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DIRECTION GÉNÉRALE DES ÉTUDES ET DES RELATIONS INTERNATIONALES

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Abstract

Based on a dynamic open-economy macroeconomic model, this paper aims at understanding the contribution of domestic financial underdevelopment to foreign reserve accumulation in some emerging market economies, especially in China. It is argued that foreign reserve accumulation is part and parcel of a growth strategy based on strong capital investment in a financially constrained economy. It is further proved using a Ramsey problem that purchasing international reserves is a welfare-improving policy in terms of production efficiency gains if it is jointly used with capital controls. In fact, when domestic firms are occasionally credit-constrained and they do not have a direct access to international financial market, they need domestic saving instruments to increase their retained earnings so that they can sufficiently invest in capital. The central bank plays the role of financial intermediary and provides domestic firms with liquid public bonds, thus relaxing domestic financial constraints. The proceeds of domestic public bonds are invested abroad due to the limited scope of domestic financial market and a depressed domestic interest rate, leading to foreign reserve stockpiling. The speed of foreign reserve accumulation would slow down once the economic growth rate decelerates and the domestic financial market develops.

Keywords: Foreign reserves, capital controls, credit constraints, domestic savings, capital investment, economic growth, Chinese economy
JEL: E22, F31, F41, F43

Resumé

A l'aide d'un modèle dynamique en économie ouverte, cet article explore le lien entre le sous-développement du marché financier national et l'accumulation des réserves de change dans des pays émergents. Partie intégrante d'une stratégie de rattrapage économique, l'achat des actifs en devise étrangère par les autorités monétaires résulte autant des contraintes financières que d'une forte croissance basée sur l'investissement. Accumuler des réserves de change permettrait ainsi d'améliorer les conditions de financement des entreprises et d'accroître l'investissement. Par ailleurs, cet article montre que l'accumulation de réserves de change pourrait être une politique optimale accélérant la transition économique vers un nouvel état stationnaire plus élevé si la banque centrale contrôlait également les flux de capitaux. Enfin, la vitesse d'accumulation des réserves change, selon cette approche, ralentirait au fur et à mesure que la croissance économique décélère et que le marché financier national se développe.

Mot clés: Réserves de change, contrôles de capitaux, contraintes financières, épargnes, investissement, croissance économique, économie chinoise
JEL: E22, F31, F41, F43

1 Introduction

Over the last decade, one can observe a surge in foreign reserve accumulation throughout the world and in particular in emerging market economies. The latter has become the main driving force of reserve stockpiling, possessing more than one half of the world total foreign reserves since 2005 (Figure 1). This phenomenon is also of a great importance for research in international finance and macroeconomics because reserve accumulation can substantially account for the 'allocation puzzle' advanced by Gourinchas and Jeanne (2007). As documented by several empirical papers, such as Alfaro et al. (2011), public capital outflows in the form of foreign reserve purchasing, along with international aid flows, contribute largely to capital flows from poor countries to rich ones.

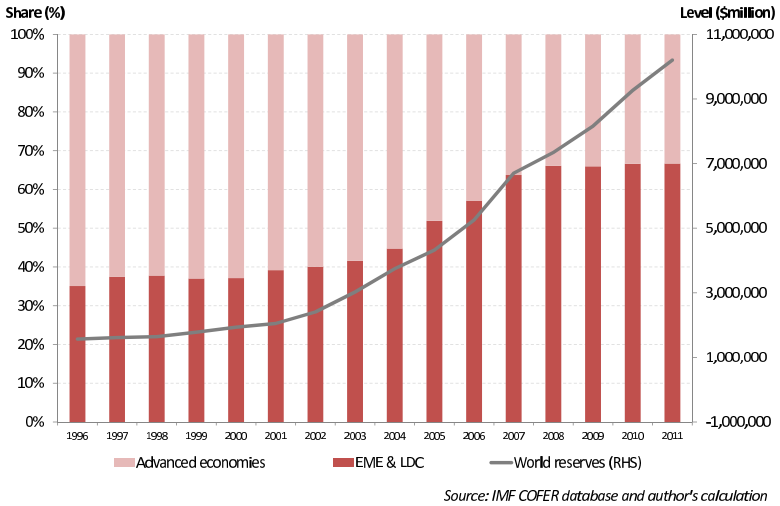


Figure 1: Evolution of the world reserves

Facing this phenomenon of large scale, a renewed literature revived the debate regarding the determinants of the demand for foreign assets. Since very recently, there seems to be a consensus that the motives for holding foreign reserves can be time-varying and specific to individual countries or country groups. An empirical paper by Ghosh et al. (2012) and a report of the Independent Evaluation Office (IEO) of the International Monetary Fund (IMF) (IEO (2012)) confirmed this broad view.

As a complement to the existing motives for foreign reserve accumulation, I shall argue in this paper that foreign reserve accumulation stems from - at least partially - the joint force of fast economic growth and financial frictions. Despite the extensive academic and

policy debates regarding the demand for international reserves, none has examined the relationship between foreign reserve holding and domestic capital formation in a growth model. This paper is especially inspired by two facts that one can observe in the data.

Fact 1 *Countries which had accumulated foreign reserves experienced on average a faster economic growth*

Figure 2 shows a clear positive relationship between the average growth rate of real GDP per capita and that of foreign reserve stocks between 1980 and 2010 for 24 emerging economies.

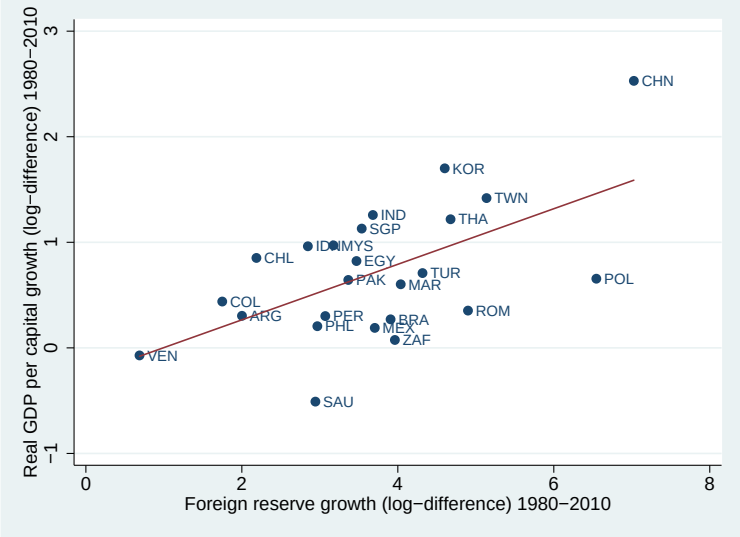


Figure 2: Foreign reserve accumulation vs. real GDP growth Data source: World Bank WDI database for real GDP series and IMF IFS database for foreign reserves series (total reserves minus gold, line 1 l.)

Most foreign reserve holders are middle income countries [1], thus in the phase of fast economic catch-up. This positive relationship between real economic growth rate and demand for foreign reserves suggests that holding reserves may be part and parcel of the catch-up strategy adopted in a number of developing countries.

Fact 2 *Among the middle income countries which have accumulated foreign reserves in the past decades, many have a relatively underdeveloped domestic financial market.*

[1]Ranked in terms of the average reserves to GDP ratio between 2000 and 2008, 14 out of the 20 biggest reserve holders are middle income countries (upper and lower middle income countries all combined according to the World Bank's classification).

Financial market underdevelopment can be understood as strong market frictions which impede smooth allocation of funds from savers to borrowers. On the one hand, bank loans constitute the major source of domestic financing short of other financial instruments (e.g. direct finance on capital markets). On the other hand, banks may have strong selection bias when allocating funding, preferring certain types of firms (e.g. state-owned enterprises) to others (e.g. private firms). Figure 3 illustrates public and private bond to GDP ratios in comparison with foreign reserves to GDP ratio in selected countries. It seems like the lower is the supply of domestic private bond and the higher is the supply of domestic public bond, the more important is the level of foreign reserves. One natural conjecture about this is that public bond constitutes an important source of domestic financing when the private market is very constrained. With a limited scope of the domestic financial market, public bond proceeds are highly likely invested abroad in the form of foreign reserves.

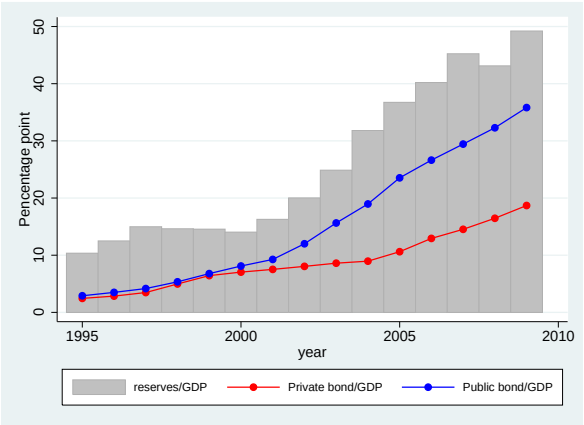
Based on these observations, I shall make three points in this paper: 1) Financial repression [2] is Pareto sub-optimal for fast-growing emerging market economies with credit constraints. As some domestic firms heavily rely on retained earnings for financing their production, a low domestic interest rate would reduce the value of their saving revenues. 2) Foreign reserve accumulation stems from the central bank's policy to ease domestic financial constraints by issuing domestic liquid assets. 3) Adopting a policy of reserve accumulation while keeping the capital account controlled is welfare-improving in terms of productivity efficiency gains, accelerating domestic capital formation and economic transition towards a higher steady state.

In the scope of this paper, I rule out the aspect of international trade and exchange rate policies, as my objective is to provide an alternative explanation of large foreign reserve accumulation in emerging market economies, which is related to domestic financial conditions. Obviously, having an exchange rate target might strengthen the demand for

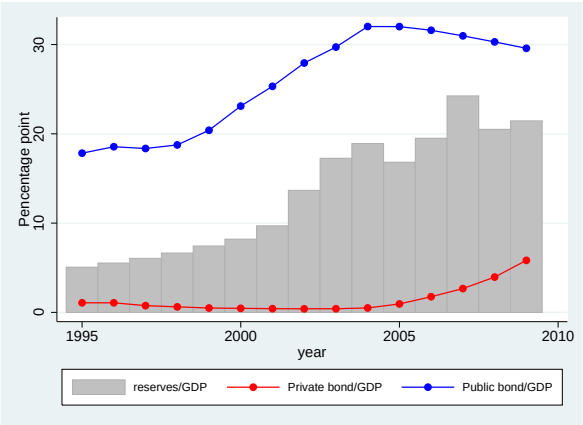
[2]Financial repression in this paper is understood in its original sense according to the theory developed by Edward Shaw, Ronald McKinnon and others in the 1970s when they studied financial conditions and economic development in developing countries. The main characteristic of the financial repression that I talk about in this paper is a very low domestic interest rate induced by distortions on financial market.

foreign reserves; however, the question about the relative weight of different motives for foreign reserve accumulation is out of the scrutiny of this paper.

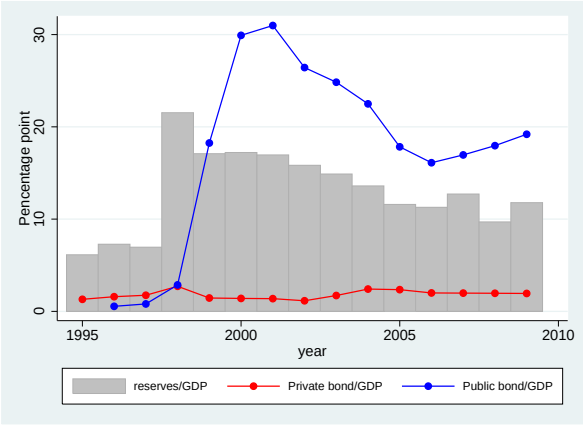
(a) China



(b) India



(c) Indonesia



(d) Philippines

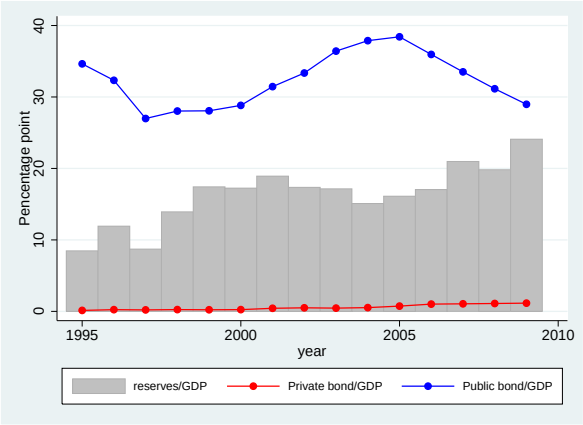


Figure 3: Domestic bond market vs. foreign reserves in selected countries Data source: World Bank WDI database

My paper is related to the literature on the motives of foreign reserve accumulation, on excessive domestic savings and on growth model with financial constraints.

A country may accumulate foreign reserves for reasons of different nature: to constitute a buffer stock for the sake of crisis prevention or/and crisis management (*precautionary approach* [3]), to intervene on foreign exchange market for defending a fixed exchange rate or for preventing the domestic currency from appreciation (*mercantilist*

[3]See Aizenman and Lee (2007), Aizenman and Hutchison (2010), Jeanne and Rancière (2011), Benigno and Fornaro (2012), Bianchi et al. (2013)

approach [4]), or to hold foreign reserves due to domestic structural deficiencies, especially financial market conditions (*structural approach*) [5]. This paper is related in particular to the last approach. In fact, countries accumulate foreign assets as a natural consequence of a wedge between domestic savings and investment. As Gourinchas and Jeanne (2007) put it, 'the allocation puzzle is a saving puzzle.' According to this paper, the distortions in the domestic financial sector may induce domestic precautionary savings. The distortions can be understood either in terms of the capacity of a country to supply safe assets (Caballero et al. (2008)) or in terms of credit constraints facing domestic firms. In the presence of credit constraints, the economy in its whole needs to make precautionary savings, which generate a wedge between aggregate savings and investment, source of capital outflows. Bacchetta and Benhima (2010), Wen (2011) and Song et al. (2011) are the most relevant references with this regard. Based on an open economy model with a borrowing constraint, Bacchetta and Benhima (2010) show that the demand for foreign bonds is a complement to domestic investment rather than a substitute. The reason behind is that foreign bonds constitute supplementary corporate saving assets for credit-constrained firms. The retained earnings can be used to invest in capital even if firms' borrowing capacity is constrained. Therefore, in a fast-growing economy, the demand for foreign bonds increases with domestic capital formation. In the same vein, Song et al. (2011) shed light on how a constrained access to bank loans of private firms compared to state-owned firms can lead to a large level of corporate precautionary savings, and thus a persistent current account surplus. As a result of privatization, the overall precautionary savings increase, so do foreign reserves. Finally, Wen (2011) tries to 'make sense of China's excessive foreign reserves' by looking at household savings. He argues that a large uninsured risk (e.g. the lack of a sound social safety net), stringent borrowing constraints, and rapid income growth can jointly generate high household saving rates and large current account surpluses in emerging economies. Therefore, foreign reserve accumulation is a natural consequence of a country's fast economic catch-up with an

[4] See Dooley et al. (2003), Korinek and Serven (2010), Benigno and Fornaro (2012)

[5] The empirical research under this approach is rather scarce. To my knowledge, Dominguez (2010) is one of the few papers which look at the role of domestic financial underdevelopment in reserve accumulation. The literature review below will mainly focus on theoretical papers.

inefficient financial system. As all the three papers look at the demand for foreign assets in a small open economy setting with no capital controls, the government/central bank is absent. The public and private capital flows are thus perfect substitutes. What they examine is rather the change in a country's net foreign assets rather than the central bank's foreign reserves *per se*. My current work differs from theirs by targeting to the central bank's reserve assets. Moreover, I will adopt a Ramsey problem to assess the optimality of the central bank's reserve policy.

In parallel, there is a rich literature examining economic growth in the presence of financial constraints. As for the central bank/government's policy in this context, a seminal paper by Woodford (1990) demonstrates the welfare-improving effects of public debts in promoting domestic capital investment in an economy with credit constraints. Woodford's paper is however based on a financial autarky setting.

The contribution of my paper is to combine the above-mentioned two strands of literature and the main features of both a small open economy and a closed economy setting. This setting gives the central bank the choice of accumulating foreign reserves and eventually set capital controls [6]

This paper is however not the first one which analyzes the central bank's optimal policy of reserve accumulation as well as its relationship with capital account policy. Bacchetta et al. (2012) and Benigno and Fornaro (2012) are among the precursors. My paper differs from Bacchetta et al. (2012) as I nest foreign reserve accumulation in a growth model and focus on the contribution of reserves to domestic capital formation, driver of economic catch-up in emerging market economies. The introduction of capital is very important for two reasons. First, this allows me to examine production efficiency gains instead of mere redistributive effects and consumption smoothing gains when examining the central bank's optimal policy. According to the literature on growth theory, the gains from production efficiency have a much more persistent and stronger impact on growth than from consumption smoothing (focus of Bacchetta et al. (2012)). Secondly, the introduction of capital formation makes my model more relevant for fast-growing

[6] As I will show later on in Section 4, capital controls are not frictions but a policy choice of the central bank.

countries with large stock of foreign reserves, such as China. The growth in these countries is largely driven by positive technology shocks and resultant capital accumulation (Nelson and Pack (1999), Bond et al. (2010) and Ahuja and Nabar (2012)) [7]. Finally, in my model, workers are allowed to save and finance directly entrepreneurs' capital investment, a missing feature in Bacchetta and Benhima (2010).

My paper differs from Benigno and Fornaro (2012) as I model in a different manner the imperfect substitutability between public and private capital flows on the one hand and I consider a different motive of foreign reserve accumulation from their paper. While they study the precautionary and mercantilist motives of foreign reserve accumulation, I focus on the imperfection of domestic financial market.

This paper is organized as follows. Section 2 describes the model setting. Section 3 analyzes reserve accumulation and capital formation in a decentralized economy. Section 4 introduces the optimal policy of the central bank and presents numerical results. Section 5 provides some empirical evidence about domestic financial conditions, foreign reserve accumulation and capital formation in China. Section 6 concludes.

[7] Notice that this view challenges somehow the widespread view on Chinese export-led growth. However, one can easily calculate the contribution to Chinese GDP growth. Investment is by far the most important contributor (more than 40% since 2000).

2 Model setting

The model setting I use is an extension of Bacchetta and Benhima (2010)'s model by taking the central bank's behavior into account. This is a standard way to model an open economy with heterogeneous agents and a central bank [8]. There is no aggregate uncertainty and I assume a perfect foresight of the future.

2.1 Policy regimes

The benchmark economy is however different from a standard small open economy setting, because the private sector does not have access to external financial market but only the central bank does. The benchmark economy can be called 'semi-open economy' à la Bacchetta et al. (2012). This policy regime mimics well the situation that one can observe in countries like China: the central bank there actively accumulates foreign reserves but forbids, with capital controls, the private sector to freely access to external financing. In the following sections, I will examine how a semi-open economy differs from a fully open economy and a closed economy, and which policy regime is optimal in terms of welfare. The main features of these different regimes are summarized in Table 1 (B^* stands for foreign reserves; r and r^* designate respectively domestic and world interest rate.).

Table 1: Policy regimes

		Policy Instruments	
Policy regime	Characteristics	Foreign Reserves	Interest Rate
Financial autarky	Domestic policy only	$B^* = 0$	$r \in \Re^+$
Open economy	Free capital mobility	$B^* \in \Re^+$	$r = r^*$
Semi-open economy	Possible capital controls	$B^* \in \Re^+$	$r \in \Re^+$

[8] Similar models with a representative agent can be found in Korinek (2011), Jeanne (2012) etc.

2.2 Family businesses

The private sector in this paper is modeled by two symmetric family businesses. Each of them is made up of a continuum of individuals of measure one. Family members in each family business are either a worker or an entrepreneur. Within the family business, the worker provides the labor force to the entrepreneur who in turn pays the worker at the marginal product of labor. Importantly, I assume that the family business pools together the incomes of both types of family members and optimally make consumption and investment decisions at the family level. This is a parsimonious way to model households and firms all combined in contrast with Bacchetta and Benhima (2010). The advantage of doing so is twofold: it simplifies the program of the private sector and renders the Ramsey program neater; it also allows both the worker and the entrepreneur to save and thus to contribute to physical capital investment [9]. Notice that this idea of family business (or 'representative family') composed of two types of members who pool together their income come from models in labor economics (Merz (1995), Ljungqvist and Sargent (2007)).

I assume that each family business is infinitely lived and capital is invested every two periods. As a result, any family produces in one period and invests in the other and so on so forth. That is, each of the two families changes its status every two periods, alternating between a 'producing-saving' period (denoted S) and an 'investing-borrowing' period (denoted I). The assumption of two symmetric family businesses is to guarantee that in each period there is always one family in its 'producing-saving' period and the other one in its 'investing-borrowing' period, so that, as we will see soon, the credit constraint facing the 'investing-borrowing' family might be at work in every period.

Family business' program

As the family businesses are symmetric, it is sufficient to look at the program of one of them. Let's consider the family who starts at time t with a 'producing-saving' period. It faces a standard intertemporal utility function with a discount factor β:

[9] Bacchetta and Benhima (2010) only allow the corporate sector to save as the worker in their model is 'hand-to-month' and consumes all the labor income every period.

$$\sum_{t=0}^{\infty} \beta^t \Big(U(c_t^S) + \beta U(c_{t+1}^I) \Big) \tag{1}$$

It has the following budget constraints every two periods:

$$\text{At } t: F(A_t, K_t, N_t) - r_t L_t = c_t^S + S_{t+1} + \frac{T_t}{2} \tag{2}$$

$$\text{At } t{+}1: r_{t+1} S_{t+1} + L_{t+2} = c_{t+1}^I + K_{t+2} + \frac{T_{t+1}}{2} \tag{3}$$

A typical family business in its 'producing-saving' period, here at time t, harvests an output (produced with inputs previously chosen) $F(A_t, K_t, N_t)$, and makes the decision between current consumption c_t^S and savings S_{t+1}. The motive for saving is explained by the fact that the output is only harvested every two periods. Namely, at $t+1$ (the family's 'investing-borrowing' period), the family business has to rely on retained earnings $r_{t+1}S_{t+1}$ and domestic borrowing L_{t+2} to invest in physical capital K_{t+2} and to consume c_{t+1}^I. r_t denotes the domestic *gross* interest rate ($r_t > 1$, for all t). T_t denotes lump-sum taxes (transfers) from an implicit government.

The production function $F(A_t, K_t, N_t)$ is a standard neoclassical production function: increasing in all arguments, concave and homogeneous of degree one. I use $F_{K,t}$ and $F_{N,t}$ to denote the marginal product of capital and of labor respectively. A_t stands for production technology. As we will see later on, the only shock in this model is a productivity shock so as to mimic the fast economic catch-up in emerging market economies.

Notice that the wage payment does not appear in the above budget constraints; this is because I assume that the wage payment between the worker and the entrepreneur is carried out internally with $w = F_{N,t}$ while the allocation decision is made by the head of the family at the family level. It is further assumed that the labor supply is inelastic and $N_t = 1$.

Similarly, the family business which starts at t with an 'investing-producing' period

has the following budget constraints:

$$\text{At } t: r_t S_t + L_{t+1} = c_t^I + K_{t+1} + \frac{T_t}{2}$$
$$\text{At } t+1: F(A_{t+1}, K_{t+1}, N_{t+1}) - r_{t+1} L_{t+1} = c_{t+1}^S + S_{t+2} + \frac{T_{t+1}}{2}$$

Credit constraint and demand for liquid asset

Most importantly, there is a credit constraint facing the family in its 'investing-borrowing' period:

$$L_{t+2} \leq \frac{\psi F(A_{t+2}, K_{t+2}, N_{t+2})}{r_{t+2}} \tag{4}$$

The maximum loan that an investing family can get is conditional on the discounted value of its next period output. ψ denotes the tightness of credit constraint with $\psi \in [0, 1]$. The smaller is ψ, the tighter the constraint is.

The form of the credit constraint follows the standard setting in the literature of financial accelerator (e.g. Bernanke et al. (1999)). I argue that the credit constraint is institutional and cannot be removed in the short-run. The reason behind stems from what I observe on Chinese financial market (details in Section 5). On the one hand, Chinese households have a large level of savings that they are willing to lend to firms in need but fail to do so due to the lack of a complete financial market (Chamon and Prasad (2010)). On the other hand, the borrowing capacity of private firms which need funding is limited following the standard argument of contract enforcement. This constraint is even stronger in countries like China where major commercial banks are state-owned and thus prefer (and sometimes are obliged) to lend to public firms [10]. Domestic private firms, albeit more productive, are unable to get enough loans from commercial banks (see Song et al. (2011)). The credit constraint constitutes thus the only market friction in our model.

[10]Notice that in the current model, the banking sector is absent. However, the results that I derive in the subsequent sections will not change if a competitive banking sector is introduced. Therefore, to keep the model tractable, I decide to leave the financial sector aside.

Due to the credit constraint presented above, total bank loans demanded are conditional on the borrower's capacity to borrow. As a result, savings that the producing family is willing to provide cannot be fully absorbed, driving down the domestic interest rate. As one can see in Section 3.3, this situation is not optimal for domestic investment and production, leaving room for central bank intervention.

If the central bank provides supplementary domestic saving instruments, the 'producing-saving' family will be able to save more. The demand for saving assets in this paper is motivated by the joint effect of a credit constraint and a rapid economic growth. In fact, this paper assumes that there is a strong technology growth A_t in this economy, which increases the marginal product of capital and encourages family businesses to invest more in physical capital. This requires extra savings as families' borrowing capacity is limited. What motivates the demand for liquid assets is thus very different from Bacchetta and Benhima (2010) where the liquidity demand is explained by the need to pay the labor force in advance.

2.3 The central bank's behavior

General setting

The central bank in our model issues domestic saving bonds, B_{t+1}, which aim at filling out the gap between domestic savings and borrowing. Due to the limited scope of domestic financial market, the central bank then invests domestic bond proceeds in foreign assets, B^*_{t+1}. The central bank's flow budget constraint is described as follows:

$$r^*_t B^*_t + B_{t+1} + T_t = r_t B_t + B^*_{t+1} \tag{5}$$

r^*_t is the world interest rate; it is assumed that the world interest rate is constant and equals to the inverse of world time preference discount factor β, namely $r^*\beta = 1$ (assuming the same time preference in the domestic economy and abroad). T_t is a lump-sum transfer from the government which taxes households to pay this transfer (this can be a transfer to the government too if $T < 0$). To focus on the central bank's policy, it is

assumed that the transfer from (to) the government is exogenous, used only to balance the central bank's flow budget constraint each period; it captures the financial gains (losses) from the central bank's investment, which are transfered to the government as the government is assumed to be the stakeholder of the central bank.

The supply of the liquid bond by the central bank is determined by the demand on domestic market (difference between domestic savings and borrowings):

$$\underbrace{B_t}_{\text{supply of liquid bonds}} = \underbrace{S_t - L_t}_{\text{demand for liquid bonds}} \tag{6}$$

A positive B_t is motivated by the fact that the investing family is financially constrained and the savings cannot be fully absorbed. If $B_t = 0$, the domestic interest rate would decrease to reduce savings, leading the economy to financial repression.

The interaction between the domestic private sector and the central bank in this model can be summarized in Figure 4.

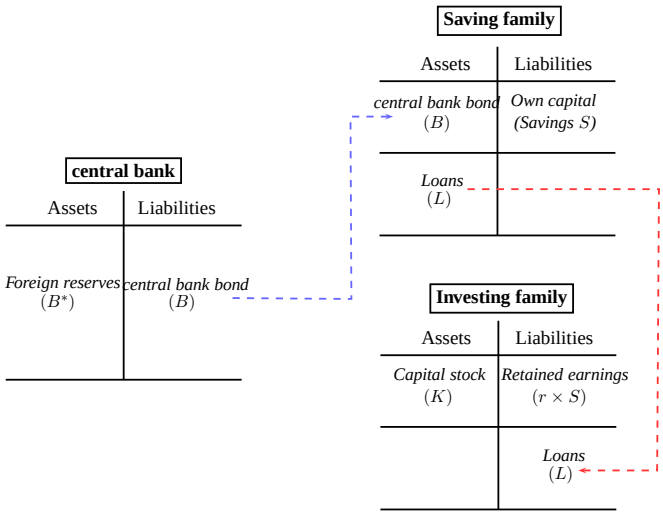


Figure 4: Balance sheet of the central bank and the private sector

Central bank's purchases of foreign reserves

The use of public debt, e.g. government or central bank bonds, has proved to be welfare-improving in the presence of market frictions, as Woodford (1990) demonstrates. In this paper, I argue that in order to provide this liquid bond, the central bank needs to invest domestic bond proceeds abroad. One reason is that in some emerging market

economies the domestic financial market being underdeveloped there are not sufficient domestic assets available to absorb the central bank's bond proceeds. Another reason is that foreign assets generate a higher return than domestic assets when the domestic economy is financially repressed. Moreover, the job of liquidity provision cannot be undertaken by the government as government financing is usually more closely regulated by law than central bank bonds. As a result, foreign reserve accumulation is the counterpart of the domestic bond provision in a financially constrained fast-growing economy where the central bank aims at ease domestic financial conditions.

To close the model setting, I present below the aggregate budget constraint of the model economy by combining the private sector's and the central bank's budget constraints:

$$F(A_t, K_t) = c_t^S + c_t^I + K_{t+1} - r_t^* B_t^* + B_{t+1}^* \tag{7}$$

Output produced by the family in its 'producing-saving' period is consumed by both families, invested in capital, and used to buy foreign assets.

3 Competitive market equilibrium

3.1 Conditions for the competitive market equilibrium

In this section, I will present fundamental features of the model in a competitive market equilibrium, given a set of central bank policy variables. It can be seen that a decentralized open economy, even in the presence of credit constraints, can achieve the unconstrained level of capital formation in the long-run. I define this situation as the first best in our model economy. Moreover, I will show how different sets of public policies can affect the level of domestic interest rate and domestic capital formation. In order to derive analytical results, I use log-utility and Cobb-Douglas production function henceforth, namely $U(c_t^i) = log(c_t^i), i \in (S, I)$ and $F(K_t, A_t) = K_t^{\alpha} A_t^{1-\alpha}$ [11]. For simplicity, I assume $T = 0$, that is the central bank balances its balance sheet with domestic bond and foreign assets every period.

Definition 1 *A perfect foresight market equilibrium in a decentralized economy is a sequence of allocation* $\{c_t^S, c_t^I, K_{t+1}, S_{t+1}, L_{t+1}\}$*, for a given sequence of price* $\{r_t\}$*, policy set* $\{B_t^*, B_t, T_t\}$*, and initial conditions* $\{K_0, L_0, S_0, B_0, B_0^*, T_0\}$*, such that for all* $t > 0$*:*

1. *The utility function* (1) *is maximized subject to the family business' budget constraints* (2) *and* (3)

2. *The borrowing constraint* (4) *is verified*

3. *Government budget constraint* (5)*, financial and good markets clearing conditions* (6) *and* (7) *are all verified*

The following first order conditions for the family business (in its 'producing-saving' stage at time t) can be derived:

[11] Log-utility is only required to derive the uniqueness of the constrained steady state. The qualitative results do not change with a more general form of utility function (e.g. Constant relative risk aversion (CRRA).

$$(1+\psi\lambda_t)F_{K,t} = r_t(1+\lambda_t) \tag{8}$$

$$u'(c_t^S) = \beta r_{t+1} u'(c_{t+1}^I) \tag{9}$$

$$u'(c_{t+1}^I) = \beta r_{t+2} u'(c_{t+2}^S)(1+\lambda_{t+2}) \tag{10}$$

$\lambda_t u'(c_t^S)$ denotes the Lagrange multiplier associated with the credit constraint at time t. Equation (8) tells us that the marginal product of capital is equal to the cost of capital r_t augmented by a coefficient $\frac{1+\lambda_t}{1+\psi\lambda_t}$, which is related to the credit constraint. Equations (9) and (10) are Euler equations. The Euler equation (10), which relates the marginal utility of consumption of an investing family to that of the same family in its 'producing-saving' stage a period later, depends clearly on the credit constraint, λ_{t+2}.

According to the Kuhn-Tucker theorem, the Lagrange multiplier with respect to the credit constraint verifies $\lambda_t u'(c_t^S)[r_t L_t - \psi F(A_t, K_t)] = 0$. Therefore, the credit constraint may be binding ($r_t L_t = \psi F(A_t, K_t)$) or non binding ($\lambda_t = 0$).

Using the log-utility, I can rewrite the consumptions as a share of the wealth: $c_t^S = (1-\beta)\big[F(A_t, K_t) - r_t L_t\big]$ and $c_{t+1}^I = (1-\beta) r_{t+1} S_{t+1}$.

This gives:

$$S_{t+1} = \beta[F(A_t, K_t) - r_t L_t] \tag{11}$$

$$K_{t+2} - L_{t+2} = \beta r_{t+1} S_{t+1} \tag{12}$$

3.2 Uniqueness in steady state

I now demonstrate that the steady state is uniquely determined when the credit constraint is binding. This result is important for us to analyze capital formation and other features of the model later on.

For this purpose, I normalize all endogenous variables by output $F(A_t, K_t)$, namely I define $b^* = \frac{B^*}{F(A,K)}$, $b = \frac{B}{F(A,K)}$ and $\tau = \frac{T}{F(A,K)}$.

The central bank's budget constraint (5) becomes:

$$(r^* - 1)b^* + \tau = (r - 1)b \tag{13}$$

Given international and domestic interest rates as well as the level of government transfer, one can observe from Equation (13) that if the central bank wants to provide liquid assets in the domestic economy, it can achieve this objective by purchasing foreign reserves.

Proposition 1 *Considering the simplest case where $\tau = 0$ (central bank do not have financial gains/losses in the long-run), the financially constrained steady state is uniquely determined if $0 \leq b < \beta(1 - 2\psi)$.*

Proof. See Appendix A ■

Proposition 1 implies that the credit constraint is not binding when $b \geq \beta(1-2\psi)$; the central bank can achieve this goal by raising b^*. When the credit constraint is binding, the domestic interest rate is equal to the world interest rate $r = r^* = \frac{1}{\beta}$; with a binding credit constraint, the domestic interest rate is repressed and lower than its international counterpart. In fact, when the central bank provides liquid bonds which are scarce in the economy, it lowers the price of the liquid bonds and pushes the domestic interest rate up to the world interest rate level. r is indeed increasing in b: $r = \frac{\psi}{\beta(1-\psi)-b}$.

It can be further proved that the coefficient of the tightness of the credit constraint is important for whether the credit constraint is binding in the steady state. When $\psi > \frac{1}{2}$, the credit constraint is unbinding in the steady state. When $\psi \leq \frac{1}{2}$, the credit constraint is strictly binding in the steady state (See Appendix B for details).

We can further look at different steady states in different policy regimes described in Table 1.

Financial autarky In a closed economy where $B^* = 0$, if $\psi > 1/2$, the credit constraint does not bind in the steady state. In this case, $\beta r = 1$, namely, the domestic interest rate is equal to the world interest rate. $K = A(\alpha\beta)^{\frac{1}{1-\alpha}}$, the capital stock achieves its first-best level. And consumptions are perfectly smoothed, namely the consumption in the saving stage and that in the investing stage are equalized $c^I = c^S$.

If $\psi \leq 1/2$, the credit constraint binds in the steady state. The Lagrange multiplier associated with the credit constraint is thus positive, $\lambda = \frac{1}{\beta^2 r^2} - 1 > 0$. The domestic interest rate is lower than the world interest rate, namely $\beta r = \frac{\psi}{1-\psi} < 1$. Capital stock cannot achieve the first-best level, $K = A\Big[\alpha\big((1-\psi)\beta^2 r + \frac{\psi}{r}\big)\Big]^{\frac{1}{1-\alpha}}$. As for consumptions, that of the investing stage is always smaller than that of the saving stage, $\frac{c^I}{c^S} = \frac{\psi}{1-\psi} < 1$.

In fact, in a financial autarky, with the productivity growth, the more capital the better. There is thus a strong demand for liquid assets. However, investing families are constrained and the central bank does not have sufficient resources to provide domestic assets without distorting the domestic economy. Therefore, the economy may be repressed if the credit constraint is very tight.

Open economy One big difference here compared to the financial autarky is that the domestic interest rate is always equal to the world interest rate, $r = r^* = \frac{1}{\beta}$. In this case the credit constraint is never binding, leading to $b = b^* = \beta(1-2\psi)$. In an open economy, both the central bank and the private sector have access to international financial market, therefore, the saving family can make deposits abroad. In aggregate, there are sufficient foreign assets to increase saving revenues of domestic family businesses. Therefore, in steady state, the economy is unconstrained. Notice that in this situation, one can only talk about the net foreign assets of the aggregate economy; it is impossible to distinguish public and private capital flows as they are perfectly substitutes.

The unconstrained steady state constitutes the first-best benchmark. A full open economy can always achieve the first best in the long-run (Table 2 provides a summary).

Table 2: Decentralized economy

Economic Structure	Steady State	ψ
Financial autarky	constrained	$\psi \leq \frac{1}{2}$
	unconstrained	$\psi > \frac{1}{2}$
Open Economy	unconstrained	$\forall \psi$

3.3 Capital formation

Law of motion

How does the capital stock evolve over time? From (11) and (12), capital accumulation which is consistent with optimal consumptions in a financially constrained economy satisfies:

$$K_{t+2} = \underbrace{\beta r_{t+1} S_{t+1}}_{\text{retained earnings}} + \underbrace{L_{t+2}}_{\text{loans}} \tag{14}$$

$$= \beta^2 r_{t+1}(1-\psi)F(A_t, K_t) + \frac{\psi F(A_{t+2}, K_{t+2})}{r_{t+2}} \tag{15}$$

Notice that retained earnings and loans are reversely affected by interest rates. An increase in domestic interest rate raises the retained earnings of the saving family (revenue effect) but reduces the loans that the same family may make next period during its investing period (borrowing effect). Whether capital stock increases or decreases depends on which effect dominates.

A feedback loop effect can be observed in equation (15). Namely, more capital invested provides a stronger collateral allowing the investing family to borrow more during the investing stage and leading to a looser credit constraint. For example, an initial positive shock on retained earnings (either an increase in r_{t+1} or an increase in initial capital stock K_t) will lead to an increase in capital formation at $t+2$, which in turn drives loans up. Accordingly, capital formation K_{t+2} further increases, generating a virtuous cycle.

From (15), one can easily show that *ceteris paribus*,

$$\frac{\partial K_{t+2}}{\partial r_{t+1}} > 0$$
$$\frac{\partial K_{t+2}}{\partial r_{t+2}} < 0$$

In the steady state

In the steady state, how does capital stock evolves? It depends on the trade-off between the revenue effect and of the borrowing effect.

Proposition 2 *Capital stock represents a U-shape curve as a function of βr. Purchasing foreign reserves, which leads to a higher supply of domestic liquid assets and thus a higher interest rate, helps increase capital stock and raise production in the economy.*

Proof. See Appendix C ■

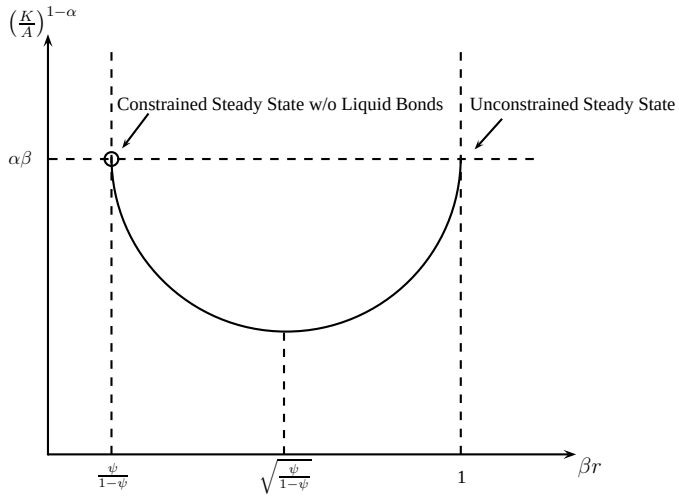


Figure 5: U-shape capital stock

From Figure 5, we observe that the capital stock can achieve its first-best level if $\beta r = 1$ (i.e. in an open economy steady state) or if $\beta r = \frac{\psi}{1-\psi}$. However, the latter point is not feasible as long as the *gross* interest rate $r > 1$, which is the case here (see Appendix A for details).

When $\frac{\psi}{1-\psi} < \beta r \leq \sqrt{\frac{\psi}{1-\psi}}$, the curve is decreasing. That is, when financial constraints are strong, domestic interest rate is very low, any increase in interest rate will decrease the value of the collateral, leading to a decrease of borrowing. The domestic savings are also very low, an increase in retained earnings (due to a raise of interest rate) cannot offset the decrease in borrowing. Therefore, on this part of the curve, the borrowing effect of the interest rate dominates the revenue effect, leading to a decrease in capital stock. I argue that the initial steady state (before the realization of any technology shock) should be located on the lower part of this decreasing curve as the upper left part assumes an extremely stringent credit constraint with no domestic bond supply and an interest rate near zero. This is hardly the case in emerging market economies.

When $\sqrt{\frac{\psi}{1-\psi}} \leq \beta r \leq 1$, the curve is increasing. This is because when the central bank provides more and more liquid assets, it decreases the price of liquidity previously

overpriced. Any family in its saving stage can save up to the first-best level and the retained earnings increase with the interest rate too. Bear in mind that the retained earnings are used for capital investment a period later when the family faces investment opportunities. The revenue effect dominates the borrowing effect on this part of the curve.

The equation below presents a more precise insight on how capital formation evolves with the interest rate (derivation in Appendix C):

$$F_K = \frac{r}{(\beta r)^2 + \psi[1 - (\beta r)^2]} \tag{16}$$

Three effects which affect capital stock can be identified from Equation (16):

- Opportunity cost of capital, r
- Revenue effect, $(\beta r)^2 = \frac{1}{1+\lambda}$
- Borrowing effect, $\psi[1 - (\beta r)^2]$

The first effect, opportunity cost of capital, is standard: the higher is r, the higher is the cost of investing in capital stock, the less capital is thus demanded. The other two effects are particular in our model.

The second term $(\beta r)^2$ is related to families' saving revenues. As families are occasionally credit-constrained, they have incentives to partly rely on retained earnings to invest in capital. As the retained earnings are increasing in interest rate, the revenue effect has a positive impact on capital stock. Notice that the square appears as the capital is invested every two periods. In an extreme case, when $\psi = 0$ (i.e. no borrowing is possible), families rely entirely on retained earnings for capital investment. The revenue effect clearly dominates the opportunity cost. As a result, a higher supply of liquid bonds leads to a higher domestic interest rate which unambiguously increases capital stock.

The third effect stems from the fact that the loans that an investing family can ask are conditional on the discounted valued of the future production. An increase in r decreases the borrowing capacity of the investing family, as it lowers the discounted value of the

collateral. The interest rate has thus a negative borrowing effect on capital stock. As described above, the borrowing effect dominates the revenue effect only when the supply of liquidity is very low. Moreover, with a credit constraint becoming less stringent, the borrowing effect cancels out with the revenue effect, leading to a competitive equilibrium without financial frictions (e.g. extreme case: $\psi = 1$).

3.4 Comparative statics

As long as the central bank provides liquid bonds B financed by foreign reserves B^*, it can raise domestic interest rate to the world interest rate level and to push domestic capital formation to its first-best level. From Figure 6, one can see that if the central bank accumulates more foreign reserves, it has a greater capacity of providing domestic liquid bonds, driving up the supply of liquid bonds and the domestic interest rate. This will ultimately lead to an increase in domestic capital formation, as Figure 5 shows, given that the initial equilibrium interest rate is greater than $\frac{1}{\beta}\sqrt{\frac{\psi}{1-\psi}}$.

On the other hand, Figure 7 shows that when the domestic credit constraint becomes tighter, namely ψ decreases, the demand for domestic liquid bonds becomes higher as the gap between domestic savings and domestic borrowing gets bigger. This drives the domestic interest rate to fall. The economy is further financially repressed.

Next, I will show how the central bank can actively choose its policies to improve conditions on domestic financial market and capital formation.

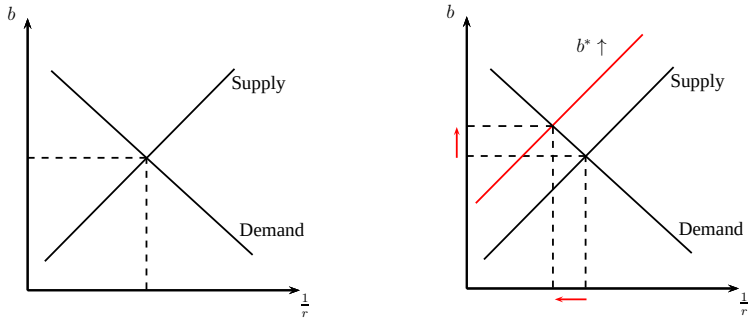


Figure 6: Increasing foreign reserve holding

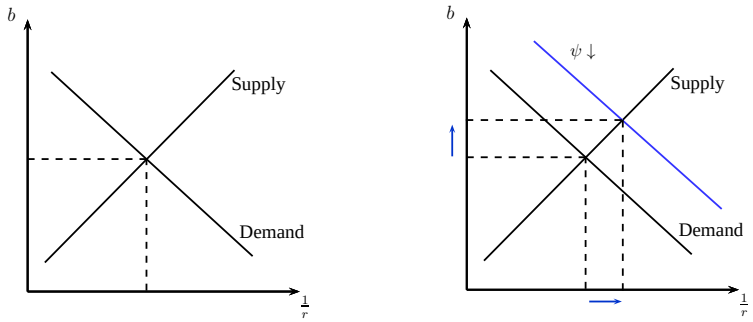


Figure 7: Tightening the domestic credit constraint

4 The central bank's optimal policy

Section 3 demonstrates that in a decentralized economy a fully open regime can always achieve the first-best situation in steady state in terms of capital formation and production. However, we also know that during economic transition, there might be a strong demand for liquid saving instruments due to a persistent and positive technology shock and credit frictions. Is the open economy optimal during economic transition? As we will see in this section with a Ramsey problem, an open economy is dominated by a semi-open economy on the convergence path. This policy of jointly accumulating foreign reserves and controlling capital flows (so as to control domestic interest rates) can be welfare-improving.

4.1 General setting

The central bank optimizes the consumptions of contemporaneous families in their different stages according to the following objective function.

$$\max_{\{S_{t+1},L_{t+1},K_{t+1},c_t^S,c_t^I,B_{t+1}^*,r_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \left\{ u(c_t^S) + u(c_t^I) \right\} \tag{17}$$

Definition 2 (Ramsey problem) *The central bank's optimal policy consists of choosing a sequence of policy variables* $\{B_{t+1}^*, B_{t+1}, r_{t+1}\}$ *and a sequence of endogenous private sector's variables* $\{c_t^S, c_t^I, K_{t+1}, L_{t+1}, S_{t+1}\}$ *for all* $t > 0$ *such that:*

- *The corresponding competitive equilibrium allocation* (8) *to* (10) *maximize the welfare function* (17)
- *Individual entrepreneurs' budget constraints,* (2) *and* (3)*, are verified*
- *Government budget constraint* (5)*, bond market clearing* (6) *and resource constraint* (7) *are all respected*

The way to solve the Ramsey problem follows the 'Primal Approach' described by Erosa and Gervais (2001).

The full Ramsey problem is presented below and the details about the first order conditions derived from the program can be found in Appendix D.

$$\begin{aligned}
\max_{\{S_{t+1},L_{t+1},K_{t+1},c_t^S,c_t^I,B_{t+1}^*,r_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \Bigg\{ & u(c_t^S) + u(c_t^I) \\
& + \eta_t^S \left[F(A_t, K_t) - r_t L_t - c_t^S - S_{t+1} - \frac{T_t}{2} \right] \\
& + \eta_t^I \left[r_t S_t + L_{t+1} - c_t^I - K_{t+1} - \frac{T_t}{2} \right] \\
& + \eta_t^G \left[F(A_t, K_t) - c_t^S - c_t^I - K_{t+1} - B_{t+1}^* + r_t^* B_t^* \right] \\
& + \theta_t^S \left[\frac{1}{c_t^S} - \beta r_{t+1} \frac{1}{c_{t+1}^I} \right] \\
& + \theta_t^I \left[\frac{1}{c_t^I} - \beta r_{t+1} (1 + \lambda_{t+1}) \frac{1}{c_{t+1}^S} \right] \\
& + \rho_t \left[(1 + \psi \lambda_t) F_{K,t} - r_t (1 + \lambda_t) \right] \\
& + \Lambda_t \left[\psi F(A_t, K_t) - r_t L_t \right] \Bigg\}
\end{aligned}$$

4.2 Steady state

First, it can be shown that the central bank can do as well as the decentralized economy in steady state regardless of the policy regime.

Proposition 3 *In financial autarky, the central bank can achieve the steady state with an unbinding credit constraint when the coefficient of the tightness of the credit constraint* $\psi > \frac{1}{2}$*. That is,* $\lambda = \Lambda = 0$*,* $\beta r = 1$ *and* $c^S = c^I$*. When* $\psi \leq \frac{1}{2}$*, the steady state with the Ramsey policy is constrained. That is,* $\lambda > 0$*,* $\Lambda > 0$*,* $\beta r < 1$ *and* $c^I < c^S$*.*

Proposition 4 *The central bank can achieve the steady state with an unbinding credit constraint in an open economy by using an optimal policy à la Ramsey. That is, the central bank can achieve the same steady state as the decentralized economy does in an open economy.*

Proof. See Appendix E ■

4.3 Economic transition in a semi-open economy

The major difference between an open economy and a semi-open economy is that in the latter setting the central bank has a full control of domestic interest rates. It can achieve this goal by imposing capital controls so that private capital flows cannot offset the effect of public capital flows. As a result, the central bank has two instruments in hands: foreign reserves and domestic interest rates. It has thus sufficient funding to provide liquid assets in the country and at the same time it is able to adjust domestic interest rates to follow savers/investors dynamics. Capital controls are thus motivated only when the central bank find a combined use of reserves and domestic interest rates optimal.

Based on the first order condition with respect to domestic interest rate (Equation (33) in Appendix D), it can be showed that when the central bank increases domestic interest rates, this is beneficial for the saver of the current period as the revenue from savings next period will be higher (first term). However, this will be harmful for the investor as it has to pay higher interests next period on loans it makes during the current investment stage (second term). It will also change intertemporal behavior of entrepreneurs, leading to more savings and less consumptions (third and fourth terms). Finally, a higher domestic interest rate implies higher opportunity cost (the last term). The first two terms translate a redistributive trade-off that the central bank has to face. This redistributive effects also imply welfare costs to the economy. Indeed, if the current investors are not very constrained and the central bank thinks the long-run steady state can be achieved soon, it has incentives to decrease the interest rate to alleviate the interest burden of investing families. If on the contrary the central bank believes that the economy will still be

constrained for a long period, it has incentives to rise domestic interest rate, so that the savers may transfer a larger amount of interest revenue to the future when they will be constrained.

Depending on the tightness of the credit constraint and the speed of convergence to the steady state, the central bank can decide to favor saving or investing families in order to enhance the social welfare.

To see why an open economy is not optimal during transition, I rearrange Equation (33). Denote $\mathcal{H}_t$ the left hand side of Equation (33). In an open economy with a binding credit constraint I can prove (see Appendix F):

$$\mathcal{H}_t = \left(\sum_{i=1}^{\infty} \Lambda_{t+2i}\right) S_{t+1} - \left(\sum_{i=0}^{\infty} \Lambda_{t+2i+1}\right) L_{t+1} \tag{18}$$

Whenever $\mathcal{H}_t$ is different from 0, the central bank has incentives to intervene and the open economy is not optimal. If $\mathcal{H}_t > 0$, the central bank has incentives to raise interest rate to favor savers. If $\mathcal{H}_t < 0$, the central bank has incentives to lower interest rate to favor investors. In general, as Table 3 summarizes, with financial frictions an open economy is suboptimal during transition while an economy with closed capital account and sufficient foreign reserves can achieve higher social welfare during transition. Bear in mind that capital controls stem from the central bank's optimal policy [12] so as to control domestic interest rate. Capital controls are thus not market frictions.

Table 3: Steady state vs. Transition

	Central Bank Intervention	
Economic Structure	Steady state	Dynamics
Financial autarky	depending on ψ	depending on ψ
Open Economy	First-best	suboptimal
Semi-open	Second-best	optimal

I further illustrate the optimality of the central bank's policy with numerical simulations.

[12] Notice that in this paper I derive the optimal policy of the central bank in a real economy without any explicit monetary policy objectives (e.g. inflation targeting, exchange rate stabilization, etc.). The combined use of foreign reserves and capital controls can also be motivated by specific policy preferences facing the 'impossible trinity.' For an insight on the trade-off between foreign reserve accumulation and other monetary policy objectives, which is not analyzed in this paper, please refer to Aizenman et al. (2010).

4.4 Numerical results

Calibration

The model can be simulated using Dynare 4.3.2 (see Adjemian et al. (2011)). The shock comes from an unanticipated and permanent productivity shock which follows an AR(1) process: $A_t = (1 + g_t)A_{t-1}$ and $g_t = \sigma g_{t-1}$. The accelerator of the growth rate, σ, is less than 1, ensuring the growth rate converges to 0. The shock is triggered at the beginning of period 1 with an initial growth rate $g_0 = 10\%$. This setting is to mimic the fast economic catch-up in emerging market economies.

I calibrate the model based on the recent literature on economic growth and foreign reserve accumulation in a financially constrained economy. Song et al. (2011), Bacchetta et al. (2012) and Benigno and Fornaro (2012) constitute the major references. Notice that due to the parsimony of the model, it is very difficult to calibrate the model based on a specific country. Therefore, the numerical results I present below aim to illustrate the qualitative results derived from the model instead of giving quantitative interpretations.

Table 4: Benchmark calibration

Symbol	Meaning	Value
α	share of capital in the production	0.33
β	discount factor	0.95
ψ	credit constraint tightness	0.10
g_0	initial shock on technology growth	0.10
σ	technology growth autoregressive coefficient	0.60

The impact of the credit constraint

I first simulate the model to visualize the impact of the tightness of the domestic credit constraint. To do so, I use the values of parameters as described in Table 4 and vary the parameter of the credit constraint ψ from 0.1 to 0.4 ($\psi = 0.1$ is the benchmark setting). Bear in mind that the tightness of the credit constraint needs to be less than 0.5 for the constraint to bind. In Figure 8, the solid line represent the situation where the credit

constraint is very stringent ($\psi = 0.1$) and the dashed line represents the opposite situation with a loose credit constraint ($\psi = 0.4$). First, we observe that the more stringent is the credit constraint, the less is the level of domestic loans (upper right panel). In fact, a tighter credit constraint limits the borrowing capacity of investing families which are thus encouraged to save more (upper left panel) in order to have enough funding to invest. For this, the central bank has to provide a larger level of domestic liquid bond (lower left panel). Due to the constrained domestic financial market, the central bank invests the bond proceeds abroad in the form of foreign reserves (lower right panel). On the contrary, if the domestic economy is not very constrained (e.g. the solid line), domestic loans match domestic savings; the demand for supplementary central bank bonds and foreign assets is thus very low.

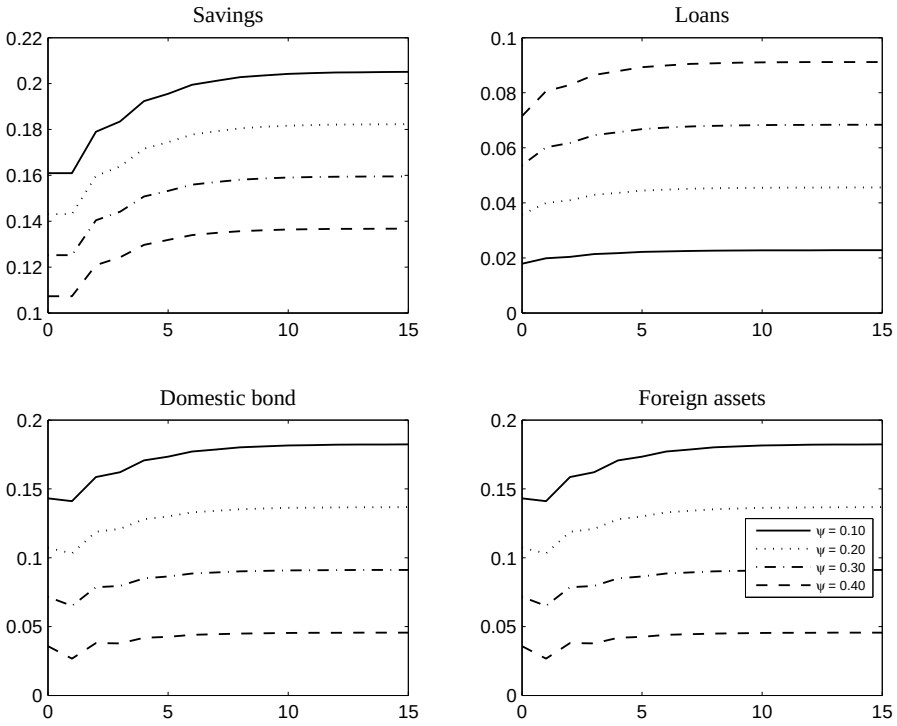


Figure 8: Responses to the credit constraint tightness in an open economy

Semi-open economy vs. open-economy

I now proceed to a numerical analysis of the impulse response functions of the benchmark semi-open economy following a positive productivity shock (parameters in Table 4). The purpose of this exercise is twofold: to show how the central bank reacts to an unanticipated permanent productivity shock on the one hand, and to examine whether the central bank policy is optimal by comparing the welfare gains in a semi-open economy with respect to a decentralized open economy.

The only shock in this economy is a positive productivity shock starting from the beginning of period 1 as the top left panel in Figure 9 illustrates. A positive productivity shock will immediately strengthens the demand for liquidity in the economy. This is because the productivity shock raises the marginal product of capital and thus incites investment in capital while the economy is financially constrained. The central bank provides extra saving instruments by issuing more domestic central bank bonds (bottom left panel) and raises domestic interest rate (top right panel) at period 1. An increased interest rate leads to a higher level of domestic savings (middle left panel) with which constrained firms can invest in physical capital. The impact of such a policy on loans is mixed: it results immediately in a decrease in domestic loans (middle right panel) because of a higher repayment rate; however, as the domestic capital will increase and it serves as a collateral, the demand for loans will ultimately increase. Notice that to finance domestic bond issuance, the central bank needs to invest in foreign assets, driving up its international reserves holding (bottom right panel). Turning to the analysis of domestic economy, Figure 10 shows that the policy of providing extra domestic bonds financed by international reserves leads to an acceleration of capital formation and output production (upper left and right panels). Moreover, both types of families are better off immediately in terms of their respective consumptions (lower left and right panels).

Most importantly, the same policy is more effective in a semi-open economy, that is, when only the central bank has access to foreign financial market. This is because the central bank effectively controls domestic interest rate (thanks to capital controls). Comparing the simulation in a semi-open economy (red solid line) and in an open economy

(black dashed line) in Figure 9 and 10, the economic transition is obviously much faster in the first case.

Finally, we can observe that the situation stabilizes when the economy reaches a new steady state (after 15 periods of simulation) which is common to the open economy and semi-open economy. This suggests that the central bank's policy of providing domestic liquid bonds and purchasing foreign reserves while controlling capital account can be welfare-improving only during transition. Once the growth rate slows down in these economies, the demand for liquid bonds and for foreign reserves stabilizes.

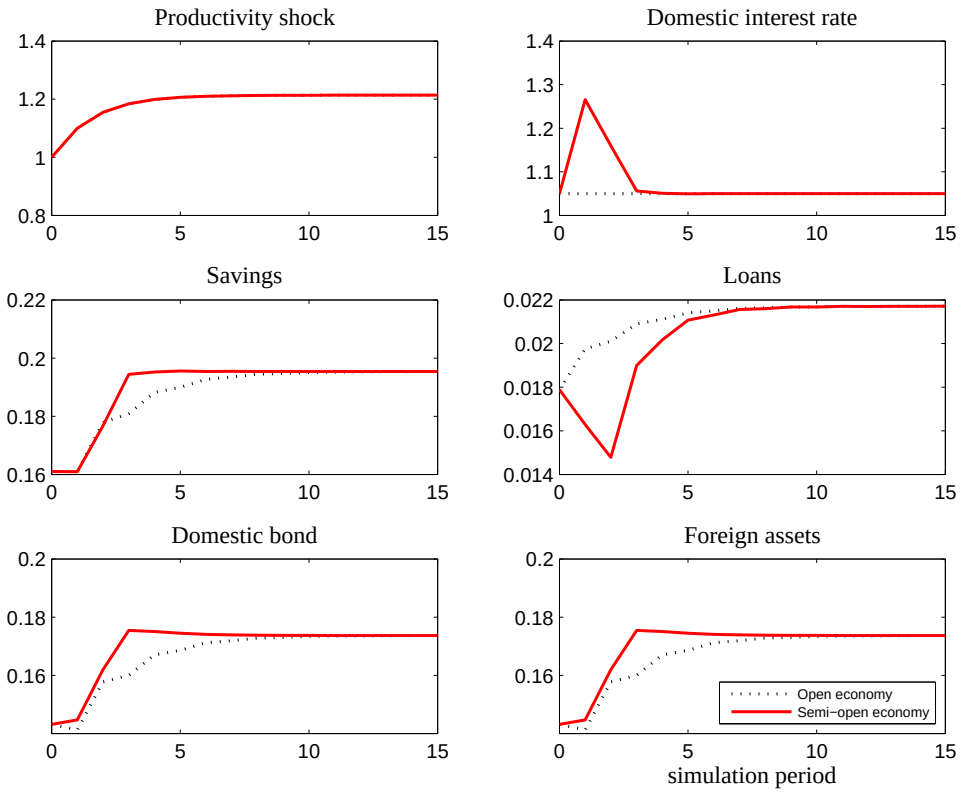


Figure 9: Simulation: shock and policy responses

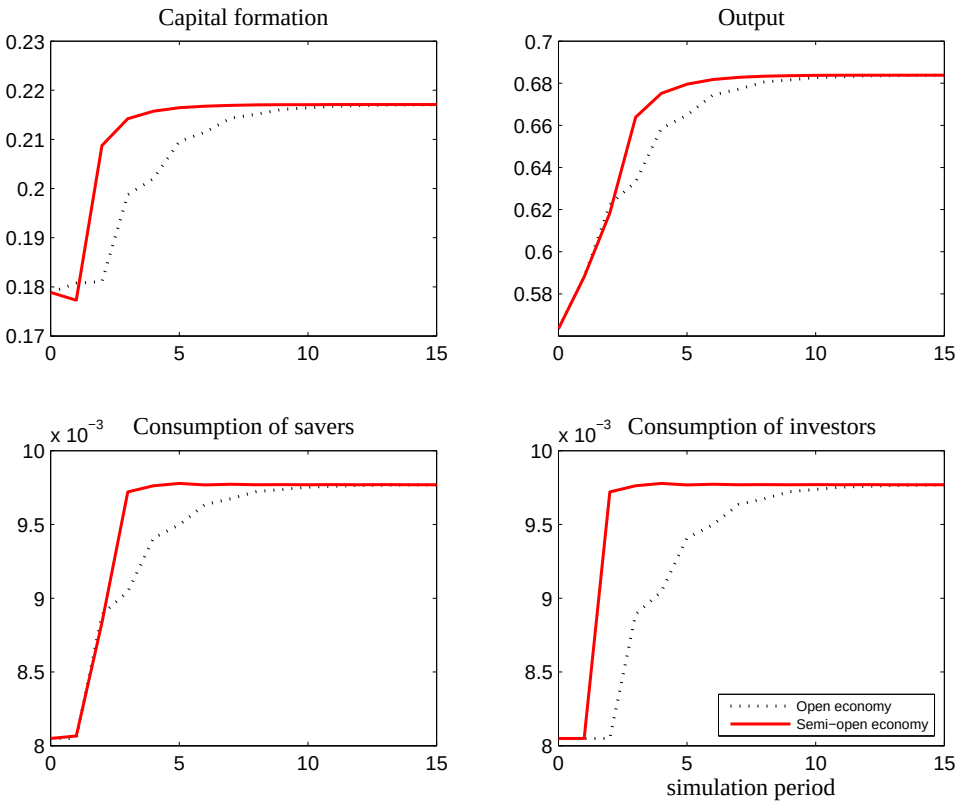


Figure 10: Simulation: welfare effects

5 Observations of foreign reserve accumulation in China

My current model is very much inspired by the fast foreign reserve accumulation in China and some fundamental features of the Chinese economy. In this section, I try to provide some empirical evidence to show how my model is related to the Chinese economy and how Chinese foreign reserve accumulation fits in this model.

5.1 Financial market failure in China

First of all, the model captures several important features of the Chinese economy. This is a fast-growing economy where stringent market frictions, especially on financial market, still exist. China has also accumulated a very large stock of foreign reserves since 2000s; its foreign reserves represent more than a third of the world total reserve assets. Moreover, in addition to foreign reserve accumulation, Chinese authorities control closely its capital account (both on capital inflows and outflows (Jeanne (2012)).

Let's focus on the financial market frictions in China. The financial intermediation in China is inefficient and underdeveloped according to the definition in Section 2.2. First, one can observe a gap between domestic deposits and domestic credit. Using a database on development of financial institutions constructed by Beck et al. (2009), I show in Figure 11 that the bank credit to deposits ratio keeps decreasing since the end 1980s when the Chinese financial market was much more market-oriented. Beck et al. (2009) use this ratio to indicate the inefficiency in financial intermediaries' fund allocation. The decline of bank credits relative to bank deposits also coincides with the period of massive reserve accumulation, as one can see from Figure 11. Following Song et al. (2011), the wedge between bank deposits and bank credit can be largely explained by capital misallocation between state-owned enterprises and private firms. The latter mainly rely on corporate savings for investment. Moreover, this credit misallocation seems to be an institutional constraint which can be hardly removed in the short-run. As Walter and Howie (2012) argue, the preference of granting loans to public enterprises is inherent to the political

regime in China. Many bank loans have been made by administrative orders to public firms for non-economic reasons (e.g. social stability). The lack of uniform accounting standards for small private firms and fiscal fraud also accentuate the difficulties of small firms to get funding from banks.

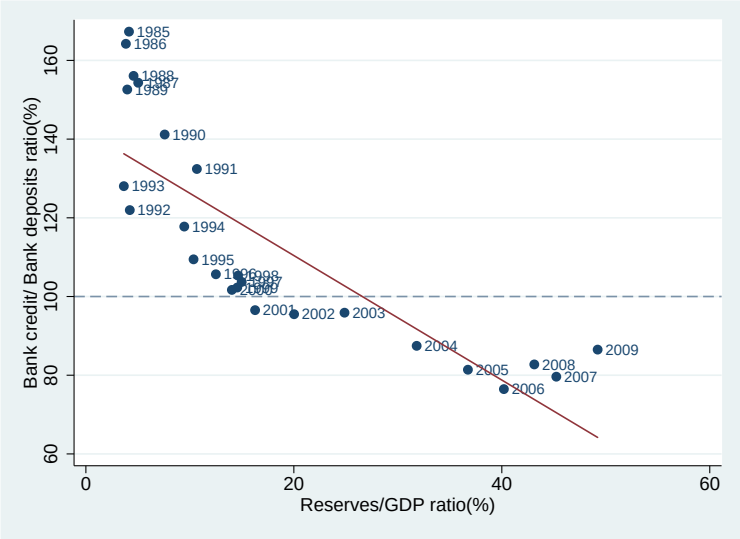


Figure 11: Inefficient fund allocation in China

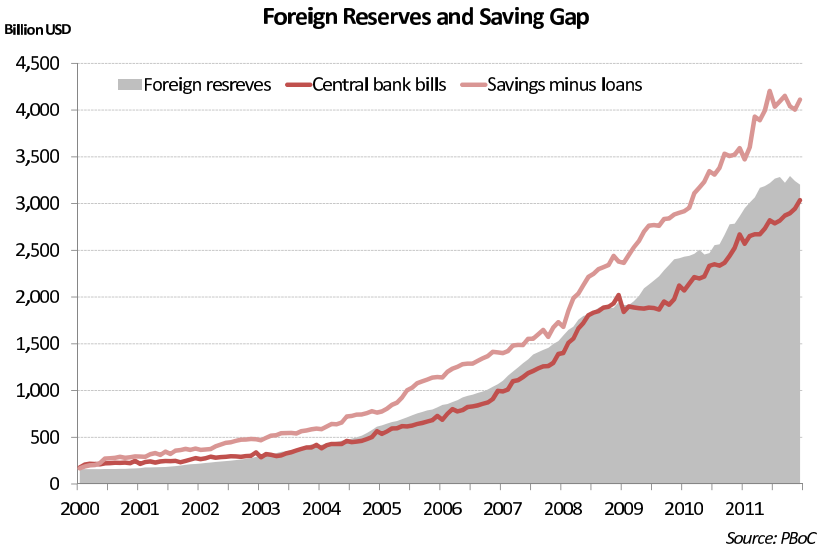


Figure 12: Foreign reserves and saving gap

On the other hand, if we look at the relationship between foreign reserve accumulation and central bank bond issuance, we observe in Figure 12 that they co-move closely together. In fact, as documented by Walter and Howie (2012), domestic private financial market (bond and equity) in China is very limited in scope and thus domestic investment

opportunities are scarce. As a result, if the central bank supplies domestic liquid saving assets, the proceeds cannot be reinvested domestically. According to the same authors, in China 'it is bank lending and [public] bond issuance that keep the engine of China's state-owned economy revving at high speed.' Besides the bank deposits and loans, the private bond and equity markets are very small. For example, in 2007 which was a record year for Chinese equity financing, more than 123 billion of U.S. dollar was raised on the equity market, but in the same year, banks extended new loans totaling 530 billion of dollars and total debt issues in the bond market accounted for another 355 billion (Walter and Howie (2012)). Now, if we look more carefully at the Chinese bond market, we observe that this market is dominated by public obligations, i.e. government debt, central bank bond and policy bank bond issuance. Corporate and commercial papers are very recent financial products and their weight on domestic bond market is negligible (around 5%) compared to the public issuance (Figure 13). Figure 14 shows the decomposition of the Chinese bond market, the proportion of the central bank bonds has steadily increased from 2002 to 2010, a period when Chinese foreign reserves increased rapidly and the government's capacity of issuing new debts was limited [13].

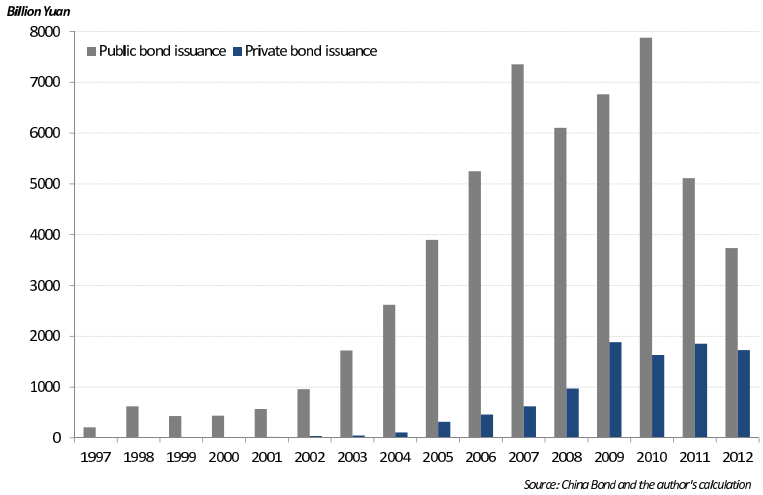


Figure 13: Public bonds vs. private bonds

[13]The government bond issuance is closely regulated by law in China. The Chinese government can only issue bonds for infrastructure construction or reducing the government's deficit.

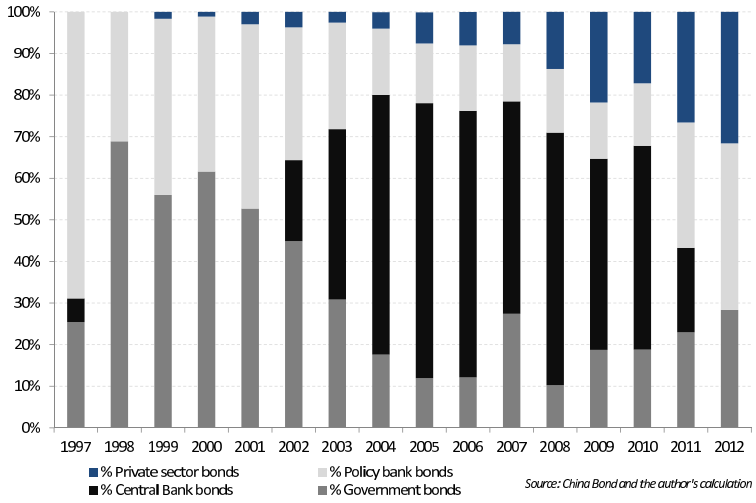


Figure 14: Decomposition of the Chinese bond market

5.2 Chinese foreign reserve accumulation and capital formation

I provide here some simple empirical observations on the bivariate relationship between foreign reserve accumulation and capital formation in China. Capital accumulation is proxied by a macroeconomic variable *gross fixed capital formation* (GFCF).

Figure 15 shows a positive relationship between foreign reserves to GDP ratio and GFCF to GDP ratio in China from 1980 to 2010. Figure 16 illustrates that the year-on-year growth rate of foreign reserves and that of gross fixed capital formation coincide very closely since the late 1990s.

Obviously, these simple results only show correlations between foreign reserve accumulation and domestic capital formation. But they give us some hints about this bivariate relationship which merits further exploration using advanced time-series econometrics.

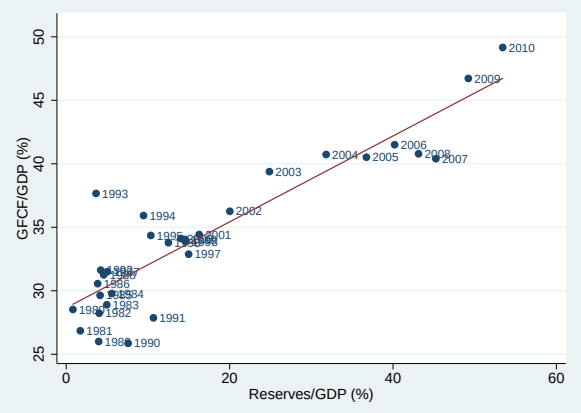


Figure 15: Foreign reserves vs. GFCF (%GDP)

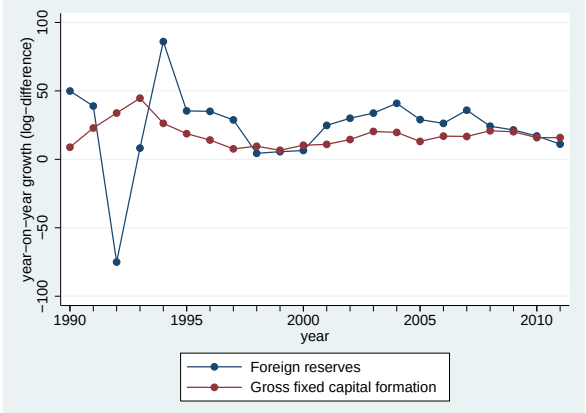


Figure 16: Annual growth rate of foreign reserves and of GFCF

6 Conclusion

Based on the model presented in this paper, I can partly attribute the rapid accumulation of foreign exchange reserves in China (and some other emerging economies) to the joint force of fast economic catch-up and domestic financial market frictions. During transition, the private sector in these countries has a strong demand for liquid assets to support capital investment. In the absence of a sound financial market with sufficient domestic assets, the central bank has to serve as a financial intermediary to provide domestic liquid assets and thus alleviate the financial repression. This raises the private sector's retained earnings, leading to a higher capital investment. The central bank proceeds are in return invested abroad in the form of foreign exchange reserves short of domestic investment opportunities.

This paper also advocates the (temporary) use of capital controls (especially in terms of controls on outflows) in addition to foreign reserve accumulation. It is proved that a combined use of capital controls and foreign reserve accumulation allows the central bank to adjust domestic interest rates and insure that investing firms have sufficient retained earnings to invest in fixed capital, pushing domestic production to its first-best level. This 'policy mix' is optimal in terms of productivity gains during economic transition. Reserves held under this view will diminish when the economy approaches its steady state and when the domestic financial market can provide more financial products.

Notice that this model is based on the assumption of the neoclassical growth model: the growth rate stems from a high capital investment. In the aftermath of the global financial crises, many countries started to rethink of their growth model. In this context, if accumulating foreign reserves and controlling capital flows is useful during economic transition, they should not hinder any structural reforms aiming at strengthening domestic financial market. Indeed, as this paper shows, an open economy can always achieve the first-best situation in steady state. In the long-run, having a sound domestic financial market so that the capital account can be liberalized would be the most desirable choice for these fast-growing emerging economies.

Finally, further theoretical research is needed to incorporate the monetary policy into

analysis; this is important to study the costs related to foreign reserve accumulation (sterilization cost, inflation risks, etc.). The other possibility of extension is to extend the model into a two-country framework. The current model finds it optimal for the domestic economy to accumulate reserves and set capital controls, it might not be the case if cross-country spill-over effects are taken into account. It will be interesting to study the same question in an international context [14].

[14]There are a couple of new papers studying the impact of foreign reserves using a two-country model in the context of bilateral relations between China and the United States (e.g. Bonatti and Fracasso (2013)); but the motives of foreign reserve accumulation that these papers have examined are different from the vision I presented in this paper.

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A Proof of Proposition 1

Proof. In a decentralized economy, the central bank bonds and the government transfer are considered as exogenous. The central bank has to have a balanced budget (5) and the financial market has to be cleared (6). The steady state of the model with a binding credit constraint is determined by the following conditions:

- Central Bank's budget balance
- Financial market clearing

Using (6), (11), the binding credit constraint and normalized policy variable, one can derive:

$$B = S - L \tag{19}$$

$$\Rightarrow B = \beta[F(A,K) - rL] - L \tag{20}$$

$$\Rightarrow B = \beta(1-\psi)F(A,K) - \frac{\psi F(A,K)}{r} \tag{21}$$

$$\Rightarrow b = \beta(1-\psi) - \frac{\psi}{r} \tag{22}$$

The financial market clearing shows a negative linear relationship between b and $\frac{1}{r}$.

Moreover, in the simplest case without any government transfer, using (13) gives,

$$(r-1)b = (r^*-1)b^* \tag{23}$$

$$\Rightarrow b = \frac{1}{r-1}(r^*-1)b^* \tag{24}$$

$$\Rightarrow b = \left(\frac{1}{1-\frac{1}{r}} - 1\right)(r^*-1)b^* \tag{25}$$

The balanced central bank's budget shows a non-linear and positive relationship between b and $\frac{1}{r}$.

The uniqueness of the steady state can be thus demonstrated graphically (Figure 17).

Figure 17 shows that there may be two solutions of $\frac{1}{r}$, corresponding respectively to $\frac{1}{r} < 1$ (equivalent to $b > 0$) and $\frac{1}{r} > 1$ (equivalent to $b < 0$). I focus here on the first case, as r being a gross interest rate is greater than 1. This equilibrium corresponds to $b > 0$ (central bank issues bonds). The interest rate is uniquely determined, $\beta < \frac{1}{r} < \min(1, \frac{\beta(1-\psi)}{\psi})$. For $\psi \leq \frac{1}{2}$, $\frac{1}{\beta}\frac{\psi}{1-\psi} < r < \frac{1}{\beta}$.

Therefore, when $0 < b < \beta(1-2\psi)$, the economy has a unique binding steady state, provided that $\psi \leq \frac{1}{2}$ (I will show in Appendix B that $\psi \leq \frac{1}{2}$ is the condition for the credit constraint to bind.). ■

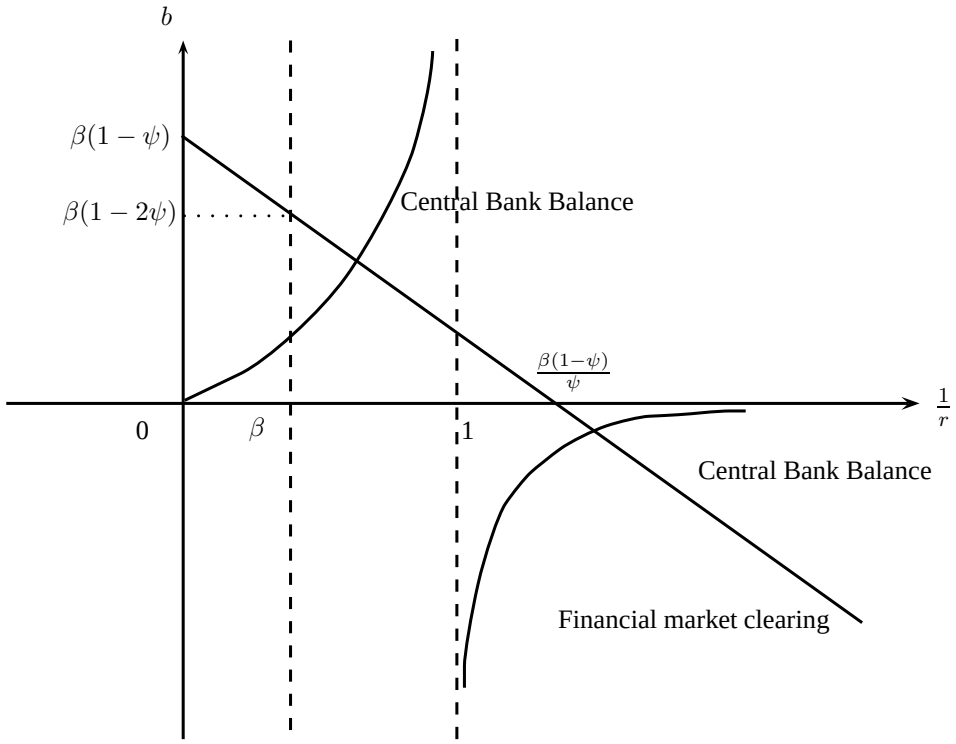

Figure 17: Uniqueness of binding steady state

B Deriving the critical value of ψ

Proof. In a steady state when the credit constraint is not binding, $S = L$ and $\beta r = 1$.

From (11) and (12):

$$S = \beta[F(A,K) - rL]$$
$$\Rightarrow S = L = \frac{1}{2}\beta F(A,K)$$

When the credit constraint is not binding, $rL < \psi F(A,K)$, namely $\frac{1}{2}r\beta F(A,K) < \psi F(A,K)$.

Therefore, in a steady state with an unbinding credit constraint, I obtain:

$$\psi > \frac{1}{2}$$

Accordingly, the steady state is binding if $\psi \leq \frac{1}{2}$ ■

C Proof of Proposition 2

Proof. In a constrained state steady, (9) and (10) generate:

$$1 + \lambda = \frac{1}{\beta^2 r^2} \tag{26}$$

Using this result, along with the first order condition of capital (8), gives Equation (16):

$$F_K = r\frac{1+\lambda}{1+\psi\lambda}$$
$$= \frac{r}{(\beta r)^2 + \psi[1-(\beta r)^2]}$$

Graphically (Figure 5), K first decreases in βr then increases to go back to the first-best level. ■

D Details on the Ramsey program

I derive first order conditions with respect to all endogenous variables from the Ramsey program described in Section 4.1.

$$FOC(S_{t+1}) : \eta_t^S = \beta r_{t+1} \eta_{t+1}^I \tag{27}$$

$$FOC(L_{t+1}) : \eta_t^I = \beta r_{t+1} \left(\eta_{t+1}^S + \Lambda_{t+1}\right) \tag{28}$$

$$FOC(B_{t+1}^*) : \eta_t^G = \beta r_{t+1}^* \eta_{t+1}^G \tag{29}$$

$$FOC(K_{t+1}) : \beta \eta_{t+1}^S F_{K,t+1} - \eta_t^I - \eta_t^G + \beta \eta_{t+1}^G F_{K,t+1} + \beta \psi \Lambda_{t+1} F_{K,t+1} + \beta \rho_{t+1} (\alpha - 1) \frac{F_{K,t+1}}{K_{t+1}} (1 + \psi\lambda) = 0 \tag{30}$$

$$FOC(c_t^S) : \frac{1}{c_t^S} - \eta_t^S - \eta_t^G - \frac{\theta_t^S}{(c_t^S)^2} + \frac{\theta_{t-1}^I r_t (1 + \lambda_t)}{(c_t^S)^2} = 0 \tag{31}$$

$$FOC(c_t^I) : \frac{1}{c_t^I} - \eta_t^I - \eta_t^G - \frac{\theta_t^I}{(c_t^I)^2} + \frac{\theta_{t-1}^S r_t}{(c_t^I)^2} = 0 \tag{32}$$

$$FOC(r_{t+1}) : \beta \eta_{t+1}^I S_{t+1} - \beta \left(\eta_{t+1}^S + \Lambda_{t+1}\right) L_{t+1} - \beta \frac{\theta_t^S}{c_{t+1}^I} - \beta \theta_t^I \frac{(1 + \lambda_{t+1})}{c_{t+1}^S} - \beta \rho_{t+1} (1 + \lambda_{t+1}) = 0 \tag{33}$$

$$FOC(\lambda_{t+1}) : \frac{-\theta_t^I \beta r_{t+1}}{c_{t+1}^S} + \beta \rho_{t+1} (\psi F_{k,t+1} - r_{t+1}) = 0 \tag{34}$$

Notice that in a financial autarky, Equation (29) is not relevant as $B^* = 0$ and in an open economy, Equation (33) is not relevant as the domestic interest rate is equal to the world interest rate $r^* = \frac{1}{\beta}$.

E Proof of Proposition 4

Proof. In the open economy steady state, (29) gives $r^* = \frac{1}{\beta}$, leading to $r = r^* = \frac{1}{\beta}$, namely $\beta r = \beta r^* = 1$.

Ramsey first order conditions (27) and (28) give $\eta^S = \eta^I$ and $\Lambda = 0$.

The definition of the Ramsey problem requires that the private first order conditions (8) to (10) are fulfilled, namely $c^S = c^I$ and $\lambda = 0$.

Substitute these results into (31) and (32), one obtains $\theta^S = \theta^I = 0$.

Therefore, the steady state is not credit-constrained. All the savings of the saving family go to the investing family in the form of private loans.

In the long run, the economy is not financially constrained in an open economy with the intervention of the central bank.

■

F Deriving Equation (18)

Proof. As long as the Ramsey first order condition on the credit constraint (34) is verified, $\theta_t^I = 0$, as $F_{K,t+1} = r_{t+1}$ in an open economy.

In a fully open economy, $r\beta = r^*\beta = 1$ and $u'(c_t^S) = u'(c_{t+1}^I)$. From (27), (28) and (29) one can derive:

$$\eta_t^S = \eta_{t+1}^I$$
$$\eta_t^I = \eta_{t+1}^S + \Lambda_{t+1}$$
$$\eta_t^G = \eta_{t+1}^G$$

From (31) valued at t and (32) valued at $t+1$:

$$\theta_t^S u''(c_t^S)(1 + r^*) = 0$$

As $u''(c_t^S)(1 + r^*) < 0$, $\theta_t^S = 0$.

Substitute $\theta_t^S = \theta_t^I = 0$ in (33) and solve it forward, one obtains Equation (18).

■

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