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Macroeconomic forecasting during the Great Recession: The return of non-linearity?

Laurent Ferrara¹

MASSIMILIANO MARCELLINO²

MATTEO MOGLIANI³

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¹Banque de France, DGEI - DERIE - International Macroeconomics Division

²European University Institute, Bocconi University and CEPR

 $^{^3{\}rm Banque}$ de France, DGEI - DCPM - Business Conditions and Macroeconomic Forecasting Division. email: matteo.mogliani@banque-france.fr

Abstract: The debate on the forecasting ability in economics of non-linear models has a long history, and the $Great\ Recession$ provides us with an opportunity for a re-assessment of the forecasting performance of several classes of non-linear models, widely used in applied macroeconomic research. In this paper, we carry out an extensive analysis over a large quarterly database consisting of major real, nominal and financial variables for a large panel of OECD member countries. It turns out that, on average, non-linear models do not outperform standard linear specifications, even during the $Great\ Recession\$ period. In spite of this result, non-linear models enable to improve forecast accuracy in almost 40% of cases. Especially some countries and/or variables appear to be more adapted to non-linear forecasting.

Keywords: Forecasting, Non-linear models, Great Recession.

JEL classification: C22, C53, E37.

Résumé: La capacité de prévision des modèles non-linéaires en économie fait débat parmi les prévisionnistes depuis de nombreuses années. La récession de 2008-09, qui a touché de nombreux pays à travers le monde et qui est connue dans la littérature sous le nom de *Grande Récession*, fournit une opportunité d'évaluer à nouveau la performance en prévision d'une large classe de modèles non-linéaires, largement utilisés en macroéconomie appliquée. Dans cet article, nous menons une analyse à grande échelle sur une base de données trimestrielles recouvrant les principales variables réelles, nominales et financières pour plusieurs pays de l'OCDE. Les résultats obtenus montrent que, en moyenne, les modèles non-linéaires ne permettent pas d'améliorer les prévisions issues de modèles linéaires standards, y compris pendant la période de la *Grande Récession*. Toutefois, dans environ 40% des cas, il existe un gain en prévision pour les modèles non-linéaires. En particulier, certains pays et /ou variables semblent être plus adaptés à la prévision non-linéaire.

Mots-Clés: Prévisions, Modèles non-linéaires, Grande Récession

Classification JEL: C22, C53, E37.

1 Introduction

Non-linear models have been extensively used in empirical economics, especially in the attempt of reproducing the stylized facts of the business cycle. The underlying idea is that the dynamics of a variable can be influenced by the past behaviour of the variable itself or of other economic variables, rather than being stable overtime. Markov-Switching Auto-Regressive (MSAR) models, popularized by Hamilton (1989), have been widely used in the literature. Other non-linear models based on observable transition variables, such as Threshold Auto-Regressive (TAR) models introduced by Tong and Lim (1980) or Smooth Transition Auto-Regressive (STAR) models introduced by Teräsvirta and Anderson (1992), have been also largely implemented in macroeconomic modelling and have proved their usefulness to reproduce stylized facts. Time-Varying Auto-Regressive (TVAR) models introduce more flexibility in modelling by allowing for continuous parameter evolution overtime (see Nicholls and Pagan, 1985, for a review).⁴

In spite of the large "in-sample" empirical evidence, the forecasting ability of non-linear models is less clear-cut. From the empirical literature, it turns out that the improvements of non-linear models over linear ones, or even over random walk processes, are rather mixed. For example, regarding threshold models, Tiao and Tsay (1994) show evidence of forecasting improvements for the US GDP, while Clements and Smith (2001) show that SETAR forecasts are not more accurate than those obtained with a random walk process for a set of exchange rates series. Teräsvirta and Anderson (1992), Sarantis (1999) or Boero and Marrocu (2002) compare forecasting performances of STAR models relative to linear alternatives for industrial production, nominal, and real exchange rates, respectively. On the basis of the MSFE criterion, results reported by these authors are quite mixed and they do not show significant forecasting improvements. Stock and Watson (1999) consider more than 200 real and financial variables for the US and show that LSTAR methods rarely improve forecasts, compared to a linear AR model. However, they point out that nonlinear methods have more success for wage, employment and exchange rate, especially for longer forecasting horizons. At the same time, Kilian and Taylor (2003) and Teräsvirta, van Dijk, and Medeiros (2005) find evidence of improvement in forecast accuracy when using smooth transition models for a set a monthly variables. Markov-Switching approaches have been also considered in the literature. However, Montgomery et al. (1998), Sarantis (1999) or Clements and Krolzig (1998) showed that Markov-Switching methods perform poorly by comparison with linear and other non-linear models. Last, time-varying linear models and other non-linear specifications have been implemented by Marcellino (2005) for a large data set of almost 500 variables for euro area countries. While in terms of average performance across variables and horizons it is not possible to beat linear benchmarks, for a subset of variables nonlinear specifications do perform better, and TVAR models perform particularly well (see also Stock and Watson, 1996).

⁴Other classes of non-linear models consider changes in the conditional variance of the variables (see Cogley and Sargent, 2005, and Clark, 2011, for a VAR approach with stochastic volatility). Since in the present paper we focus on quarterly macroeconomic variables, we assume that the conditional variances are constant, possibly after allowing for changes in the conditional means.

All in all, there is no clear consensus in the empirical literature on the forecasting performance of non-linear models compared to linear challengers. In fact, it turns out that results strongly depend on various factors, such as the estimation and forecasting periods, the type of model implemented, the macroeconomic variables considered or the forecasting horizon. In addition, only few studies put a comparative forecasting exercise with non-linear models into an international perspective. We refer, for example, to Marcellino (2005), Rapach and Wohar (2006) or Teräsvirta, van Dijk, and Medeiros (2005) for a comparative analysis over G7 and euro area countries.

The aim of this paper is to assess whether non-linear autoregressive models would have been useful to predict the latest business cycle episodes across main industrialized economies, in particular the strong negative growth rates in major macroeconomic and financial variables that occurred during the 2008-2009 recession, sometimes referred to as the *Great Recession*, in many countries. We focus on univariate models mainly because multivariate non-linear specifications, such as non-linear VAR or exogenous-augmented non-linear autoregressions, can suffer from heavy over-parametrization and display biased forecasts. More specifically, we evaluate the forecasting performance of a set of univariate non-linear models (STAR, TAR, TVAR, MS) over the period ranging from 2004q1 to 2009q4, also split as 2004q1-2006q4 and 2007q1-2009q4, through an extensive analysis over a large quarterly database consisting of major real and financial variables for main OECD member and non-member economies, covering overall 19 countries and 23 variables for each of them. In addition, results are assessed through the rolling scheme, as well as for various forecast horizons, and evaluated through a battery of usual loss functions (mean squared forecast error) and tests for predictive ability.

Our main findings are as follows. Non-linear specifications outperform the linear benchmark for about 40% of the macroeconomic series analyzed over both the pre-crisis and the recession periods, with a slight improvement during the first window. The outcome for both the short- and medium-term forecast horizons appears quite similar. However, forecasts for a few specific variables (interest rates, prices) and countries (Japan, overall) show a systematic improvement under non-linear specifications. Time-varying specifications outperform the other non-linear models, suggesting that flexible parametrization based on evolutionary dynamics of autoregressive coefficients can more efficiently deal with large macroeconomic shocks.

Summing up, our aggregate results are, to our knowledge, the first cross-variables and cross-countries evidence of moderate predictive ability of non-linear models. Our findings appear, by and large, in line with those reported in the existing disaggregated literature. However, the novelty of our analysis relies on the implementation a large-scale study, suggesting, on the one hand, the absence of a systematic forecast gain for non-linear models and, on the other hand, interesting predictive patterns for a bunch of models, countries and macroeconomic series.

The paper is structured as follows. Forecasting models are presented in Section 2, while the design of the experiment is detailed in Section 3. Empirical results are discussed in section 4. Section 5 summarizes and concludes.

2 Forecasting models

In this section, we provide a brief description of the models that are considered in our analysis. We assume that, for a country of interest, we observe a given variable y_t , for t = 1, ..., T. Thus, for a given horizon h, the objective is to evaluate y_{t+h} through the h-step-ahead forecast given by $\hat{y}_{t+h} = E(y_{t+h}|\mathfrak{F}_t)$, where \mathfrak{F}_t contains the history of the process up to time t and the shape of the conditional expectation depends on the model under review.

We consider two horizons of interest, namely h=1 and h=4. In the literature on forecasting with linear models, the issue of multi-step forecasting, that is h>1, is generally tackled using two alternative approaches. The most common choice is to iterate one-step-ahead forecasts, referred to as iterated forecasting. The second possibility, referred to as direct forecasting, is to directly model the relationship between y_{t+h} and past values $y_t, y_{t-1}, y_{t-2} \dots$ and then to forecast \hat{y}_{t+h} based on this modeling. The idea lying behind the direct forecasting is that the potential impact of specification errors on the one-step-ahead model can be reduced by using the same loss function for estimation as for forecasting. In a general framework, it is not clear whether direct multi-step forecasting can strongly improve forecast accuracy (see Marcellino, Stock, and Watson, 2006, or Chevillon, 2007, for a review). Within our non-linear framework with many variables, direct forecasting has the additional great advantage of simplicity, since the counterpart of the iterated approach would require complex procedures based on numerical integration (see for example Teräsvirta, van Dijk, and Medeiros, 2005, for forecasting with non-linear models). To get a fair comparison, benchmark predictions are also obtained via the direct procedure.

We focus on three types of non-linear models often used in the literature. Fist, we consider piecewise linear models whose transition from one state to the other is governed by an observable variable. In the description below, we assume that only two regimes exist, but the extension to more regimes is straightforward. The most general case is the STAR model, for which the shape is parameterized according to either a logistic (LSTAR) or an exponential (ESTAR) function. When the transition goes directly from one state to the other, the model is referred to as a threshold model (TAR). As we do not include additional explanatory variables in our models, we assume that the transition variable is a past value of the variable under review. Second, we consider a piece-wise linear model for which the transition is governed by an unobservable variable that is supposed to follow in turn a first order Markov chain with two regimes (MSAR). Last, we also integrate in our comparative analysis a simple AR model for which parameters are supposed to be time-varying (TVAR). The model description below starts with the presentation of the standard AR model that will be used as a benchmark.

2.1 Linear AR model

The Auto-Regressive model is given by the following equation, for a given h > 0 and for all t,

$$y_{t+h} = \alpha + \beta x_t + \varepsilon_{t+h},\tag{1}$$

where $x_t = (y_t, y_{t-1}, \dots, y_{t-p+1})'$, $\{\varepsilon_t\}_{t=1}^T$ is supposed to have finite variance σ^2 , α is a constant and β is a p-vector of parameters. Forecasts obtained from equation (1) have performed well when compared to alternative and more sophisticated univariate and multivariate models, as documented in the literature (Meese and Geweke, 1984; Marcellino et al., 2006). In this respect, we define the AR class as the benchmark model in our forecasting exercise.

Forecasts with the linear AR model are carried out in the following way. First, we select the appropriate autoregressive degree p^* over the range $p = (1, ..., p_{\text{max}})$, with $p_{\text{max}} = 6$, where p^* optimizes the BIC criterion. Second, we estimate the AR model (1) by using the ordinary least-squares (OLS) method. Finally, the h-step direct forecast $\hat{y}_{t+h} = E(y_{t+h}|\mathfrak{F}_t)$ is obtained as

$$\hat{y}_{t+h} = \hat{\alpha} + \hat{\beta}x_t, \tag{2}$$

where $x_t = (y_t, y_{t-1}, \dots, y_{t-p^*+1})'$.

2.2 Smooth-Transition AR (STAR) model

We consider the following smooth-transition model given by, for all t,

$$y_{t+h} = (\alpha_1 + \beta_1 x_t)(1 - G(z_{t-d}; \gamma, c)) + (\alpha_2 + \beta_2 x_t)G(z_{t-d}; \gamma, c) + \varepsilon_{t+h}, \tag{3}$$

where $(\alpha_1, \beta_1)'$ and $(\alpha_2, \beta_2)'$ are p + 1-vectors of parameters, $x_t = (y_t, y_{t-1}, \dots, y_{t-p+1})'$, $\{\varepsilon_t\}_{t=1}^T$ is supposed to have finite variance σ^2 , and $G(\cdot)$ is the smooth-transition function described either by

$$G(z_{t-d}; \gamma, c) = \left[1 + \exp\left(\frac{\gamma}{\sigma_{z_{t-d}}}(z_{t-d} - c)\right)\right]^{-1}$$

$$\tag{4}$$

or by

$$G(z_{t-d}; \gamma, c) = 1 - \exp\left(\frac{\gamma}{\sigma_{z_{t-d}}} (z_{t-d} - c)^2\right).$$

$$(5)$$

The model embedding the function in (4) is referred to as the logistic smooth-transition autoregression (LSTAR), while the model described using function (5) is referred to as the exponential smooth-transition autoregression (ESTAR). In both models, γ is the smoothing parameter controlling for the shape of regime changes, z_{t-d} is the transition variable, where d is the delay parameter, $\sigma_{z_{t-d}}$ is the standard-deviation of the transition variable used as scaling factor and estimated through empirical moments (Granger and Teräsvirta, 1993), and c is the threshold parameter.

Specification and parameter estimation for STAR models are carried out in the following way. First, we select the appropriate autoregressive degree p^* over the range $p = (1, ..., p_{\text{max}})$, with $p_{\text{max}} = 6$, the delay parameter d^* in the set $d = (0, ..., d_{\text{max}})$, with $d_{\text{max}} = 3$, and the transition variable $(z_{t-d})_t$ as the optimizers of the BIC criterion. The transition variable is selected among a set of pre-determined indicators: $z_{t-d} = y_{t-d}$, which represents the past level or the past growth rate of the dependent variable (depending on the transformations for unit-roots applied to each series), and $z_{t-d} = y_{t-d} - y_{t-d-j}$, which represents the past growth rate or the past acceleration rate of the dependent variable, with

 $j = (1, ..., j_{max})$ and $j_{max} = 2$. It is worth noticing that d is not restricted to be lower than the autoregressive order p.

Second, we estimate the STAR models (3)-(4) and (3)-(5) by constrained quasi-maximum likelihood (NLLS).⁵ Following Van Dijk, Teräsvirta, and Franses (2002), we initialize the optimization algorithm with a set of non-linear parameters obtained by a two-dimensional grid-search over (γ, c) and minimizing the sum of squares function

$$Q(\gamma, c) = \sum_{t=1}^{T-h} \left\{ y_{t+h} - \alpha(\gamma, c)' - \beta(\gamma, c)' x_t(\gamma, c) \right\}^2,$$

where $x_t(\gamma, c) = (x'_t(1 - G(z_{t-d}; \gamma, c)), x'_tG(z_{t-d}; \gamma, c))'$, $\alpha(\gamma, c) = (\tilde{\alpha}_1(\gamma, c), \tilde{\alpha}_2(\gamma, c))'$, with $\tilde{\alpha}_1(\gamma, c) = \alpha_1(\gamma, c)(1 - G(z_{t-d}; \gamma, c))$ and $\tilde{\alpha}_2(\gamma, c) = \alpha_2(\gamma, c)G(z_{t-d}; \gamma, c)$, and $\beta(\gamma, c) = (\beta_1(\gamma, c), \beta_2(\gamma, c))'$ denote the conditional vector of regressors and the conditional vector of parameters, respectively.⁶ Once initial parameter values are selected, estimation of STAR models is obtained by concentrating again the sum of squares function $Q(\gamma, c)$, *i.e.*, by concentrating the maximum-likelihood function on γ and c (using Newton-Raphson or BFGS), and estimating the remaining parameters through OLS (Leybourne et al., 1998).

Finally, the h-step direct forecast (\hat{y}_{t+h}) is obtained as

$$\hat{y}_{t+h} = (\hat{\alpha}_1 + \hat{\beta}_1 x_t)(1 - G(z_{t-d^*}; \hat{\gamma}, \hat{c})) + (\hat{\alpha}_2 + \hat{\beta}_2 x_t)G(z_{t-d^*}; \hat{\gamma}, \hat{c}). \tag{6}$$

2.3 Threshold AR model

Consider the following threshold model, for all t,

$$y_{t+h} = (\alpha_1 + \beta_1 x_t)(1 - I(z_{t-d}; c)) + (\alpha_2 + \beta_2 x_t)I(z_{t-d}; c) + \varepsilon_{t+h}, \tag{7}$$

where $x_t = (y_t, y_{t-1}, \dots, y_{t-p+1})', \{\varepsilon_t\}_{t=1}^T$ is supposed to have finite variance σ^2 and

$$I(z_{t-d};c) = \mathbb{1}(z_{t-d} \le c)$$

is the indicator function, where z_{t-d} is the transition variable, d is the delay parameter and c is the threshold. Specification and parameter estimation for the TAR model are carried out in the following way. First, we select the appropriate autoregressive degree p^* over $p = (1, ..., p_{\text{max}})$, with $p_{\text{max}} = 6$, and the delay parameter d^* over $d = (0, ..., d_{\text{max}})$, with $d_{\text{max}} = 3$, as the optimizers of the BIC criterion. Note that d is not restricted to be lower than the autoregressive order p. As for the STAR model, the transition variable is selected among a set of pre-determined indicators (see Section 2.2).

⁵Non-linear constraints are imposed on smooth-transition parameters ($\gamma > 0$, and $c \in [z_{t-d}|_{t=T*2.5\%}, z_{t-d}|_{t=T*97.5\%}]$) to ensure economic interpretability of the estimated model.

⁶The algorithm is designed to seek optimal initial values for the quasi-maximum likelihood estimation by solving $(\hat{\gamma}, \hat{c}) = \operatorname{argmin} Q(\gamma, c)$ through a two-dimensional grid-search over γ and c, spanning from $\gamma_{\min} = 0$ to $\gamma_{\max} = 20$, with a step of 0.2, and from $c_{\min} = z_{t-d}|_{t=T*2.5\%}$ to $c_{\max} = z_{t-d}|_{t=T*97.5\%}$, with a step of $(c_{\max} - c_{\min})/50$, respectively.

It is nevertheless worth noticing that when $(z_{t-d} = y_{t-d})$, the TAR model reduces to a self-exciting threshold model (SETAR).

Second, we estimate the TAR model (7) by concentrated least-squares. Following Hansen (1999), we estimate the optimal threshold parameter through a one-dimensional search over c and minimizing the sum of squares function

$$Q(c) = \sum_{t=1}^{T-h} (y_{t+h} - \alpha(c)' - \beta(c)' x_t(c))^2,$$

where $x_t(c) = (x'_t(1 - I(z_{t-d}; c)), x'_tI(z_{t-d}; c))'$, $\alpha(c) = (\tilde{\alpha}_1(c), \tilde{\alpha}_2(c))'$, with $\tilde{\alpha}_1(c) = \alpha_1(c)(1 - I(z_{t-d}; c))$ and $\tilde{\alpha}_2(c) = \alpha_2(c)I(z_{t-d}; c)$, and $\beta(c) = (\beta_1(c), \beta_2(c))'$ denote the conditional vector of regressors and the conditional vector of parameters, respectively. Once the optimal threshold parameter is selected, estimation of the TAR model is obtained by OLS.

Finally, the h-step direct forecast (\hat{y}_{t+h}) is obtained as

$$\hat{y}_{t+h} = (\hat{\alpha}_1 + \hat{\beta}_1 x_t)(1 - I(z_{t-d^*}; \hat{c})) + (\hat{\alpha}_2 + \hat{\beta}_2 x_t)I(z_{t-d^*}; \hat{c}). \tag{8}$$

2.4 Time-varying AR model

We consider the following time-varying (TVAR hereafter) parameter model, for all t,

$$y_{t+h} = \alpha_t + \beta_t x_t + \varepsilon_{t+h},\tag{9}$$

where $x_t = (y_t, y_{t-1}, \dots, y_{t-p+1})'$, $\{\varepsilon_t\}_{t=1}^T$ is supposed to have finite variance σ^2 , and $\phi_t = (\alpha_t, \beta_t)'$ is the p+1-vector of time-varying parameters, which are allowed to evolve according to the following multivariate random-walk process:

$$\phi_t = \phi_{t-1} + \epsilon_t,$$

where ϵ_t are iid $N(0, \lambda^2 \sigma^2 Q)$, with $Q = (E(z_t'z_t))^{-1}$ being the covariance matrix (Nyblom, 1989), $z_t = (1, x_t)'$ and λ^2 the ratio of the variance of each parameter disturbance to the variance of the regression error (ε_{t+h}) .

Forecasts with the time-varying AR model are carried out in the following way. First, we select the appropriate autoregressive degree p^* over $p = (1, ..., p_{\text{max}})$ and variance-ratio parameter λ^* over $\lambda = (0.0025, ..., \lambda_{\text{max}})$, with $p_{\text{max}} = 6$ and $\lambda_{\text{max}} = 0.02$, as the optimizers of the BIC criterion. Second, we estimate the time-varying AR model (9) cast into its state-space form by using the Kalman filter. Finally, the h-step direct forecast (\hat{y}_{t+h}) is obtained as

$$\hat{y}_{t+h} = \hat{\alpha}_t + \hat{\beta}_t x_t. \tag{10}$$

⁷The algorithm is designed to solve $\hat{c}= \operatorname{argmin} Q(c)$ through a one-dimensional search, spanning from $c_{\min}=y_{t-d}|_{t=T*10\%}$ to $c_{\max}=y_{t-d}|_{t=T*90\%}$, with a step of $(c_{\max}-c_{\min})/100$.

2.5 Markov-Switching AR model

We consider the following Markov-switching (MSAR hereafter) parameter model, for all t,

$$y_{t+h} = \alpha_{s_{t+h}} + \beta_{s_{t+h}} x_t + \varepsilon_{t+h}, \tag{11}$$

where $x_t = (y_t, y_{t-1}, \dots, y_{t-p+1})'$, $\{\varepsilon_t\}_{t=1}^T$ is supposed to have finite variance σ^2 , and s_t is an unobservable two-state random variable, such as $s_t \in \{1, 2\}$, assumed to follow a strictly stationary, temporally homogeneous, first-order Markov chain with transition probabilities $p_{ij} = P\{s_{t+h} = j | s_t = i\}$, for i, j = 1, 2 (Hamilton, 1989).

Forecasts with the Markov-switching AR model are carried out in the following way. First, we select the appropriate autoregressive degree p^* such that $p = (1, ..., p_{\text{max}})$, with $p_{\text{max}} = 6$, as the optimizer of the BIC criterion. Second, we estimate the MSAR model (11) by the expectation-maximization (EM) algorithm. We initialize the algorithm with a set of parameter values obtained by estimating the model through OLS and adding/subtracting to the estimated values their respective standard errors. In addition, we get estimates for the transition matrix \mathbf{P} of the two-state Markov chain, the smoothed-probabilities vector $\hat{\xi}_{t|t}$ and its forecast value at date t+h, i.e., $\hat{\xi}_{t+h|t} = \mathbf{P} \cdot \hat{\xi}_{t|t}$, with $\hat{\xi}_{t|t} = P\{s_t = j|Y_t;\theta\}$ for j = 1, 2. Finally, the h-step direct forecast $\hat{y}_{t+h} = E(y_{t+h}|s_{t+h} = j, Y_t;\theta)$ is obtained as

$$\hat{y}_{t+h} = (\hat{\alpha}_{s_{t+h}} + \hat{\beta}_{s_{t+h}} x_t) \cdot \hat{\xi}_{t+h|t}. \tag{12}$$

3 Forecasting experiment

In this section the database is first described, then we detail the various steps of the forecasting experiment.

3.1 The database

In this experiment, we use quarterly data for a large sample of real and financial series (23 variables) of 18 OECD member economies plus one non-member country (South Africa). The dataset generally spans from 1970Q1 to 2009Q4, although a few series are not available for the complete sample size. This leads to a total amount of 383 usable series $(23 \times 19 = 437, \text{ minus } 54 \text{ missing variables})$. In order to keep the dataset as homogeneous as possible, the main sources for our selected series are the OECD Economic Outlook and the OECD Main Economic Indicators databases. Tables 1 and 2 summarize the main characteristics of our dataset (availability of data, first observation available, treatment for expected non-stationarity, full description and sources). Regarding data stationarity, it turns out that business surveys (industrial and consumer confidence indexes), rates (unemployment rate, capacity utilization rate, long-term and short-term interest rates) and variables expressed as percentage of GDP (Government primary balance and Government net lending) are found to be stationary, by

⁸It is worth noticing that the conditional variance does not depend on the hidden state s_{t+h} .

Table 1: Main sources

Series	Code	Description	Source
GDP	GDPV	Gross domestic product, volume, market prices	OECD EO^a
Investment	ITV	Gross fixed capital formation, total, volume	OECD EO^a
Housing	IHV	Gross fixed capital formation, housing, volume	OECD EO
Imports	MGSV	Imports of goods and services, volume, national accounts basis	OECD EO^a
Exports	XGSV	Exports of goods and services, volume, national accounts basis	OECD EO^a
Consumption	CPV	Private final consumption expenditure, volume	OECD EO^a
Unemployment	UNR	Unemployment rate	OECD EO^a
Hours worked	HRS	Hours worked per employee, total economy	OECD EO
Inflation - CPI	CPI	Inflation, Consumer price index	OECD $EO^{a,*}$
Inflation - GDP	PGDP	Inflation, GDP deflator	OECD EO^a
Short-term rate	IRS	Short-term interest rate on government bonds	OECD EO^a
Long-term rate	IRL	Long-term interest rate on government bonds	OECD EO^a
NER	NXCH	Nominal exchange rate, USD per National currency	OECD EO
NEER	NEXCH	Nominal effective exchange rate, chain-linked, overall weights	OECD EO
REER	REXCH	Real effective exchange rate, CPI Based	OECD MEI
Gov. primary balance	NLGXQ	Government primary balance, as a percentage of GDP	OECD EO
Gov. net lending	NLGQ	Government net lending, as a percentage of GDP	OECD EO
Gov. consumption	CGQ	Government final consumption expenditure, volume	OECD EO^a
Industrial production	IPI	Industrial production index, total industry excluding construction	OECD MEI
Capacity utilization	CUR	Capacity utilization rate, manufacturing	OECD MEI**
Industrial confidence	ICI	Industrial confidence indicator, manufacturing	OECD MEI
Consumer confidence	CCI	Consumer confidence indicator	OECD MEI
Stock market index	SMI	Stock market share prices index, all shares or main stock index	OECD MEI

Note: EO = Economic Outlook, MEI = Main Economic Indicators. ^a Bundesbank for Germany, *IMF-IFS data for the United Kingdom, **FED database for the USA.

using standard stationarity tests. Other variables are taken in log-differences by comparison with the previous quarter.

3.2 Forecasting design

The forecasting period of interest is the *Great Recession* period that has affected simultaneously the main OECD countries over the years 2007-2009. We chose 2004q1 - 2009q4 as the complete forecasting period (24 quarters). This period can be fruitfully split into 2 balanced sub-periods of interest, namely a pre-crisis sub-period from 2004q1 to 2006q4 and a crisis sub-period from 2007q1 to 2009q4, allowing thus an interesting temporal decomposition for analyzing the results.

We assume that, for a country of interest, we observe a given variable y_t , for t = 1, ..., T, where date T corresponds to 2009q4. As described above, we split this sample into two sub-samples: a learning sub-sample from t = 1 to $t = t_0$, t_0 corresponding to 2003q4, and a forecasting period for $t = t_0 + 1$ to t = T. Thus for both horizons h = 1 and h = 4, the objective is to compare the actual values y_{t+h} , for $t = t_0$ to t = T - h, with the h-step-ahead direct forecasts given by $\hat{y}_{t+h} = E(y_{t+h}|\mathfrak{F}_t)$, where the shape of the conditional expectation depends on the model under review.

To evaluate the empirical performance of our set of non-linear models, we compute rolling forecasts, where parameter estimation is based on a moving window of the data, discarding thus older observations. More precisely, date t = 1 corresponds to 1970Q1, whenever possible, and forecasts are computed with a forecast horizon of h = 1, 4 (i.e., \hat{y}_{t_0+h}). When an additional data point is added after the first forecast,

Table 2: The dataset

Treatment	Δlog	Level	Δlog	Δlog	Δlog	Level	Level	Δlog	Δlog	Δlog	Level	Level	Δlog	Δlog	Level	Level	Level	Δlog					
$I(\cdot)$	I(1)	I(1)	1(1)	I(1)	I(1)	I(1)	(0)1	I(1)	I(1)	I(1)	(0)1	(0)1	I(1)	I(1)	I(1)	(0)1	(0)1	1(1)	I(1)	(0)1	(0)1	(0)1	I(1)
$_{ m USA}$	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1		70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	78Q1	70Q1
SWE	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	82Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1				70Q1
POR	70Q1	70Q1		70Q1	70Q1	70Q1	70Q1		70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1			70Q1	70Q1	77Q1	87Q1	$86Q_{2}$	88Q1
POL	90Q1	90Q1		90Q1	90Q1	90Q1	92Q2	93Q1	89Q1	90Q1	$91Q_{3}$		91Q1	85Q1	86Q1			90Q1	85Q1	$92Q_2$			$91Q_{3}$
NZL	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	74Q1	70Q1	70Q1	70Q1	70Q1			70Q1	77Q2		70Q1	88Q2	70Q1
NOR	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	79Q1	70Q1	70Q1	70Q1	70Q1			70Q1	70Q1		87Q1		86Q1
NLD	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	80Q1	70Q1	70Q1	70Q1	71Q4	85Q1	73Q1	70Q1
MEX	70Q1	70Q1		70Q1	70Q1	70Q1			70Q1	70Q1	90Q1	90Q1	70Q1	70Q1	70Q1			70Q1	80Q1				70Q1
KOR	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1		70Q1	70Q1	91Q1	82Q3	70Q1	70Q1	70Q1			70Q1	90Q1		91Q3		81Q1
$_{ m SAF}$	70Q1	70Q1		70Q1	70Q1	70Q1			70Q1	71Q1	81Q1	70Q1	70Q1	85Q1	86Q1			70Q1	90Q1	86Q1	74Q3	$90Q_{2}$	70Q1
JPN	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1		74Q2	82Q1	70Q1
ITA	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	70Q1	71Q1	70Q1	70Q1	70Q1	70Q1			70Q1	70Q1	70Q1	85Q1	73Q1	70Q1
	70Q1															70Q1	70Q1	70Q1	70Q1	85Q1	85Q1	74Q1	70Q1
	70Q1 '															•			70Q1 ,				
	70Q1																						85Q1
	70Q1	•	•	•	•	•	•		•	•	•	•	•	•	•			•	70Q1	•			
	70Q1 7															0Q1					ω		70Q1 7
	70Q1 - 7															7			70Q1 7		5Q1	w	1-
	70Q1 7	•		•	•	•	•			•	•	•	•	•	•	.0Q1					70Q1 8		~
\vdash	GDPV 7																				-		

Note: In the same order, countries are Australia, Belgium, Canada, Germany, Spain, France, UK, Italy, Japan, South Africa, South Korea, Mexico, The Netherlands, Norway, New Zealand, Poland, Portugal, Sweden and USA. Figures denote the date of the first available observation (the dataset spanning up to 2009Q4).

the first period of the initial estimation sample is also deleted (data indexed $2, \ldots, t_0 + 1$), then the model is re-estimated, forecasts are computed (i.e., \hat{y}_{t_0+1+h}) and so on until the last available data point. This leads to a fixed rolling window of at least 44 quarters and at most 135 quarters, depending on the variable and the country. The rolling forecasting approach might be preferable to a recursive scheme if some sort of structural change occurs within the sample (Giacomini and White, 2006). A comparison with the recursive approach is also undertaken, since the latter could provide more efficient estimations of the non-linear models, if they manage to properly capture parameter time-variation.

It is worth noticing that while the relevant parameters (i.e., α , β , γ , c, depending on the model) are re-estimated at each step of the out-of-sample recursion, the model specification is supposed to be unchanged throughout the forecasting exercise. That is, for each series, we select the best specification (i.e., p^* , d^* , λ^* , depending on the model) by optimizing the BIC criterion over the relevant parameters, using the first estimation sample, i.e., from t = 1 to $t = t_0$. Theoretical support for information criteria-based specification of non-linear models can be found in Kapetanios (2001) and Psaradakis et al. (2009), among others. In principle, the specification could be changed at each step, but in practice this would be computationally very intense, since we consider 383 different variables, 24 pseudo out-of-sample recursions and two forecast horizons.

3.3 Forecast Evaluation

We use several methods to evaluate the forecasts. First, for a given horizon h and a given variable y_t , out-of-sample forecast accuracy of a given model j is measured in terms of root mean squared forecast error (RMSFE):

$$RMSFE_{j}(h) = \sqrt{\frac{1}{T - t_{0} + 1} \sum_{t=t_{0}}^{T - h} (y_{t+h} - \hat{y}_{t+h})^{2}},$$
(13)

where \hat{y}_{t+h} is the forecast value estimated from the model and y_{t+h} is the observed value.

In order to present results in a parsimonious way, we propose to use ratios of RMSFE by comparison with the benchmark AR model. That is, for each non-linear forecast model j such as $j \in \{\text{LSTAR,ESTAR,TVAR,MSAR}\}$, and for a given horizon h, we report the RMSFE $_j(h)$ relative to the linear benchmark model AR, denoted by RMSFE $_{AR}(h)$, as defined by:

$$R_j(h) = \frac{RMSFE_j(h)}{RMSFE_{AR}(h)}.$$
(14)

Thus, a ratio $R_j(h)$ lower than one indicates the non-linear model j outperforms the standard AR model, and conversely.

In addition to the empirical analysis of ratios $R_j(h)$, we compute standard statistical tests to formally assess the predictive ability of non-linear models. We report results for the Diebold and Mariano (1995, DM hereafter) test, which posits the null hypothesis of unconditional equal forecast accuracy

⁹To dispel any doubts, we conducted a small-scale empirical analysis, where we consider a representative and limited number of models, variables and countries. Aggregate results (not reported) appear qualitatively similar to those obtained through our fixed-specification scheme.

between forecasts stemming from a non-linear model j and the benchmark AR model, $E[L(y_{t+h}, f_{AR}) - L(y_{t+h}, f_j)] = 0$, with $L(y_{t+h}, f_j)$ being a loss function (in the present paper, the mean squared forecast error). The DM statistic takes the following form:

$$DM_{j}(h) = \sqrt{\frac{n+1-2\bar{h}+n^{-1}\bar{h}(\bar{h}-1)}{n}} \times \frac{\bar{d}}{\sqrt{\hat{\omega}^{2}}} \sim t_{n-1},$$
 (15)

where n is the number of observations (forecast periods), the first term is the small sample correction proposed by Harvey, Leybourne, and Newbold (1997), \bar{d} is the estimated sample mean from a regression of $L(y_{t+h}, f_{AR}) - L(y_{t+h}, f_j)$ over a constant and $\hat{\omega}^2$ is a consistent HAC estimator of the asymptotic variance ω^2 .¹⁰

Since the forecast exercise is computed through a rolling window scheme, we also report results for the Giacomini and White (2006, GW hereafter) test, which posits the null hypothesis of conditional equal forecast accuracy between a non-linear model j and the AR model, $E[L(y_{t+h}, f_j) - L(y_{t+h}, f_{AR})|\mathfrak{F}_t] = 0$. The GW statistic takes the following form:

$$GW_{j}(h) = n \left(n^{-1} \sum_{t=t_{0}}^{T-h} \zeta_{t} \Delta L_{t_{0},t+h} \right)' \hat{\omega}^{-1} \left(n^{-1} \sum_{t=t_{0}}^{T-h} \zeta_{t} \Delta L_{t_{0},t+h} \right) \sim \chi_{q}^{2}, \tag{16}$$

where ζ_t is the $(q \times 1)$ test function and $\hat{\omega}^2$ is a consistent HACC estimator of the asymptotic variance matrix ω^2 . In our approach, the test function is supposed to be a constant, leading thus to χ_1^2 asymptotic distribution for the GW statistic.¹¹

Then, in addition to the point forecasts assessment through the DM test, we implement the sign forecast test of Pesaran and Timmermann (1992, PT hereafter), which posits the null hypothesis of distributional independency (no predictive power) for y_{t+h} and \hat{y}_{t+h} and takes the following form:

$$PT_{j}(h) = \frac{P_{y_{t+h}\hat{y}_{t+h}} - P_{y_{t+h}\hat{y}_{t+h}}^{\star}}{\sqrt{V(P_{y_{t+h}\hat{y}_{t+h}}) - V(P_{y_{t+h}\hat{y}_{t+h}}^{\star})}} \sim N(0, 1), \tag{17}$$

where $P_{y_{t+h}\hat{y}_{t+h}} = E[\mathbb{1}(y_{t+h} \cdot \hat{y}_{t+h} > 0)]$, with $E[\cdot]$ being the expectation (mean) operator, $P_{y_{t+h}\hat{y}_{t+h}}^{\star} = Pr(y_{t+h} \cdot \hat{y}_{t+h} > 0)$, and $V(P_{y_{t+h}\hat{y}_{t+h}})$ and $V(P_{y_{t+h}\hat{y}_{t+h}}^{\star})$ are their sample variances.¹²

A common practice in the literature on large forecast comparisons is to mimic the behavior of a true forecaster by setting an automatic *insanity filter* (Stock and Watson, 1999). The filter mechanically

 $^{^{10}}$ It is worth noticing that the direct h-step ahead forecast design implies $\bar{h}=1$. Further, in the present paper we use the HACC estimator proposed by Newey and West (1994), with an automatic bandwidth selection $b=(4(T/100)^{2/9})$.

¹¹In the present paper, we follow Giacomini and White (2006) and we use the HACC estimator proposed by Newey and West (1994) with a fixed bandwidth b = h - 1.

¹²The Pesaran and Timmermann (1992) being a sign test, we take first-differences of actual values and predictions, that is $y_{t+h} \equiv y_{t+h} - y_{t+h-1}$ and $\hat{y}_{t+h} \equiv \hat{y}_{t+h} - y_{t+h-1}$. Further, it is worth noticing that the test can produce non-definite results, such as when $Pr(y_{t+h} \cdot \hat{y}_{t+h} > 0) = 0$ and/or the denominator of (17) is equal to 0. In the present work, we do not consider these extreme results, leading to a total number of test statistics available for the analysis potentially lower than the number of forecasted series.

discards each forecast value exceeding, in absolute value, some given threshold and replaces it with some reasonable value. In this way, the logical process dictated by the good-sense of the human forecaster is virtually reproduced and automated. In general, the filter is set to replace forecasts whose variations exceed any change previously observed with a no-change forecast. However, the implementation of such a filter does not seem to be recommended for our study. Indeed, large deviations in forecasts are plausible, as well as supposed to occur, due to the presence of the *Great Recession* episode in our sample. Further, the aim of this paper is to identify forecasting models which show some degree of forecast accuracy during the downturn of the crisis, while we still want to penalize explosive responses of non-linear models, an issue well documented in the literature. Thus, to avoid that *non-sense* forecasts contaminate the analysis, we proceed by trimming our results as follows. We first merge the RMSFE ratios, $R_j(h)$ for all models, and we compute the empirical distribution. Then, we discard the RMSFE values greater than the 95% quantile. These truncated data are finally used for the subsequent analysis.¹³

4 Results

4.1 Aggregate results

Summary descriptive statistics for ratios of RMSFE, $R_i(h)$, are presented for all models, variables and countries in Table 3. Results are presented for each forecast horizon, namely h=1 and h=4, as well as for each forecasting period, namely the complete period 2004q1-2009q4 and the two sub-periods 2004q1-2006q4 and 2007q1-2009q4 (this latter sub-period being referred to as the Great Recession period in the remaining). In addition, the last column (referred to as "Criterion") reports some useful measures enabling to compare results obtained from non-linear models to those from the linear benchmark model. In particular, for the RMSFE ratios, the column "Criterion" reports the percentage of $R_i(h)$ lower than unity. Thus a ratio higher than 50% indicates that non-linear models outperform more frequently linear models. Regarding the corrected DM test, as this test is two-sided, the column "Criterion" reports the percentage of the number of rejections stemming from the right side (the non-linear model outperforms significantly at 5%) by comparison with the number of rejections stemming from the left side (the linear model outperforms significantly at 5%). Thus a value higher than 100% indicates more frequent ratios within the right tail of the DM statistics distribution (non-linear models outperform), and conversely. For the PT sign test, we report the percentage of statistics rejecting at 5% level the null of distributional independency between the actual series and its non-linear predictor. Thus a ratio close to 100% for this test indicates an excellent performance of the non-linear predictor. Finally, for the GW test, we report

Table 3: Results for all models

					h	=1			
	Fcst. Window	N.Obs	Mean	Median	Min	Max	StDev	Skew	Crit. $(\%)$
	2004q1-2009q4	1825	1.043	1.015	0.429	1.591	0.133	1.068	39.78
R(h)	2004q1-2006q4	1825	1.049	1.007	0.347	3.190	0.213	2.879	44.00
	2007q1-2009q4	1825	1.041	1.011	0.377	2.533	0.159	1.909	42.52
	2004q1-2009q4	1825	-0.258	-0.492	-5.048	7.639	1.270	0.563	67.50
DM(h)	2004q1-2006q4	1825	-0.134	-0.276	-5.649	6.530	1.519	0.341	84.46
	2007q1-2009q4	1825	-0.205	-0.367	-5.441	5.472	1.217	0.126	63.69
	2004q1-2009q4	1605	1.810	1.910	-2.396	4.904	1.110	-0.468	58.50
PT(h)	2004q1-2006q4	1605	1.407	1.560	-2.400	3.479	1.130	-0.551	40.06
	2007q1-2009q4	1605	1.209	1.205	-2.333	3.479	0.983	-0.299	27.41
	2004q1-2009q4	1825	1.367	0.871	0.000	14.06	1.644	2.706	7.340
GW(h)	2004q1-2006q4	1825	1.410	0.919	0.000	8.861	1.532	1.674	7.890
	2007q1-2009q4	1825	1.154	0.824	0.000	9.129	1.203	1.735	3.840
						=4			
	Fcst Window	N.Obs	Mean	Median	Min	Max	StDev	Skew	Crit. (%)
	2004q1-2009q4	1816	1.025	1.011	0.376	1.427	0.111	-0.110	38.22
R(h)	2004q1-2006q4	1816	1.052	1.013	0.173	9.403	0.305	12.04	42.68
	2007q1-2009q4	1816	1.024	1.007	0.429	2.314	0.129	1.505	41.69
	2004q1-2009q4	1816	-0.243	-0.470	-5.458	13.17	1.555	1.459	59.93
DM(h)	2004q1-2006q4	1816	-0.071	-0.352	-10.34	15.38	2.058	1.061	93.73
	2007q1-2009q4	1816	-0.225	-0.340	-14.08	7.856	1.522	-0.315	62.55
D. (1)	2004q1-2009q4	1655	1.344	1.636	-3.468	4.904	1.416	-0.771	49.06
PT(h)	2004q1-2006q4	1655	1.269	1.560	-3.479	3.479	1.159	-0.646	36.44
	2007q1-2009q4	1655	0.842	1.004	-2.818	3.479	1.174	-0.639	19.70
	2004 4 2005 :	1010	2 5 2 5	4 404	0.000	100.6		4400	1001
Q****/* `	2004q1-2009q4	1816	2.585	1.134	0.000	180.9	7.464	14.82	16.24
GW(h)	2004q1-2006q4	1816	5.012	1.602	0.000	273.2	13.41	10.04	28.30
	2007q1-2009q4	1816	2.928	1.349	0.000	340.8	10.56	22.26	17.51

Note: The training sample starts in 1970Q1. h=1 and h=4 denote forecast horizons at 1 and 4 quarters, respectively. R(h) is the ratio of the root mean squared error, relative to the benchmark AR(p). DM(h), PT(h) and GW(h) are the corrected Diebold and Mariano (1995), the Pesaran and Timmermann (1992) and the Giacomini and White (2006) statistics, respectively. Skew is the unbiased measure of sample skewness. For the R(h) statistic, "Criterion" denotes the percentage number of values lower than 1. For the DM(h) test, the percentage number of right-tail rejections of H0 (h) over the left-tail rejections of h0 (h0). For both the h0 and the h0 tests, the percentage number of right-tail rejections of h0. h0. h1. h2 is the number of total available variables times the number of non-linear models minus the trimming and few variables for which a specific non-linear model could not be estimated due to convergence issues.

the percentage of statistics rejecting at 5% level the null of equal conditional forecast accuracy between linear and non-linear models.

From Table 3, based on RMSFE ratios R(h), we observe that non-linear models allow an improvement of forecast accuracy in almost 40% of cases (39.78% for h = 1 and 38.22% for h = 4), with respect to the benchmark linear model. The counterpart of this quite good result is that on average non-linear models do not enable a systematic increase in forecasting accuracy, for any horizon. Indeed, for both forecast horizons, on average there is no clear improvement due to non-linear models, the mean ratio being close but greater than one (1.043 for h = 1 and 1.025 for h = 4), even during the *Great Recession* sub-period (equal to 1.041 for h = 1 and 1.024 for h = 4). These mean values are a bit influenced by some very strong positive values, the median values actually being even closer to 1.

The corrected DM test confirms this observation and does not allow to discriminate, on average, between non-linear and linear models. However, during the pre-crisis period, DM criteria tend to reach 100%, especially as regards the medium-term horizon (h = 4).

Moreover, the GW test cannot reject at the 5% level the null of no differences in conditional forecasting ability between non-linear and linear models very frequently (the rejection frequency is slightly lower than 8% for h=1 and reaches a maximum of 28.3% for h=4 during the pre-crisis period).

The results for the PT sign test indicates that for about 58% of cases, the null of independency between realizations and one-step ahead forecasts, at the usual 5% level, is rejected over the whole forecasting period, meaning thus that predictions and realizations go most of the time in the same direction. The value is only slightly lower for h = 4, at about 49%.

Let us have a closer look at the two sub-periods under consideration, namely the pre-crisis (2004-06) and the *Great Recession* (2007-09) periods. In Figure 1, we present the distributions of RMSFE ratios R(h), estimated using a non-parametric Kernel approach.¹⁴

For each of the two subperiods under consideration, we split the results by type of non-linear model, for both h=1 and h=4. It turns out that for the short-term horizon h=1, results are in general quite similar for both sub-periods, with a slightly larger variance during 2004-06, except for the TVAR model for which the standard deviation in 2004-06 is twice the one in 2007-09. Thus for h=1, during the pre-crisis sub-period the TVAR model simultaneously presents very good results for some variables, as well as very bad results for others. In fact, it turns out that good results are more frequent than bad results, leading to an average ratio R(h=1) of 60.31% (see Table 6 in the Appendix). On the contrary, differences are smaller during the *Great Recession*, reflecting perhaps common global shocks that have

¹³We checked whether the choice of the threshold, as well as the trimming method itself, leads to a selection bias due to the systematic exclusion of specific variables and countries. Results (not reported) suggest that the exclusion probability is almost uniform for both countries and variables (with some unavoidable exception due to sampling issues), leading to a rejection of the selection bias problem for our empirical analysis.

¹⁴A Gaussian kernel has been adopted along with the standard bandwidth equal to $1.06 \times s \times n^{-1/5}$ where s is the standard error of data and n is the number of observations.

affected simultaneously variables and countries diminishing the ability of the TVAR model to produce accurate forecasts. Regarding the longer horizon h = 4, results by models are more heterogeneous, but we get that the variance of RMSFE ratios is generally larger during the first sub-period 2004-06. In

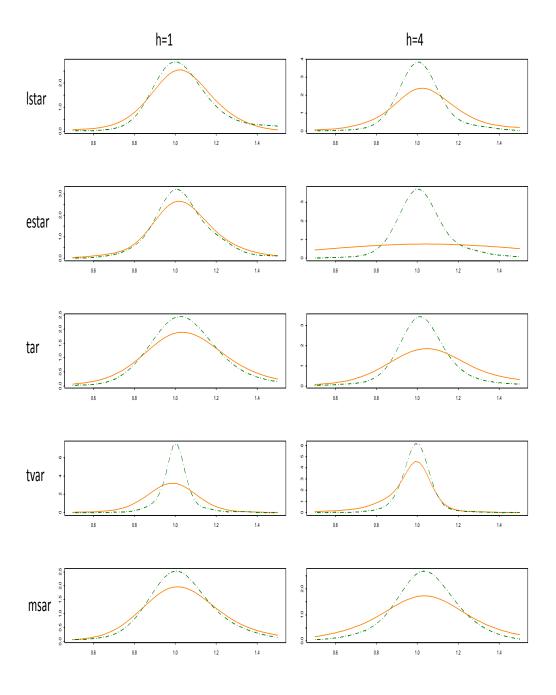


Figure 1: Distribution across variables of RMSFE ratios (relative to the AR benchmark) for the 2 sub-periods 2004-2006 (full line) and 2007-2009 (dotted line), estimated using a Kernel approach, for both h=1 and h=4, and for each of the 5 types of non-linear models.

particular, the ESTAR model exhibits a standard error of 0.492 over the pre-crisis that falls to 0.136 during the *Great Recession*. This latter result is mainly driven by the presence of forecast outliers, that is an extremely small number of very good and very bad forecasts which stretch the empirical distribution of relative RMSFEs. We also note at this point that the distribution of RMSFE ratios obtained using the TVAR is slightly skewed to the left, meaning good forecasting performances over the entire forecasting period, but specifically during the pre-crisis period.

Overall, Figure 1 indicates that there is more heterogeneity in forecasting performance during the pre-crisis period than during the crisis. Hence, during the pre-crisis period, it should be easier to identify variables and/or countries that have a highly non-linear behavior. In contrast, during the *Great Recession* growth rates were so extreme and volatile, and likely generated by global common shocks, that it is more difficult to clearly discriminate for which countries and variables non-linear models perform better.

In summary, this analysis of the aggregate results suggests that, on average, the linear benchmark model is not outperformed by non-linear alternatives, even during the *Great Recession* period. A possible explanation is that the recession was so different from previous events that even non-linear models could not capture it appropriately. On the positive side, it is however noteworthy that non-linear specifications lead to a forecasting gain in almost 40% of cases, according to the RMSFE ratio measure. This result calls for a more disaggregate analysis. Hence, we are now going to assess first the forecasting accuracy of each type of non-linear model, and then their performance for specific variables and countries.

4.2 Results by type of non-linear model

Results are now split by each of the five types of non-linear models for all variables and countries, with details presented in Tables 5 and 6 in the Appendix and empirical distributions of RMSFE ratios in Figure 2 for h = 1 and h = 4.

It is noteworthy that the overall rather poor results in terms of average RMSFE ratios are in fact driven by models with transition based on either an observable variable (ESTAR, LSTAR, TAR) or an unobservable variable (MSAR), for both horizons. In contrast, the TVAR produces accurate forecasts on average: it outperforms the AR model for h = 4 (average ratio of RMSFE is equal to 0.984 for the entire period). Further evidence is provided by the corrected DM statistics, for which the "Criterion" previously defined is largely over 100%.

For the short-term horizon (h = 1), results for the non-linear models based on transition variables, either observable or non-observable, are quite similar in the sense that they do not outperform on average the benchmark linear model. This pattern can also be seen in figure 2 (top) where the distributions are unimodal with a similar variance, but are all slightly skewed to the right. The only exception is the TVAR model for which average ratios of RMSFE are lower than one, although close. The ratio estimated from the DM statistics also lies over the 100% threshold. This idiosyncratic behavior of the

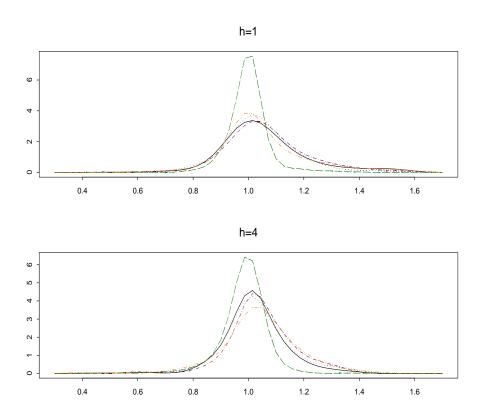


Figure 2: Distribution of RMSFE ratios for each model for h = 1 and h = 4. Models are the LSTAR (black full line), ESTAR (blue dotted line), TAR (red dashed-dotted line), TVAR (green dashed line) and MSAR (orange dashed-large dotted line).

TVAR model is also visible in figure 2, where we observe that the standard error is much smaller than for other models (0.069 against values greater that 0.100 for other models) and that the distribution appears more symmetric around the mean value equal to 0.989. As we already noted, this standard error is especially small during the *Great Recession* period.

As regards the medium-term horizon h=4, Figure 2 underlines the asymmetries to the right in the estimated distributions, showing frequent ratios of RMSFE higher than one, leading thus to a poor performance on average. The TVAR model is again the only specification whose distribution has a central position close to unity (median equal to 0.999), but skewed to the left (skewness equal to -1.38), indicating thus a higher probability of having a ratio of RMSFE lower than one. Thus, on average, the TVAR model clearly outperforms other non-linear models. This is particularly visible during the pre-crisis period (also shown in Figure 1).

The relative poor performance of non-linear models, either based on observed or unobserved transition variables, may be partly due to the lack of long enough data samples. This might render more tricky the specification and estimation steps. In addition, we have used information criteria-based spec-

ifications, while non-linear models based on transition variables may need a more detailed specification procedure that integrates statistical tests, regarding, for instance, the degree of non-linearity or the number of regimes. By contrast, TVAR models appear more flexible and the specification step is perhaps less crucial to get a good forecasting performance.

4.3 Country-specific and variable-specific results

Are they any variables and/or countries for which non-linear modeling could be more relevant for forecasting? To answer this question, we plot in Figure 3 the distribution of RMSFE ratios $R_j(h = 1)$ split by country, estimated using a non-parametric Kernel approach. We do not spot any major asymmetry in the distribution, but we note a kind of bubble located around a ratio close to 0.7, indicating very accurate non-linear forecasts for certain variables across countries. On the other hand, non-accurate non-linear forecasts are represented by a second bubble located close to 1.4.

The same distribution is presented in Figure 4 for the medium-term horizon (h = 4). Here, we spot a median higher than 1, but also a clear asymmetry to the left of the distribution, leading to the conclusion that ratios lower than one are more frequent for a longer horizon. More precisely, there exists a peak close to 0.5, meaning that some variables in some countries present a very non-linear pattern.

In order to identify specific variables and countries that strongly contribute to the left part of the distribution, we focus only on RMSFE ratios obtained in the most favorable case of our forecasting

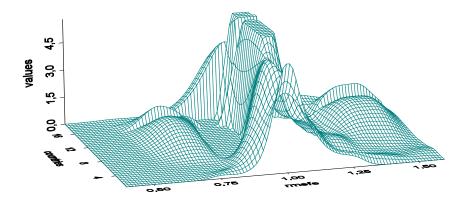


Figure 3: 3D-plot distribution of RMSFE ratios split by countries, estimated using a Kernel approach. For each country, the h = 1 rolling forecasts are considered.

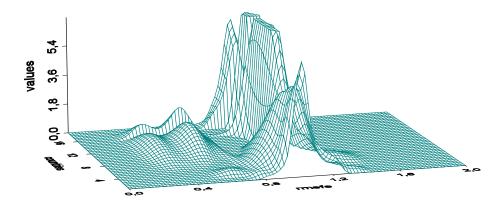


Figure 4: 3D-plot distribution of RMSFE ratios split by countries, estimated using a Kernel approach. For each variable, the h=4 rolling forecasts are considered.

experiment, namely variables or countries for which we have found a significant improvement of at least 10% in forecasting by comparison with linear models (*i.e.*, for RMSFE ratios < 0.90), over the complete forecasting period. We are going to assess the contribution of variables and countries to those low RMSFE ratios. Frequency results for both variables and countries are graphically presented in Figures 5 (for h = 1) and 6 (h = 4) and in Figures 7 (for h = 1) and 8 (for h = 4), respectively.¹⁵

It turns out that some variables are particularly suited for non-linear forecasting. Indeed, short-term and long-term interest rates are the variables that contribute most to explain low RMSFE ratios for, respectively, both short and medium-term horizons. Overall, there is a clear discrimination between variables as regards the medium-term horizon, while results are more mixed for the short-term horizon. Indeed, for h = 4, we get that interest rates (both short and long) and price deflators are clearly the variables that seem to be nicely adapted to non-linear forecasting, posting a contribution larger than 14% each.

In contrast, standard macroeconomic variables like industrial production, GDP or GDP-components (private investment, private consumption, exports and imports) do not appear to be particularly suitable for non-linear forecasting, in the sense that their contributions are very low. Financial market variables, like the stock market indexes and exchange rates, also do not show any improvement when non-linear models are used to forecast them at quarterly frequency. Overall, these results seem to be in line with

¹⁵Results are also presented under a tabular form in Tables 7 and 8 in the Appendix, where the last column contains the sum of frequencies over countries for a given variable and the last row contains the sum over variables for a given country.

the literature underlying that it appears extremely difficult to outperform very basic models to forecast exchange rates or financial variables.

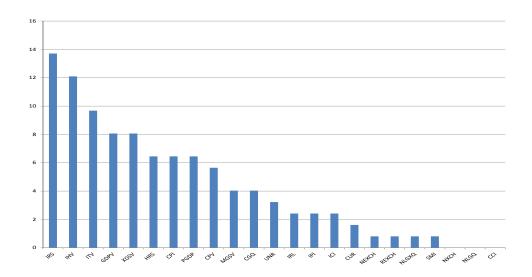


Figure 5: Contributions of variables to RMSFE ratios lower than 0.90 for h=1

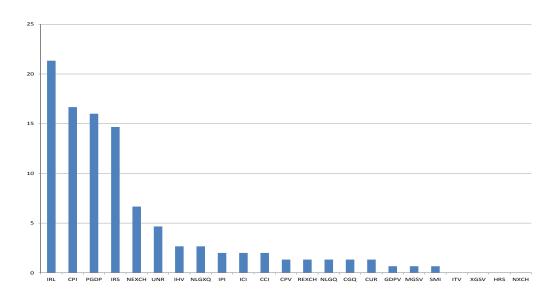


Figure 6: Contributions of variables to RMSFE ratios lower than 0.90 for h=4

As regards the countries, the discrimination is less clear-cut, but a ranking however emerges. Notably, Japan has the major contribution for both prediction horizons. Otherwise, European countries

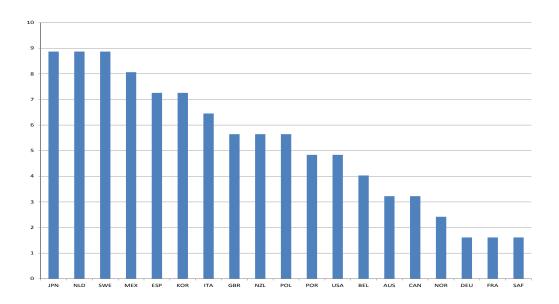


Figure 7: Contributions of countries to RMSFE ratios lower than 0.90 for h = 1

like Spain and Italy obtain also a good rank, while Mexico obtains the best results among emerging countries.

It is striking to observe that some industrialized countries that have been strongly affected during the *Great Recession* are strongly contributing to RMSFE ratios lower than 0.9. For example, in the case of Japan the gain for h=1 is mainly due to low ratios during the 2007-09 period, especially for variables such as exports and imports which were hardly affected by the large drop in international

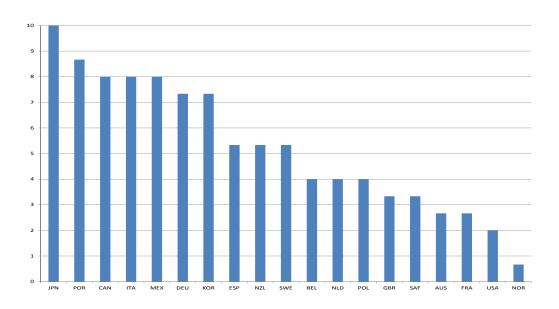


Figure 8: Contributions of countries to RMSFE ratios lower than 0.90 for h=4

trade, sometimes referred to as the *Great Collapse*. In opposition, the Japanese short-term interest rate appears to be more accurately forecast by non-linear models during the pre-crisis period. Regarding the medium-term horizon, the gain of non-linear models comes mainly from the more accurate forecasts for interest rates and prices during the pre-crisis period. The fact that interest rates are found to be suitable for non-linear forecasting, for both horizons, may be linked to the specific dynamic pattern of those series that present structural breaks in the sense that those rates are now constant at historical lowest levels since the end of the 90s. As another example, the Netherlands are also one of the industrialized countries that strongly contributes to low RMSFE ratios. As regards the short-term horizon, the improvements relate to housing prices, GDP and investment, for which non-linear forecasts are largely better during the *Great Recession*. Regarding the medium-term horizon, and similarly to Japan, the gain of non-linear models comes mainly from the more accurate forecasts for interest rates and prices during the pre-crisis period.

To draw conclusions from this section, we cannot exhibit systematic countries or variables that could be said to be convenient for non-linear forecasting. Results are mixed and strongly depend on the period of evaluation or from the prediction horizon. However, we note that gains in forecasting stemming from non-linear parameterizations can either arise from variables or countries that have experienced large drops during the recession or from variables that present clear structural breaks over the sample, like for example Japanese interest rates.

4.4 Comparing rolling-window to expanding-window

It can be argued that the use of a rolling-window scheme, which is usually implemented to deal with structural breaks, is not appropriate in our empirical application, because non-linear models are potentially able to incorporate such changes. Further, freezing the number of in-sample observations throughout the out-of-sample exercise could appear penalizing the forecasting performance of non-linear models, which usually require longer training-sample than linear models in order to get efficient parameter estimates. As far as we are concerned, an expanding-window scheme could be seen as more appealing and appropriate with respect to our forecast comparison exercise, which involves mainly non-linear models. To shed light on this issue, we compare in Table 4 the main aggregate results from the rolling-window recursion scheme (Table 3) to those from the expanding-window scheme.

Looking at RMSFE ratios R(h), it can be easily stressed that both schemes lead to similar results, which can be summarized as an improvement of the forecast accuracy for non-linear models in the range of 40-46% of cases when h=1 and 38-43% when h=4, with respect to the benchmark linear model. A small though significant difference can be observed for the short-term forecasts during the *Great Recession* period, where the forecast accuracy in the expanding-window exercise improves by 4% of cases with respect to the rolling-window exercise. Test statistics tend to confirm the evidence of similar forecast accuracy pointed out by the RMSFEs: both the corrected DM test and the PT sign test are very close in terms of rejection rates across schemes, except for some differences in the DM test for the

Table 4: Results for all models - Rolling and expanding windows

				h	n = 1		
	E+ W:1	N	Mean	S	tDev	Cr	it. (%)
	Fcst. Window	Rolling	Expanding	Rolling	Expanding	Rolling	Expanding
	2004q1-2009q4	1.043	1.028	0.133	0.124	39.78	41.83
R(h)	2004q1-2006q4	1.049	1.041	0.213	0.196	44.00	43.37
	2007q1-2009q4	1.041	1.025	0.159	0.142	42.52	46.28
	2004q1-2009q4	-0.258	-0.157	1.270	1.397	67.50	82.72
DM(h)	2004q1-2006q4	-0.134	-0.117	1.519	1.601	84.46	88.62
` ,	2007q1-2009q4	-0.205	-0.113	1.217	1.303	63.69	88.28
	2004q1-2009q4	1.810	1.858	1.110	1.102	58.50	60.67
PT(h)	2004q1-2006q4	1.407	1.434	1.130	1.138	40.06	40.59
	2007q1-2009q4	1.209	1.255	0.964	0.983	27.41	28.91
				ŀ	n = 4		
	D / 1177 1	N	Mean		tDev	Cr	it. (%)
	Fest. Window	Rolling	Expanding	Rolling	Expanding	Rolling	Expanding
	2004q1-2009q4	1.025	1.019	0.111	0.110	38.22	38.67
R(h)	2004q1-2006q4	1.052	1.044	0.305	0.289	42.68	43.62
	2007q1-2009q4	1.024	1.017	0.129	0.125	41.69	42.79
	2004q1-2009q4	-0.243	-0.202	1.555	1.620	59.93	76.03
DM(h)	2004q1-2006q4	-0.071	-0.057	2.058	2.104	93.73	92.81
` '	2007q1-2009q4	-0.225	-0.169	1.522	1.657	62.55	75.00
	2004q1-2009q4	1.344	1.351	1.416	1.434	49.06	49.12
PT(h)	2004q1-2006q4	1.269	1.282	1.159	1.170	36.44	37.30
` '	2007q1-2009q4	0.842	0.842	1.174	1.171	19.70	19.67

Note: See Table 3.

full sample and the *Great Recession* period. Splitting the results by each of the five types of non-linear models (not reported, but available upon request) does not substantially change the picture described above. Hence, we have strong (aggregated and disaggregated) evidence in favor of the robustness of our results with respect to the type of estimation scheme chosen for the out-of-sample exercise.

5 Conclusions

In this paper, we evaluate in a systematic way the forecasting ability of the set of most-common nonlinear models for a large group of macroeconomic variables and countries, before and during the *Great Recession* period. We compare the non-linear forecasts with those provided by a benchmark linear autoregressive model and we assess the results through a set of standard forecasting tests.

From this large empirical analysis, we get improvements in forecasting ability from non-linear models in about 40% of the cases, based on RMSFE ratios. Some of these gains could be due to the use of a large

set of alternative models. We point out that (i) some specific countries, like Japan or Mexico, appear to be more suitable for non-linear forecasting than others and that (ii) variables which exhibit either large drops during the recession or structural breaks within the sample, lead sometimes to more accurate forecasts when using non-linear alternatives. Examples are provided by long- and short-term interest rates and price series, in particular for h=4. Among the classes of non-linear models implemented, it turns out that TVAR models provide the best performance, especially in the medium-term forecasting horizon of one year. This may be due to a greater robustness to specification and estimation issues by comparison with other non-linear models. With respect to variables and countries, we cannot provide any strong evidence of systematic forecasting gains.

In spite of these encouraging results in favor of non-linear approaches, it turns out that, on average, the linear benchmark model is not outperformed by non-linear alternatives, even during the *Great Recession* period. This poorer than expected result may be due to several reasons that need to be further investigated. For instance, the small size of the learning sample for some variables and/or countries can clearly lead to misspecification and estimation issues. Further, it can be argued that non-linear models based on a transition variable, either observable or unobservable, require a more careful, test-based, specification, for example for the selection of the number of regimes. As a consequence, the automatic implementation of non-linear models cannot be recommended in general, and a deeper analysis of the specification step has to be taken into account during the modeling procedure.

Intuitively, we could expect more accurate results from non-linear models during the *Great Recession*, provided that large shocks introduce a non-linear pattern across macroeconomic and financial variables. However, it turns out that the magnitude of this shock was so large that such an event was barely observed in the past, even for the series with the longest learning sample. Thus, this sudden large non-linearity was absolutely not anticipated even by the non-linear models, which are simply autoprojective. It is likely that using exogenous information, conveyed for example by leading indicators of recession, would be a useful forecasting strategy.

A last argument that could explain the (on average) outperformance of linear models is that the gains in forecasting with non-linear models would not be fully reflected in the point forecasts, but rather in higher moments of the forecast distributions (density forecasts), which may show for example an asymmetric tail behavior.

Overall, we believe that our analysis has highlighted interesting patterns on the forecasting performance of non-linear models, providing the first systematic evidence on their behavior during the *Great Recession*. Many questions deserve additional investigation, and these are interesting topics for future research in this area.

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APPENDIX

Table 5: Results by models

LSTAR	Fest Window	N.Obs	Mean	Median	h = Min	= 1 Max	StDev	Skew	Crit. (%)	N.Obs	Mean	Median	h. Min	h = 4 Max	StDev	Skew	Crit. (%)
R(h)	2004q1-2009q4 2004q1-2006q4 2007q1-2009q4	365 365 365	1.057 1.039 1.057	1.027 1.027 1.025	0.467 0.467 0.377	1.566 2.331 1.720	0.154 0.189 0.181	0.860 2.116 1.044	37.81 39.18 43.56	364 364 364	1.023 1.058 1.019	1.013 1.028 1.009	0.438 0.441 0.429	1.395 2.092 1.789	0.105 0.209 0.123	-0.274 0.946 0.906	39.84 39.29 42.86
$\mathrm{DM}(h)$	2004q1-2009q4 2004q1-2006q4 2007q1-2009q4	365 365 365	-0.273 -0.240 -0.179	-0.618 -0.387 -0.433	-4.176 -4.978 -5.395	7.639 4.976 3.782	1.283 1.468 1.199	1.109 0.193 0.182	88.00 74.51 69.23	364 364 364	-0.315 -0.330 -0.237	-0.492 -0.539 -0.310	-5.458 -10.343 -7.578	9.957 12.681 6.535	1.611 2.069 1.510	1.202 0.464 -0.039	48.48 71.23 47.69
PT(h)	2004q1-2009q4 2004q1-2006q4 2007q1-2009q4	331 331 331	1.835 1.384 1.250	1.909 1.497 1.205	-1.535 -2.169 -1.455	4.904 3.479 3.479	1.047 1.083 0.976	-0.241 -0.443 -0.101	57.10 37.76 29.31	327 327 327	1.376 1.272 0.878	1.636 1.560 1.004	-2.890 -2.880 -2.678	4.904 3.479 3.479	1.355 1.169 1.123	-0.836 -0.750 -0.608	49.54 37.61 19.88
GW(h)	2004q1-2009q4 2004q1-2006q4 2007q1-2009q4	365 365 365	1.315 1.366 1.111	0.875 0.826 0.846	0.000	12.662 7.315 6.635	1.529 1.515 1.089	2.782 1.664 1.700	6.03 7.67 3.01	364 364 364	2.803 5.199 2.898	1.111 1.682 1.406	0.000	103.449 185.818 91.875	7.086 13.799 6.529	9.781 8.362 8.670	18.68 29.12 20.88
ESTAR	Fcst Window	N.Obs	Mean	Median	$^{\mu}$	= 1 Max	StDev	Skew	Crit. (%)	N.Obs	Mean	Median	h	= 4 Max	StDev	Skew	Crit. (%)
$\mathrm{R}(h)$	2004q1-2009q4 2004q1-2006q4 2007q1-2009q4	357 357 357	1.042 1.044 1.038	$\begin{array}{c} 1.026 \\ 1.025 \\ 1.017 \end{array}$	0.599 0.407 0.430	$\begin{array}{c} 1.565 \\ 1.972 \\ 1.700 \end{array}$	$0.127 \\ 0.165 \\ 0.147$	0.732 0.678 0.771	36.69 41.18 42.58	360 360 360	$ \begin{array}{c} 1.032 \\ 1.077 \\ 1.032 \end{array} $	1.018 1.032 1.007	0.394 0.173 0.588	1.419 9.403 1.924	0.118 0.492 0.136	-0.054 13.599 2.182	36.39 40.00 43.89
$\mathrm{DM}(h)$	2004q1-2009q4 2004q1-2006q4 2007q1-2009q4	357 357 357	-0.338 -0.258 -0.209	-0.602 -0.538 -0.377	-3.810 -5.649 -3.117	4.143 5.227 4.266	1.219 1.445 1.183	0.279 0.335 0.331	47.06 71.11 57.14	360 360 360	-0.276 -0.210 -0.175	-0.483 -0.415 -0.271	-3.473 -7.153 -14.082	4.555 9.115 4.639	1.401 1.881 1.511	0.668 0.511 -1.936	50.91 62.50 82.05
PT(h)	2004q1-2009q4 2004q1-2006q4 2007q1-2009q4	318 318 318	$1.839 \\ 1.393 \\ 1.254$	1.932 1.560 1.240	-1.052 -2.333 -1.796	4.904 3.479 3.479	1.031 1.098 0.953	-0.151 -0.556 -0.277	59.75 39.31 29.25	324 324 324	1.424 1.338 0.881	1.686 1.560 1.042	-2.786 -2.333 -2.400	4.904 3.479 3.479	$ \begin{array}{c} 1.370 \\ 1.145 \\ 1.160 \end{array} $	-0.804 -0.620 -0.658	50.93 39.51 20.37
GW(h)	2004q1-2009q4 2004q1-2006q4 2007q1-2009q4	357 357 357	1.248 1.306 1.063	0.853 0.872 0.793	0.000	8.552 8.861 6.294	1.338 1.410 1.064	2.047 1.986 1.520	5.04 5.60 1.40	360 360 360	2.121 4.225 3.063	1.090 1.647 1.249	0.000	21.648 96.951 340.765	2.975 8.717 18.076	2.828 5.755 18.272	16.39 25.00 15.00
TAR	Fcst Window	N.Obs	Mean	Median	h = Min	= 1 Max	StDev	Skew	Crit. (%)	N.Obs	Mean	Median	h . Min	= 4 Max	StDev	Skew	Crit. (%)
$\mathrm{R}(h)$	2004q1-2009q4 2004q1-2006q4 2007q1-2009q4	369 369 369	1.065 1.088 1.060	$ \begin{array}{c} 1.042 \\ 1.041 \\ 1.038 \end{array} $	0.429 0.347 0.505	$ \begin{array}{c} 1.591 \\ 2.887 \\ 2.494 \end{array} $	0.142 0.254 0.177	0.641 2.135 2.321	30.35 36.31 36.04	359 359 359	$1.048 \\ 1.103 \\ 1.034$	1.034 1.072 1.016	$0.591 \\ 0.431 \\ 0.441$	1.389 2.808 1.799	$0.113 \\ 0.267 \\ 0.145$	-0.105 1.935 0.710	27.58 34.54 37.60
$\mathrm{DM}(h)$	2004q1-2009q4 2004q1-2006q4 2007q1-2009q4	369 369 369	-0.490 -0.439 -0.358	-0.735 -0.619 -0.566	-3.649 -4.570 -4.994	6.555 4.884 5.472	1.199 1.493 1.195	0.980 0.295 0.268	31.11 45.95 57.58	359 359 359	-0.556 -0.434 -0.409	-0.803 -0.718 -0.488	-4.606 -10.306 -11.459	13.170 15.115 7.756	1.526 1.898 1.586	2.637 1.385 -0.485	28.57 53.33 37.88
$\operatorname{PT}(h)$	2004q1-2009q4 2004q1-2006q4 2007q1-2009q4	333 333 333	1.688 1.286 1.152	1.725 1.455 1.080	-2.396 -2.400 -2.333	4.492 3.479 3.479	1.180 1.192 1.023	-0.506 -0.476 -0.432	52.85 36.94 25.23	327 327 327	1.345 1.238 0.896	1.535 1.497 1.080	-2.786 -3.479 -2.400	4.118 3.479 3.479	1.391 1.159 1.127	-0.745 -0.869 -0.632	48.01 37.00 18.04
GW(h)	2004q1-2009q4 2004q1-2006q4 2007q1-2009q4	369 369 369	1.299 1.465 1.104	0.808 1.100 0.761	0.000	13.899 7.597 7.502	1.575 1.442 1.210	2.940 1.307 1.972	5.96 8.40 4.07	359 359 359	2.747 4.486 3.375	1.201 1.649 1.377	0.000	180.986 273.168 189.643	10.292 15.749 11.432	15.144 14.492 13.020	14.76 29.53 18.94
Note: sec	Note: see Table 3.																

Table 6: Results by models (continued)

TVAB					h. ==	-							4	h = 4			
1	Fest Window	N.Obs	Mean	Median	Min	– ř Max	StDev	Skew	Crit. (%)	N.Obs	Mean	Median	Min	Max	StDev	\mathbf{Skew}	Crit. (%)
	2004q1-2009q4	383	1.000	0.999	0.773	1.399	990.0	1.394	50.91	385	0.984	0.999	0.508	1.376	0.069	-1.377	54.55
$\mathrm{R}(h)$	2004q1-2006q4	383	0.989	0.997	0.513	2.493	0.132	5.265	60.31	385	0.96.0	0.997	0.392	1.400	0.120	-1.263	57.92
	2007q1-2009q4	383	1.004	1.001	0.720	1.345	0.071	0.828	47.26	385	0.995	1.000	0.612	1.551	0.069	0.955	51.17
	900421 900024	383	890 0	0.139	7367	206	1 975	0.450	999 99	о я	0 333	0 109	7 377	11 649	1.671	1 905	98 986
	200441-200944	200	0.000	0.132	-4.00.	007.0	L-4-1	0.420	2.00.00	000	0.00	0.132	-4.0.4-	11.042	1.0.1	1.200	260.30
DM(n)	2004q1-2006q4	303	0.489	0.413	-3.480	0.282 4 F.G.E.	1.504	0.383	312.50 66.67	585 905	0.082	0.412	-0.838	15.38U	7.300	0.220	199 56
	2007q1-2009q4	000	-0.113	-0.085	-3.441	4.303	1.240	-0.00/	70.00	000	0.098	0.073	-0.031	000.7	1.0.1	0.010	132.30
	2004q1-2009q4	333	1.889	1.948	-2.179	4.904	1.112	-0.593	62.46	349	1.428	1.665	-2.533	4.904	1.391	-0.696	51.29
PT(h)	2004q1-2006q4	333	1.518	1.560	-1.945	3.479	1.126	-0.636	42.94	349	1.366	1.560	-1.945	3.479	1.109	-0.526	37.54
	2007q1-2009q4	333	1.220	1.240	-1.884	3.479	0.987	-0.368	26.43	349	0.874	1.004	-2.678	3.479	1.182	-0.667	21.49
	200481-200984	383	1 308	0 78 178	000	14.06	1 701	9886	0 17	ς. α	3 093	1 131	000	171	XX	11 50	18.41
CW/(b)	200441-200344	363	1.350	0.001	0000	200.51	1.656	1.670	9.14	000 000 000 000	6.647	1.101	000.0	100.8	15.000 18.000	6 370	32.9K
(1) 10	2007q1-2009q4 2007q1-2009q4	383	1.204	0.749	0.000	9.129	1.351	1.858	5.22	385	2.892	1.318	0.000	82.77	6.476	8.241	17.92
MSAR					h:	 							h	= 4			
	Fest Window	N.Obs	Mean	Median	Min	Max	StDev	\mathbf{Skew}	Crit. (%)	N.Obs	Mean	Median	Min	Max	StDev	\mathbf{Skew}	Crit. (%)
	2004q1-2009q4	351	1.054	1.012	0.585	1.589	0.152	1.058	42.74	348	1.042	1.032	0.376	1.427	0.129	-0.435	31.32
$\mathrm{R}(h)$	2004q1-2006q4	351	1.090	1.011	0.429	3.190	0.278	3.043	42.17	348	1.069	1.030	0.177	2.686	0.298	1.778	40.52
	2007q1-2009q4	351	1.047	1.010	0.476	2.533	0.185	2.154	43.02	348	1.044	1.030	0.472	2.314	0.152	1.450	31.90
	2004q1-2009q4	351	-0.271	-0.432	-5.048	4.864	1.308	0.073	51.22	348	-0.450	-0.730	-4.127	10.008	1.377	1.709	31.48
$\mathrm{DM}(h)$	2004q1-2006q4	351	-0.259	-0.457	-5.552	6.530	1.510	0.519	57.89	348	-0.113	-0.400	-5.485	12.622	1.884	1.525	92.31
	2007q1-2009q4	351	-0.163	-0.280	-4.399	2.768	1.252	-0.047	67.50	348	-0.432	-0.728	-3.399	7.252	1.358	1.115	40.74
	2004q1-2009q4	290	1.800	1.921	-2.037	4.904	1.169	-0.671	69.09	328	1.144	1.535	-3.468	4.492	1.554	-0.720	45.43
PT(h)	2004q1-2006q4	290	1.459	1.560	-2.333	3.479	1.142	-0.652	43.79	328	1.128	1.272	-2.818	3.479	1.205	-0.453	30.49
	2007q1-2009q4	290	1.167	1.080	-1.945	3.479	0.972	-0.276	26.90	328	0.680	0.928	-2.818	3.479	1.265	-0.574	18.60
	2004q1-2009q4	351	1.582	0.928	0.000	11.897	1.912	2.379	10.54	348	2.184	1.163	0.000	104.509	6.020	14.432	12.64
GW(h)	2004q1-2006q4	351	1.463	0.930	0.000	8.221	1.617	1.702	8.55	348	4.364	1.346	0.000	170.575	11.690	9.380	24.14
	2007q1-2009q4	351	1.287	0.923	0.000	6.998	1.260	1.368	5.41	348	2.399	1.416	0.000	58.195	4.308	7.467	14.66
Note: set	Note: see Table 3.																

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Table 7: Frequency results: percentage of RMSFE ratios lower than 0.9 (All models, 2004q1-2009q4 and h=1)

Series	AUS	BEL	CAN	DEU	ESP	FRA	GBR	ITA	JPN	$_{ m SAF}$	KOR	MEX	NLD	NOR	NZL	$_{\rm POL}$	POR	SWE	USA	Sum	N.Obs
GDPV	0.00	0.00	0.00	0.00	3.23	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.42	0.00	0.00	0.81	0.00	1.61	0.00	8.06	10
ATI	0.00	0.81	0.00	0.00	0.81	0.00	0.00	0.00	0.00	0.00	2.42	0.00	2.42	0.00	0.00	0.81	1.61	0.81	0.00	89.6	12
IHV	0.00	0.00	0.00	0.00	1.61	1.61	0.00	0.81	0.00	0.00	0.81	0.00	4.03	0.81	0.00	0.00	0.00	0.00	2.42	12.10	15
MGSV	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.81	0.81	0.00	0.00	0.00	0.00	0.81	0.00	0.00	0.00	0.00	1.61	4.03	20
XGSV	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.42	0.00	0.00	0.81	0.00	0.00	0.81	0.81	0.00	3.23	0.00	8.06	10
CPV	0.00	0.00	0.00	0.00	0.00	0.00	2.42	0.00	0.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00	0.81	1.61	0.00	5.65	7
UNR	0.00	0.00	0.00	0.00	1.61	0.00	0.00	0.00	0.00	0.00	0.81	0.00	0.00	0.00	0.00	0.81	0.00	0.00	0.00	3.23	4
HRS	0.81	0.00	0.81	0.81	0.00	0.00	1.61	0.00	0.00	0.00	0.00	0.00	0.00	0.81	0.00	0.81	0.00	0.81	0.00	6.45	œ
CPI	0.00	0.00	0.00	0.00	0.00	0.00	0.81	0.00	0.00	0.00	0.00	2.42	0.00	0.00	0.00	1.61	1.61	0.00	0.00	6.45	∞
PGDP	0.81	1.61	0.81	0.00	0.00	0.00	0.00	1.61	0.81	0.81	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	6.45	∞
IRS	1.61	0.00	0.81	0.81	0.00	0.00	0.00	0.81	3.23	0.00	1.61	2.42	0.00	0.00	1.61	0.00	0.00	0.81	0.00	13.71	17
IRL	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.42	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.42	က
NXCH	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0
NEXCH	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.81	1
REXCH	0.00	0.00	0.00	0.00	0.00	0.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.81	1
NLGXQ	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.81	1
NLGQ	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0
CGO	0.00	0.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.23	0.00	0.00	0.00	0.00	4.03	ಬ
IPI	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.61	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.81	2.42	က
CUR	0.00	0.81	0.00	0.00	0.00	0.00	0.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.61	7
ICI	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.81	0.00	0.00	0.81	0.00	0.00	0.00	0.00	0.00	0.81	0.00	0.00	2.42	က
CCI	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0
SMI	0.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.81	1
Sum	3.23	4.03	3.23	1.61	7.26	1.61	5.65	6.45	8.87	1.61	7.26	8.06	8.87	2.42	5.65	5.65	4.84	8.87	4.84	100.00	
N.Obs	4	ಸು	4	2	6	2	7	∞	11	2	6	10	11	က	7	7	9	11	9		124
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Note: Results are expressed in percentage. In the same order, countries are Australia, Belgium, Canada, Germany, Spain, France, UK, Italy, Japan, South Africa, South Korea, Mexico, The Netherlands, Norway, New Zealand, Portugal, Sweden and USA.

Table 8: Frequency results: percentage of RMSFE ratios lower than 0.9 (All models, 2004q1-2009q4 and h=4)

Note: Results are expressed in percentage. In the same order, countries are Australia, Belgium, Canada, Germany, Spain, France, UK, Italy, Japan, South Africa, South Korea, Mexico, The Netherlands, Norway, New Zealand, Portugal, Sweden and USA.

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