F(\beta,\kappa,\lambda,0,D) = \frac{\kappa^2(D-1)}{\kappa^2+\lambda}$
κ	$\lim_{\kappa\longrightarrow 0^+} F(\beta,\kappa,\lambda,0,D) = 0$	$\frac{\partial F}{\partial \kappa} > 0$	$\lim_{\kappa\longrightarrow +\infty} F(\beta,\kappa,\lambda,0,D) = \frac{\beta^2\left(1-\beta^{D-1}\right)}{1-\beta}$
λ	$\lim_{\lambda\longrightarrow 0^+} F(\beta,\kappa,\lambda,0,D) = \frac{\beta^2\left(1-\beta^{D-1}\right)}{1-\beta}$	$\frac{\partial F}{\partial \lambda} < 0$	$\lim_{\lambda\longrightarrow +\infty} F(\beta,\kappa,\lambda,0,D) = 0$
D	$F(\beta,\kappa,\lambda,0,1) = 0$	$\frac{\Delta F}{\Delta D} > 0$	$\lim_{D\longrightarrow +\infty} F(\beta,\kappa,\lambda,0,D) = \frac{\beta^2\lambda(1-\omega)^2}{(1-\beta)(\kappa^2+\lambda)\omega}$

These results are in accordance with conventional wisdom: the precommitment equilibrium is all the more likely to be a reputational equilibrium as λ is low[18] and β, κ, D are large, *i.e.* as the central bank is patient and conservative, the short-run Phillips curve is steep (for given expectations) and the punishment interval is long. In particular, the result $\lim_{D\longrightarrow +\infty}\lim_{\beta\longrightarrow 1^-} F(\beta,\kappa,\lambda,0,D) = \lim_{\beta\longrightarrow 1^-}\lim_{D\longrightarrow +\infty} F(\beta,\kappa,\lambda,0,D) = +\infty$ is a direct consequence of the folk theorem, even though the discount factor β also appears in the Phillips curve.

Let $\underline{D}_r \in \mathbb{N}^*$ for $r \geq 1$ denote the value of $\underline{D}$ when $\frac{\overline{\varepsilon}^2}{V_\varepsilon}$ is equal to r. We focus on the following cases: i) $\frac{\overline{\varepsilon}^2}{V_\varepsilon} = 1$, corresponding to the limit case of a Dirac distribution; ii) $\frac{\overline{\varepsilon}^2}{V_\varepsilon} = 3$, corresponding to the uniform distribution; iii) $\frac{\overline{\varepsilon}^2}{V_\varepsilon} = 6$, corresponding to an "isoceles triangle distribution". The consideration of various calibrations found in the literature (detailed in **appendix J**) then leads to the numerical results reported in **table 3**, where the notation NA stands for "non-available".

[18] We proceed here as if λ were independent of the other parameters, that is to say as if the central bank sought to minimize an *ad hoc* loss function instead of the social loss function, in order to be able to define the central bank's degree of conservatism and to assess its effect of the central bank's reputation. **Appendix J** makes clear however that all the calibrations considered in this paper set λ to its model-consistent value, so that the corresponding loss functions are the social loss functions.

Table 3: numerical results for calibrations 2-6.

No.	ω	Π^c	Π^{tp}	$\lim_{D \longrightarrow +\infty} F(\beta, \kappa, \lambda, \rho, D)$	$\underline{D}_1$	$\underline{D}_3$	$\underline{D}_6$
2a	$0,64$	NA	NA	$16,04$	8	22	48
2b	$0,64$	$0,26$	$0,26$	$16,04$	8	22	48
2c	$0,64$	$0,43$	$0,43$	$16,04$	8	22	48
3	$0,66$	$1,23$	$1,22$	$13,19$	10	28	63
4	$0,51$	$0,48$	$0,48$	$24,55$	7	15	30
5	$0,64$	NA	NA	$11,66$	≤ 14	≤ 35	≤ 77
6	$0,64$	NA	NA	$10,05$	≤ 17	≤ 42	≤ 97

Three points are worth noting about these results. First, when they exist Π^c and Π^{tp} are very close to each other, which implies that the timeless perspective equilibrium hardly reduces welfare compared to the precommitment equilibrium. Second, when they exist Π^c and Π^{tp} can substantially vary from one calibration to another, ranging from $0,26\%$ to $1,23\%$ for Π^c and from $0,26\%$ to $1,22\%$ for Π^{tp}, but they are always sizeable (except arguably for calibration 2b) so that the welfare gain from commitment is never negligible. Third, whatever the calibration considered $\underline{D}_1$, $\underline{D}_3$ and $\underline{D}_6$ exist and are respectively found in-between 7 and 17 quarters ($1\frac{3}{4}$ year and $4\frac{1}{4}$ years), 15 and 42 quarters ($3\frac{3}{4}$ years and $10\frac{1}{2}$ years), 30 and 97 quarters ($7\frac{1}{2}$ years and $24\frac{1}{4}$ years).

Though some of these figures are clearly beyond the order of a few years, we argue that the precommitment equilibrium should nonetheless qualify as a reputational equilibrium for three reasons. First, if we limit ourselves to the calibrations for which the exact values of $\underline{D}_1$, $\underline{D}_3$ and $\underline{D}_6$ are known (*i.e.* the calibrations 2-4, which set $\rho \leq \frac{1}{2}$), then the upper bound falls from 17 to 10 quarters (from $4\frac{1}{4}$ to $2\frac{1}{2}$ years) for $\underline{D}_1$, from 42 to 28 quarters (from $10\frac{1}{2}$ to 7 years) for $\underline{D}_3$ and from 97 to 63 quarters (from $24\frac{1}{4}$ to $15\frac{3}{4}$ years) for $\underline{D}_6$. Second, the "isoceles triangle distribution", corresponding to $\underline{D} = \underline{D}_6$, for which $\underline{D}$ takes its maximal values, and to a lesser extent the uniform distribution, corresponding to $\underline{D} = \underline{D}_3$, for which $\underline{D}$ takes intermediate values, might not be the most relevant distributions to consider in our context. Indeed, for the stabilization bias to be overcome we require that the central bank should prefer not to deviate even in the most tempting situation, which corresponds to an infinite sequence of shocks equal to $\bar{\varepsilon}$. Now the probability that this situation should occur is equal to zero whatever the distribution considered, and any situation nearby (such as a long sequence of shocks close to $\bar{\varepsilon}$) is all the more unlikely as the probability of ε being close to $\bar{\varepsilon}$ is low. This implies that our condition for the stabilization bias to be overcome is demanding for the uniform distribution and even more demanding for the "isoceles triangle distribution". Third, if in reality the private agents are not initially coordinated on a given value of D, then our results underestimate the social cost of deviating from the precommitment equilibrium and hence overestimate the value taken by $\underline{D}_r$ for any $r \geq 1$, as shown in **appendix**

$\mathbf{F}$[19].

Conclusion

This paper examines whether reputation concerns can induce the central bank to implement the time-inconsistent optimal monetary policy in a standard New Keynesian model. Our analysis rests on a simple grim-trigger mechanism assumption in an infinite-horizon repeated game with complete information. This grim-trigger mechanism assumption is all the more relevant in our framework as the forward-looking nature of our standard New Keynesian model greatly facilitates the coordination of the private agents on the punishment length − except in the particular case of serially uncorrelated cost-push shocks. Our results suggest that the inflation bias and the stabilization bias can be overcome for the calibrations used in the literature. These results enable us to endogenize Woodford's (1999) timeless perspective and tend to weaken the case for monetary policy delegation shortly presented in the introduction of this article.

Examining the issue of central bank reputation in a dynamic and possibly stochastic model with a finite punishment length raises some practical difficulties. In this paper we have overcome these difficulties for a standard New Keynesian model by focusing on the most tempting situation for the central bank, thus determining a necessary and sufficient condition for the central bank never to deviate from the precommitment equilibrium, rather than a necessary and sufficient condition for the central bank not to deviate from this equilibrium in a given situation. Now this method can easily be generalized to many other dynamic and possibly stochastic models for which it should provide a useful indicator of whether to consider the discretionary equilibrium or the timeless perspective equilibrium in the presence of a sizeable inflation or stabilization bias. In particular, to apply our method to the popular New Keynesian model with structural inflation inertia first considered by Clarida, Galí and Gertler (1999)[20] would be interesting for two reasons: first, because the stabilization bias may be even larger in this model than in our model with $\gamma = 0$, as shown by Dennis and Söderström (2002), even though this bias disappears in the limit case of a purely backward-looking Phillips curve; second, because the stabilization bias remains sizeable

[19] In addition, $\underline{D}_r$ for any $r \geq 1$ is affected by two assumptions about the timing of the model. On the one hand, our assumption that the central bank observes the current shock before deciding whether to deviate from the precommitment equilibrium tends to bias $\underline{D}_r$ for any $r \geq 1$ upwards, as clear from **appendix H**. On the other hand, our assumption that the private agents can instantly observe and react to a deviation from the precommitment equilibrium (as mentioned in subsection 1.2) tends to bias $\underline{D}_r$ for any $r \geq 1$ downwards.

[20] We are referring to the model with a Phillips curve of the form $\pi_t = a\widetilde{E}_t\{\pi_{t+1}\} + b\pi_{t-1} + cx_t + \eta_t$ (sometimes called the "hybrid" New Keynesian Phillips curve for its partly forward-looking, partly backward-looking nature in terms of the inflation rate) and a social loss function of the form $L_t = E_t\left\{\sum_{k=0}^{+\infty}\beta^k\left[(\pi_{t+k})^2 + d\,(x_{t+k})^2\right]\right\}$, where a, b, c and d are strictly positive real numbers. This work could be carried out only in the form of numerical computations (since no tractable analytical results would then be available) and would notably require the use of a software programme maximizing numerically a quadratic function $(\widehat{L}_n^A - \widehat{L}_n^B)$ of a large number of bounded variables ($\varepsilon_i \in [-\overline{\varepsilon}, \overline{\varepsilon}]$ for $0 \leq i \leq n$ with $n \to +\infty$).

when information and/or transmission lags[21] are introduced into this model, as shown by Dennis and Söderström (2002) and Lam and Pelgrin (2004)[22].

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[21] The presence of information lags in our framework would mean that the central bank and the private agents observe variables and shocks with delay, but the private agents could still observe the central bank's decision (about whether to deviate from the precommitment equilibrium) without delay if the central bank measures variables and shocks and follows its interest rate rule in a transparent way.

[22] Depending on the calibration and on the nature of the lag(s) considered, Π^c ranges indeed from $0,24\%$ to $0,86\%$ in Dennis and Söderström (2002) and from $0,59\%$ to $0,92\%$ in Lam and Pelgrin (2004). As emphasized by Lam and Pelgrin (2004), the stabilization bias is however dramatically reduced by the introduction of information and/or transmission lags into the model when the gain from commitment is measured by the percentage decrease in the unconditional mean of the loss function from its value under discretion, *i.e.* by $\Omega^c \equiv 100\left(1-\frac{L^c}{L^d}\right)$ and (if $L^{tp} < L^d$) $\Omega^{tp} \equiv 100\left(1-\frac{L^{tp}}{L^d}\right)$. But the relevance of this measure of the gain from commitment can be questioned on two grounds: first, on the ground that the social loss function is linearly related but not proportional to the second-order approximation of the representative household's utility function, as shown by Woodford (2003a, chapter 6); second, on the ground that the representative household's utility is ordinal, not cardinal.

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Appendix

In this appendix, we adopt the conventions $\sum_{j=0}^{-1}(.) = \sum_{j=1}^{0}(.) = 0$ and $0^0 = 1$.

A Determination of the discretionary equilibrium

At each date t considered the central bank chooses z_t and x_t so as to minimize L_t subject to the Phillips curve taken at date t. Since z_{t+k} and x_{t+k} for $k \geq 1$ will be chosen in the future and since today's choice of z_t and x_t will not influence tomorrow's choice of z_{t+k} and x_{t+k} (as the model is purely forward-looking in terms of the quasi-differenced inflation rate z), the private agents' expectations $E_t\{z_{t+k}\}$ and $E_t\{x_{t+k}\}$ do not depend on the choice of z_t and x_t, so that the central bank considers these expectations as given when minimizing L_t subject to the Phillips curve taken at date t.

The first-order condition of the minimization programme at date t is $\kappa z_t + \lambda x_t = \lambda x^*$, from which we derive $E_t\{z_{t+1}\} = \frac{\kappa^2+\lambda}{\beta\lambda} z_t - \frac{\kappa x^*}{\beta} - \frac{\delta\eta_t}{\beta}$ with the Phillips curve taken at date t. For $k \geq 1$ similarly, the first-order condition of the minimization programme at date $t+k$ taken in expectations $E_t\{.\}$ is $\kappa E_t\{z_{t+k}\} + \lambda E_t\{x_{t+k}\} = \lambda x^*$, from which we derive the recurrence equation $E_t\{z_{t+k+1}\} = \frac{\kappa^2+\lambda}{\beta\lambda} E_t\{z_{t+k}\} - \frac{\kappa x^*}{\beta} - \frac{\delta\rho^k\eta_t}{\beta}$ with the Phillips curve taken in expectations $E_t\{.\}$ at date $t+k$. These two equations lead in turn to

$$\begin{aligned} E_t\{z_{t+k}\} &= \left(\frac{\kappa^2+\lambda}{\beta\lambda}\right)^k \left[z_t - \frac{\kappa\lambda x^*}{\kappa^2+\lambda(1-\beta)} - \frac{\delta\lambda\eta_t}{\kappa^2+\lambda(1-\beta\rho)}\right] \\ &\quad + \frac{\kappa\lambda x^*}{\kappa^2+\lambda(1-\beta)} + \frac{\delta\lambda\rho^k\eta_t}{\kappa^2+\lambda(1-\beta\rho)} \end{aligned}$$

for $k \geq 1$. The solution to the optimization programme satisfies therefore

$$z_t = \frac{\kappa \lambda x^*}{\kappa^2 + \lambda (1-\beta)} + \frac{\delta \lambda \eta_t}{\kappa^2 + \lambda (1-\beta\rho)},$$

since L_t would take an infinite value otherwise. The condition $\kappa z_t + \lambda x_t = \lambda x^*$ then leads to

$$x_t = \frac{\lambda (1-\beta) x^*}{\kappa^2 + \lambda (1-\beta)} - \frac{\delta \kappa \eta_t}{\kappa^2 + \lambda (1-\beta\rho)}.$$

B Determination of the 0-commitment equilibrium

We follow the undetermined coefficients method to solve analytically the central bank's optimization problem. Since the central bank has observed shocks ε_{-i} for $i \geq 0$ at date 0[23], the variables can be rewritten in the following way prior to the minimization of L_0:

$$z_k \equiv \sum_{j=0}^{k-1} a_{j,k} \varepsilon_{k-j} + g_k \text{ and } x_k \equiv \sum_{j=0}^{k-1} b_{j,k} \varepsilon_{k-j} + h_k$$

for $k \geq 0$. We look for the coefficients $a_{j,k}$, $b_{j,k}$, g_k and h_k for $k \geq 0$ and $0 \leq j \leq k-1$ which minimize L_0 subject to the Phillips curve considered at all dates, *i.e.* which minimize the following Lagrangian:

$$E_0 \left\{ \sum_{k=0}^{+\infty} \beta^k \left[(z_k)^2 + \lambda (x_k - x^*)^2 \right] \right\} - \sum_{k=0}^{+\infty} \mu_k \left(z_k - \beta E_k \{z_{k+1}\} - \kappa x_k - \delta \eta_k \right).$$

The first-order conditions of the Lagrangian's minimization with respect to $a_{0,k}$ for $k \geq 1$, $a_{j,k}$ for $k \geq 2$ and $j \in \{1, ..., k-1\}$, $b_{j,k}$ for $k \geq 1$ and $j \in \{0, ..., k-1\}$, g_0, g_k for $k \geq 1$, h_k for $k \geq 0$ can be respectively written in the following way:

$$\begin{aligned}
2\beta^k V_\varepsilon a_{0,k} - \mu_k \varepsilon_k &= 0 \text{ for } k \geq 1, \\
2\beta^k V_\varepsilon a_{j,k} - \mu_k \varepsilon_{k-j} + \beta \mu_{k-1} \varepsilon_{k-j} &= 0 \text{ for } k \geq 2 \text{ and } j \in \{1, ..., k-1\}, \\
2\beta^k \lambda V_\varepsilon b_{j,k} + \kappa \mu_k \varepsilon_{k-j} &= 0 \text{ for } k \geq 1 \text{ and } j \in \{0, ..., k-1\}, \\
2g_0 - \mu_0 &= 0, \\
2\beta^k g_k - \mu_k + \beta \mu_{k-1} &= 0 \text{ for } k \geq 1, \\
2\beta^k \lambda (h_k - x^*) + \kappa \mu_k &= 0 \text{ for } k \geq 0,
\end{aligned}$$

and the Phillips curve considered at all dates provides the following two additional equations:

$$\begin{aligned}
\beta a_{j+1,k+1} - a_{j,k} + \kappa b_{j,k} &= -\delta \rho^j \text{ for } k \geq 1 \text{ and } j \in \{0, ..., k-1\}, \\
\beta g_{k+1} - g_k + \kappa h_k &= -\delta \rho^k \eta_0 \text{ for } k \geq 0.
\end{aligned}$$

[23] Similar computations show that the 0-commitment equilibrium would be unchanged under the alternative assumption that the central bank has observed shocks ε_{-i} for $i \geq 1$ but not ε_0 when committing at date 0.

Let us note $u \equiv k - j$, $v \equiv j$, $A_{u,v} \equiv a_{j,k}$ and $B_{u,v} \equiv b_{j,k}$, so that $A_{u,v}$ and $B_{u,v}$ characterize respectively the responses of z_{u+v} and x_{u+v} to ε_u. Our eight equations are then equivalent to the following systems of equations:

$$\begin{cases} \kappa g_0 + \lambda h_0 = \lambda x^* \\ \kappa g_{k+1} + \lambda h_{k+1} - \lambda h_k = 0 & \text{for } k \geq 0 \\ \beta g_{k+1} - g_k + \kappa h_k = -\delta \rho^k \eta_0 & \text{for } k \geq 0 \end{cases} \quad (6)$$

$$\text{and} \begin{cases} \kappa A_{u,0} + \lambda B_{u,0} = 0 & \text{for } u \geq 1 \\ \kappa A_{u,v+1} + \lambda B_{u,v+1} - \lambda B_{u,v} = 0 & \text{for } u \geq 1 \text{ and } v \geq 0 \\ \beta A_{u,v+1} - A_{u,v} + \kappa B_{u,v} = -\delta \rho^v & \text{for } u \geq 1 \text{ and } v \geq 0 \end{cases} . \quad (7)$$

System (6) implies that the coefficients g_k satisfy the following equations:

$$\begin{aligned} \beta\lambda g_1 - \left(\kappa^2 + \lambda\right) g_0 &= -\kappa\lambda x^* - \delta\lambda\eta_0, \\ \beta\lambda g_{k+2} - \left(\beta\lambda + \kappa^2 + \lambda\right) g_{k+1} + \lambda g_k &= \delta\lambda \left(1 - \rho\right) \rho^k \eta_0 \text{ for } k \geq 0. \end{aligned}$$

The latter equation corresponds to a recurrence equation on the g_k for $k \geq 0$. The corresponding characteristic polynomial has three positive real roots ρ, ω and ω' with:

$$\begin{aligned} \omega &\equiv \frac{\left(\beta\lambda + \kappa^2 + \lambda\right) - \sqrt{\left(\beta\lambda + \kappa^2 + \lambda\right)^2 - 4\beta\lambda^2}}{2\beta\lambda} < 1, \\ \omega' &\equiv \frac{\left(\beta\lambda + \kappa^2 + \lambda\right) + \sqrt{\left(\beta\lambda + \kappa^2 + \lambda\right)^2 - 4\beta\lambda^2}}{2\beta\lambda} > 1. \end{aligned}$$

The general form of the solution to the recurrence equation is therefore $g_k = p_1 \rho^k + p_2 \omega^k + p_3 \omega'^k$ for $k \geq 0$, where $(p_1, p_2, p_3) \in \mathbb{R}^3$. Three equations are then needed to determine (p_1, p_2, p_3). Two are provided by the initial conditions $\beta\lambda g_1 - \left(\kappa^2 + \lambda\right) g_0 = -\kappa\lambda x^* - \delta\lambda\eta_0$ and $\beta\lambda g_2 - \left(\beta\lambda + \kappa^2 + \lambda\right) g_1 + \lambda g_0 = \delta\lambda \left(1 - \rho\right) \eta_0$. The third one is simply $p_3 = 0$ and comes from the fact that $\beta\omega'^2 \geq 1$, as can be readily checked, so that no solution with $p_3 \neq 0$ would fit the bill as L_0 would then be infinite. We thus eventually obtain the following solution for system (6):

$$\begin{aligned} g_k &= \frac{\lambda \left(1 - \omega\right) \omega^k x^*}{\kappa} + \frac{\delta\omega \left[\left(1 - \rho\right) \rho^k - \left(1 - \omega\right) \omega^k\right] \eta_0}{\left(1 - \beta\rho\omega\right) \left(\omega - \rho\right)} \text{ for } k \geq 0, \\ h_k &= \omega^{k+1} x^* + \frac{\delta\kappa\omega \left(\rho^{k+1} - \omega^{k+1}\right) \eta_0}{\lambda \left(1 - \beta\rho\omega\right) \left(\omega - \rho\right)} \text{ for } k \geq 0. \end{aligned}$$

The similarity between systems (6) and (7) enables us to derive the solution of system (7) from the solution of system (6) in a straightforward way:

$$A_{u,v} = \frac{\delta\omega\left[(1-\rho)\rho^v - (1-\omega)\omega^v\ \right]}{(1-\beta\rho\omega)(\omega-\rho)} \text{ for } u \geq 1 \text{ and } v \geq 0,$$
$$B_{u,v} = \frac{\delta\kappa\omega\left(\rho^{v+1} - \omega^{v+1}\right)}{\lambda(1-\beta\rho\omega)(\omega-\rho)} \text{ for } u \geq 1 \text{ and } v \geq 0,$$

so that we eventually obtain the following results for $k \geq 0$:

$$z_k = \frac{\lambda(1-\omega)\omega^k x^*}{\kappa} + \frac{\delta\omega\left[(1-\rho)\eta_k - (1-\omega)\xi_k\right]}{(1-\beta\rho\omega)(\omega-\rho)},$$
$$x_k = \omega^{k+1}x^* + \frac{\delta\kappa\omega(\rho\eta_k - \omega\xi_k)}{\lambda(1-\beta\rho\omega)(\omega-\rho)}.$$

where $\xi_k \equiv \sum_{j=0}^{k} \omega^j \varepsilon_{k-j} + \omega^k \rho\eta_{-1}$.

C Determination of the punishment equilibrium

The central bank acts in a discretionary way from date n to date $n+D-1$ included since it cannot influence the private agents' expectations during this punishment interval. The first-order condition of the minimization programme at date $n+k$ for $0 \leq k \leq D-1$ is $\kappa z_{n+k} + \lambda x_{n+k} = \lambda x^*$, from which we derive $z_{n+k} = \frac{\beta\lambda}{\kappa^2+\lambda}\widetilde{E}_{n+k}\{z_{n+k+1}\} + \frac{\kappa\lambda x^*}{\kappa^2+\lambda} + \frac{\delta\lambda\eta_{n+k}}{\kappa^2+\lambda}$ with the Phillips curve taken at date $n+k$. The latter equation leads to

$$z_{n+k} = \left(\frac{\beta\lambda}{\kappa^2+\lambda}\right)^{D-k}\widetilde{E}_{n+k}\{z_{n+D}\} + \left[1 - \left(\frac{\beta\lambda}{\kappa^2+\lambda}\right)^{D-k}\right]\frac{\kappa\lambda x^*}{\kappa^2+\lambda(1-\beta)} + \left[1 - \left(\frac{\beta\lambda\rho}{\kappa^2+\lambda}\right)^{D-k}\right]\frac{\delta\lambda\eta_{n+k}}{\kappa^2+\lambda(1-\beta\rho)}$$

for $0 \leq k \leq D-1$, so that by replacing $\widetilde{E}_{n+k}\{z_{n+D}\}$ by

$$E_{n+k}\left\{z_{n+D}^{c,n+D}\right\} = \frac{\lambda(1-\omega)x^*}{\kappa} + \frac{\delta\omega\rho^{D-k}\eta_{n+k}}{1-\beta\rho\omega}$$

we eventually get for $0 \leq k \leq D-1$

$$z_{n+k} = \left[\kappa^2 - \beta\lambda(1-\omega)^2\left(\frac{\beta\lambda}{\kappa^2+\lambda}\right)^{D-k}\right]\frac{\lambda x^*}{\kappa\left[\kappa^2+\lambda(1-\beta)\right]} + \left[1 - \frac{\beta\omega(1-\omega)}{1-\beta\rho\omega}\left(\frac{\beta\lambda\rho}{\kappa^2+\lambda}\right)^{D-k}\right]\frac{\delta\lambda\eta_{n+k}}{\kappa^2+\lambda(1-\beta\rho)}$$

and, with the first-order condition $\kappa z_{n+k} + \lambda x_{n+k} = \lambda x^*$,

$$\begin{aligned} x_{n+k} &= \left[(1-\beta)+\beta(1-\omega)^2\left(\frac{\beta\lambda}{\kappa^2+\lambda}\right)^{D-k}\right]\frac{\lambda x^*}{\kappa^2+\lambda(1-\beta)} \\ &\quad -\left[1-\frac{\beta\omega(1-\omega)}{1-\beta\rho\omega}\left(\frac{\beta\lambda\rho}{\kappa^2+\lambda}\right)^{D-k}\right]\frac{\delta\kappa\eta_{n+k}}{\kappa^2+\lambda(1-\beta\rho)}. \end{aligned}$$

D Proof of proposition (2)

Suppose that $S \leq 0$ and (if it exists) $M < 0$. The only admissible values for the rational expectations of $\left(z^A_{n+k}, x^A_{n+k}\right)_{k\geq 1}$ and $\left(z^B_{n+k}, x^B_{n+k}\right)_{k\geq D}$ are then respectively $\left(z^{c,0}_{n+k}, x^{c,0}_{n+k}\right)_{k\geq 1}$ and $\left(z^{c,n+D}_{n+k}, x^{c,n+D}_{n+k}\right)_{k\geq D}$. Therefore $S = \widehat{S}$ and (if they exist) $M = \widehat{M}$, so that $\widehat{S} \leq 0$ and (if it exists) $\widehat{M} < 0$.

Now suppose that $\widehat{S} \leq 0$ and (if it exists) $\widehat{M} < 0$. Because $L^{c,n+D}_{n+D}$ is the minimal value of L_{n+D}, whatever $\left(\eta_{-1}, n, \varepsilon_0, ..., \varepsilon_n\right)$ we have $\beta^D E_n\left\{L^{c,n+D}_{n+D}\right\} \leq \beta^D E_n\left\{L_{n+D}\right\}$ and hence

$$E_n\left\{\sum_{k=D}^{+\infty}\beta^k\left[\left(z^{c,n+D}_{n+k}\right)^2+\lambda\left(x^{c,n+D}_{n+k}-x^*\right)^2\right]\right\} \leq E_n\left\{\sum_{k=D}^{+\infty}\beta^k\left[\left(z^B_{n+k}\right)^2+\lambda\left(x^B_{n+k}-x^*\right)^2\right]\right\},$$

so that $\widehat{L}^B_n \leq L^B_n$ whatever $\left(n, \eta_{-1}, \varepsilon_0, ..., \varepsilon_n\right)$. The inequalities $\widehat{S} \leq 0$ and (if it exists) $\widehat{M} < 0$ imply therefore that

$$\underset{\substack{n,\ \eta_{-1} \text{ and } \varepsilon_i \\ \text{for } 0\leq i\leq n}}{Sup}\left(\widehat{L}^A_n - L^B_n\right) \leq 0 \text{ and, if it exists, } \underset{\substack{n,\ \eta_{-1} \text{ and } \varepsilon_i \\ \text{for } 0\leq i\leq n}}{Max}\left(\widehat{L}^A_n - L^B_n\right) < 0. \tag{8}$$

The central bank may alternatively at date n: not deviate from the 0-commitment equilibrium and plan never to deviate from it (option $A1$); not deviate from this equilibrium and consider deviating from it at a later date (option $A2$); deviate from this equilibrium (option B). Inequalities (8) imply that L_n is lower with option $A1$ than with option B whatever $\left(n, \eta_{-1}, \varepsilon_0, ..., \varepsilon_n\right)$. Option $A2$, which implies that option B might be chosen at a later date, is therefore not time-consistent (*i.e.* not compatible with rational expectations), contrary to option $A1$. As a consequence, the only admissible values for the rational expectations of $\left(z^A_{n+k}, x^A_{n+k}\right)_{k\geq 1}$ are $\left(z^{c,0}_{n+k}, x^{c,0}_{n+k}\right)_{k\geq 1}$ and therefore $L^A_n = \widehat{L}^A_n$ whatever $\left(\eta_{-1}, n, \varepsilon_0, ..., \varepsilon_n\right)$. Inequalities (8) then imply that $S \leq 0$ and, if it exists, $M < 0$.

E Determination of the sign of $\frac{\Delta\widehat{S}}{\Delta D}$ when $\delta = 0$ or $x^* = 0$

Whatever $\left(\eta_{-1}, n, \varepsilon_0, ..., \varepsilon_n\right)$, $\widehat{L}^A_n$ does not depend on D, while

$$
\begin{aligned}
\widehat{L}_n^B &= E_n\left\{\sum\nolimits_{k=0}^{D-1}\beta^k\left[\left(z_{n+k}^{d,n+D}\right)^2+\lambda\left(x_{n+k}^{d,n+D}-x^*\right)^2\right]\right\}\\
&\quad +E_n\left\{\sum\nolimits_{k=D}^{+\infty}\beta^k\left[\left(z_{n+k}^{c,n+D}\right)^2+\lambda\left(x_{n+k}^{c,n+D}\right)^2\right]\right\}\\
&= \frac{\kappa^2+\lambda}{\lambda}E_n\left\{\sum\nolimits_{k=0}^{D-1}\beta^k\left(z_{n+k}^{d,n+D}\right)^2\right\}+\beta^D E_n\left\{L_{n+D}^{c,n+D}\right\}.
\end{aligned}
$$

Let $\widehat{L}_n^{B\prime}$ denote the value which $\widehat{L}_n^B$ would take if the punishment length were equal to $D'\equiv D+1$:

$$
\widehat{L}_n^{B\prime}=\frac{\kappa^2+\lambda}{\lambda}E_n\left\{\sum\nolimits_{k=0}^{D-1}\beta^k\left(z_{n+k}^{d,n+D'}\right)^2\right\}+\beta^D E_n\left\{\frac{\kappa^2+\lambda}{\lambda}\left(z_{n+D'}^{d,n+D'}\right)^2+\beta L_{n+D'}^{c,n+D'}\right\}.
$$

Whatever $\left(\eta_{-1},\, n,\, \varepsilon_0,\, ...,\, \varepsilon_n\right)$ and $k\in\{0,...,D-1\}$, $\left(z_{n+k}^{d,n+D}\right)^2$ is an increasing function of D if $\delta=0$ or $x^*=0$, so that

$$
\frac{\kappa^2+\lambda}{\lambda}E_n\left\{\sum\nolimits_{k=0}^{D-1}\beta^k\left(z_{n+k}^{d,n+D}\right)^2\right\}\leq\frac{\kappa^2+\lambda}{\lambda}E_n\left\{\sum\nolimits_{k=0}^{D-1}\beta^k\left(z_{n+k}^{d,n+D'}\right)^2\right\}.
$$

Moreover, $L_{n+D}^{c,n+D}$ is the minimal value of L_{n+D} so that whatever $\left(\eta_{-1},\, n,\, \varepsilon_0,\, ...,\, \varepsilon_n\right)$,

$$
\beta^D E_n\left\{L_{n+D}^{c,n+D}\right\}\leq\beta^D E_n\left\{\frac{\kappa^2+\lambda}{\lambda}\left(z_{n+D'}^{d,n+D'}\right)^2+\beta L_{n+D'}^{c,n+D'}\right\}.
$$

These two inequalities imply in turn that $\widehat{L}_n^B\leq\widehat{L}_n^{B\prime}$ whatever $\left(\eta_{-1},\, n,\, \varepsilon_0,\, ...,\, \varepsilon_n\right)$, so that $\widehat{L}_n^A-\widehat{L}_n^B$ is a decreasing function of D whatever $\left(\eta_{-1},\, n,\, \varepsilon_0,\, ...,\, \varepsilon_n\right)$. As a consequence, $\widehat{S}$ is a decreasing function of D, *i.e.* $\frac{\Delta\widehat{S}}{\Delta D}\leq 0$.

F Coordination of the private agents on a punishment length when $\rho\neq 0$

Let us write $z_{n+k}^{d,n+D}$ and $x_{n+k}^{d,n+D}$ for $k\in\{0,...,D-1\}$ in the following form:

$$
z_{n+k}^{d,n+D}=c_{1,n+k}+c_{2,n+k}\chi^{D-k}\text{ and }x_{n+k}^{d,n+D}=d_{1,n+k}+d_{2,n+k}\chi^{D-k}, \tag{9}
$$

$$
\begin{aligned}
\text{where }\chi &\equiv\frac{\beta\lambda\left[1-\delta\left(1-\rho\right)\right]}{\kappa^2+\lambda},\\
c_{1,n+k} &\equiv\frac{\kappa\lambda}{\kappa^2+\lambda\left(1-\beta\right)}x^*+\frac{\lambda}{\kappa^2+\lambda\left(1-\beta\rho\right)}\delta\eta_{n+k},\\
c_{2,n+k} &\equiv\frac{-\beta\lambda^2\left(1-\omega\right)^2}{\kappa\left[\kappa^2+\lambda\left(1-\beta\right)\right]}x^*-\frac{\beta\lambda\omega\left(1-\omega\right)}{\left(1-\beta\rho\omega\right)\left[\kappa^2+\lambda\left(1-\beta\rho\right)\right]}\delta\eta_{n+k},\\
d_{1,n+k} &\equiv\frac{\left(1-\beta\right)\lambda}{\kappa^2+\lambda\left(1-\beta\right)}x^*-\frac{\kappa}{\kappa^2+\lambda\left(1-\beta\rho\right)}\delta\eta_{n+k},\\
d_{2,n+k} &\equiv\frac{\beta\lambda\left(1-\omega\right)^2}{\kappa^2+\lambda\left(1-\beta\right)}x^*+\frac{\beta\kappa\omega\left(1-\omega\right)}{\left(1-\beta\rho\omega\right)\left[\kappa^2+\lambda\left(1-\beta\rho\right)\right]}\delta\eta_{n+k}.
\end{aligned}
$$

This result holds when the value of D is known by all private agents from the start. Now let us consider the alternative case where each private agent i has his or her own initial belief D_i about the value of D. Let Γ denote the set of all private agents and Γ_t the set of the private agents who set their prices optimally at date $t \in \mathbb{Z}$. The distribution of D_i across individuals $i \in \Gamma$ is assumed to be exogenous and independent of the distribution (across individuals $i \in \Gamma$ for any date $t \in \mathbb{Z}$) of the variable which takes the value 1 if $i \in \Gamma_t$ and the value 0 if $i \notin \Gamma_t$. If n denotes the first date of the first punishment interval, then equation (9) for $k = 0$ becomes

$$z_n = c_{1,n} + \frac{c_{2,n}}{\#\Gamma_n} \sum_{i \in \Gamma_n} \chi^{D_i} \text{ and } x_n = d_{1,n} + \frac{d_{2,n}}{\#\Gamma_n} \sum_{i \in \Gamma_n} \chi^{D_i},$$

that is to say equivalently at the first order with the law of large numbers

$$z_n = c_{1,n} + \frac{c_{2,n}}{\#\Gamma} \sum_{i \in \Gamma} \chi^{D_i} \text{ and } x_n = d_{1,n} + \frac{d_{2,n}}{\#\Gamma} \sum_{i \in \Gamma} \chi^{D_i},$$

where $\#\Psi$ represents the cardinal of set Ψ (*i.e.* the number of elements of Ψ).

We assume for simplicity that each private agent: i) observes the aggregate variables z_n and x_n after prices are set at date n and before date $n+1$ (say, as official statistics are publicly disclosed); ii) believes that all other private agents are coordinated on a given value of D, whether this value coincides or not with her initial belief[24]. Since $\rho \neq 0$, the probability that $(c_{2,n}, d_{2,n}) = (0, 0)$ is equal to zero whatever the distribution of shock ε. The observation of (z_n, x_n) therefore reveals to each private agent j the value taken by $\frac{1}{\#\Gamma} \sum_{i \in \Gamma} \chi^{D_i}$, so that she deduces that all other private agents are coordinated on

$$D^* = \frac{\ln\left(\frac{1}{\#\Gamma} \sum_{i \in \Gamma} \chi^{D_i}\right)}{\ln \chi},$$

which is well defined[25] since $\chi \in]0;1[$ when $\rho \neq 0$, and as an "expectations-taker" she will therefore revise her initial belief D_j accordingly at the onset of period $n+1$. This implies that all private agents are coordinated on D^* as soon as the second date of the punishment interval, that is to say that the coordination problem is limited to the first date of the punishment interval. Moreover, the aggregate variables behave as if all private agents were coordinated on D^* from the start: $z_{n+k} = z_{n+k}^{d,n+D^*}$ and $x_{n+k} = x_{n+k}^{d,n+D^*}$ for $k \in \{0, ..., D^* - 1\}$, while the increase in price

[24] Assumption ii) requires in particular that each private agent does not observe the disaggregated variables, *i.e.* the price and quantity of each differentiated good (say, because of prohibitive data collection costs), since he or she could otherwise learn from this observation the distribution of D_i across individuals $i \in \Gamma$. Without assumption ii), *i.e.* if each private agent were aware of the possibility of an initial general disagreement about the value of D, the coordination problem would require a more complicated treatment which is beyond the scope of this paper.

[25] We choose to disregard the problem raised by the fact that D^* is generally not an integer, for the sake of simplicity, on the ground that this problem is artificially due to the discrete nature of our model and would not arise in the continuous-time version of this model.

dispersion due the lack of coordination at date n adds the term $\alpha^{t-1}(1-\alpha)^2 V_n$ to the social loss function L_t for $t \geq n$, where α is the proportion of prices not optimally reset at each date and $V_n = c_{2,n}^2 var_{i\in\Gamma_n}\left(\chi^{D_i}\right)$ the empirical variance of the logarithm of the prices "optimally" reset at date n, as can easily be shown from Woodford's (2003a, chapter 6) analysis. This implies that the results of the paper, obtained under the assumption that the value of D is known by all private agents from the start, underestimate the social cost of deviation from the precommitment equilibrium and hence overestimate $\underline{D}$ if in reality the private agents do not initially know the value of D but instead behave as assumed in this appendix.

G Proof of proposition (4)

$\widehat{L}_n^B$ does not depend on n while $\widehat{L}_n^A = L_n^{c,0}$ is a strictly increasing function of n, so that

$$\begin{aligned}\widehat{S} &= \lim_{n\longrightarrow+\infty}\left(\widehat{L}_n^A - \widehat{L}_n^B\right) = \frac{\lambda x^{*2}}{1-\beta} - \frac{\lambda^2 x^{*2}}{\kappa^2\left[\kappa^2+\lambda(1-\beta)\right]^2}\Bigg[\kappa^2\left[\kappa^2+\lambda(1-\beta)^2\right]\frac{1-\beta^D}{1-\beta} \\ &\quad +\beta^2\left(\kappa^2+\lambda\right)\lambda(1-\omega)^4\left(\frac{\beta\lambda}{\kappa^2+\lambda}\right)^{2D}\frac{1-\frac{\left(\kappa^2+\lambda\right)^{2D}}{\beta^D\lambda^{2D}}}{1-\frac{\left(\kappa^2+\lambda\right)^2}{\beta\lambda^2}} \\ &\quad -2\beta^{D+2}\lambda^2(1-\omega)^2\left[1-\left(\frac{\lambda}{\kappa^2+\lambda}\right)^D\right]\Bigg] - \beta^D\frac{\lambda(1-\omega)^2\left[\kappa^2+\lambda(1-\beta\omega)^2\right]x^{*2}}{(1-\beta)\kappa^2(1-\beta\omega)\left(1-\beta\omega^2\right)}.\end{aligned}$$

Since $\widehat{S}$ is attained only asymptotically, $\widehat{M}$ does not exist. As a consequence, $\widehat{S} \leq 0$ is a necessary and sufficient condition for the precommitment equilibrium to be a reputational equilibrium. Finally, $\widehat{S} \leq 0$ is equivalent to $\frac{\kappa^2}{\lambda^2 x^{*2}}\widehat{S} \leq 0$ which is inequality (4).

H Proof of proposition (5)

Expressing $\widehat{L}_n^A - \widehat{L}_n^B$ as a function of $\left(\eta_{-1}, n, \varepsilon_0, ..., \varepsilon_n\right)$, we get:

$$\widehat{L}_n^A - \widehat{L}_n^B = k_1\left(\xi_n - \eta_n\right)^2 + k_2\left(\eta_n\right)^2 + k_3$$

$$\text{and } \widehat{S} = \underset{\substack{\eta_{-1}\in\left]\frac{-\overline{\varepsilon}}{1-\rho},\frac{\overline{\varepsilon}}{1-\rho}\right[,\ n\in\mathbb{N},\\ \varepsilon_i\in[-\overline{\varepsilon},\overline{\varepsilon}]\text{ for }0\leq i\leq n}}{Sup}\left[k_1\left(\xi_n-\eta_n\right)^2 + k_2\left(\eta_n\right)^2 + k_3\right],$$

$$\text{where } k_1 = \frac{\omega^2 (1-\omega)}{(\omega-\rho)^2 (1-\beta\rho\omega)^2},$$

$$k_2 = \frac{\left(1-\beta^D \rho^{2D}\right)\omega}{(1-\beta\rho^2)(1-\beta\rho\omega)^2} - \frac{\left(\kappa^2+\lambda\right)\lambda}{\left[\kappa^2+\lambda(1-\beta\rho)\right]^2} \sum_{i=0}^{D-1} \beta^i \rho^{2i} q_i^2$$

$$\text{and } k_3 = \frac{\beta\left(1-\beta^{D-1}\right)\omega V_\varepsilon}{(1-\beta)(1-\beta\rho^2)(1-\beta\rho\omega)^2} - \frac{\left(\kappa^2+\lambda\right)V_\varepsilon}{\lambda(1-\rho^2)} \sum_{i=1}^{D-1} \beta^i \left(1-\rho^{2i}\right) q_i^2$$

$$\text{with } q_i = 1 - \frac{\beta\omega(1-\omega)}{1-\beta\rho\omega}\left(\frac{\beta\lambda\rho}{\kappa^2+\lambda}\right)^{D-i} \text{ for } 0 \le i \le D-1.$$

Now consider the upper bound $\overline{\widehat{S}} \geq \widehat{S}$ of function $\widehat{L}_n^A - \widehat{L}_n^B$ when ε_n is artificially allowed to be higher than $\overline{\varepsilon}$ or lower than $-\overline{\varepsilon}$:

$$\overline{\widehat{S}} \equiv \underset{\substack{\eta_{-1} \in \left]\frac{-\overline{\varepsilon}}{1-\rho}, \frac{\overline{\varepsilon}}{1-\rho}\right[,\ n \in \mathbb{N}, \\ \varepsilon_n \in \mathbb{R},\ \varepsilon_i \in [-\overline{\varepsilon}, \overline{\varepsilon}] \\ \text{for } 0 \le i \le n-1 \text{ if } n \ge 1}}{Sup} \left[k_1 (\xi_n - \eta_n)^2 + k_2 (\eta_n)^2 + k_3\right].$$

The value of $\varepsilon_n \in \mathbb{R}$ maximizing $\widehat{L}_n^A - \widehat{L}_n^B$ is $\varepsilon_n = -\rho\eta_{n-1}$ (so that $\eta_n = 0$) because $\widehat{L}_n^A - \widehat{L}_n^B$ depends on ε_n only *via* its term $k_2 (\eta_n)^2$ and because

$$k_2 \le \frac{-\beta\lambda\left(1-\beta^D\rho^{2D}\right)\omega(1-\omega)}{(\kappa^2+\lambda)(1-\beta\rho^2)(1-\beta\rho\omega)^2} < 0$$

$$\text{since } q_i \ge q_{D-1} = \frac{\kappa^2+\lambda(1-\beta\rho)}{(\kappa^2+\lambda)(1-\beta\rho\omega)} > 0 \text{ for } i \in \{0,...,D-1\}.$$

Since $k_1 > 0$, $\overline{\widehat{S}}$ is attained for $\eta_{-1} \longrightarrow \frac{\overline{\varepsilon}}{1-\rho}$ and (if $n \ge 1$) $(\varepsilon_0, ..., \varepsilon_{n-1}) = (\overline{\varepsilon}, ..., \overline{\varepsilon})$ or equivalently $\eta_{-1} \longrightarrow \frac{-\overline{\varepsilon}}{1-\rho}$ and (if $n \ge 1$) $(\varepsilon_0, ..., \varepsilon_{n-1}) = (-\overline{\varepsilon}, ..., -\overline{\varepsilon})$, so that we eventually obtain:

$$\overline{\widehat{S}} = \underset{n\in\mathbb{N}}{Sup}\left[k_1 \frac{(\omega-\rho)^2(1-\omega^n)^2\overline{\varepsilon}^2}{(1-\rho)^2(1-\omega)^2} + k_3\right] = k_1 \frac{(\omega-\rho)^2\overline{\varepsilon}^2}{(1-\rho)^2(1-\omega)^2} + k_3$$

which is attained only asymptotically (for $n \longrightarrow +\infty$). In the general case $0 \le \rho < 1$, we thus have $\widehat{S} \le \overline{\widehat{S}}$ and $\widehat{M} < \overline{\widehat{S}}$ (if $\widehat{M}$ exists) since $\overline{\widehat{S}}$ is attained only asymptotically, so that $\overline{\widehat{S}} \le 0 \Longrightarrow (\widehat{S} \le 0$ and $\widehat{M} < 0$ if $\widehat{M}$ exists), *i.e.* $\overline{\widehat{S}} \le 0$ is a sufficient condition for the precommitment equilibrium to be a reputational equilibrium. In the specific case $0 \le \rho \le \frac{1}{2}$, the value $\lim_{n\longrightarrow+\infty}\left(-\rho\eta_{n-1}\right) = \mp\frac{\rho\overline{\varepsilon}}{1-\rho}$ of $\varepsilon_n \in \mathbb{R}$ maximizing $\widehat{L}_n^A - \widehat{L}_n^B$ belongs to $[-\overline{\varepsilon}, \overline{\varepsilon}]$, so that $\widehat{S} = \overline{\widehat{S}}$ and therefore ($\widehat{S} \le 0$ and $\widehat{M} < 0$ if $\widehat{M}$ exists) $\Longrightarrow \overline{\widehat{S}} \le 0$, *i.e.* $\overline{\widehat{S}} \le 0$ is a necessary condition for the precommitment equilibrium to be a reputational equilibrium.

We have thus shown that $\overline{\widehat{S}} \leq 0$ is a necessary and sufficient condition when $0 \leq \rho \leq \frac{1}{2}$ and a sufficient condition when $\frac{1}{2} < \rho < 1$ for the precommitment equilibrium to be a reputational equilibrium. Finally, $\overline{\widehat{S}} \leq 0$ is easily shown to be equivalent to inequality (5).

I Determination of function F's variations and limits when $\rho = 0$

Equation $\beta\lambda\omega^2 - \left(\beta\lambda + \kappa^2 + \lambda\right)\omega + \lambda = 0$ implies

$$\frac{\partial\omega}{\partial\beta} = \frac{-\omega^2(1-\omega)}{1-\beta\omega^2} < 0,\ \frac{\partial\omega}{\partial\kappa} = \frac{-2\kappa\omega^2}{\lambda(1-\beta\omega^2)} < 0 \text{ and } \frac{\partial\omega}{\partial\lambda} = \frac{(1-\omega)(1-\beta\omega)}{\lambda(1-\beta\omega^2)} > 0,$$

from which we easily get

$$\frac{\partial F}{\partial\beta} > 0,\ \frac{\partial F}{\partial\kappa} = \frac{\beta^2\left(1-\beta^{D-1}\right)\lambda(1-\omega)\left[(1-\beta)+1+\beta\omega\right]}{(1-\beta)\left(\kappa^2+\lambda\right)^2\left(1-\beta\omega^2\right)} > 0$$

$$\text{and } \frac{\partial F}{\partial\lambda} = -\frac{\beta^2\left(1-\beta^{D-1}\right)\kappa^2(1-\omega)\left[\beta\lambda\omega^2(1-\omega)+2\lambda\omega+\kappa^2\omega+\kappa^2\right]}{(1-\beta)\left(\kappa^2+\lambda\right)^2\lambda\omega\left(1-\beta\omega^2\right)} < 0.$$

Moreover, the limits

$$\lim_{\beta\longrightarrow 0^+}\omega = \frac{\lambda}{\kappa^2+\lambda},\ \lim_{\beta\longrightarrow 1^-}\omega = \frac{\left(\kappa^2+2\lambda\right)-\sqrt{\left(\kappa^2+2\lambda\right)^2-4\lambda^2}}{2\lambda},\ \lim_{\kappa\longrightarrow 0^+}\omega = 1 \text{ and } \lim_{\lambda\longrightarrow +\infty}\omega = 1$$

imply respectively

$$\lim_{\beta\longrightarrow 0^+}F(\beta,\kappa,\lambda,D) = 0,\ \lim_{\beta\longrightarrow 1^-}F(\beta,\kappa,\lambda,D) = \frac{\kappa^2(D-1)}{\kappa^2+\lambda},$$
$$\lim_{\kappa\longrightarrow 0^+}F(\beta,\kappa,\lambda,D) = 0 \text{ and } \lim_{\lambda\longrightarrow +\infty}F(\beta,\kappa,\lambda,D) = 0.$$

Finally, the equivalences

$$\omega \underset{\kappa\longrightarrow +\infty}{\sim} \frac{\lambda}{\kappa^2} \text{ and } \omega \underset{\lambda\longrightarrow 0^+}{\sim} \frac{\lambda}{\kappa^2}$$

imply respectively

$$\lim_{\kappa\longrightarrow +\infty}F(\beta,\kappa,\lambda,D) = \frac{\beta^2\left(1-\beta^{D-1}\right)}{1-\beta} \text{ and } \lim_{\lambda\longrightarrow 0^+}F(\beta,\kappa,\lambda,D) = \frac{\beta^2\left(1-\beta^{D-1}\right)}{1-\beta}$$

with l'Hôpital's rule.

J Calibrations used in the literature

Table 4 presents a few calibrations of our standard New Keynesian model used in the literature, for quarterly data with the inflation rate measured as an annualized percentage. Most of them are calibrations of the canonical New Keynesian model as they set γ to zero. We retain only the calibrations with a model-consistent value of λ, *i.e.* such that $\lambda = \frac{4\kappa}{\theta}$ where θ is the elasticity of substitution between differentiated goods[26]. Most studies choose for κ Rotemberg and Woodford's (1997) estimated value, roughly equal to $0,10$, and all studies use the value $\theta = 8$ taken from Rotemberg and Woodford (1997) to derive λ from κ, except Aoki and Nikolov (2004) who implicitly use the value $\theta = 6$.

Table 4: calibrations used in the literature.

No.	Study	β	κ	λ	ρ	γ	$\sqrt{V_\varepsilon}$	x^*
1a	Woodford (2003a, chapter 7)	$0,99$	$0,10$	$0,05$	NE	$0,0$	NE	$0,2$
1b	Woodford (2003a, chapter 7)	$0,99$	$0,10$	$0,05$	NE	$0,5$	NE	$0,2$
2a	{ Woodford (2003a, chapter 7) Schaumburg and Tambalotti (2003)	$0,99$	$0,10$	$0,05$	$0,00$	$*$	NS	$0,0$
2b	Adam and Billi (2004a, 2004b)	$0,99$	$0,10$	$0,05$	$0,00$	$0,0$	$0,006$	$0,0$
2c	Woodford (1999)	$0,99$	$0,10$	$0,05$	$0,00$	$0,0$	$0,010$	$0,0$
3	Aoki and Nikolov (2004)	$0,99$	$0,12$	$0,08$	$0,35$	$0,0$	$0,015$	$0,0$
4	Adam and Billi (2004a, 2004b)	$0,99$	$0,23$	$0,11$	$0,36$	$0,0$	$0,007$	$0,0$
5	Schaumburg and Tambalotti (2003)	$0,99$	$0,10$	$0,05$	$0,70$	$0,0$	NS	$0,0$
6	Woodford (2003a, chapter 7)	$0,99$	$0,10$	$0,05$	$0,80$	$0,0$	NS	$0,0$

The notations NE, NS and $*$ stand respectively for "non-existent, "non-specified" and "$\gamma \in \{0,0; 0,5; 0,8; 1,0\}$".

[26] We therefore do not consider the calibrations used by Evans and Honkapohja (2002), McCallum and Nelson (2000), Vestin (2000) and Walsh (2003a, chapter 11; 2003b) in particular.

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