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DIRECTION GÉNÉRALE DES ÉTUDES ET DES RELATIONS INTERNATIONALES

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*This title is stolen from Delamare (1994). I thank David Bounie, Alain Bresson, François de Callataÿ, Arnaud Manas, and Henri Pagès for discussions, and Arnaud Clairand, Jérôme Jambu, and Jean-Yves Kind for access to their data. All errors remain mine. The views presented here do not necessarily reflect those of the Banque de France or the Eurosystem.

Abstract

I measure the parameters of coin wear using data collected in the 19th century. A comparison across denominations and countries shows that coin wear (in relative terms) is linear in the logarithm of coin value. Data from coin hoards of the 18th and early 19th centuries yield similar estimates of mean coin wear, showing that hoards provide useful information. Finally, under assumptions of normality for initial coin weights and coin loss I use maximum likelihood estimation to recover the parameters of the wear process from a sample of coins whose age is unknown. The method performs well on the hoard data (for which the age is known and can serve as a check).

Keywords: coin circulation, coin wear, abrasion, velocity, denomination, gold, silver (JEL N10).

Je mesure les paramètres du frai des monnaies à partir de données rassemblées au XIXe siècle. La comparaison des résultats pour divers pays et dénominations montre que le frai relatif est une fonction linéaire du logarithme de la valeur. Les trésors monétaires des XVIIIe et XIXe siècles donnent des estimations proches, ce qui montre que les trésors fournissent des informations exploitables. Enfin, en supposant que les distributions des poids initiaux et du frai sont gaussiennes le maximum de vraisemblance permet d'en estimer les paramètres même lorsque l'âge des monnaies est inconnu. La méthode donne de bons résultats sur les trésors pour lesquels l'âge est connu et sert de contrôle.

Mots-clés: circulation de monnaies, frai, vélocité, dénomination, or, argent (JEL N10).

1 Introduction

Coins tell stories: that is why numismatists and coin collectors study them. Much of what they say is in words and images: their sides (and at times their edges) speak to who had them made, and for what purpose. But coins also speak in numbers.

Numismatists have, of course, long recognized this fact, and quantitative approaches are not new. In this paper, I propose that the study of coin weights can tell us about how they were made, and how they were used. Specifically, I study coin wear (the French language has a specific word, *frai*). Coin wear, for reasons I explain in the rest of this introduction, was an important topic in the last decades of the gold standard, and for this reason large-scale surveys of coin wear were carried out on the circulating medium of exchange.

In the rest of the paper, I measure the parameters of coin wear using data collected in the 19th century.[1] Statistical methods allow me to estimate the mean as well as the variance of coin wear, both of which appear to be constant over time for a given denomination. Then, I compare estimates of (mean) coin wear across denominations and countries and show that, as a percentage of coin weight, coin wear is linear in the logarithm of coin value. This regularity holds across time and countries. I then use data from coin hoards from the 18th and early 19th centuries, and show that they yield estimates of mean coin wear that are very close to those derived from the contemporary large-scale surveys. Hoards thus provide useful information. Finally, I estimate the parameters of coin wear from a sample of coins whose vintage is unknown: the method is tested on 18th and 19th century coin hoards, for which the vintage is known, and shown to perform quite well. This method could prove very fruitful for ancient and medieval coins, where vintages are not precisely known and information about the conditions of circulation, and even the original weights of coins, is scarce.

Why coin wear mattered

In a fiat money regime, coins are typically made at a low cost relative to their face value, so that the question of estimating and accounting for weight loss on coinage is a fairly minor and technical one, and the costs are born by the State. In a commodity money regime, the stock of capital in the form of coins has a substantial value. In 1870 the coin stock represented 8% of GDP in Britain, and 38% in France; by 1900 these figures were

[1] See Spurr (1965), Cope (1969), Ruscoe (1987, 1988) for studies of 20th century coinage and alloys.

still 6% and 22%.[2]

The consequence of coin depreciation manifests itself among other ways through external trade and exchange rates. Suppose that the current account is balanced on average, but alternates between periods of surplus and periods of deficit. In times of surplus, trade will be balanced by importation of precious metal, which will be converted into new, undepreciated coins, thus increasing the average weight of circulating coins. In times of deficit, if worn and unworn coins are equivalent in domestic circulation but taken by weight at foreign mints, the heavier coins will be exported, thus reducing the average weight of coins in circulation. This effect will tend to worsen the exchange rates, since the best coins in circulation will be progressively lighter. The gold points are, in effect, not fixed but altered by the balance of trade. More generally, any movement of metal out of coined form due to a higher price of the relevant metal in uncoined form will have the same effect if it is selectively done because of conversion to industrial uses, or, in a bimetallic system, flows induced by changing relative supplies of gold and silver.

A good illustration of the phenomenon is provided by Britain in the 18th century, when gold was already the dominant medium of exchange. The minting of gold coin, after substantial amounts between 1695 and 1720, declined to about £0.5m on average. During that time, the mint was buying standard (22k) gold at a fixed price of £3 17s 10.5d per troy ounce or £46.725 per troy lb, providing a floor for the price of gold. The gold content of coins provided a ceiling for that price: if all coins in circulation had been full weight, then any set of 44.5 guineas (worth £46.725 at face value) would have weighed exactly 1 troy lb. The extent to which the market price of gold rose above the par of £46.725 per troy lb indicates the extent to which coins in circulation were underweight. Figure 1 shows that the price of gold bullion in London varied between increasingly wide limits as time passed, suggestive of a regular process of weight loss, until a recoinage was initiated in 1773.

Who should bear the cost of this loss and maintain the physical stock of coins? There is a potential externality, since wear on coins is the result of many transactions between different parties. The owner of a depreciated coin can take it to the mint and convert it into a new coin: if the mint takes the worn coin by weight, the owner bears the cost; if it takes it by tale, the State does. Traditionally, going back to medieval times, costs were born by the bearer: the mint were profit centers and simply bought metal by weight. Progressively, the public policy question emerged as an important one,

[2] By 1900 the UK: Coin stock from Capie and Webber (1985, 199), GDP at current prices from Feinstein (1972); France: coin stock from Flandreau (1995) and Sicsic (1989), GDP from Lévy-Leboyer and Bourguignon (1990).

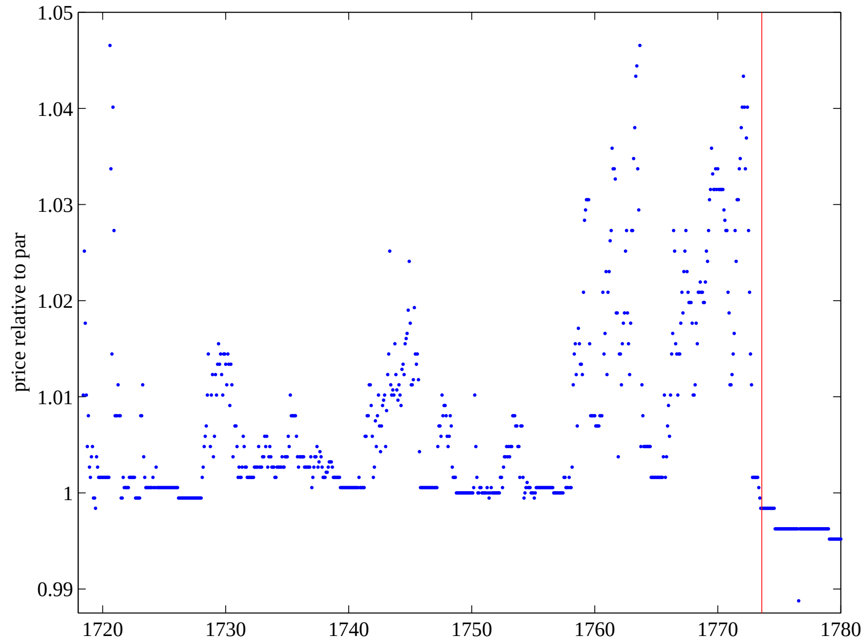


Figure 1: London Price of standard gold in bars, relative to par (defined as a troy ounce of full-weight guineas valued at 21s), first quotation of each month, Mar 1718 to Jan 1780. The vertical line indicates when the Bank of England began to buy underweight coins at face value, in August 1773. Source: Parliamentary papers 1810-11(43) x.197.

particularly on the occasion of general recoinages. A general principle was eventually recognized, and it is stated succinctly by the Director of the US Mint in 1873 (U.S. Secretary of the Treasury 1873–1914, 1873:472):

> The loss from natural abrasion should be defrayed by the Government and not by the last holder, for the reason that it has occurred while the coins were performing the function of a circulating medium. This principle has been fully recognized in the recent coinage laws of the German Empire, Denmark, Sweden and Norway. Provision should of course be made for excluding coins which may have been artificially reduced in weight or violently injured, and the reception of worn coins should be confined to the Mints, where all necessary precautions against receiving fraudulently reduced coins can be effectively observed.

The principle, however, took some time to be implemented.

In Britain, the question rose to prominence during the Great Recoinage of 1695–96. The stock of specie, particularly silver, was badly depreciated, and a decision was made to remint all silver coins. The introduction in 1662 of a superior technology that produced coins less susceptible to clipping had proved ineffective, because the new coins could not compete at the same face value with the older, worn and clipped coins. A general recoinage was needed, but since it was clear that it would fail if individual coin owners were asked to bear the cost, a compromise was adopted, putting most of the cost on the State (Sargent and Velde 2002). The next major recoinage, of gold this time, took place in 1774 (14 Geo III. c. 70). The ensuing proclamation of 24 Jun 1774 set limits of 5% for pre-1760 guineas, 2.65% for guineas minted between 1760 and 1771, and 1.1% for guineas minted after 1771: below these limits the coins were not legal tender and every person was required to "cut, break or deface" such coin tendered to him in payment, and the person tendering it was to bear the loss. A short window of a few weeks was provided during which light coins were legal tender in payment of taxes and dues at face value, effectively putting the cost of the recoinage at the taxpayer's expense. When the sovereign was issued, a legal limit of 0.43% was set by proclamation of 1 Jul 1817, revised to 0.63% by proclamation of 6 Feb 1821; the requirement to cut light coins was renewed, as it was in 1842 and 1843 after a partial recoinage of the gold currency, and by the Coinage Act of 1870 (33 Vict. c. 10), but without any penalty for ignoring the law. In practice only the Bank of England systematically enforced the law, so that bankers were in the habit of selecting coins before sending them as deposits to the Bank. Jevons (1868) made an attempt at measuring the extent of worn coinage, concluding that a third of the sovereigns and half of the half-sovereigns were below the legal tender limit in weight, with the proportion being higher in agricultural districts. In 1888 the Royal Mint carried out its own study, and legislative action ensued. When a recoinage was undertaken in 1889, the government decided to accept worn gold coins minted before 1837 at face value. The provision was extended in 1892 to all worn gold coins (52 & 53 Vict. c. 58, 54 & 55 Vict. c. 72, and Order in Council of 16 Mar 1892; Royal Mint 1870–1913, 20:82, 22:84).

Other countries had reached this solution earlier. In Germany and Austria, the treaty of monetary union of 1857 had established the obligation for each member state to redeem its large silver coinage at face value, no matter how worn, but explicitly left states free as far as gold coinage was concerned, only making all gold coinage current when worn no more than 0.25%. Upon unification, Germany's new monetary system (laws of 4 Dec 1871 and 9 Jul 1873) set weight limits for legal tender coins but clearly placed the costs on the federal state (the Empire).[3] The Scandinavian states adopted similar rules in 1873, as did Austria in 1892, the same year as the United Kingdom.

France and the parties to the Latin Monetary Union never reached as clear-cut a decision on the question. In France the depreciation of coins through use had been noticed and measured during the Revolution. When Napoleon established the French monetary laws of 1803, a legal weight limit was proposed, but the finance minister preferred to postpone the question. A partial recoinage ensued, with gold taken at weight but silver taken at face value as long as the imprint was still legible (Thuillier 1993, 368, 379, 724, 740). Various recoinages took place over time: the pre-Revolutionary coinage in 1829–34, then the copper coinage in 1845, all at the cost of the State (Costes 1885).

When France formed the Latin Monetary Union with Belgium, Switzerland, and Italy in 1865, a legal weight limit was set (0.7% for gold, 1.3% for silver), but the question of cost, although repeatedly raised in international conferences, was not addressed. Only in 1889 did the French legislature appropriate funds from the Mint's profits to retire worn coins, and an arrangement was set up with the Bank of France to retire underweight coins as they came through the Bank (Administration des monnaies et médailles 1896–1914, 1,xiii). The arrangement, which followed earlier ones, was ad-hoc, limited by the appropriations, and extended annually by the legislature.

In the United States, the minting Act of 1873 (17 Statutes at Large 424, section 14) allowed a loss on gold coins by abrasion of 0.5% after 20 years, and a ratable proportion for less than 20 years (that is, 250 parts per million, or ppm, per year). Coins whose weight loss did not exceed the allowance were received at face value by the Treasury. The loss on silver coins was born by the government (U.S. Secretary of the Treasury 1897, 270–1).

The main concern that hindered the adoption of retiring worn coins at the expense of the State was the possibility of moral hazard, in the form of voluntary reductions in the coins' weight. Those countries that explicitly adopted this policy typically retained the right to refuse coins that were unlawfully altered (Germany, Bundesrat decision of 12 Dec 1877). This option was also implicit in the less formal arrangements adopted by countries like France.

[3]The weight limit was 0.5% on gold crowns and double crowns, 0.8% on half-crowns. The limit of 0.5% for the main gold coin was also adopted by the Scandinavian countries.

Another concern was the reluctance of states to bear the costs, especially the upfront costs of a recoinage. Estimating the size of this potential liability required a knowledge of the rates of wear. For my purposes, a useful consequence of these debates has been a number of studies of the weight loss of coins in various countries, data which I will use in this paper.

2 Existing models

The loss on coins will depend on the physical characteristics of the coins and on their intensity of use. I want to recover information about the latter from the loss on coins, so it will prove useful to parse the influence of the various factors. For this, we turn to the physics of solids.

2.1 *Monetary frictions: a little tribology*

Based on notions from materials sciences, Delamare (1994, 99) establishes that the main source of weight loss is through abrasion, or removal of material by friction. Corrosion also contributes to weight loss by creating a layer of material on the surface of the coin that is also removed through abrasion.

A physical model of coin wear

To understand the factors involved in determining weight loss through physical contact, Delamare considers the three-body contact of two coins and a particle (the two-body contact of coin against coin results in transfer of matter from one coin to the other, which washes out statistically). When a particle is pushed along the surface of a coin by an external force, matter on the coin will be pushed aside or shorn off depending on the angle of attack. The weight removed ΔP depends on the metal's density ρ and the volume removed ΔV, which in turn is the product of the area of contact A by the length of contact $\Delta \ell$ and a coefficient of abrasion K_a (specific to the material). The area of contact is the ratio of the force exerted F to the hardness H of the alloy. The length of contact is assumed to be proportional to time through a constant C reflecting intensity of use. An additional proportional factor K_C captures the additional effect of corrosion.

The model he proposes can thus be summarized as:

$$\Delta P = \left(K_a \frac{\rho}{H} K_C \right) (CF) \, \Delta t = K \bar{F} \Delta t \tag{1}$$

where K (measured in s^2/m^2) combines the relevant physical characteristics of the coin's metal or alloy: abrasion, density, corrodibility, and hardness, and $\bar{F}$ (measured in watts) a factor encapsulating the forces applied to the coin. Delamare notes that the weight loss does not depend directly either on the coin's area or weight, although the latter can enter indirectly through $\bar{F}$, either because the coin is handled with other coins of the same weight, or because its weight influences its intensity of use.

Experimental evidence

Delamare (1994, 107–124) reviews the experimental literature, which dates back to 1798. All experiments show that weight loss is linear in time. Hardness is inversely related to weight loss. The experiments have also shown that coin size, weight, and relief generally do not matter. The one case where weight matters is the case of an oscillating table on which coins rest without touching each other: then, the force exerted on the coin is obviously proportional to its own weight. Both Nanteuil (1928) and Delamare (1994), however, dismiss the experiment as a poor representation of the actual process of wear.

The term K represents the relevant characteristics of the alloy. Two of the parameters are readily quantifiable, namely density (ρ) and hardness (H), which enter into Delamare's model as ratio of each other (ρ/H_v). Table 1 provides some numbers for gold, silver, and bronze, relevant for the 19th century.[4] They show that the density/hardness ratio can vary substantially among alloys, but that the alloys used in practice in the 19th century had a similar ratio, with gold being 25–33% more susceptible to abrasion than silver, and bronze 10% less. Those factors that we do measure, therefore, do not vary substantially across alloys.

Corrodibility has been measured, but for metals other than the typical 19th century alloys. Experiments have shown that corrosion can increase by a factor of 4 for a coin plated with aluminium rather than nickel (Delamare 1994, 121). Little is known about the coefficient of abrasion K_a.

For my purposes, the model implies that the factors K and F are additively separable

[4] Some of the measurements reported were carried out at the CNAM, Paris in February and March 2012. I thank Pr. Ikhlef for his assistance. The following coins were used: a French 5F silver piece 1868, a French 1F silver piece 1869, a French 5 centimes bronze piece 1853, a British 1s silver piece 1899, a Prussian 1/6 Thaler piece 1812, and a Prussian Thaler of 1831. A dozen measurements were made on various parts of the coins, which showed an average deviation of up to 9%. Measurements made on the reliefs were 13 to 20% lower than on the incuse parts of the coins; this is due to the effect of the strike, which hardens the surface of the metal. For understanding coin wear, the relief measurements are probably the more relevant ones, but I nevertheless used the average measurements when calculating the ρ/H_v ratio in the last column.

Alloys	proportions (‰)	density	Vickers hardness			ρ/H_v	countries
			high	low	mean		
gold							
Au/Cu	997/3	19.25			52	0.370	
Au/Ag	917/83	18.06			67	0.270	
Au/Cu	917/83	17.62			139	0.127	UK
Au/Ag/Cu	917/43/40	17.85			110	0.162	US (before 1834)
Au-Cu	900/100	17.31			147	0.118	France, US (after 1837), Switzerland
silver							
Ag/Cu	990/10	10.47			100	0.105	
Ag/Cu	925/75	10.36	102	87	93	0.111	UK
Ag-Cu	900/100	10.31	112	80	94	0.110	France, US
Ag-Cu	835/165	10.20	120	92	105	0.097	France (after 1866)
Ag-Cu	750/250	10.05	123	99	121	0.083	Germany (until 1871)
Ag-Cu	521/479	9.69	132	111	123	0.079	Germany (until 1871)
bronze							
Cu/Sn/Zn	955/30/15	8.85			102	0.087	UK (after 1860), Canada (after 1878)
Cu/Sn/Zn	950/40/10	8.84	120	99	112	0.079	France (after 1852), Canada (1858–59)

Table 1: Values of density and hardness for some alloys. Source: Delamare (1994, 116, 269) for gold and UK bronze, and measurements by the author (see footnote 4 for explanations).

when weight loss is in logs. Moreover, holding constant the metal or alloy of the coin, only intensity of use will affect weight loss. Conversely, for a given alloy, the patterns of weight loss allow us to measure directly the intensity of use, or in economic terms the velocity of the coin. This paper is concerned with estimating and analyzing the velocity of coins of different denominations from weight loss data. This will shed light on the $\bar{F}$ factor, which has not been investigated to my knowledge.

The magnitude or variable under study

Delamare (1994, 25) notes that the loss of weight of a coin ΔP can be studied under various transformations. The two main one are (a) ΔP or $\Delta P/\Delta t$, namely weight loss (absolute or per unit of time), preferred by physicists, and (b) $\Delta P/P_o$ or $\Delta P/P_o/\Delta t$, the weight loss relative to the original or standard weight (also absolute or per unit of time), preferred by economists and most numismatists. He followed the physicist's approach.

As an economist, I will be interested in the *relative* loss per unit of time, because I think of the loss on coins as the depreciation of a physical capital. As my results will show, this is also the appropriate transformation to use when comparing coins of various sizes.

2.2 Statistical models and estimates of coin wear

The first statistical models of coin wear date back to Nanteuil (1928) and Kosambi (1942). Müller (1977) provided a formal elaboration, based on Brownian motion. His paper, little noticed by numismatists, tried to estimate the original standard of a coin from a surviving population of undated coins. The model posits that the weight of coins issued by the mint follow a normal distribution that is invariant over time, and that weight loss per unit of time follows another invariant normal distribution. The model implies that at any time t, the distribution of weights for coins of vintage i is

$$N(\mu_0 + (t-i)\alpha, \sigma_0^2 + (t-i)\beta) \quad (2)$$

where μ_0 and σ_0^2 characterize the initial distribution of weights as the coins come out of the mint, and α and β characterize the process of wear.

In the next two sections, I will use evidence from coins whose vintage is known, and that are sampled at known dates. This will allow me to test the fit of the model, recover the parameters α and β, and compare them across denominations and countries. In the last section, I will return to the problem of estimating coin wear when the vintage is unknown.

3 Measuring coin wear: data from 19th century surveys

I begin with the data collected and published by the French mint in the late 1880s, because it is of the highest quality. Not only were the surveys extensive and repeated, but the publications (mainly Ruau 1885 and Ruau 1888) included many useful statistics. The mint sampled coins from circulation and computed first moments of the weight distribution by vintage. It also published additional data which permits inference about second moments. I begin with a description of the sample collection.

3.1 *The Paris mint data*

The Paris mint's interest in coin wear arose from the public policy discussions I described in the first section. In 1878 the monetary conference of the Latin Monetary Union expressed the wish to see data collected on wear or weight loss of coins in circulation (Malou 1871–1880, 3(5):93). A few years later, in 1884, surveys were carried out by the director of the French mint on all current gold and silver denominations.

For each denomination, the Banque de France collected every few days samples of 5,000 coins in various locations in France. The coins were presumably taken from the tills of the Bank of France branches. Coins were sorted by date and nationality,[5] and individually weighed to a precision of 1mg. Further samples were collected until the sample average of the denomination's weight varied by less than the weighing precision.

The operation was repeated in 1888 and each year from 1892 to 1896, but only for 20F and (from 1892 to 1894) 10F gold pieces.[6] Summary statistics for the different surveys are reported in Table 2.

Date	denomination	number of samples	total sample size	range of years
1884	20F (gold)	21	89,531	1803–79
	10F (gold)	10	49,027	1850–68
	5F (gold)	5	24,756	1855–69
	5F (silver)	17	64,529	1795–1878
	2F (silver)		4,316	1866–73
	1F (silver)		4,390	1866–72
	$\frac{1}{2}$F (silver)		5,386	1864–73
1888	20F (gold)	10	41,826	1803–87

Table 2: Samples of French coins analyzed by the French mint (1884–1896).

[5]The Latin Monetary Union, to which France belonged, made each member state's coinage legal tender in the other member states. I only analyze the data on French coins.

[6]The 1884 survey is reported in Ruau (1885) and the 1888 survey, which also included a sample of bronze denominations, in Ruau (1888); the surveys in 1892–96 were reported by the *Commission de contrôle monétaire* (Centre des Archives économiques et financières (CAEF), Archives de la monnaie, K-1, 8). I do not use the later surveys because the reminting program begun in 1889 affected the distribution of circulating coins by removing the lighter ones.

Denomination structure in 19th c. France

France adopted the metric system during the Revolution, and its monetary laws of 1795 and 1803 adjusted the pre-Revolutionary monetary units to conform with it. The metals used were gold, silver, and copper (replaced in 1852 by bronze).

The gold coinage initially consisted of 40F and 20F pieces. Some changes took place in mid–century: in 1849 a 10F denomination was introduced, in 1854 the 40F piece was discontinued and new gold denominations of 100F, 50F, and 5F were introduced. Throughout the century, the mainstay of the gold coinage remained the 20F piece, weighing 6.45g and 90% fine. The large silver coin, rated at 5 francs, weighed 5g and 90% fine. Its fractions were 2F, 1F, ½F, ¼F (replaced in 1848 by a 0.20F piece), all silver. This structure closely resembled the pre-Revolution set of denominations which remained legal tender until 1834.[7]

Only one change in alloy took place, as part of the formation of the Latin Monetary Union: in 1864, the fineness of the silver fractions of 0.20F and 0.50F was lowered from 900‰ to 835‰ (this was extended to the 2F and 1F pieces in 1866) and the earlier coins were recalled.

As a result of this history, only two denominations had a long history of circulation by the time of the 1884 survey, namely the 20F gold piece and the 5F silver piece; but they constituted the mainstay of French coin circulation, accounting for 51% and 35% of all coinage between 1803 and 1884. For the other denominations, the time series are rather short.

First moments: linear trend

Figure 2 shows the average weight loss of each vintage (year of minting) for the various denominations in the 1884 survey. Figure 3 shows the data for the 20F gold pieces and the 5F silver pieces separately, along with the number of coins sampled. Because of variations in minting volumes, the sample size is quite small for some years, sometimes a single coin. Nevertheless, a linear trend is quite noticeable for all denominations. It appears that, for the 20F gold coins, the weight limit below which a coin ceased to be

[7]The franc was defined by the law of 28 thermidor III (15 Aug 1795) to be 4.5g of pure silver. The equivalence between the pre-1795 livre (L) and the franc (F) was set at 1F = 1L 3d (or 1F = 1.0125L) by the law of 24 germinal IV (Braesch 1936, 36). The legal tender value of old 6L (écu) and 3L (small écu) coins was lowered to 5.80F and 3.90F in 1810 (Thuillier 1983, 115). The pre-Revolutionary gold and silver coinage was demonetized in 1835, low-grade silver coinage in 1845, and copper coinage in 1852.

legal tender prevented the wear process from continuing indefinitely, whereas that limit had no effect on the distribution of 5F coins.

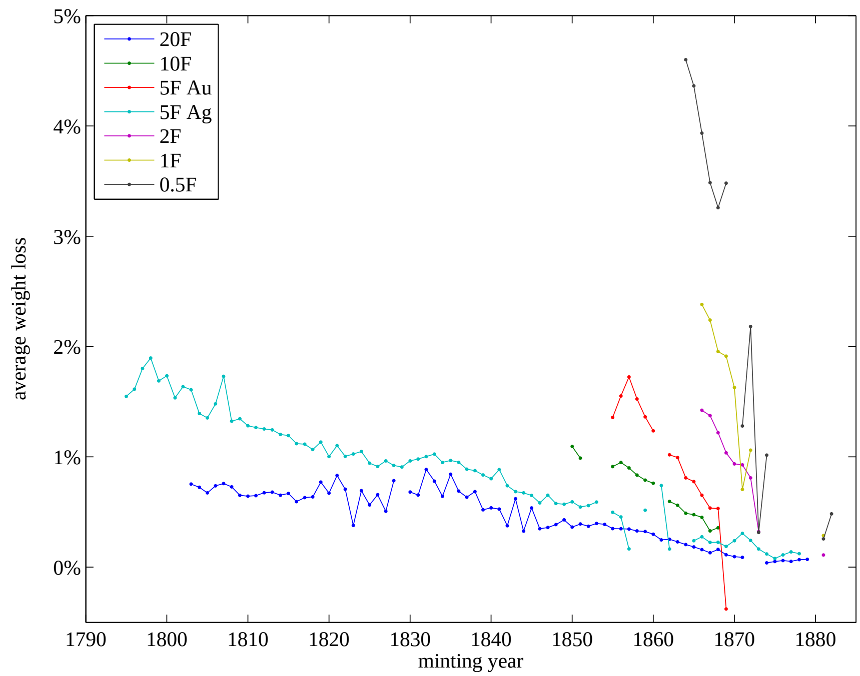


Figure 2: Average weight loss of French coinage by vintage, 1884 survey. Source: Ruau (1885).

To account for varying sample sizes, I use weighted least squares to estimate the time trend in weight loss for each denomination. The results are shown in Table 3. The coefficient on age is the parameter α in the Kosambi-Müller model, expressed in the last column as an annual rate of loss (ppm per year). The intercepts are significant and negative, reminiscent of a pattern of low wear in the first few years of circulation that has been noticed in many other samples (Delamare 1994, 62–65). One explanation for this pattern is that coins may not circulate as soon as they are minted, but remain out of circulation in the central bank's vaults for several years (Foville 1886, 11). The intercepts, if divided by the estimates of the slopes, would correspond to initial periods of 3 to 10 years out of circulation.

It is possible to relate the estimates of Table 3 to the physical process of coin manipulation through one interesting observation. When 400,000 pieces were analyzed in 1838, they were weighed in bulk before being sorted and weighed individually. The experimenters noted that the sum of the individual weights after manipulation fell

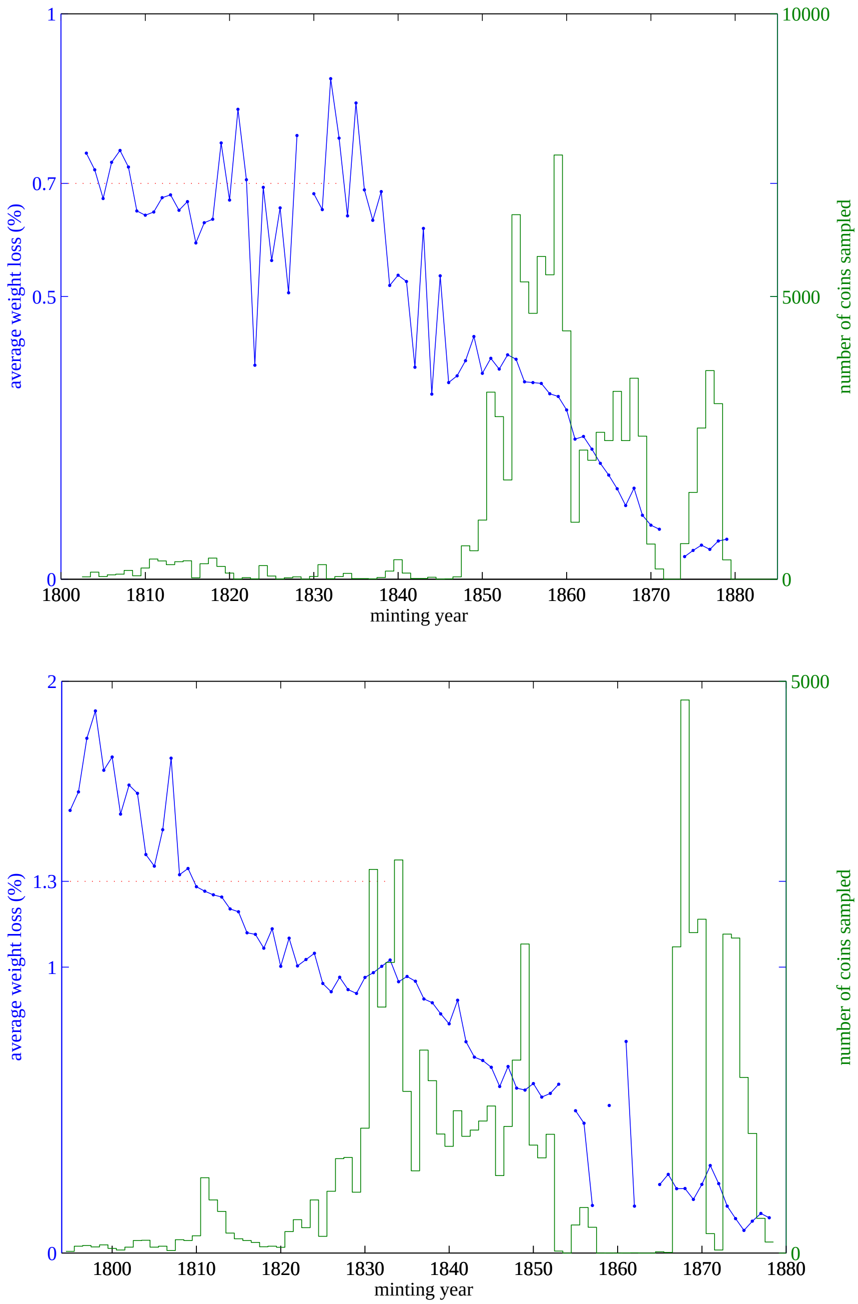


Figure 3: Average weight loss by vintage (left scale) and number of coins sampled (right scale), 20F gold pieces(top graph) and 5F silver piece (bottom graph), 1884 survey. The dotted lines indicate the legal tender limit on weight loss. Source: Ruau (1885).

denomination	age	constant	obs	R^2	annual weight loss (ppm)
20F	0.92 ***	−3.79 ***	44	0.992	142
	(0.03)	(0.76)			
10F	1.53 ***	−13.26 ***	15	0.997	473
	(0.09)	(2.21)			
5F Au	1.63 ***	−18.63 ***	14	0.997	1008
	(0.08)	(1.80)			
5F Ag	4.88 ***	−18.06 ***	79	0.994	195
	(0.09)	(3.60)			
2F	9.95 ***	−38.20 **	9	0.995	995
	(1.02)	(15.77)			
1F	9.12 ***	−47.99 *	8	0.985	1824
	(1.56)	(24.79)			
0.5F	6.43 ***	−19.78 *	12	0.978	2571
	(0.72)	(11.69)			

Table 3: Regressions of annual weight loss on age, 1884 survey. The dependent variable is the average weight loss (in mg) of each vintage, the regressors are age (1884 less minting year) and a constant. For the 20F gold piece, only data after 1833 was used (see text). One, two and three stars denote significance at the 10% (resp. 5%, 1%) level.

short of the total weight before manipulation by 13.26ppm per coin.[8] This loss is of the same magnitude as the monthly loss on 5F silver pieces estimated from the 1884 survey (195ppm per year, or 16.25ppm per month). It would follow that the average 5F piece was handled a few times per month, unless the figure in the 1838 report is for some reason over-estimated,.

Second moments

The original reports of the Paris mint surveys contain additional information that has never been exploited. It allows me to estimate second moments of the weight distributions by vintages. Two different sets of data are amenable to two distinct statistical methods of estimation. I describe them in turn, then use the results to assess the time variation in second moments.

[8] Académie des Sciences, Paris, papiers Dumas, carton 22.

The CLT method

As described earlier, the method of sampling used in 1884 and 1888 consisted in taking successive batches of 5,000 coins of a given denomination, until the samples' total average weight had converged.

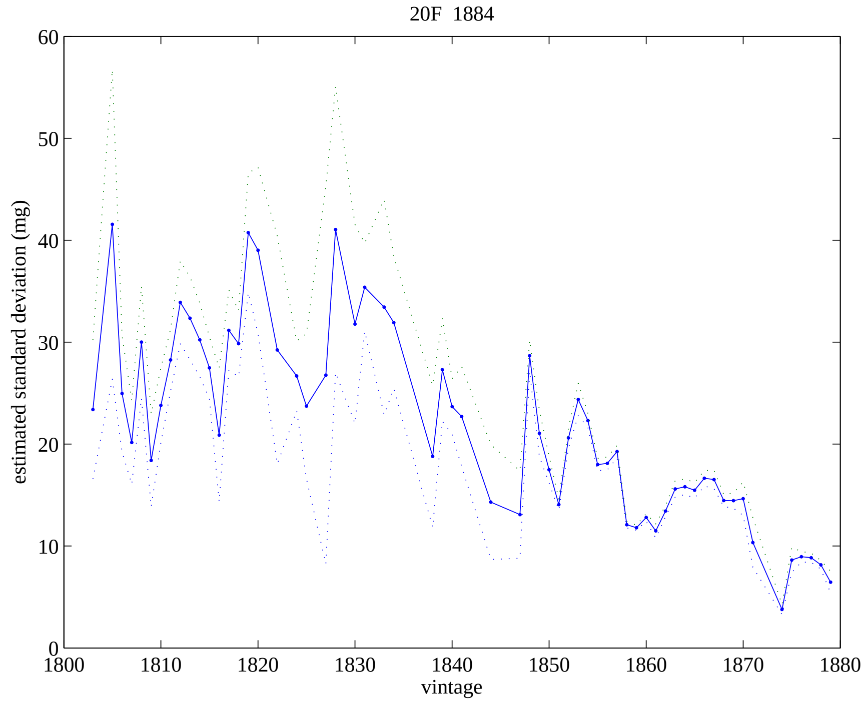
For some denominations (the 20F gold piece and the 5F silver piece), the report provides for each sample j of 5,000 coins and for each vintage i, the number of coins N_{ij} and the total weight of the coins. Assume that the population weight x_i of each vintage i has a mean μ_i and a variance σ_i^2, the report gives us a set of observations $\{N_{ij}, \bar{x}_{ij}\}_{j=1}^{N_i}$. The central limit theorem (CLT) says that the statistic $y_{ij} = \sqrt{N_{ij}}(\bar{x}_{ij} - \mu_i)$ converges in distribution to $N(0, \sigma_i)$ as j grows. Estimating μ_i with the sample average of $\sum_j \bar{x}_{ij} / \sum_j N_{ij}$, I can estimate σ_i^2 as the sample variance of $\{y_{ij}\}_{i=1}^{N_i}$ (see the Appendix).

The GMM method

Because of the interest in the proportion of coins that fell above or below tolerance levels, the reports also provided the number and total weight of coins of each denomination and vintage falling within four pre-determined bins: heavier than the tolerance at production set in 1803 (+0.2% for gold coins, +0.3% for silver coins), within the tolerance at production, less than the tolerance at production but above the weight loss tolerance (−0.7% for gold, −1.3% for silver), and lighter than the weight loss tolerance. What makes these statistics particularly interesting is that the bins are not symmetric around the averages of the annual distributions.

There are two ways to use this information in order to make inferences about the vintage-specific weight distributions. Both are parametric and assume that the distributions are normal. The first method is to use only the number of coins falling in each bin and maximize the log-likelihood of the multinomial distribution; the other is to use both number and weight of coins, using the generalized method of moments (GMM), each bin-specific average providing a moment condition (see the Appendix). I use results for the second, which proves itself to be more efficient in simulations of a Gaussian data-generating mechanism. The results are shown for two denominations in Figure 4.

The two methods yield comparable results, as shown in Figure 5.



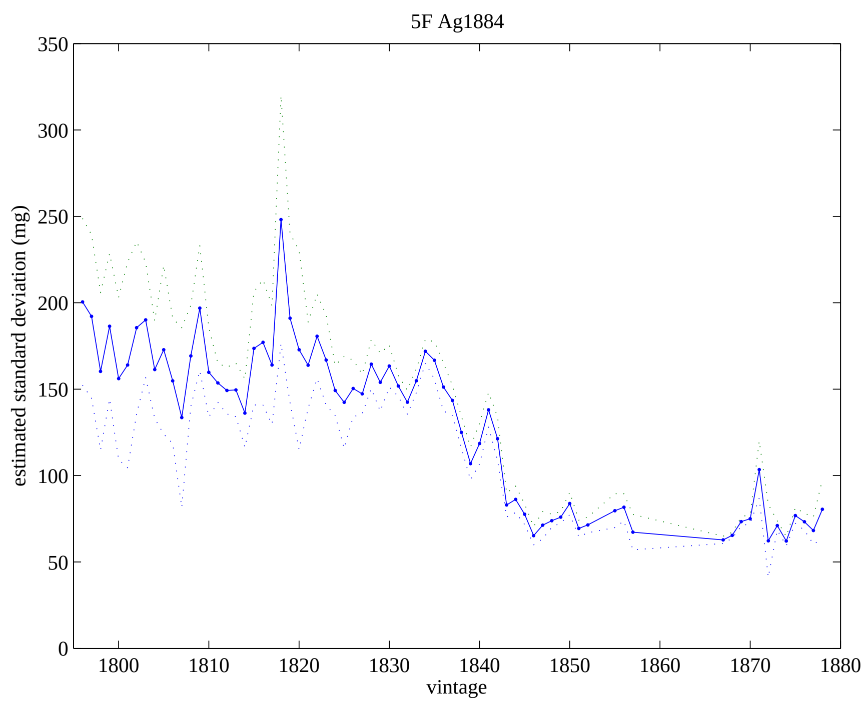


Figure 4: Estimated standard deviation of coin weights by vintage, 20F gold piece (top) and 5F silver piece (bottom), 1884 data. Estimation by GMM. The dotted lines indicate the 2-standard error bands.

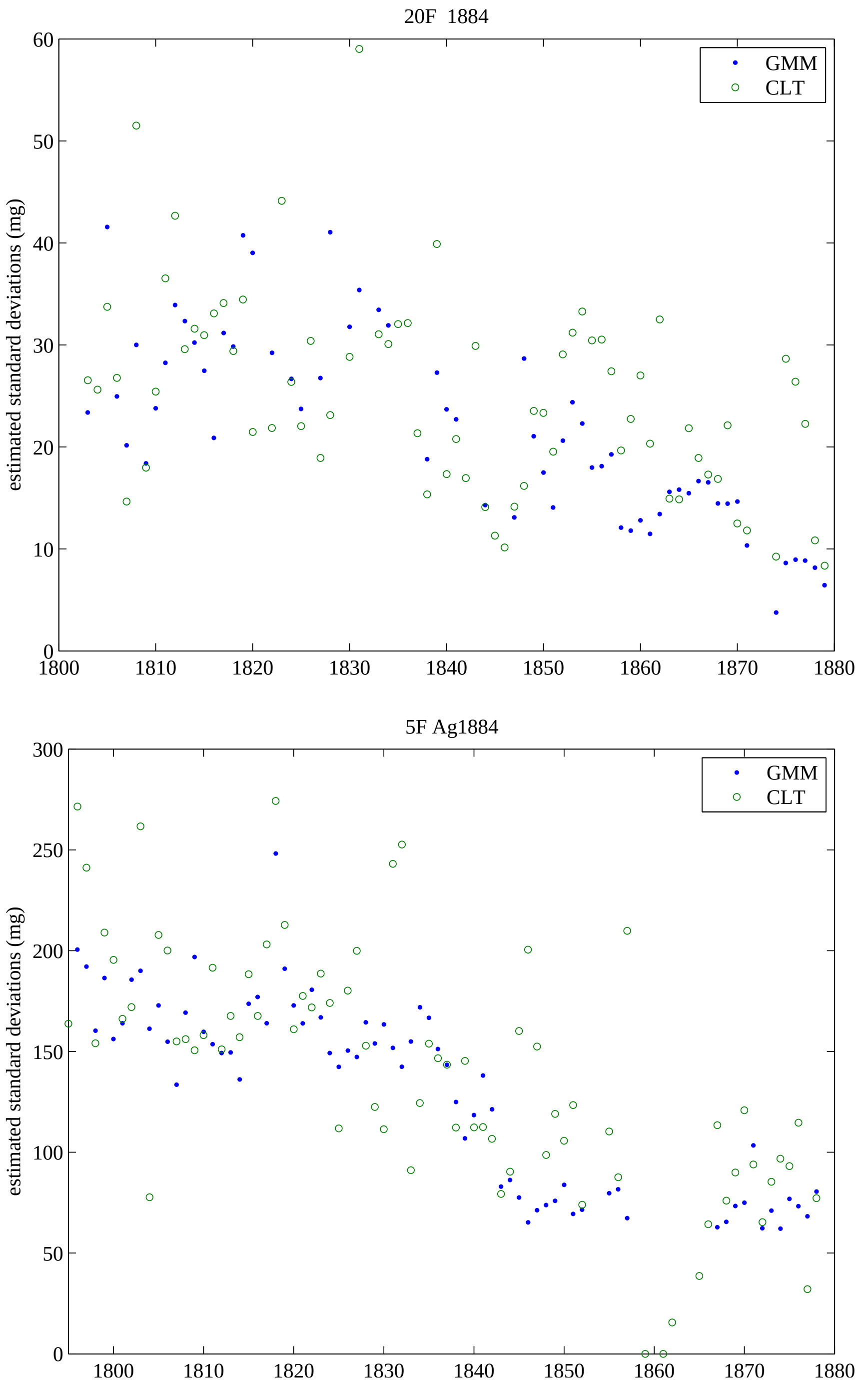


Figure 5: Comparison of the CLT and GMM estimates.

	CLT estimates				GMM estimates			
denomination	age	constant	obs	R^2	age	constant	obs	R^2
20F Au (1884)	14.1 ***	279.7 ***	74	0.82	13.7 ***	−53.2 *	62	0.87
	(2.9)	(80.2)			(1.3)	(37.2)		
20F Au (1887)	15.2 ***	33.8	73	0.61	15.7 ***	−81.9 *	58	0.89
	(3.7)	(117.6)			(1.6)	(51.4)		
10F Au	102.6 *	−1773.3	16	0.31	2.4	105.0 *	15	0.94
	(69.5)	(1688.9)			(2.6)	(63.3)		
5F Au	−14.2	840.8	14	0.30	4.3 *	−0.8	12	0.91
	(55.4)	(1264.9)			(2.5)	(57.8)		
5F Ag	470.9 ***	1861.4	79	0.70	434.8 ***	−2351.5 **	72	0.90
	(82.0)	(3326.4)			(33.4)	(1352.9)		

Table 4: Regressions of estimated variances of coin weights on age, 1884 and 1888 surveys. The dependent variable is the variance of coin weights estimated by the CLT and GMM method, in mg^2. One, two and three stars denote significance at the 10% (resp. 5%, 1%) level.

Time variation in second moments

Figure 5 shows that there is a clear time trend in the estimated standard deviations. Table 4 presents the results of regressing the variances (as suggested by the Kosambi-Müller model) on coin age. The results are not significant for the coins with short time series (10F and 5F gold pieces). For the 20F gold and 5F silver pieces, the estimated time trends are consistent across the two methods. Only one intercept appears significant, for the 20F gold coin in the 1884 sample: its square root of 0.26% seems rather high compared with the official tolerance at production of 0.2%.[9] These results validate the findings of Kosambi (1942) and the assumption of Müller (1977), and indicate that, for each denomination, the process of wear is stable through time.

[9]There is little direct evidence on the complete distribution of coins as they come out of the mint, because only certain statistics, namely the proportion of coins falling above or below the legal remedy, were of concern. In a few instances additional information has been published, which allows some inference. A sample of 50 coins taken from the mint's production in 1893 had a standard deviation of 0.7% (Archives économiques et financières, K-7(8), *Rapport de la commission de contrôle de la circulation monétaire*, 1893). For silver, we have the following observation. The chemist Jean-Baptiste Dumas, who served in the late 1830s on a parliamentary commission on coinage, noted in his papers the average, 25 lowest and 25 highest weights out of a sample of 1000 coins minted in Lille in 1823 (Académie des Sciences, Paris, papiers Dumas, carton 22). From these observations one can estimate the standard deviation of the sample to have been 0.37% (the reported mean was 0.3% below standard).

Variation in parameters over time

The 1884 and 1888 surveys allowed us to estimate the parameters of the Kosambi-Müller model. But were the parameters stable over time?

Fig 3 shows some fluctuations in weight loss from the linear trend. There are two possible sources of around the linear time trend. One is that the sample is biased because some vintages have been subjected to selective culling. The other is that the mean weight loss experienced by coins varies over time, and the sample accurately reflects this variation.

Culling

France's coinage was bimetallic and, as gold and silver prices varied, one or the other metal tended to be taken out of or into coined form. As described in the introduction, these processes tended to be selective: for two coins passing at the same face value, the exporter or melter would prefer the heavier one.

Flandreau (1995) has studied the history of the French money stock up to 1879, using the results of a census of coin vintages carried out in France in 1878.[10] He estimates a constant, "normal" rate of absorption (1.1% for gold, a surprisingly low 0.15% for silver), as well as (positive) deviations due to bimetallic arbitrage and other events. At first glance, they do not appear to have affected the weight distribution of coins shown in Figure 3. In particular, one such event was the improvement in refining around 1830 that allowed to extract more gold from old silver coinage. Flandreau estimates that it resulted in 84% of the coinage being melted down. Although the impact of the ensuing speculation appears in the very low survival rates of pre-1830 silver coinage, it doesn't appear in the weight distribution: if anything, the surviving pre-1830 coins are heavier than normal given their age. Further work is needed here.

Other surveys

To find out if the weight loss process $N(\alpha, \beta)$ varied over time, one would want data of similar quality as the 1884 and 1888 surveys from different time periods. The surveys of 1892–96 on gold coinage show lower rates of wear, but, as described in the first section, France had begun in 1889 to recoin underweight gold pieces, a process that was intended

[10]The survey involved a much larger number of coins (over two millions), but only numbers per vintage were recorded. The vintage-specific survival rates are quite similar to those of the 1884 mint survey.

to change the weight distribution of coinage. As for earlier periods, although some surveys were carried out, I have found only partial information summarized below.

Inquiries in coin wear are first mentioned in the late 18th century. In a speech to the Convention in February 1793, the finance minister Clavière mentions that "various measurements" had shown a loss of 6 grains (0.32g) on each 6L écu (1.08%), and 10 grains (5.31g) on 3L écus (3.60%) (Archives parlementaires, 1re série 1862–, 58:272). These numbers, cited again by Angot des Rotours a few years later (Thuillier 1993, 161), must be averages over all vintages. Given known minting volumes, and assuming a constant disappearance rate of 1% per year, the average coin was about 31 years old, yielding an average annual loss of 350ppm and 1200ppm. Costes (1885, 57) reports experiments made in the Paris mint, showing that pre-Revolution écus had lost 1.95% in 1816, 2.19% in 1822, and 2.44% in 1828, suggesting an annual loss of 410ppm per year for the period from 1816 to 1828, double the 195ppm I estimated from the 1884 survey.

In 1838 a commission was appointed by the minister of finance to inquire into the recoinage of copper coinage. Two members, the chemist Jean-Baptiste Dumas and the finance inspector Louis-Auguste Saint-Julle de Colmont, were given further instructions to inquire into the state of coinage and minting institutions. Among other experiments they sorted by age and mint location, and individually weighed 400,000 5F pieces; they concluded that weight loss was linear in time and averaged 4mg per year (160 ppm). Their 1839 report remained confidential and the appendix on coin wear appears to be lost.[11]

In 1847 the French government considered recoining all écus minted before 1825, because imperfect refining methods had left a gold content of about 0.08% in the silver: with a gold-silver ratio, this meant that the intrinsic content of the écu was 1.25% too high, and speculators had begun melting down the coins to extract the gold. To properly estimate the potential profit to the government of doing so itself, it was necessary to know the actual weight of pre-1825 coins in circulation. Samples were taken, sorted by reign, and weighted. The results are included in Table 5.[12]

Dumas returned to the subject thirty years later, by then a Senator and chair of the currency commission. In a speech to the Senate in 1870 he cited his 1839 experiments,

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[11]Page 34 of the *Rapport à la Commission instituée par arrêté de M. le Ministre des finances du 14 juillet 1838, pour étudier les questions relatives à la refonte des monnaies*, December 1839. Contrary to the assertion of Thuillier (1983, 305), at least one printed copy exists in France: Bibliothèque de l'Institut, 4° M 1171. The report's appendices were not circulated and remained in the Ministry of Finance, which was destroyed by fire in 1871. Dumas's papers (Académie des Sciences, papiers Dumas, carton 22) contain a few relevant documents, but not the appendices.

[12]CAEF, Archives de la monnaie, H-2, 14.

and reported measurements made in 1868 on 5F silver pieces.[13] I have found a summary of these measurements in Dumas's papers: they were made on 2,602,345 pieces of 5F melted down from 1864 to 1866 to provide silver for the new subsidiary coinage: vintages prior to 1824 had suffered an annual loss of 120 ppm, those from 1824 to 1830 had suffered 140ppm, and those from 1831 to 1864 had suffered 144ppm. It is not clear that the coins were chosen at random. Dumas also reported that the gold 10F piece wore twice as fast as the gold 20F piece, and the gold 5F piece four times as fast. The results for the 20F gold pieces were reported to an international monetary conference (Malou 1871–1880, 3(5):121) and are included in Table 5.

In summary, the earlier surveys provide estimates of annual loss of 5F silver pieces during the first half of the 19th century that are of the same magnitude as that derived from the 1884 survey, although somewhat smaller. We will see that evidence from hoards fits quite well with the latter. The estimates of annual loss on pre-Revolution 6L pieces appear to be twice as large; but Dumas and Colmont note that prior to the adoption of the Droz collar in 1807, coins were irregular and susceptible to clipping. A misshapen coin offers a one-time opportunity for clipping, so clipping would affect the level of coin loss independently of age, that is, introduce an intercept. The estimates of annual loss based on averages could be substantially biased upward.

Weight loss by denomination

Having estimates of annual weight loss by denomination, it seemed interesting to compare them. This is where I depart from the physicist's approach: Figure 6 plots relative rates of loss against denomination (in log scale). For a given metal (fixing the K factor in Delamare's model) the relative weight loss appears to be linear in the log of coin value, and the slopes appear to be nearly identical across metals. The difference in alloy (gold, silver, and bronze) appears to affect only the intercept; in other words, the factors affecting K in Delamare's model are additive in logs.

There are clearly too few points in the graph to assert a more general relationship. I now turn to evidence from other countries.

3.2 Variation across time and space

Throughout time, monetary authorities in various countries have been interested in determining the extent of weight loss on coins. Grierson (1963) mentions that prior to

[13] *Journal Officiel*, 19 Jan 1870, p. 121.

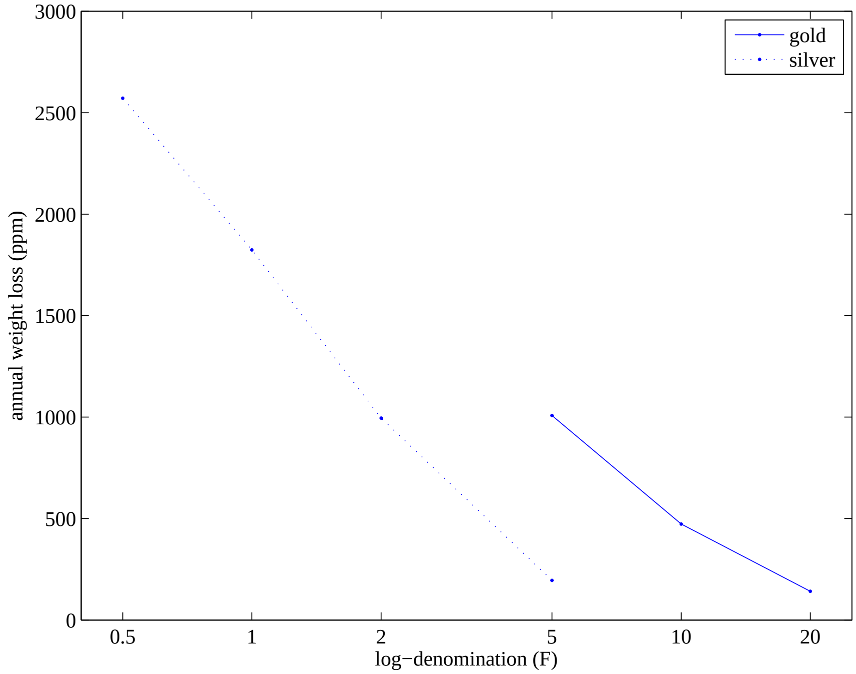

Figure 6: Relative weight loss by denomination (1884 data).

the English recoinage of 1561/62, it was proposed to send agents to thirty-six butcher shops in London where, on pretence of settling a wager as to whether more money was received during the mornings or the afternoons, they were to get permission to examine the day's takings and so could classify its contents roughly into good, bad and indifferent coins. Whether the plan was carried out we do not know.

The first estimates that I can find date from the late 18th century and numerous studies have been carried out in various countries during the 19th and 20th centuries. The manner of estimating annual weight loss varies by publication: sometimes the average weight loss per year is reported for each age or vintage, sometimes only the mean annual loss over all vintages. Computing a mean annual loss implicitly assumes no intercept (which, as will be apparent, is not a safe assumption). Also, it is a consistent estimator but less efficient than weighted least squares on vintage-specific losses, which is feasible when the published results include the number of coins in each vintage, and which is the same as ordinary least squares regression of the individual coin weights. The first to use least squares to estimate the rate of wear was Kosambi (1942).[14]

[14]Nanteuil (1928) divides his sample in two halves and effectively computes least squares with two weights and no intercept.

Britain

Liverpool (1805, 187) reports the results of two experiments made on silver coinage in 1787 and 1798 by the London Mint: only average weights are reported, for both experiments. This allows to compute the weight loss rate between the two dates. During the Bank Suspension period of 1797–1819 questions related to metallic coinage were moot. In 1816 the United Kingdom effectively adopted the gold standard, by allowing free minting of a new coin worth £1, called the sovereign, and indefinitely postponing free minting of the silver coins. In December 1826 and July 1833, all denominations commonly produced, both gold and silver, were examined for wear, by selecting samples of fixed size for various vintages and measuring the average wear. The question lay dormant for several decades until Jevons (1868) brought it to the fore, with a paper that attempted to estimate the wear on gold coins as well as the total existing stock of coins, based on survival rates of coins by vintage. Martin (1882) replicated the measurements, on a larger scale. The Royal Mint carried out its own investigation in January 1888, sampling coins at 300 post offices throughout the kingdom, and repeated the experiment in 1895. Finally, one should note the analyses of gold coins after 1891, as well as the examination of worn silver coins carried out in 1908.

Figure 7 shows the weight loss and survival rates for the sovereigns and half-sovereigns.[15] The survival rates of British gold coinage display less variability than those of French coinage, and estimation of the absorption rate by least-squares regression of the logs of survival rates on age yield somewhat higher absorption rates (5 to 6%) than found by Flandreau.

United States and other countries

One study of subsidiary coinage (silver half and quarter dollars) was reported by the Director of the Mint in 1885. It was similar in size to the contemporary European surveys. A plot of weight loss by vintage shows a sharp discontinuity at the time of the Civil War, when the greenback replaced silver coinage and most US silver coinage was shipped to Latin America or melted down. After 1865, both denominations show a regular linear trend in weight loss by age (Figure 8).

I have found a few other studies, one from Switzerland and a rather old report for (pre-unification) German and Austrian coinage.

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[15]The minting data are found in the British Parliamentary Papers: "Account of gold, silver, and copper monies..." 1847–48 (601) XXXIX.289; "Account of gold, silver, and copper monies..." 1854(2) XXXIX.473;

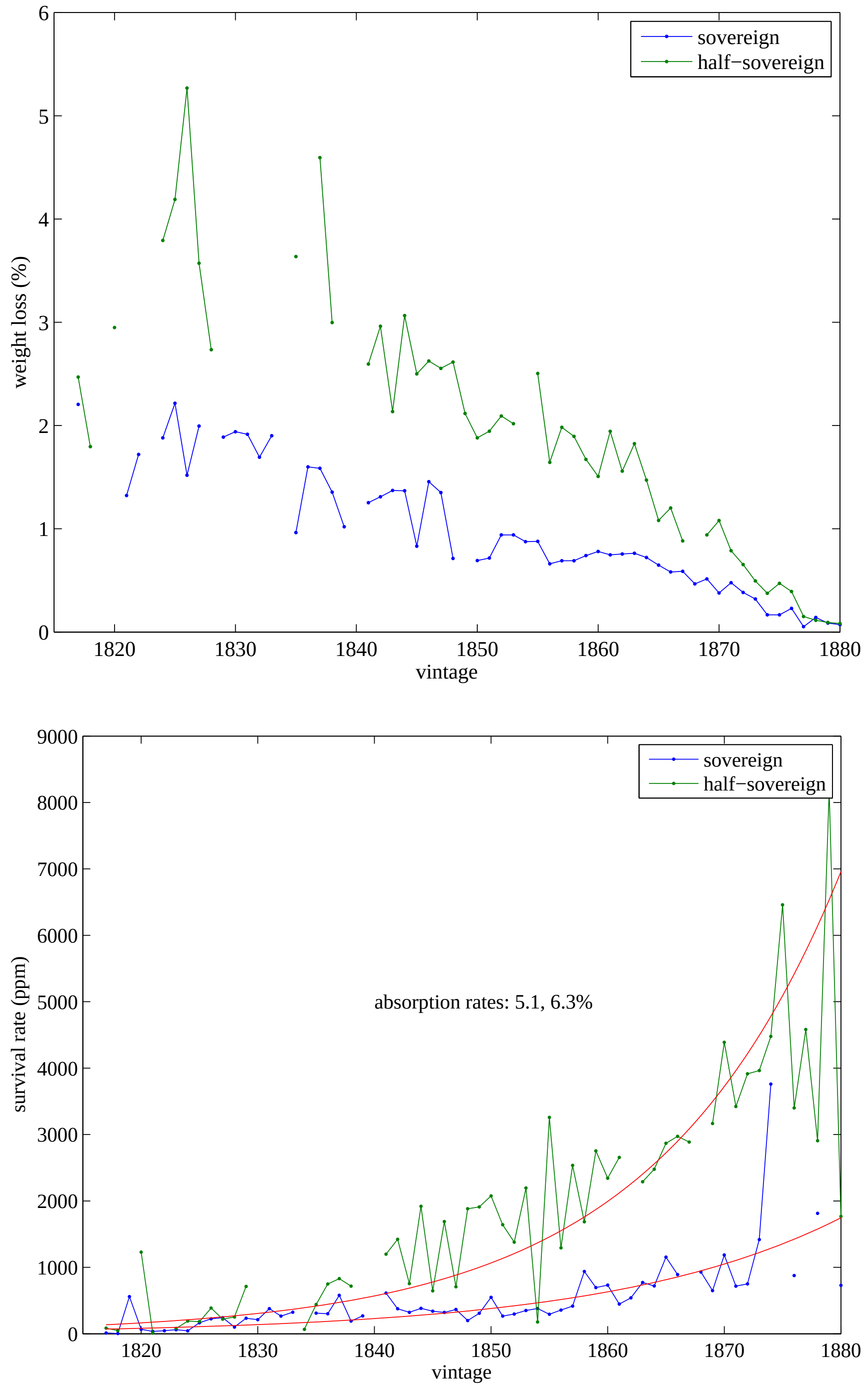

Figure 7: Average weight loss and survival rates, 1882 survey of British gold coinage. Source: Martin (1882).

value (local)	value (F)	weight (g)	diam (mm)	fine (‰)	annual loss (ppm): avg	OLS	date of measures	range of vintages	sample size
UK, gold									
1	25.2	7.99	22.05	917	251.1	208.8	1826	1817-25	1,350
					375.9	231.8	1833	1817-29	1,200
					366.7	352.6	1868	1817-67	280
					350.8	316.2	1882	1817-80	600
					313.7	315.8	1888	1817-87	32,854
					291.6	191.4	1895	1838-95	23,469
0.50	12.6	3.99	19.25	917	312.4	182.4	1826	1817-25	830
					676.3	384.6	1833	1817-29	1,200
					1130.4	981.2	1868	1817-67	178
					710.4	579.8	1882	1817-80	600
					851.3	933.2	1888	1817-87	23,643
					844.4	599.0	1895	1838-95	16,766
UK, silver									
0.25	6.3	28.28	38.81	925	169.5	—	1798	n.a.	-
0.13	3.2	14.14	32.31	925	1658.6	—	1798	n.a.	-
					1045.3	1153.5	1826	1817-25	900
					1034.5	1919.8	1833	1817-29	1,200
					1439.5	—	1906, 09	n.a.	1,600
0.10	2.5	11.31	30.04	925	1891.0	—	1906, 09	n.a.	2,000
0.05	1.3	5.66	23.60	925	4362.4	—	1798	n.a.	-
					2420.3	2568.6	1826	1817-25	900
					2309.0	3589.6	1833	1817-29	1,200
					2585.5	—	1906, 09	n.a.	4,000
0.03	0.6	2.83	19.41	925	2737.3	—	1798	n.a.	-
					3964.2	4258.7	1826	1817-25	900
					3605.3	3351.0	1833	1817-29	1,200
					4002.5	—	1906, 09	n.a.	2,000
0.01	0.3	1.41	16.21	925	2689.5	—	1906, 09	n.a.	1,000

Table 5: Annual abrasion rates and coin denominations for various countries (1). The denominations are converted to French francs. Sources, UK: Liverpool (1805, 187), Jacob (1831, 2:380), Porter (1834, 3:16), Jevons (1868), Martin (1882), Royal Mint (1870–1913, 20:95–109, 26:80–106, 39:58–61), Ansell (1870, 57) (for coin specifications). France: Feer-Herzog (1870, 101), Malou (1871–1880, 3(5):121), Ruau (1885). Germany: Karmarsch (1851, 1:575), Karmarsch (1846, 235) (coin specifications). Switzerland: Feer-Herzog (1870, 98). US: U.S. Secretary of the Treasury (1863, 48), U.S. Secretary of the Treasury (1873–1914, 14:119). Canada: Royal Mint (1870–1913, 43:143, 45:168).

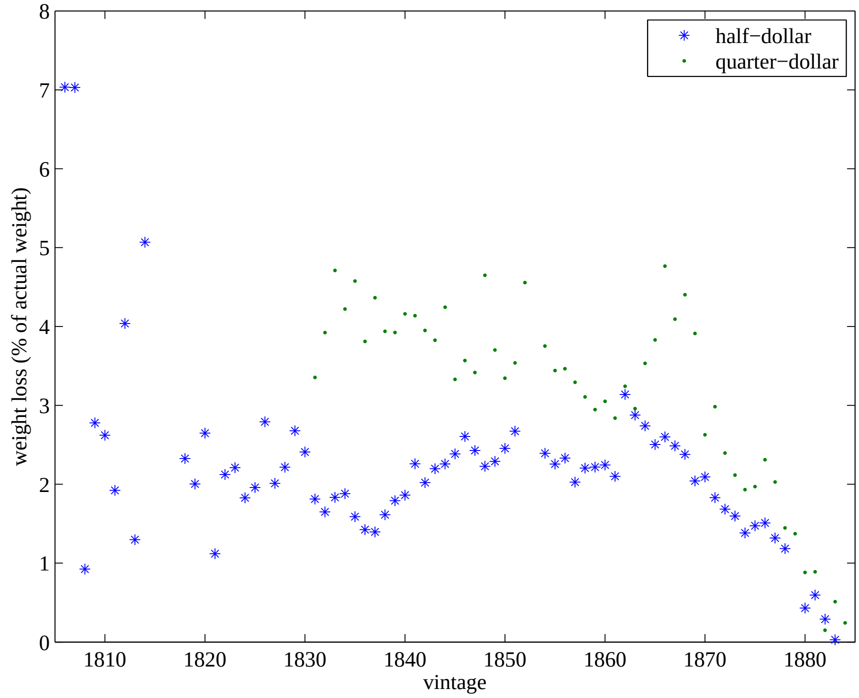

Figure 8: Weight loss of silver half-dollars and quarter-dollars by vintage (1885 sample). Source: U.S. Secretary of the Treasury (1886, 498–499).

"Account of all gold, silver, and copper monies…" 1864(516) xxxii.281; and in Royal Mint (1870–1913).

value (local)	value (F)	weight (g)	diam (mm)	fine (‰)	annual loss (ppm): avg	OLS	date of measures	range of vintages	sample size
France, gold									
20	20.0	6.45	21.00	900	210.2	103.0	1868	1851-60	30,000
					180.2	159.9	1868	1831-67	10,000
					153.5	141.9	1884	1833-79	89,531
10	10.0	3.23	19.00	900	516.8	473.1	1884	1851-69	49,189
5	5.0	1.61	17.00	900	1179.9	1007.8	1884	1855-69	24,756
France, silver									
5	5.0	25.00	37.00	900	218.5	—		1796-1824	560,000
					202.8	195.3	1884	1795-1878	64,915
2	2.0	10.00	27.00	900	933.5	—		1804-24	18,000
					1121.4	998.8	1884	1866-81	4,316
1	1.0	5.00	23.00	900	1680.7	—		1803-24	32,000
					2218.8	1823.6	1884	1866-81	4,390
0.50	0.5	2.50	18.00	900	2550.7	—		1803-24	8,000
					2496.0	2571.4	1884	1864-82	5,386
Germany/Austria, silver									
1	3.7	22.27	34.00	750	270.0	—	n.a.	n.a.	-
0.33	0.9	6.68	26.50	583	560.0	—	n.a.	n.a.	-
0.17	0.6	4.80	23.00	521	480.0	—	n.a.	n.a.	-
0.08	0.3	3.25	20.70	375	780.0	—	n.a.	n.a.	-
Switzerland, gold									
20	20.0	6.45	21.00	900	173.0	110.9	1868	1806-67	1,877
10	10.0	3.23	19.00	900	441.8	—	1868	1850-67	-
5	5.0	1.61	19.00	900	675.8	—	1868	1850-67	-

Table 5: Annual abrasion rates and coin denominations for various countries.

value (local)	value (F)	weight (g)	diam (mm)	fine (‰)	annual loss (ppm): avg	OLS	date of measures	range of vintages	sample size
US, gold									
20	103.7	33.44	34.29	900	111.1	—	1862	n.a.	-
5	25.9	8.36	21.59	900	281.7	—	1862	n.a.	-
2.50	13.0	4.18	19.05	900	165.9	—	1873	n.a.	-
1	5.2	1.67	13.97	900	399.2	—	1873	n.a.	-
US, silver									
1	5.2	26.73	38.10	900	1587.3	—	1862	n.a.	-
0.50	2.6	12.50	30.48	900	1109.4	1036.3	1885	1862-78	132,834
0.25	1.3	6.25	24.13	900	1420.4	1405.8	1885	1862-78	187,795
Canada, silver									
0.50	2.6	11.62	29.72	925	1491.5	1292.7	1912, 14	1858-1914	1,178
0.25	1.3	5.81	23.62	925	2152.8	1955.0	1912, 14	1858-1914	14,115
0.10	0.5	2.32	18.03	925	3473.1	3786.9	1912, 14	1858-1914	3,638
0.05	0.3	1.16	15.49	925	2979.0	2735.7	1912, 14	1858-1914	5,111

Table 5: Annual abrasion rates and coin denominations for various countries.

Weight loss across denominations and countries, or, a general law

I collect estimates of annual weight losses together in Table 5. Where possible, I use least squares to estimate the rate of loss. To make the figures comparable, I report relative rates of loss in ppm rather than absolute rates in grams. I also provide a conversion of the monetary units into francs, using metal parities. I exclude bronze and copper coinage, since I have found 19th century data for no country but France.

	regressor	gold dummy	R^2	AIC
*dependent variable: 1000*log of absolute weight loss (mg)*				
log of area (mm^2)	544.6 ***	−1576.9 ***	0.897	12.89
	(185.3)	(180.9)		
log of weight (mg)	298.6 ***	−1722.6 ***	0.896	12.90
	(104.6)	(166.8)		
log of value (F)	337.2 ***	−2621.7 ***	0.904	12.82
	(92.7)	(275.2)		
gold dummy alone		−1802.4 ***	0.881	13.04
		(174.7)		
dependent variable: relative weight loss (ppm)				
log of area (mm^2)	−1359.2 ***	−2092.0 ***	0.831	13.27
	(224.0)	(218.7)		
log of weight (mg)	−744.4 ***	−1728.2 ***	0.825	13.30
	(128.0)	(204.3)		
log of value (F)	−572.1 ***	−139.4	0.792	13.48
	(128.5)	(381.7)		
gold dummy alone		−1529.3 ***	0.718	13.78
		(253.6)		

Table 6: Regressions of annual weight loss on various coin characteristics. Each regression also includes a constant. AIC: Akaike Information Criterion (a lower value indicates a better fit).

Delamare's model can be rewritten as

$$\log(\Delta P/\Delta t) = \log(K) + \log(\bar{F}) \qquad (3)$$

so that a dummy for the metal should pick up the $\log(K)$ term, and any other factor that is picked up in a linear regression should correspond to the $\log(\bar{F})$ term, or intensity of use.

The top panel of Table 6 presents the results of some regressions of annual (absolute) weight loss on a gold dummy and coin area, coin weight, and coin value. The coefficients on the coin characteristics are significant, but comparison with the regression on the

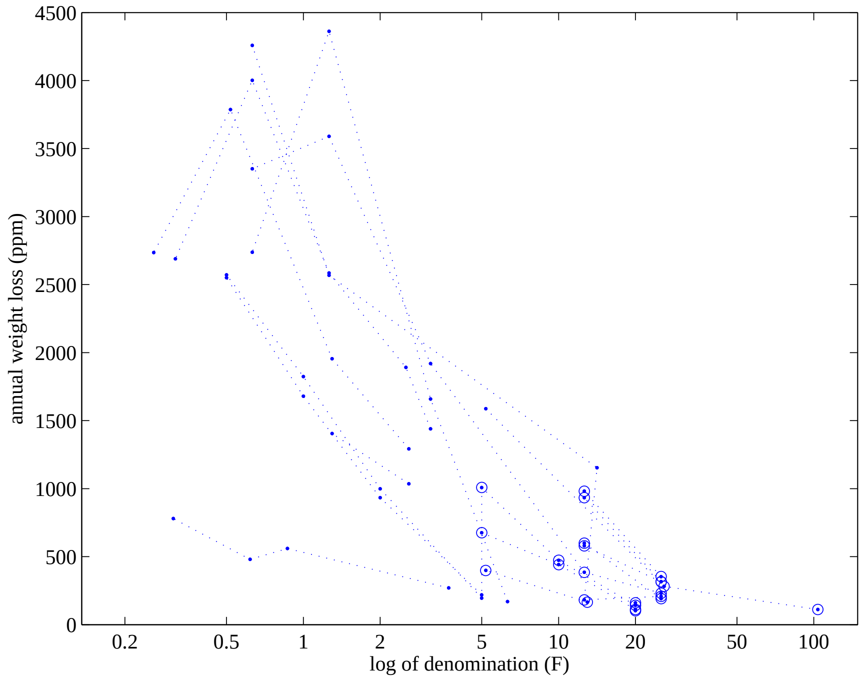

Figure 9: Relative weight loss and denomination. The dotted lines connect estimates from the same country and survey. Circles indicate gold denominations. Source: Table 5.

gold dummy alone show that they explain very little of the variation. Not only does the gold dummy explain most of the variation alone: its coefficient seems too large, as it would imply a K coefficient for gold between $e^{1.58} \sim 5$ and $e^{2.74} \sim 15$ times smaller than for silver, a difference that is much larger than those that can be accounted for by known physical characteristics of the alloys (Table 1). Gold and silver coins have markedly different rates of wear, but it is not because of the alloy.

If we move from the physicist's point of view to the economist's point of view, the right transformation of the weight loss data is a relative loss. The bottom panel of Table 6 tries the same explanatory variables. The results are striking: while area or weight again add little to the gold dummy, coin value makes the gold dummy insignificant, and explains just as much.

Figure 9 plots the estimates of annual relative weight loss as a function of the logarithm of value or denomination. In spite of varying data quality, there is a reasonably clear linear relationship between the log of coin value and the annual weight loss, especially for given countries. Moreover, the linear relationship holds roughly independently

dependent variable: weight loss (ppm)		
log of value (F)	−610.5 ***	
	(73.3)	
constant	2107.7 ***	
	(145.7)	
log-value interacted with country dummies		
UK		−588.8 ***
		(92.0)
France		−766.5 ***
		(139.5)
Germany/Austria		50.1
		(449.1)
Switzerland		−732.4 ***
		(209.3)
US		−566.8 ***
		(132.2)
Canada		−848.7 **
		(458.2)
constant		2141.1 ***
		(151.6)
adj R2	0.788	0.786
p-value of F-test:		0.511

Table 7: Regressions of annual weight loss on log of value.

of the metal, and the slope is the same for all countries except Germany (see Table 7).[16] This suggests that the linear relation between the rate of weight loss and the log of coin value is quite robust, except perhaps at the lowest end of the denomination scale.

Taking the rate of weight loss as a measure of intensity of use, or velocity, I propose that velocity V is inversely related to a coin's monetary value v, and more specifically:

$$V_i \sim a - \log(v_i)$$

where the constant a will depend on the unit of monetary value.

I admit that this striking empirical pattern cannot be easily reconciled with Delamare's model (which is in terms of absolute, rather than relative, wear) unless one assumes that the factor $\bar{F}$ is a product of the coin's weight and its intensity of use. I will

[16]When the data from Germany and Austria is excluded, the p-value for the F-test that country-specific intercepts are identical is 3.1%.

leave this hypothesis for tribologists.

4 Measuring coin wear: data from hoards and the numismatic trade

4.1 Hoards

The previous section has analyzed all the available data derived from samples of actual coin circulation, the earliest of which dates from 1798. For earlier time periods, numismatists rely on another source, namely hoards. It seems particularly interesting to test whether they constitute a reliable source of data to estimate coin wear. For this reason, I concentrate on French hoards of the 18th and early 19th century, close in time to the samples I have analyzed, and composed of the same coins (5F pieces) or very similar ones (see Callataÿ 1994 for such a comparison on one 18th century hoard). As described above, the monetary system before and after the Revolution was quite similar, and pre-Revolution gold and silver coins remained legal tender until 1834. One can hope that the patterns of circulation are not too dissimilar.

Monetary hoards have existed as long as money has. Numismatists tend to think of hoards as samples taken from circulation, distinguishing between two main types of hoards: those that have been accumulated over a long period of time, with some possible care exercised in the choice of coins, and emergency hoards constituted quickly during an emergency, when there was presumably less time to pick and choose coins. The latter type will be closer to a random sample taken from the existing circulation, although in practice it is often difficult to tell the two types apart, unless the hoard's archaeological context or composition can date it to a known period of unrest and uncertainty. That would seem to be the case for several hoards dating from the early years of the French Revolution in my sample.

Nineteenth century hoards

The CGB website provides data on four 19th c. hoards, briefly described in Table 8, to which I added post-1796 coins from the Tirepied hoard. All consist of 5-franc silver pieces or equivalents, minted after 1796.[17] For each coin, I compute its "relative age" as

[17]I thank Jérôme Jambu for making the Tirepied data (Jambu 2012) available to me. Coins minted by Napoleon in Italy, and by neighboring countries like Piedmont and Belgium in the 19th century, were identical in size, weight, and composition to the French coins.

hoard and location	number of coins	range of dates
"Amélie"s hoard' (near Rouen)	329	1795–1846
Gimont (Gers)	128	1797–1839
Soignies (Belgium)	71	1799–1839
Tirepied (Manche)	331	1796–1824
Lagny	148	1796–1824
dependent variable: weight loss (mg)		
relative age	6.328***	6.576***
	(0.528)	(0.557)
constant	59.430***	24.096*
	(9.289)	(13.949)
hoard dummies:		
G		-3.364
		(17.942)
S		32.392
		(22.654)
T		97.945***
		(13.760)
L		-14.944
		(18.453)
centered R^2	0.125	0.187
observations	1007	

Table 8: Nineteenth century hoards of silver 5F coins, and regression of weight loss on age.

the difference between the minting date of the most recent coin in the same hoard and the coin's own minting date.

The corrosion on buried coins probably explains the significant positive intercept. There is, however, no reason to expect corrosion to be correlated with the age of the coin. In fact, the estimate of annual weight loss of 263ppm is reasonably close to the 195ppm found in the 1884 survey, but rather larger than the 160ppm reported by Dumas in 1838.

Eighteenth century hoards

In this section I combine data on 18th century coins from twelve French and Belgian hoards (Table 9).[18] These coins consist exclusively of French écus, a large silver coin

[18] I thank Arnaud Clairand for providing me the data of the Montigny and Saint-Avold hoards, and Jérôme Jambu for the Tirepied hoard. Savigné and Karl's hoard are from the CGB website (Trésors

minted at a standard of 29.489g and valued at 6 livres. The regression in Table 9 estimates the annual weight loss to be 157ppm, very close to Dumas's 1838 estimate of 160ppm. The results from the 18th and 19th century hoards indicate that hoards can be a pretty good proxy for the general mass of circulating coins, yielding estimates of annual wear that are within ±20%.

hoard and location	number of coins	range of dates	
Montigny (Deux-Sèvres)	155	1726–92	
Saint-Avold (Moselle)	348	1726–91	
Tirepied (Manche)	115	1726–93	
"Karl's hoard" (Loiret)	146	1726–91	
Savigné L'évêque (Sarthe)	70	1726–87	
Grand-Halleux IV (Belgium)	117	1726–86	
Nieuwekerken (Belgium)	185	1726–92	
Grand-Halleux V (Belgium)	14	1726–43	
Châtelet (Belgium)	385	1726–91	
Louvre (Paris)	81	1726–74	
Lagny (Seine-et-Marne)	277	1726–90	
Corswarem (Belgium)	119	1726–92	
pooled regressions with hoard dummies			
dependent variable: weight loss of écus (mg)			
relative age	4.626	4.998	4.969
	(0.266)	(0.269)	(0.267)
distance to Paris (km)		0.217	
		(0.033)	
distance to hoard (km)			0.181
			(0.025)
observations	2012	2009	
centered R^2	0.529	0.540	0.541
dependent variable: weight loss of half-écus (mg)			
relative age	12.384		
	(2.452)		
observations	103		
centered R^2	0.334		

Table 9: Eighteenth-century hoards of silver 6L coins, and regression of weight loss on age (with hoard dummies).

Smaller denominations are scarce in the hoards. The only one that is present in

2 and 19); Dengis (1987). The other hoards are published (Lallemand 1961, Dengis 1987, Beeckmans 2001, Dengis 1994, Callataÿ 1994, Trombetta and Foucray 1996). The publication of the Louvre hoard (Trombetta 1996) is not usable; I thank Jean-Yves Kind for letting me weigh the coins myself.

substantial numbers (123) is the half écu. The regression on age yields a coefficient of 12.38mg per year (840ppm), with a standard error of 2.45. This is consistent with the loss of 995ppm estimated on the 2F silver piece (1/2.5 of the 5F piece) in the 19th c.

Distance and wear

Having individual coin data allows me to throw in another explanatory variable based on the location where each coin was produced. In the eighteenth century, up to nineteen mints operated throughout France, and the origin of each coin (unless too worn) can be identified from mint marks. Interestingly, distance from the mint to Paris carries a significant coefficient: its magnitude would mean having been produced 100km from Paris is equivalent to more than four years' wear. This is suggestive of a spatial dimension to coin wear, although the fact that most of the hoards were found in Northern France means that it is difficult to tell whether distance to Paris or distance to the hoard location is what matters (the two have a correlation of 69% in my sample, and using distance from origin to hoard yields a similar estimate).

The result does not appear to be a small-sample fluke. Notes from a 1838 sample of 400,000 5F coins provide weight by mint of origin, although without breakdown by age.[19] The coefficient (Table 10) is very similar.[20]

dependent variable: average weight loss by mint (mg)	
distance to Paris (km)	0.12
	(0.045)
constant	69.1
	(21.4)
observations	14
centered R^2	0.370

Table 10: Regression of weight loss in a sample of 5F coins on distance between the minting place and Paris.

[19]Académie des Sciences, Paris, papiers Dumas, carton 22. The sample contained 400,000 coins.

[20]A regression of the hoard data on distance from Paris and hoard dummies alone yields an estimate of 0.09 (with standard error of 0.034).

4.2 Coins in the numismatic trade

In many cases there are no hoards available, or those hoards that have been published are too small to make inferences. Another potential source of information is to take as large a sample of coins as possible from the numismatic trade, and treat it as one large hoard, or sample of the circulation of a given type of coinage. By comparing results from this procedure in cases where hoards are also available, we may tell how useful that procedure is. An added benefit is that the sample contains smaller denominations: hoards consist almost exclusively of large silver pieces.

dependent variable: weight loss (ppm)	
constant	6065.5 ***
	(1177.4)
relative age interacted with denomination:	
0.3L	426.6 ***
	(22.8)
0.6L	244.4 ***
	(21.3)
1.2L	204.8 ***
	(24.7)
3L	185.4 ***
	(20.9)
6L	100.4 ***
	(15.7)
12L	89.8 ***
	(29.6)
24L	−8.6
	(19.6)
48L	−12.9
	(29.6)
observations	2793
centered R^2	0.245

Table 11: Regression of weight loss on age in a sample of French coins (1726–91) available at www.cgb.fr.

The French numismatics firm CGB provides on its website information about the coins it has handled and sold over the past twelve years.[21] I have collected data on the French 18th century coinage, namely all coins minted between 1726 and 1791. For such coins, we do not know at which point the coins were taken out of circulation. I simply

[21] www.cgb.fr, accessed in February 2011.

assume that all coins were taken out of circulation in 1834, when the pre-Revolution coinage was demonetized. The more serious problem is selection bias: the coin trade is mostly interested in high-quality specimens, and badly worn coins are less likely to appear in the sample.[22]

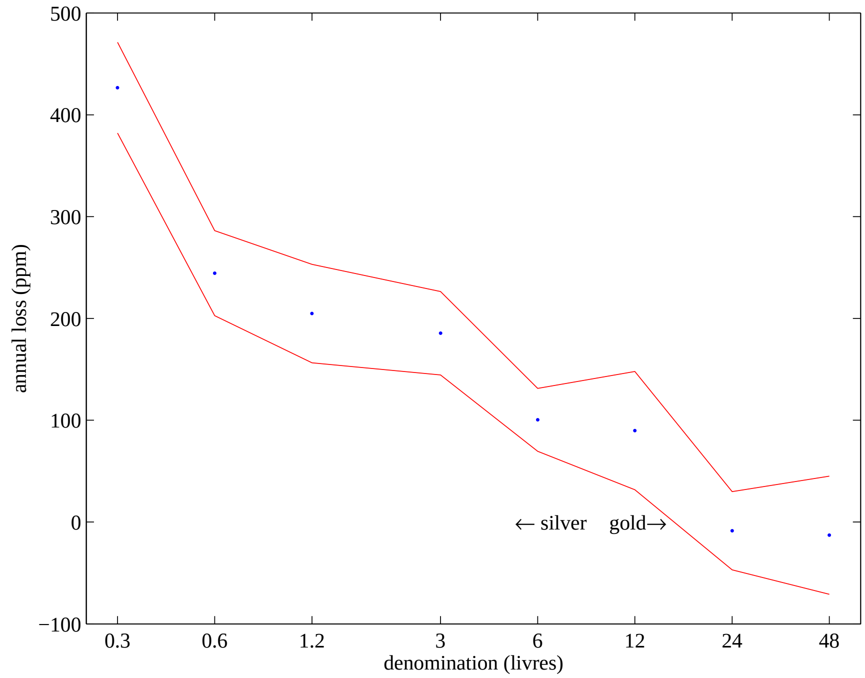

Figure 10: Estimates of annual wear for coins in the numismatic trade, by denomination (the lines show the two-standard deviation bands).

The results in Table 11 show significant effects of age on coin wear, increasing as the coin denomination decreases, but only for silver. The annual weight loss for the 6L coin (which represents 53% of my sample) is about a third smaller than the estimate from hoards, and the magnitudes of weight loss for the smaller denomination are considerably smaller than those for similar denominations in the 19th century (Table 3; recall that the livre and the franc had nearly the same silver content). I suspect that the selection bias is at work here: the mean weight loss of 6L coins in the numismatic trade sample is half of that in the hoards sample, even as the age profiles of the two samples are very similar, and roughly consistent with actual minting volumes. Nevertheless, the

[22]The CGB web site lists another 700 18th c. French coins of lesser quality in its "royales" section, but the coins were not weighed like the higher quality ones.

inverse relation of relative weight loss with denomination remains, and is still roughly linear in the log of denomination (Figure 10).

5 Estimating coin wear from undated coins

In this section I return to the problem posed by Müller (1977). Before the 16th century, coins were not dated by year. Can we measure the parameters of coin wear from a sample of coins whose vintage is unknown? Müller's approach is simple and elegant. With a few modeling assumptions (constant Gaussian distributions for initial weights and for wear and a constant loss rate of coins over time) the distribution of surviving coins can be derived analytically. He used a method of moments to estimate a parameter of interest (the initial mean weight, or standard) under certain simplifying assumptions. I show that the simplifying assumptions are not satisfied in practice and that the resulting estimate is (slightly) biased, but the approach remains valid: the distribution can still be derived analytically, and the parameters estimated by maximum-likelihood.

Specifically, Müller's model is that coins are continuously issued from the mint for $t \geqslant 0$, at a constant rate, with a fixed normal distribution of weights $N(m, \sigma^2)$. The parameter m represents the standard at which the coins are minted. The coins are then subject to two processes over time. The first, wear, is modeled as a weight loss with mean and variance linear in time; in other words the weight of each coin is subject to a shock $\sim N(-\alpha\Delta t, \beta\Delta t)$ during the interval of time Δt. The second process, absorption or disappearance of whole coins through random losses or melting, is assumed to have a constant hazard rate, that is, Poisson with parameter λ.

At time t, the distribution of coins by age $0 \leqslant s \leqslant t$ is

$$\frac{\lambda}{1 - e^{\lambda t}} e^{-\lambda s}$$

and the probability distribution of weight x at time t is:

$$F_t(x) = \int_0^t \frac{\lambda e^{-\lambda s}}{1 - e^{\lambda t}} \frac{1}{\sqrt{2\pi(\sigma^2 + \beta s)}} \exp\left(-\frac{1}{2}\frac{(x - m + \alpha s)^2}{\sigma^2 + \beta s}\right) ds. \tag{4}$$

Müller (1977) shows that the limiting distribution of weight loss as $t \to \infty$ (assumption A) is

$$F(x) = \lim_{t \to \infty} F_t(x) = (a/c) e^{-(\mathrm{sign}(x)c - b)x} \tag{5}$$

with $a = \lambda/\alpha$, $b = \beta/\alpha$, and $c = \sqrt{1 + 2ab}$. This distribution applies to a population of coins that have been issued from a mint at a constant rate for a very long period of time, or to a population of coins issued at one point in time and sampled regularly and equally over a long period of time. The first interpretation seems the appropriate one for a hoard of coins drawn from circulation at a point in time.

The second simplifying assumption (which I call assumption B) is that $\lambda\beta << \alpha^2$, so that the first three moments[23] of the distribution of weights are $M_1 = m - \alpha/\lambda$, $M_2 = \sigma^2 + (\alpha/\lambda)^2$ and $M_3 = -2(\alpha/\lambda)^3$. From the mean and skewness one can recover the initial mean weight, or standard of the coin, as:

$$m \simeq M_1 + \sqrt[3]{-M_3/2} \tag{6}$$

which is relatively easy to calculate.[24] I call this Müller's method 1.

Assumption B may not be satisfied in practice. My estimates for 5F silver coins are $\alpha \simeq 5$mg and $\beta \simeq 500$mg, implying $\lambda << 5\%$, which is not the case empirically. It is possible to relax assumption B and compute exact formulas for the four moments of $F(x)$, maintaining assumption A: the raw moments are $m_0 = 1$, $m_1 = 1/a$ and

$$m_k = \frac{k}{2a}\left(2m_{k-1} + (k-1)m_{k-2}\right)$$

for $k \geqslant 2$, while the centered moments are

$$\begin{aligned}
\mu_2 &= \frac{1+ab}{a^2} \\
\mu_3 &= \frac{2+3ab}{a^3} \\
\mu_4 &= \frac{3}{a^4}(3 + 6ab + 2a^2b^2)
\end{aligned}$$

and the observable moments of the distribution of weights are $M_1 = m - m_1$, $M_2 = \sigma^2 + \mu_2$, $M_3 = -\mu_3$ and $M_4 = 3\sigma^4 + \mu_4 + 6\mu_2\sigma^2$. The system $\{M_i\}_{i=1,\dots 4}$ can be solved for the parameters m, σ^2, $a = \lambda/\alpha$ and $b = \beta/\alpha$: I call this Müller's method 2.

More importantly, the use of $F(x)$, with time integrated out, relies on the assumption that for empirically relevant time lapses the distribution of coin weights is close to its asymptotic limit. That turns out not to be the case.

[23]Without the assumption, it is necessary to compute the fourth moment as well in order to identify the initial mean weight.

[24]Müller (1977) also shows how to compute the mean and skewness when weights are only given in bins, as is often the case in numismatic publications.

To assess Müller's method 2 (with assumption A only), I simulate data, using the values estimated for the 5F silver pieces ($\alpha = 5\text{mg}$ and $\beta = 500\text{mg}^2$). I use an absorption rate of $\lambda = 1\%$ per year. The standard initial weight of coins was $25g$, and I choose a variance such that 95% of newly minted coins fall within the official weight tolerance of 0.3%. The evolution of the weight distribution over time is shown in Figure 11.

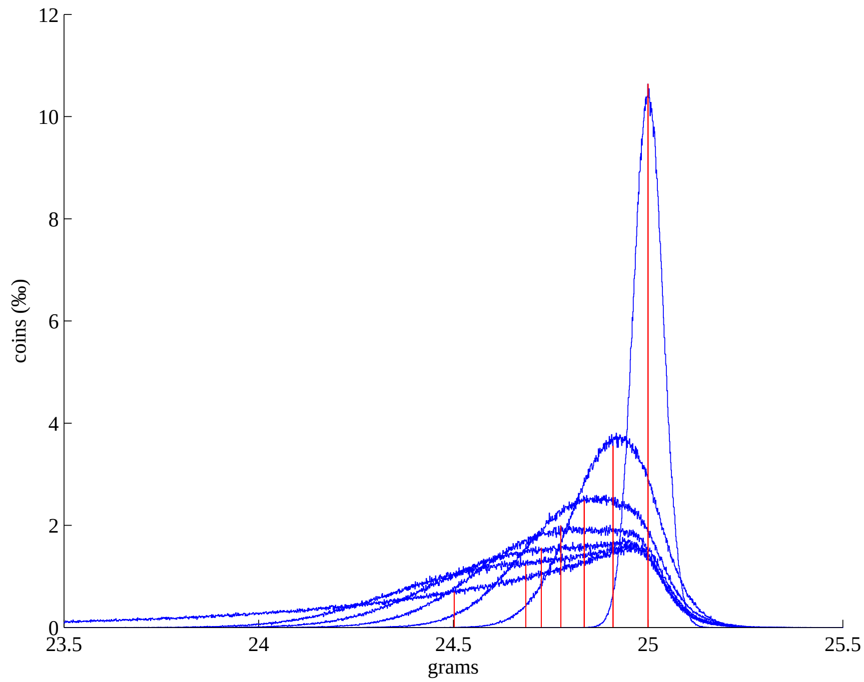

Figure 11: Simulated distribution of coin weights in the Müller model after 0, 20, 40, 60, 80, 100, and 1000 years. The vertical bars indicate the means of each distribution.

The results in Table 12 show that Müller's method 2 provides a good estimator of the original weight, but performs poorly for the other parameters, at least for plausible values of t (a hundred years or less). The point estimates for β are even negative.

years	m	σ^2	λ/α	β/α
truth	25.000	0.00146	2.00	0.100
0	25.005	0.00142	190.68	0.004
20	24.949	0.00933	25.87	0.007
40	24.915	0.01850	12.64	−0.040
60	24.880	0.02603	9.63	−0.026
80	24.859	0.03397	7.55	−0.023
100	24.850	0.04419	6.10	−0.042
1000	25.032	0.00111	1.89	0.027

Table 12: Comparison between true parameters and parameter estimates using Müller's method 2 on simulated data.

5.1 Extending Müller's results

To extend Müller's results for the case of finite t, we return to (4), use the change of variable $u = \sigma^2 + \beta s$ and the fact that

$$\int \frac{1}{\sqrt{x}} e^{-bx - \frac{a}{x}} dx = \frac{\sqrt{\pi}}{2\sqrt{b}} \left[e^{-2\sqrt{ab}} \left(1 - \text{erf}(\sqrt{\frac{a}{x}} - \sqrt{bx}) \right) + e^{+2\sqrt{ab}} \left(\text{erf}(\sqrt{\frac{a}{x}} + \sqrt{bx}) - 1 \right) \right] \tag{7}$$

to arrive at the following expression for the distribution of weights: $F_t(x) = \hat{F}_t(m + \sigma^2/b - x)$ with

$$\hat{F}_t(x) = \frac{\frac{a}{2c}}{e^{-\frac{a}{b}\sigma^2} - e^{-(\frac{a}{b}\sigma^2 + \lambda t)}} \left\{ e^{\frac{1-c}{b}x} \left[\text{erf}\left(\frac{x - \frac{c}{b}\sigma^2}{\sqrt{2\sigma^2}} \right) - \text{erf}\left(\frac{x - \frac{c}{a}(\frac{a}{b}\sigma^2 + \lambda t)}{\sqrt{2(\sigma^2 + \frac{b}{a}\lambda t)}} \right) \right] + e^{\frac{1+c}{b}x} \left[\text{erf}\left(\frac{x + \frac{c}{a}(\frac{a}{b}\sigma^2 + \lambda t)}{\sqrt{2(\sigma^2 + \frac{b}{a}\lambda t)}} \right) - \text{erf}\left(\frac{x + \frac{c}{b}\sigma^2}{\sqrt{2\sigma^2}} \right) \right] \right\} \tag{8}$$

with $\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$ and the same notations a, b, and c as before. When $\sigma^2 \to 0$ and $t \to +\infty$, this simplifies to (5).

The function $F_t(x)$ is the likelihood that a coin of any vintage $s \leqslant t$ has weight x. The log-likelihood of a given sample of coin weights is a function of the five parameters m, σ^2, $a = \lambda/\alpha$, $b = \beta/\alpha$, and λt. Maximizing the log-likelihood provides estimates of these five parameters conditional on the sample.[25]

To provide starting values for the maximization, the first five moments of the function $F_t(x)$ can be computed as functions of the five parameters, extending Müller's method of moments to the case of finite t. We find that $\int_{-\infty}^{+\infty} x^k F_t(x)dx = m_{kt}$ with:

$$
\begin{aligned}
m_{1t} &= \frac{1}{a}(1 + \frac{a}{b}z_{1t}) \\
m_{2t} &= \frac{2+ab}{a^2}(1 + \frac{a}{b}z_{1t}) + \frac{1}{b^2}z_{2t} \\
m_{3t} &= 6\frac{1+ab}{a^3}(1 + \frac{a}{b}z_{1t} + \frac{1}{2}\frac{a^2}{b^2}z_{2t}) + \frac{1}{b^3}z_{3t} \\
m_{4t} &= 6\frac{4+6ab+a^2b^2}{a^4}(1 + \frac{a}{b}z_{1t} + \frac{1}{2}\frac{a^2}{b^2}z_{2t}) + 2\frac{2+3ab}{ab^3}z_{4t} + \frac{1}{b^4}z_{4t} \\
m_{5t} &= 30\frac{4+8ab+3a^2b^2}{a^5}(1 + \frac{a}{b}z_{1t} + \frac{1}{2}\frac{a^2}{b^2}z_{2t} + \frac{1}{6}\frac{a^3}{b^3}z_{3t}) + 5\frac{1+2ab}{ab^4}z_{4t} + \frac{1}{b^5}z_{5t}
\end{aligned}
$$

with

$$z_{it} = \frac{(\sigma^2)^i - (\sigma^2 + \beta t)^i e^{-\lambda t}}{1 - e^{-\lambda t}}$$

and mean and centered moments of $F_t(x)$ can be computed by the usual formulas:

$$
\begin{aligned}
\mu_{1t} &= m + \frac{s_o^2}{b} - m_{1t} \\
\mu_{2t} &= m_{2t} - m_{1t}^2 \\
\mu_{3t} &= -(m_{3t} - 3m_{1t}m_{2t} + 2m_{3t}) \\
\mu_{4t} &= m_{4t} - 4m_{1t}m_{3t} + 6m_{1t}^2 m_{2t} - 3m_{1t}^4 \\
\mu_{5t} &= -(m_{5t} - 5m_{4t}m_{1t} + 10m_{3t}m_{1t}^2 - 10m_{2t}m_{1t}^3 + 4m_{1t}^5)
\end{aligned}
$$

I use simulated data in the same way as before to assess the precision of the estimates. I simulate a population of about 10 million coins evolving over 50 years, and for draw samples of varying sizes, from 250 to 2000 coins. For each sample size, I draw 100 samples and use the ML estimator. The results are shown in Figure 12, the thick line plotting the mean estimate and the dotted lines the 5% 95% quantiles.

Finally, I use the ML estimator on the hoard data described in the previous section. The results are shown in Table 13. By way of comparison I also report the results of Müller's two methods of moments.

[25]In practice, imprecise measurement of coin weights can be a problem for small coins: for example, when coins weighing 5g and less are measured to the second decimal (or cg) only ten or twenty different values will be recorded. In that case, one can maximize the likelihood of coin weights falling within bins.

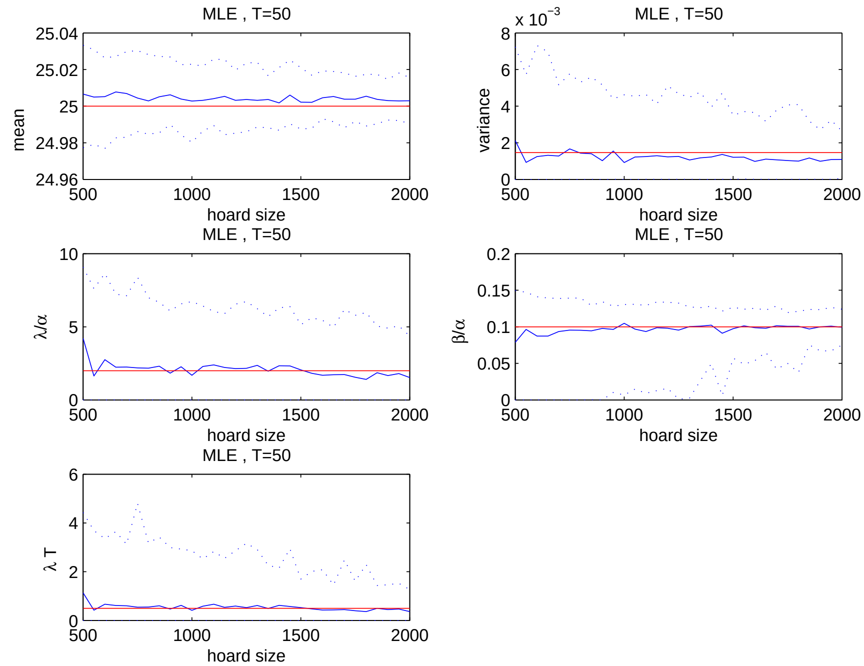

Figure 12: Maximum-likelihood estimates on simulated data, varying the hoard size. For each hoard size, 100 samples were drawn; the solid line indicates the median, and the dotted lines the 10% and 90% quantiles.

For 18th c. coins the estimated weight is 1.2% below the official weight of 29.489g. For 19th c. coins, the estimate is exactly at the official weight of 25g. The estimates of standard deviation at issue are 152mg (0.5% of official weight) and 97mg (0.4%) respectively, which seem a bit large compared to the official tolerances at the mint (0.78% in the 18th c. and 0.3% in the 19th c.), but are nevertheless of the right order of magnitude.

To interpret the remaining estimates, an assumption must be made on the length of time during which the coins circulated, which is not identified. The reason is simple: a stock of coins that suffered x% annual absorption and y g annual weight loss will look no different after 100 years than a stock of coins suffering $2x$% annual loss and $2y$ g annual weight loss after 50 years. My hoards are, fortunately, rather homogeneous: they were buried around the same time, and were drawn from a coinage that began at a well-specified date (1726 for the 18th c. coins, 1795 for the 19th c. coins; see Tables 8 and 9). The values of t that I choose are 65 and 40 years, respectively.

Once the assumption on t is made, I can convert the direct estimates into estimates of the parameters: average annual weight loss α, variance of annual weight loss β, and absorption rate λ. The results, in the bottom panel of Table 13, are very much in line with the estimates found earlier for 19th c. silver coinage: about 5mg for α and $450mg^2$ for β. The estimate of α for 18th c. hoards is nearly identical to the estimate found using the vintage of the coins (Table 9), while it is twice as high for 19th c. hoards (Table 8). Moreover, the absorption rate, between 1 and 3%, is also quite reasonable.

Müller's method 1 under-estimates the original weight by 1.5% to 3.5%. Method 2 performs better, in fact almost as well as maximum-likelihood, for the original weight, a feature that I have confirmed with simulated data;[26] but the estimates for the other parameters differ considerably, and can be impossible to interpret (for example, the negative variance in 18th c. hoards).

	m (g)	σ^2	λ/α	β/α	λt	
18th c. hoards						
ML	28.970	0.0153	2.696	0.076	0.525	
	(0.000)	(0.0000)	(0.000)	(0.000)	(0.001)	
mean of sample	28.866					
mode of sample	29.060					
Müller 1	29.216					
Müller 2	29.197	0.028	3.022	0.028		
19th c. hoards						
ML	25.028	0.0069	2.472	0.046	1.091	
	(0.000)	(0.0000)	(0.000)	(0.000)	(0.000)	
mean of sample	24.848					
mode of sample	24.810					
Müller 1	25.031					
Müller 2	24.948	−0.007	10.383	0.347		
			α (mg)	β (mg^2)	λ (%)	t
18th c. hoards			3.0	226	0.8	*65*
19th c. hoards			11.0	509	2.7	*40*

Table 13: Estimates of the parameters of the Müller model by maximum likelihood (ML) and by Müller's two methods, from 18th and 19th c. hoards; standard errors in parentheses.

6 Conclusion

My results confirm, extend, and modify those found in the existing literature. Coin wear (in commodity money regimes) can be modeled as a Brownian motion, with weight loss

[26] The estimator of the original weight appears biased, but only by about −0.1%.

	m (g)	σ^2	λ/α	β/α	λt	
staters (N=93)						
ML	14.299	0.0564	0.758	0.212	0.791	
	(0.404)	(0.1461)	(5.737)	(0.283)	(6.371)	
mean of sample	13.885					
mode of sample	13.950					
trites (N=361)						
ML	4.767	0.0024	3.202	0.048	0.803	
	(0.017)	(0.0014)	(1.813)	(0.002)	(0.420)	
mean of sample	4.670					
mode of sample	4.720					
Lydian trites (N=290)						
ML	4.738	0.0003	11.331	0.012	0.909	
	(0.007)	(0.0001)	(13.729)	(0.002)	(1.127)	
mean of sample	4.706					
mode of sample	4.720					
			α (mg)	β (mg²)	λ (%)	t
staters			10.4	22133	0.8	*100*
trites			2.5	1209	0.8	*100*
Lydian trites			0.80	100	0.9	*100*

per unit of time approximately normal with constant mean and variance. The difference in coin material between silver and gold coinage is not quantitatively important in explaining variation in coin wear across denominations. The appropriate measure appears to be relative weight loss, not absolute weight loss. Under that transformation, a striking relation appears between annual coin wear and the log of value. If coin wear is taken as a proxy for velocity, then velocity is inversely related to value. That seems to be the right transformation: when absolute weight loss was regressed on value, the coefficient was positive, with the counter-intuitive implication that higher denominations are more intensively used.

This relation appears stable across 19th century countries. This suggests that considering only quantities of denominations produced is not a good way to assess whether small coins were produced in sufficient quantities for the needs of trade, since velocity varies significantly across denominations. Coin hoards are a good proxy for the general circulating medium, at least with large samples (a thousand or more). Coins from the numismatic trade offer good data for estimating coin wear, but the selection problem can be an attenuating factor. Further work could be done to investigate the extent of this bias. For older (and scarcer) coins, that bias is likely to be less important. Coin wear also appears to incorporate a spatial dimension.

Finally, I have derived an estimator of the parameters of coin circulation processes (mean and variance of weight loss, and rate of coin loss) that does not rely on knowing the vintages of the coins. This method could have wide applicability to hoard data from ancient and medieval periods.

Appendix: Estimation

Consider a sample of size N drawn from a distribution $x \sim N(\mu, \sigma)$. We do not observe all the realizations $\{x_i\}_{i=1}^{N}$. Instead, we are given statistics based on the location of the realizations in various pre-determined bins with edges $\{a_k\}_{k=1}^{K}$, the bins forming a partition of the distribution's support. For bins $k \in A$ we are given the count of realizations in the bin n_k and the average value in that bin $\bar{x}_k = \frac{1}{n_k} \sum_{a_k \geqslant x_i < a_{k+1}} x_i$. For the other bins $k \in B$ we are given the realizations $\{x_i\}_B = \{x_i\}_{a_k \leqslant x_i < a_{k+1}}$. Define $n_A = \sum_{k \in A} n_k$ and likewise n_B.

MLE estimation

The likelihood function (David and Nagajara 2003, 12, 191–195) is

$$L = n_A! \prod_{k \in A} \frac{1}{n_k!} [F(a_{k+1}, \mu, \sigma) - F(a_k, \mu, \sigma)]^{n_k} \prod_{\{x_i\}_B} f(x_i, \mu, \sigma)$$

where

$$f(x, \mu, \sigma) = (2\pi\sigma^2)^{-\frac{1}{2}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$$

and

$$\Phi(x, \mu, \sigma) = \int_{-\infty}^{x} f(s, \mu, \sigma) ds.$$

The log-likelihood function is then

$$\log(L) = C + \sum_{k \in A} (n_k) \log(F(a_{k+1}, \mu, \sigma) - n_B \log(\sigma) - (F(a_k, \mu, \sigma)) - \frac{1}{\sigma^2} \sum_{\{x_i\}_B} (x_i - \mu)$$

with $e^C = \log(n_A! / \prod_{k \in A} n_k!) - n_B \log(2\pi)/2$.

A slightly different case is that in which the n_1 lowest realizations and the n_2 highest realizations are given, along with the sample size N. Let x_{n_1} be the highest of the n_1 lowest realizations, and x_{n_2} be the lowest of the n_2 highest realizations. The log-likelihood function is then:

$$\log(L) \frac{(N - n_1 - n_2)!}{(n_1 + n_2)!} [F(x_{n_2}) - F(x_{n_1})]^{n_1 + n_2} - \frac{N - n_1 - n_2}{\sqrt{2\pi\sigma}} \sum_{x_{n_1} > x, x > x_{n_2}} \frac{1}{2} \left(\frac{x - \mu}{\sigma} \right)^2.$$

GMM estimation

Another way to use the available information is to employ the generalized method of moments (GMM). For each bin k, define the moment

$$f_k(\mu, \sigma) = \sqrt{N_k} (\bar{x}_k - \mu_k)$$

where

$$\mu_k = \frac{\int_{a_k}^{a_{k+1}} x f(x, \mu, \sigma) dx}{\int_{a_k}^{a_{k+1}} f(x, \mu, \sigma) dx}.$$

The central limit theorem says that $f_k(\mu, \sigma) \to N(0, \sigma_k)$ as $N \to +\infty$ with $\sigma_k(\mu, \sigma) = \int_{a_k}^{a_{k+1}} x^2 f(x, \mu, \sigma) dx / \int_{a_k}^{a_{k+1}} f(x, \mu, \sigma) dx$. To use GMM, I minimize

$$f(\mu, \sigma) = \sum_k f_k(\mu, \sigma)' W f_k(\mu, \sigma)$$

where $W = I_K$ in the first stage and the diagonal matrix formed with $\{\sigma_k(\hat{\mu}, \hat{\sigma}_k)\}_k^{-1}$ in the second stage. The asymptotic variance is given by $(D'WD)^{-1}$ where D is the gradient of $\{f_k\}_k$. The J-test statistic I use is $f(\hat{\mu}, \hat{\sigma})/\sqrt{N}$.

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