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Private Uncertainty and Multiplicity

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Résumé: Ce document démontre que sous des conditions générales une petite incertitude privé sur un état endogène global de l'économie peut générer une multiplicité d'équilibres dans des modèles qui sinon auraient un équilibre unique. Le principal résultat est présenté dans un modèle macroéconomique entièrement micro fondé où les agents apprennent à partir des prix d'équilibre. Les résultats s'appliquent à une large classe de problèmes statiques d'extraction de signal où la corrélation fondamentale et les externalités stratégiques contribuent conjointement à la multiplicité d'équilibres. Les cas où une seule de ces deux elements est suffisante pour une multiplicité sont également isolés et discutés.

Classification JEL: D82, D83, E3.

Mots-clés: information dispersée, contenu informatif des prix, croyances du second ordre.

Abstract: This paper shows that under general conditions small enough private uncertainty on an aggregate endogenous state of the economy can generate a multiplicity of equilibria in otherwise unique-equilibrium models. The main result is presented in a fully microfounded macroeconomic model where agents learn from equilibrium prices. The findings apply to a broad class of static signal extraction problems where both fundamental correlation and pay-off externalities jointly contribute to a multiplicity of equilibria. The cases where only one of these two determinants is sufficient for a multiplicity are also isolated and discussed.

JEL Classification: D82, D83, E3.

Keywords: dispersed information, informational content of prices, second-order beliefs.

1 Introduction

This paper shows that a small enough degree of private uncertainty can generate a multiplicity of equilibria in macro-models that have a unique equilibrium under perfect knowledge. This finding contrasts with a classical message of the global games literature (Hans and van Damme, 1993) maintaining that multiplicity can be solved by perturbing the model away from perfect information. In an influential paper Morris and Shin (2000) put forward this view in macroeconomics. They propose 'rethinking' multiplicity as the artifact of two extreme assumptions: "First, [a] economic fundamentals are assumed to be common knowledge; and second, [b] economic agents are assumed to be certain about each other's behavior in equilibrium"[1]. Arbitrarily small private uncertainty on the fundamentals would lead therefore to uniqueness. Nevertheless, later works have showed that when agents have access to public information generated by market transactions - typically a system of prices - private uncertainty on fundamentals is generally not enough to pin down the number of equilibria[2]. That is, when agents have available information about others' beliefs then the original multiplicity is restored.

This paper makes a step forward in looking at the possibility that *private* - rather than public - endogenous signals can confuse agents and be a source of multiplicity. It identifies the conditions under which an endogenous information structure allowing for infinitesimal departures from the assumptions [a] and [b] can indeed invalidate the uniqueness of the equilibrium. In particular, it shows that a *small enough* private uncertainty on an endogenous aggregate state of the economy can generate three rational expectation equilibria (REE) in models where if this uncertainty is null or large enough then a unique equilibrium exists. This result reverses Morris and Shin's argument on the effect of a marginal relaxation of perfect information. At the same time it still maintains that less information prevents multiplicity since uniqueness is restored as uncertainty increases. The paper also isolates a particular case where a multiplicity arises with perfect knowledge of fundamentals, that is, when [a] holds but [b] does not. This case further emphasizes the crucial role of uncertainty about others' beliefs in sustaining a multiplicity of equilibria.

To provide microfoundations for the informational frictions of interest, I present a RBC economy where a system of prices generates a static signal extraction problem which renews each period. The economy is segmented in a continuum of islands, each inhabited by a three types of agents: consumer, intermediate and final producer. The final producer hires island-specific inputs - labor and capital - at local prices to produce an homogenous good which is

[1] Morris and Shin (2000), pag. 140, square brackets added.

[2] Angeletos and Werning (2006), and Hellwig, Mukherji and Tsyvinski (2006). See the discussion below.

traded across islands at the end of the period. The consumer supplies island-specific labor and sells one unit of an endowment in a global market to intermediate producers of local capital. Consumers also enjoy the services of money which is available at a fixed supply on each island. There are three sources of fundamental randomness. An idiosyncratic productivity shock hits the production of the local capital; and consumer-workers are subject to a preference shock constituted by an aggregate and an island-specific stochastic component. Hence global prices - for the endowment and the consumption good - move with the aggregate shock, whereas local prices - for labor and capital - also react to island-specific shocks.

An equilibrium requires that agents' actions and the information conveyed by the prices they observe be mutually consistent. Therefore final producers make decisions without knowing the value of their production. They only observe the local prices arising in the input markets. The local wage coveys information about the island-specific shock which is a *private exogenous signal* of the aggregate one. The price of the local capital instead constitutes a *private endogenous signal,* being equal to the price of the endowment plus an idiosyncratic productivity disturbance. Consumers and intermediate producers instead are perfectly informed since they can directly observe the price of the endowment which reveals the aggregate shock. Without idiosyncratic productivity, i.e. under perfect knowledge - there exists only one equilibrium where all global prices move together at the same rate as the aggregate shock induces a pure inflationary effect.

The main proposition of the paper demonstrates that small enough private uncertainty about the price of the endowment can generate a multiplicity of equilibria, whereas when this uncertainty is large enough or null a unique equilibrium exists. More specifically, if the cross-sectional variance of the preference shocks is above a certain threshold, then for any small-enough cross-sectional variance of the productivity shocks, three rational expectation equilibria exist, otherwise there is a unique equilibrium. That is, precise-enough endogenous information generates a multiplicity when the exogenous one is sufficiently loose.

The microfoundation of the information structure clarifies the conditions under which private uncertainty matters; namely, when it concerns an endogenous aggregate state that responds in opposite ways to an aggregate shock in the two extreme scenarios of no information and perfect foresight. The price of the endowment is the only one that has this feature in the model. In particular, when the price for local capital is not very informative[3] its *allocational effect* dominates. That is, if firms cannot be confident in their price predictions then, when the local price for capital increases they just demand less capital; as a consequence the price for the endowment goes down until the fixed supply clears.

[3] That is at the limit of infinite cross-sectional variance of local prices.

With a small variance in the productivity shocks, the *informational effect* prevails. In this case an increase in the local price for capital informs a rising consumption price and so firms demand more capital; as a consequence the price for the endowment goes up with the other prices.

This feature leads straight to the intuition for the result. The price for the endowment is always highly reactive to production decisions. But, only when the local price for capital is sufficiently informative - i.e. for small-enough private uncertainty - then also production decisions are in turn highly responsive to the price for the endowment, which constitutes the average endogenous signal received by firms. This informational feedback closes a circle of high complementarity allowing for the possibility of a multiplicity of equilibria. Nevertheless, only with small-enough private uncertainty - and not with full information - the interaction between firms' demand for capital and the endogenous informativeness of the local price generates a non-linearity which self-fulfills a multiplicity of equilibria. In other words, the signal extraction problem introduces a non-linearity in an otherwise (log)linear model, so when the informational problem vanishes only one equilibrium exists. Finally, the more precise is exogenous information the less powerful is such non-linear interaction and so the less easier a multiplicity arises. This intuition translates into the cross-sectional variance of the preference shocks being larger than an index measuring the elasticity of the incentives that move endogenous variables.

In the process of proving the main result, the paper presents a fairly general analysis of static signal extraction problems that goes beyond the specific restrictions of the model. I show that in presence of endogenous uncertainty multiple equilibria can arise also when agents forecast a purely exogenous realization. When instead the uncertain variable moves with the aggregate expectation, negative pay-off externalities (strategic substitutability) encourage multiplicity. Nevertheless with strong positive pay-off externalities a multiplicity can be sustained by partially-correlated endogenous signals even when agents know the fundamentals. In the last section I discuss this possibility allowing for an ad-hoc variation of the model.

This paper contributes to the debate on the robustness of multiplicity of equilibria in the classic currency attack model. Morris and Shin (1998) first noticed that small private uncertainty on fundamentals leads to a unique equilibrium. In fact, this possibility relies on the exogenous nature of the information structure; when market transactions generate public information then the original multiplicity is restored (Angeletos and Werning (2006), and Hellwig, Mukherji and Tsyvinski (2006)). The present finding completes the picture showing how private - rather than public - endogenous signals can even generate a multiplicity in the context of models that exhibit equilibrium uniqueness - rather than a multiplicity - with perfect knowledge.

This work adds to a large body of literature about dispersed information in macro-models dating back to Lucas (1973). Examples of recent applications of that approach include Angeletos and La'O (2012), Hellwig (2005), Hellwig and Venkateswaran (2009), Lorenzoni (2009) among others. In all of these works a unique equilibrium exists whose welfare properties are challenged by the interaction of public and private signals as in Morris and Shin (2002). Angeletos, Lorenzoni and Pavan (APL, 2010) and Amador and Weill (AW, 2010) instead also show the possibility that signal extraction problems can generate multiple equilibria, although multiplicity is not their main focus. They both find that when agents condition to endogenous signals which are highly reactive to the average expectation than three determinate REE arise.[4] Nevertheless, in contrast to the main result of the present paper, their multiplicity vanishes as the model approaches the perfect information scenario. That is, they do not meet the conditions for which a multiplicity arises because of a *marginal* perturbation of common knowledge of fundamentals. The main modelling difference is that I study a signal extraction problem where the aggregate state underlying the endogenous signals, namely the price for the endowment, reacts more than one-to-one to the average expectation: an unfeasible parameter region in both AW and ALP. This feature allows for the existence of an equilibrium where the signal and the uncertain shock are negatively correlated. However, the conditions sustaining a multiplicity as the one in AW and APL are recovered in a dedicated section of this paper as an additional result of the general analysis.

A different form of multiplicity is presented by Benhabib, Wang and Wen (2012). They look at a model in which a partly revealing REE can arise, beyond a fully revealing one, when agents weight an hod-hoc exogenous signal embodying a non-fundamental component. The sunspot equilibrium obtains for a point-wise specific calibration of the exogenous parameters of the model, so that, ceteris paribus, uniqueness is restored for a marginal variation of the cross-sectional variance of the signals. Their case is related to the possibility of the emergence of multiple equilibria with private endogenous signals and perfect knowledge of fundamentals which is laid out in the last section of this paper.

2 A microfounded macro-model

This section presents a dynamic macro-model encompassing the reduced form of the seminal Cobweb model (Muth, 1961). The model's main objective is to microfound in the most transparent way the whole class of signal extraction problems studied in the next section in more abstract terms.

[4]Other examples of interest are Ganguli and Yang (2009), and Desgranges and Rochon (2012) who find a determinate multiplicity vanishing for small uncertainty in Grossman-Stiglitz asset pricing models with endogenous information structures. In comparable environments cases of indeterminacy of the equilibrium has been studied by Barlevy and Veronesi (2003).

Preferences and technology

Consider an endowment economy composed of a continuum of islands with unit mass. Each island $i \in I \equiv [0,1]$ is inhabited by a representative consumer and a representative producer. The utility of the representative consumer in island i is

$$U_{i,t} \equiv \sum_{t=0}^{\infty} \delta^t \left(\Phi_{i,t} \left(\frac{C_{i,t}^{1-\psi}}{1-\psi} - \frac{\left(L_{i,t}^s\right)^{1+\gamma}}{1+\gamma} \right) + \log \frac{M_{i,t-1}}{P_t} \right), \tag{1}$$

subject to a budget constraint for each period

$$\frac{R_t}{P_t} + \frac{W_{i,t}}{P_t} L_{i,t}^s + \frac{M_{i,t-1}}{P_t} = \tau_t C_{i,t} + \frac{M_{i,t}}{P_t} + \frac{T_{i,t}}{P_t}, \tag{2}$$

where ψ and γ are positive constants, R is the return on one unit of an endowment that expires in one period and is renewed each time in each island, W_i is a island-specific wage, L_i^s is supply of island-specific (local) labor, C_i is the consumption of the final good whose price is P and M_i is the money demand on island i. T_i is a redistributive nominal transfer such that[5] $(1-\tau) \int C_i di - \int T_i di = 0$ where τ is a gross consumption tax/subsidy which will play a role in section 4; for the moment I assume $\tau_t = 1$ at each t. Finally $\Phi_{i,t}$ is a consumption-leisure preference shock whose properties will be defined below.

The endowment is acquired in an inter-island market to be transformed in island-specific capital K_i. The transformation is operated by competitive firms maximizing profits

$$R_{i,t} K_{i,t}^s - R_t Z_{i,t}, \tag{3}$$

under the constraint of the following linear technology

$$K_{i,t}^s \equiv e^{-\hat{\eta}_{i,t}} Z_{i,t}, \tag{4}$$

where $e^{-\hat{\eta}_{i,t}}$ is stochastic island-specific productivity factor. The local capital K_i^s is produced using Z_i units of the endowment which are acquired in a inter-islands market at a price R.

Local capital and local labor are used by the representative final producer in island i to produce an homogeneous consumption good that is consumed across islands. Competitive firms maximizes profits

$$P_t Y_{i,t} - W_{i,t} L_{i,t} - R_{i,t} K_{i,t}, \tag{5}$$

under the constraint of a Cobb-Douglas technology with constant return to scale

$$Y_{i,t}\left(K_{i,t}, L_{i,t}\right) \equiv K_{i,t}^{1-\alpha} L_{i,t}^{\alpha}, \tag{6}$$

[5] The only scope of the transfer is ensuring that in equilibrium the budget constraints holds for each i.

with $\alpha \in (0,1)$, where K_i, L_i, and Y_i denote respectively the demand for local capital, the demand for local labor and the produced quantity of the consumption good, generated in island i. Notice that production is island-specific, that is, each representative producer hires labor and capital from his own island only. Input markets are segmented and there is one different price for each input on each island.

Shocks

At each time the economy is hit by i.i.d. aggregate and island-specific disturbances. The productivity of the intermediate sector is affected by the stochastic noise

$$\hat{\eta}_{i,t} \sim N(0, \hat{\sigma}), \tag{7}$$

where $\hat{\eta}_i$ is an island-specific realization distributed independently across the islands. A second source of randomness concerns the utility of consumption and leisure in each island. It is determined by a shock

$$\log \Phi_{i,t} = \varepsilon_t + \hat{\phi}_{i,t}, \tag{8}$$

composed by an aggregate component $\varepsilon_t \sim N(0,1)$ drawn from a white noise distribution, and an island-specific component $\hat{\phi}_{i,t} \sim N(0, \sigma_\phi)$ which is a white noise disturbance independently distributed across the islands.

Equilibrium

Each period consists of two stages. In stage one the shocks hit and all input markets - two local for labor and capital on each island, and one global for the endowment - open and clear simultaneously. The production of the consumption good is implemented at the end of the first stage. In the second stage, the final market operates so that the consumption good clears across the islands and its price emerges. All agents in the economy have the same unbiased prior belief about the distribution of the shocks and acquire information through the equilibrium prices with which they deal. This means that in equilibrium each agent must take actions that are consistent with the information content of the prices she observes at the stage of the action.

As usual in the literature on noisy rational expectations from Grossman (1975) and Hellwig (1980) onward, I restrict attention to equilibria with a log-linear representation which, as we will see, obtain with no approximation in the case of the model at hand. A formal definition of an equilibrium follows.

Definition 1 *At each period t, given the stochastic realizations $(\varepsilon_t, \{\hat{\phi}_{i,t}, \hat{\eta}_{i,t}\}_I)$, a log-linear rational expectation equilibrium is a distribution of local prices $\{R_i, W_i\}_I$, global prices (P, R) and relative individual and aggregate quantities such that:*

- *(optimality) agents optimize their actions conditional to the prices they observe;*

- *(market clearing) demand and supply in local markets match,* $L_i = L_i^s$ *and* $K_i = K_i^s$*; the money market clears,* $M_{i,t}^d = \bar{M}_i^s$*; and the endowment market clears* $\int Z_i di = 1$*;*

- *(log-linearity) and log-deviations of individual actions from their equilibrium steady state are linear functions of the shocks.*

The first condition requires agents' actions to be optimal conditional to the information agents infer from the equilibrium prices they observe. Concerning market clearing conditions, notice that I assume that there is a constant amount of island-specific money $\bar{M}_i^s$ available in each island. This implies that the market condition for the consumption good $\int Y_i di = \int C_i di$ obtains from the aggregation of the budget constraints (2). The requirement of a log-linear equilibrium allows for the tractability of the aggregate relations and more importantly, ensures that deviations of global prices from the equilibrium steady state are one-to-one functions of the aggregate shock only. Therefore observing a global price is informationally equivalent to observing the aggregate shock.

Recovering the information sets

Now let us spell out what each type of agent can learn from equilibrium prices. To start, notice that at the first stage the consumer-workers are able to point-wise predict the price of the consumption good that is not observable yet. In fact, at the first stage the consumer-workers and the intermediate producers trade the endowment on a global market, so they are able to infer the only aggregate shock perturbing global prices.

Final producers instead do not trade on any global market in the first stage. Hence they will be uncertain about the consumption price at the time of planning production[6]. In particular, a firm type i acquires input quantities $K_{i,t}(E_t^i\left(P_t\right),$ $W_{i,t}, R_{\,i,t})$ and $L_{i,t}\left(E_t^i\left(P_t\right), W_{i,t}, R_{\,i,t}\right)$ and implements a production $Y_{i,t}$, conditional to the local prices and

$$E_t^i\left(P_t\right) \equiv \mathbf{E}[P_t|\omega_{i,t}], \tag{9}$$

which denotes producer i's expectation about the price of the final good conditional on the information set $\omega_{i,t}$

$$\omega_{i,t} = \{R_{i,t}, W_{i,t}\}, \tag{10}$$

[6]The consumption price does not reveal simultaneously to the production choice. Lack of simultaneity is what makes informational frictions matter. For a deep analysis of the issue see Hellwig and Venkateswaran (2011).

consisting of the equilibrium prices arising from the transactions they carry out during the first stage: local wages and the price for local capital. Therefore the accuracy of the final producers' decisions depends on the informativeness of local equilibrium prices.

Nevertheless producers' uncertainty is solved at the end of the second stage. Once they sell the quantity of consumption good produced in the first stage, the price P is finally observed. Therefore, at the end of the second stage all agents have the same information so that all periods are informationally independent. Moreover, given that shocks are i.i.d. and the supply of money is fixed in each island, consumers' expectations at time t over the future course of the economy is the unique stochastic steady state at each future period. Hence, as in Amador and Weill (2010) the only intertemporal first-order condition - the one for money - collapses to the one-period equilibrium relation

$$\frac{\Lambda_{i,t}}{P_t} = \delta \mathbf{E}\left[\frac{\Lambda_{t+1,i}}{P_{t+1}}\right] + \delta \frac{1}{\bar{M}_i^s} = \frac{\delta}{1-\delta}\frac{1}{\bar{M}_i^s} = 1 \tag{11}$$

where I substituted the market clearing condition $M_{i,t}^d = \bar{M}_i^s$ and normalized to one without loss of generality. This means the signal extraction problem firms face is a static one[7] and only concerns the price of consumption which moves with the aggregate shock. From here onward I will omit time indices as the following relations are all simultaneous.

Part of the information that consumers and intermediate producers hold is transmitted to the final producers through local market transactions. The optimal supply of local labor moves with the preference shock and the nominal wage to satisfy

$$W_i = \Phi_i \left(L_i^s\right)^{\gamma} \tag{12}$$

where the real value of the island-specific multiplier $\Lambda_i/P = 1$ is fixed by (11). Hence, in equilibrium the wage observed by firms type i hiring $L_i = L_i^s$ reveals the preference shock Φ_i affecting the consumer-worker type i. That is, the local wage conveys a *private exogenous signal* of the aggregate shock. Notice that the quantities arising in the local markets type i can be expressed as functions of $E^i(P)$, Φ_i and R_i.[8] In other words, all the observables are measurable with respect to Φ_i and R_i that constitute therefore the finest available information set.

[7]Here a repeated static signal extraction problem is embedded in a dynamic macro model by the only mean of a price system. This avoids the adoption of less natural assumptions usually made in literature like: one-period permanent shocks (as in Amador and Weill (2010)), empty intertemporal markets (as in Angeletos and La'O (2012) where in equilibrium there are no transactions in the bond market), or the usual worker-shopper metaphor (inspired by Lucas (1980)).

[8]To see this one can work with (44a), (44b),(44d) and (44f) in appendix A.1.1.

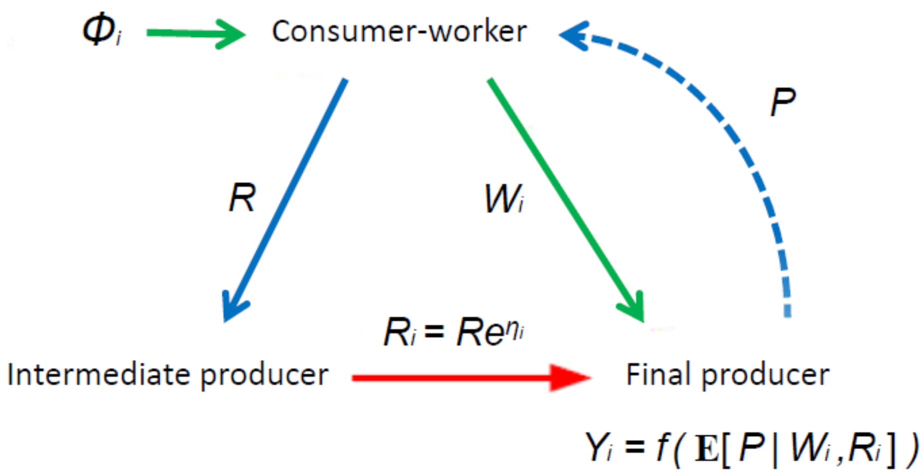

Figure 1: Flows of information in the economy.

The two pieces of information are different in nature. In particular, firm i hires local capital at the equilibrium price

$$R_i = Re^{\hat{\eta}_i} \tag{13}$$

which is a noisy signal of the price for the endowment.[9] In contrast to Φ_i, the price of local capital transmits a *private endogenous signal*, that is a noisy island-specific observation of the price for the endowment which embodies information about both the aggregate shock and producers' expectations which cannot be untangled. Figure 1 summarizes the flows of information in the economy.

Characterization of an equilibrium

All first order conditions in the model have a multiplicative form, so they can be log-linearized and solved without any approximation. In particular, the requirement of a log-rational equilibrium implies that the price (as any other variable in the model) is distributed lognormally according to

$$P = \bar{P}e^{p-\sigma(p)/2}$$

where $\sigma(\cdot)$ denotes the variance operator and $p \sim N(0, \sigma(p))$ is the stochastic log-component of a deviation of P from its stochastic steady state $\bar{P}$, obtained

[9] Any quantity of the island-specific capital is supplied at a price equal to (or more precisely, at the minimum price not smaller than) the global price of the endowment augmented for an i.i.d. productivity disturbance.

as a linear combination of all the shocks. To better enlighten the origins of the multiplicity, let me state the following proposition which enables a characterization of the equilibrium in terms of a profile of firms' expectations about p.

Proposition 2 *Given a profile of weights* $\{\mathbf{e}_{i,1}, \mathbf{e}_{i,2}, \mathbf{e}_{i,3}\}_I$ *such that log-linear expectations for final producers are described by*

$$E^i(P) = \bar{P} e^{E^i(p) - \sigma\left(E^i(p)\right)/2} \tag{14}$$

with[10]

$$E^i(p) = \mathbf{e}_{i,1}\varepsilon + \mathbf{e}_{i,2}\hat{\phi}_i + \mathbf{e}_{i,3}\hat{\eta}_i, \tag{15}$$

then there exists a unique log-linear conditional deviation and a unique steady state for each variable in the model.

Proof. Appendix A.1.2. ■

The characterization of an equilibrium follows straightway once the requirement of rational expectations is imposed.

Definition 3 *A log-linear rational expectation equilibrium is characterized by a profile of weights* $\{\mathbf{e}_{i,1}, \mathbf{e}_{i,2}, \mathbf{e}_{i,3}\}_I$ *such that (14) are rational expectations conditional to the information set (10).*

In practice, an equilibrium is characterized by a distribution of firms' expectations about p, the stochastic component of the consumption price P. Each individual price expectation type i is conditioned to the observation of $\log \Phi_i$ and r_i, denoting the stochastic log-components of respectively Φ_i and R_i observed in the input markets. Both are log-linear functions of the shocks. Hence, a profile of optimal weights given to these two pieces of information maps into a profile of weights $\{\mathbf{e}_{i,1}, \mathbf{e}_{i,2}, \mathbf{e}_{i,3}\}$. That is, the number of equilibria of the model corresponds to the number of solutions to the signal extraction problem.

Before looking at how the two endogenous elements of the inference problem p and r_i move, it is useful to observe that under perfect information there exists a unique equilibrium where the aggregate shock has a pure inflationary effect. This is due to the fact that the aggregate shock alters the ratio between the marginal utility of consumption and money holdings, but not the one between

[10]An other way to see (14) is to start from a log-normal price distribution $P = P^m e^p$ where $p \sim N(0, \sigma(p))$ is normally distributed and P^m is the unconditional median. Then the correct conditional expectation is

$$\mathbf{E}(P\,|\omega_i) = P^m e^{\mathbf{E}(p|\omega_i) + \frac{\mathbf{E}(\sigma(p)|\omega_i)}{2}},$$

which can be rewritten as (14) using the law of total variance $\mathbf{E}(\sigma(p)\,|\omega_i) = \sigma(p) - \sigma(\mathbf{E}(p|\omega_i))$ where remeber $E^i(\cdot) = \mathbf{E}(\cdot|\omega_i)$ and the expression for the steady state is $\bar{P} = P^m e^{\sigma(p)/2}$.

consumption and leisure. When a positive aggregate shock hits, the price must increase in order to decrease the real value of money so that the marginal utility of cash holding can match the increased marginal utility of consumption and leisure. In fact, the stochastic log-linear component of the consumption price is given by

$$p = \varepsilon + \beta \left(E \left(p \right) - \varepsilon \right), \tag{16}$$

with

$$\beta \equiv -\frac{\alpha\psi}{1+\gamma-\alpha},$$

measuring the impact of the aggregate expectation

$$E \left(p \right) \equiv \int E^i \left(p \right) \; di, \tag{17}$$

on the consumption price (for details see appendix A.1.3). Under perfect information $E^i \left(p \right) = p = \varepsilon$, that is the consumption price reacts one-to-one to an aggregate demand shock. In the opposite case of no information, that is with $E^i \left(p \right) = 0$, the labor supply shrinks, but the demand for local labor does not increase since final producers do not foresee any increase in the consumption price. What happens is that the consumption price $p = \left(1 - \beta \right) \varepsilon$ overreacts to the aggregate shock to clear a suboptimal production. Notice that $\beta < 0$, that is, the model replicates the same reduced form of the celebrated Cobweb model (Muth, 1961) for which the impact of the aggregate expectation on the actual price is strictly negative. This occurs because the only uninformed type in the economy are the final producers. As a consequence the aggregate preference shock induces the dynamics of a supply-side shock: an average expectation of an higher consumption price stimulates production which will decrease the actual consumption price.

In analogy with the consumption price, one can express a stochastic log-deviation r_i of the price for local capital R_i from its steady state as

$$r_i = \varepsilon + \varkappa \left(E \left(p \right) - \varepsilon \right) + \hat{\eta}_i \tag{18}$$

with

$$\varkappa \equiv \frac{1+\gamma}{1+\gamma-\alpha}$$

(for details see appendix A.1.3). Notice $\varkappa > 1$. This means that the price of the endowment overreacts to an aggregate expected departure from the perfect information outcome, meaning to an aggregate change in production. In particular, it exhibits opposite reactions in the extreme cases of perfect information ($r = \varepsilon$) or no information ($r = \left(1 - \varkappa \right) \varepsilon$). The underlying economic intuition is simple. When the aggregate shock is perfectly observed, as said above, the demand shock produces a neutral inflationary effect: it moves all global prices at the same rate whereas leaves quantities unchanged. If instead final producers do not expect a positive shock to occur, labor supply shrinks in front of an unchanged labor demand as producers do not foresee variations in the consumption price. As a consequence, in equilibrium final producers hire less labour and the local wage increases less fast than under perfect knowledge. This also determines a fall in the productivity of local capital and indirectly a reduction in the price for the endowment.

Multiplicity

Note that the cases of no information and full information obtain only at the limits of respectively infinite and null volatility of the island-specific shocks. At both limits the economy has a unique equilibrium. The following proposition states a multiplicity result for finite degrees of private uncertainty.

Proposition 4 *If the variance of productivity shocks σ_ϕ satisfies*

$$\sigma_\phi > \frac{1+\gamma}{\alpha} - (1-\psi)\,, \tag{19}$$

then there exists a finite threshold of the cross-sectional variance of preference shocks $\hat{\sigma}_*$ such that for any $\hat{\sigma} \in (0, \hat{\sigma}_*)$ the economy has three determinate REE. A unique equilibrium obtains otherwise for a small enough $\hat{\sigma}$.

Proof. The proof directly follows from proposition 9 which is derived in the next section. That proposition establishes that the fix point equation of the signal extraction problem on (16) when observing (8) and (18) has three distinct fix points $(\mathbf{a}, \mathbf{b})$ which pin down three different triples ($\mathbf{e}_{i,1} = (\mathbf{a}+\mathbf{b})\,(1-\varkappa)/\,(\varkappa - \mathbf{b}\varkappa)$, $\mathbf{e}_{i,2} = \mathbf{a}\,(1-\varkappa)\,/\varkappa$, $\mathbf{e}_{i,3} = \mathbf{b}/\varkappa$) for each i, where $\mathbf{a}$ and $\mathbf{b}$ are rescaled optimal weights put respectively on (8) and (18) yielding rational expectations of (16). ■

The proposition states that, provided the precision of the exogenous information is below a certain threshold, *small enough* private uncertainty on the price of the endowment generates a multiplicity of equilibria. In other words, a marginal perturbation away from perfect knowledge invalidates the uniqueness of the equilibrium. Moreover the *more precise* is endogenous information conveyed by the price of the local capital, the more likely is that the economy exhibits a multiplicity of equilibria, whereas uniqueness is restored with large private uncertainty.

Explaining a multiplicity: allocation vs. information

To grasp intuition on the economic mechanism let us focus on the local market for capital. Remember that the aggregate quantity exchanged on the market is fixed by the constant supply of endowment. Hence, a higher (or lower) price reflects just a higher (or lower) desire for local capital. For the rest of this section consider a positive aggregate shock. Imagine firms see on average their local price initially increasing. On one hand, a negative *allocational* effect is always in play: an increasing cost discourages capital demand. On the other hand, when a rise in the price for local capital is positively correlated with an increase in the value of production[11], a positive *informational* effect shifts capital demand onwards contrasting the allocational effect . Nevertheless the sign of the correlation is an equilibrium outcome depending on the extent to which the local price reflects islands-specific disturbances rather than the underlying price of the endowment.

In fact, there exists an equilibrium where the informational effect is negative, that is the prices of consumption and local capital are negatively correlated. It arises when local prices reflect mainly islands-specific disturbances. In this case, despite a positive aggregate shock, the demand for capital - and as a consequence its price - decreases since firms are not confident in their price predictions, that is, the allocational effect dominates. Moreover this equilibrium always exists no matter how small is the variance of the productivity shocks. In particular, for a high-enough allocational reaction the variance of the endowment price squeezes so much that the price for local capital becomes sufficiently uninformative and the allocational effect self-fulfills its (first-order) dominance. Only at the limit of zero variance of the productivity shocks - where by definition the allocational effect cannot prevail as the informational effect is maximal (full revelation) - this equilibrium vanishes.

A positive informational effect arises instead for sufficiently low variance of the productivity shocks - i.e. with small-enough private uncertainty. In this case, there are two equilibria where the endowment price rises because on average firms are confident in an increase of their production value and so they want to increase their production. These two equilibria differ for which effect drives the response to a marginal increase in the local price. In one equilibrium the positive informational effect prevails: firms would ask more capital pushing the local price further away. In the other one instead the allocational effect (second-order) dominates: the opposite stabilizing marginal response occurs. In other words, whereas in the latter a further increase in informational precision is not paid back by an higher average increase in the price for local capital, in the former it is. Of course, the unique equilibrium surviving in the limit of complete information - i.e. at the limit of zero cross-sectional variance of productivity shocks - is the one where the informational effect dominates. In fact, only when the assumption of perfect information is marginally relaxed the allocational effect can play a (first or second-order) dominant role yielding the kind of non-linearity needed for a multiplicity.

[11] A positive aggregate shock always increases the consumption price because of $\beta < 1$.

Finally, condition (19) is a restriction concerning the presence of exogenous information flowing form the market for local labor. It implies that a multiplicity is made easier as the exogenous information is *less precise*, specifically, when the cross-sectional volatility of the exogenous signal σ_ϕ is above a certain threshold. Notice that such threshold is defined as a simple combination of the CES parameters of the utility function and technology process. In particular, this combination is an index of the convexity of the problem: it is 0 with $\alpha = 1$ and $\psi = \gamma = 0$, whereas it approaches infinity when $\alpha = 0$, or $\psi \to \infty$ or $\gamma \to 0$. The informational effect is reinforced as the model approaches linearity because along this direction endogenous variables become more reactive to shocks. This implies that, ceteris paribus, the less convex the problem, the higher the lowest cross-sectional variance of endogenous signals at which a multiplicity arises. This condition must be interpreted as a condition on the relative informativeness of exogenous versus endogenous information where the latter depends on the shape of incentives underlying agents' choices.

3 Analysis of the signal extraction problem

This section analyzes the general class of linear and static signal extraction problems embodied in the model above. The exposition aims to be self-contained so that no reference to the specific model is strictly needed.

The problem is the following: a continuum of agents $i \in (0, 1)$ have to forecast an aggregate endogenous state, let say a price

$$p = \varepsilon + \beta \left(E \left(p \right) - \varepsilon \right), \tag{20}$$

reacting linearly to an exogenous normally distributed disturbance $\varepsilon \sim N(0, 1)$ and the aggregate expectation $E(p) \equiv \int E^i(p)\ di$ where $E^i_t(p_t) \equiv \mathbf{E}[p_t|\omega_i]$ is an individual expectation conditional to the set of signals held by agent i.

The parameter β measures the nature and impact of the payoff externalities. For $\beta = 0$ the price process is completely exogenous. In this case an incentive to use signals of the aggregate supply shock can only concern its fundamental content. Examples of signal extraction problems of this kind are found in Amador and Weill (2010), Desgranges and Rochon (2011) and Ganguli and Yang (2009). For $\beta \neq 0$ instead, the price moves with the average expectation, so that also pay-off externalities are involved as in Morris and Shin (2002) and subsequent literature[12]. With this in mind, I consider all values $\beta < 1$ in order to provide

[12]To reshape the problem in the usual coordination framework with quadratic utility function, assume there is a continuum of agents divided into two types i and j choosing an action in the real domain, respectively x_i and x_j. Type i is a fraction β of the population and has utility $U_i = -(x_i - \bar{x})^2$ where $\bar{x}$ is the average action across the whole population. Agents type

results that are directly applicable to a larger class of economies. Cases of extreme degrees of expectational complementarity providing for $\beta > 1$ are finally discussed in the last section of this paper.

Agents have available some endogenous and exogenous information to forecast this price. To better illuminate the role played by each one I will proceed in two subsequent steps. First, I will consider the presence of a single private signal of an endogenous state. Second, I will build on this result extending the analysis to the case where both endogenous and exogenous information are available. Finally, I will provide additional results which put in relation the main finding of this paper with others in the literature.

3.1 Endogenous information

A private endogenous signal

For the moment, suppose each agent holds a single private endogenous signal[13]

$$r_i = \varepsilon + \varkappa \left(E\left(p\right) - \varepsilon\right) + \hat{\eta}_i \tag{21}$$

representing a noisy observation of an aggregate endogenous state, let say an other price. This can be rescaled by $\varkappa$ to obtain and equivalent signal

$$\omega_i = \{E\left(p\right) + \zeta^{-1}\varepsilon + \eta_i\}, \tag{22}$$

which for the purpose of this section represents the information set of agent i. The equivalence obtains defining $\eta_i \equiv \varkappa^{-1}\hat{\eta}_i \sim N\left(0, \zeta^{-2}\sigma\right)$, so that $\sigma = \left(1 - \varkappa\right)^{-2}\hat{\sigma}$, and $\zeta \equiv \varkappa / \left(1 - \varkappa\right)$. In particular notice that $\varkappa > 1$ implies $\zeta < -1$. This notation is particular convenient to directly enlighten two parameters of crucial importance: ζ and σ. The latter represents the variance of the private noise whereas the former the covariance of the fundamental component with the aggregate shock ε, both expressed in terms of the variance of the fundamental component ζ^{-2}. The limit values $\sigma \to \infty$ and $\sigma \to 0$ entail the extreme situations where informational heterogeneity vanishes and agents have respectively no information and perfect information on the fundamental realization. All the intermediate cases consist of dispersed information.

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j have utility $U_j = -\left(x_j - \varepsilon\right)^2$ where ε is an exogenous realization drawn from a normal prior. Assume that only agents j know the realization ε. In equilibrium, the average action is $\bar{x} = \varepsilon + \beta\left(\bar{x}^e - \varepsilon\right)$, where $\bar{x}^e$ is the average expectation of the average action across agents type i.

[13] In terms of the previous footnote this assumption is equivalent to assume that agents type j see a signal $s_i = \varepsilon + \varkappa\left(\bar{x}^e - \varepsilon\right) + \eta_i$, where η_i is a private white noise disturbance. This signal can also be rewritten as a private signal $r + \eta_i$ of the aggregate action $r = \bar{x} + \phi\left(\bar{x} - \varepsilon\right)$, with ϕ such that $\left(1 + \phi\right)\beta = \varkappa$.

The actual law of motion

A private signal about an aggregate endogenous variable provides information on the unknown fundamental but also on second-order agents' beliefs. The two pieces of information cannot be identified separately because when agents use heterogeneous information - that in this case means they put weight on the signal itself - then the problem of forecasting the forecasts of others is in play. As a result, the signal generates non trivial feedback informational effects. Once agents collectively put weight on the signal then the aggregate expectation reacts to it, and so the precision of the signal itself is affected by the use of the signal.

To impose order in the analysis let us fix a liner forecasting strategy. Notice that, when the random variable to be forecasted is normally distributed - and here it is the case - a linear forecasting strategy is the optimal one as it correctly identifies the first and second moment of the objective conditional distribution. Agent i's forecast is written as

$$E^i(p) = b_i \left(E(p) + \zeta^{-1}\varepsilon + \eta_i \right), \tag{23}$$

where b_i is a constant coefficient to be determined that weights the expectational signal. In other words, agent type i expects a displacement of the actual price from the deterministic equilibrium that is proportional to the signal as defined in (22). If all agents use the rule above then by definition (17) the aggregate expectation is

$$E(p) = \frac{\mathbf{b}}{1-\mathbf{b}}\zeta^{-1}\varepsilon, \tag{24}$$

where $\mathbf{b} \equiv \int b_i di$ is the average weight across agents. Therefore an individual expectation can be rewritten as

$$E^i(p) = b_i \left(\frac{1}{1-\mathbf{b}}\zeta^{-1}\varepsilon + \eta_i \right), \tag{25}$$

where the signal is now expressed as a function of exogenous shocks depending on the average weight. In fact, the collective strategy of weighing the expectational signal according to (23) has a non-linear effect on the variance of the fundamental component of the signal as it is shown by (25). This happens because the aggregate shock does not vanish in the aggregation feeding back into the aggregate expectation and in turn into the signal, coming full circle. Nevertheless, the variance of the private component is never affected. Hence, the informativeness of the signal of the fundamental innovation - as well as the overall variance of the signal itself - changes non linearly with the average weight.

Plugging (25) in (20), we finally obtain the actual law of motion of the price

$$p = \varepsilon + \beta \left(\frac{\mathbf{b}}{1-\mathbf{b}}\zeta^{-1}\varepsilon - \varepsilon \right), \tag{26}$$

as functions of the average weight and the aggregate shock only. Importantly the correlation between the signal and the price can take either sign depending on the extent of $\mathbf{b}$. Therefore, different combinations of variances and correlation are in principle possible depending on the average weight given to the signal. This very feature creates room for the emergence of a multiplicity of equilibria.

It is worth noticing here that the law of motion for the price has been obtained without guessing any a-priori form, but just using definitions and temporary equilibrium conditions. This means that these relations are still valid for *disequilibrium* beliefs, that is they entail the price course given an arbitrary profile of weights restricting agents' expectations.

The set of equilibria

The course of the economy is entirely determined by (25)-(26) for a given profile of individual weights. A rational expectation equilibrium (REE) obtains when agents' beliefs are consistent with the actual conditional distribution of price fluctuations according to (9). In other words, the forecast error of each agent has to be orthogonal to the available information. This implies a restriction on the profile of individual weights $\{b_i\}_I$. The orthogonality restriction[14] entails what I will call the *best individual weight function*

$$b_i(\mathbf{b}) = \underbrace{\frac{\zeta(1-\mathbf{b})}{1+\sigma(1-\mathbf{b})^2}}_{\text{fundamental determinant}} + \underbrace{\frac{\beta(\mathbf{b}-\zeta(1-\mathbf{b}))}{1+\sigma(1-\mathbf{b})^2}}_{\text{pay-off determinant}} \tag{27}$$

provided $\mathbf{b} \neq 1$, that is the optimal weight that each agent must put on his own expectational signal as a function of the average weight $\mathbf{b}$. It is instructive to distinguish between two components determining the best individual weight. The first and second term on the right hand side reflect the informativeness of the signal about respectively the fundamental shock and a deviation of the average expectation from the perfect information outcome. The latter interacts with the former when the price moves with the aggregate expectation (in the case $\beta \neq 0$) and so generates pay-off externalities in the signal extraction problem. Notice that both this effects are stronger as the precision of the signal increases, that is as σ decreases.

An equilibrium requires that (27) holds for each agent so that an optimal value obtains imposing $b_i = \mathbf{b}$. In particular an equilibrium value of $\mathbf{b}$ determines an aggregate expectation and in turn an equilibrium in the economy as the shocks unfold. The set of the REE of the economy is therefore characterized by the following fix point equation

$$\mathbf{b}^3 - 2\mathbf{b}^2 + \left(1+\sigma^{-1}(1-\beta)(1+\zeta)\right)\mathbf{b} - \sigma^{-1}(1-\beta)\zeta = 0 \tag{28}$$

[14]Which is $\mathbf{E}\left[\left(\frac{1}{1-\mathbf{b}}\zeta^{-1}\varepsilon+\eta_i\right)\left((1-\beta)\varepsilon+\frac{\beta\mathbf{b}}{1-\mathbf{b}}\zeta^{-1}\varepsilon - b_i\left(\frac{1}{1-\mathbf{b}}\zeta^{-1}\varepsilon+\eta_i\right)\right)\right]=0$.

that is, the the locus of the fix points of (27). The following proposition states analytical conditions for a multiplicity of REE.

Proposition 5 *For a $\beta < 1$, if $\zeta < -1$ (that is with $\varkappa > 1$) then there always exists a threshold σ_* that is monotonically decreasing in both ζ and β such that for any $\sigma \in (0, \sigma_*)$ equation (28) has three real solutions, whereas it has a unique real solution otherwise.*

Proof. Postponed in Appendix A.2.2. ■

The result just obtained does not consider the effects of exogenous signals, nevertheless, it embodies all the key determinants beyond the multiplicity result in the previous section. In what follows I will spell out the core insights originated by the presence of endogenous private uncertainty. The next subsection will show instead how the joint presence of exogenous uncertainty changes the picture.

The fundamental determinant

The core mechanism beyond multiplicity can be discussed looking at the particular case $\beta = 0$ when a pure fundamental determinant is at work and pay-off externalities do not matter. Figure 2 plots the individual weight function $b_i(\mathbf{b})$ in for $\beta = 0$, $\zeta = -4$ (that is $\varkappa = 4/3$) and different values of σ namely 0.5 (dashed line), 1 (solid line) and 5 (dotted line). It is a cubic function taking the value 0 at $\mathbf{b} = 1$ and in the limits of $\mathbf{b} \to \pm\infty$. It is positive with $\mathbf{b} > 1$ and negative otherwise. The first derivative is zero at $\mathbf{b} \to \pm\infty$ and $-\zeta$ at $\mathbf{b}$ =1. In particular notice as $\sigma \to \infty$ then the curve approaches the x-axis, whereas as $\sigma \to 0$ the curve approaches the line $\zeta - \zeta\mathbf{b}$.

When the private endogenous signal is very noisy - that is when private uncertainty is high enough - the optimal weight put on such a signal must be negative because the average signal and the aggregate shock go in opposite directions. This shows up in the picture for a σ high enough: the curve is sufficiently close to the x-axis so that an equilibrium $\mathbf{b}$ only arises in the negative quadrant. As σ marginally shrinks, this weight must further decrease as the signal becomes marginally more informative. That is, the best individual weight function is further away from the x-axis and closer to the line $\zeta - \zeta\mathbf{b}$ before approaching zero as $\mathbf{b} \to \pm\infty$. The latter intersects the bisector in the positive quadrant only if $-\zeta > 1$ - i.e. $\varkappa > 1$ - that is therefore a necessary condition to obtain a multiplicity. In particular, two other equilibria exist for a σ small enough where the average signal is positively correlated with the shock. Such equilibria feature high expectational complementarity as the average expectation co-moves with the average signal which in turn feeds back into expectations.

With perfect knowledge, one positive and one negative equilibrium are fated to vanish and only one positive equilibrium survives in which the average signal

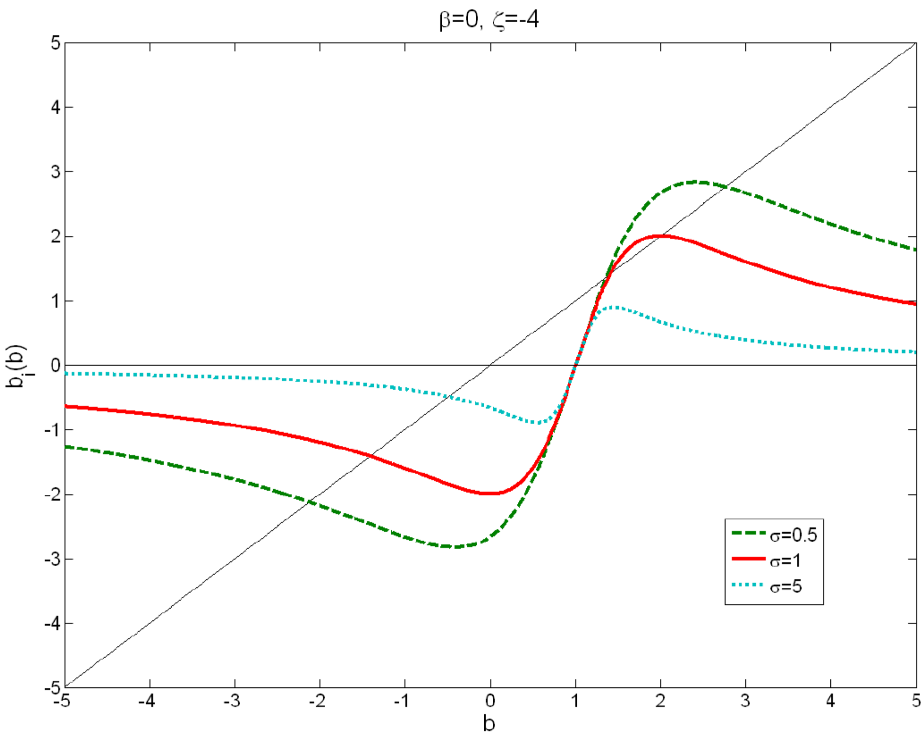

Figure 2: Plot of the best individual weight for different values of σ.

increases at the rate ε. To sum up, whenever the average endogenous private signal reacts in opposite ways in the unique-equilibrium limits $\sigma \to \infty$ and $\sigma \to 0$, a continuous transition between the two necessarily occurs with a bifurcation leading to a multiplicity in a neighborhood of $\sigma \to 0$. This implies the following remark.

Remark 6 *The fundamental determinant alone, that is when pay-off externalities are null ($\beta = 0$), can be sufficient to sustain a multiplicity of equilibria provided $\zeta < -1$ ($\varkappa > 1$).*

Figure 2 gives also insights on the out-of-equilibrium properties of the multiple equilibria generated by the signal extraction problem. In particular, with a small enough degree of private uncertainty, the rational expectation equilibrium in the middle is not strongly rationalizable in the sense of Guesnerie (1992, 2005). In fact, if agents expect that the average weight on the endogenous signal is in a neighborhood of that equilibrium then their best individual weight must be further away from the equilibrium (notice $\lim_{\sigma \to 0} b_i' (\mathbf{b}) = -\zeta > 1$). But since this is common knowledge, a second-order rational belief on the average weight must equally lie further away from the equilibrium, etc. In other words, this equilibrium cannot be obtained as a singleton from a rationalizability process. Importantly notice that this equilibrium is the one surviving under perfect knowledge. Rationalizability and multiplicity are two deeply interconnected phenomena. Small degrees of private uncertainty could generate two new equilibria which could work instead as absorbing points of a rationalizability dynamic. This conjecture surely deserves a closer assessment that will be the object of a future work. I prefer to keep the present work focused on the issue of the existence of multiple equilibria. At the same time it is worth alerting the reader of the possibility that the closest equilibrium to the one under perfect knowledge could not be the one selected by a process of iterated deletion of never best replies.

Interaction with the pay-off determinant

Consider now how changes in β can affect the number of equilibria. The existence of a unique fix point in the region $\mathbf{b} < 1$ is not affected because the curve has the same qualitative behavior for $\beta < 1$. To assess the existence of a multiplicity one has to check how the best individual weight function moves with β for values $\mathbf{b} > 1$. Since $\zeta < -1$ the quantity $\mathbf{b} - \zeta (1 - \mathbf{b})$ is lower than one for $\mathbf{b} > 1$ and it is linearly decreasing in $\mathbf{b}$. Hence, for a positive β this determinant is pro-multiplicity (increases $b_i (\mathbf{b})$) only for moderate increase of $\mathbf{b}$ beyond one. Nevertheless, for the case $\beta \to 1$, where this impact is maximal, the two multiple equilibria collapses on the single unfeasible limit point $b_i (\mathbf{b}) = \mathbf{b} = 1$. Therefore, given the monotonicity of the pay-off determinant, no strictly positive value of β below the unity can sustain a multiplicity unless the fundamental determinant is not already sufficient to generate it. On the other hand, for negative values of β, provided $\mathbf{b}$ is large enough the determinant becomes pro-multiplicity, and more importantly, can be arbitrarily large for a β low enough. That is, a multiplicity can arise for a β low enough even in the case the fundamental determinant is not sufficient alone. This feature is illustrated in figure 3 whose discussion is postponed to next subsection. The following remark finally summarizes the contribution of the pay-off determinant.

Remark 7 *Pay-off substitutability ($\beta < 0$) promotes multiplicity whereas sub-unitary ($\beta \in (0,1)$) pay-off complementarity does not.*

3.2 Endogenous and exogenous information

Adding a private exogenous signal

This section extends the previous analysis to the original case presented in the microfounded model where agents deal with both endogenous and exogenous imperfect information. Consider the case agents observe both an endogenous signal (21) and an exogenous signal $\varepsilon + \hat{\phi}$ which constitute a new information set

$$\omega_i = \{E(p) + \zeta^{-1}\varepsilon + \eta_i,\ \zeta^{-1}\varepsilon + \phi_i\} \tag{29}$$

where $\phi_i \equiv \zeta^{-1}\hat{\phi}_i \sim N\left(0, \zeta^{-2}\sigma_\phi\right)$. The strategy of the analysis mimics the one already discussed, so I will proceed more quickly through the same steps: I fix a linear forecasting rule, I recover the law of motion for the price, and finally I characterize the conditions for the existence of a multiplicity of REE.

The actual law of motion

In this case, agents have two possibly correlated pieces of information. Their forecasting strategy is written as

$$E^i(p) = a_i\left(\zeta^{-1}\varepsilon + \phi_i\right) + b_i\left(E(p) + \zeta^{-1}\varepsilon + \eta_i\right), \tag{30}$$

where a_i and b_i are constants weighting respectively the exogenous and the endogenous signal. Since all agents use the rule above then by definition of aggregate expectation it is

$$E(p) = \frac{\mathbf{a} + \mathbf{b}}{1 - \mathbf{b}}\zeta^{-1}\varepsilon, \tag{31}$$

where $\mathbf{a} \equiv \int a_i di$ and $\mathbf{b} \equiv \int b_i di$ are the average weight across agents. An individual expectation can be rewritten as

$$E^i(p) = a_i\left(\zeta^{-1}\varepsilon + \phi_i\right) + b_i\left(\frac{\mathbf{a} + 1}{1 - \mathbf{b}}\zeta^{-1}\varepsilon + \eta_i\right) \tag{32}$$

and the actual law of motion of the market price (16) is given by

$$p = (1-\beta)\,\varepsilon + \beta \frac{\mathbf{a}+\mathbf{b}}{1-\mathbf{b}} \zeta^{-1} \varepsilon, \tag{33}$$

as functions of weights and exogenous shocks only. As before, the law of motion for the price has been recovered without using any a-priori guess on the form of the aggregate law.

The set of equilibria

Rational expectations imply restrictions on both the profile of $\{a_i\}_I$ and $\{b_i\}_I$. Spelling out the orthogonality conditions we can express the locus of REE as the profile $\{a_i = \mathbf{a}, b_i = \mathbf{b}\}_{i \in I}$ that is a solution to the following fix point equation

$$\begin{aligned}(1-\beta)\,(\mathbf{a}+1)\,(1-\mathbf{b})\,\zeta + \beta\,(\mathbf{a}+\mathbf{b})\,(\mathbf{a}+1) + \\ -\,\mathbf{a}\,(\mathbf{a}+1)\,(1-\mathbf{b}) - \mathbf{b}\left((\mathbf{a}+1)^2 + (1-\mathbf{b})^2\,\sigma\right) = 0,\end{aligned}$$

with

$$\mathbf{a} = \frac{(1-\beta)\,(\zeta\,(1-\mathbf{b}) - \mathbf{b})}{\mathbf{b} - \beta + (1+\sigma_\phi)\,(1-\mathbf{b})}.$$

The relation above entails an equation of the fifth degree in $\mathbf{b}$. Nevertheless one can divide the both sides by $-(1-\mathbf{b})^2\,(1-\beta+(1-\mathbf{b})\,\sigma_\phi)^{-2}$, ruling out the unfeasible solution $\mathbf{b}=1$ and reducing the problem to the study of the following cubic fixed-point equation

$$\Phi_1 \mathbf{b}^3 + \Phi_2 \mathbf{b}^2 + \Phi_3 \mathbf{b} + \Phi_4 = 0, \tag{34}$$

with

$$\begin{aligned}\Phi_1 &\equiv \sigma\sigma_\phi^2, \\ \Phi_2 &\equiv -\,(1+\sigma_\phi-\beta)\,2\sigma\sigma_\phi, \\ \Phi_3 &\equiv (1-\beta)\,(1+\zeta)\,\sigma_\phi\,((1-\beta)\,(1+\zeta)+\sigma_\phi) + (1-\beta+\sigma_\phi)^2\,\sigma, \\ \Phi_4 &\equiv -\,((1-\beta)\,(1+\zeta)+\sigma_\phi)\,(1-\beta)\,\zeta\sigma_\phi.\end{aligned}$$

So, as before, an eventual multiplicity would concern the existence of not more than three equilibria. Moreover, as expected, the fix equation (34) is equivalent to the one previously studied in the limit case of $\sigma_\phi \to \infty$ in which the exogenous signal is not informative on the aggregate shock. Therefore, we can state the following as a corollary of proposition 5.

Corollary 8 *At the limit $\sigma_\phi \to \infty$ the parameter region where a multiplicity of equilibria arises corresponds to the one characterized in proposition 5.*

At the two limit cases $\sigma_\phi \to 0$ and $(\sigma, \sigma_\phi) \to (0,0)$ it is easy to check that the system has a unique equilibrium respectively at $(\mathbf{a} = \zeta, \mathbf{b} = 0)$ and $(\mathbf{a} = 0, \mathbf{b} = \zeta / (1+\zeta))$, where both solutions entail a unitary weight on the aggregate shock. The proposition below establishes all the intermediate cases.

Proposition 9 *For a $\beta < 1$, if $\zeta < -1$ (that is with $\varkappa > 1$) and*

$$\sigma_\phi > -(1+\zeta)(1-\beta), \tag{35}$$

then there exists a threshold σ_* that is monotonically decreasing in ζ and β such that for any $\sigma \in (0, \sigma_*)$ equation (34) has three real and distinct solutions. A unique real solutions obtains otherwise for a small enough σ.

Proof. Postponed in Appendix A.2.3. ■

The proposition rules the original case entailed by the microfounded model and can be just transposed in terms of proposition 4 as explained in that proof. It confirms the intuition that the introduction of exogenous information makes the conditions for a multiplicity more stringent. Still the potential of an endogenous signal to generate multiple equilibria increases with its precision. That is, the lower is σ the larger is the area were a multiplicity arises, or equivalently, the conditions for a multiplicity of equilibria are tighter with more endogenous information.

Figure 3 shows a numerical exploration of the parametric space $\zeta < -1$ and $\beta < 1$ for some calibration of σ and σ_ϕ. Each box illustrates one different case among $\sigma = (1, 0.1, 0.01)$ whereas in all are considered the values $\sigma_\phi = (10, 20, 30, \infty)$. The white area is the one where a multiplicity arises for any of these σ_ϕ-values. With darker grey is denoted the area in which a multiplicity arises for increasing values of σ_ϕ, with the exception of the darkest one where a multiplicity never arises. The border line between the white and the darkest region represents the locus of calibrations for which a strictly smaller σ is necessary to obtain a multiplicity. All the other borders instead denote the lower bounds of the multiplicity area for the different σ_ϕ values.

3.3 Additional results

The framework developed above can used to investigate a full range of static signal extraction problems beyond the specific case provided by the model. In particular, it is possible to show that the same fixed-point equation (34) can deliver a multiplicity also for values $\zeta \geq -1$. Nevertheless in such a case the cross-sectional variance of endogenous signals must lie in between two strictly positive boundaries. That is, for $\zeta \geq -1$ a multiplicity cannot obtain as a marginal perturbation from perfect knowledge. The following states this result.

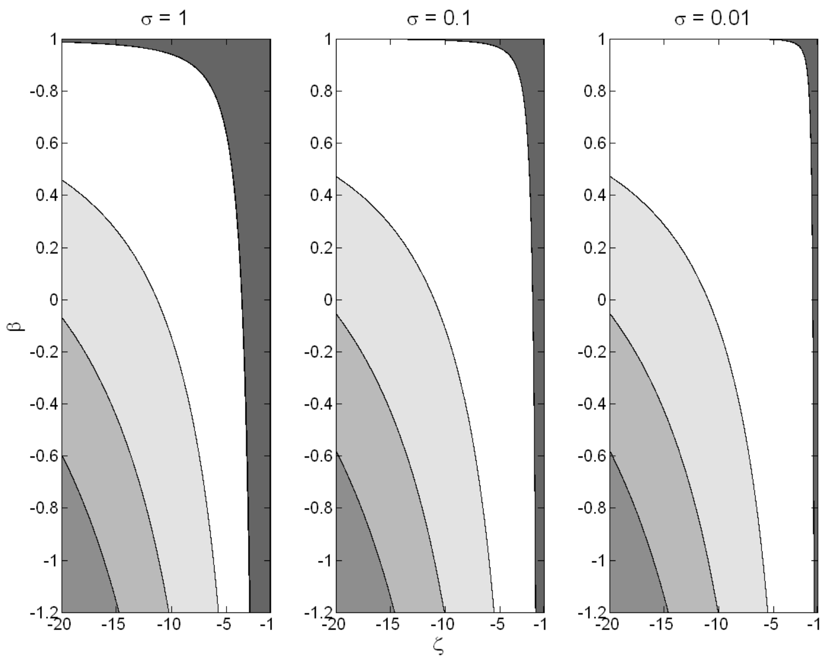

Figure 3: Multiplicity regions in the space $\zeta < -1$ and $\beta < 1$ for different calibrations of σ and σ_ϕ.

Proposition 10 *For a $\beta < 1$ and a $\zeta \geq -1$ (that is with $\varkappa \leq 1$), if σ_ϕ is such that*

$$\sigma_\phi > \frac{8\left(1+\zeta\right)}{\left(\zeta - 8\right)}\left(1-\beta\right), \tag{36}$$

then there exists a compact region such that three determinate REE exist for any σ lying in between two strictly positive boundaries

$$0 < m\left(\zeta, \beta, \sigma_\phi\right) < \sigma < M\left(\zeta, \beta, \sigma_\phi\right), \tag{37}$$

where $\lim_{\sigma_\phi \to \infty} m\left(\zeta, \beta, \sigma_\phi\right) = 3\left(1+\zeta\right)\left(1-\beta\right)$ *and* $\lim_{\sigma_\phi \to \infty} M\left(\zeta, \beta, \sigma_\phi\right) = 27\zeta\left(1-\beta\right)/8$.

Proof. Postponed in Appendix A.2.4. ■

This region is the one to which equilibria of the kind found in literature by Angeletos, Lorenzoni and Pavan (2010), and Amador and Weill (2010) and others belong to. Differently from the equilibria characterized in proposition 8 and 9 these equilibria *disappear* for a small enough degree of cross-sectional variance of endogenous signals. Moreover given σ the region in the parametric space $\left(\zeta, \beta, \sigma_\phi\right)$ that satisfies (36)-(37) is wider as σ increases, that is, conditions for a multiplicity weaken as private uncertainty increases. Notice that (36) requires $\zeta > 8$ that is $\varkappa \in \left(8/9, 1\right)$. This situation corresponds to one in which

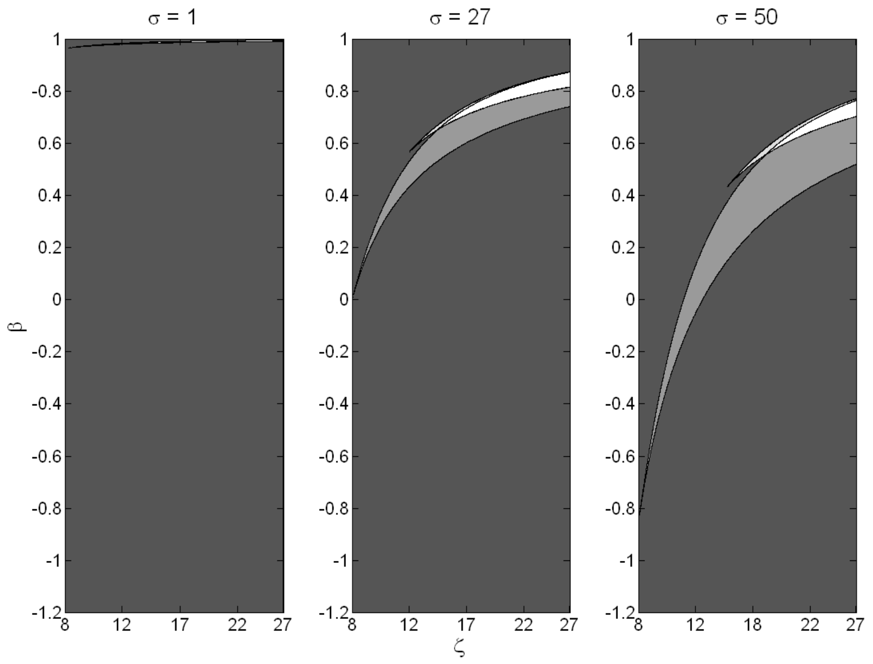

Figure 4: Multiplciity regions in the space $\zeta > 8$ and $\beta < 1$ for different calibrations of σ and σ_ϕ.

agents have available noisy private signals about an aggregate state that is very sensitive - *but does not overreact* - to an aggregate expected departure from the perfect information outcome[15].

Figure 4 shows a numerical exploration of the parametric space $\zeta > 8$ and $\beta > 1$ for some calibration of σ and σ_ϕ. Each box illustrates one different case among $\sigma = (1, 27, 50)$. The light dark area is the one where a multiplicity arises for $\sigma_\phi \to \infty$, the white one is where a multiplicity arises for $\sigma_\phi = 10$, the darkest one that is reserved for the area where a multiplicity never arises.

4 Multiplicity with perfect knowledge of fundamentals

In this section I will extend the model to isolate an extreme case in which endogenous signals can generate a multiplicity of equilibria even with common

[15]In ALP and AW the endogenous signal reacts to the average expectation (of the type of agent observing the signal) according to respectively $\lambda \in (0, 1)$, the probability of a liquidity shock, and $\delta / (1 + \delta)$ where $\delta > 0$ is the Frisch elasticity.

knowledge of the aggregate shock ε. This result is due to strong pay-off externalities that arise with $\beta > 1$, a region that concludes the possible range of cases for the class of static and linear signal extraction problems studied in this paper.

A fiscal transfer on current consumption

In this section I will introduce a particular rule for a fiscal transfer on current consumption with the only aim of studying a particular case of signal extraction problem. The consumption acquired at time t are taxed/subsidized by

$$\tau_t = \left(\frac{P_t}{\bar{P}}\right)^{-\varphi}$$

where φ is a policy parameter and $\bar{P}$ is the steady state of the consumption price. For $\varphi = 1$ for example $\tau_t C_i P_t = C_i \bar{P}$, that is the authority guarantees a subsidy in case of an increase of the consumption price such that the actual nominal cost of consumption at time t is constant and always equal to its steady state. With $\varphi > 0$ the policy is destabilizing as it increases demand when the price is higher than the steady state, so that the price is pushed even further up. With this specification, the actual law of motion of the consumption price is given by (see Appendix A.1.3.)

$$p = \frac{1}{1-\varphi}\varepsilon + \beta_\varphi \left(E\left(p\right) - \frac{1}{1-\varphi}\varepsilon\right), \tag{38}$$

where, crucially, the expectational feedback

$$\beta = \frac{-\alpha\psi}{\left(1+\gamma-\alpha\right)\left(1-\varphi\right)},$$

can now be positive and specifically greater than one in the case

$$1 < \varphi < 1 + \frac{\alpha\psi}{1+\gamma-\alpha}. \tag{39}$$

With $\beta > 1$ the consumption price itself reacts in opposite directions to an aggregate nominal shock in the two cases of no information ($E\left(p\right) = 0$) and perfect foresight ($E\left(p\right) = \left(1+\varphi\right)^{-1}\varepsilon$). In other words the consumption tax amplifies the effect of aggregate expected deviations from the perfect information outcome on the actual price.

Timing and information

To illuminate the main point I will assume some extreme informational assumptions. Suppose that now producers cannot simultaneously condition their expectations to the prices arising in the market; information is sticky so they must fix a demanded quantity before the input markets open. Nevertheless, assume now that firms know the aggregate realization of the preference shock ε and also they can look at some qualitative expectation surveys on price expectations of the kind often published or commented by monetary authorities. In particular, I model the latter as an endogenous signal of producers' forecasts written as $s_i \equiv \{E(p) + \xi + \eta_i\}$, where $\xi \sim N(0,1)$ is an independently-drawn noise that is common to private signals across agents, whereas $\eta_i \sim N(0,\sigma)$ is an idiosyncratic individual-specific component. The common shock represents a statistical measurement error in the survey and the individual-specific one could be the result of genuine private interpretation. The information set can be now written as

$$\omega_i = \{\varepsilon,\ E(p) + \xi + \eta_i\}.$$

Two features matter here: first, both observational shocks are mutually independent and identically distributed in time and across agents; second, ξ represents a non-fundamental component. The aggregate component of the monetary shock ε is actually all the fundamental information agents need. In other words, there is no uncertainty on fundamentals in the model. Nevertheless, I will demonstrate that expectational complementarities can be so strong that non-fundamental equilibria are self-fulfilled by the collective use of correlated private endogenous signals.

The actual law of motion

As before, let us consider a linear forecasting rule,

$$E^i(\pi) = b_i\left(E(\pi) + \xi + \eta_i\right), \tag{40}$$

where $\pi \equiv p - (1-\varphi)^{-1}\varepsilon$ labels the distance of the actual price from $(1-\varphi)^{-1}\varepsilon$, the fundamental value that is now known. Again b_i denotes the coefficient weighting the expectational signal. Given that all agents follow the same strategy, by definition, one can write the average expectation as

$$E(\pi) = \frac{\mathbf{b}}{1-\mathbf{b}}\xi, \tag{41}$$

with $\mathbf{b} \neq 1$, where again $\mathbf{b}$ is the average weight across the population. We can then rewrite the forecasting rule of the agents

$$E^i(\pi) = b_i\left(\frac{1}{1-\mathbf{b}}\xi + \eta_i\right), \tag{42}$$

and the actual law of motion for the consumption price

$$\pi = \beta\frac{\mathbf{b}}{1-\mathbf{b}}\xi,$$

as non-linear functions of the exogenous shocks and the parameters only. Exactly as in the first case discussed in this paper, the equilibrium of the economy is entirely determined by a profile $\{b_i\}_I$.

The set of equilibria

The equilibrium is now the same as stated by definition 1 with due correspondences and it is now characterized by a profile of weights $\{b_i\}_I$ such that (40) are rational expectations of p. It is easy to check that the *fundamental equilibrium* - that is the one in which everybody puts a zero weight on the expectational signal - is always a REE. That is intuitive because firms already have all the fundamental information they need. Nevertheless equilibria different from the fundamental one - for which it is optimal for producers to put weight on the expectational signal - cannot be a-priori excluded. To obtain a closed form for the optimal b_i for a given $\mathbf{b}$ one needs to spell out the orthogonality conditions to pin down the set of optimal $\{b_i\}_I$. Imposing zero covariance between the expectational signal and the forecast error, one obtains that the best individual weight function

$$b_i\left(\mathbf{b}\right) = \frac{\beta \mathbf{b}}{1 + \sigma \left(1 - \mathbf{b}\right)^2}, \tag{43}$$

in response to an average weight across the population. Notice that (43) corresponds to (27) with $\zeta = 0$. An equilibrium obtains when (43) holds for every i, so that every agent puts the same weight $b_i = \mathbf{b}$ on the expectational signal. The following proposition states a simple analytical result.

Proposition 11 *The fundamental equilibrium entailed by $b_i = \mathbf{b} = 0$ for each $i \in I$ is always an equilibrium of the economy. Two distinct non-fundamental equilibria exist for $b_i = \mathbf{b}_{\pm}$ for each $i \in I$ taking values*

$$\mathbf{b}_{\pm} = 1 \pm \sqrt{\left(\beta - 1\right)/\sigma},$$

if and only if $\beta > 1$.

This case is one in which a multiplicity of equilibria is sustained by a pure pay-off externality determinant arising for strong values of expectational complementarity $\beta > 1$. Even if expectational signals are not informative on fundamentals, private signals provide information on the average second-order belief of others when the price overreacts to the average forecasting mistake. Non-fundamental equilibria necessarily arise with partially correlated expectational signals and disappear in the limit cases of perfect correlation or independence. Hence, as before the multiplicity of equilibria are strictly linked to arbitrarily small amount of private uncertainty due to the idiosyncratic observational noises. In particular, non-fundamental fluctuations are sustained by a signal extraction problem on an additional imperfect information being a common non-fundamental shock. This implies that non-fundamental equilibria exhibit necessarily aggregate non-fundamental volatility driven by the correlated component in private expectational signals.

The key mechanism beyond this result is similar to the one underlying an exogenous sunspot equilibrium. To see this notice that for $\beta_\varphi > 1$ it is possible to build up private sunspot equilibria when agents observe truly exogenous signals for an ad-hoc value of cross-sectional correlation. Suppose to replace the private endogenous signal s_i with an exogenous white noise signals $\varsigma_i = \{\xi + \eta_i\}$ where the shocks have the same characteristics as before. For the specific value of private uncertainty $\sigma = \beta_\varphi - 1$ - that requires necessarily $\beta_\varphi > 1$ - we obtain a continuum of *indeterminate* equilibria exhibiting non-fundamental aggregate volatility driven by ξ.[16] In our original case, instead the non-linearity introduced by endogenous signals prevents indeterminacy and generates only two non-zero *determined* equilibria values $\mathbf{b}$ for which non-fundamental volatility shows up. In other words, the non-linearity of the optimal individual weight (27) is key to determinacy.

5 Conclusions

This paper has laid out the general conditions under which private uncertainty on a certain endogenous state of the economy determines a multiplicity of equilibria in models that have a unique equilibrium under perfect knowledge or absence of endogenous signals. This occurs when agents are privately uncertain about a global price that exhibits opposite reactions to an aggregate shock in the scenarios of no information and perfect foresight. The results hold even in absence of pay-off complementarities, that is, when the unobserved variable to be forecasted is purely exogenous. Nevertheless, the paper also discussed a case in which a multiplicity arises with strong pay-off externalities when the fundamentals are perfectly known.

Some important theoretical questions are left in the background. The existence of a multiplicity of determinate equilibria raises the issue of agents' coordination. Different approaches inquiring the out-of-equilibrium dynamics of agents' beliefs have been implemented to answer this question. In particular, a selection on a unique equilibrium could occur conditionally to a given learning scheme. Some preliminary work in this direction has been done in Gaballo (2011). Another important issue concerns the extent to which endogenous signals can originate a multiplicity in dynamic settings. The model I presented here is essentially static in nature, as the realization of the shock is not informative about the future course of the economy. When this is not the case agents accumulate additional correlated information through time. As showed by Angeletos, Hellwig and Pavan (2007) the dynamic interaction between exogenous and endogenous information can still sustain a multiplicity of equilibria in the

[16] This mechanism is similar to the one in Benhabib, Wang and Wen (2012). Nevertheless in their paper the sunspot equilibrium is determinate because the signal includes private fundamental disturbances.

context of coordination games encompassing the currency attack model. How this can survive in a microfounded macro-model with a unique equilibrium under complete information is a question that hopefully the analysis in this paper can help to address in a near future.

Appendix

A.1 Relations in the model

A.1.1 First-order conditions and the deterministic steady state

The whole list of first-order conditions of the model are

$$\begin{aligned}
W_i &= \alpha E^i\left(P\right) L_i^{\alpha-1} K_i^{1-\alpha} && \text{(44a)}\\
R_i &= \left(1-\alpha\right) E^i\left(P\right) L_i^{\alpha} K_i^{-\alpha} && \text{(44b)}\\
Y_i &= L_i^{\alpha} K_i^{1-\alpha} && \text{(44c)}\\
\Phi_i \left(L_i^s\right)^{\gamma} &= W_i \frac{\Lambda_i}{P} && \text{(44d)}\\
\Phi_i C_i^{-\psi} &= \Lambda_i && \text{(44e)}\\
\frac{\Lambda_i}{P} &= \frac{\delta}{1-\delta}\frac{1}{\bar{M}_i} = 1 && \text{(44f)}\\
R &= e^{-\hat{\eta}_i} R_i && \text{(44g)}\\
K_i &= e^{-\hat{\eta}_i} Z_i && \text{(44h)}
\end{aligned}$$

where the first three refer to the problem of final producers, the last two to the problem of intermediate producers and the rest to the consumer's problem.

The unique deterministic price and aggregate production obtain respectively as $P^* = \alpha^{\frac{-\alpha\psi}{1+\gamma-\alpha+\alpha\psi}}$ and $Y^* = \alpha^{\frac{\alpha}{1+\gamma-\alpha+\alpha\psi}}$ after solving the system for $\varepsilon = 0$, $\hat{\phi} = 0$, $\hat{\eta}_i = 0$ and constant actions across agents.

A.1.2. proof. proposition 2

Preliminaries. The requirement that the equilibrium has a symmetric log-linear representation requires that any variable X_i in the model has the form

$$X_i = \bar{X} e^{x_i - \frac{1}{2}\sigma(x_i)}$$

where $\bar{X}$ is the steady state and $\sigma\left(x_i\right)$ is the variance of x_i, namely a log-deviation from the steady state is composed by a stochastic component

$$x_i = \mathbf{x}_1 \varepsilon + \mathbf{x}_2 \hat{\phi}_i + \mathbf{x}_3 \hat{\eta}_i$$

which is a linear combination of the shocks and a constant. Here I want to show that for each variable in the model there exists a unique steady state and a unique log-linear deviation implied by fixing a profile of coefficients $\left(\mathbf{e}_{i,1}, \mathbf{e}_{i,2}, \mathbf{e}_{i,3}\right)$ in (15). For the sake of notational convenience, I analyze symmetric equilibria, that is ones in which individual weights $\left(\mathbf{e}_{i,1}, \mathbf{e}_{i,2}, \mathbf{e}_{i,3}\right)$ for each i are equal to the average ones $\left(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\right)$. This choice is without loss of generality as the average weights are sufficient statistics of a profile of individual weights $\{\mathbf{e}_{i,1},\mathbf{e}_{i,2},\mathbf{e}_{i,3}\}_I$.

Production side. The aggregate demand for the endowment

$$\int Z_i di = \int \bar{Z} e^{\mathbf{z}_1\varepsilon+\mathbf{z}_2\hat{\phi}_i+\mathbf{z}_3\hat{\eta}_i-\frac{1}{2}\sigma(z_i)} di = \bar{Z} e^{\mathbf{z}_1\varepsilon-\frac{1}{2}\sigma(\mathbf{z}_1\varepsilon)},$$

satisfies the market clearing condition $\int Z_i di = 1$ for any ε so that necessarily $\mathbf{z}_1 = 0$ and $\bar{Z} = 1$. Using (44h) and the relation above we obtain

$$K_i = \bar{K} e^{\mathbf{k}_1\varepsilon+\mathbf{k}_2\hat{\phi}_i+\mathbf{k}_3\hat{\eta}_i-\frac{1}{2}\sigma(k_i)} = e^{-\hat{\eta}_i} Z_i = e^{\mathbf{z}_2\hat{\phi}_i+(\mathbf{z}_3-1)\hat{\eta}_i-\frac{1}{2}\sigma(z_i)},$$

where therefore $\mathbf{k}_1 = \mathbf{z}_1 = 0$, $\mathbf{k}_2 = \mathbf{z}_2$, $\mathbf{k}_3 = \mathbf{z}_3 - 1$ and $\bar{K} = \bar{Z} = 1$ is the only possibility for the equality to hold for any $(\varepsilon,\hat{\phi}_i,\hat{\eta}_i)$ realization. According to (44g)

$$R = e^{-\hat{\eta}_i} R_i = \bar{R} e^{\mathbf{r}_1\varepsilon+\mathbf{r}_2\hat{\phi}_i+(\mathbf{r}_3-1)\hat{\eta}_i-\frac{1}{2}\sigma(r_i)}$$

but also integrating both side across islands we have,

$$R = \bar{R} e^{\mathbf{r}_1\varepsilon+\frac{\mathbf{r}_2^2\sigma_\phi+(\mathbf{r}_3-1)^2\hat{\sigma}_i}{2}-\frac{1}{2}\sigma(r_i)}, \tag{45}$$

so that $\bar{R} = R$, $\mathbf{r}_2 = 0$ and $\mathbf{r}_3 = 1$ and only $\mathbf{r}_1$ is left to be determined. Plugging (44f) in (44d) and the resulting in (44a) we get

$$L_i = \alpha^{\frac{1}{1-\alpha+\gamma}} e^{-\frac{\varepsilon+\hat{\phi}_i}{1-\alpha+\gamma}} E^i(P)^{\frac{1}{1-\alpha+\gamma}} K_i^{\frac{1-\alpha}{1-\alpha+\gamma}} \tag{46}$$

and substituting the expression above in (44b) we have

$$R_i = (1-\alpha)\,\alpha^{\frac{\alpha}{1-\alpha+\gamma}} e^{-\frac{\alpha\varepsilon+\alpha\hat{\phi}_i}{1-\alpha+\gamma}} E^i(P)^{\frac{1+\gamma}{1-\alpha+\gamma}} K_i^{-\frac{\alpha\gamma}{1-\alpha+\gamma}}$$

where $R_i = \bar{R} e^{\mathbf{r}_1\varepsilon+\hat{\eta}_i-\frac{1}{2}\sigma(r_i)}$. Hence the following restrictions on log-deviations must hold for any $(\varepsilon,\hat{\phi}_i,\hat{\eta}_i)$ realization

$$\mathbf{r}_1 = \frac{1+\gamma}{1-\alpha+\gamma}\mathbf{e}_1 - \frac{\alpha}{1-\alpha+\gamma}, \tag{47}$$

$$0 = \frac{1+\gamma}{1-\alpha+\gamma}\mathbf{e}_2 - \frac{\alpha\gamma}{1-\alpha+\gamma}\mathbf{k}_2 - \frac{\alpha}{1-\alpha+\gamma}, \tag{48}$$

$$1 = \frac{1+\gamma}{1-\alpha+\gamma}\mathbf{e}_3 - \frac{\alpha\gamma}{1-\alpha+\gamma}\mathbf{k}_3 \tag{49}$$

which pin down $\mathbf{r}_1$, $\mathbf{k}_2$ and $\mathbf{k}_3$ as functions of $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$. Concerning the steady state of $\bar{R}$ it is related to $\bar{P}$ by

$$\bar{R} e^{-\frac{1}{2}\sigma(r_i)} = (1-\alpha)\,\alpha^{\frac{\alpha}{1-\alpha+\gamma}} \bar{P}^{\frac{1+\gamma}{1-\alpha+\gamma}} e^{\frac{1}{2}\left(\frac{\alpha\gamma}{1-\alpha+\gamma}\sigma(k_i)-\frac{1+\gamma}{1-\alpha+\gamma}\sigma\left(E^i(p)\right)\right)}$$

where all the exponentials are constant terms depending on the variance of the shocks and the coefficients $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$; these are zero in the deterministic case when the median and the average action coincide. Therefore for *given* $\mathbf{e}_1$,$\mathbf{e}_2$ and $\mathbf{e}_3$ there exist unique steady state value of R, R_i, K_i, Z_i and unique relative deviations defined by the relations above. Once K_i is uniquely defined then also L_i is according to (46) with steady state $\bar{L}$ determined by

$$\bar{L}e^{-\frac{1}{2}\sigma(l_i)} = \alpha^{\frac{1}{1-\alpha+\gamma}}\bar{P}^{\frac{1}{1-\alpha+\gamma}}e^{-\frac{1}{2}\left(\frac{1-\alpha}{1-\alpha+\gamma}\sigma(k_i)+\frac{1}{1-\alpha+\gamma}\sigma\left(E^i(p)\right)\right)}.$$

Analogously we can find the unique implied steady state and log-deviation of W_i and Y_i working respectively on (44d) (after plugging (44f) in) and (44c). In particular

$$Y_i = \alpha^{\frac{\alpha}{1-\alpha+\gamma}}e^{\frac{-\alpha\left(\varepsilon+\hat{\phi}_i\right)}{1-\alpha+\gamma}}E^i\left(P\right)^{\frac{\alpha}{1-\alpha+\gamma}}K_i^{\frac{(1-\alpha)(1+\gamma)}{1-\alpha+\gamma}}$$

which implies

$$\begin{aligned}
\mathbf{y}_1 &= \frac{\alpha}{1-\alpha+\gamma}\mathbf{e}_1 - \frac{\alpha}{1-\alpha+\gamma}, \\
\mathbf{y}_2 &= \frac{\alpha}{1-\alpha+\gamma}\mathbf{e}_2 + \frac{(1-\alpha)(1+\gamma)}{1-\alpha+\gamma}\mathbf{k}_2 - \frac{\alpha}{1-\alpha+\gamma}, \\
\mathbf{y}_3 &= \frac{\alpha\mathbf{e}_3}{1-\alpha+\gamma} + \frac{(1-\alpha)(1+\gamma)}{1-\alpha+\gamma}\mathbf{k}_3 + \frac{(1-\alpha)(1+\gamma)}{1-\alpha+\gamma},
\end{aligned}$$

and steady state

$$\bar{Y}e^{-\frac{1}{2}\sigma(y_i)} = \alpha^{\frac{\alpha}{1-\alpha+\gamma}}\bar{P}^{\frac{\alpha}{1-\alpha+\gamma}}e^{-\frac{1}{2}\left(\frac{(1-\alpha)(1+\gamma)}{1-\alpha+\gamma}\sigma(k_i)+\frac{\alpha}{1-\alpha+\gamma}\sigma\left(E^i(p)\right)\right)}.$$

This concludes the description of the supply side which is completely determined for a given profile of individual weights $\{\mathbf{e}_{i,1}, \mathbf{e}_{i,2}, \mathbf{e}_{i,3}\}_I$.

Demand side. From (44e) we have

$$C_i = P^{-\frac{1}{\psi}}e^{\frac{1}{\psi}\left(\varepsilon+\hat{\phi}_i\right)}$$

using (44f) after substituting for (44e), which gives the restrictions $\mathbf{c}_1 = -\frac{1}{\psi}(\mathbf{p}_1 - 1)$, $\mathbf{c}_2 = \frac{1}{\psi}$, $\mathbf{p}_2 = \mathbf{p}_3 = \mathbf{c}_3 = 0$ and steady state $\bar{C}e^{-\frac{\sigma(c_i)}{2}} = \bar{P}^{-\frac{1}{\psi}}e^{\frac{1}{2\psi}\sigma(p)}$ for each $(\varepsilon, \hat{\phi}_i, \hat{\eta}_i)$ realization. The clearing condition for the good market is therefore

$$\int Y_i di = \bar{Y}e^{\mathbf{y}_1\varepsilon-\frac{1}{2}\sigma(\mathbf{y}_1\varepsilon)} = \int C_i di = \bar{P}^{-\frac{1}{\psi}}e^{-\frac{1}{\psi}(\mathbf{p}_1-1)\varepsilon+\frac{1}{2\psi}\sigma(p)}$$

from which one can determine the unique price process

$$P = \bar{P}e^{\mathbf{p}_1\varepsilon-\frac{1}{2}\sigma(p)}$$

such that the stochastic price deviation is pinned down by the relation $-\frac{1}{\psi}(\mathbf{p}_1 - 1) = \mathbf{y}_1$ yielding

$$\mathbf{p}_1 = 1 - \psi \mathbf{y}_1 = -\frac{\alpha\psi}{1-\alpha+\gamma}\mathbf{e}_1 + \frac{1-\alpha+\gamma+\alpha\psi}{1-\alpha+\gamma}$$

and a steady state

$$\bar{P} = \alpha^{\frac{-\alpha\psi}{1-\alpha+\gamma+\alpha\psi}} \Phi^{-\frac{(1-\alpha+\gamma)\psi}{1-\alpha+\gamma+\alpha\psi}}$$

where $\Phi = e^{-\frac{1}{2}\left(-\frac{(1-\alpha)(1+\gamma)}{1-\alpha+\gamma}\sigma(k_i) - \frac{\alpha}{1-\alpha+\gamma}\sigma\left(E^i(p)\right) - \sigma(y_i) + \frac{1}{\psi}\sigma(p)\right)}$ obtained substituting for $\bar{Y}$. Notice $\bar{P} = P^*$ in the deterministic case.

A.1.3. The linear system of stochastic log-deviations from the steady state

Here we consider the aggregate stochastic log-deviations from a steady state in the model including the special case of section 4 with $\varphi \neq 0$. In equilibrium, these must satisfy the following system of first order conditions

$$\begin{bmatrix} w \\ r \\ l \\ \lambda \\ y \\ p \end{bmatrix} = \begin{bmatrix} 0 & 0 & \alpha-1 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha^{-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{1}{\psi} & 0 & \frac{\varphi}{\psi} \\ 1 & 0 & -\gamma & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} w \\ r \\ l \\ \lambda \\ y \\ p \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{1}{\psi} \\ 0 & -1 \end{bmatrix} \begin{bmatrix} E(p) \\ \varepsilon \end{bmatrix}$$

obtained from (44) where notice log-constants must cancel out each other from both sides of the equalities. The system is solved as

$$\begin{bmatrix} w \\ r \\ l \\ \lambda \\ y \\ p \end{bmatrix} = \begin{pmatrix} \frac{\gamma}{1+\gamma-\alpha} & \frac{1-\alpha}{1+\gamma-\alpha} \\ \frac{1+\gamma}{1+\gamma-\alpha} & \frac{-\alpha}{1+\gamma-\alpha} \\ \frac{1}{1+\gamma-\alpha} & \frac{-1}{1+\gamma-\alpha} \\ \frac{-\alpha\psi}{(1-\varphi)(\gamma-\alpha+1)} & \frac{1+\gamma-\alpha+\alpha\psi}{(1+\gamma-\alpha)(1-\varphi)} \\ \frac{\alpha}{1+\gamma-\alpha} & \frac{-\alpha}{1+\gamma-\alpha} \\ \frac{-\alpha\psi}{(1+\gamma-\alpha)(1-\varphi)} & \frac{1+\gamma-\alpha+\alpha\psi}{(1+\gamma-\alpha)(1-\varphi)} \end{pmatrix} \begin{bmatrix} E(p) \\ \varepsilon \end{bmatrix},$$

giving the relations (16) and (18) for $\varphi = 0$, and (38) for $\varphi \neq 0$.

A.2 Proof. of propositions 5, 9 and 10

A.2.1. Preliminaries

The fixed-point equations (28) and (34) have the same structure. Here I will prove a lemma that will be useful in the following proofs.

Lemma 12 *The equation*

$$x^3 - 2\vartheta x^2 + \left(\vartheta^2 - \mu\right) x - \kappa = 0, \tag{50}$$

with $\vartheta > 0$ *and* κ, μ *real scalars, has three real roots if and only if*

$$\mu > -\vartheta^2/3, \tag{51}$$

and

$$\kappa \in [k_-, k_+], \tag{52}$$

where

$$\begin{aligned} k_- &= -\left(\frac{2}{9}\mu + \frac{2}{27}\vartheta^2\right)\sqrt{\vartheta^2 + 3\mu} - \frac{2}{3}\mu\vartheta + \frac{2}{27}\vartheta^3, && (53)\\ k_+ &= \left(\frac{2}{9}\mu + \frac{2}{27}\vartheta^2\right)\sqrt{\vartheta^2 + 3\mu} - \frac{2}{3}\mu\vartheta + \frac{2}{27}\vartheta^3. && (54)\end{aligned}$$

Proof. Consider the equation rewritten as

$$\underbrace{x\left(\vartheta - x\right)^2}_{\equiv y(x)} = \underbrace{\kappa + \mu x}_{\equiv z(b)}, \tag{55}$$

where $y(x)$ and $z(x)$ are two real continuous and differentiable functions defined as respectively the left-hand term and the right-hand term of the equation above. The latter is a line with intercept κ and slope μ, whereas the former is a cubic passing through the origin with roots at $(0,0)$ and $(\vartheta, 0)$, and with local maximum and minimum respectively at $\left(\vartheta/3, 4\vartheta^3/27\right)$ and $(\vartheta, 0)$.

Consider two points $\{\hat{x}_\pm(\mu), y(\hat{x}_\pm(\mu))\}$ such that the slope of the curve is equal to a given constant μ. These are

$$\begin{aligned} \hat{x}_\pm(\mu) &= \frac{2\vartheta \pm \sqrt{\vartheta^2 + 3\mu}}{3}, \\ y(\hat{x}_\pm(\mu)) &= \left(\frac{2\vartheta \pm \sqrt{\vartheta^2 + 3\mu}}{3}\right)\left(\vartheta - \frac{2\vartheta \pm \sqrt{\vartheta^2 + 3\mu}}{3}\right)^2, \end{aligned}$$

where $\hat{x}_\pm(\mu)$ solves $y'(\hat{x}_\pm(\mu)) = \mu$.

From the theorem of the mean value we know that if there exist at least two distinct values x_1 and x_2 with $x_1 < x_2$ such that $y(x_1) = z(x_1)$ and $y(x_2) =$

$z(x_2)$ (that is multiple intersections exist) then it also exists an intermediate value $x_3 \in [x_1, x_2]$ such that $y'(x_3) = \mu$. Therefore if the latter does not exist then the former condition is violated. Hence a second restriction for $z(x)$ having three intersections with $y(x)$ provides for (51). In this region we have to assess whether or not $\kappa \in [k_-, k_+]$ where

$$\begin{aligned} k_+ &= y(\hat{x}_-(\mu)) - \mu \hat{x}_-(\mu), \\ k_- &= y(\hat{x}_+(\mu)) - \mu \hat{x}_+(\mu), \end{aligned}$$

are the intercepts of the two lines having slope μ and being tangents at a point of $y(x)$. This is a necessary and intersection with $y(x)$. ■

Remark 13 *Notice that within the parameter region $\mu > -\vartheta^2/3$ with $\vartheta > 0$ it is true that:*

i *k_+ is always positive with a minimum at 0 and $\partial k_+/\partial \mu > 0$ for $\mu > \vartheta^2$;*

ii *k_- is negative for $\mu > 0$ and has a maximum at $8\vartheta^3/27$ corresponding to the lower parameter bound $\mu = -\vartheta^2/3$;*

iii *k_- is a decreasing and concave in μ, that is*

$$\frac{\partial k_-}{\partial \mu} = -\frac{1}{3}\left(\sqrt{\vartheta^2 + 3\mu} + 2\vartheta\right) < 0 \ \textit{and} \ \frac{\partial k_-}{\partial \mu^2} = -\frac{1}{2\sqrt{\vartheta^2 + 3\mu}} < 0,$$

where in particular $\partial k_-/\partial \mu > -\vartheta$ for $\mu < 0$.

A.2.2. Proposition 5

Proof. The fixed-point equation (28) corresponds to (50) with

$$\begin{aligned} \vartheta &= 1, \\ \kappa &= \zeta \frac{(1-\beta)}{\sigma}, \\ \mu &= -(1+\zeta)\frac{(1-\beta)}{\sigma}. \end{aligned}$$

To check the existence of multiple solutions we need to investigate when (52) holds with $\zeta < -1$ and $\beta < 1$, that is, in the case $\mu > 0$, $\kappa < 0$. We need to prove that given a couple $\left(\bar{\zeta}, \bar{\beta}\right)$ there exists a small enough σ such that a (52) is satisfied. Firstly, let us write down the derivative of k_- and κ with respect to σ, given respectively by

$$\frac{\partial k_-}{\partial \mu}\frac{\partial \mu}{\partial \sigma} = \frac{\partial k_-}{\partial \mu}(1+\zeta)\frac{(1-\beta)}{\sigma^2} \quad \text{and} \quad \frac{\partial \kappa}{\partial \sigma} = -\zeta \frac{(1-\beta)}{\sigma^2}, \tag{56}$$

so that we have

$$\frac{\partial k_-}{\partial \sigma} > \frac{\partial \kappa}{\partial \sigma} \quad \text{whenever} \quad \frac{\partial k_-}{\partial \mu}(1+\zeta) > -\zeta, \tag{57}$$

where notice $\partial k_- \backslash \partial \sigma$ and $\partial \kappa \backslash \partial \sigma$ are both positive since $\partial k_- \backslash \partial \mu$ is always negative (remark 13.iii). Now consider a point $\left(\bar{\zeta}, \bar{\sigma}, \bar{\beta}\right)$ such that

$$\bar{\kappa}\left(\bar{\zeta}, \bar{\sigma}, \bar{\beta}\right) \leq \bar{k}_-\left(\bar{\zeta}, \bar{\sigma}, \bar{\beta}\right) \leq 0 < \bar{k}_+\left(\bar{\zeta}, \bar{\sigma}, \bar{\beta}\right),$$

that is (52) is violated. Given that

$$\lim_{\sigma \to 0} \frac{\partial k_-}{\partial \mu} = -\infty \leq -\frac{\bar{\zeta}}{\bar{\zeta}+1} \tag{58}$$

it is always true, then (57) holds and so k_- always decreases monotonically faster than κ as σ approaches its lower bound. Since k_- is negative whereas k_+ remains always positive (remark 13.i and 13.ii), there must exist a σ_* small enough such that (52) holds for any $\sigma < \sigma_*$. Finally notice that such threshold σ_* must increase with decreasing $\bar{\beta}$ (and $\bar{\zeta}$) because ceteris paribus it increases μ (and at the same time relaxes the constraint (58)). For a proof that a multiplicity does not obtain for $\sigma \to 0$ in the case of $\zeta \geq -1$ see A.2.4. ■

A.2.3. Proposition 9

Proof. The fixed-point equation (34) corresponds to (50) with

$$\begin{aligned}
\vartheta &= \frac{1-\beta+\sigma_\phi}{\sigma_\phi}, \\
\kappa &= \zeta \frac{1-\beta}{\sigma}\left(\frac{(1-\beta)(1+\zeta)+\sigma_\phi}{\sigma_\phi}\right), \\
\mu &= -(1+\zeta)\frac{1-\beta}{\sigma}\left(\frac{(1-\beta)(1+\zeta)+\sigma_\phi}{\sigma_\phi}\right)
\end{aligned}$$

and it coincides with (28) in the limit of $\sigma_\phi \to \infty$. To check the existence of multiple solutions for $\zeta < -1$ and $\beta < 1$ we need to assess how the new parameter σ_ϕ changes the existence conditions uncovered before. Observe that

$$\frac{\partial k_\pm}{\partial \mu}\frac{\partial \mu}{\partial \sigma} = \frac{\partial k_\pm}{\partial \mu}(1+\zeta)\frac{(1-\beta)}{\sigma^2}\left(\frac{(1-\beta)(1+\zeta)+\sigma_\phi}{\sigma_\phi}\right) \tag{59a}$$

and

$$\frac{\partial \kappa}{\partial \sigma} = -\zeta\frac{(1-\beta)}{\sigma^2}\left(\frac{(1-\beta)(1+\zeta)+\sigma_\phi}{\sigma_\phi}\right),$$

so that

$$\frac{\partial k_-}{\partial \sigma} > \frac{\partial \kappa}{\partial \sigma} \text{ if } \frac{\partial k_-}{\partial \mu}(\zeta+1) > -\zeta \text{ provided } \sigma_\phi > -(1-\beta)(1+\zeta), \quad \text{(60a)}$$

$$\frac{\partial k_-}{\partial \sigma} > \frac{\partial \kappa}{\partial \sigma} \text{ if } \frac{\partial k_-}{\partial \mu}(\zeta+1) < -\zeta \text{ provided } \sigma_\phi < -(1-\beta)(1+\zeta). \quad \text{(60b)}$$

The argument put forward for the proof of proposition 5 can be exactly replicated for a given $(\bar{\beta}, \bar{\zeta}, \bar{\sigma}_\phi)$ such that $\bar{\sigma}_\phi > -(1-\bar{\beta})(1+\bar{\zeta})$. The latter is therefore a sufficient condition for a multiplicity.

Let us focus therefore on the case $\bar{\sigma}_\phi < -(1-\bar{\beta})(1+\bar{\zeta})$ for which $\mu < 0$, that is κ and k_- are positive (remark 13.ii). Notice that for small enough σ now (51) becomes binding so that the following restriction applies

$$-\frac{(1-\beta+\sigma_\phi)^2}{3\sigma_\phi^2} < -\frac{(1-\beta)(1+\zeta)((1-\beta)(1+\zeta)+\sigma_\phi)}{\sigma\sigma_\phi}, \quad (61)$$

which entails a lower bound to σ

$$\sigma > m(\zeta, \beta, \sigma_\phi) \equiv \frac{3\sigma_\phi (1-\beta)(1+\zeta)((1-\beta)(1+\zeta)+\sigma_\phi)}{(1-\beta+\sigma_\phi)^2}, \quad (62)$$

constituting a necessary condition for the existence of a multiplicity. In particular, notice that

$$\frac{\partial m(\zeta,\beta,\sigma_\phi)}{\partial \sigma_\phi} = \frac{3(\beta-1)^2(\zeta+1)((1+\zeta)(1-\beta)+(1-\zeta)\sigma_\phi)}{(1-\beta+\sigma_\phi)^3} < 0,$$

for

$$0 < -\frac{1}{1-\zeta}(1+\zeta)(1-\beta) < \sigma_\phi < -(1+\zeta)(1-\beta),$$

that is as σ_ϕ increases m lowers. Therefore for σ_ϕ sufficiently small m decreases until reaches $m = 0$ for $\sigma_\phi = 0$. Nevertheless at that limit we know that a unique equilibrium exists has the exogenous signal fully reveals the aggregate shock. The lower bound m also reaches 0 at $\sigma_\phi = -(1+\zeta)(1-\beta)$ and then becomes negative for higher values of σ_ϕ. Hence, we can conclude that $\sigma_\phi > -(1-\beta)(1+\zeta)$ is the condition for a multiplicity to arise for a σ small enough, that is, for *any* σ under a certain threshold. For a proof that a multiplicity does not obtain for $\sigma \to 0$ in the case of $\zeta \geq -1$ see A.2.4. ■

A.2.4. Proposition 10

Proof. Here we consider the fixed-point equation (34) for $\zeta \geq -1$. First of all notice that (52) cannot be satisfied for $\zeta \in [-1, 0]$ that is for $\mu < 0$ and $\kappa < 0$ because $y(x)$ is non-monotone only in the first quadrant. Hence, a multiplicity may eventually arise for $\zeta > 0$ for which $\mu < 0$ and $\kappa > 0$. In this case the restriction (51) is also binding and it implies (62)as lower bound to σ, but now the bound is always positive. In particular reduces to

$$\lim_{\sigma_\phi \to \infty} m\left(\zeta, \beta, \sigma_\phi\right) = 3\left(1+\zeta\right)\left(1-\beta\right), \tag{63}$$

in the limit $\sigma_\phi \to \infty$. Notice that for $\sigma \to \infty$ we have $\lim_{\sigma\to\infty} k_- = \kappa = 0$ and $\lim_{\sigma\to\infty} k_+ = 4\vartheta^3/27$, whereas

$$\lim_{\sigma\to\infty} \frac{\partial k_-}{\partial \mu} = -\theta < -1 < -\frac{\zeta}{\zeta+1}, \tag{64}$$

for whatever $\zeta > 0$. According to (60a) the latter implies that $0 > \partial\kappa/\partial\sigma > \partial k_-/\partial\sigma$ so that, for decreasing σ, k_- increases initially faster than κ. Therefore at least locally there does not exist any multiplicity region in the limit $\sigma \to \infty$.

Nevertheless, (64) can be eventually reverted for smaller σ. Suppose now we start from a point $\left(\bar{\zeta}, \bar{\sigma}, \bar{\beta}, \bar{\sigma}_\phi\right)$ such that $\mu = -\vartheta^2/3$. At this point, $\lim_{\mu\to-\vartheta^2/3} k_\pm = 8\vartheta^3/27$. By continuity of κ with respect to σ and the conditions (60a), we can conclude that if and only if

$$\kappa \geq \ 8\,\vartheta^3/27, \tag{65}$$

for some σ, then there exists a compact region of the parameter space such that (52) is satisfied whereas such a region does not exist otherwise. Disequality (65) corresponds to

$$\frac{\left(1-\beta\right)\zeta\left(\left(1-\beta\right)\left(1+\zeta\right)+\sigma_\phi\right)}{\sigma\sigma_\phi} > \frac{8}{27}\left(\frac{1-\beta+\sigma_\phi}{\sigma_\phi}\right)^3,$$

that is

$$\sigma < M\left(\zeta, \beta, \sigma_\phi\right) \equiv \frac{27\sigma_\phi^2\left(1-\beta\right)\zeta\left(\left(1-\beta\right)\left(1+\zeta\right)+\sigma_\phi\right)}{8\left(1-\beta+\sigma_\phi\right)^3}, \tag{66}$$

that provides a higher bound to σ with

$$\lim_{\sigma_\phi\to\infty} M\left(\zeta, \beta, \sigma_\phi\right) = \frac{27}{8}\left(1-\beta\right)\zeta,$$

in the limit of $\sigma_\phi \to \infty$. Finally from intersection of (62) and (66), one obtains a necessary condition for a multiplicity of multiple real solutions as

$$\sigma \in \left(m\left(\zeta, \beta, \sigma_\phi\right), M\left(\zeta, \beta, \sigma_\phi\right)\right),$$

that is a non empty interval if and only if

$$\sigma_\phi > \frac{8\left(1+\zeta\right)}{\left(\zeta-8\right)}\left(1-\beta\right),$$

with $\zeta > 8$. ■

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