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A COMPLEMENTARY APPROACH**

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# Measuring the NAIRU: a complementary approach\*

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## Abstract

Estimates of the Nairu generally suffer from a large uncertainty, which can be reduced by adopting a bivariate framework and assuming that shifts of the Phillips curve share a common trend with the unemployment rate. We consider in this paper if this common trend assumption is empirically relevant or not for seven economies over the sample 1973-2010. First, it appears that the Nairu can substantially differ from the unemployment trend. Second, relaxing the common trend assumption improves the fit of the inflation equation. Third, this assumption is necessary for getting an important reduction of uncertainty in a bivariate framework.

Keywords: Nairu, inflation, uncertainty.

JEL codes: C32, E31, E24.

## Résumé

Les estimations du Nairu sont généralement entourées d'une grande incertitude, qui peut être réduite en adoptant un cadre bivarié et en supposant que la pente de la courbe de Phillips partage une tendance commune avec le taux de chômage. Nous étudions dans cet article si cette dernière hypothèse est empiriquement pertinente pour sept économies sur la période 1973-2010. Il apparaît d'abord que le Nairu peut différer substantiellement de la tendance du chômage. Ensuite, nous montrons que l'ajustement de l'équation d'inflation est amélioré, lorsque l'on relâche cette hypothèse. Enfin, cette hypothèse s'avère nécessaire pour réduire fortement l'incertitude dans un cadre bivarié.

Mots-clés: Nairu, inflation, incertitude.

Classification JEL: C32, E31, E24.

# 1 Introduction

The time-varying non-accelerating inflation rate of unemployment (tv-Nairu) plays a crucial role in the design of economic policies. Major international institutions, e.g. IMF, OECD or European Commission<sup>1</sup>, use them as a component of potential output for assessing inflation tensions, estimating Taylor rules and computing structural deficits.

The standard model of the tv-Nairu, based on an expectation-augmented Phillips curve<sup>2</sup>, considers that inflation changes are driven by the gap between the unemployment rate and the tv-Nairu, an unobserved stochastic trend inferred by the Kalman filter (Gordon 1997). As explained by Ball and Mankiw (2002), in such a model, we can reinterpret the tv-Nairu as the long-run trend of shifts in the Phillips curve. Laubach (2001) has extended the model in a bivariate framework, by incorporating in the model a trend-cycle decomposition of unemployment and by assuming that shifts in the Phillips curve share a common trend with the unemployment rate<sup>3</sup>. We can simultaneously interpret this common trend as an unemployment trend or as a tv-Nairu. Using information about the behavior of unemployment, in addition to inflation, seems appealing, as it reduces the wide uncertainty, which generally surrounds estimates of the tv-Nairu<sup>4</sup>. However, we consider in this paper if the common trend assumption is empirically relevant or not. Should we always interpret an increase of the unemployment trend as an increase of the tv-Nairu? Or is it possible in such a case to observe a disinflation trend consistent with the Phillips slope, from which we would infer that the tv-Nairu would remain stable?

In this paper, we answer these questions with a double-trend bivariate model (DTB), an extended version of the Laubach model where we relax the common trend assumption. This model incorporates a time-varying shift (tv-shift) in the inflation equation<sup>5</sup>. A combination of the unemployment trend and the Phillips tv-shift produces the tv-Nairu and this model allows for the occurrence of disinflation periods driven by the Phillips tv-shift. We compare tv-Nairu estimates of this model with those of a single-trend univariate model (STU) in the spirit of Gordon (1997) and of a single-trend bivariate one (STB) in the spirit of Laubach (2001) for seven economies over the sample 1973-2010.

Our empirical results lead to three conclusions. First, DTB estimates of the tv-Nairu substantially differ from those of the unemployment trend for France and Germany in the 1970s and the 1980s. Second, relaxing the common trend assumption improves the fit of the inflation equation: the coefficient of determination increases by one to four points in the DTB model compared to the STB model. Third, this assumption is necessary for

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<sup>1</sup>See Benes et al. (2010), Beffy et al. (2007) and Denis et al. (2006).

<sup>2</sup>Although expectations are specified with distributed lags in an old-fashioned style, this tool can still not be replaced by DSGE based output gaps, because of the lack of consensus about the identification of shocks that generate the potential output. This lack of consensus is related to the observational equivalence in the model of Smets and Wouters (2007) between labour supply shocks and wage markup shocks (see Chari et al. 2009). While the first one are efficient and should be included in the potential output, the second one are inefficient and should not.

<sup>3</sup>The tv-Nairu is modelled in a similar way as Kuttner (1994) did with a bivariate model of potential output. Other papers followed a similar approach. Basistha and Startz (2008) used a larger multivariate model. Planas et al. (2008) adopted a Bayesian framework. Laubach and Williams (2003) extended the model for measuring the natural rate of interest. Harvey (2008) and Kajuth (2010) estimated bivariate models of the unemployment rate and the inflation level, instead of inflation changes: the inflation level was directly related to an inflation trend and to the unemployment cycle.

<sup>4</sup>See Staiger et al. (1997).

<sup>5</sup>Staiger et al. (2001) estimated a similar equation for the US, but they did not incorporate it in a joint bivariate model.

getting an important reduction of uncertainty in a bivariate framework: the uncertainty around the tv-Nairu is reduced for all countries, by 33% to 73%, with the STB model compared to the STU one, while it is reduced for only 3 countries, by 7% to 38%, with the DTB model.

Section 2 presents the specification of the double-trend bivariate model. Section 3 presents our estimation strategy. Section 4 comments on the empirical results. Section 5 concludes.

## 2 Model specification

We present in this section the specification of the DTB model, a bivariate model of the unemployment rate and the inflation rate, where the tv-Nairu is not assumed equal to the unemployment trend. We describe successively its two blocks: a trend-cycle decomposition of the unemployment rate and a time-varying parameter model of the inflation rate.

### 2.1 Trend-cycle decomposition of the unemployment rate

We specify the long-run trend and the short-run cycle of the unemployment rate, with the following trend-cycle decomposition of Clark (1987):

$$u_t = u_t^* + (u_t - u_t^*) \quad (1)$$

$$u_t^* = u_{t-1}^* + \mu_{t-1} + \sigma_v v_t \quad (2)$$

$$\mu_t = \mu_{t-1} + \sigma_\zeta \zeta_t \quad (3)$$

$$u_t - u_t^* = \phi_1 (u_{t-1} - u_{t-1}^*) + \phi_2 (u_{t-2} - u_{t-2}^*) + \sigma_w w_t \quad (4)$$

Equation (1) decomposes the unemployment rate into a trend  $u_t^*$  and a cycle  $u_t - u_t^*$ . Equations (2) and (3) specify the unemployment trend  $u_t^*$  as a random walk with drift, the drift  $\mu_t$  itself being assumed to be random walk. Equation (4) specifies the unemployment cycle  $u_t - u_t^*$  as an AR(2) process, which is assumed to be stationary.

### 2.2 Time-varying parameter model of the inflation rate

Following a vast literature initiated by Friedman (1968) and Phelps (1968), we can model the inflation rate with the following time-varying parameter model in the spirit of Gordon (1997)<sup>6</sup>:

$$\Delta\pi_t = \beta(L)\Delta\pi_{t-1} + \gamma (u_{t-1} - u_{t-1}^N) + \delta(L)x_t + \sigma_\varepsilon \varepsilon_t$$

In this expectation-augmented Phillips curve, the inflation  $\pi_t$  is driven by backward expectations<sup>7</sup>, the lag of the gap between the unemployment rate  $u_t$  and a tv-intercept  $u_{t-1}^N$  (called thereafter the unemployment gap), a control variable  $x_t$  and a temporary supply shock  $\varepsilon_t$ . Here, the control variable  $x_t$  is specified equal to terms of trade (the difference between import price inflation and consumption price inflation). The tv-Nairu is equal to  $u_t^N$ : in the absence of temporary supply shocks,  $\Delta\pi_t$  converges toward 0, when

<sup>6</sup>The concept of tv-Nairu considered here should not be confused with the natural rate of unemployment, a notion arising in New Keynesian - DSGE models.

<sup>7</sup>We have made this choice for matter of comparability with Laubach (2001), but we could include in further research explicit inflation expectations based on survey data (see Driver et al. 2006).

the unemployment stays equal to  $u_t^N$ . The Phillips slope  $\gamma$  is assumed to be negative, which is necessary for identifying the Nairu<sup>8</sup>.

If we do not make the common trend assumption of Laubach (2001), the tv-Nairu  $u_t^N$  should be distinguished from the unemployment trend  $u_t^*$ . Thus, the inflation equation based on the unemployment cycle  $u_{t-1} - u_{t-1}^*$  should have some shifts  $\gamma(u_{t-1}^* - u_{t-1}^N)$ :

$$\Delta\pi_t = \gamma(u_{t-1}^* - u_{t-1}^N) + \beta(L)\Delta\pi_{t-1} + \gamma(u_{t-1} - u_{t-1}^*) + \delta(L)x_t + \sigma_\varepsilon\varepsilon_t.$$

We model these shifts with a tv-intercept  $\alpha_t$ :

$$\Delta\pi_t = \alpha_{t-1} + \beta(L)\Delta\pi_{t-1} + \gamma(u_{t-1} - u_{t-1}^*) + \delta(L)x_t + \sigma_\varepsilon\varepsilon_t \quad (5)$$

As usually done in tv-parameters regressions, we specify  $\alpha_t$  called thereafter the Phillips tv-shift, as a random walk:

$$\alpha_t = \alpha_{t-1} + \sigma_\eta\eta_t \quad (6)$$

Equations (5) and (6) imply that inflation is I(2). If we do not believe that inflation is theoretically I(2), such a specification allows to approximate a structural change of the inflation equation.

Finally, we define the DTB model as a state-space model, with the measure equations (1), (5) and the transition equations (2), (3), (4), (6). Innovations  $\varepsilon_t, v_t, \zeta_t, w_t, \eta_t$  are i.i.d.  $\mathbf{N}(0,1)$  processes, which are assumed independent from each other. The tv-Nairu  $u_t^N$  is related to the unemployment trend and the Phillips tv-shift in the following way:

$$u_t^N = u_t^* - \frac{\alpha_t}{\gamma}. \quad (7)$$

### 3 Estimation strategy

We estimate the DTB model for seven economies with quarterly time series in the period 1973Q1-2010Q3<sup>9</sup>: the United States (US), the United Kingdom (UK), Canada (CA), Australia (AU), France (FR), Germany (GE) and Italy (IT). All series come from OECD and BIS databases.

For matter of comparison, we also estimate two benchmark models: the STU model defined by equations (2), (3), (5), and (6) with the constraints  $\alpha_0 = 0, \sigma_\eta = 0$ , which ensure the identity  $u_t^N = u_t^*$ ; the STB model defined as a restricted version of the DTB model with the same constraints  $\alpha_0 = 0, \sigma_\eta = 0$ . We write the three models in state-space form and use an (approximately) diffuse initialization for non-stationary state variables (see Durbin and Koopman 2001). We initialize other state variables with their unconditional distribution. We transform constrained parameters, in order to perform maximization with respect to unconstrained quantities. Then, we estimate parameters by maximizing the diffuse log-likelihood using the Kalman filter and the Expectation-Maximization algorithm<sup>10</sup>. We select for all models four lags of inflation and two lags of terms of trade using the diffuse Bayesian Information Criterion (BIC).

<sup>8</sup>Since Friedman (1968) and Phelps (1968), theories underlying the Phillips curve can only explain a negative relationship between inflation and unemployment.

<sup>9</sup>See the technical appendix for details about the state-space form of the DTB model, the estimation strategy and the dataset.

<sup>10</sup>All estimation procedures are performed with Matlab. Some programmes come from the Kalman filter toolbox of Kevin Murphy available from the website <http://www.cs.ubc.ca/~murphyk/>.

As shown in Stock and Watson (1998), the likelihood of such models has a point-mass at zero for variances of tv-parameters innovations. Therefore, there is a non-null probability, called the pile-up probability, that estimates of these variances are strictly equal to zero. Here, innovation variances of stochastic trends are kept fixed to get results, which can be compared to those of Laubach (2001). For  $\sigma_v$  and  $\sigma_\zeta$ , we use the same values as he did:  $\sigma_v$  equal to 0.2 for the United States, Canada, Australia and Italy, and to 0.1 for other countries;  $\sigma_\zeta$  equal to 0.015. For  $\sigma_\eta$ , the value 0.03 ensures a degree of smoothness for DTB estimate of the tv-Nairu, which is similar to that of STU and DTB estimates<sup>11</sup>.

Then, following Hamilton (1986), we compute by simulation the standard errors of state variables<sup>12</sup>. Such a procedure allows to take into account the two sources of uncertainty surrounding state variables in a state-space model: the filter standard error associated with the Kalman smoother (given known parameters); the parameter standard error associated with the estimation of unknown parameters with a finite sample.

## 4 Empirical results

Estimation results are reported in tables 1, 2 and 3. The first part of each table contains estimates and standard errors of parameters for each model. The second part contains indicators of the fit of each model: the  $R^2$  of the inflation equation and the diffuse log-likelihood. The last part contains standard errors surrounding the tv-Nairu estimates. Figures 1 to 4 present smoothed estimates of the tv-Nairu, the unemployment trend and the Phillips tv-shift in the DTB model. Figures 5 and 6 present smoothed estimates of the tv-Nairu in STU and STB models.

### 4.1 Parameter estimates

Except for Italy where a null Phillips slope has a higher likelihood than any negative value<sup>13</sup>, the size of  $\gamma$  is larger in the DTB model (from -0.12 for the UK to -0.35 for the US) than in the STU model (from -0.07 for the GE to -0.23 for the US). It is also more significant with higher t-statistics for a majority of countries (US, CA, UK and GE). Conversely, these estimates are similar in the DTB model and in the STB model. Thus, the result of Laubach (2001) is confirmed and reinforced: adding a law of motion for the unemployment gap increases the size and the significance of the Phillips slope, even when the tv-Nairu is not assumed equal to the unemployment trend.

The fourth lag coefficient ( $\beta_4$ ) of inflation is significant for all countries and models. The second lag coefficient ( $\delta_2$ ) of terms of trade is significant for all models in the United States, France and Italy. For both bivariate models (STB and DTB), the unemployment cycle has a high persistence ( $\phi_1 + \phi_2$  ranging from 0.93 for the US to 0.995 for IT). When the unemployment cycle has complex roots, the estimate of the average frequency  $\lambda_u$  is

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<sup>11</sup>If we interpret the STU model as a Butterworth filter of order 2, the theoretical gain of this filter, provided in Harvey and Trimbur (2003), has a frequency cutoff of approximately  $\pi/50$  (25 years) for  $\sigma_\zeta$  equal to 0.015. If we interpret the DTB model as a Butterworth filter of order 1, such a frequency cutoff is obtained for  $\sigma_\eta$  equal to 0.03.

<sup>12</sup>Parameters are simulated 2000 times from Gaussian distributions with a mean and a variance equal to their point estimate and the variance of their estimate.

<sup>13</sup>As explained by Laubach (2001), the unconstrained estimate of this parameter would even be positive and this might be related to the wage setting, which depends only on unemployment in northern and central regions.



in the usual range of business cycle frequencies (corresponding periods of 6.0 years for US, 7.3 years for CA, 9.0 years for GE).

## 4.2 Performance criteria

Then, we notice that relaxing the common trend assumption improves the fit of the inflation equation<sup>14</sup>. Indeed, the coefficient of determination is larger by one to four points for the DTB model (from 28.01 for GE to 57.40 for IT), which includes a Phillips tv-shift, than for the STB model (from 24.09 for GE to 55.49 for IT), which do not. As illustrated below with graphs, the Phillips tv-shift allows the unemployment gap to stay positive (negative) during prolonged disinflationary (resp. inflationary) periods.

Finally, the common trend assumption is necessary for getting an important reduction of uncertainty surrounding smoothed estimates of the tv-Nairu in a bivariate framework:<sup>15</sup> the average total standard error is reduced by 33% for Canada to 73% for Germany with the STB model compared to the STU one (from 0.72 to 0.48 for Canada, from 1.58 to 0.43 for Germany). Conversely, the average total standard error of the tv-Nairu is smaller with the DTB model than with the STU one only for the United States (7% smaller), Australia (resp. 11%) and Germany (resp. 38%). Relaxing the common trend assumption enlarges uncertainty around the tv-Nairu, because it adds to the uncertainty around the unemployment trend the one surrounding the Phillips tv-shift.

## 4.3 Smoothed estimates of the tv-Nairu

Except for Italy where the tv-Nairu is not identified because the Phillips slope is equal to zero, figures 1, 2, 3 and 4 show how the Phillips tv-shift implies a distinct evolution of the tv-Nairu relative to the unemployment trend. Differences are more substantial for European countries, like France and Germany, and almost equal to zero for the US and Canada. In France, the tv-Nairu is higher than the unemployment trend in the 1970s, due to the progressive rise in inflation<sup>16</sup>. Conversely, the unemployment trend is persistently higher than the tv-Nairu in the 1980s. This difference is related to the disinflation that occurred in this period. In Germany, the tv-Nairu is almost one point lower than the unemployment trend from the 1970s to the mid-1980s. This difference corresponds to the progressive disinflation that happened during this period. Then, the tv-Nairu and the unemployment trend are almost equal since the mid-1980s.

Results of the STU model are qualitatively similar to the DTB model (figures 5 and 1). For France, the unemployment gap appears on average negative in the 1970s, positive in the 1980s and 1990s and equal to zero in the 2000s. For Germany, the unemployment gap appears on average positive from the 1970s to the mid-1980s and equal to zero until 2010. However, contrary to the DTB model, the STU model does not allow one to

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<sup>14</sup>Here, we limit our discussion on in-sample properties of each model regarding inflation. Indeed, Stock and Watson (2008) has shown that pseudo out-of-sample forecasts of inflation of any Phillips curve model cannot beat on average univariate ones.

<sup>15</sup>We do not comment diffuse log-likelihood and BIC, which can not be compared across models. Although the STB model is a constrained version of the DTB one, we can not compare their diffuse log-likelihood (neither their diffuse BIC), because the number of diffuse variables is different in each model. As the STU model is not nested in other models, its diffuse log-likelihood and BIC can not be directly compared to other ones.

<sup>16</sup>This occurred in spite of the import price variable, which takes into account the impact of oil shocks.

distinguish the impact of the unemployment cycle on inflation from that of the Phillips tv-shift.

Contrary to DTB and STU models, the unemployment gap is on average centered in all periods and countries for the STB model (figure 6). As the unemployment trend of the DTB model (figure 2) is similar to that of the STB model, we see that the difference between these results come from the Phillips tv-shift, which is constrained equal to zero in the STB model.

## 5 Conclusion

The paper proposes a new bivariate model of unemployment and inflation with an unemployment trend and a Phillips tv-shift, which appears in times of disinflation. The estimation of this model for seven economies allows one to assess the empirical relevance of the common trend assumption used previously. Our empirical results lead to three conclusions. First, DTB estimates of the tv-Nairu substantially differs from those of the unemployment trend for France and Germany in the 1970s and the 1980s. Second, relaxing the common trend assumption improves the fit of the inflation equation: the coefficient of determination increases by one to four points in the DTB model compared to the STB model. Third, this assumption is necessary for getting an important reduction of uncertainty in a bivariate framework: the uncertainty around the tv-Nairu is reduced for all countries, by 33% to 73%, with the STB model compared to the STU one, while it is reduced for only three countries, by 7% to 38%, with the DTB model.

In further research, we could address a similar issue for potential output, by extending the Kuttner model with a Phillips tv-shift in the same way as we did for the tv-Nairu. We could also apply this approach to the measure of the natural rate of interest. Finally, we could look at the sensitivity of tv-Nairu estimates to the specification of inflation expectations, as they could have become more anchored in the 1990s than in previous decades.

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# A Tables and figures

Table 1. Estimation results of the DTB Model

Parameter	US	CA	AU	UK	GE	FR	IT
<i>Phillips curve equation</i>							
$\beta_1$	0.15 (0.10)	0.14 (0.08)	0.21 (0.07)	0.34 (0.08)	0.17 (0.08)	0.30 (0.08)	0.27 (0.07)
$\beta_2$	-0.20 (0.09)	-0.06 (0.07)	0.07 (0.08)	0.03 (0.08)	-0.05 (0.08)	-0.05 (0.08)	-0.04 (0.07)
$\beta_3$	0.16 (0.08)	-0.02 (0.07)	0.21 (0.08)	0.11 (0.08)	0.10 (0.08)	0.08 (0.08)	0.12 (0.08)
$\beta_4$	-0.61 (0.09)	-0.51 (0.08)	-0.46 (0.08)	-0.36 (0.07)	-0.41 (0.08)	-0.41 (0.08)	-0.46 (0.07)
$\gamma$	-0.35 (0.09)	-0.27 (0.07)	-0.14 (0.10)	-0.12 (0.06)	-0.13 (0.05)	-0.15 (0.07)	0.00 (-)
$\delta_1$	0.06 (0.02)	0.02 (0.02)	0.02 (0.02)	0.03 (0.02)	0.02 (0.02)	0.05 (0.01)	0.08 (0.02)
$\delta_2$	-0.05 (0.02)	0.00 (0.02)	-0.02 (0.02)	0.01 (0.02)	-0.01 (0.02)	-0.04 (0.01)	-0.05 (0.02)
$\sigma_\varepsilon^2$	0.33	0.33	0.73	0.59	0.18	0.20	0.46
$\sigma_\eta^2$	9.00E-04	9.00E-04	9.00E-04	9.00E-04	9.00E-04	9.00E-04	9.00E-04
<i>Unemployment trend-cycle decomposition</i>							
$\phi_1$	1.76 (0.10)	1.66 (0.14)	1.64 (0.12)	1.83 (0.07)	1.88 (0.06)	1.61 (0.13)	1.85 (0.10)
$\phi_2$	-0.83 (0.06)	-0.72 (0.07)	-0.67 (0.10)	-0.83 (0.07)	-0.91 (0.04)	-0.64 (0.10)	-0.85 (0.09)
$\sigma_v^2$	4.00E-02	4.00E-02	4.00E-02	1.00E-02	1.00E-02	1.00E-02	4.00E-02
$\sigma_\zeta^2$	2.25E-04	2.25E-04	2.25E-04	2.25E-04	2.25E-04	2.25E-04	2.25E-04
$\sigma_w^2$	2.59E-02	4.52E-02	3.51E-02	1.41E-02	6.56E-03	2.22E-02	5.32E-03
$R^2$	50.57	39.50	32.32	43.34	28.01	48.56	57.40
Log-lik.	-161.49	-180.61	-223.94	-123.83	-9.05	-70.13	-168.62
Total SE	0.61	0.78	1.91	2.37	0.98	1.24	-
Param. SE	0.14	0.15	0.54	0.31	0.22	0.29	-
Filter SE	0.59	0.76	1.82	2.35	0.95	1.19	-

Legend: standard errors of parameters are written in parenthesis.

Table 2. Estimation results of the STU Model

Parameter	US	CA	AU	UK	GE	FR	IT
<i>Phillips curve equation</i>							
$\beta_1$	0.19 (0.10)	0.16 (0.08)	0.20 (0.07)	0.36 (0.08)	0.21 (0.08)	0.31 (0.08)	0.27 (0.07)
$\beta_2$	-0.15 (0.09)	-0.05 (0.08)	0.07 (0.08)	0.04 (0.08)	-0.01 (0.08)	-0.04 (0.08)	-0.04 (0.07)
$\beta_3$	0.21 (0.08)	-0.01 (0.08)	0.21 (0.07)	0.11 (0.08)	0.13 (0.08)	0.08 (0.08)	0.12 (0.08)
$\beta_4$	-0.56 (0.09)	-0.50 (0.08)	-0.46 (0.07)	-0.36 (0.07)	-0.37 (0.08)	-0.40 (0.08)	-0.45 (0.07)
$\gamma$	-0.23 (0.06)	-0.19 (0.06)	-0.10 (0.07)	-0.08 (0.05)	-0.07 (0.04)	-0.11 (0.05)	0.00 (-)
$\delta_1$	0.05 (0.03)	0.02 (0.02)	0.02 (0.02)	0.03 (0.02)	0.02 (0.02)	0.04 (0.01)	0.09 (0.02)
$\delta_2$	-0.05 (0.02)	0.00 (0.02)	-0.02 (0.02)	0.01 (0.02)	-0.01 (0.02)	-0.04 (0.01)	-0.05 (0.02)
$\sigma_\varepsilon^2$	0.35	0.34	0.73	0.61	0.19	0.21	0.47
$\sigma_v^2$	4.00E-02	4.00E-02	4.00E-02	1.00E-02	1.00E-02	1.00E-02	0.00E+00
$\sigma_\zeta^2$	2.25E-04	2.25E-04	2.25E-04	2.25E-04	2.25E-04	2.25E-04	0.00E+00
$R^2$	47.27	35.97	31.85	41.85	22.51	47.02	56.18
Log-lik.	-149.36	-148.40	-201.33	-187.88	-102.98	-110.18	-168.21
Total SE	0.65	0.72	2.14	2.02	1.58	0.90	-
Param. SE	0.23	0.21	0.97	0.56	0.65	0.38	-
Filter SE	0.60	0.68	1.88	1.94	1.43	0.79	-

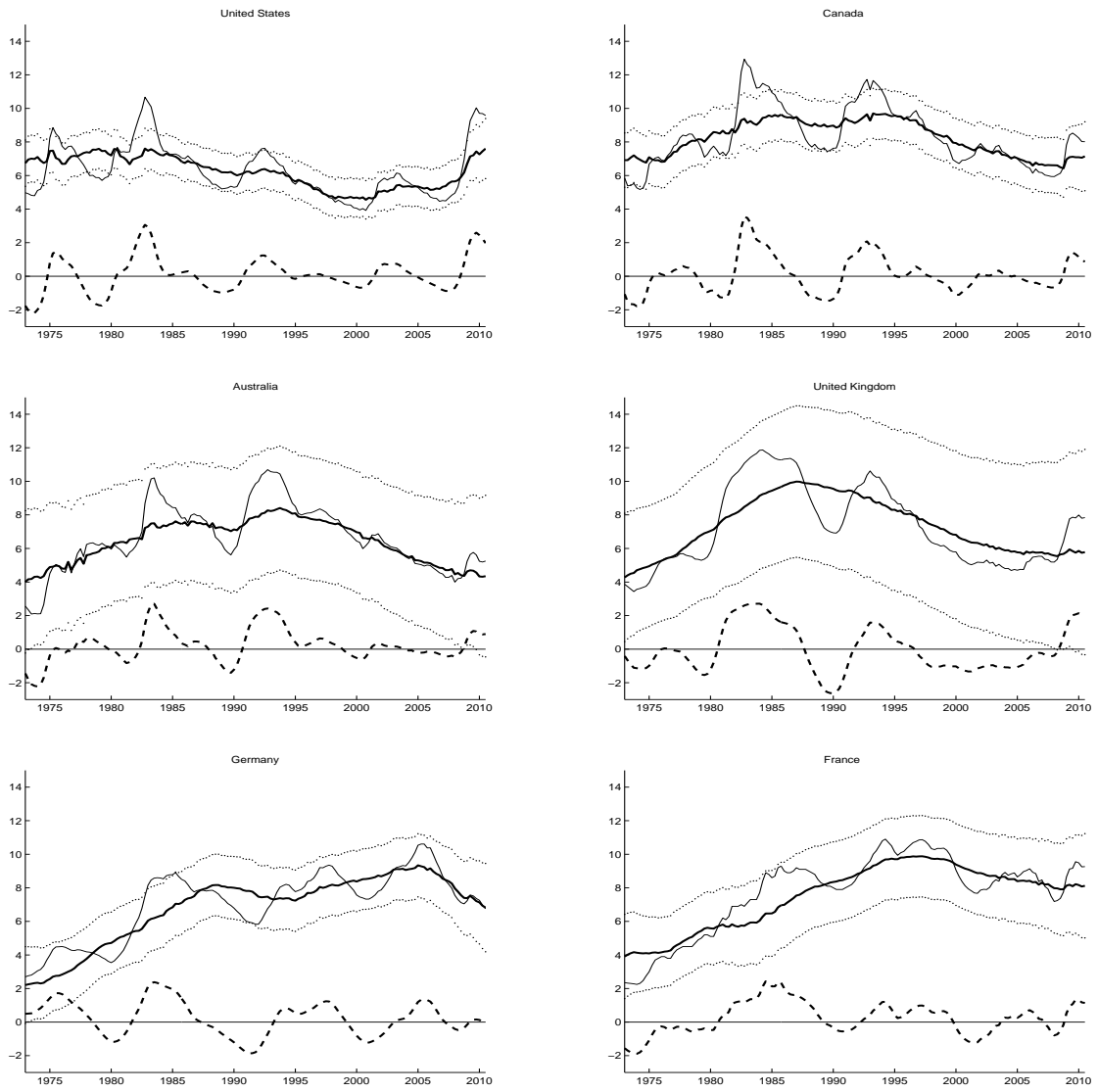
Legend: standard errors of parameters are written in parenthesis.

Table 3. Estimation results of the STB Model

Parameter	US	CA	AU	UK	GE	FR	IT
<i>Phillips curve equation</i>							
$\beta_1$	0.17 (0.10)	0.16 (0.08)	0.21 (0.07)	0.35 (0.08)	0.19 (0.08)	0.31 (0.08)	0.28 (0.07)
$\beta_2$	-0.20 (0.09)	-0.04 (0.08)	0.08 (0.08)	0.03 (0.08)	-0.03 (0.08)	-0.05 (0.08)	-0.04 (0.07)
$\beta_3$	0.15 (0.08)	-0.02 (0.08)	0.22 (0.08)	0.12 (0.08)	0.12 (0.08)	0.09 (0.08)	0.12 (0.08)
$\beta_4$	-0.61 (0.09)	-0.48 (0.08)	-0.45 (0.08)	-0.36 (0.07)	-0.39 (0.08)	-0.39 (0.08)	-0.44 (0.07)
$\gamma$	-0.34 (0.09)	-0.24 (0.07)	-0.10 (0.09)	-0.09 (0.07)	-0.11 (0.05)	-0.15 (0.07)	0.00 (-)
$\delta_1$	0.06 (0.02)	0.02 (0.02)	0.02 (0.02)	0.03 (0.02)	0.02 (0.02)	0.05 (0.01)	0.09 (0.02)
$\delta_2$	-0.04 (0.02)	0.00 (0.02)	-0.02 (0.02)	0.01 (0.02)	-0.01 (0.02)	-0.04 (0.01)	-0.05 (0.02)
$\sigma_\varepsilon^2$	0.33	0.34	0.74	0.60	0.19	0.21	0.47
$\sigma_\eta^2$	0.00	0.00	0.00	0.00	0.00	0.00	0.00
<i>Unemployment trend-cycle decomposition</i>							
$\phi_1$	1.76 (0.10)	1.66 (0.14)	1.64 (0.12)	1.82 (0.08)	1.88 (0.06)	1.59 (0.12)	1.85 (0.10)
$\phi_2$	-0.84 (0.06)	-0.73 (0.07)	-0.67 (0.10)	-0.82 (0.07)	-0.91 (0.04)	-0.63 (0.10)	-0.85 (0.09)
$\sigma_v^2$	4.00E-02	4.00E-02	4.00E-02	1.00E-02	1.00E-02	1.00E-02	4.00E-02
$\sigma_\zeta^2$	2.25E-04	2.25E-04	2.25E-04	2.25E-04	2.25E-04	2.25E-04	2.25E-04
$\sigma_w^2$	2.52E-02	4.48E-02	3.50E-02	1.42E-02	6.46E-03	2.21E-02	5.32E-03
$R^2$	48.77	36.32	29.40	41.88	24.09	44.62	55.49
Log-lik.	-158.51	-179.15	-222.79	-121.71	-4.63	-67.89	-165.60
Total SE	0.40	0.48	0.83	1.14	0.43	0.50	2.04
Param. SE	0.12	0.13	0.22	0.41	0.12	0.17	0.65
Filter SE	0.38	0.46	0.79	1.06	0.41	0.46	1.91

Legend: standard errors of parameters are written in parenthesis.

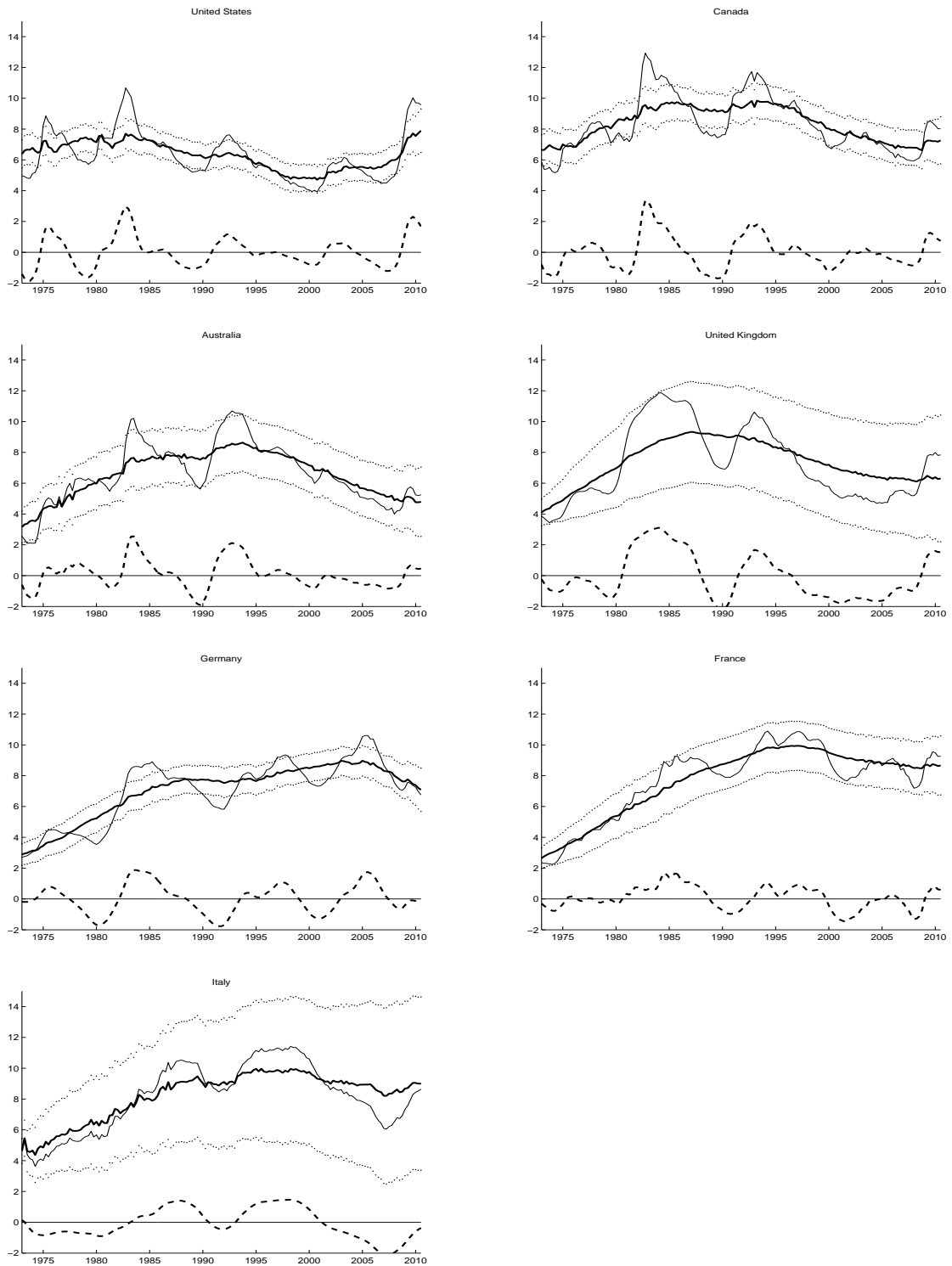
Figure 1: Smoothed estimates of the tv-Nairu in the DTB model



Thin line: unemployment rate; bold line: tv-Nairu; dotted lines: 95%-confidence interval; dashed line: unemployment gap.

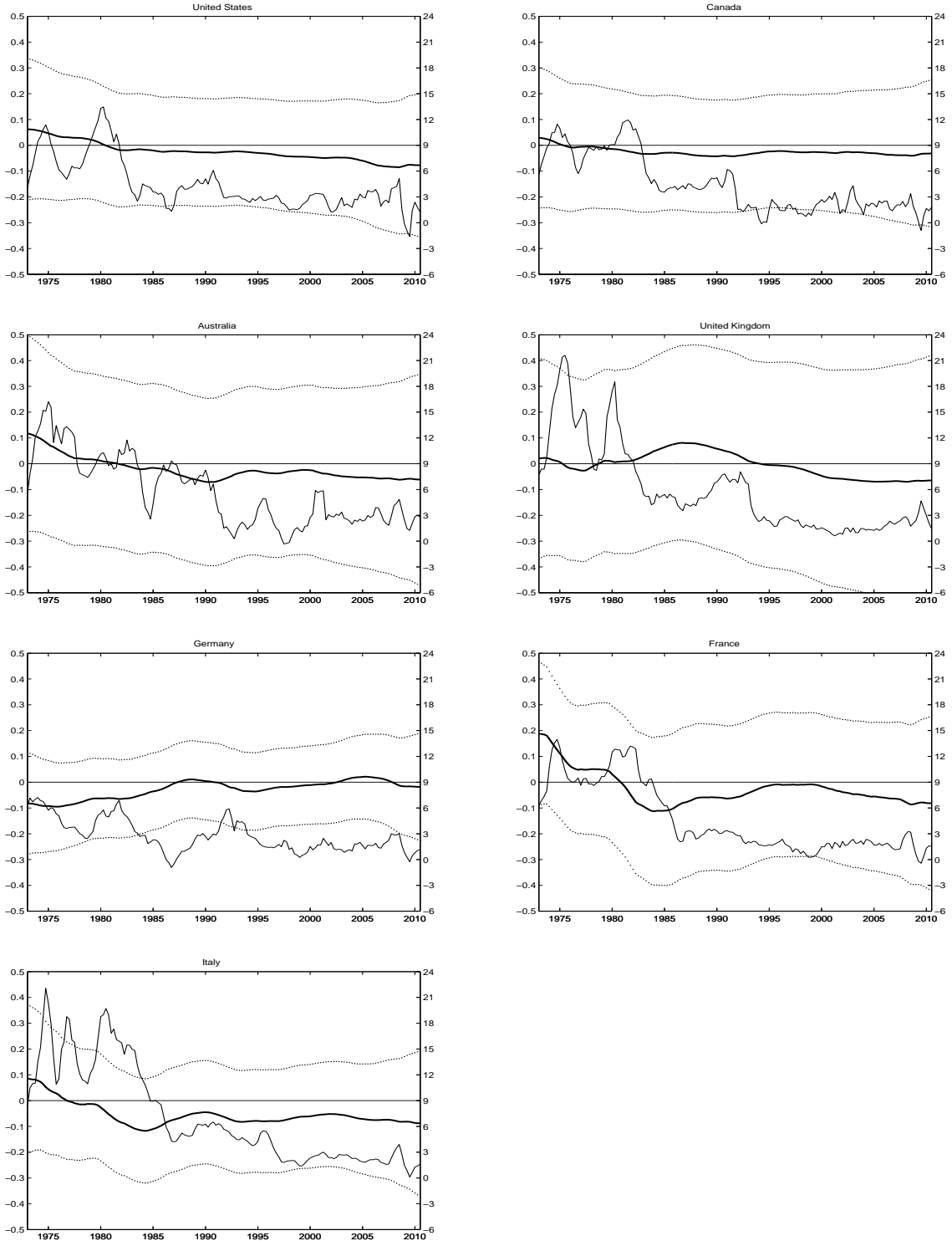


Figure 2: Smoothed estimates of the unemployment trend in the DTB model



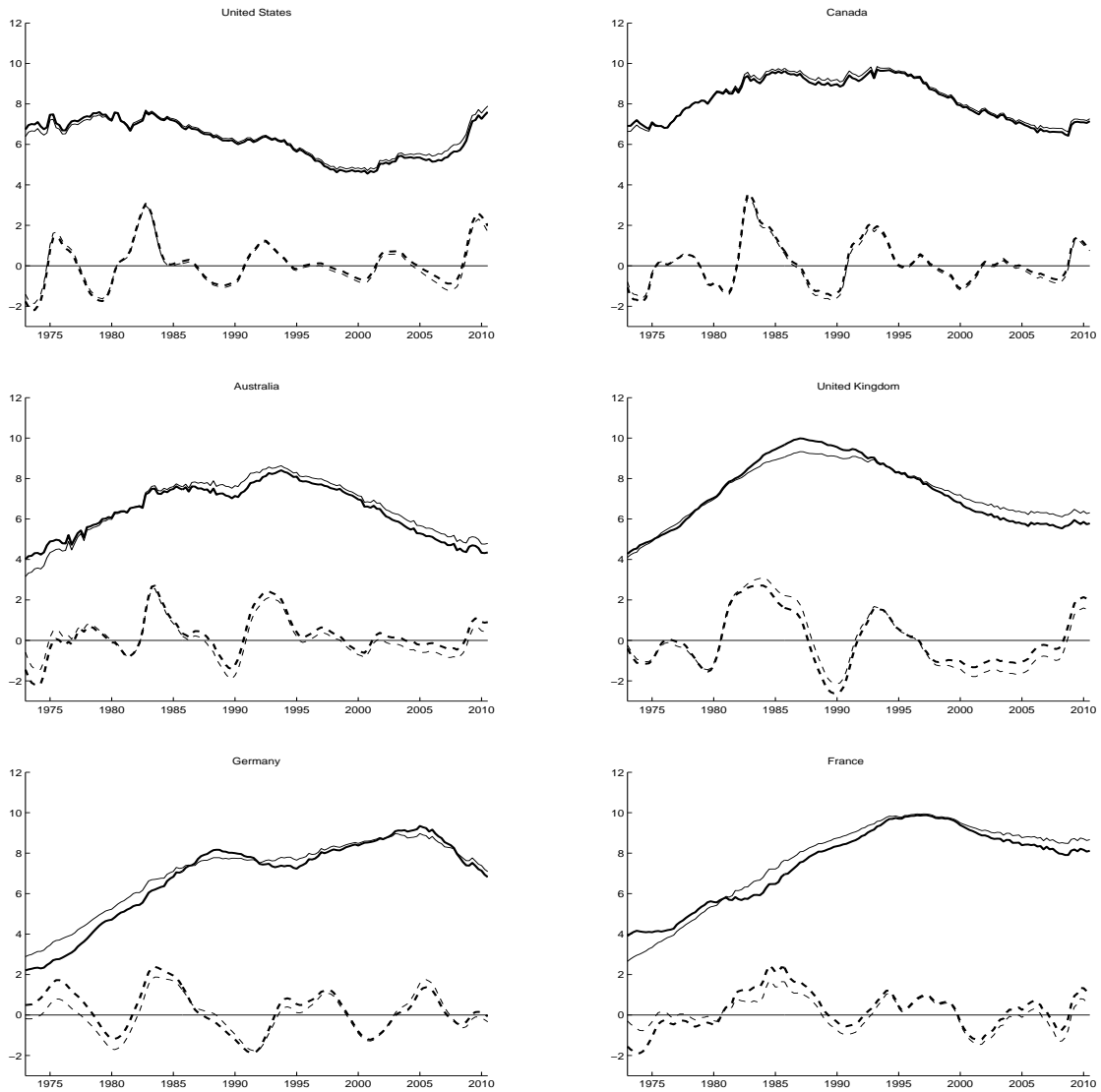
Thin line: unemployment rate; bold line: unemployment trend; dotted lines: 95%-confidence interval; dashed line: unemployment cycle.

Figure 3: Smoothed estimates of the Phillips tv-shift in the DTB model



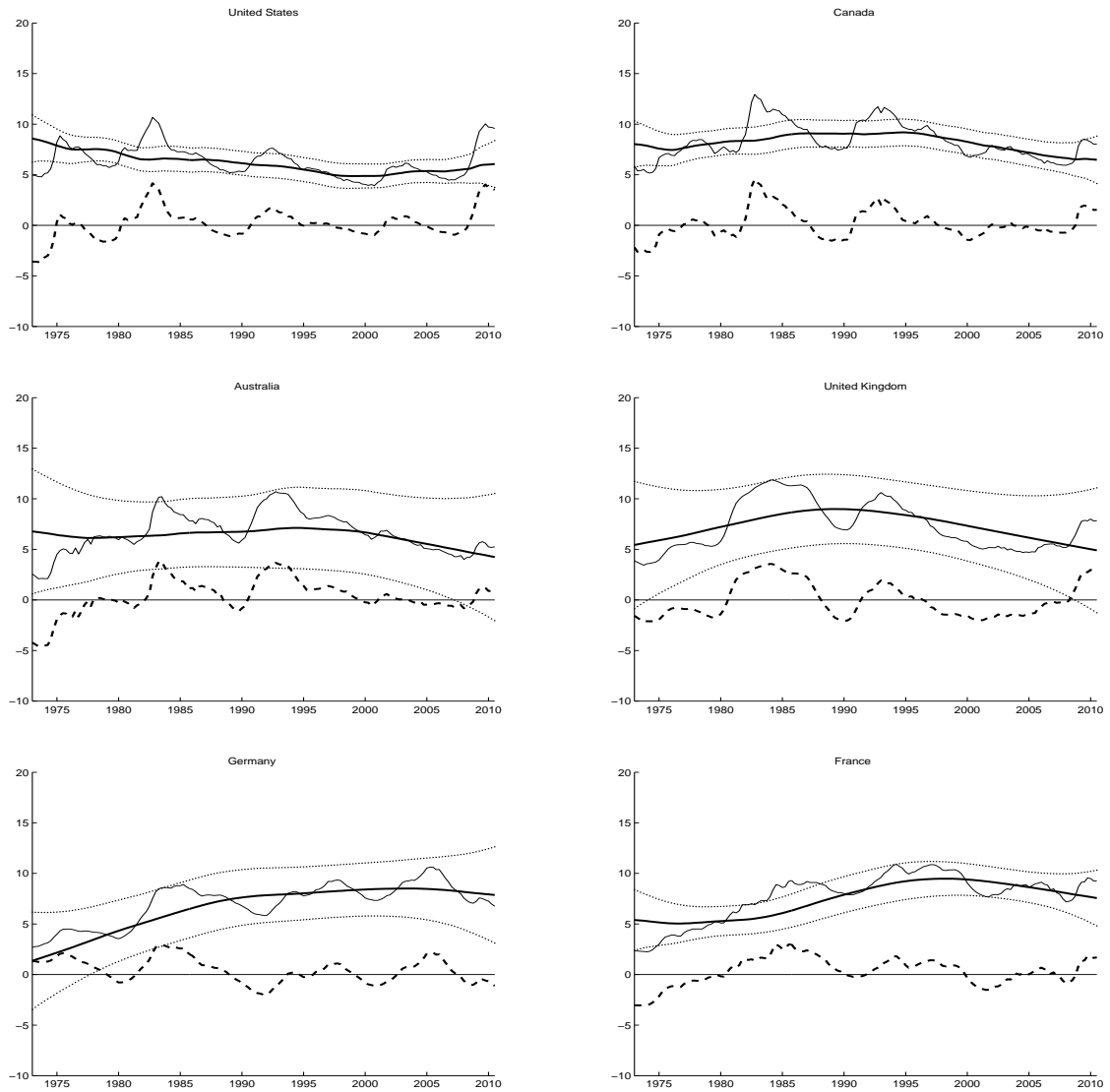
Thin line: inflation rate (right axis); bold line: tv-shift; dotted lines: 95%-confidence interval.

Figure 4: Trend-cycle and Nairu-gap decompositions in the DTB model



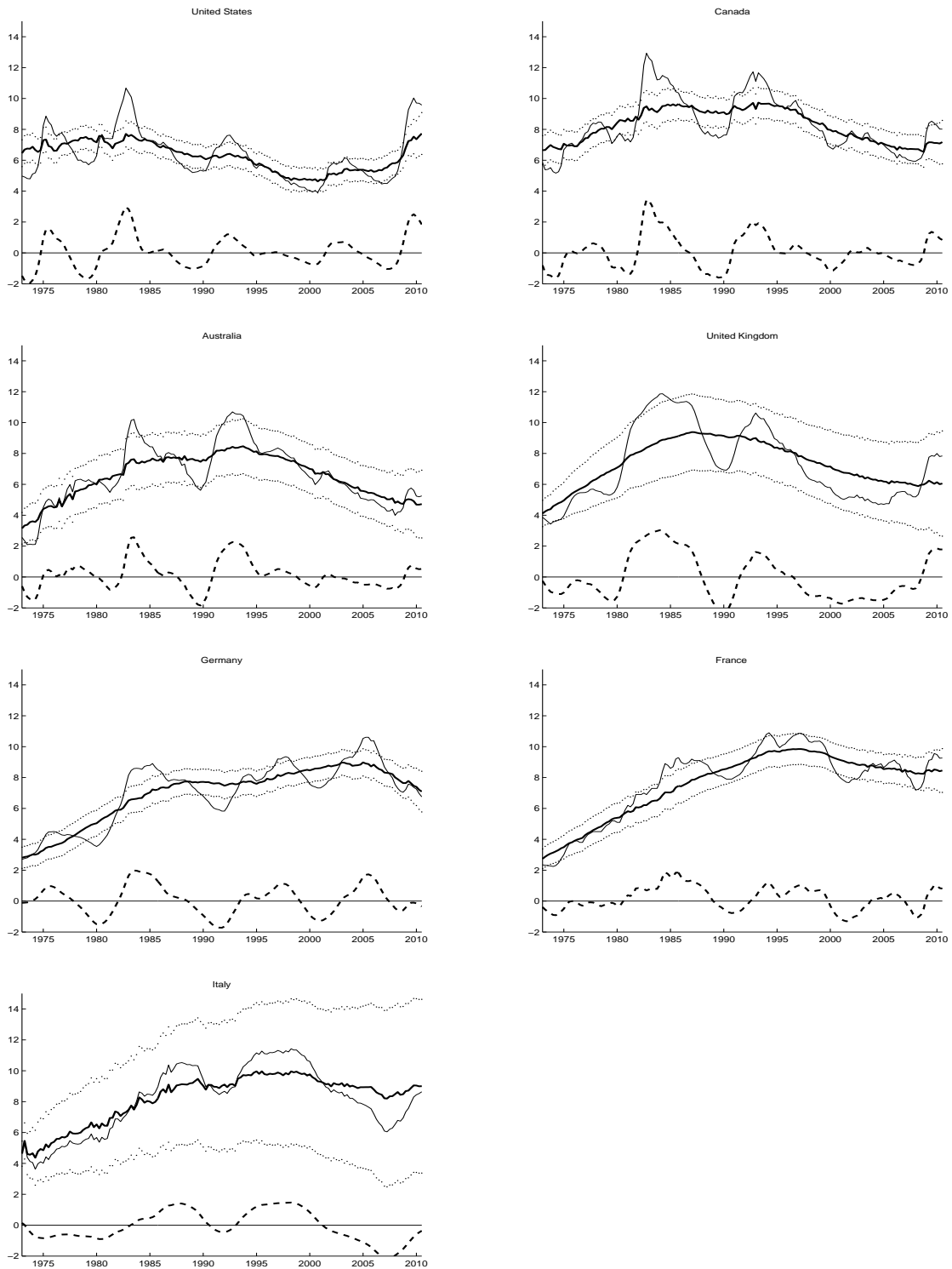
Thin line: unemployment trend; bold line: tv-Nairu; dashed thin line: unemployment cycle; dashed bold line: unemployment gap.

Figure 5: Smoothed estimates of the tv-Nairu in the STU model



Thin line: unemployment rate; bold line: tv-Nairu; dotted lines: 95%-confidence interval; dashed line: unemployment gap.

Figure 6: Smoothed estimates of the tv-Nairu in the STB model



Thin line: unemployment rate; bold line: tv-Nairu; dotted lines: 95%-confidence interval; dashed line: unemployment gap.

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