MACRO STRESS TESTING WITH A
MACROECONOMIC CREDIT RISK MODEL:
APPLICATION TO
THE FRENCH MANUFACTURING SECTOR

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Application to the French manufacturing sector

AVOUYI-DOVI Sanvi†   JARDET Caroline‡   KENDAOUI Ludovic§
MOQUET Jeremy¶   BARDOS Mireille∥

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†Banque de France, DGEI-DIR-RECFIN, 31 rue croix des petits champs, 75049 Paris cedex 01, sanvi.avouyi-dovi@banque-france.fr

‡Banque de France, DGEI-DIR-RECFIN, 31 rue croix des petits champs, 75049 Paris cedex 01, caroline.jardet@banque-france.fr [corresponding author]

§Banque de France, DE-Observatoire des Entreprises, 31 rue croix des petits champs, 75049 Paris cedex 01, ludovic.kendaoui@banque-france.fr

¶Banque de France, DE-Observatoire des Entreprises, when the paper was drafted

∥Banque de France, DE-Observatoire des Entreprises, when the paper was drafted.
Abstract

The aim of this paper is to build and estimate a macroeconomic model of credit risk for the French manufacturing sector. This model is based on Wilson’s CreditPortfolioView model (1997a, 1997b); it enables us to simulate loss distributions for a credit portfolio for several macroeconomic scenarios. We implement two simulation procedures based on two assumptions relative to probabilities of default (PDs): in the first procedure, firms are assumed to have identical default probabilities; in the second, individual risk is taken into account. The empirical results indicate that these simulation procedures lead to quite different loss distributions. For instance, a negative one standard deviation shock on output leads to a maximum loss of 3.07% of the financial debt of the French manufacturing sector, with a probability of 99%, under the identical default probability hypothesis versus 2.61% with individual default probabilities.

Keywords: macro stress test, credit risk model, loss distribution.

JEL codes: G32, C22, C53

Résumé

Cet article présente un modèle macroéconomique de risque de crédit pour le secteur manufacturier français, fondé sur le modèle "CreditPortfolioView" de Wilson (1997a, 1997b). A partir du modèle, des distributions de perte d’un portefeuille de crédit sont simulées pour différents scénarios macroéconomiques. Deux procédures de simulation sont mises en oeuvre. Pour la première, toutes les firmes sont supposées avoir la même PD alors que la seconde tient compte du risque individuel de défaut. Les résultats empiriques montrent que ces deux procédures conduisent à des distributions de perte assez différentes. Par exemple, une baisse d’un écart type du taux de croissance du produit conduit, avec une probabilité de 99%, à une perte de 3.07% de la dette financière du portefeuille lorsque la première procedure de simulation est mise en oeuvre, alors que cette perte s’élève à 2.61% avec la seconde.

Mots-clés : macro stress tests, modèle de risque de crédit, distribution de perte.

Codes JEL : G32, C22, C53.
Non-technical summary

The aim of this paper is to build and estimate a macroeconomic model of credit risk for the French manufacturing sector. We investigate the model in the following way:

a) First, we set up an extended version of Wilson’s model (a so-called augmented version) by imposing feedback effects between default rates and macroeconomic variables. In this manner, default rates are affected by macroeconomic factors, and, in their turn, these factors are themselves impacted by default rates. More precisely, the model contains four major variables: real GDP, corporate spreads, the short-term interest rate and default rates. Real GDP allows us to investigate the effect of business cycle on default rates; corporate spreads represent financial risks and the short-term interest rate can be interpreted as the monetary policy indicator. Each variable may depend on each of the others. Of course, only relevant relationships will be collated in order to build the model. The model proposed here seems more realistic than the previous one (Boss (2002), Virolainen (2004)). The consequence of the interdependence hypothesis is that the relationships between default rates and macroeconomic factors have to be estimated jointly in a multivariate framework. Empirical results support the choice of a multivariate framework: for instance, the short-term interest rate is significantly affected by the lagged values of default rates.

b) Second, we improve the simulation procedures for loss distributions (i.e. the methods for their calculation) introducing the hypothesis of assigning probabilities of default according to company rating grades. The simulation procedures available in the literature (Boss (2002), Virolainen (2004)) are based on the assumption that companies have identical probabilities of default. It is clearly unacceptable that default probabilities are not firm-specific; this hypothesis is too strong and somewhat unrealistic. Thus, in this paper, we assume that the loss distribution is generated using the hypothesis that default probability is dependent on the rating grades of companies. In this way, we adopt two alternative procedures in order to simulate the loss distribution for a given macroeconomic scenario.

We show that the two simulation procedures adopted in this paper lead to quite different results. For example, in the benchmark, or basic, scenario (where there is no shock, Tables 3 and 4), the expected loss (the mean of the total loss) obtained with the traditional approach (identical default probabilities irrespective of rating grades) is equal to 1.05% of total financial debt whereas it equals 1.30% in the alternative simulation procedure (default probability related to the rating grade). In contrast, extreme values of the loss distribution, especially unexpected loss (maximum loss to occur with a probability
99%), are higher in the case of the traditional approach: it reaches 2.91%, while it stands at 2.38% in the alternative approach.
1 Introduction

Research on credit risk, i.e., the risk that borrowers fail to meet their obligations linked to credit extended to them is now abundantly documented. The main findings of this body of literature are the results concerning the dynamic behaviour of credit risk, especially its relationships with the macroeconomic environment of firms. Among others, we can mention the work of Amato and Furfine (2004), Allen and Saunders (2004), Wilson (1997a and b), Nickell et al. (2000), Bangia et al. (2002), and Pesaran et al. (2005).

The development of models of credit risk has been accompanied by their increasingly frequent use in order to perform macroeconomic stress-testing. These tests, which are highly useful for risk managers, allow the latter to examine how the system reacts under extreme conditions. At the firm level, stress tests encompass various techniques used by financial firms in order to gauge their potential vulnerability to rare events. These approaches can be extended to macro-prudential analysis. In this context, by convention, we call them "aggregate" or "macro" stress tests. They can be viewed and interpreted as a measure of the risk of a group of firms subject to an exceptional but plausible stress scenario.

Current credit risk models, especially those implemented in financial institutions, incorporate few macroeconomic factors. For example, these models do not necessarily take into account the impact of the business cycle on credit risk while decision-makers need to examine the impact of economic activity on financial institutions’ credit risk as a part of financial stability analysis. As a consequence, models relating credit risk to macroeconomic factors seem a better and more useful framework to assess how severe macroeconomic shocks impact on credit risk in a given economic sector.

Following the findings of Sorge and Virolainen (2006), two approaches may be derived:

1) estimating the sensitivities of banking sector balance sheets to dramatic changes in the main macroeconomic variables. The estimated model is used to assess the impact of forward-looking stress scenarios on the financial system (Kalirai and Scheicher (2002), Hoggarth and Zicchino (2004), Delgado and Saurina (2004), de Bandt and Oung (2004));

2) implementing the Value-at-Risk (VaR) concept (already used at a micro-level) in order to generate a conditional probability distribution of loss. Diverse distributions are expected to be associated with different simulated economic environments. Here the objective consists in assessing the sensitivities of portfolios to different sources of risk. In this area, two alternative approaches may be adopted: (i) the
The first one is based on the work of Merton (1974), which consists in modelling the response of equity prices to macroeconomic factors. Asset price movements are then mapped into default probabilities (Drehmann and Manning (2004), Pesaran et al. (2004)); (ii) the second approach is linked to the work of Wilson (1997a and 1997b) in which the default rate of an economic sector is directly related to macroeconomic factors (Boss (2002), Virolainen (2004), Choi, Fong and Wong (2006)). The second approach is adopted in this paper. Indeed, we relate some macroeconomic factors directly to default rates of companies. In this way, taking advantage of the availability of an abundant and well documented database at the Banque de France (the FIBEN Database\footnote{FIBEN: Fichier Bancaire des Entreprises}), a macroeconomic credit risk model is proposed and estimated for the French manufacturing sector.

We investigate the model in the following way:

a) First, we set up an extended version of Wilson’s model (a so-called augmented version) by imposing feedback effects between default rates and macroeconomic variables. In this manner, default rates are affected by macroeconomic factors, and, in their turn, these factors are themselves impacted by default rates. More precisely, the model contains four major variables: real GDP, corporate spreads, the short-term interest rate and default rates. Real GDP allows us to investigate the effect of business cycle on default rates; corporate spreads represent financial risks and the short-term interest rate can be interpreted as the monetary policy indicator. Each variable may depend on each of the others. Of course, only relevant relationships will be collated in order to build the model. The model proposed here seems more realistic than the previous one (Boss (2002), Virolainen (2004)). The consequence of the interdependence hypothesis is that the relationships between default rates and macroeconomic factors have to be estimated jointly in a multivariate framework. Empirical results support the choice of a multivariate framework: for instance, the short-term interest rate is significantly affected by the lagged values of default rates.

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The paper is organized as follows. Section 2 provides a brief description of credit risk in the French manufacturing sector. Section 3 describes the macroeconomic credit risk model. Section 4 focuses on macroeconomic stress testing. Section 5 sets out the conclusions.

2 Credit risk in the French manufacturing sector

In the banking sector, credit risk remains the most important source of risk: more than 80% of French banks’ overall capital requirements are traceable to credit operations. Before investigating a model designed to perform stress tests on credit risk in the French manufacturing sector (i.e. the food, consumer goods, car, capital goods and intermediate goods industries), it seems pertinent to shed some light on these kinds of risks and how they can be assessed, especially using the Banque de France’s tools.

One of the main advantages of our analysis stems from the availability and the ability to use a specific Banque de France banking database (the FIBEN database). In this database, information is collected on firms subject to the BIC/BRN\(^2\) (tax regime under which firms file a complete balance sheet) and report turnover of more than $750,000 or carry debt in excess of $380,000. Around 40,000 balance sheets (covering 84% of bank loans among BIC/BRN firms of this sector) are available across the manufacturing sector per year. Now, let us show how these data are used to gauge credit risk in the French manufacturing sector by describing: the relevant measure of companies’ exposure and the measure of the individual probability of default.

\(^2\)The total number of firms in the French manufacturing sector subject to the BIC/BRN is around 80,000.
2.1 Financial debt in the French manufacturing sector

Assessing credit risk in this sector requires us to define the measures used to gauge companies’ exposure. The bank debt of a firm is a traditional measure of firms’ exposure. However, a recent noteworthy stylised fact of the French economy is the growth of groups with, in particular, the expansion of holding companies responsible for their own financing. This increase in the number of holding companies requires special attention regarding the analysis of the financial statements of companies, due to the existence of intragroup flows, although the role of holding companies is to manage most of the financing flows at group level. For instance, in 2005, the financing by the group and its partners, which is a major feature of large firms, accounted for half of total financial debt. More precisely, in 2005, 37,392 firms were selected for this study; their financial debt (FD) amounted to €112 billion (end-December 2005); bank debt accounted for 40% of financial debt whereas financing by the group and its partners reached 50% of FD; it should be noted that 10% of financial debt is essentially made up of bond debt (see appendix). Therefore, the analysis of the credit risk borne by the French manufacturing sector cannot only rely on individual bank debt data. That is why we will focus on financial debt rather than bank debt, which includes bank loans, bond debt, leasing commitments, discounted trade bills and financing by the group and its partners. In other words, our estimation of credit risk will be based on individual financial debt data.

Fig. 1. Bank debt of the French manufacturing sector in 2005
Figure 1 shows the distribution of the bank credit portfolio for the volume recorded at December 2005. The median bank credit volume is € 0.33 million. The highest credit exposure exceeds € 4.33 billion. This skewed distribution plays a crucial role in portfolio risk: if borrowers fail, the fact that their exposure is average or very large makes a significant difference in terms of loss. This evidence leads to a recommendation: to be able to capture portfolio risk, the volumes of individual exposures need to be taken into account.

2.2 Assessing individual default risk

The Banque de France scores\(^3\) are designed for the early detection of default risk at the individual company level. The concept of default used in the assessment of the Banque de France scores differs from the Basel Committee definition. For the Banque de France, a firm is defaulting if it is subject to legal proceedings (an exhaustive database of legal incidents is available at Banque de France). As a consequence, the Banque de France’s definition of default is more restrictive than that of the Basel Committee. Note that the concept of company default is associated with a complex phenomenon; several events of default can be considered (see for example Basel II concepts) and they evolve at different rates. The Banque de France’s scores are obtained via a multi criteria statistical approach and offer an overall picture of a firm’s position regarding a number of areas of analysis including productive structure, financial debt (level, structure and cost), balance sheet structure, inter-company loans, other debt, profitability, solvency and growth (Bardos et al., 2004).

One of the key stages in risk analysis is to match each score value to a probability of default which is used to refine the analysis by providing a measure of risk. In practice, default probabilities are not supplied for each score value but for score intervals, denoted by \( r \), that group together companies with similar risk profiles.

We distinguish two types probability of default: posterior and prior probabilities of default. The posterior probability is determined with reference to a known score value. In this sense, it differs from the prior

\(^3\)The score is a statistical tool for the detection of company failure. For example, the BDFI2 score is a linear combination of eight ratios: profit margin; size of financial charges relative to overall surplus; proportion of tax and social security liabilities; size of trade payables; proportion of financial debt; size of the net cash position; the size of net overall working capital; proportion of bad or contested debts.

It is distinct from the Banque de France rating, which corresponds to an assessment by a financial expert. However, the score acts as an analytical tool for the financial expert responsible for assigning a rating to a company.
probability of default (also called the default rate) for which there is no information other than that about the company’s sector; it is estimated using the default rate across the entire population. In contrast, estimates of posterior probabilities of default are based on the empirical score distributions for each group (i.e. defaulting and non-defaulting firms). Here, our calculations of the posterior probability is linked to empirical estimates given by Bayes’ theorem\(^4\). This empirical approach is made possible by the abundance and the quality of the observations available at the Banque de France.

More precisely, let us denote \( P_r \) the posterior probability of default of a firm which is in score interval \( r \). \( P_{dr} \) is the probability of the score being in interval \( r \) given that the company is defaulting. \( P_{nr} \) is the probability of the score being in interval \( r \) given that the company is not defaulting. Both probabilities are obtained from the empirical score distribution for each group. We denote \( \pi_d \) and \( \pi_n \) the prior probabilities to default and not default respectively (\( \pi_n = 1 - \pi_d \)). Then, using Bayes’ theorem, we have:

\[
P_r = \frac{\pi_d P_{dr}}{\pi_d P_{dr} + \pi_n P_{nr}}
\]

Risk classes are defined according to these intervals and their associated default probabilities insuring homogeneity and temporal stability of risk in each class. Thus, the value of default probability in each risk classes is directly linked to the default rate.

The BDFI2 score covers the French manufacturing sector with six risk classes or rating grades at a one-year horizon. The ratings distribution (Fig. 2) highlights the fact that there are few company defaults and that only 20% companies are in the three highest-risk rating grades.

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\(^4\) Alternatively, we could calculate posterior probability by means of a theoretical formula corresponding to the linear classification analysis model. However, our methodology, based on Bayes’ theorem, is better at capturing the actual economic situation (Bardos, 2007).
3 A macroeconomic credit risk model

3.1 The model

The macroeconomic credit risk model is based on the CreditPortfolioView model proposed by Wilson (1997a and 1997b) and developed by McKinsey. This approach is well suited to macro-stress tests because it relates the default rate in a given economic sector to macroeconomic factors. Hence, when the model is estimated, the default rate can be simulated through the effects of macroeconomic shocks applied to the system. In turn, these default rates can be used to simulate the loss distribution for a given credit portfolio.

We shall briefly describe the main ingredients of the Wilson’s approach.

(i) First, for a given sector, the average default rate is modelled by a logistic function:

$$p_t = \frac{1}{1 + \exp(-y_t)}$$  \hspace{1cm} (2)

Note that in Wilson’s original specification, the average default rate is given by $p_t = \frac{1}{1 + \exp(y_t)}$. Our specification slightly departs from this by taking into account the variable $-y_t$ instead of $y_t$. Using this transformation, the macroeconomic index $y_t$ and the default rate $p_t$ are positively correlated.
where $p_t$ is the default rate and $y_t$ is a macroeconomic index which is assumed to be related to a set of macroeconomic factors, according to the following linear specification:

\[ y_t = \beta_0 + \beta_1 x_{1,t} + \beta_2 x_{2,t} + \ldots + \beta_n x_{n,t} + \varepsilon_t \]  

(3)

where $x_{i,t}$ ($i = 1, \ldots, n$) is a set of macroeconomic variables, and $\beta_i$ ($i = 1, \ldots, n$) a set of unknown coefficients which need to be estimated. Error terms $\varepsilon_t$ are assumed to be normally distributed and independent and identically distributed (i.i.d.).

(ii) Second, each macroeconomic variable is assumed to follow an autoregressive process of order $q$:

\[ x_{i,t} = \rho_{i,0} + \sum_{j=1}^{q} \rho_{i,j} x_{i,t-j} + \nu_{i,t} \]  

(4)

for $i = 1, \ldots, n$.

where $\rho_{i,j}$, for $j=1,\ldots, q$ and $i=1,\ldots,n$, are the unknown coefficients that have to be estimated; for each $i$, $\nu_{i,t}$ is an i.i.d error term.

The system of equations ((2) to (4)) describes the joint evolution of the default rate and macroeconomic factors. The estimated model can be used to simulate the future path of the default rate for given values of the macroeconomic factors. Using Monte Carlo methods, it is then possible to estimate the credit loss portfolio for the underlying macroeconomic environment.

Our macroeconomic credit risk model is very close to the Wilson’s one. However, two differences should be mentioned:

– first, unit root tests reveal that the macroeconomic index is not stationary. Therefore a consistent estimation of parameters $\beta_i$ ($i = 1, \ldots, n$) would not be possible. To address this issue, we deal with the first difference of the macroeconomic index ($\Delta y_t = y_t - y_{t-1}$ instead of $y_t$). This transformation has also been performed in the paper of Boss (2002) for aggregate corporate default rates in Austria;

– second, in the original specification, current and past values of $y_t$ are not included in (4). Therefore, default rates are assumed to be affected by macroeconomic factors, whereas the inverse interaction is not allowed. This restriction is slightly unrealistic and could bias the results of macro stress tests. Indeed, we might expect that large deviations in the default rate would affect variables such as GDP, the unemployment rate or interest rates. For this, the relationship between default rates and macroeconomic factors should interact and be estimated jointly in a multivariate framework that allows interdependence
between macroeconomic factors and the default rate (spillover effects). Besides, as $y_t$ is not stationary, cointegrating relations could exist between macroeconomic variables and the index. This argument reinforces the use of a multivariate model (i.e. a VECM model) which is more consistent in detecting and estimating long-run relationships between variables (Granger representation theorem). Note that if there is no cointegration relation, a more parsimonious model, the first difference VAR model, may be used.

To sum up, the macroeconomic credit risk model investigated here is:

$$p_t = 1 + \exp\left(-\left(y_{t-1} + \Delta y_t\right)\right)$$

(5)

$$\left( \begin{array}{c} \Delta X_t \\ \Delta y_t \end{array} \right) = \mu + \alpha'\lambda \left( \begin{array}{c} X_{t-1} \\ y_{t-1} \end{array} \right) + \sum_{i=1}^{p} \varphi_i \left( \begin{array}{c} \Delta X_{t-i} \\ \Delta y_{t-i} \end{array} \right) + \omega_t$$

(6)

where $X_t$ is the $(n \times 1)$ vector of macroeconomic variables at time $t$ ($X_t = (x_{1,t}, x_{2,t}, ..., x_{n,t})'$), $\mu$ is the $(n + 1 \times 1)$ vector of constant, $\lambda$ is the $(k \times n + 1)$ cointegrating matrix, $k$ is the number of cointegrating relations, $\alpha$ is a $(k \times n + 1)$ matrix and $\varphi_i$ are $(n+1 \times n+1)$ matrices, for $i = 1, ..., p$. Finally, $\omega_t$ is a $(n+1 \times 1)$ vector of error terms. We assume that $E(\omega_t) = 0_{(n+1,1)}$, $E(\omega_t\omega_t') = \Omega$ and $E(\omega_t\omega_{t+i}') = 0_{(n+1,n+1)}$ for $i \neq 0$.

### 3.2 Estimation procedure

The database contains default rates for the French manufacturing sector (see figure 3) and some macroeconomic variables, at quarterly frequency over the period 1995q1 to 2006q4. Given the observed values of default rates, we construct the corresponding macroeconomic index by inverting equation (5):

$$y_t = \log\left( \frac{p_t}{1 - p_t} \right)$$

(7)

We then select a set of relevant macroeconomic factors in order to explain the default rates. We perform traditional cointegration tests (Johansen (1991)) in order to identify long-run relations between the macroeconomic variables and $y_t$. If one or more cointegrating relations are detected, the VECM model (6) is estimated. Otherwise, we estimate the first difference VAR. Parameters are obtained by maximum likelihood estimators. The number of lags in this multivariate model, $p$, is based on conventional criteria but, due to the size of our dataset, we impose some constraints on the system: for $p$, we do not go further
than 2. We check, of course, whether the residuals of our equations corroborate the required properties by applying the Ljung and Box white noise test.

Fig. 3. Default rate of the manufacturing industry sector

3.3 Results of the estimation

The choice of macroeconomic factors is crucial for macroeconomic stress testing. In previous studies on the determinants of corporate default rates, the set of macroeconomic variables has typically included measures of profitability, indebtedness and interest rates. Boss (2002) examines a large set of variables falling into the following categories: business cycle determinant indicators, price stability indicators, some specific household indicators, corporate indicators, financial market indicators and external variables. However, his model includes: manufacturing production, the inflation rate, Austrian traded index, the nominal short-term interest rate and oil prices. In contrast, the model for the Finnish corporate default rate (Virolainen (2004)) contains only three macroeconomic factors: seasonally adjusted real GDP, the short-term interest rate and corporate indebtedness.

Our goal mainly consists in estimating a parsimonious model, with a limited number of factors. In
addition, many combinations of macroeconomic factors have been tested. Two criteria are applied for selecting the model used for the analysis: (i) the signs of parameters estimates have to be, of course, in accordance with economic intuition and interpretation; (ii) we keep the model that outperforms the others regarding out-of-sample simulations of default rates.

To be more precise, our macroeconomic credit risk model involves three variables: seasonally adjusted real GDP (or, rather, in logarithmic terms, $\log GDP_t$), the 3-month nominal interest rate ($R3m_t$), and the difference between the corporate bond interest rate and the 10-year government bond interest rate the so-called credit corporate spread ($spread_t$). Real GDP describes the effect of economic activity. The short-term interest rate provides a view of the stance of the monetary policy, while the corporate spread may be interpreted as a market measure of risk; for example, the spread increases if corporate bonds are expected to be more risky.

Among the three macroeconomic variables, only real GDP is generated by a non-stationary process (the presence of a unit root is detected). In addition, cointegration tests indicate that there is no cointegrating relation between real GDP and the macroeconomic index $y_t$. As a consequence, a VAR model, with $y_t$, $spread_t$, $R3m_t$, and $\log GDP_t$ (included in first difference, $\Delta \log GDP_t$), is estimated. Furthermore, residuals tests show that one lag is sufficient to produce white noise residuals.

Therefore, the empirical macroeconomic credit risk model is as follows:

$$
\begin{pmatrix}
\Delta \log GDP_t \\
R3m_t \\
spread_t \\
\Delta y_t
\end{pmatrix}
= \mu + \varphi_1
\begin{pmatrix}
\Delta \log GDP_{t-1} \\
R3m_{t-1} \\
spread_{t-1} \\
\Delta y_{t-1}
\end{pmatrix}
+ \omega_t \quad (8)
$$

Equations (9) to (12) report the results of the estimation.

$$
\Delta y_t = -0.010^{(0.016)} + 0.529^{***}(0.113) \Delta y_{t-1} - 6.964^{***}(2.874) \Delta \log GDP_{t-1} \\
+ 0.020^{**}(0.0104) R3m_{t-1} + 0.006^{**}(0.003) spread_{t-1} \quad (9)
$$

\( ^6 \) Average rate of French bonds issued by the private sector (Source: INSEE).
\[
\Delta \log GDP_t = 0.002^{***} - 0.0005 \Delta y_{t-1} + 0.171^{**} \Delta \log GDP_{t-1}
\]
\[
-0.0002 R3m_{t-1} + 0.0008 spread_{t-1}
\]

\[
R3m_t = 0.315^{**} - 2.610^{**} \Delta y_{t-1} + 40.13 \Delta \log GDP_{t-1}
\]
\[
-0.851^{***} R3m_{t-1} + 0.331 spread_{t-1}
\]

\[
spread_t = 0.019 - 1.057 \Delta y_{t-1} + 3.19 \Delta \log GDP_{t-1}
\]
\[
-0.007 R3m_{t-1} + 0.673^{***} spread_{t-1}
\]

Standard errors of parameters are given in brackets; ** and *** indicate significant at the 5% and 1% levels respectively.

Let us begin the analysis of the estimation results by focusing on the interpretation of equation 11 which describes the evolution of \( R3m_t \). Note that the coefficient of \( \Delta y_{t-1} \) in equation 11 is negative and significantly different from 0. This result shows that an increase in the default rate is followed by a sharp cut in the short-term interest rate. Consequently, the lagged values of the default rate affect the future path of some macroeconomic factors. This is an ex-post corroboration of the use of a multivariate framework to model macroeconomic factors and the default rate. In addition, the negative correlation between the short-term interest rate and the lagged default rate could stem from the assumption that a higher default rate reflects a bad economic environment. So, even though output is not formally one of the target variables of the European Central Bank, the position of activity in the business cycle seems to influence the conduct of the monetary policy; for instance, a cut in the short-term interest rate could be viewed as the response of the monetary authorities to the deterioration of the macroeconomic environment.

Now, regarding the evolution of \( \Delta y_t \) (see equation 9), we observe a marked impact of the quarterly growth rate of GDP, \( \Delta \log GDP_{t-1} \). Indeed, the coefficient associated with real GDP is negative and strongly significant. Hence, a decrease in the growth rate of GDP tends to be followed by an increase in
the default rate in the next quarter ($\Delta y_t$). For instance, a decrease of 1% in GDP corresponds to an increase of around 7% in the default rate. This result confirms the widely documented and highly intuitive observation (Amato et al. (2004)) that in a period of recession (expansion), the default rate tends to increase (decrease). In addition, the coefficient of $R3m_{t-1}$ in the equation describing the evolution of $\Delta y_t$ (see equation 9) is positive and significant, so that it confirms that an increase in the short-term interest rate leads to an increase in the default rate. This result is also in line with expectations that the default rate increases along with interest rates owing to higher borrowing costs. In this way, there is a feedback effect between the default rate and the nominal interest rate. This is a second argument in favour of a multivariate framework. Finally, the coefficient of $spread_{t-1}$ is positive and significant. Once again, the estimated parameter has the expected sign: an increase in the corporate spread indicates that corporate debt instruments are viewed as more risky by market participants. This could be a sign of a weakening of the situation of the corporate sector, leading to an increase in the default rate.

The dynamics of output growth, $\Delta \log GDP_t$, (equation 10) and the corporate spread (equation 12) are less varied than those of the previous variables. Indeed, output growth is mainly generated by its own lagged values, with a coefficient smaller than one in absolute value indicating that output growth is mean reverting. The corporate spread is also driven by its own past values.

The empirical results bear out to a large extent the use of a multivariate framework even though GDP growth and the corporate spread are essentially driven by autoregressive processes. The results confirm the hypothesis concerning the interaction between some macroeconomic factors and the default rate.

### 3.4 Forecasting performances of the macroeconomic credit risk model

Figure 4 shows the observed default rate on a quarterly sample from 1995 to 2007 and the corresponding forecast of the default rate obtained from our macroeconomic credit risk model. On average, the in-sample simulation of the default rate fits the observed series quite well.
One of the main features of a model is its ability to provide good out-of-sample simulations. Actually, over-parameterized models usually perform very well in in-sample tests, but their out-of-sample performances are often rather weak.

Given past values of macroeconomics factors and default rates, and the parameters of our macroeconomic credit risk model, we can conduct forecasts of all the variables of the model iterating their equations forward. In addition, parameter estimates can be revised if new data for the variables become available. In this way, we assume that the model can be revised at a quarterly or annual frequency.

Using the assumption that the model is estimated on a quarterly sample from 1995q1 to 2001q4, we perform the following out-of-sample forecasting exercise:

- for the quarterly revision, the model is re-estimated for each additional quarter and the forecasted value of default rate for the next quarter is computed;
• for the annual revision, the model is re-estimated at the end of each additional year and forecasted values of default rates over a four-quarter horizon are computed.

Figures 5 and 6 show the observed and predicted values of default rates with quarterly and annual revisions of the model respectively. The quarterly revision leads to better forecasts of the default rate. However, the model tends to slightly underestimate the default rate during periods of rises in the default rate whereas it slightly overestimates the default rate in phases when this rate is falling. In contrast, in the case of the annual revision, the differences between the forecasted and observed default rates are generally smaller.

In the case of our macroeconomic credit risk model, a key question is whether including macroeconomic factors improves the predictions of default rates. To address this issue, we compare predictions from our model with ones derived from a simple AR(1) specification for the default rate (equation 13 below). The AR(1) specification is a reasonable benchmark, also used for instance by Stock and Watson (2001) in the context of forecasting macro series. The benchmark AR(1) model is:

$$\Delta y_t = \rho_0 + \rho_1 \Delta y_{t-1} + \eta_t$$

This equation is also estimated in a quarterly sample from 1995q1 to 2001q4, and it is revised quarterly and annually. In both cases, quarterly forecasts of the default rate are computed. The forecasting performances of the two models are assessed by comparing their Root Mean Square Errors (RMSE\(^7\)). We note that the two RMSEs corresponding to the macroeconomic credit risk model are smaller than the ones obtained with our benchmark model: for the quarterly revision, 0.035 for the macroeconomic model versus 0.037 in the case of the benchmark model; for the annual revision, 0.041 for the macroeconomic model against 0.044 for the benchmark model. These results mean that macroeconomic variables yield information about the evolution of default rates and slightly improve default rate predictions.

\(^7\) RMSE = \(\sqrt{\frac{1}{n} \sum_{j=1}^{n} (p_j - \hat{p}_j)^2}\), where \(p_j\) is the observed default rate, and \(\hat{p}_j\) the forecasted default rate.
Fig. 5 - Quarterly out-of-sample forecasts of the default rate with a quarterly revision of the model (as a %)
Fig. 6 - Quarterly out-of-sample forecasts of the default rate
with an annual revision of the model (as a %)
4 Macroeconomic stress testing

The goal of this section is to examine the response of the default rate to a macroeconomic shock. Here, we focus on the effects of an output shock. We consider negative shocks on output of one and three standard deviations respectively. We assume that these shocks occur only in 2005q4 and we analyze their consequences on default rates and the loss distribution for the four quarters of 2006. In particular, we compare the default rates and loss distributions derived from the simulations carried out under the assumption of the presence of shocks with the ones obtained with the basic scenario (in which a shock does not occur in 2005q4).

For each macro crisis scenario (presence of shocks), we proceed in two steps: (i) using the macroeconomic credit risk model, we compute the responses of the default rate over 2006; (ii) the corresponding simulated default rates are used to assess loss distributions. In what follows we detail this two-step procedure.

4.1 Simulation methodology

4.1.1 Responses of default rates to an output shock

Let us consider our macroeconomic credit risk model (8):

\[ X_t = \mu + \varphi_1 X_{t-1} + \omega_t \]

where \( X_t = (\Delta \log GDP_t, R3m_t, spread_t, \Delta y_t)' \), and \( E(\omega_t \omega_t') = \Omega \).

Iterating the model forward, we have:

\[ E_t(X_{t+k}) = (I_4 + \varphi_1 + ... + \varphi_k)\mu + \varphi_k^T X_{t-1} + \varphi_k^T \omega_t \]

where \( I_4 \) is the \( 4 \times 4 \) identity matrix.

Let us assume that the economy is subjected to a shock of size \( \omega_t = \omega = (\omega_1, ..., \omega_4) \) at period \( t \). Therefore the (expected) response of \( X_t \) at date \( t + k \) (\( X_{t+k} \)) is given by:

\[ \frac{\partial E_t(X_{t+k})}{\partial \omega_t} \bigg|_{\omega_t=\omega} = \varphi_k^T \omega \]
As \( E(\omega_t\omega'_t) = \Omega \) where \( \Omega \) is generally non diagonal, the choice of \( \omega \) cannot be made arbitrarily. The approach suggested by Sims (1980) is to solve the problem surrounding the choice of \( \omega \) by using the Cholesky decomposition of \( \Omega \):

\[
\Omega = PP'
\]

where \( P \) is an \( 4 \times 4 \) lower triangular matrix. Then, (14) can be rewritten as:

\[
X_t = \mu + \varphi_1 X_{t-1} + P \xi_t
\]

such that \( \xi_t = P^{-1} \omega_t \) are orthogonalized, that is \( E(\xi_t\xi'_t) = I_4 \).

Hence, the expected response of \( X_{t+k} \) at time \( t + k \) to a unit shock to the \( j \)th equation is given by:

\[
\left. \frac{\partial E_t(X_{t+k})}{\partial \omega_t} \right|_{\omega_t = \omega} = \varphi_1^k \xi
\]

where \( \xi \) is the \( 4 \times 1 \) vector with unity as its \( j \)th element and zeros elsewhere.

It should be recalled that the responses are not invariant to the ordering of the variables in \( X_t \) in the case of the Cholesky decomposition. When ordering \( \Delta \log GDP_t \) first, we implicitly assume that shocks to other variables have no instantaneous effect on output growth. In doing so, we are in line with the literature, in which it is usually assumed that output only reacts to changes in nominal variables after at least one quarter (see Blanchard and Watson (1986), Bernanke (1986)). Here we go further by assuming that changes in the default rate also have no impact on output within the quarter. Therefore, what we identify as "output shock" can be deemed to be only that shock that instantaneously affects output growth.

Given this definition of "output shock", and with (16), we assess three profiles for expected default rates in 2006:

- the first one, the benchmark scenario, in which there is no shock (in other words, the "output shock" equals to zero over the sub-period 2005q4-2006q4);
- the second for a negative "output shock" is equal to one standard deviation in 2005q4 and zero otherwise;
- the third for a negative "output shock" is equal to three standard deviations in 2005q4 and zero otherwise.
Table 1 - Expected annual default rate for each macroeconomic scenario (%)

<table>
<thead>
<tr>
<th></th>
<th>Basic scenario</th>
<th>scenario 1</th>
<th>scenario 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006q1</td>
<td>2.21</td>
<td>2.23</td>
<td>2.28</td>
</tr>
<tr>
<td>2006q2</td>
<td>2.17</td>
<td>2.25</td>
<td>2.41</td>
</tr>
<tr>
<td>2006q3</td>
<td>2.14</td>
<td>2.28</td>
<td>2.63</td>
</tr>
<tr>
<td>2006q4</td>
<td>2.10</td>
<td>2.32</td>
<td>2.88</td>
</tr>
</tbody>
</table>

Basic scenario: no shock
Scenario 1: One standard deviation output shock
Scenario 2: Three standard deviation output shock
Table 1 and Figure 7 report the default rates corresponding to the two scenarios. In the basic scenario (no shock), default rates are very close to the observed ones, which is a sign of a quite good performance by the model in terms of predicting default rate profiles. Intuitively and empirically (see previous empirical results), a negative shock on output leads to an increase in the default rate. A negative output shock of one standard deviation corresponds to a decrease in the annual output growth rate equal to 69 basis points (bp). The corresponding contemporaneous increase in the default rate is equal to 2 bp (compared with the benchmark scenario). With a three standard deviation negative output shock (207 bp decline in annual output growth), the default rate is expected to increase instantaneously by 7 bp. A year later, the increase in the default rate is equal to 22 bp and 78 bp for scenarios 1 and 2 respectively.

4.1.2 Loss distribution simulation

The loss distribution of a given portfolio over time horizon $H$ can be determined by means of a Monte Carlo simulation. For each step of the simulation, we assume that a borrower $i$ can default at time $t$ with probability $p_{i,t}$. If the borrower defaults, its loss ($loss_{i,t}$) is as follows:

$$loss_{i,t} = \text{Volume of Financial Debt}_{i,t} \times (1 - \delta_{i,t})$$

(18)

where $\delta_{i,t}$ is the recovery rate of $i$ at time $t$. For the sake of simplicity, we assume that the recovery rate is constant for all borrowers and equal to 0.50.

Applying this procedure to each borrower allows us to compute the total loss, $loss_t$, as the sum of individual losses, that is:

$$loss_t = \sum_i loss_{i,t}$$

In order to estimate the loss distribution, the simulation is replicated 20000 times.

In the procedure described above, one key element is the choice of the probability of default for each borrower at each step of the simulation. We suggest two procedures:

i) The first follows the traditional approach (see Boss (2002), Virolainen (2004)) assuming that the default probability is identical for all borrowers, i.e. $p_{i,t} = p_t$. The average default rate $p_t$ used for simulations is then the default rate expected for each macroeconomic scenario. Therefore, given the results in Table 6, we take $p_t = 2.10\%$ to simulate the loss distribution of the basic scenario and $p_t = 2.32\%$ and $2.88\%$ for
the simulations of the loss distributions for scenarios 1 and 2, respectively. The drawback of this approach is that it does not take into account the individual risk of companies. Indeed, we assume that borrowers are subjected to identical probability of default. This assumption is unrealistic for several reasons: a) for a given economic environment, probabilities of default differ significantly at different rating grades (see figure 2); b) in addition, financial debts are not identically distributed for each rating grade (see appendix). Therefore, the loss distribution obtained from the traditional approach is likely to be biased.

ii) To address this issue, we propose an alternative procedure based on loss distribution simulations that explicitly take into account the rating grades of each firm. For a given macroeconomic scenario, we compute the expected default rate especially for 2006q4. This rate is then used as the default rate, that is, the prior probability of default. Using formula 1 (previously defined using Bayes’ theorem, see above), it is possible to compute the posterior probability of default for a company for which the score is known. Table 7 in section 4.2.1 shows that these procedures yield loss distributions that are significantly different. In what follows, we present the loss distribution obtained from both methods for each macroeconomic scenario.

4.2 Results of the macro stress tests

We conduct a Monte Carlo simulation of each model (with 20,000 repeated simulations for each scenario). We should recall that the basic scenario is simulated without a macroeconomic shock whereas the two crisis scenarios involve a negative output shock of one standard deviation and a negative output shock of three standard deviations respectively.

4.2.1 Basic scenario

First, let us consider the basic scenario. Given the results of the macroeconomic model, the annual default rate of the French manufacturing sector stands at 2.10% at the end of 2006 (Table 1). So, on the one hand, we compute Monte Carlo simulations based on companies’ financial debt data in 2005 by assuming that the default probability of each issuer is unchanging \( p_t = 2.10\% \). The corresponding distribution of total loss is reported in columns 2 (amount of the loss) and 3 (loss as a percentage of the financial debt) of Table 2. On the other hand, we adopt the alternative methodology presented earlier. In doing so, individual risk is taken into account in our simulations: we assume that the prior probability of default is derived from the macroeconomic model (2.10%), then we compute corresponding individual probabilities.
of default for each company applying the methodology presented in section 2.2. The corresponding loss
distribution is reported in columns 4 and 5 (Table 2).

| 1<sup>st</sup> percentile | 598 | 0.53 | 881 | 0.79 |
| 5<sup>th</sup> percentile | 699 | 0.62 | 984 | 0.88 |
| 10<sup>th</sup> percentile | 761 | 0.68 | 1,059 | 0.94 |
| 25<sup>th</sup> percentile | 885 | 0.79 | 1,192 | 1.06 |
| 50<sup>th</sup> percentile | 1,071 | 0.96 | 1,384 | 1.24 |
| 75<sup>th</sup> percentile | 1,325 | 1.18 | 1,635 | 1.46 |
| 90<sup>th</sup> percentile | 1,671 | 1.49 | 1,984 | 1.77 |
| 95<sup>th</sup> percentile | 2,003 | 1.79 | 2,212 | 1.97 |
| 99<sup>th</sup> percentile | 3,266 | 2.91 | 2,667 | 2.38 |
| Minimum | 457 | 0.41 | 581 | 0.52 |
| Mean | 1,179 | 1.05 | 1,462 | 1.40 |
| Maximum | 4,419 | 4.08 | 4,702 | 4.19 |

The two simulation procedures yield significantly different results. If issuers have identical default prob-
abilities, the expected loss (mean) for the year 2006 amounts to €1,179 million, while when individual
default probabilities are used the expected loss is larger (€1,462 million). This difference in expected
losses does not lead to a shift in loss distributions. In fact the extreme values of the loss distributions,
especially the unexpected losses (99<sup>th</sup> percentile) are obviously much higher with a common default
probability because the hypothesis of identical default probabilities is not relevant.

This reflects two factors acting in opposite directions, but the first one is the main driver of the results
obtained here. On the one hand, compared with the other ones, large companies have very high finan-
cial debt but mainly have low-risk rating grades (that is, with low individual probabilities of default).
Therefore, when we consider the identical default probability for each rating grade, we tend to assign a higher probability of default to companies who have large financial debt. Of course, this tends ultimately to over-estimate the loss for these companies. On the other hand, the mean values of financial debt are higher in high-risk rating grades. These two factors also explain the fact that the loss distribution is more concentrated on a smaller range of values when individual default probabilities are used.

4.2.2 Crisis scenarios

Since the loss distributions obtained by the two simulation methods are not directly comparable, we will show the impact of negative output shocks on both expected and unexpected loss obtained with each method.

First of all, let us consider that all companies have the identical default probabilities defined as the default probabilities of the French manufacturing sector expected for each macroeconomic model (Table 3).

<table>
<thead>
<tr>
<th>Annual default probability (%)</th>
<th>2.10</th>
<th>2.32</th>
<th>2.88</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected loss</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Amount (EUR millions)</td>
<td>1,179</td>
<td>1,299</td>
<td>1,614</td>
</tr>
<tr>
<td>% of financial debt</td>
<td>1.05</td>
<td>1.16</td>
<td>1.44</td>
</tr>
<tr>
<td>Unexpected loss</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Amount (EUR millions)</td>
<td>3,266</td>
<td>3,441</td>
<td>3,773</td>
</tr>
<tr>
<td>% of financial debt</td>
<td>2.91</td>
<td>3.07</td>
<td>3.37</td>
</tr>
</tbody>
</table>

The simulations show that expected and unexpected losses tend to increase with the decrease in output. Expected loss ranges from 1.05% to 1.44% of total financial debt, while unexpected loss ranges from 2.91% to 3.37%. These results seem to indicate that changes in activity have a significant impact on portfolio credit losses through their impact on default rates. For example, in the worst crisis scenario (scenario 2), the output shock causes the unexpected loss to increase by 15% relative to the basic scenario. The impact is greater on the value of expected losses in such a crisis scenario as the mean of the losses increases by almost 37%.
Now let us assume that individual default probabilities are taken into account in the simulations. The results are reported in Table 4. Once again, we observe that losses tend to increase when the macroeconomic scenario worsens, that is, when the decrease in output is sharper. By comparing the basic scenario and scenario 2, we see that unexpected losses increase by 25% when output decreases by three standard deviations in 2005q4. In addition, the increase in expected losses is 33%.

Two facts are worth noting when we compare these results with those obtained from the standard simulation procedure (Table 3):

a) as noted previously, taking into account individual risk in the simulation leads to higher expected losses and to smaller unexpected losses, whatever the macroeconomic scenario. This is a consequence of the distribution of financial debt across risk classes;

b) in addition, when comparing the crisis scenarios with the basic scenario, on average, the increase in the loss is greater (expected loss) when individual probabilities of default are considered.

<table>
<thead>
<tr>
<th>Table 4 - Crisis scenarios with individual default probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic scenario</td>
</tr>
<tr>
<td>Annual default probability</td>
</tr>
<tr>
<td>Expected loss</td>
</tr>
<tr>
<td>Amount (EUR millions)</td>
</tr>
<tr>
<td>% of financial debt</td>
</tr>
<tr>
<td>Unexpected loss</td>
</tr>
<tr>
<td>Amount (EUR millions)</td>
</tr>
<tr>
<td>% of financial debt</td>
</tr>
</tbody>
</table>

5 Conclusion

In this paper, a macroeconomic credit risk model for the French manufacturing sector has been modelled and estimated. The main results are as follows:

- There are significant and robust relationships between default rates and macroeconomic factors like
real GDP, corporate spreads and the short-term interest rate. These relationships are jointly estimated in a multivariate framework. This point is a marked improvement on existing results.

- Our macroeconomic credit risk model with explicit links between default rates and macro factors is well suited for macro stress testing purposes. We use the model to analyze the impact of stress scenarios on the credit risk of an aggregated French manufacturing sector credit portfolio. The results of the stress tests suggest that the economic environment impacts significantly on the evolution of loss distribution. The effect of the economic context on credit risk is not negligible. As expected, the hypothesis of identical default probabilities has to be rejected. It is a very rough approximation of reality. In fact, there is likely to be quite a lot of heterogeneity in default probabilities across firms within an industry. This is taken into account here by including company-level rating information from the Banque de France’s scores database.

This study has been conducted on the French manufacturing sector; it would be interesting to extend it to include services or the whole of the industrial sector. The robustness of the results could be also checked by including recent observations in the historical sample. As this study assumes a constant recovery rate of 50%, it will henceforth also be possible to improve modelling with regard to actual loss given default.
### Appendices

**Distribution of financial debt across rating grades in 2005 (Euro millions)**

<table>
<thead>
<tr>
<th>Rating</th>
<th>Financial debt</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median</td>
<td>0.19 0.38 0.53 0.47 0.63 0.71 0.33</td>
</tr>
<tr>
<td>Mean</td>
<td>1.86 3.51 3.74 3.50 5.28 4.95 2.97</td>
</tr>
<tr>
<td>99(^{th}) percentile</td>
<td>21.57 46.10 51.47 54.84 75.90 62.66 40.99</td>
</tr>
<tr>
<td>Maximum</td>
<td>2,200 4,326 1,542 548 1,562 593 4,326</td>
</tr>
<tr>
<td>Total</td>
<td>29,700 38,129 17,086 11,749 10,754 942 112,078</td>
</tr>
<tr>
<td>Number of companies</td>
<td>15,923 10,865 4,568 3,357 2,037 942 37,692</td>
</tr>
</tbody>
</table>

**Decomposition of financial debt in the sample (Euro millions)**

<table>
<thead>
<tr>
<th>Category</th>
<th>Mean</th>
<th>Standard deviation</th>
<th>Max</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial debt</td>
<td>2.97</td>
<td>37.70</td>
<td>4,326</td>
<td>112,078</td>
</tr>
<tr>
<td>Financing by the group and its partners</td>
<td>1.62</td>
<td>25.15</td>
<td>3,225</td>
<td>57,648</td>
</tr>
<tr>
<td>Bond debt</td>
<td>0.12</td>
<td>7.50</td>
<td>1,000</td>
<td>4,437</td>
</tr>
<tr>
<td>Bank debt</td>
<td>1.33</td>
<td>18.82</td>
<td>2,365</td>
<td>49,993</td>
</tr>
<tr>
<td>- Bank loans</td>
<td>0.99</td>
<td>18.10</td>
<td>2,365</td>
<td>37,262</td>
</tr>
<tr>
<td>- Leasing commitments</td>
<td>0.26</td>
<td>1.85</td>
<td>183</td>
<td>9,757</td>
</tr>
<tr>
<td>- Trade bills discounted</td>
<td>0.08</td>
<td>0.66</td>
<td>46</td>
<td>2,974</td>
</tr>
</tbody>
</table>
References


233. R. Cooper, H. Kempf and D. Peled, “Monetary rules and the spillover of regional fiscal policies in a federation” February 2009


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fax :0033 (0)1 42 92 62 92
email : thierry.demoulin@banque-france.fr
jeannine.agoutin@banque-france.fr
veronique.jan-antuoro@banque-france.fr
nathalie.bataille-salle@banque-france.fr