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Domino Effects when Banks Hoard Liquidity: the French network

Valère Fourel‡ Jean-Cyprien Héam§ Dilyara Salakhova¶ Santiago Tavolaro∥

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Résumé: Afin d’analyser les effets systémiques de la thésaurisation des banques sur la stabilité d’un réseau financier, cet article propose un nouveau modèle de contagion bancaire. Cette dernière est étudiée suivant deux canaux : les expositions bilatérales directes entre établissements de crédit et les difficultés de financement à court-terme auxquelles les banques peuvent éventuellement faire face si une crise de confiance vient à se matérialiser (comportement préventif de thésaurisation). En s’inspirant du rôle majeur joué par la thésaurisation pendant la crise financière de 2007-2009, le modèle développé dans cet article se distingue du traditionnel algorithme séquentiel de calcul de défauts en cascade largement évoqué et employé dans la littérature sur le risque systémique pour mesurer les effets d’un choc de marché sur l’ensemble d’un système bancaire, en introduisant le comportement de thésaurisation des banques. Au-delà de la simple contagion via les expositions bilatérales, un tel phénomène initié par certaines banques peut entraîner des problèmes de financement à court-terme pour d’autres. En s’appuyant sur les données d’expositions bilatérales des banques françaises collectées par l’Autorité de Contrôle Prudentiel, le secteur bancaire français semble être relativement robuste lorsqu’il est à la fois soumis à un risque de marché (pertes sur les actifs de marché détenus par les banques dans leur portefeuille) et aux effets induits de la contagion par insolvabilité et par difficulté de financement à court-terme. Les résultats obtenus en termes de poids relatifs de chacun des canaux de contagion étudiés sur les pertes totales estimées mettent en exergue le caractère fondamental et complexe des effets de la thésaurisation.

Mots-clés: Thésaurisation, contagion par insolvabilité et par difficulté de financement, réseaux financiers, risque systémique

JEL classification: G01, G21, G28

Abstract: We investigate the consequences of banks’ liquidity hoarding behaviour for the stability of the financial system by proposing a new model of banking contagion through two channels, bilateral exposures and funding shortage. Inspired by the key role of liquidity hoarding in the 2007-2009 financial crisis, we incorporate banks’ hoarding behaviour in a standard Iterative Default Cascade algorithm to compute the propagation of a common market shock through a banking system. In addition to potential solvency contagion, a market shock leads to banks liquidity hoarding that may generate problems of short-term funding for other banks. As an empirical exercise, we apply this model to the French banking system. Relying on data on banks bilateral exposures collected by France’ Prudential Supervisory Authority, the French banking sector appears resilient to the combination of an initial market shock (losses on marked-to-market assets) and the resulting solvency and liquidity contagion. Moreover, the model gauges the relative weight of the various factors in the total loss.

Keywords: Liquidity hoarding, solvency and funding contagion, financial networks, systemic risk

JEL classification: G01, G21, G28
1 Introduction

The recent financial crisis has challenged the traditional view of the characteristics of system financial stability and the channels of propagation of losses. Whereas capital levels were closely monitored, heavy reliance of financial institutions on wholesale funding was overlooked. And banks that seemed to be safe experienced creditor runs and significant cash outflows that ultimately led to their bail-outs or defaults. For instance, in September 2008, Lehman Brothers Holding Inc. filed for bankruptcy protection despite the reassuring conclusions of a report affirming its solvency; Dexia faced liquidity shortages leading to a restructuring of the bank in spite of successfully passing the stress test run by the European Banking Authority in July 2011 with a Tier 1 Capital representing 12.1% of its risk-weighted assets.

Liquidity constraints played a key role during the crisis, since the whole interbank market on both sides of the Atlantic froze, requiring central banks to intervene as a lender of last resort. A high level of uncertainty and increased counterparty risk were the main reasons for this sudden decrease in interbank market activity and for the observed banks’ liquidity hoarding behaviour. Indeed, faced with uncertainty about the future availability of liquidity and fearing insolvency of their counterparties, banks stopped lending to each other and started withdrawing their short-term positions. The consequences of such a behaviour were especially adverse given that many banks were highly leveraged and heavily reliant on short-term wholesale funding in the run-up to the crisis.

Whereas the literature on the propagation of losses through bilateral interbank exposures (solvent contagion) is abundant and shows, in general, scarce evidence of contagion, solvency contagion in a joint framework with banks’ liquidity hoarding behaviour (funding shortage) has been hardly studied. Therefore, our paper aims to fill this gap and proposes a simple model that allows us to study how banks’ preemptive actions to secure their liquidity needs can affect the system in the context of a common market shock and potential solvency contagion.

In this paper, we model a banking system as a weighted directed network, explicitly taking into account the bilateral relationships between banks. Since linkages among financial institutions vary as to the ease with which the link can be broken, we distinguish between short- and long-term commitments. On the one hand, only short-term links such as overnight loans play a significant role in the liquidity shortage that can materialise within the system as they can be easily reduced in size or even cut in a short period of time. On the other

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1 For more detail on Dexia’s situation, see Bank of International Settlements 2013.
hand, long-term exposures are a channel of solvency contagion. A description of the lending and borrowing activities of Lehman Brothers proposed by the financial industry think-tank "Committee on Capital Markets Regulation" (2012) gives an intuitive difference between the two types of exposure and therefore sources of risk: "Lehman was extensively reliant on short-term funding, particularly through repos, and consequently it suffered a liquidity crisis when this short-term funding became unavailable. Had Lehman been more reliant on long-term, unsecured debt, it may have been less likely to fail in the first place, although third-party exposure would have been greater in that event."

Intuitively, the mechanism is as follows. We consider events occurring within a week. Initially, a system is hit by a market shock that impacts many banks at the same time. A market shock weakens the system and makes it vulnerable to contagion. Some banks may default immediately if they are not sufficiently capitalised to absorb the losses due to this market shock. As a result, defaulting banks do not honour their commitments and impose direct losses on their counterparties, thus potentially triggering solvency contagion. At the same time, banks that have to write down losses after the market shock perceive the entire system as being in distress and may start hoarding liquidity, thus generating cash outflows for their counterparties and exacerbating their funding problems. Eventually, the banks will suffer from a liquidity shortage and file for bankruptcy due to illiquidity. Both channels may subsequently lead to multiple rounds of contagion.

This paper is closely related to the strand of literature that studies solvency contagion using a network approach (to name but a few, Furfine, 2003; Mistrulli, 2011; Upper, 2004). All these papers share a common framework: financial institutions are linked through their bilateral exposures, thus forming a financial network, and a shock to one bank can propagate through the system via the existing linkages. Most of the papers on financial networks only examine solvency contagion under an idiosyncratic (Mistrulli, 2011) or market shock (Cont et al., 2010), whereas we model the propagation of losses through both solvency and liquidity.

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3Officially, a bank files for bankruptcy with no distinction made between defaults due to insolvency or illiquidity. However, in our model, we underline the reason of bankruptcy: lack of capital or lack of liquidity. As evidence that defaults due to the lack of liquid assets took place during the recent crisis, we cite Christopher Cox, the Chairman of the Securities and Exchange Commission (SEC), explaining the background and the circumstances of the run on Bear Stearns in March 2008: "The fate of Bear Stearns was the result of a lack of confidence, not a lack of capital. When the tumult began last week, and at all times until its agreement to be acquired by JP Morgan Chase during the weekend, the firm had a capital cushion well above what is required to meet supervisory standards calculated using the Basel II standard. Specifically, even at the time of its sale on Sunday, Bear Stearns’ capital, and its broker-dealers’ capital, exceeded supervisory standards. Counterparty withdrawals and credit denials, resulting in a loss of liquidity - not inadequate capital - caused Bear’s demise.” (Letter to the Chairman of the Basel Committee on Banking Supervision, March 20, 2008: www.sec.gov/news/press/2008/2008-48.htm)
This paper contributes to the literature in several ways. First, it delivers a theoretical framework to analyse how banks’ actions in hoarding liquidity may lead to liquidity shortages in the interbank market, and eventually, the default of their counterparties. We also derive a measure of losses propagating through this specific channel. Second, we design an operational approach to implement a realistic market shock, either a shock common to all banks or driven by the fall of one specific asset class. Third, the empirical part of the paper is a direct application of the model to the French banking system. It measures the resilience of the French banking system to financial contagion, both of the solvency and liquidity variety, thus enriching the evidence available for most of the industrialised countries.

From a different point of view, our paper also contributes to the emerging literature on multilayer financial networks. In this framework, each layer is a network in one particular market, e.g. interbank exposures in a CDS market represent one network, whereas exposures in the interbank money market can represent another network and so on (see Barigozzi et al., 2010). All these layers are interconnected: a shock can affect all the networks at the same time or pass from one layer to another. In our basic model, we consider two networks, namely, that of long-term interbank exposures and that of interbank short-term exposures. These two networks propagate different types of contagion. However, a deeper analysis of the possible interactions between these two networks remains beyond the scope of this paper.

At the same time, we are aware of the limits of our empirical exercise. First of all, the French banking system is highly integrated into the EU and the world systems, and banks are not only exposed to other French banks but also to foreign ones, which we cannot take into account in our analysis due to data limitations. Second, we analyse the system at one date, on 31st December 2011, therefore the results may be very specific to this date, and it would be very interesting to look at how the network and the results change over time. Third, we use exogenous recovery rates, while testing the results for a range of recovery rates.

The paper is organised as follows. Section 2 presents the model implemented here which aims to propose a rationale to explain how solvency defaults and liquidity hoarding can occur in a banking network when the system is affected by a market shock. Section 3 provides an application of our methodology to the French banking system with a comprehensive set of results. Section 4 discusses avenues for future work and concludes.

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4Gai et al. (2010) build up a stylised model to study propagation of a liquidity shock through a banking system. Gauthier et al. (2010) take both propagation channels into account, though the authors assume an exogenous probability of creditors’ run and do not take into account the fact that banks may create liquidity problems in the system by their own actions.
2 The model

In this section, we provide a framework to model liquidity and solvency contagion using the sequential default approach used in the network literature, as well as proposing realistic common market shocks.

We consider a set of $N$ banks that are exposed to each other. We distinguish short-term exposures from long-term exposures. Liquidity contagion only spreads through short-term exposures whereas long-term exposures are a channel of solvency contagion. We denote $E^{LT}$ (resp. $E^{ST}$) the matrix of long-term (respectively short-term) exposures, where $E^{LT}(i,j)$ (resp. $E^{ST}(i,j)$) represents the gross (resp. netted) exposure of bank $i$ towards bank $j$ (for $(i,j) \in [1; N]^2$). Exposures encompass loans and securities. The asset side of bank $i$ is decomposed into several items: interbank exposures ($E^{LT}(i,j)$ and $E^{ST}(i,j)$ for $j \in [1; N]$), cash $Ca(i)$ and other assets $OA(i)$. We denote total assets by $TA(i)$. The liability side of bank $i$ consists of equity $C(i)$ (hereafter capital), interbank exposures ($E^{LT}(j,i)$ and $E^{ST}(j,i)$ for $j \in [1; N]$ and $j \neq i$) and all other liabilities gathered in $OL(i)$. The market shocks affect the $OA$ component of banks’ balance sheets.

Banks start hoarding liquidity when they regard a situation as distressed, and we use a shock to banks’ economic capital as a signal of the distress in the system. We denote the economic capital of bank $i$ as $EC(i)$, and we interpret it as the overall level of capital that is considered by the bank as the capital mandatory to run its business optimally in the long run.\footnote{In our application, we used the required capital as a proxy of the economic capital.} At the same time, bank’s leverage ratio gives a public signal of a bank’s fragility. If a bank acts preemptively by withdrawing liquidity, it hoards more from its riskier, more leveraged, counterparty.

Lastly, as we consider an iterative approach with multiple rounds, we have to keep track of the different values taken by the different items of the network at each step of the algorithm. To do so, the variables are indexed by $t$ for the round of contagion and upper-indexed by $k$ for the algorithmic steps.

A schematic balance sheet of bank $i$ is represented in Table 1 page 5.
As discussed in the literature, contagion usually only spreads through a system that is weakened by a market shock, so that realistic market shocks are a crucial element in correctly assessing the significance of contagion during an episode of distress. We provide the details of the market shocks proposed in this paper, which differ from the shocks used so far in the literature.

These elements are presented in the following subsections. We follow the process of the algorithm step by step: we describe the market shocks that trigger initial losses, then we analyse the contagion mechanisms by disentangling solvency contagion from liquidity contagion, and finally, we introduce the indicators of fragility designed to reflect the systemic risk inherent in the system. In the last subsection, we illustrate how the model works using a simplified network.

### 2.1 Market shocks

Before describing the contagion mechanisms, we present the market shocks implemented. To assess the impact of the default of a specific bank on the resilience of a banking system under adverse conditions, we need to define an external event that will affect the system stability and the specific bank in question. As noted in Upper (2011), contagion is likely to occur only when the entire system is under stress.

Papers differ in the types of shocks they consider. The basic premise is to envisage idiosyncratic shocks. For instance, Upper and Worms (2004) for Germany, Mistrulli (2011) for Italy, van Lelyveld and Liedorp (2006) for the Netherlands, Toivanen (2009) for Finland, Furfine (2002) for the USA each considers the effect of the default of one bank. However,
as underlined in Elsinger et al. (2006a,b), a large common market shock impacting all the credit institutions of the system at the same time appears to be a necessary condition to observe contagion propagation. Therefore, several papers, such as Cont et al. (2010) and Elsinger et al. (2006a,b), analyse the resilience of the system by applying shocks with one systematic component affecting all the banks in the network.

In this paper, we provide an explicit formulation of the common shocks implemented, which affect the "Other Assets" category held in the portfolio of each bank (OA) at the initial date. We define two types of common shocks corresponding to different ways of considering stress episodes. In the first exercise, a general common shock is simulated so that the whole banking sector is in distress. This general common market shock enables us to assess the resilience of the network from an overall perspective. In another exercise, we consider asset class-specific common market shocks. This asset class specific approach simulates a large range of scenarios in which a sudden and dramatic price drop is observed in a particular financial market. The dotcom bubble and, more recently, the mortgage crises can be an illustration of such asset specific shocks.

We have to emphasise that the shocks differ from the usual ones presented in stress-test exercises. In fact, the paper focuses on the risk of the interbank market seizing up which is a phenomenon that can materialise in just a few days. The triggering event has to be very dynamic, unexpected in a sense. We therefore consider very short-term shocks (occurring within a week\(^6\)) on marked-to-market assets whereas traditional stress-scenarios last for several years and involves the deterioration of the banking book. Moreover, this very short-term perspective is compatible with the fact that the initial exposures are taken as given.

### 2.1.1 Common market shock

Similarly to Elsinger et al. (2006a), we define a common market shock as losses on banks’ balance sheet component "other assets" (OA) due to a correlated deterioration in asset prices.

The first step is to define and simulate the joint distribution of banks’ other assets (OA). We consider that banks’ OA are composed of four types of assets: equities, corporate debt, insurance debt and sovereign debt. We do not analyse retail activity, even though retail assets represent a significant share of banks’ assets. The main reason for this is that retail assets are not priced in the same way, and we cannot obtain relevant estimates.

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\(^6\)A time span of one week has been chosen for simplicity; considering 3 days or 10 days does not change our results.

\(^7\)Strictly speaking, this class groups together all financial institutions except banks.
of marked-to-market value of retail assets. On the other hand, retail assets are also less volatile, for instance, the probability of default of real estate assets hardly changed during the crisis. The French real estate market has some specific features. The vast majority of retail activity corresponds to real estate loans taken out by households. Unlike most countries (especially the USA and the UK), French households rarely enter into mortgages. French real estate loans depend on the household’s past income and not on the expected future value of the house purchased. Retail activity is therefore barely sensitive to the business cycle and unlikely to suffer from a real estate price collapse. Furthermore, French banks have mitigated their individual retail activity risk with a risk-pooling mechanism for real estate loans. Lastly, the time horizon in the housing market is hugely longer than one week. For each bank in the scope of the analysis, we identify the weight that each type of asset in the category "other assets" OA represent in the bank’s portfolio.

To have an accurate idea of how asset values jointly comove in order to properly simulate the shocks that affect banks’ balance sheets, we collect time series for these financial variables. We retain four price series for the period from 02/01/2001 to 30/05/2012: Eurostoxx 50 for equities, JPM Insurance Senior All Index for Insurance, JPM Euro Area Government Bond All Index and JPM Large Corporate Bond Index. Table 2, page 7 reports a standard statistical analysis of the daily returns.

<table>
<thead>
<tr>
<th></th>
<th>Returns</th>
<th>Variance</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Equity</strong></td>
<td>-0.83%</td>
<td>1.07%</td>
<td></td>
</tr>
<tr>
<td><strong>Government Bonds</strong></td>
<td>1.15%</td>
<td>0.04%</td>
<td>-0.28, 1</td>
</tr>
<tr>
<td><strong>Insurance Bonds</strong></td>
<td>1.28%</td>
<td>0.06%</td>
<td>0.23, 0.43, 1</td>
</tr>
<tr>
<td><strong>Corporate Bonds</strong></td>
<td>1.44%</td>
<td>0.03%</td>
<td>0.19, 0.50, 0.86, 1</td>
</tr>
</tbody>
</table>

Table 2: Statistics of daily returns (02/01/2001-30/05/2011). The average daily return on equities is −0.83%, its variance is 1.07% and its correlation with Sovereign daily return is −0.28.

To obtain the joint distribution of the four assets, we estimate a t-Student copula of those four time series using weekly returns between 02/01/2001 and 30/05/2011. The marginal probability distributions are estimated non parametrically with the kernel density method. By aggregating the "other assets" of all the banks, we construct an imaginary consolidated French banking system portfolio. Afterwards, using the correlated returns simulated for each asset type and the weights of these assets in the aggregated portfolio, we compute the profit-and-loss of the imaginary bank. Our ultimate shocks are the left tail of the profit-and-loss distribution for the entire system represented by the aggregated bank.
2.1.2 Asset-class-specific shock

Several crises seem to have been ignited by concerns arising from one particular asset type, thus we might be interested in measuring the resilience of the network after a significant asset price drop. Although one could interpret it as the bursting of a bubble, in this paper we do not pretend that our shocks represent perfectly the way a bubble occurs, we simply consider that one asset class suffers from a sizeable drop in value. We adapt the framework presented for the "common market shock" methodology to this perspective.

We have four asset classes (equities, corporate debt, insurance debt and sovereign debt), thus, we analyse the effects of one asset class-specific shock at a time. We keep the same data and the same estimation methodology as for the common market shock. But, instead of considering the profit-and-loss of the French banking system, we consider blocks of joint realisations. In each block, we select the realisation in which the considered asset has the lowest values. We obtain, at the same time, the joint distribution of the three other assets conditionally on the specific asset being at its lowest return.

Based on this simulated distribution of the specific asset, we apply the contagion algorithm presented in the following section.

2.2 Mechanisms of default contagion

The mechanisms of default contagion combine solvency and liquidity default cascades. We first consider a round of pure solvency contagion occurring during the period \((t = 1)\) right after the initial shock at date \((t = 0)\). Then, we consider several periods \((t = 2, t = 3...)\) during which there is room for liquidity hoarding. Liquidity hoarding at period \(t\) can lead to new defaults. These defaults might involve new losses due to solvency contagion and a new wave of liquidity hoarding that will take place at time \(t + 1\).

The timing of the model is explained in detail in Table 3 page 10. As for the variables characterising the nodes of the network, they are updated at the end of each period \(t\).

The whole process is presented schematically on Figure 1 page 9.

2.2.1 Solvency contagion

There exist two strands in the literature that attempt to address the issue of solvency contagion. The first one is often called the "Clearing Vector Approach" based on the seminal paper by Eisenberg and Noe (2001). This approach, extended in Gauthier et al. (2010) or
Solvency Contagion

Liquidity Contagion and following its defaults

Initial situation

$t = 0$

Initial shock

$t = 1$

Fundamental defaults

$t = 2$

There is at least one bank whose losses are bigger than its capital

Starting from $k = 0$

$k ← k + 1$

Solveny Contagation

Final situation

Bank(s) start hoarding liquidity

There is a bank whose liquidity condition is violated or it does not have enough capital

Starting from $k = 0$

$k ← k + 1$

Liquidity Contagion and following its defaults

All its counterparts lose their exposures

All its counterparts lose their exposures

Figure 1: Scheme of Default Contagion.
| $t = 0$ | - Initial situation |
| $t = 1$ | - An initial shock hits the system.  
- Banks account for the fundamental losses due to this shock. |
| $t = 2$ | - Solvency defaults propagate through the system until there are no more defaults.  
- Banks record all the losses due to the solvency default contagion. |
| $t = 3$ | - Solvent banks for which the capital requirement condition is violated start hoarding liquidity from their solvent counterparties.  
- This generates reallocation of resources and possible liquidity defaults, which in its turn may trigger solvency defaults contagion.  
- Liquidity contagion cascades stop when there are no more defaults, and banks record all the losses. |
| $t = 4, ...$ | - New waves of liquidity contagion may take place if there are banks whose capital is lower than the required one.  
- And the same process as at $t = 3$ takes place.  
- All the rounds of $t \geq 4$ stop when liquidity hoarding leads only to reallocation of resources and no defaults because of the violation of liquidity conditions. |

Table 3: Timing of the model

in Gouriéroux et al. (2012, 2013), establishes the existence and uniqueness of the debt repayment among banks: it provides the endogenous recovery rate on interbank assets. The second strand refers to the ”Iterative Default Cascade” developed by Furfine (2003). This method proposes an algorithm that mimics domino effects: instead of looking for a joint vector of debt repayment, this algorithm writes down the losses step by step as it might happen during a default cascade. Since we consider events within one week, we argue that the second approach is more realistic in replicating banks’ behaviour. Intuitively, the algorithm relies on two essential rules. First, we specify that a bank is in default when its capital has been wiped out. Second, once a bank has failed on its commitments, all its counterparties incur losses equal to their exposure to that defaulted bank and have to absorb the losses using their capital. We consider an exogenous recovery rate for solvency contagion denoted $R^S$. We run a case-sensitive analysis for different levels of $R^S$ in order to ensure the robustness of our results.

At time $t = 1$, the ”other assets” of the $N$ banks are impacted by a shock according to the methodology previously described. If the initial losses are larger than a bank’s capital, the latter goes into bankruptcy. We can therefore define the set of all banks defaulting due

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8One might argue that it takes several months for the official resolution of the bankruptcy, but we rather focus on how the information spreads: as soon as a bank files for bankruptcy, all the agents in the market are aware of it and immediately take action.
to a market shock, referred to as "fundamental defaults", as

$$\text{FD}(C) = \left\{ i \in \mathbb{N} : C_0(i) + OA_0(i) - OA_1(i) \leq 0 \right\}$$

(2.1)

$$= \{ i \in \mathbb{N} : C_1(i) = 0 \},$$

where \( C_1(i) = (C_0(i) + OA_0(i) - OA_1(i))^+ \) is the capital of bank \( i \) just after the initial shock.

From this situation, we can define a Solvency Default cascade (to use Amini et al.’s terminology) as a sequence of capital levels \((C_2^k(i), i \in \mathbb{N})_{k \geq 0}\) (where \( k \) represents the algorithmic step) occurring at time \( t = 2 \) and corresponding to defaults due to insolvency:

$$\begin{cases} 
C_2^0(i) = C_1(i) \\
C_2^k(i) = \max(C_2^{k-1}(i) - \sum_{j, C_2^{k-1}(j) = 0} (1 - R^S) \times E_0(i, j); 0), \text{ for } k \geq 1.
\end{cases}$$

(2.2)

The sequence converges (in at most \( n \) steps) since \((C_2^k)_{k}\) is a component-wise decreasing sequence of positive real numbers. Note that subscripts are used for periods of time and superscripts for rounds of cascades. By "period", we mean the sequential spread of losses through different channels. It does not refer to a time line interpretation: we consider that all the events occurred jointly within week.

Comparison of the banks initially in default (that is \( \text{FD}(C) \)) and the banks in default at the end of \( t = 2 \) corresponds to the set of institutions that defaulted only due to solvency default contagion. We label this set \( S_2 \).

2.2.2 Liquidity hoarding mechanism

The liquidity contagion has been scarcely studied in the literature on financial networks. However, two main strands of research can be mentioned. The first channel looks at asset or market liquidity, and losses in asset value (deterioration of the bank’s balance sheet and ultimate insolvency) driven by massive sales of the asset. The studies interested in this so called fire-sales phenomenon aim to model the adverse effects of massive asset sales initiated by one or more financial institutions on the whole financial network. Banks hit by a shock will attempt to improve their leverage ratio by massively selling their assets. Other banks may hold the same assets in their portfolio and these large sales will deteriorate the value of these assets and contaminate the balance sheet of other banks, which will in turn start selling the same assets, causing the whole system to enter to a vicious circle. Cifuentes et al. (2005) model banks’ reaction as a mechanical rule of selling assets in order to improve
their solvency ratios. Asset prices decrease with the growing volume of assets sold.

The other approach to liquidity contagion tackles the issue of funding liquidity, that is, the issues arising on the liability side. The seminal paper by Allen and Gale (2001) analyses several types of stylised networks with an exogenous probability of funding difficulties. Their study provides interesting insights about the propagation of a liquidity shock through a network, though it remains unapplied due to the simplicity of the networks used. Some interesting exceptions studying the propagation of liquidity shocks are the papers by Gai et al. (2010) who build up a stylised model to study how banks’ hoarding behaviour leads to the propagation of a liquidity shock through the system and by Gauthier et al. (2010) that disentangle credit and liquidity risks in a game theory framework proposed by Morris and Shin (2010). Our paper is closely related to the second strand of the literature, in that we also model liquidity risk: a bank may experience funding problems when its counterparties start hoarding liquidity during a crisis. And we propose a mechanism that endogenously takes into account funding shortages.

After a round of solvency contagion, some banks are in default while others have enough capital to absorb their losses. These banks will consider themselves in distress if their new level of capital no longer satisfies the supervisory requirement. As explained by Acharya and Skeie (2011) and Brunnermeier (2009), these banks may fear that their counterparties are also in distress, so that the perception of counterparty risk increases and the banks, as a pre-emptive defensive action, start hoarding liquidity. One may argue that banks can borrow from the money market if they experience a liquidity shortage. However, during a crisis - and we are only looking at periods of stress- banks will find hard to raise funds from a private investor for many reasons: an investor will request high quality collateral, high interest rates and haircuts or simply reject the transaction. In this simple model, therefore, we assume that banks can only stop rolling over existing short-term loans when they need liquidity and increase their cash (or alternatively put the received cash in the central bank’s deposit facility). The liquidity obtained is used to improve their liquidity position in view of potential future problems on the interbank market and to reimburse their creditors which have started hoarding liquidity too. If a bank fails to satisfy its short-term commitments, it defaults due to illiquidity.

To know how much liquidity a bank hoards in total, and how much it hoards from each counterparty, we make some assumptions. First of all, the total amount of liquidity withdrawn depends on the size of the shock to the bank’s capital: the bigger the losses due to

Note, that in our model, the only reason for hoarding is a precautionary one. In other words, we exclude any predatory behaviour.

We exclude central banks’ policy tools from our analysis in order to study what may happen in the absence of any public intervention.
the market shock, the more the bank hoards liquidity. The proportion of liquidity hoarded by bank $i$ is $\lambda(i) \in [0; 1]$. It is assumed to depend on the gap between the institution’s capital $C(i)$ and its economic capital $EC(i)$ of the institution: at time $t$, we denote $\lambda_t(i) = \phi_{(\theta_1, \theta_2)}\left(\frac{(EC_t(i) - C_t(i))^+}{EC(i)}\right)$, where $\phi_{(\theta_1, \theta_2)}(x)$ is the cumulative density function of a Gaussian law with mean of $\theta_1$ and variance of $\theta_2$. \footnote{We assume that bank $i$ curtails its positions in the short-term interbank market by stopping rolling over debt for a total amount $\lambda_t(i) E_{ST,t}^{k-1}(i)$ where $E_{ST,t}^{k-1}(i) = \sum_{j \in S_{t-1}} E_{ST,t-1}^{k-1}(i, j)$ and $S_{t-1}$ is the set of non-defaulted banks at the end of period $t - 1$.}

Second, the amount of liquidity the bank hoards from each counterparty depends on the market perception of counterparty risk, for which the leverage ratio can be used as a proxy. The higher the leverage, the riskier a bank is perceived to be and the more its counterparties will hoard from it. Defining $\mu_t(j)$ as $\mu_t(j) = 1 - C_t(j)/TA_t(j)$, we can decompose the total amount of liquidity hoarded by bank $i$ with respect to the counterparties:

$$\lambda_t(i) E_{ST,t}^{k-1}(i) = \lambda_t(i) E_{ST,t}^{k-1}(i) \sum_{j, C_{t-1}^{k-1}(j) \geq 0} \frac{\mu_t(j) E_{ST,t}^{k-1}(i, j)}{\sum_{h} \mu_t(k) E_{ST,t}^{k-1}(i, h)}.$$  \hfill (2.3)

When a bank hoards liquidity, it improves its liquidity position, whereas liquidity withdrawn by its counterparties deteriorates it. Therefore, the following liquidity condition simply says if bank $i$ has enough liquid assets, either interbank or non-interbank, to pay its short-term debt:

$$Ca_i + i \text{hoarding inflows} - i \text{hoarding outflows} > 0.$$  \hfill (2.4)

The above stated rule for modelling liquidity hoarding and the liquidity condition is a direct extension of usual rules applied in the literature. For instance, Gai and Kapadia (2011) assume that a constant exogenous proportion of liquidity is hoarded in case of distress. With our notations, it would be expressed as $\lambda_t(i) = \lambda$. We contribute to the literature by proposing a hoarding rule that accounts for the magnitude of liquidity hoarding (driven by a capital gap) and the distribution of it among the counterparties (driven by the respective individual leverage ratios).
In line with the solvency contagion algorithm, we state that a bank is in default when its capital has been wiped out (solvency condition) or when it cannot satisfy its short-term commitments (liquidity condition).

\[
C^0_t(i) = C_{t-1}(i) \\
\text{for } k \geq 1,
\]

**Solvency condition:**
\[
C^k_t(i) = C^0_t(i) - \sum_{j, c^k_{t-1}(j) = 0} (1 - R^L) E^{ST}(i, j)
\]

**Liquidity condition:**
\[
C^{nk}_t(i) = \begin{cases} 
0 & \text{if } Ca_t(i) + \lambda_t(i) E_t^{ST,k-1}(i) - \\
\sum_{h, c^{k-1}(h) \geq 0} \lambda_t(h) E_t^{ST,k-1}(h) \frac{\mu_t(i) E_t^{ST,k-1}(h,i)}{\Sigma \mu_t(l) E_t^{ST,k-1}(h,l)} < 0 & \\
C^j_t(i) & \text{otherwise}
\end{cases}
\]

**Updating equation:**
\[
C^k_t(i) = \max(C^{nk}_t(i); C^{mk}_t(i); 0)
\]

We denote the recovery rate in the liquidity cascade $R^L$. In general, one can distinguish a recovery rate in the case of a default due to illiquidity from a recovery rate of a default due to insolvency ($R^S$). And one might argue that the former recovery rate should be higher since the asset side of an illiquid bank is not impaired. In the proposed algorithm, all banks that do not satisfy the liquidity condition have their total assets higher than their total debts (which makes them solvent). Thus, $R^L$ is used to represent bankruptcy costs that do not reflect insolvency but costs associated with the liquidation of an illiquid bank \footnote{Based on a survey for U.S. banks, James (1991) establishes a bankruptcy cost of 10%}.

At the end of period $t$, the algorithm provides the status of each bank (alive or in default), their capital and their short-term exposures. Some banks may have defaulted during period $t$, thus some non-defaulted banks have recorded losses on their capital levels. If their capital is then lower than their economic capital, another round of liquidity hoarding dealt with in period $t + 1$ will take place.

### 2.3 Indicators

In the paper, we perform several types of shocks (common market shocks and asset class-specific shocks). As previously explained, we are interested in tail events, therefore we report the effect distribution through Value-at-Risk ($VaR$) and Expected Shortfall ($ES$). $VaR$ and $ES$ are usually risk measures but they are very informative concerning the tail of a distribution. In our framework, the $VaR(q)$ is defined as the level of total loss as a
percentage of total capital at quantile \( q \) (for instance, "\( \text{VaR}(1\%) = 0.1\% \)" means that the 1% worst losses are greater than 0.1% of total capital) while the \( \text{ES}(q) \) is the average total loss as a percentage of total capital over the worst \( q \) cases (for instance, "\( \text{ES}(1\%) = 0.2\% \)" means that over the 1% worst cases, the losses represent on average 0.2% of total capital).

As we are interested in very adverse situations, we only consider the following levels: 5%, 1%, 0.1% and 0.01% (which correspond to statistical events occurring once every 5 months, 2 years, 20 years and 200 years respectively).

Since we adopt a purely sequential approach we can easily decompose the indicator in three terms: the effects of the "fundamental shock" (prior to any contagion), the effects of solvency contagion (subsequent to the shock and prior to liquidity contagion) and the effects of liquidity contagion (subsequent to solvency contagion). Comparing the relative weight of each term is very informative about the mechanisms at work during stressed situations.

### 2.4 Illustration of contagion mechanisms for a simple network

Let us consider a basic network composed of six banks as represented in Figure 2. We assume that both recovery rates, \( R^S \) and \( R^L \), are set to 0 for simplicity.

![Network Diagram](image)

**Balance-sheet info**

<table>
<thead>
<tr>
<th>Bank</th>
<th>( C )</th>
<th>( EC )</th>
<th>( Cash )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B1 )</td>
<td>1</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>( B2 )</td>
<td>12</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>( B3 )</td>
<td>32</td>
<td>30</td>
<td>2</td>
</tr>
<tr>
<td>( B4 )</td>
<td>22</td>
<td>5</td>
<td>1.5</td>
</tr>
<tr>
<td>( B5 )</td>
<td>32</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>( B6 )</td>
<td>22</td>
<td>5</td>
<td>2</td>
</tr>
</tbody>
</table>

**Legend**

\( B_i \) is exposed to bank \( j \) for \( LT \) in the long term and \( ST \) in the short term.

Figure 2: Initial Network
2.4.1 Round of solvency contagion

Let us consider that at $t = 0$ an initial shock erodes the capital of all the banks by the order of 2. All of the banks can absorb this shock except bank $B_1$ which goes into bankruptcy. The fundamental default set is limited to bank 1. The solvency algorithm takes place and its various steps are represented in Figure 3 page 17.

The default of bank $B_1$ results in losses for its counterparties: banks $B_2$, $B_3$ and $B_5$. For each bank in question, the losses incurred and which correspond to its total exposure to bank $B_1$, are absorbed by its capital. Bank $B_2$ does not have enough capital to absorb its exposures, so it goes into default. Banks $B_3$ and $B_5$ are sufficiently capitalised to stay alive. Bank $B_2$’s default is characterised as a ”solvency default”.

The default of bank 2 results in losses for banks 3 and 4. This second step of solvency contagion is the last one since all the banks exposed to bank 2 have enough capital to absorb the losses.

The solvency equilibrium (the last step of this round of solvency contagion) is represented in Figure 4 page 18.
(a) Fundamental Default

(b) Round of Solvency contagion, First step

(c) Round of Solvency contagion, Second step

Figure 3: Solvency Contagion
2.4.2 First Liquidity Contagion Round

Since the solvency contagion is over, banks $B_3$, $B_4$ and $B_5$ are solvent. But when they compare their capital (in column $C$) with their economic capital (in column $EC$), banks $B_3$ and $B_5$ start hoarding liquidity whereas bank $B_4$ does not modify its behaviour since its capital is substantially greater than its economic capital.

In this example, we consider simply that $\lambda_i = (EC_i - C_i)^+ / EC_i$ and that this proportion of liquidity hoarding is uniformly applied to all short-term exposures (or equivalently, that all banks have the same leverage ratio). Therefore, banks $B_3$ and $B_5$ reduce their total short-term exposures by 66% ($= (30 - 10)^+/30$) and 50% ($= (4 - 2)^+/4$) respectively. Consequently, the cash outflows are:

Cash outflow for bank $B_3 = 1.5 = \frac{0.5 \times 3}{3}$ toward bank $B_5$

Cash outflow for bank $B_4 = 1.66 = \frac{0.5 \times 2}{2} + \frac{0.66 \times 1}{3}$ toward bank $B_5$ toward bank $B_3$

Cash outflow for bank $B_5 = 0.$
Symmetrically, the cash inflows are:

\[
\begin{align*}
\text{Cash inflow for bank } B3 &= 0.66 = \frac{0.66 \times 1}{\text{from bank } B4} \\
\text{Cash inflow for bank } B4 &= 0 \\
\text{Cash inflow for bank } B5 &= 2.5 = \frac{0.5 \times 3}{\text{from bank } B3} + \frac{0.5 \times 2}{\text{from bank } B4}
\end{align*}
\]

For all the steps in the liquidity hoarding phenomenon, the algorithm will consider that a bank is in default if it does not satisfy one of the two conditions (solvency and liquidity conditions) previously referred to in the theoretical model. A bank remains alive if its capital is above zero (solvency condition) and if its cash position allows it to honour its short-term commitments (liquidity condition). At each step, these two conditions are simultaneously checked for every bank. For the sake of clarity, we will consider them sequentially and first look at whether each bank satisfies its liquidity condition and then check if each bank is still solvent.

For this first step, as the network is initially in equilibrium in terms of solvency contagion, only the liquidity condition is checked. Combining the cash inflow, the cash outflow and the cash holdings of each bank, we can check if each bank fulfills its liquidity condition. For bank \( B5 \), we have a positive value since there is no cash outflow. Although bank \( B3 \) has a bigger cash outflow (1.5) than its cash inflow (0.66), its cash holding (2) can absorb the difference \((2 + 0.66 > 1.5)\). By contrast, bank \( B4 \) is short in terms of liquidity. Its cash outflow (1.66) is higher than its cash inflow (0) and it does not have enough liquid assets (1.5) to pay back its creditors. In other words, banks \( B5 \) and \( B3 \)'s behaviour which consists in stopping rolling over the short-term debt issued by bank \( B4 \) generates a cash outflow for bank \( B4 \) that it cannot cope with. We consider that bank \( B4 \) is in default due to illiquidity.

Note that in this particular case, the action of bank \( B5 \) or bank \( B3 \) alone would have not led bank \( B4 \) to have a liquidity shortfall since each component of the cash outflow of bank \( B4 \) is lower than its cash holdings. The situation after this first step of liquidity hoarding is represented in the top network of Figure 5, page 21.

In this new step, the check on whether the liquidity condition for each bank is fulfilled or not only concerns banks \( B3 \) and \( B5 \), and is represented in the middle network of Figure 5, page 21. We can easily see that bank \( B5 \) satisfies its liquidity condition in the sense that bank \( B5 \) is a pure short-term lender. For bank \( B3 \), the cash inflow is now 0 since \( B4 \), the only initial short-term debtor of bank \( B3 \), is in distress and its cash outflow (towards bank \( B5 \)) is 1.5. Since bank \( B3 \)'s cash position is 2, bank \( B3 \) fulfills its liquidity condition. The situation at the end of the liquidity contagion step (middle network of Figure 5, page 21)
is not a solvency equilibrium since the losses due to bank $B_4$’s default have not been taken into account. Thus, during this second step there is an additional check with respect to the solvency condition in the algorithm of contagion represented in the lowest plot of Figure 5, page 21. Bank $B_5$ suffers a loss of 20 ($= 18 + 2$) while bank $B_3$ suffers a loss of 1 ($= 0 + 1$). Bank $B_5$ does not have enough capital to absorb this loss while $B_3$ has. This fact triggers solvency contagion: bank $B_6$ is able to absorb the losses incurred corresponding to its exposure to bank $B_5$.

The situation after this first round of solvency contagion and this first round of liquidity contagion is stable from a solvency point of view (all remaining capital levels are strictly positive) and from a liquidity point of view (all cash holdings are sufficiently high). Note that the two remaining banks, $B_3$ and $B_6$, have their capital lower than their economic capital; but since they are not short-term lenders, this cannot lead them to stop short-term lending. We therefore consider that the final situation, represented in Figure 6, page 22, is the equilibrium situation reached within a week.

In this example, the equilibrium is reached with only one round of solvency contagion and one round of liquidity contagion. With more complex networks, several rounds of liquidity contagion are easily conceivable.
Figure 5: First Round of Liquidity Contagion
3 Application to the French banking system

This section gives us an empirical application of our model to a real network. We first introduce the data used in our framework followed by some descriptive statistics on the French banking system. Lastly the results are presented.

3.1 Data

French credit institutions are required to report to the Autorité de Contrôle Prudentiel (French Prudential Supervisory Authority) a full and detailed description of their balance sheets (FINREP Report, CEBS, 2009a) and all the large bilateral exposures that they may have to either other credit institutions or even a country or a company (Large Exposure Report, CEBS, 2009b). Such data allow the French Prudential Supervisory Authority to closely and continuously monitor the developments in the network and banks’ counterparty risks. Data on balance sheets are collected on a semi-annual basis whereas information on large exposures is reported quarterly. In the Large Exposure report, each credit institution is obliged to communicate all its exposures amounting to more than 10% of its capital or more than 300 millions of Euros. We use this unique data set on bilateral exposures and
balance sheet composition to reconstruct the French banking network in December 2011.

Each bilateral exposure corresponds to the gross bilateral sum of both securities and loans that a bank holds in its portfolio with respect to a certain counterparty. Given that there is no information about the maturity of the assets held by each bank in the Large Exposure reports, we extract the ratio of short term over long term assets from their balance sheet. We then apply this ratio to the amount of bilateral exposure of the corresponding bank reported in the Large Exposure dataset to obtain an estimation of long-term and short-term bilateral exposures.

The French banking system consists of more than 300 financial institutions at the solo level. Nevertheless, the French banking system is highly clustered with five major banking groups at the consolidated level accounting for more than 80% of the total assets of the system. We select the 11 largest banking groups such that our study constitutes an almost complete representation of the French banking sector, both in terms of size and business models. Indeed, it is composed of several major universal banks (either mutual banks or purely commercial banks) but also specialised banks (such as those engaged in consumer-loan activity). French banks also differ in terms of their degree of cross-border activities: some of them have intensive international activities while others have mainly, not to say only, domestic activities. In the end, the sample of banks selected here enables us to consider all of these heterogeneities (bank size, business model, global/local activity).

While some French banks are active globally, a national level analysis is still relevant from a macroprudential perspective since French banks have the bulk of their activity in France. Moreover, international level analyses (see for instance Alves et al., 2013) complement national ones. They make it possible to consider the major role that some ”hub” banks can play under stressed conditions. Lastly, while source of funding considerations are central in contagion phenomena, other contagion channels should not be underestimated (behavioural dynamics, fire-sales, reinvestment decisions...) Indeed, a project of further research is to complement our model with the other sources of funding (such as foreign banks and insurance companies).

Given that we study banks at the group-consolidated level, we do not consider exposures between subsidiaries within a group. Since a group will likely try to avoid any failure of

---

13 Amounts are taken before any type of deduction, provision or risk mitigation and without any netting. We exclude derivatives, off-balance sheet exposures and marked-to-market short-long positions. Note that the items that we select account for almost of all balance sheet instruments.

14 Short-term is defined as ”less than 1 month” while long-term is defined as ”more than 1 month”. Comparing the one-week base for a shock with this threshold of 1-month maturity for exposures introduces a conservative bias for the effect of the liquidity condition.

any of its subsidiaries, by reallocating profits and losses between subsidiaries for instance, considering the banking sector at a solo level would considerably bias any contagion analysis by artificially counting defaults that would not occur in reality.

As documented in almost all the studies on real financial networks, the latter are usually scale-free, meaning that a few banks are connected to many other banks. This scale-free characteristic commonly observed for financial networks does not hold when we consider a small number of banks: in this sense, the network of the French banking system is rather special, since it is an almost complete one, as we can see in Figure 7, page 24.

Figure 7: The French Banking Network in December 2011. The nodes correspond to the 11 largest French credit institutions while the edges represent the exposures (loans and securities) between the credit institutions. The widths of edges are proportional to the exposures.

\[16\] There are few banking systems characterised by a very limited number of banking groups, another example being Canada.
3.2 Descriptive statistics

Table 4, page 25 reports some descriptive statistics on the French banking system.

<table>
<thead>
<tr>
<th>Description</th>
<th>as a %</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interbank Exposures / TA</td>
<td>2.2</td>
<td>2.1</td>
<td>1.6</td>
<td>1.6</td>
</tr>
<tr>
<td>Interbank Exposures / Cap</td>
<td>34.2</td>
<td>28.2</td>
<td>27.9</td>
<td></td>
</tr>
<tr>
<td>Net position / Cash</td>
<td>171.1</td>
<td>557.1</td>
<td>0.0</td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Descriptive statistics on the French banking network. "Exposures/TA" corresponds to the sum of all reported exposures by a bank expressed as a percentage of its total assets. "Exposures/Cap" is the sum of all reported exposures by a bank expressed as a percentage of its capital. "Net position/Cash" is the ratio of the difference between all the short-term assets owned by a bank and all its short-term liabilities vis-a-vis its lenders over its cash holdings.

The amount of money that the 11 largest French banks lend to each other, corresponding to the sum of the interbank exposures, represents about 2.2% of their total assets. Even if this number may seem small at first glance, the sum of exposures amounts on average to 34.2% of total capital. The exposure distribution is asymmetric with most banks reporting small exposures while a few banks declare larger ones.

The ratio of total asset interbank exposures over total capital can be an indicator of a bank’s sensitivity to solvency contagion, with the capital representing a safety cushion when losses are recorded. This ratio has a mean of 34.2% with a standard deviation of 28.2%. As the distribution is asymmetric, we may consider the median as a "central" indicator. With a 27.9% median for interbank exposure over capital, a generalised spread of solvency contagion seems unlikely from a very first analysis based on descriptive statistics.

In addition, another indicator that may help to measure a bank’s sensitivity to liquidity contagion is its net position (all credit granted minus all loans borrowed) with respect to cash. Depending on its net position, a bank will not only start hoarding liquidity but will also be able to cope with a liquidity shock. A large amount of cash will reduce its probability of becoming illiquid. The range of values observed for this indicator within the sample is ample. This finding can be explained by the fact that this indicator takes into account two characteristics of a bank: its position vis-a-vis the rest of the network and the amount of cash it holds as expressed in our model. The greater the value, the lower the likelihood that the bank will suffer from liquidity hoarding. Comparing a positive mean and a zero median indicates that, generally speaking, the liquidity hoarding would not massively impact the network but that a few banks may be sensitive to this phenomenon.
3.3 Results

Since our model relies on several parameters, we have checked the robustness of our results by running simulations for a large set of parameters (see Appendix for details). For the sake of simplicity, we report the values for one set of parameters that seems either representative or conservative. First, we present the results for a solvency recovery rate of 40% which is conservative with respect to the EBA’s stress-tests\(^{17}\). Second, as becoming illiquid does not necessarily mean that the value of its assets is subject to a large deterioration, the recovery rate for banks whose counterparty defaulted due to illiquidity issues is assumed to be equal to 80% corresponding to twice the bankruptcy cost estimated in James (1991). Third, among the various functional forms of lambda we tested, the results displayed correspond to cases in which banks start hoarding liquidity when their capital falls below 120% of their economic capital (however, the choice of lambda does not impact the results).

Lastly, the aim of this paper is to assess the emergence of potential contagion phenomena from a macroprudential perspective; we do not focus on bank’s individual exposures to specific risk factors (which relates to microprudential concerns). We therefore decompose the effect of a shock into three terms: the direct effect of a shock before any contagion mechanism takes place, the incremental effect of solvency contagion and the incremental effect of liquidity contagion.

3.3.1 General and Asset Class-Specific Market Shocks

In this subsection, the analysis is based on two different exercises where in the first one the system is impacted by a common market shock and in the second one, asset class-specific shocks are envisaged.

The first result is that whether a general shock or an asset class-specific shock is considered, no bank goes into default. This finding is the direct consequence of two facts: first, in December 2011, French banks were well capitalized and substantially above the required capital levels; second, banks’ exposures to the considered asset classes are limited, and negative shocks to the value of any of these assets are not large enough to trigger the default of any of the banks. Extremely large financial market shocks are required to observe the default of at least one bank, though in our exercise we use weekly returns to compute market shocks. The system appears resilient to these shocks. Solvency contagion is then absent by definition. Liquidity contagion can emerge, though it is rather limited. This last result points to the fact that liquidity contagion appears even when there is no solvency

\(^{17}\)The 2011 EBA’s stress-test exercise reports that recovery rates for banks vary between 85% and 55%.
contagion. We could explain such small losses by the state of the banking system at the date of the analysis: as mentioned above, banks’ capital is substantially above than required and they also have enough liquid assets compared to their interbank funding. Therefore, after all the efforts made by the authorities, it is quite meaningful that banks are resilient to the market shock and that the liquidity contagion is limited.

The second result of such stress-test exercises is that the losses recorded are mainly if not solely caused by the initial shock, except in the extreme left tail of the distribution (see Table 5, page 34). In the extreme case for VaR(0.01%) of a general shock, there is 0.82% of capital loss due to liquidity hoarding phenomena. This almost 1% loss must nonetheless be compared to a shock with a magnitude of 46.31%.

We underline the fact that we obtain similar and therefore robust results for different recovery rates and for other specifications of the liquidity hoarding rule.

We also note that our results are in line with past stress-tests of French banks with respect to market risk (see e.g. IMF 2012), even though the performed stress-tests are much smaller in scope than in our study.

3.3.2 Idiosyncratic shocks combined with common market shocks

We may also be interested in a scenario where there is a common market shock and a bank goes into default (for reasons other than the market shock). To deal with this concern, we perform a second set of stress-test exercises where we force one bank at a time to default in the presence of the same market shocks. The figures presented in the following tables are averages over the 11 individual idiosyncratic scenarios.

These complementary exercises give the following results: either a general shock or an asset class-specific shock, combined with an idiosyncratic default leads to no defaults due to solvency contagion, and liquidity contagion is present but limited. The absence of the domino effect when even the biggest banks default can be explained by banks’ high levels of capitalization as well as by the small interbank exposures in total and, especially, to one specific counterparty. The size of losses measured as a percentage of the system’s capital is entirely due to the fact that banks lose their exposures to the defaulted bank. As seen in Table 6, page 35, these losses are equal to 1.18% over all types of shocks and all quantiles. The number shown is the average losses over the defaults of 11 banks, though losses vary among defaults of different banks depending on the exposure of the system to this bank.

The results of liquidity hoarding are similar to those of market shocks alone without idiosyncratic shocks.

The effects of solvency contagion and those of liquidity contagion are of the same
magnitude, each of them triggering losses accounting for about 1.8% of total capital under the most adverse scenarios \((VaR(0.1\%)\) and \(VaR(0.01\%))\). At the same time, the direct losses due to the initial shock (see Table 6, page 35) are 30 times larger than those occurring via each of these channels.

The robustness check exercises (see Appendix for more details) underline the quality and the stability of our results with respect to different specifications. When the solvency recovery rate varies from 0% to 100%, the figures keep the same approximate magnitude (see Table 7, page 36). We report results only for general market shock with and without idiosyncratic shocks, since results of asset-specific market shock are very similar and of the same order\(^{18}\).

### 4 Conclusion

This paper develops a model that allows us to take into account the losses of the system due to solvency and liquidity contagion after an initial correlated shock impacting the system. This is one of the first papers that makes it possible to disentangle between the losses caused by different sources of risk. We also propose a toolkit to simulate market shocks in line with liquidity hoarding phenomena. We use this model to evaluate the resilience of the French banking system to systemic market shocks.

The literature on the pure default contagion is much vaster, though the results are rather controversial. In a similar framework, Cont \textit{et al.} (2010) studying Brazilian banking system find evidence of sizeable domino effects after an initial shock. At the same time, Elsinger \textit{et al.} (2006a), who analyse the Austrian banking network, record rare occurrences of such effects. Other studies conducted on network contagion without any initial stress on the whole banking system document very limited consequences (Amundsen and Arnt (2005), Upper and Worms (2004)). What seems to be really essential for the existence of domino effects is not only the initial market shock, but also its magnitude.

Our results, which complement the study of pure solvency default contagion with the liquidity channel, shed light on four points. First, we clearly identify that for the French banking system on 31 December 2011, the contagion effects appear to be significantly smaller than the initial shock. Second, we find that losses due to solvency and liquidity contagion are of similar magnitude. One would therefore underestimate the losses in the system if one

\(^{18}\)Available on demand

28
did not take into account the distress propagated via funding shortages.

The real case analysis shows the high resilience of the French banking system to market risks as well as contagion effects. The losses due to market shocks lead, under extremely adverse simulations, to very few defaults and result in low capital losses due to contagion. They are much smaller than those from market shocks. Such results suggest that the actions undertaken by authorities for the recapitalization of banks as well as the exceptional credit lines offered to them to avoid any liquidity shortage appear to have had a positive effect on the financial stability of the system.
5 Bibliography

Acharya, V. and D. Skeie, 2011, ”A Model of Liquidity Hoarding and Term Premia in Inter-Bank Markets”, Federal Reserve Bank of New York Staff Reports, 498.


CEBS, 2009a: Committee of European Banking Supervisors, Guidelines for implementation of the framework for consolidated financial reporting, European Banking Authority, December 15th 2009

CEBS, 2009b: Committee of European Banking Supervisors, Guidelines on reporting requirements for the revised large exposures regime, European Banking Authority, December 11th 2009


Gourieroux, C., J.C. Heam and A. Monfort, 2012, ”Bilateral Exposures and Systemic


6 Appendix: Robustness Check

Running the simulation needs to define 3 mains specifications:

- The solency recovery rate, $R^S$, is varying from 0.1 (only 10% of exposures it repaid) to 1 (absence of loss). Indeed, the results vary with the solvency recovery rate but keep the same approximate size, as explained in the discussion of the paper.

- The liquidity recovery rate $R^L$ is set to 0.8: in case of default due to liquidity shortage, 20% of exposures are lost. Using another recovery rate do not change our results since the liquidity contagion spread is not overwhelming. As said before, 20% is a conservative setting since it is twice the bankruptcy cost estimated in James (1991).

- The hoarding function $\lambda(.)$ is a more sophisticated figure. We consider the inverse of a Gaussian c.d.f. as baseline shape. We run the simulations with 9 couples for the mean and the variance. The magnitudes of results are stable across specifications. However, the results are more sensitive to the mean than to the variance. In fact, the mean parameter acts as a threshold for triggering hoarding phenomena; therefore, it is logical than an easy triggering threshold leads to a more effective liquidity hoarding phenomenon.

As illustration, Table 7, page 36 reports the effect of a general market shock (with and without idiosyncratic shocks) for various solvency recovery rates.
Table 5: Capital loss in a French banking system (as a % of the total capital of the system) after being impacted by different market shocks (a general market shock and an asset class-specific shocks) decomposed into the source of losses: due the general market shock itself, and due to solvency and liquidity contagion. The solvency recovery rate is equal 40%; the liquidity recovery rate is equal to 80%. $\lambda$ is such that banks start hoarding liquidity when their capital falls below 120% of required capital. For example, at $VaR(0.01\%)$ the system loss is 40.51% of capital, of which 39.8% is due to the general market shock. Solvency contagion is absent by definition.
General Market Shock + Idiosyncratic shock (Capital Loss, % of the Total Capital)

<table>
<thead>
<tr>
<th></th>
<th>VaR(5%)</th>
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<th>VaR(0.01%)</th>
<th>ES(5%)</th>
<th>ES(1%)</th>
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<td>1.18</td>
<td>1.18</td>
<td>1.18</td>
<td>1.18</td>
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<tr>
<td>Liquidity Contagion (C)</td>
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<td>0.65</td>
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</tr>
<tr>
<td>Total (=A+B+C)</td>
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<td>53.02</td>
<td>36.63</td>
<td>44.88</td>
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Bubble Shock on Equity + Idiosyncratic shock (Capital Loss, % of the Total Capital)

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<th>VaR(5%)</th>
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<th>VaR(0.01%)</th>
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<td>44.88</td>
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Bubble Shock on Financial Institutions Bonds + Idiosyncratic shock (Capital Loss, % of the Total Capital)

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Bubble Shock on Sovereign Bonds + Idiosyncratic shock (Capital Loss, % of the Total Capital)

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Bubble Shock on Large Corporate Bonds + Idiosyncratic shock (Capital Loss, % of the Total Capital)

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Table 6: Capital loss in a French banking system (as a % of the total capital of the system) after being impacted by different market shocks (a general market shock and an asset class-specific shocks) and an idiosyncratic shock, decomposed into the source of the losses: due the general market shock itself, and due to solvency and liquidity contagion. The solvency recovery rate is equal 40%; the liquidity recovery rate is equal to 80%. $\lambda$ is such that banks start hoarding liquidity when their capital falls below 120% of required capital.
<table>
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<th>ES(5%)</th>
<th>ES(1%)</th>
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<th>VaR(1%)</th>
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<th>Recovery Rate</th>
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</table>

Table 7: Capital loss in a French banking system (as a % of the total capital of the system) due to liquidity hoarding after being hit by a general market shock with and without an idiosyncratic shock, for different recovery rates. \( \lambda \) is so that banks start hoarding liquidity when their capital falls below 120% (left column) and 150% (right column) of required capital.
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