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Which size and evolution of the government expenditure multiplier in France (1980-2010)?

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Résumé

L'importance des plans de relance budgétaire qui ont été mis en oeuvre dans la plupart des économies avancées depuis le déclenchement de la crise financière et la vitesse à laquelle les budgets des États sont maintenant consolidés en Europe a donné une nouvelle actualité au débat sur le multiplicateur de dépenses publiques. Cette étude est centrée sur l'effet des dépenses publiques en biens et services. En utilisant la procédure d'identification des chocs de dépenses publiques proposée par Blanchard et Perotti (2002), nous parvenons à la conclusion que le multiplicateur est statistiquement significatif et peu éloigné de 1 à l'impact et qu'il devient non significatif après environ 3 ans en France. Nous effectuons de nombreux tests de robustesse liés à la définition des dépenses publiques, aux hypothèses de stationnarité des variables considérées, à l'effet des anticipations et au choix de l'échantillon de données. Par ailleurs, en estimant un modèle VAR structurel à coefficients variables dans le temps, nous trouvons (1) que le multiplicateur n'a pas significativement évolué, quel que soit l'horizon temporel considéré, depuis le début des années 1980 et (2) que la variance des chocs affectant l'économie évolue bien davantage que les coefficients autorégressifs du modèle. Même en ayant recours à des spécifications alternatives où les priors bayésiens favorisent l'évolution des coefficients dans le temps, l'évolution principale que nous mettons en évidence est une réduction (non significative) du multiplicateur de moyen terme, en lien avec une politique monétaire plus réactive depuis les années 1990. Nous ne trouvons pas d'indice d'une augmentation systématique du multiplicateur de dépenses publiques lors de chaque récession en France, contrairement à la conclusion d'Auerbach et Gorodnichenko (2012) pour les États-Unis. En tout cas, la position dans le cycle économique ne semble pas être le facteur principal ayant influencé le multiplicateur de dépenses publiques en France au cours des 30 dernières années.

Mots clés : Multiplicateur de dépenses publiques, Évolution, TV-SVAR.
Codes JEL : E62, C54.

Abstract

The importance of the stimulus packages that were injected in most advanced economies from the start of the financial crisis and the speed at which budgets are now being consolidated in Europe has revived the long-lasting debate on the size of fiscal multipliers. In this study, we focus on government expenditures on goods and services. Our conclusion following Blanchard and Perotti (2002) for the identification of government spending shocks is that the multiplier is significant and not far from 1 on impact and becomes statistically insignificant after about 3 years in France. We provide numerous robustness checks concerning the definition of expenditures, assumptions about data stationarity, the role of expectations and the choice of the sample. Moreover, using a time-varying SVAR model, our main findings are (1) that the multiplier did not evolve significantly at any horizon since the beginning of the 1980s and (2) that the variance of shocks hitting the economy evolves a lot more than the model autoregressive parameters. Even in alternative specifications where the Bayesian priors are pushed towards time-variation, the main evolution that we uncover is a (non significant) decrease of the medium term expenditure multiplier, partly linked to a more aggressive monetary policy since the 1990s. We do not find evidence of an increase of the multiplier during every recession in France, contrary to the finding of Auerbach and Gorodnichenko (2012) for the United States. At least, business cycle conditions do not seem to be the main driver of the evolution of the expenditure multiplier in the last 30 years in France.

Keywords: Government expenditure multiplier, Evolution, TV-SVAR.

JEL codes: E62, C54.

1 Introduction

The importance of the stimulus packages that were injected in most advanced economies from the start of the financial crisis and the speed at which budgets are now being consolidated in Europe has revived the long-lasting debate on the size of fiscal multipliers. There is no clear theoretical answer to this question. These multipliers may depend, among other factors, on the expenditures, transfers and taxes that are being considered, on the degree of openness of the economy, and on the reaction of monetary policy. For these reasons, these multipliers may not be unique and may change over time.

This paper focuses on the evolution of the expenditure multiplier in France during the last 30 years. Our definition of expenditures is more restrictive than the sum of government consumption and investment computed in national accounts. It only corresponds to the purchase of goods and services by the government and does not include the compensation of civil servants (see Appendix B). Although the statistical model developed in this paper entails a measure of government net receipts, the focus on expenditures is motivated by two reasons. First, government net receipts include very different kinds of taxes and their evolution may be explained by modifications of marginal tax rates or of tax bases, whose effect on economic activity could be very different. Second, a shock on government expenditures is easier to identify than a shock on government receipts, especially in a time-varying context (see *infra*).

Very few papers try to assess if the size of fiscal multipliers has evolved over time. Our work is closely related to Kirchner et al. (2010) and to Auerbach and Gorodnichenko (2012) which are the two main contributions in this field. Kirchner et al. (2010) estimate a time-varying structural VAR (TV-SVAR) on Euro-Area data starting in 1980Q1 whereas Auerbach and Gorodnichenko (2012) estimate a regime-switching VAR model on U.S. post-WWII quarterly data[1]. Both of these studies also focus on the evolution of the expenditure multiplier.

Kirchner et al. (2010) conclude that the short run effectiveness of government spending in stimulating GDP in the Euro-Area (as a whole) increased until the end-1980s and continuously decreased afterwards. Their impact spending multiplier has a value of 0.7 in 1980 and 0.5 in 2008, with an intermediary value slightly above 1 at the end of the 1980s. Their long run spending multiplier continuously decreases from -0.7 to -1.7: this continuous evolution in the long run is the main evolution that they uncover. At this stage, we can already notice that the multipliers computed by Kirchner et al. (2010) reach their lowest value at the beginning of the Great Recession. However, they do not

[1] Auerbach and Gorodnichenko (2012) impose that the regimes are linked to the state of the business cycle (expansion or recession)

report the shape of the probability distribution around their median estimates.

Auerbach and Gorodnichenko (2012) report that the spending multiplier in the United States is significantly higher during recessions (2.2 after 5 years) than it is during expansions (-0.3 after 5 years). However, this result is driven by defense expenditures only whose share in total government expenditures is much higher in the United Stated than in France[2]. The non-defense spending multipliers estimated by Auerbach et al. (2012) are not significantly different during recessions and expansions[3] [4]. Moreover, using a longer sample (1890q1-2010q4) and relying on a different econometric methodology (local projections rather than VARs), Owyang et al. (2013) do not observe higher multipliers during times of slack in the United States. Given that VARs cannot be considered as always superior to local projections from a theoretical point of view (Jorda (2005)), there seems to be room for further econometric studies on the evolution of governement spending multipliers.

There are arguments in favor of a continuous evolution of the size of the multiplier, independently of business cycle conditions. For instance, the French economy has become more open since the beginning of the 1980s and this could have led to a growing leakage effect, an argument often heard in the French public debate on the effectiveness of a fiscal stimulus. Moreover, the monetary policy reaction to a fiscal stimulus might have changed. Disinflation became a priority of the French central bank at the beginning of the 1990s, in order for France to qualify for the euro; the French central bank became independent in 1993 and its prerogatives were partly transferred to the European Central Bank (ECB) in 1999. Finally, opposite effects may have influenced the size of the multiplier during the Great Recession: on the one hand, the zero lower bound may have muted the response of monetary policy and pushed multipliers above one (Christiano et al. 2011, Woodford 2011) but on the other hand, the rapid increase of French public debt may have led consumers to behave in a more Ricardian way (Sutherland 1997). Hence, we do not

[2]In the United States, defense expenditures represent 25% of government consumption and investment from 1995 to 2010 and 35% from 1960 to 1994 (NIPA, Table 3.15.5). The breakdown of government expenditures by function is only available since 1995 in France. From 1995 to 2010, defense expenditures represent only 6% of all government expenditures in France. This share is roughly the same for the compensation of civil servants (D1) and for the expenditures on goods and services (P2+P51+D631A).

[3]When they focus on non-defense spending, Auerbach et al. (2012) compute a multiplier after 5 years equal to 1.0 during expansions and to 1.1 during recessions.

[4]For France, Bouthevillain and Dufrénot (2011) adopt a similar approach and find that the short run elasticity of output growth to government spending is 13 percentage points higher during recessions than they are during expansions, but this difference is not significant. Using a Neo-Keynesian macroeconometric model with time-varying hysteresis effects, Creel et al. (2011) find for France that the short run expenditure multiplier is slightly above one whatever the cycle position, whereas the long run multiplier would stay above one for a shock impulsed at a trough and would fall close to zero for a shock impulsed at a peak.

impose *a priori*, like Auerbach and Gorodnichenko (2012), that the value of multipliers should only depend on whether the economy is in a recessionary or in an expansionary state.

Since time-variation could come from multiple sources, a flexible non-linear model is the appropriate tool to use. Here, we rely on a time-varying structural VAR (TV-SVAR) model (see Appendix D). The same kind of model has been used by Cogley and Sargent (2001), Cogley and Sargent (2005) and Primiceri (2005) to assess the evolution of monetary policy in the United States and its impact on the economy. All coefficients of the model, including those of the variance-covariance matrix, are left free to vary over time. This means that the size of the policy shocks identified by the model, as well as their contemporaneous and lagged impacts on the economy, are time-dependent. Since this model contains a lot of parameters, it is estimated using Bayesian techniques. Indeed, the likelihood function in highly-parameterized models tends to be very complicated. It typically contains many narrow peaks, possibly in regions where parameter values are incredible, so that it is difficult for a maximization algorithm to discriminate between them. The use of Bayesian priors allows focusing particularly on certain regions of the likelihood function.

We rely on a quarterly VAR with five variables (real government expenditures, real government net receipts[5], real GDP, GDP deflator and nominal 3-month interest rate). The first three variables are those proposed by Blanchard and Perotti (2002). Like (Perotti 2002), we add the other two variables (GDP deflator and short term interest rate), in order to take into account the feedback of prices and monetary policy. For the identification of the structural shocks, we assume that government spending does not react to activity nor to net receipts or interest rates within a quarter. This assumption is motivated by the fact that there is no evidence of an automatic response of government spending to business cycle conditions and that any discretionary response is necessarily delayed due to the existence of decision and implementation lags[6]. Finally, we assume that government spending is fixed in nominal terms, so that the volume of expenditures reacts negatively, with a unitary elasticity, to unexpected inflation within a quarter. This is the only difference with Blanchard and Perotti (2002), motivated by the fact that we include prices and interest rates in our model[7].

[5]Government net receipts are defined as the sum of government financing capacity and government expenditures (see Appendix B). Hence, for instance, unemployment benefits are treated as a negative receipt.

[6]Net receipts can react contemporaneously to business cycle conditions due to the existence of automatic stabilizers but the same lags explain why they cannot react to business cycle conditions in a discretionary manner within a quarter. However, these considerations would only be important if we tried to identify structural shocks on government net receipts (see Appendix C).

[7]This identification scheme has already been implemented on French data using a fixed coefficients

Two main objections have been raised in the literature against the use of aggregate data and SVAR models in order to assess the effectiveness of fiscal policy.

First of all, the effect of government spending may vary from one kind of expenditures to another. For instance, government consumption as measured by national accounts is composed of both direct purchases of goods and services and compensation of civil servants, two expenditures that do not affect the rest of the economy in the same way. Some goods and services, if they generate externalities (e.g.: education or research expenditures) may also have an indirect impact on the utility of consumers or on the production function of firms. This is why some authors favor the use of defense expenditures only (Barro and Redlick 2009). Another advantage generally put forth to support the use of defense expenditures is the fact that they depend on geostrategic considerations rather than on business cycle conditions. Thus, they are more easily considered as exogenous.

However, this particular choice of data has two drawbacks. First, it is not sure that results obtained with defense expenditures can be extended to non-defense spending. Second, these results, on U.S. data, are mainly driven by what happened during WWII or the Korean war (Hall 2009) and are not necessarily relevant in the present context (e.g.: goods rationing and capacity constraints during wars, cf. Perotti (2011)). Moreover, a multivariate model can effectively address the issue of endogeneity, contrary to the univariate regressions used by Barro and Redlick (2009). The only identifying assumption we need is to suppose that the value of government spending cannot adjust to business conditions within a quarter, which seems reasonable. Finally, we distinguish direct purchases of goods and services from compensation of civil servants, hence limiting the heterogeneity issue.

The second objection raised against the use of SVAR models in the context of fiscal policy concerns the identification scheme and the treatment of expectations. Ramey (2011b) shows that government spending shocks identified following a Blanchard and Perotti procedure on U.S. data are Granger-caused by forecasts based on exogenous information. She relies on the median forecast from the Survey of Professional Forecasters (SPF) available since 1969 or on a military spending news variable constructed from press releases and available since 1939[8]. In other words, innovations computed by an econome-

SVAR model in a study by Biau and Girard (2005). Their results are compatible with ours although they define government spending as the sum of public consumption and public investment and their estimation sample ends up in 2003. They compute that a government spending shock has a positive short term effect on GDP, with an impact multiplier of 1.4, and a statistically non significant effect in the medium term.

[8]The defense news variable is based on episodes where *Business Week* began to forecast large rises in defense spending: the Ramey-Shapiro variable identifies three major episodes (the Korean war, the Vietnam war and the Carter-Reagan buildup); another richer variable contains, for 31 dates, the magnitude of increase in spending for the next years.

trician who estimates a SVAR model *à la* Blanchard and Perotti are not innovations for economic agents because their information set is larger than the set of past and present values of the variables included in the model. In this case, estimated parameters and impulse-response functions (IRF) computed using them are inconsistent.

In order to address this issue, Ramey (2011b) embeds one of her news variable into a standard SVAR used for the analysis of fiscal policy. This news variable is ordered first in the model, just before government spending, and a Cholesky identification scheme is used. In practice, this identification scheme amounts to regress reduced-form residuals from the government spending equation on residuals from the news equation in order to identify unanticipated government spending shocks. When she compares responses to a usual government spending shock (Blanchard and Perotti 2002, Perotti 2007) and to an unanticipated government spending shock, Ramey (2011b) identifies two main differences. Private consumption increases and private investment decreases on impact in the first case but the opposite result holds in the second case. A recent controversy (Ramey 2011a, Perotti 2011) highlighted that these results were sensitive to the inclusion of particular observations[9] and that IRFs for private consumption and GDP in the two cases were probably not significantly different from each other (see Figure 1 in Ramey (2011a)). Using European data and relying on forecasts from the European Commission, Beetsma and Giuliodori (2011) do not find significant evidence of an anticipation effect either.

Although the empirical relevance of anticipation effects remains controversial, controlling for these effects provides a good robustness check. A first way to deal with anticipation effects, advocated by Sims (2009) and by Forni and Gambetti (2010), is to include forward-looking variables, like short term interest rates, consumer prices and stock prices, in the model. In our paper, short term interest rates and prices (GDP deflator) are actually included in the model. Following Ramey's methodology, we have also assembled a database of government investment forecasts made by the forecasting department of the French statistical institute (INSEE)[10]. We use it in order to control for expectations not already absorbed by the VAR model. However, its impact on the estimation of the government investment multiplier is only marginal.

Our conclusion using a TV-SVAR model and following Blanchard and Perotti (2002) for the identification of government expenditure shocks is that the size of the expenditure multiplier in France did not evolve statistically significantly at any horizon since the beginning of the 1980s. The impact multiplier is significant and not far from 1 whereas the medium term multiplier is not significantly different from 0. The variance of shocks

[9]For example, Perotti (2011) shows that the results of Ramey (2011b) are reversed by dummying out two quarters (1950Q4 and 1951Q1) following a wave of panic buying and corresponding to the introduction of specific regulations by the Federal Reserve.

[10]The same could not be done for governement consumption of goods and services.

hitting the economy evolves a lot more than the model autoregressive parameters. The same kind of conclusion has also been reached by Primiceri (2005) with a TV-SVAR focusing on monetary policy in the United States[11].

Even in alternative specifications where the Bayesian priors are pushed towards time-variation, the main evolution that we uncover is a (non significant) decrease of the medium term expenditure multiplier, partly linked to a more aggressive monetary policy since the 1990s. We do not find evidence of an increase of the multiplier during every recession in France, contrary to the finding of Auerbach and Gorodnichenko (2012) for the United States. At least, business cycle conditions do not seem to be the main driver of the evolution of the expenditure multiplier in the last 30 years in France. One possible explanation could be that the unemployment rate, generally considered as an indicator of slack, remained high in France since the middle of the 1980s[12]. But if one thinks that the last 30 years of data only enable measuring the government spending multiplier in bad times for France, the practical relevance of conclusions reached on U.S. data regarding the evolution of the expenditure multiplier seems to be limited for the French economy.

[11]His main findings are the following: 1. Time-variation is mostly located in the variance of shocks hitting the economy: shocks to inflation and unemployment but also discretionary monetary policy shocks; 2. Impulse response functions of inflation and unemployment to a monetary policy shock are hardly modified since the beginning of the 1970s; 3. Even if the systematic part of monetary policy (i.e.: the parameters of the Taylor rule) displays some time-variation, this evolution does not explain at all why inflation rose and fell in the United States in the 1970s and 1980s.

[12]France is not an isolated case: Owyang et al. (2013) who estimate expenditure multipliers in good and bad times on U.S. and Canadian data consider that Canada was characterized by a very long period of slack from 1975 to 2005. This is due to the steadily high Canadian unemployment rate - over 7% - on this period.

2 Constant parameter OLS estimates

2.1 Benchmark specification

The model that we consider is a quarterly VAR with 5 variables and l lags[13]. This model can be written as:

$$\begin{aligned} y_t &= \begin{pmatrix} I_5 & I_5 \otimes y'_{t-1} & ... & I_5 \otimes y'_{t-l} \end{pmatrix} \cdot \beta + A_{idtf}^{-1} A^{-1} \Sigma \cdot \varepsilon_t \\ &= Z_t \cdot \beta + A_{idtf}^{-1} A^{-1} \Sigma \cdot \varepsilon_t \end{aligned}$$

where $\varepsilon_t \sim NID(0,1)$ are structural innovations and the $(25l+5) \times 1$ vector β contains coefficients of constants and lags.

The five variables of the VAR are government expenditures, government net receipts, GDP, GDP deflator and the 3-month interest rate, ordered in this way. The first 3 variables are expressed in real terms, deflated by the GDP deflator[14]. Depending on the case, the first 4 variables may be considered in log-levels or in log-differences. The 3-month interest rate is always expressed in percentage points.

A is a lower triangular matrix and the A_{idtf} matrix implements the identification scheme. We assume that government spending is fixed in nominal terms, so that the volume of expenditures reacts negatively, with a unitary elasticity, to unexpected inflation within a quarter. We also assume that government spending does not react to any other shock within a quarter. Perotti (2002) makes similar assumptions. So, the first line of the A_{idtf} matrix is (1 0 0 1 0). No further assumption regarding the identification of other shocks has to be made in order to identify contemporaneous and lagged effects of government spending shocks (see Appendix C).

$$A_{idtf} = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

[13]In practice, AIC and BIC criteria indicate that the optimal number of lags is never higher than 2 for any of the constant parameter specifications considered in the following. Hence, we will always choose $l = 2$.

[14]Using the GDP deflator to express these variables in real term allows us to compute the impulse responses as shares of GDP. Blanchard and Perotti (2002), as well as Auerbach and Gorodnichenko (2012), made the same choice.

Here and in the remainder of this paper, the government spending multiplier M_t following a shock equal to 1[15] at date *1* is defined, at date t, as a ratio between the cumulated increase in GDP from date *1* to date t and the cumulated increase in government spending from date *1* to date t:

$$M_t = \frac{\sum_{i=1}^{t} Y_i}{\sum_{i=1}^{t} G_i} = \frac{\sum_{i=1}^{t}\sum_{j=1}^{i} \Delta Y_j}{\sum_{i=1}^{t}\sum_{j=1}^{i} \Delta G_j}$$

This definition is also used by Auerbach and Gorodnichenko (2012) but they only report the value of the multiplier at a 5-year horizon (i.e.: M_{20}) in their paper.

In the benchmark case, we estimate this 5-variable SVAR model with the following specification: we log-difference the first 4 variables and filter out low frequencies of all variables (specification 3 below), we exclude the compensation of civil servants from government expenditures, we assume that government expenditures shocks are unexpected and VAR parameters are constant over the whole sample (1980Q1-2010Q4).

We will explain these assumptions and check their impact on our results in the following sections. With this specification, the point-estimate of the government multiplier is equal to 1.1 on impact and to 0.5 after 20 quarters (figure 1). The impact multiplier is significantly positive, but it is not the case of the medium run one. Appendix G shows that the medium run expenditure multiplier remains significantly positive (with a point-estimate above 1) in a SVAR model without prices or interest rates as endogenous variables, while it reverts to zero with our 5-variable VAR. This medium run behavior of the 3-variable SVAR model is due to the omission of a feedback loop. For various OECD countries over the period 1980-2000, Perotti (2002) also finds that 3-variable models tend to deliver larger government spending multipliers than 5-variable ones in the medium run.

[15]The unit of measurement is the percentage point of GDP. We convert increases in G into percentage points of GDP using the average share of public expenditures in GDP over the sample (about 14%).

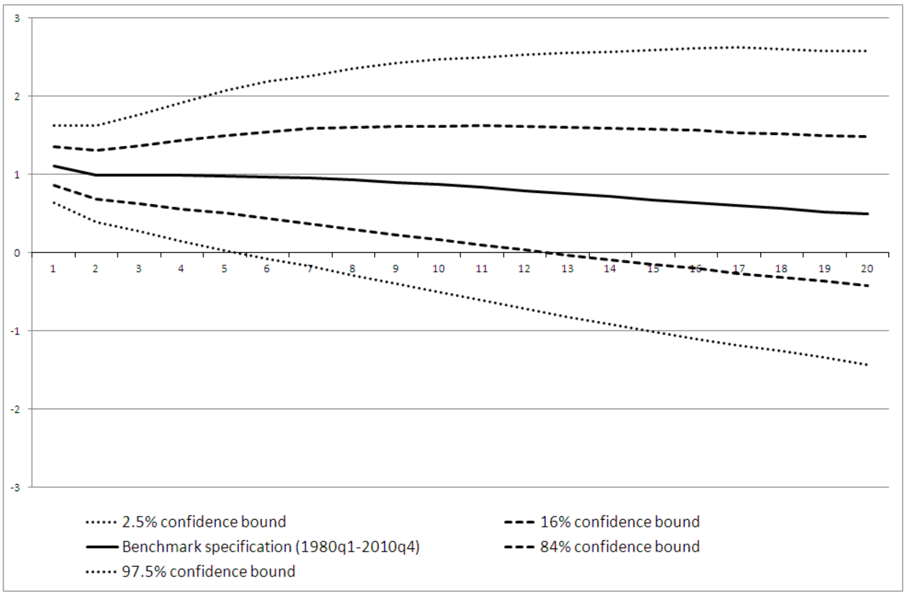

Figure 1: Government expenditure multiplier for the benchmark specification (specification 3, see below). Model with 2 lags. IRFs, 68% and 95% confidence intervals are computed using Kilian's (1998) bootstrap-after-bootstrap.

2.2 Data handling: to difference or not, to detrend or not

2.2.1 Data persistence

Auerbach and Gorodnichenko (2012) estimate the equations of their model with (U.S.) data in log-levels, without differencing or filtering them although they are nonstationary or, at least, very persistent[16]. They consider it as a simple way to preserve possible cointegrating relationships among the variables. They acknowledge that an alternative, but more difficult, way of handling data would have been to estimate the equations in log differences and to include error correction terms. In fact, there is a long tradition of estimating SVAR models, especially those measuring the effects of monetary policy, with all data in log-levels[17].

Another reason could justify the practice of estimating VAR models in log-levels. Even though there is a spurious regression problem when one regresses a nonstationary variable on another independent nonstationary variable, leading to a regression coefficient that converges to a stochastic variable rather than to 0, the same problem does not necessarily occur in VAR models. Sims et al. (1990) show that some linear combinations of the VAR coefficients have the usual asymptotic distribution: standard theory applies if coefficients of interest can be rewritten as coefficients on stationary, mean zero, variables. A practical consequence of this result, when the vector of autoregressive coefficients in equation i at lag l is noted ϕ_{il}, is that $\sqrt{T}\left(\hat{\Phi}_{il} - \Phi_{il}\right)$ is asymptotically Gaussian for each p as long as the number of lags in the model is sufficient. The <u>asymptotic</u> distribution of the VAR coefficients does not have any bias in this case.

However, small sample results, with sample sizes usually available in macroeconomics, may be noticeably different from what these asymptotic results suggest. For instance, the estimation of autoregressive coefficients in (V)AR models when data are very persistent is affected by finite sample bias. Kilian (1998) proposes a way to deal with this issue for the computation of IRFs. In the following, whenever we do not rely on Bayesian

[16]The variables considered by Auerbach and Gorodnichenko (2012) are real government purchases (consumption + investment), real government net receipts and real GDP.

[17]Here we name just a few prominent examples in this field. Sims (1992) estimates a 6-variable VAR model for different countries including a short term interest rate, a monetary aggregate, a consumer price index, an industrial production index, an index of the foreign exchange value of domestic currency and a commodity price index. All variables but the interest rate enter the model as log-levels while this variable is entered as a percent. Bernanke and Gertler (1995) estimate a 4-variable VAR model including the log of real GDP, the log of the GDP deflator, the log of an index of commodity prices and the federal funds rate in percentage points. Finally, Eichenbaum and Evans (1995) estimate a 5-variable VAR model that includes the U.S. industrial production index, the U.S. consumer price index, the ratio of non-borrowed to total reserves, a measure of the difference between U.S. and foreign short term interest rates and the real exchange rate. All variables are in log-levels except the interest rates.

estimation techniques, we always adopt his bootstrap-after-bootstrap method to compute IRFs and the corresponding confidence intervals. Concerning IRFs, Kilian and Chang (2000) show that confidence bands may have poor coverage properties in small samples in the presence of very persistent variables, even if standard methods of inference are justified asymptotically. In the following, we follow their recommendation to discount interval estimates for higher horizons and report IRFs at a horizon of only 20 quarters (the minimum required to compare our results with those of Auerbach and Gorodnichenko 2012).

2.2.2 Low-frequency evolutions: the case of the 1980s in France

French data on the last 30 years are characterized by low-frequency evolutions that are difficult to explain using only the 5 endogenous variables of the model. For instance, the inflation rate (log-difference of the GDP deflator) continuously decreased during the 1980s (see figure 2). The second oil shock in 1979 and the counter-oil shock in 1985, but also decisions taken by the French government to achieve disinflation are exogenous events, beyond the short term interest rate movements that we have in the model, that may explain why the inflation rate rose and fell in the 1980s.

Ignoring these exogenous events would most certainly bias the coefficients of our 5-variable model. Hence, three different data specifications will be considered in order to deal with these low-frequency evolutions:

- Specification 1: Our first data set consists in real government expenditures, real government net receipts, real GDP, prices (GDP deflator) and the 3-month interest rate. All series are specified in log-level, except the 3-month interest rate which is specified in level (% points). In order to take into account that prices (GDP deflator) have an important low-frequency component (the inflation rate is steadily decreasing, see figure 2 and appendix A), we allow for linear and quadratic trends in each of the equations of the VAR. This specification is intended to be closest to the specifications in Auerbach and Gorodnichenko (2012) and Kirchner et al. (2010) and to the specification of usual monetary policy VARs.
- Specification 2: In order to apply a standard multivariate cointegration analysis on I(1) variables, we consider the previous data set with a single modification: prices are replaced by price inflation. Hence, the cointegration analysis is done with the first three series (real government expenditures, real government net receipts and real GDP) remaining specified in log-level, price inflation being defined as the log-difference of the GDP deflator and the 3-month interest rate remaining specified in level. In this way, all variables may be considered as I(1), even around a linear

deterministic trend (ADF or ERS tests).

In order to be consistent with our first specification, we allow for a linear deterministic trend in the cointegration space[18]. Both the trace and the maximum-eigenvalue cointegration test conclude to the existence of a single cointegration relation in this case[19]. Moreover, an LR test with a 5% level does not reject the joint hypothesis that real government expenditures, real government net receipts and real GDP may be excluded from the cointegration relation and that this cointegration relation may, itself, be excluded from the inflation equation. Hence, only price inflation, the 3-month interest rate and the linear trend are linked together in the long run.

This multivariate cointegration analysis justifies a second data set with real government expenditures, real government net receipts, real GDP and prices (GDP deflator) specified in log-difference and the 3-month interest rate remaining in level. In this case, we allow for a linear trend in each of the equations of the VAR. This specification allows taking into account the long run relation between inflation and the 3-month interest rate. The fact that we are taking the log-difference of the first three variables (real government expenditures, real government net receipts and real GDP) does not imply any loss of information since these variables may be excluded from the cointegration relation.

- Specification 3: The third data set is identical to the second, but instead of allowing for a linear trend in each of the equations of the VAR (which is equivalent to previously regressing them on a linear trend), we pre-filter all data in subtracting to them a changing mean, constructed as the geometric average of their past.

Data specifications 1 and 3 are very close to those chosen by Blanchard and Perotti (2002) who are also confronted to low-frequency movements on U.S. data (see pp. 1339-1340 of their article). It leads them to present their results with two different specifications. In their first specification, all data (real government expenditures, real government net receipts and real GDP) are specified in log-levels and they allow for linear and quadratic trends in each of the equations of the VAR. In their second specification, all data are specified in log-differences from which they subtract a changing mean,

[18]This is case 4 decribed by Juselius (2006), p.100 of her book.

[19]We rely on cointegration tests at a 10% level. This is consistent with Juselius (2006), p.145 of her book: "[...] In small samples we often lack information to make a sharp distinction between unit roots, near unit roots and very stationary roots. In such cases, choosing the rank based on a small p-value like 0.05 is likely to exclusively pick up cointegration relations with relatively fast adjustment back to equilibrium. Unfortunately, the probability of excluding stationary relations characterized by slow, but nevertheless significant adjustments, is likely to be high, i.e. the probability of type 2 errors is generally high for such relations. [...]"

constructed as the geometric average of past log-differences[20].

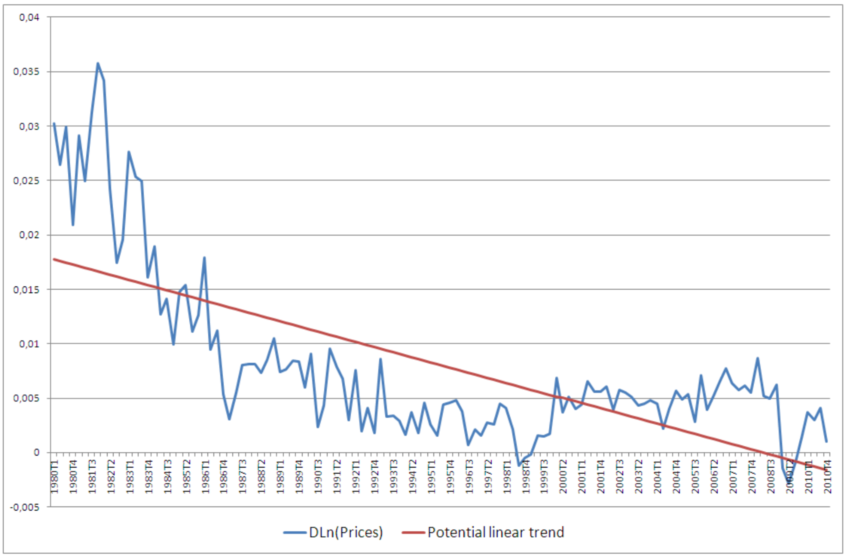

Figure 2: Quarterly inflation rate in France (log-difference of the GDP deflator)

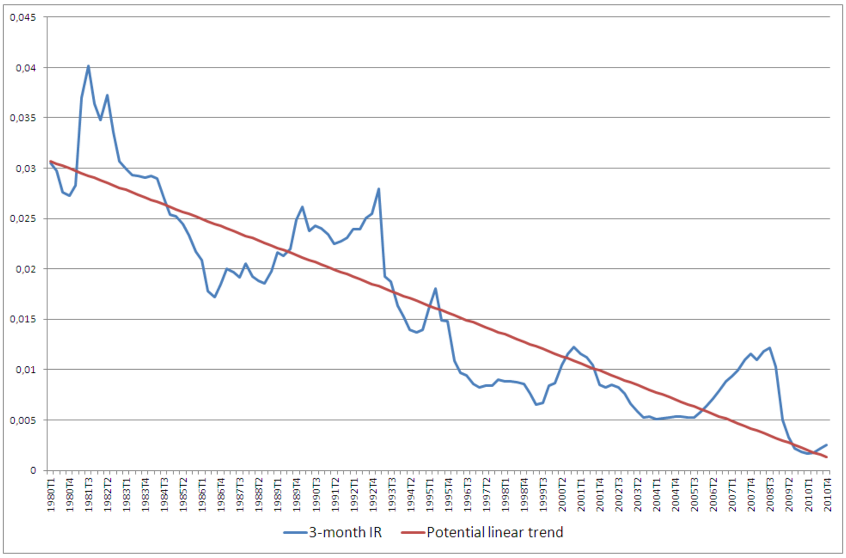

Figure 3: nominal 3-month interest rate in France

[20]Blanchard and Perotti (2002) construct their geometric average using a decay parameter equal to 2.5% per quarter and verify that varying this parameter between 1 and 5% makes little difference to their results. In our third specification, the filter is defined as a weighted arithmetic mean of the original data in log-differences: $\Delta \widetilde{\log x_t} = \delta \cdot \sum_{i=0}^{+\infty} (1-\delta)^i \Delta \log x_{t-i}$. The decay parameter δ is computed such that the filter has a cutoff period of 15 years: $\delta \sim 0.1$.

2.2.3 Comparison of results for various pre-treatments of the series

Given that IRFs may depend on whether data has been differenced or detrended, we choose to present results with the three specifications that we previously described. First we compare results on the whole sample. We also report results on the 1989Q1-2010Q4 sample with no previous detrending or filtering. Indeed, detrending and filtering are especially justified to deal with low-frequency movements in the 1980s[21]. Without previous detrending or filtering, specification 2 is equivalent to specification 3.

For our main variable of interest, the government expenditure multiplier at different horizons, results are generally not significantly different at the 68% level for the first 12 quarters. The government expenditure multiplier is not significantly different from 1 on impact and becomes statistically insignificant after about 3 years.

In the following, we will generally rely on specification 3 over the whole sample (1980Q1-2010Q4).

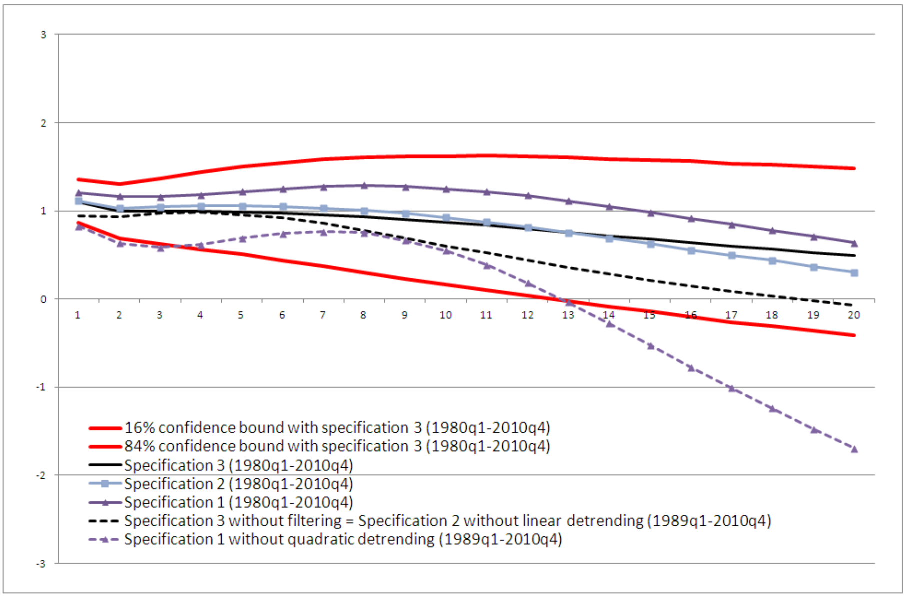

Figure 4: Government expenditure multipliers depending on data handling. Specification 3 is the benchmark. Model with 2 lags. IRFs and 68% confidence intervals are computed using Kilian's (1998) bootstrap-after-bootstrap.

[21] More precisely, the interest rate is always expressed in percentage points, not in logarithms. Even in specifications 2 and 3, the interest rate remains in level. Primiceri (2005) and Stock and Watson (2001) also use the inflation rate and the interest rate in level (along with the unemployment rate) for the estimation of Taylor rules in VAR models. This choice does not exclude a possible cointegrating relationship between the two variables. Note, however, that the interest rate is filtered is the same way as the other variables in specification 3 in order not to introduce a phase shift between them.

2.3 Different definitions of government expenditures

2.3.1 Government expenditures in goods and services, excluding the compensation of civil servants

In order to compare our results with the previous literature, we first check what our measure of government expenditures (excluding the compensation of civil servants) changes in comparison with the usual definition (sum of government consumption - P3 - and investment - P51 - in national accounts).

On the whole sample (1980Q1-2010Q4), the impact multiplier (i.e.: M_1) is higher with the usual definition of government expenditures (figure 5). The point estimate impact multiplier equals 1.6 with the usual definition whereas it equals 1.1 with our measure of expenditures. This difference is not far from being statistically significant at the 68% level. For the United States, the non-defense spending multiplier found by Auerbach and Gorodnichenko (2012) is 1.6 when they do not distinguish, like here, between expansions and recessions[22].

This difference between multipliers using different definitions of government expenditures can be linked to accounting conventions and to the fact that all expenditures are not of the same kind. Indeed, following the 1993 System of National Accounts (1993 SNA), non-market output by general government is estimated as the sum of intermediate consumption, compensation of civil servants and consumption of fixed capital in the corresponding sectors. This means that an exogenous increase in the compensation of civil servants mechanically leads to an increase in the value-added of non-market sectors, for strictly accounting reasons (see Appendix B for details). Of course, one may also observe a further increase in the value-added of market sectors due, for example, to an increase in the private consumption of civil servants. We want to exclude the first (strictly accounting) channel from our measure of the government expenditure multiplier. Moreover, this allows us to consider more homogenous expenditures.

[22]They never exclude the compensation of civil servants from their definition of government spending.

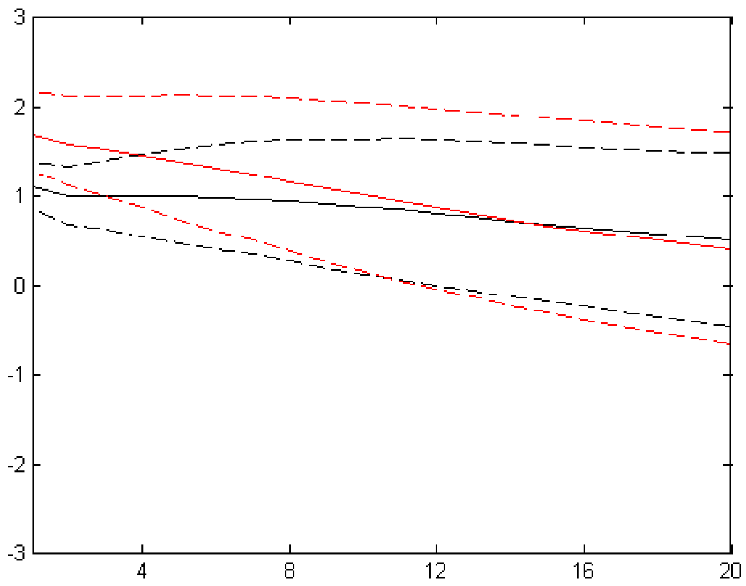

Figure 5: Government expenditure multipliers depending on the definition of expenditures: new definition (P2+D631A+P51) in black ; usual definition (P3+P51) in red. Model with 2 lags. IRFs and 68% confidence intervals are computed using Kilian's (1998) bootstrap-after-bootstrap.

2.3.2 Investment and other expenditures in goods and services

Now, we distinguish government investment (P51) from other expenditures in goods and services. Investment represents 10 to 15% of total government expenditures with the usual definition (P3+P51) and 20 to 25% with the new definition (P2+D631A+P51), both shares being roughly stable since 1980 (figure 6). Contrary to government consumption, government investment in national accounts does not entail any compensation of civil servants. Its composition by asset remains roughly stable since 1980: 45% of public investment is devoted to buildings, 30% to civil engineering structures, 10 to 15% to machines, 5% to transportation material and the rest to computers and software (3% in 1980, a little less than 10% in 2010). The stability in the share and in the composition of investment since 1980 rules out potential composition effects that could have driven the evolution of the expenditure multiplier.

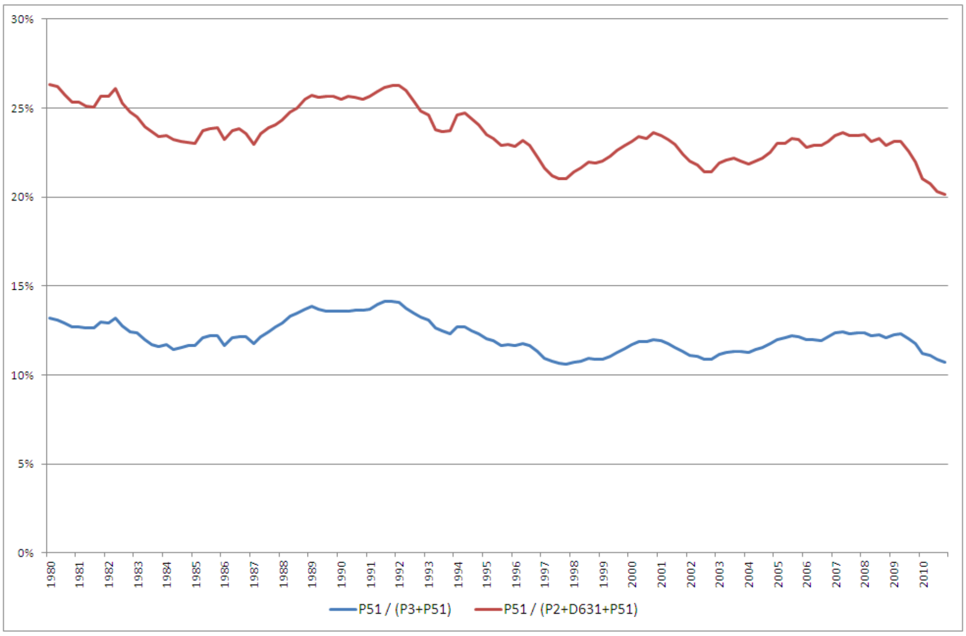

Figure 6: Share of investment in government expenditures

In order to distinguish multipliers linked to different kinds of expenditures, we extend the VAR model. Government investment (P51) is ordered first, before other expenditures (P2+D631A) and the same variables as before, for the computation of the investment multiplier[23]. Identification still relies on the assumption that the structural shock on investment is equal to the corresponding reduced-form shock plus the reduced-form shock on prices.

The point-estimate impact investment multiplier (M_1) is equal to 2.5 and the multiplier after 20 quarters (M_{20}) is equal to 2.8 (figure 7), in line with the results of Auerbach and Gorodnichenko (2012) on U.S. data ($M_{20} = 2.4$ using a linear model). It is significantly higher than the corresponding multiplier on other expenditures ($M_1 = 0.2$, $M_{20} = -1.2$). In both cases, results are similar and not significantly different if the model is estimated over the whole sample (1980Q1-2010Q4) with data specification 3 or on the second part of the sample (1989Q1-2010Q4) when data are neither filtered nor detrended (specifications 2 and 3 are equivalent in this case)[24].

[23]Due to the limited correlation between government investment and other expenditures (0.2 between 1980 and 2010), results are practically unchanged when the ordering of the first two variables is reversed.

[24]Results on the second part of the sample are interesting to bear in mind for comparison with the next part, where expectations are controlled for.

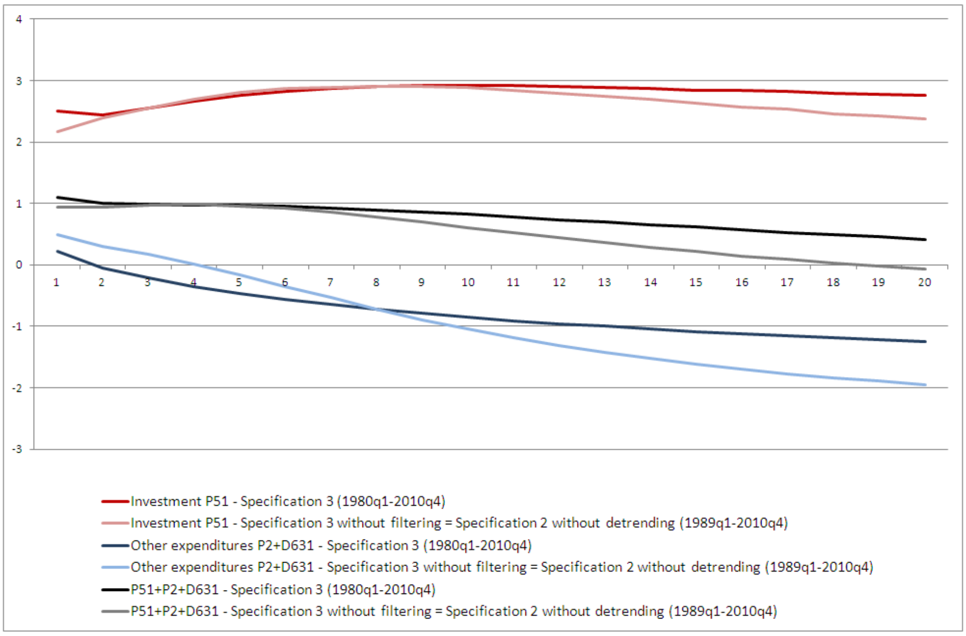

Figure 7: Multipliers linked to investment (P51) and other expenditures (P2+D631A) and aggregate multiplier (P51+P2+D631A). They are computed using Kilian's (1998) bootstrap-after-bootstrap.

2.4 Controlling for expectations

In order to identify unexpected shocks, one needs to define the observed and expected evolutions of government expenditures.

- Should the first (real-time) or the latest release of quarterly national accounts be considered as the reference series for the evolution of government expenditures? If national accounts are assumed to be built upon all available information in real-time, then the first release should be used because it is a reliable picture of what economic agents can observe in real-time and upon which they build their decisions. If, on the contrary, the first release is considered to be a blurred image of what economic agents can really observe in real-time, then the latest release should be used. Using the first release in this case leads to underestimate the unexpected component of shocks because the first release of national accounts does not contain much more information than the lastest forecast.

  For several reasons, the latest release of French national accounts gives a more accurate picture of the government expenditures that economic agents can observe in real-time. This is true for both definitions of expenditures. For instance, compensation of civil servants and intermediate consumption in the first release of quarterly national accounts are based on a moving average of indicators for central government only because there is no reliable information for local administrations available to national accountants at the time of the first release. Moreover, the decomposition of investment between firms and general government is conventional in this first release. This is why we define unexpected shocks as the difference between the latest release of quarterly national accounts and real-time forecasts made by the forecasting department of the statistical institute.

- Another difficulty in the identification of unexpected shocks arises when accounting concepts have changed between the release of the forecast and the release of national accounts. In this case, using the latest release would lead to overestimate the unexpected component. The definition of government investment has remained practically unchanged since the 1980s[25] but this is not the case for government consumption[26]. Moreover, we do not have forecasts of government intermediate consumption (P2) or social benefits in kind (D631A). Therefore, we only try to control for expected government <u>investment</u> (P51) shocks.

- The latest forecast made by the forecasting department of the French statistical institute (INSEE) before the first release of national accounts is used as an approximation of the expected evolution of government investment in France. The first available forecast in our database is the forecast for 1989Q1.

- The correlation between forecasted investment growth and investment growth in the latest release of national accounts lies between 0.2 and 0.3 and remains roughly stable on the available sample.

Following Ramey (2011a) and Auerbach and Gorodnichenko (2012), we expand the VAR model in order to control for expectations. The first three variables are now government investment forecasts, government investment itself and other expenditures (P2+D631A). The following four variables remain unchanged. The structural shock on investment is now a shock to the second variable. By construction, it is orthogonal to government investment forecasts[27]. Using this methodology, there is no evidence of an

[25] The only notable exception is the decision to treat software as an investment, rather than an intermediate consumption, at the end of the 1990s. But at that time, this asset represented only 6% of government investment.

[26] Before the introduction of the 1993 SNA at the end of the 1990s, the definition of government consumption was narrower. For instance, all the expenditures incurred on the market by the government on behalf of households were recorded as households' consumption.

[27] We consider this methodology to be definitely superior to the one consisting in the inclusion government investment <u>forecast errors</u> as a first variable in the VAR instead of government investment <u>forecasts</u>.

overlooked anticipation effect in the computation of the government investment multiplier[28] (figure 8).

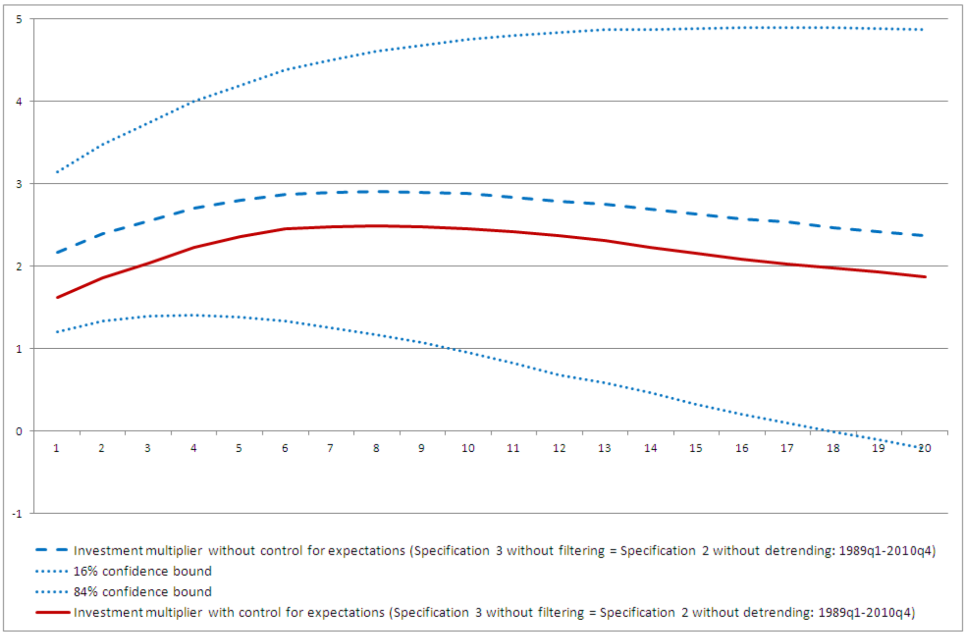

Figure 8: Investment multipliers with or without control for expectations. Multipliers and 68% confidence intervals are computed using Kilian's (1998) bootstrap-after-bootstrap.

Indeed, if investment forecasts use all available information when they are made, investment forecast errors can only be explained by news that arrived afterwards. Hence, investment and investment forecast errors are determined simultaneously and a regression of the former on the latter leads to a biased coefficient.

[28]This result for public investment is not necessarily surprising since half of this investment is devoted to buildings. The construction of buildings progresses at a regular pace and should be well forecasted using lagged investment values, already included in the VAR model.

2.5 Different estimation samples

We now focus on results obtained with our measure of government expenditures on different samples. Here are the main conclusions:

- The expenditure multiplier is slightly higher at each horizon in a sample including the Great Recession (1980Q1-2010Q4) than in a sample excluding it (1980Q1-2007Q2) but these differences are not statistically significant at the 68% level (figure 9).

- Although it is not significant at the 68% level, the most important difference on two equal subsamples (1980-1995, 1995-2010) concerns the medium-run multiplier (horizon of 20 quarters, figure 10). It is larger when estimated over the first 15 years (1980-1995). Nevertheless, the impact multipliers on the two subsamples are very close to each other and not far from 1.

- We also estimate the SVAR model with constant parameter OLS on rolling windows with a length of 15 years (figure 11). Beyond the decrease in the medium-run multiplier (which is only marginally significant around 2000), it also indicates a slight and marginally significant decrease in the impact multiplier around 2000. Datation refers to the center of the rolling windows and, therefore, is far from precise. In the following, we compare these results with those from a Time-Varying SVAR model.

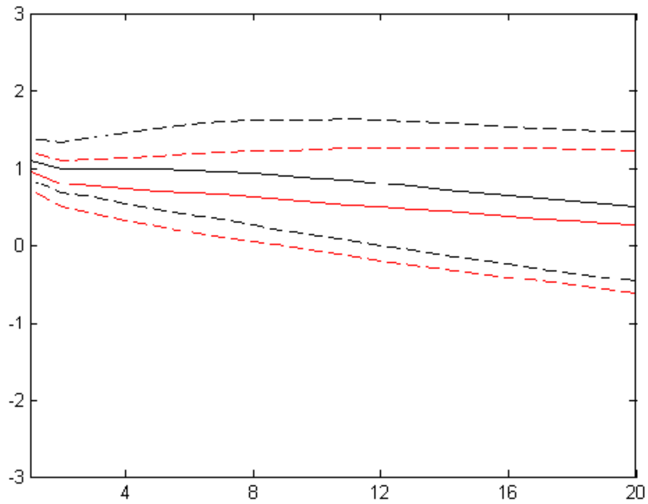

Figure 9: Government expenditure multipliers depending on the estimation sample: 1980Q1-2010Q4 in black ; 1980Q1-2007Q2 in red. New definition of expenditures (P2+D631A+P51). Model with 2 lags. IRFs and 68% confidence intervals are computed using Kilian's (1998) bootstrap-after-bootstrap.

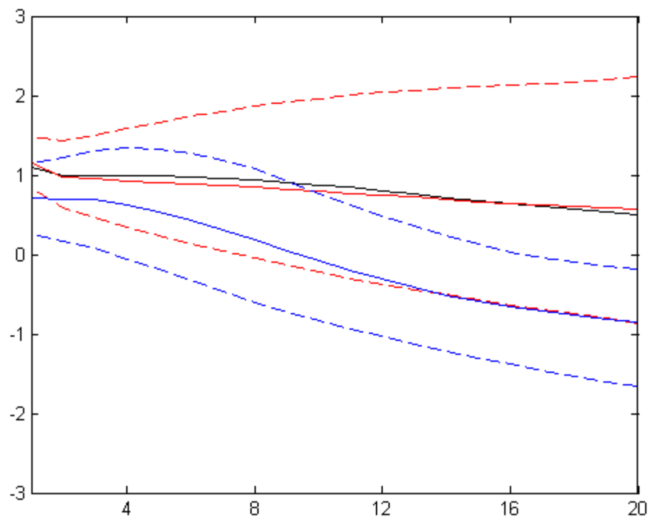

Figure 10: Government expenditure multipliers depending on the estimation sample: 1980Q1-2010Q4 (median IRF only) in black ; 1980Q1-1995Q2 in red ; 1995Q3-2010Q4 in blue. New definition of expenditures (P2+D631A+P51). Model with 2 lags. IRFs and 68% confidence intervals are computed using Kilian's (1998) bootstrap-after-bootstrap.

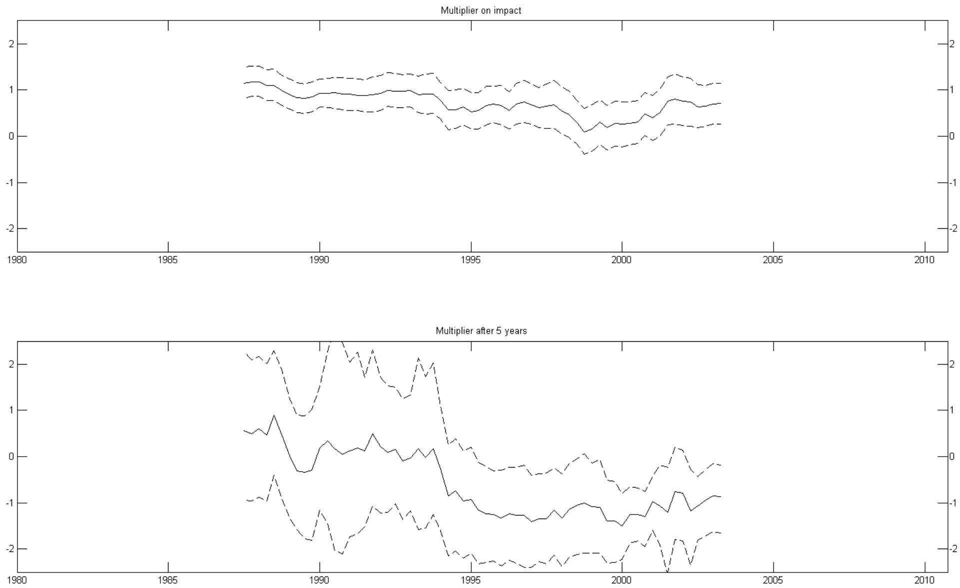

Figure 11: Government expenditure multipliers on rolling windows having a length of 15 years. New definition of expenditures (P2+D631A+P51). Model with 2 lags. IRFs and 68% confidence intervals are computed using Kilian's (1998) bootstrap-after-bootstrap.

3 Assessing the evolution of the expenditure multiplier

3.1 Econometric methodology

3.1.1 Description of the TV-SVAR model

The model that we consider is now a quarterly Time-Varying SVAR (TV-SVAR) with 5 variables and l lags. The coefficients of the VAR and the variance covariance matrix of the error term are time-varying. This model can be written as:

$$\begin{aligned} y_t &= \begin{pmatrix} I_5 & I_5 \otimes y'_{t-1} & \dots & I_5 \otimes y'_{t-l} \end{pmatrix} \cdot \beta_t + A_{idtf}^{-1} A_t^{-1} \Sigma_t \cdot \varepsilon_t \\ &= Z_t \cdot \beta_t + A_{idtf}^{-1} A_t^{-1} \Sigma_t \cdot \varepsilon_t \end{aligned}$$

where $\varepsilon_t \sim NID(0,1)$ are structural innovations and the $(25l+5) \times 1$ vector β_t contains coefficients of constants and lags.

Matrices A_{idtf}, A_t and Σ_t have the following structure:

$$\begin{aligned} A_{idtf} &= \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, A_t = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ \alpha_{21,t} & 1 & 0 & 0 & 0 \\ \alpha_{31,t} & \alpha_{32,t} & 1 & 0 & 0 \\ \alpha_{41,t} & \alpha_{42,t} & \alpha_{43,t} & 1 & 0 \\ \alpha_{51,t} & \alpha_{52,t} & \alpha_{53,t} & \alpha_{54,t} & 1 \end{pmatrix}, \\ \Sigma_t &= \begin{pmatrix} \exp\left(\frac{h_{1,t}}{2}\right) & 0 & 0 & 0 & 0 \\ 0 & \exp\left(\frac{h_{2,t}}{2}\right) & 0 & 0 & 0 \\ 0 & 0 & \exp\left(\frac{h_{3,t}}{2}\right) & 0 & 0 \\ 0 & 0 & 0 & \exp\left(\frac{h_{4,t}}{2}\right) & 0 \\ 0 & 0 & 0 & 0 & \exp\left(\frac{h_{5,t}}{2}\right) \end{pmatrix} \end{aligned}$$

The evolution of the time-varying parameters is described by the following state equations:

$$\begin{aligned} \beta_t &= \beta_{t-1} + K_{1t} \cdot \upsilon_t \\ \alpha_t &= \alpha_{t-1} + K_{2t} \cdot \zeta_t \\ h_t &= h_{t-1} + K_{3t} \cdot \eta_t \end{aligned}$$

The innovations ε_t, ν_t, ς_t and η_t are assumed to be jointly normally distributed with the following variance covariance matrix:

$$V = Var\begin{pmatrix} \varepsilon_t \\ \upsilon_t \\ \zeta_t \\ \eta_t \end{pmatrix} = \begin{pmatrix} I_5 & 0 & \dots & 0 \\ 0 & Q & \ddots & \vdots \\ \vdots & \ddots & S & 0 \\ 0 & \cdots & 0 & W \end{pmatrix}$$

These state equations are augmented with parameters K_{it} $(i = 1, 2, 3)$ which can be equal to 0 or 1. Depending on the value of K_{it}, state variables β_t, α_t and h_t evolve between date $t-1$ and date t or remain constant. When, for instance, $K_{1t} = 0$, all the components of the state vector β_t remain unchanged between date $t-1$ and date t. Koop et al. (2009) introduced this specification into the TV-SVAR framework developed by Primiceri (2005) in order to assess whether the monetary transmission mechanism in the United States changed and whether changes were gradual or abrupt.

3.1.2 Bayesian estimation strategy

Following Canova and Ciccarelli (2009), priors on α_0, β_0, h_0, Q, S and W are specified using OLS estimates on the 1980Q1-2007Q2 sample (i.e.: on a sample excluding the Great Recession).

$$\begin{aligned} \beta_0 &\sim N\left(\hat{\beta}_{OLS}, 4 \cdot \hat{V}\left(\hat{\beta}_{OLS}\right)\right) \\ \alpha_0 &\sim N\left(\hat{\alpha}_{OLS}, 4 \cdot \hat{V}\left(\hat{\alpha}_{OLS}\right)\right) \\ h_0 &\sim N\left(2 \cdot \log \hat{\sigma}_{OLS}, 4 \cdot \hat{V}\left(\hat{\sigma}_{OLS}\right)\right) \end{aligned}$$

Q, S and W are diagonal matrices with diagonal elements q_i, s_i and w_i

$$\begin{aligned} q_i &\sim IG\left(k_q^2 \cdot \nu_q \cdot \hat{V}_{i,i}\left(\hat{\beta}_{OLS}\right), \nu_q\right) \\ s_i &\sim IG\left(k_s^2 \cdot \nu_s \cdot \hat{V}_{i,i}\left(\hat{\alpha}_{OLS}\right), \nu_s\right) \\ w_i &\sim IG\left(k_w^2 \cdot \nu_w \cdot \hat{V}_{i,i}\left(\hat{\sigma}_{OLS}\right), \nu_w\right) \end{aligned}$$

$\hat{V}_{i,i}\left(\hat{\beta}_{OLS}\right)$, $\hat{V}_{i,i}\left(\hat{\alpha}_{OLS}\right)$ and $\hat{V}_{i,i}\left(\hat{\sigma}_{OLS}\right)$ correspond, respectively, to the diagonal elements of matrices $\hat{V}\left(\hat{\beta}_{OLS}\right)$, $\hat{V}\left(\hat{\alpha}_{OLS}\right)$ and $\hat{V}\left(\hat{\sigma}_{OLS}\right)$. The meaning of hyperparameters k_q, k_s, k_w, ν_q, ν_s and ν_w will be clarified in the next section.

Finally, the Gibbs sampling algorithm is used to simulate the joint posterior distribution

$$p\left(\beta^{1..T}, Q, \alpha^{1..T}, S, h^{1..T}, W, K_1^{1..T}, K_2^{1..T}, K_3^{1..T}, s^{1..T}|y^{1..T}\right)$$

where $x^{1..T}$ corresponds to $(x_1, ..., x_T)$. The implementation of this algorithm is described in Appendix D.

3.2 Results with the TV-SVAR model

3.2.1 Benchmark specification

We now estimate the TV-SVAR model presented above. In the definition of the Bayesian priors, we have to set important hyperparameters governing time variation of the model parameters.

- First, we have to set the hyperparameter p scaling the probability to observe a jump in the parameter values between date $t-1$ and date t. In the benchmark specification, this hyperparameter is defined so that the mean probability to observe a jump is 0.5 for the three classes of model parameters (autoregressive parameters, parameters of the impact matrix A_t and variances of the orthogonalized shocks).
- Second, we have to set the hyperparameters k_q, k_s and k_w governing the variance of parameter innovations. The econometric literature gives some guidance when the law of motion of parameters is a random walk with Gaussian innovations (i.e. when the probability of jump between two successive dates is equal to 1). Following Stock and Watson (1996), the choice of $k_q = 0.01$ has become standard in the literature on time-varying parameter regressions (Cogley and Sargent 2001, Cogley and Sargent 2005, Primiceri 2005). It corresponds to a standard error of the innovations in the random walk processes describing parameter evolutions equal to 1% of the standard error of the OLS estimates. Here, taking into account that the probability p of jump is not equal to 1 in our prior specification, we follow Koop et al. (2009) and we set the benchmark values for k_q, k_s and k_w at $\frac{0.01}{\sqrt{p}}$ with $p = 0.5$.
- Third, hyperparameters ν_q, ν_s and ν_w corresponding to the tightness of the priors scaling time variation are chosen so that these priors are proper and to avoid implausible behavior of the parameters. In practice, we set $\nu_s = \nu_w = \frac{1}{2}$ and $\nu_q = \frac{5}{2}$ in the benchmark specification[29].

[29]Remind that ν_q, ν_s and ν_w are homogenous to a number of points, to be compared with half the size of the sample used for the estimation of the model, $\frac{122}{2} = 61$ points here.

Recall that these choices direct but do not fully constrain time-variation *a posteriori*. Indeed, the weight given to priors in final (*a posteriori*) estimates declines with the size of the sample and asymptotically converges to 0.

With this benchmark specification, we find that posterior means for the transition probabilities are comprised between 0.35 and 0.75 ($E(p_1|y^{1\cdots T}) = 0.5$, $E(p_2|y^{1\cdots T}) = 0.35$ and $E(p_3|y^{1\cdots T}) = 0.75$). Thus, although our benchmark specification does not exclude rare abrupt changes, our results are close to those of a TV-SVAR model *à la* Primiceri. Results indicate that time variation is mostly located in the variance of (reduced-form) shocks (figure 12) [30].

Two features are worth noticing:

- a gradual decrease of the variance of residuals in the price inflation equation since the beginning of the 1980s;
- a peak of volatility in 1992-1993 in the interest rate equation, corresponding to the crisis of the European Monetary System (EMS)[31]. This means that the restrictive monetary policy at that time is partly interpreted as an interest rate shock, with more variance than usual, rather than only as a strengthening of the systematic monetary response to business cycle conditions. It seems particularly useful in such cases to estimate a model which is able to discriminate between heteroscedasticity and time-varying parameters.

However, there is only very limited variation over time of the impulse response functions (IRFs) to a government spending shock (see figure 13 showing government expenditure multipliers on 20 quarters and at 4 different dates: 1980, 1990, 2000 and 2010).

[30] Notice that the algorithm advocated by DelNegro and Primiceri (2013) leads to smoother variances than the algorithm originally advocated by Primiceri (2005), see Appendix D.

[31] Since the beginning of the 1990s, France and the other members of the EMS had chosen to anchor their currency to the Deutsche Mark. In practice, this meant that the French central bank was constrained to follow the German short term interest rate policy. Difficulties arose after the German reunification when the Bundesbank began to counteract the inflationary pressures following the choice of a rate of exchange of 1:1 for conversion of East German money to Deutschmarks and the fiscal expansion decided by the German government. Following the same monetary policy outside Germany with worse macroeconomic conditions meant positive interest rate shocks. The size of these shocks rapidly increased because other countries had to offer a growing spread relative to the German interest rate in order to maintain a fixed parity with the Deutsche Mark. Some countries (UK, Italy, Spain) chose to leave the EMS at the end of 1992. France paid an interest rate spread of 200bp until the summer 1993 when fluctuation bands inside the EMS were enlarged. The interest rate spread relative to Germany then disappeared rapidly. For more details on the EMS crisis, see Muet (1994).

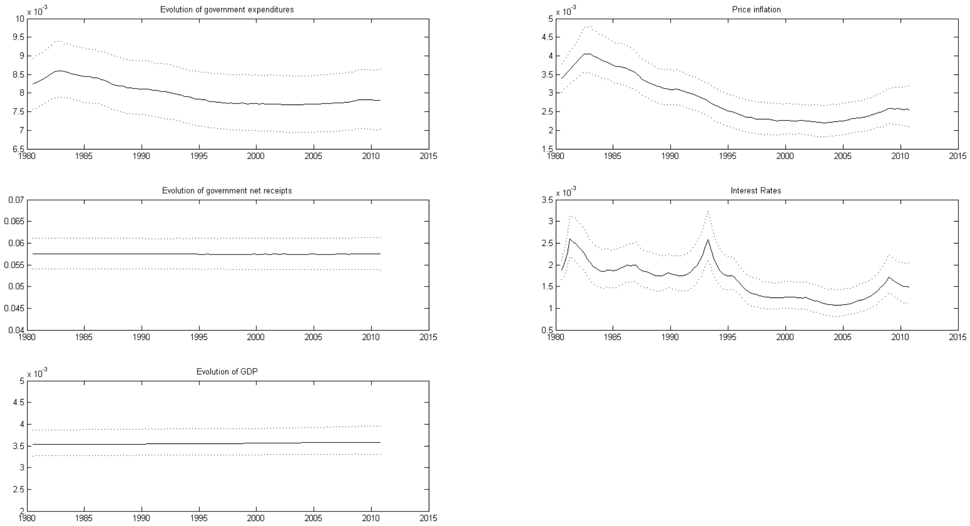

Figure 12: Posterior median of the standard deviation of shocks ; 16% and 84% confidence bounds. Benchmark specification.

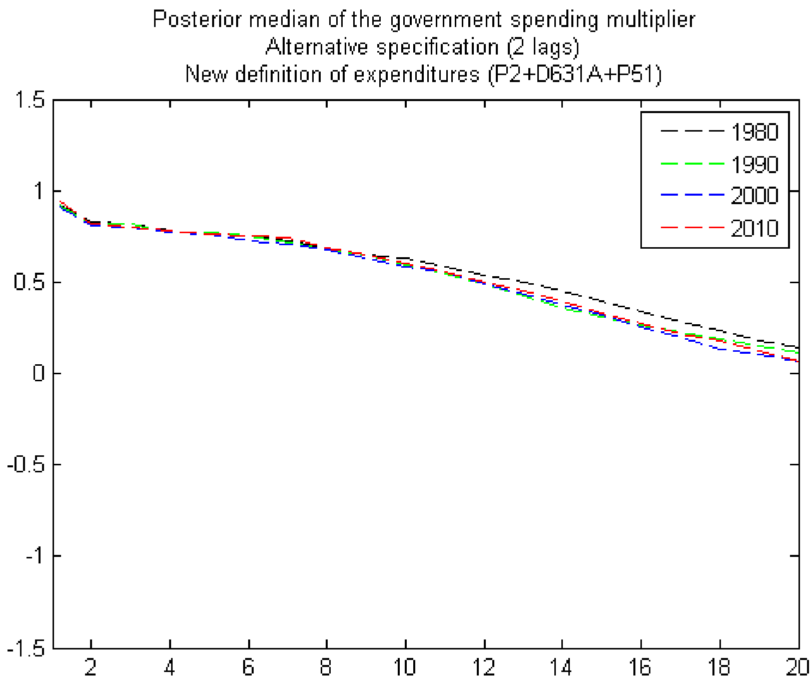

Figure 13: Government spending multiplier for a shock initiated at different dates (1980, 1990, 2000 and 2010). Benchmark specification.

3.2.2 Two alternative specifications where time variation is more favored by the Bayesian priors

The hyperparameters governing time variation of the model parameters are now set to alternative values. We consider two alternative specifications:

- In the first specification, the probability p of observing a jump in parameter values between two successive dates is set at a prior mean value of 0.01 instead of 0.5. Consequently, the hyperparameters k_Q, k_S and k_W are revised upwards ($k_Q = k_S = k_W = \frac{0.01}{\sqrt{p}}$). In this case, we set $\nu_s = \nu_w = \frac{1}{2}$ (unchanged) and $\nu_q = \frac{20}{2}$. This specification is intended to be closer to the regime-switching VAR specification adopted by Auerbach and Gorodnichenko (2012) but, contrary to them, we do not constrain the regime switch to be influenced by business cycle conditions only. We let the data decide for the timing and size of the jumps, only indicating that jumps should be rare and rather important *a priori*.

- In the second specification, the probability of observing a jump has a prior mean value of 0.5, as in the benchmark, but the hyperparameters k_Q, k_S and k_W are set at a value of $\frac{0.1}{\sqrt{0.5}}$ instead of $\frac{0.01}{\sqrt{0.5}}$ in the benchmark specification. Consequently, evolutions are assumed to be gradual but with a much higher amplitude than in the benchmark. In this case, we set $\nu_s = \nu_w = \frac{1}{2}$ (unchanged) and $\nu_q = \frac{50}{2}$. This prior specification is intended to really push the model towards time-variation but it can be considered as extreme given what is usually considered reasonable in the literature (*cf.* Stock and Watson (1996)).

Examining the results of the first alternative specification shows that even when the Bayesian priors are more in line with the Auerbach and Gorodnichenko (2012) specification, data information is strong enough to pull posterior transition probabilities upwards for the log-variances h_t ($E(p_3|y^{1...T}) = 0.25$, compared to a prior transition probability equal to 0.01). However, posterior transition probabilities for coefficients α_t and β_t remain very close to the prior probabilities ($E(p_1|y^{1...T}) = 0.02$, $E(p_2|y^{1...T}) = 0.01$), which may indicate that data is less informative for these coefficients. All in all, IRFs, multipliers and evolutions of shock variances remain practically unchanged compared to the benchmark specification (not shown).

It is only with the second alternative specification that one may see an evolution in the IRFs and in the expenditure multiplier. The most notable evolution is a decrease in the medium term multiplier between the beginning and the end of the sample. This is consistent with what we previously obtained relying on simple rolling windows OLS estimates. Another visible evolution in the IRFs concerns the systematic response of monetary policy: the upward adjustment of the short term interest rate following a government spending shock becomes more aggressive in the 1990s (figure 14). This evolution can probably be linked to the disinflation and exchange rate policy followed by the French Central Bank at that time.

This visual inspection is confirmed if one compares the observed evolution of the government expenditure multiplier with its counterfactual evolution if the systematic

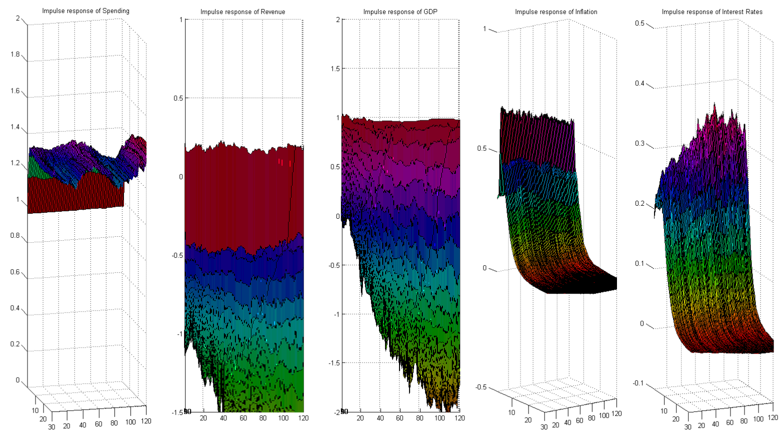

Figure 14: Government spending multiplier in the whole sample. Second alternative specification.

response of monetary policy had remained unchanged since the beginning of the 1980s (figures 15 and 16). The evolution of the expenditure multiplier after 1990 seems to be linked in major part to the evolution of the systematic monetary policy. However, the most important part of the decrease of the expenditure multiplier between 1980 and 1990 remains unexplained by the responses of interest rates, inflation and government net receipts[32].

[32]Another counterfactual analysis has been carried out. Not only parameters of the interest rate equation, but also those of the government net receipts and inflation equations, have been frozen at their 1980 value. Even like this, the most important part of the decrease of the expenditure multiplier between 1980 and 1990 remains unexplained.

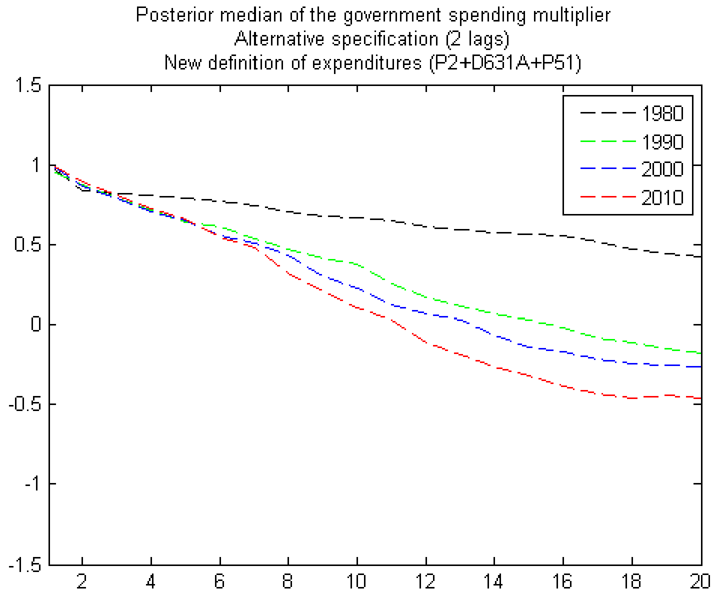

Figure 15: Government spending multiplier in 1980, 1990, 2000 and 2010, for the second alternative specification.

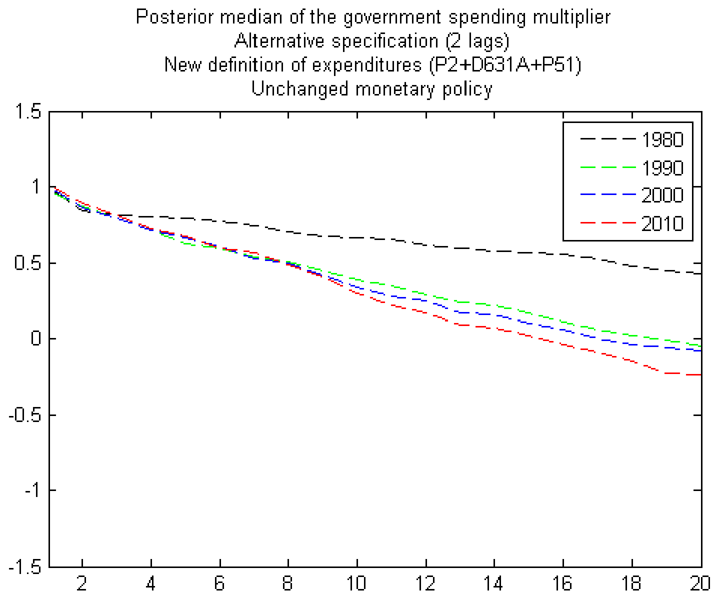

Figure 16: Government spending multiplier in 1980, 1990, 2000 and 2010, for the second alternative specification when systematic monetary policy is kept unchanged.

Even with the second alternative specification, the evolution of the expenditure multiplier cannot be considered significant given the width of the 68% credibility intervals [33] (figure 17).

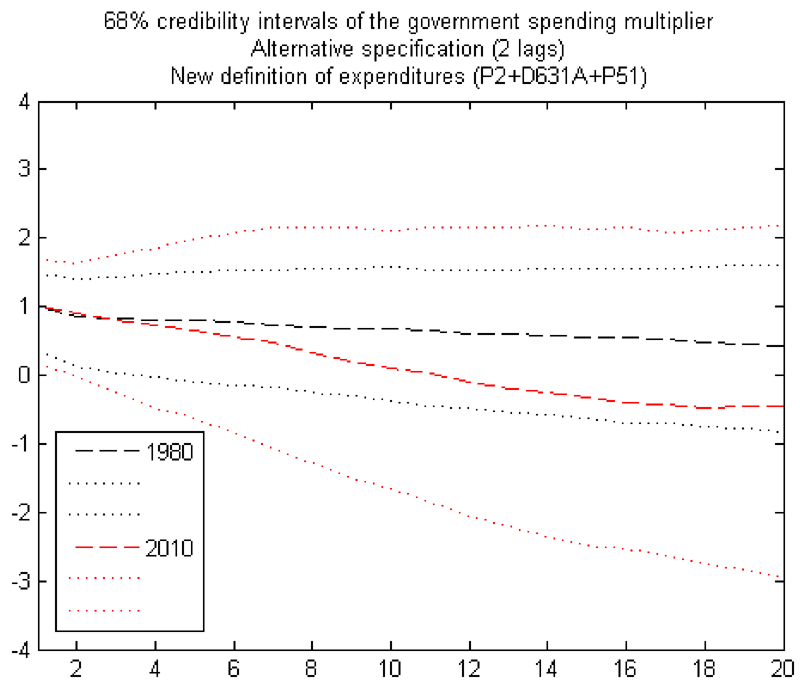

Figure 17: Government spending multiplier and their credibility intervals at two dates (1980 and 2010), for the alternative specification.

As a conclusion of this part relying on TV-SVAR models, we can notice that none of the three priors enables to detect a statistically significant evolution in the government expenditure multiplier in the last 30 years in France. In accordance with the Bayesian paradigm, we dubbed "benchmark" the prior which is most in line with external information on the problem at hand. In this case, we mainly relied on results by Stock and Watson (1996) on the probable degree of time variation in macroeconomic relationships. We could also have discriminated between prior specifications using the expected log-likelihood and the marginal likelihood as model selection criteria (see Appendix E). These are the criteria advocated by Koop et al. (2009) who estimate similar models[34]. The benchmark specification and the first alternative specification are preferred if we consider the expected log-likelihood and the two alternative specifications are preferred

[33] 16% and 84% quantiles of the posterior distributions are represented in dotted lines.

[34] See also Giannone et al. (2012) who theoretically justify the use of the marginal likelihood in order to choose prior hyperparameters in VARs.

if we consider the marginal likelihood. Nevertheless, this slight ambiguity resulting from the choice of model selection criteria does not affect the overall conclusion of the article.

4 Conclusion

Relying on OLS estimation of a SVAR model over the period 1980-2010, we find that the government expenditure multiplier is significant and not far from 1 on impact and becomes statistically insignificant after about 3 years in France. This result is based on a careful exploitation of national accounts in order to use the most relevant definition of government expenditures (excluding the compensation of civil servants). We also carry out numerous robustness checks concerning assumptions about data stationarity, the role of expectations and the choice of the sample.

We only rely on the Blanchard-Perotti identification scheme in order to identify structural shocks on government expenditures on goods and services. Hence, we do not need to know the correct value of the elasticity of government net receipts relative to business cycle conditions. We justify this partial identification scheme in appendix C.

Our second conclusion using a Time Varying-SVAR model is that the expenditure multiplier in France did not evolve significantly at any horizon since the beginning of the 1980s. The variance of shocks hitting the economy evolves a lot more than the autoregressive parameters of the model. Even in alternative specifications where the Bayesian priors are pushed towards time-variation, the main evolution that we uncover is a (non significant) decrease of the medium term expenditure multiplier, partly linked to a more aggressive monetary policy since the 1990s. We do not find evidence of an increase of the multiplier during every recession in France, contrary to the finding of Auerbach and Gorodnichenko (2012) for the United States. At least, business cycle conditions do not seem to be the main driver of the evolution of the expenditure multiplier in the last 30 years in France. One possible explanation could be that the unemployment rate, generally considered as an indicator of slack, remained high in France since the middle of the 1980s. But if one thinks that the last 30 years of data only enable measuring the government spending multiplier in bad times for France, the practical relevance of conclusions reached on U.S. data regarding the evolution of the expenditure multiplier seems to be limited for the French economy.

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A Data handling

A.1 Data in log-levels (except interest rate in % points) with quadratic trends

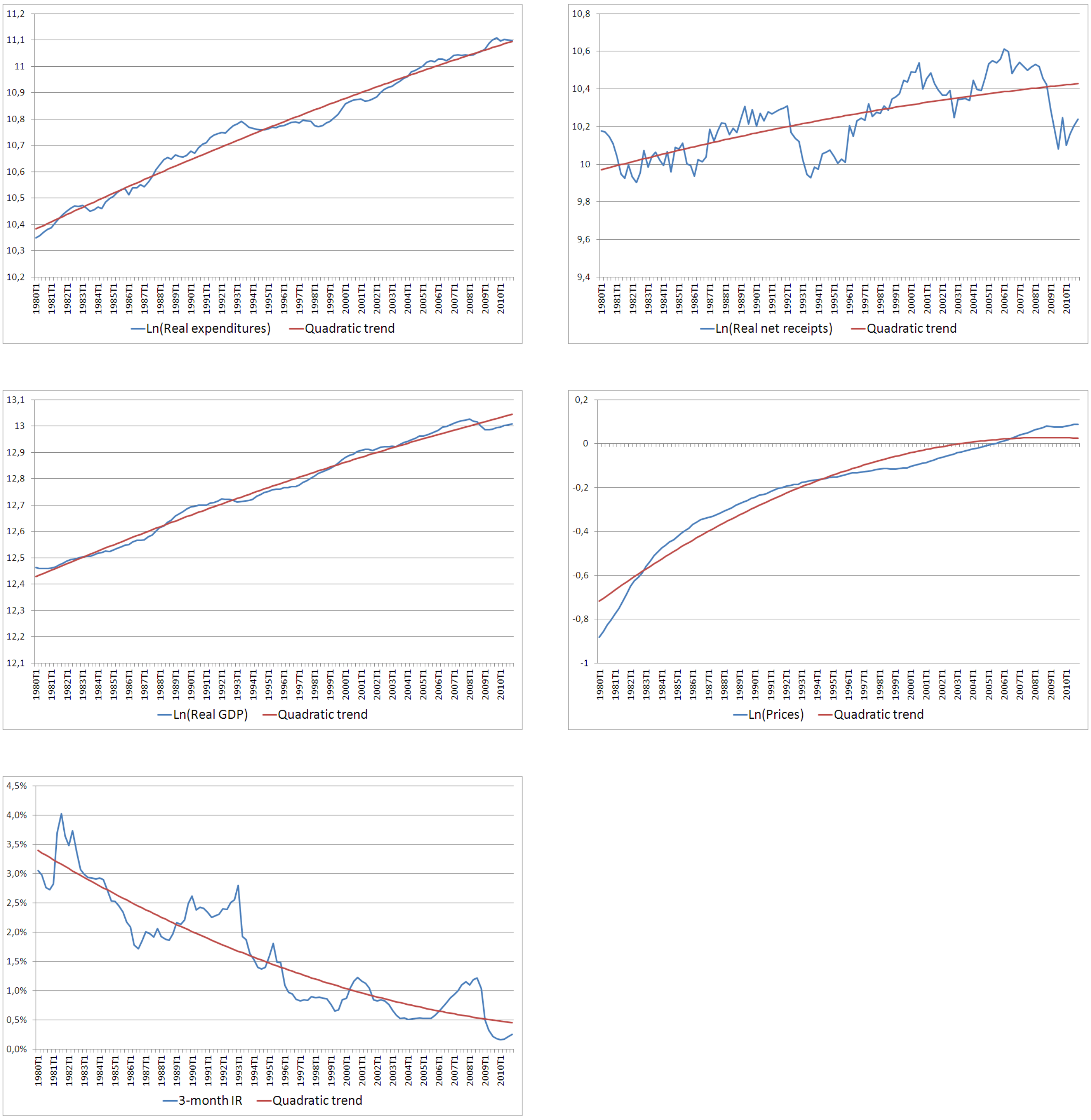

Figure 18: Data in log-levels (specif. with prices) with quadratic trends

A.2 Data in log-differences (except interest rate in % points) with linear trends

Figure 19: Data in log-levels (specif. with inflation) with linear trends

B Defining government expenditures using the SNA

The 1993 System of National Accounts (1993 SNA) is an international reference manual on national accounts[35]. It has been produced jointly by the OECD, the United Nations Statistical Division, the International Monetary Fund, the World Bank and the Commission of the European Communities. The 1995 European Accounting System (1995 ESA) directly derives from the 1993 SNA: it is the reference manual for the elaboration of national accounts all over the European Union.

The sum of general government (S13)[36] consumption (P3) and investment (i.e.: gross fixed capital formation, P51) is the most obvious definition of "government expenditures". In this case, net receipts are defined as the difference between the government financing capacity (B9A) and these expenditures. Net receipts mainly include taxes, net of subsidies and transfers.

However, this definition of government expenditures corresponds both to purchases of goods and services and to the compensation of civil servants (D1). One can criticize this definition for two reasons. First, increasing purchases of goods and services and increasing the compensation of civil servants are two different ways to stimulate the economy that should have different effects. Second, Figure 20 shows that the value of non-market production (P1) by general government is defined in national accounts as the sum of the compensation of civil servants (D1), its intermediate consumption (P2) and its consumption of fixed capital (K1). On the demand side, the main part of this non-market production is consumed by the general government itself. As a consequence, when the compensation of civil servants increases, non-market production and general government consumption simultaneously increase, as well as GDP, but for strictly accounting reasons. Of course, civil servants may spend part of their wage increase in order to buy market products: this is the economic channel that economists generally have in mind when they try to estimate the government expenditure multiplier. But when government expenditures are defined as the sum of general government consumption and investment, the economic and accounting channels are mixed up.

A simple way to measure government purchases of goods and services is to sum up intermediate consumption (P2), social benefits in kind corresponding to market products (D631A)[37] and gross fixed capital formation (P51) in the general government account[38]

[35]It is available at: http://www.oecd.org/std/nationalaccounts/systemofnationalaccounts1993.htm

[36]Codes in brackets correspond to the international classification of national accounts. They are used in 1993 SNA and 1995 ESA.

[37]We only consider reimbursements of market products. By definition, using the 1993 SNA classification, $D631A \equiv D6311 + D6312A + D6313A$.

[38]Theoretically, one should only consider intermediate consumption of market products (P_2^a) but this disaggregate series is not easily available in national accounts. Moreover, intermediate consumption of

(figure 21). Social benefits in kind correspond, for instance, to the fraction of drug purchases by households which are reimbursed by public health insurances. These insurances are part of the general government sector in national accounts.

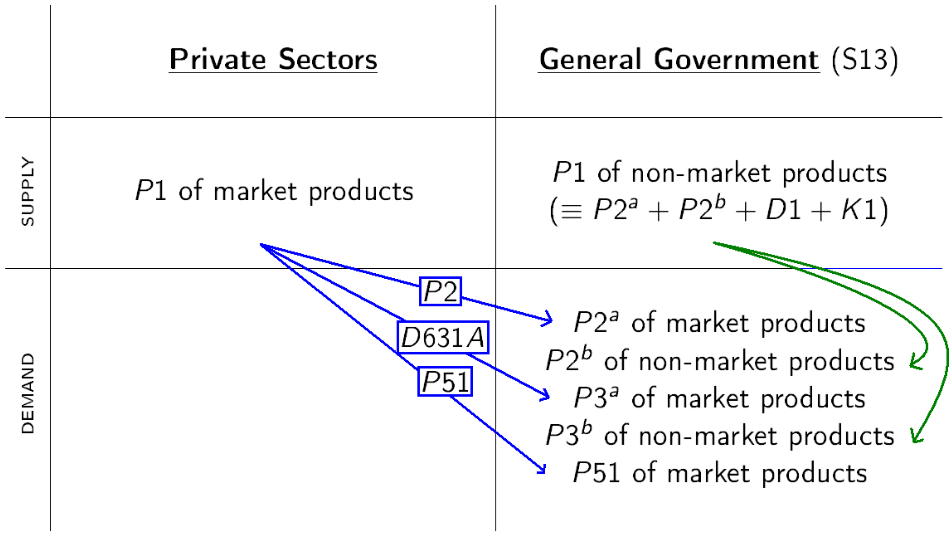

Figure 20: General government and its links to private sectors in national accounts

non-market products (P_2^b) only represents 5% of total intermediate consumption ($P_2^a + P_2^b$) by general government. Therefore, considering $P_2^a = P_2$ is only a slight approximation. And even if intermediate consumption of non-market products (P_2^b) increases non-market production, for the same accounting reasons as the compensation of civil servants, this does not mechanically translate into an increase in GDP because the value-added of general government is defined, as usual, as the difference between its production and its intermediate consumption.

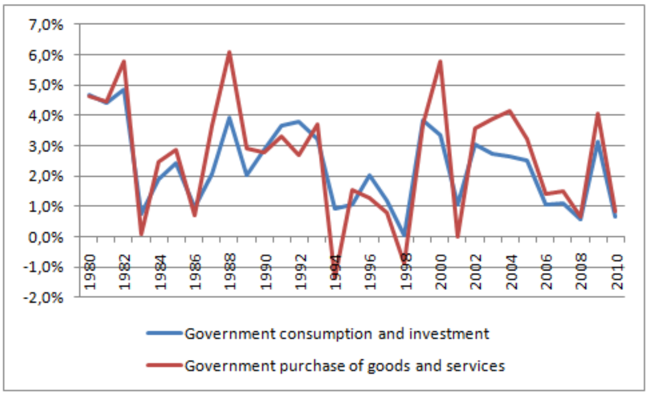

Figure 21: Annual evolution of government expenditures (deflated with the GDP deflator) following the two definitions

C Partial identification: focusing on government spending shocks

C.1 Some intuition gained from the Blanchard and Perotti (2002) identification scheme in the 3-variable model

Recall the identification scheme developed by Blanchard and Perotti (2002) for the 3-variable (government spending, government net receipts, GDP) quarterly VAR. They express reduced-form residuals ξ_t^g, ξ_t^t and ξ_t^x as a linear combination of orthogonal structural shocks e_t^g, e_t^t and e_t^x:

$$\begin{cases} \xi_t^g = e_t^g \\ \xi_t^t = a_1 \xi_t^x + a_2 e_t^g + e_t^t \\ \xi_t^x = b_1 \xi_t^g + b_2 \xi_t^t + e_t^x \end{cases}$$

where a_1 is the instantaneous elasticity of net receipts with respect to output.

Reduced-form residuals can then be expressed solely in terms of structural shocks:

$$\begin{cases} \xi_t^g = e_t^g \\ \xi_t^t = \frac{a_2 + a_1 b_1}{1 - a_1 b_2} e_t^g + f_1(e_t^t, e_t^x) \\ \xi_t^x = \frac{b_1 + a_2 b_2}{1 - a_1 b_2} e_t^g + f_2(e_t^t, e_t^x) \end{cases}$$

where f_1 and f_2 are linear functions of their arguments.

Now assume that the econometrician computes an elasticity of net receipts with respect to output $a_1^{'}$ instead of a_1. In this case, he identifies the structural shocks recursively using the following equations:

$$\begin{cases} \xi_t^g = \hat{e}_t^g \\ \xi_t^t - a_1^{'} \xi_t^x = \hat{\alpha}_2 \hat{e}_t^g + \hat{e}_t^t \\ \xi_t^x = \hat{\beta}_1 \hat{e}_t^g + \hat{\beta}_2 \hat{e}_t^t + \hat{e}_t^x \end{cases}$$

In the first equation, the structural shock on government spending is correctly identified: $\hat{e}_t^g = e_t^g$. In the second equation, the structural shock on net receipts is estimated with error: $\hat{e}_t^t \xrightarrow{p} e_t^t + \left(a_1 - a_1^{'}\right) \cdot f_2\left(e_t^t, e_t^x\right) \neq e_t^t$ and $\hat{\alpha}_2 \xrightarrow{p} a_2 + \left(a_1 - a_1^{'}\right) \cdot \frac{b_1 + a_2 b_2}{1 - a_1 b_2}$. In the third equation, determining ξ_t^x, the regression leads to:

$$\begin{pmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \end{pmatrix} = \begin{pmatrix} \sum {e_t^g}^2 & \sum e_t^g \hat{e}_t^t \\ \sum e_t^g \hat{e}_t^t & \sum {\hat{e}_t^t}^2 \end{pmatrix}^{-1} \begin{pmatrix} \sum e_t^g \xi_t^x \\ \sum \hat{e}_t^t \xi_t^x \end{pmatrix} = \begin{pmatrix} \sum {e_t^g}^2 & 0 \\ 0 & \sum {\hat{e}_t^t}^2 \end{pmatrix}^{-1} \begin{pmatrix} \sum e_t^g \xi_t^x \\ \sum \hat{e}_t^t \xi_t^x \end{pmatrix}$$

Hence $\hat{\beta}_1 = \left(\sum e_t^{g2}\right)^{-1} \left(\sum e_t^g \xi_t^x\right) = \left(\sum e_t^{g2}\right)^{-1} \left(\sum e_t^g \left[\frac{b_1+a_2b_2}{1-a_1b_2} e_t^g + f_2\left(e_t^t, e_t^x\right)\right]\right) \xrightarrow{p} \frac{b_1+a_2b_2}{1-a_1b_2}$.

Finally, the instantaneous impact of a government spending shock on net receipts can be computed in the following way: $\left(\hat{\alpha}_2 + a_1^{'}\hat{\beta}_1\right) \cdot e_t^g \xrightarrow{p} \frac{a_2+a_1b_1}{1-a_1b_2} \cdot e_t^g$, which is the true impact effect.

Even if he is unable to identify the structural shocks e_t^t and e_t^x correctly due to his error on the elasticity a_1, the econometrician recovers the true structural shock e_t^g and its instantaneous impact on government net receipts and GDP. So, the impulse response functions to a government spending shock are correct in spite of the error on the elasticity a_1.

C.2 A more general result on partial identification

Suppose that reduced-form residuals are linear combinations of orthogonal structural shocks. Once the structural shock corresponding to the first variable of the VAR has been correctly identified, its instantaneous impact on all endogenous variables can be deduced without error, whatever the (unknown) true mapping of all other structural shocks to their reduced-form counterpart.

First we show that this result holds in the case of the 5-variable VAR (government spending, net receipts, GDP, prices, interest rate) proposed by Perotti (2002) and used in the present paper. He assumes the following identification scheme:

$$\begin{cases} \xi_t^g = -\xi_t^\pi + e_t^g \\ \xi_t^t = a_1 \xi_t^x + a_2 e_t^g + e_t^t \\ \xi_t^x = b_1 \xi_t^g + b_2 \xi_t^t + e_t^x \\ \xi_t^\pi = c_1 \xi_t^g + c_2 \xi_t^t + c_3 \xi_t^x + e_t^\pi \\ \xi_t^r = d_1 e_t^g + d_2 e_t^t + d_3 \xi_t^x + d_4 \xi_t^\pi + e_t^r \end{cases}$$

Government expenditures are set in nominal terms each quarter. Shocks to inflation ξ_t^π lead to a one-to-one decline in the volume government expenditures. Due to decision and implementation lags, the innovations affecting government spending in volume ξ_t^g are independent from the other shocks affecting the economy (net receipts, output or interest rates).

We now express reduced-form residuals, except those on government spending, solely in terms of structural shocks:

$$\begin{cases} \xi_t^g = -\xi_t^\pi + e_t^g \\ \xi_t^t = \alpha_1 e_t^g + f_1(e_t^t, e_t^x, e_t^\pi) \\ \xi_t^x = \beta_1 e_t^g + f_2(e_t^t, e_t^x, e_t^\pi) \\ \xi_t^\pi = \gamma_1 e_t^g + f_3(e_t^t, e_t^x, e_t^\pi) \\ \xi_t^r = \delta_1 e_t^g + f_4(e_t^t, e_t^x, e_t^\pi, e_t^r) \end{cases}$$

where f_1, f_2, f_3 and f_4 are linear functions of their arguments.

Once the structural shock on government spending has been correctly identified (it only depends on the specification of the first equation), the orthogonality of the structural shocks and the linearity of functions f_1, f_2, f_3 and f_4 warrant that the instantaneous effect of this shock on all other endogenous variables can also be recovered.

The same result holds for a more general identification scheme in a model with n

endogenous variables and n structural shocks:

$$\left(\begin{array}{c|ccccc} 1 & b_2 & b_3 & b_4 & ... & b_n \\ \hline a_2 & & & & & \\ a_3 & & & & & \\ a_4 & & & \text{B} & & \\ ... & & & & & \\ a_n & & & & & \end{array}\right)\begin{pmatrix} \xi_t^1 \\ \xi_t^2 \\ \xi_t^3 \\ \xi_t^4 \\ ... \\ \xi_t^n \end{pmatrix} = \left(\begin{array}{c|ccccc} 1 & 0 & 0 & 0 & ... & 0 \\ \hline c_2 & & & & & \\ c_3 & & & & & \\ c_4 & & & \text{U} & & \\ ... & & & & & \\ c_n & & & & & \end{array}\right)\begin{pmatrix} e_t^1 \\ e_t^2 \\ e_t^3 \\ e_t^4 \\ ... \\ e_t^n \end{pmatrix}$$

Using partitioned inverse formulas and defining submatrix W as $W = \left[B - \begin{pmatrix} a_2 \\ a_3 \\ ... \\ a_n \end{pmatrix} \begin{pmatrix} b_2 & b_3 & ... & b_n \end{pmatrix} \right]^{-1}$, this system can be rewritten in the following form:

$$\begin{pmatrix} 1 & b_2 & b_3 & b_4 & ... & b_n \\ 0 & 1 & 0 & 0 & ... & 0 \\ 0 & 0 & 1 & 0 & ... & 0 \\ 0 & 0 & 0 & 1 & ... & 0 \\ ... & ... & ... & ... & ... & \\ 0 & 0 & 0 & 0 & ... & 1 \end{pmatrix}\begin{pmatrix} \xi_t^1 \\ \xi_t^2 \\ \xi_t^3 \\ \xi_t^4 \\ ... \\ \xi_t^n \end{pmatrix} = \left(\begin{array}{c|c} 1 & 0 \ \ 0 \ \ 0 \ \ ... \ \ 0 \\ \hline \text{-W}\begin{pmatrix} a_2 \\ a_3 \\ a_4 \\ ... \\ a_n \end{pmatrix} & \text{W} \end{array}\right)\begin{pmatrix} e_t^1 \\ e_t^2 \\ e_t^3 \\ e_t^4 \\ ... \\ e_t^n \end{pmatrix}$$

The orthogonality conditions warrant that the (n-1) elements of the column vector $W\begin{pmatrix} a_2 \\ a_3 \\ a_4 \\ ... \\ a_n \end{pmatrix}$ can be identified without error.

D Econometric methodology

D.1 Description of the TV-SVAR model

The model that we consider is a quarterly TV-SVAR with 5 variables and l lags. The coefficients of the VAR and the variance covariance matrix of the error term are time-varying. This model can be written as:

$$\begin{aligned} y_t &= \begin{pmatrix} I_5 & I_5 \otimes y'_{t-1} & \ldots & I_5 \otimes y'_{t-l} \end{pmatrix} \cdot \beta_t + A_{idtf}^{-1} A_t^{-1} \Sigma_t \cdot \varepsilon_t \\ &= Z_t \cdot \beta_t + A_{idtf}^{-1} A_t^{-1} \Sigma_t \cdot \varepsilon_t \end{aligned}$$

where $\varepsilon_t \sim NID(0,1)$ are structural innovations and the $(25l+5) \times 1$ vector β_t contains coefficients of constants and lags.

The five variables of the VAR are the growth rates of government expenditures, government net receipts and GDP, price inflation and the 3-month interest rate, ordered in this way[39]. The matrix A_{idtf} implements the identification scheme. We assume that government spending is fixed in nominal terms, so that the volume of expenditures reacts negatively, with a unitary elasticity, to unexpected inflation within a quarter. Like Blanchard and Perotti (2002), we also assume that government spending does not react contemporaneously to the current state of the business cycle.

Matrices A_{idtf}, A_t and Σ_t have the following structure:

$$A_{idtf} = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, A_t = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ \alpha_{21,t} & 1 & 0 & 0 & 0 \\ \alpha_{31,t} & \alpha_{32,t} & 1 & 0 & 0 \\ \alpha_{41,t} & \alpha_{42,t} & \alpha_{43,t} & 1 & 0 \\ \alpha_{51,t} & \alpha_{52,t} & \alpha_{53,t} & \alpha_{54,t} & 1 \end{pmatrix},$$

$$\Sigma_t = \begin{pmatrix} \exp\left(\frac{h_{1,t}}{2}\right) & 0 & 0 & 0 & 0 \\ 0 & \exp\left(\frac{h_{2,t}}{2}\right) & 0 & 0 & 0 \\ 0 & 0 & \exp\left(\frac{h_{3,t}}{2}\right) & 0 & 0 \\ 0 & 0 & 0 & \exp\left(\frac{h_{4,t}}{2}\right) & 0 \\ 0 & 0 & 0 & 0 & \exp\left(\frac{h_{5,t}}{2}\right) \end{pmatrix}$$

[39]The three first variables are expressed in real terms. We also filter out the low-frequency component of all variables using a backward exponential filter with a cutoff period of 15 years. This filtering was also implemented by Blanchard and Perotti (2002).

The evolution of the time-varying parameters is described by the following state equations:

$$\begin{aligned} \beta_t &= \beta_{t-1} + K_{1t} \cdot \upsilon_t \\ \alpha_t &= \alpha_{t-1} + K_{2t} \cdot \zeta_t \\ h_t &= h_{t-1} + K_{3t} \cdot \eta_t \end{aligned}$$

The innovations ε_t, ν_t, ς_t and η_t are assumed to be jointly normally distributed with the following variance covariance matrix:

$$V = Var \begin{pmatrix} \varepsilon_t \\ \upsilon_t \\ \zeta_t \\ \eta_t \end{pmatrix} = \begin{pmatrix} I_5 & 0 & \dots & 0 \\ 0 & Q & \ddots & \vdots \\ \vdots & \ddots & S & 0 \\ 0 & \cdots & 0 & W \end{pmatrix}$$

Contrary to Primiceri (2005), we assume that Q, S and W are diagonal matrices. Indeed, Kirchner et al. (2010) show that the Gibbs sampling algorithm recovers the true data generating process more easily with this specification and that results are then less sensitive to the choice of the priors.

Another difference with Primiceri (2005) is that these state equations are augmented with parameters K_{it} $(i = 1, 2, 3)$ which can be equal to 0 or 1. Depending on the value of K_{it}, state variables β_t, α_t and h_t evolve between date $t-1$ and date t or remain constant. When, for instance, $K_{1t} = 0$ all the components of the state vector β_t remain unchanged between date $t-1$ and date t. Koop et al. (2009) introduced this specification into the TV-SVAR framework developed by Primiceri (2005) in order to assess whether the monetary transmission mechanism in the United States changed and whether changes were gradual or abrupt.

Following Koop et al. (2009), we specify a hierarchical prior for K: $K_{it} = 1$ with probability p_i. p_i itself follows a Beta distribution: $p_i \sim Be(b_{1i}, b_{2i})$. This distribution ensures that p_i lies between 0 and 1 and, in particular, we have $E(p_i) = \frac{b_{1i}}{b_{1i}+b_{2i}}$.

D.2 Bayesian estimation strategy

Following a Bayesian approach, we use Gibbs sampling for simulating the posterior distribution $p\left(\beta^{1..T}, Q, \alpha^{1..T}, S, h^{1..T}, W, K^{1..T} | y^{1..T}\right)$ with the notation $x^{1..T}$ for a matrix $(x_1, ..., x_T)$. We simulate sequentially in 13 steps conditional posterior distributions of $\beta^{1..T}$, Q, $\alpha^{1..T}$, S, $h^{1..T}$, W and $K^{1..T}$. As recommended by DelNegro and Primiceri (2013), we take care of the order of Gibbs steps: we draw log-volatilities $h_{1:T}$ in the last step (13); we draw states s_t conditional on all other parameters, drawn themselves in steps 1 to 11. Each conditional posterior distribution is easier to draw than the joint posterior distribution. In particular, we use the simulation smoother of Carter and Kohn (1994) to draw in the conditional posterior distributions. The following 13 steps are repeated until the convergence criteria of the Gibbs sampler towards the joint posterior distribution are satisfied (see Appendix F):

- Step 1: Simulation of $p\left(\beta^{1..T}|y^{1..T}, Q, \alpha^{1..T}, S, h^{1..T}, W, K^{1..T}\right)$

Knowing all parameters and state variables, except $\beta^{1..T}$, we can write the model in the following linear state-space form:

$$\begin{cases} y_t = Z_t \cdot \beta_t + A_{idtf}^{-1} A_t^{-1} \Sigma_t \cdot \varepsilon_t \\ \beta_t = \beta_{t-1} + K_{1t} \cdot \upsilon_t \end{cases}$$

Given priors on β_0, we draw the state variables $\beta^{1..T}$ jointly from their posterior distribution using the algorithm of Carter and Kohn (1994).

- Step 2: Simulation of $p\left(K_{1t}|y^{1..T}, Q, \alpha^{1..T}, S, h^{1..T}, W, K_1^{1..T\backslash\{t\}}, K_2^{1..T}, K_3^{1..T}\right)$

Given a prior on p_1 ($p_1 \sim Be(b_{11}, b_{21})$), we draw the state vector K_{1t} from its posterior distribution using the algorithm of Gerlach et al. (2000). The same method is implemented in Koop et al. (2009). This step is repeated for each date t sequentially, allowing to progressively update K_1, but some computations are common to all dates, allowing gains in efficiency (see Gerlach et al. (2000) for details).

- Step 3: Simulation of $p\left(p_1|y^{1..T}, K_1^{1..T}\right)$

The prior $p_1 \sim Be(\underline{b_{11}}, \underline{b_{21}})$ is updated with a draw from the conditional posterior distribution $p\left(p_1|y^{1..T}, K_1^{1..T}\right) \sim Be(\overline{b_{11}}, \overline{b_{21}})$ with $\overline{b_{11}} = \underline{b_{11}} + \sum_{t=1}^{T} K_{1t}$ and $\overline{b_{21}} = \underline{b_{21}} + T - \sum_{t=1}^{T} K_{1t}$.

- Step 4: Simulation of $p\left(Q|y^{1..T}, \beta^{1..T}, K_1^{1..T}\right)$

Q is a diagonal matrix: $Q = \begin{pmatrix} q_1 & 0 & ... & 0 \\ 0 & q_2 & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & q_{25p+5} \end{pmatrix}$.

For each i, the prior $q_i \sim IG\left(\underline{q_i}, \underline{\nu_q}\right)$ is updated with a draw from the conditional posterior distribution $p\left(q_i|y^{1..T}, \beta_i^{1..T}\right) \sim IG\left(\overline{q_i}, \overline{\nu_q}\right)$ with $\overline{q_i} = \underline{q_i} + \frac{1}{2} \cdot \sum \left(\beta_{i,t+1} - \beta_{i,t}\right)^2$ and $\overline{\nu_q} = \underline{\nu_q} + \frac{1}{2} \cdot \sum_{t=1}^{T} K_{1t}$.

- Step 5: Simulation of $p\left(\alpha^{1..T}|y^{1..T}, \beta^{1..T}, \Sigma^{1..T}, S, K^{1..T}\right)$

The model is now rewritten in the form: $A_t \cdot A_{idtf} \cdot (y_t - Z_t \cdot \beta_t) = \Sigma_t \cdot \varepsilon_t$ Introducing the notation $y_t^* \equiv y_t - Z_t \cdot \beta_t$, this system of equations with 5 dependent variables may be written in a linear state-space form:

$$\begin{cases} y_{1,t}^* + y_{4,t}^* = \sigma_{1,t} \cdot \varepsilon_{1,t} \\ y_{2,t}^* - a \cdot y_{3,t}^* = -\alpha_{21,t} \cdot (y_{1,t}^* + y_{4,t}^*) + \sigma_{2,t} \cdot \varepsilon_{2,t} \\ y_{3,t}^* = -\alpha_{31,t} \cdot (y_{1,t}^* + y_{4,t}^*) - \alpha_{32,t} \cdot \left(y_{2,t}^* - a \cdot y_{3,t}^*\right) + \sigma_{3,t} \cdot \varepsilon_{3,t} \\ y_{4,t}^* = -\alpha_{41,t} \cdot (y_{1,t}^* + y_{4,t}^*) - \alpha_{42,t} \cdot (y_{2,t}^* - a \cdot y_{3,t}^*) - \alpha_{43,t} \cdot y_{3,t}^* + \sigma_{4,t} \cdot \varepsilon_{4,t} \\ y_{5,t}^* = -\alpha_{51,t} \cdot (y_{1,t}^* + y_{4,t}^*) - \alpha_{52,t} \cdot (y_{2,t}^* - a \cdot y_{3,t}^*) - \alpha_{53,t} \cdot y_{3,t}^* - \alpha_{54,t} \cdot y_{4,t}^* + \sigma_{5,t} \cdot \varepsilon_{5,t} \\ \alpha_t = \alpha_{t-1} + K_{2,t} \cdot \zeta_t \end{cases}$$

In our empirical application, we assume that S is a diagonal matrix whereas Primiceri (2005) assumes that it is block-diagonal. This assumption allows to treat each equation of the system defining y_t^* independently.

Given a prior on α_0, the Carter and Kohn (1994) algorithm allows us to draw a vector $\alpha^{1..T}$ from the conditional posterior distribution $p\left(\alpha^{1..T} | y^{1..T}, \beta^{1..T}, \Sigma^{1..T}, S, K_2^{1..T}\right)$. This vector is then recasted in matrix form to give us a draw $A^{1..T}$.

- <u>Step 6</u>: Simulation of $p\left(K_{2t} | y^{1..T}, Q, \alpha^{1..T}, S, h^{1..T}, W, K_2^{1..T \setminus \{t\}}, K_1^{1..T}, K_3^{1..T}\right)$

Given a prior on p_2 ($p_2 \sim Be(b_{12}, b_{22})$), we draw the state vector K_{2t} from its posterior distribution using the algorithm of Gerlach et al. (2000). The same method is implemented in Koop et al. (2009). This step is repeated for each date t sequentially, allowing to progressively update K_2, but some computations are common to all dates, allowing gains in efficiency (see Gerlach et al. (2000) for details).

- <u>Step 7</u>: Simulation of $p\left(p_2 | y^{1..T}, K_2^{1..T}\right)$

The prior $p_2 \sim Be(\underline{b_{12}}, \underline{b_{22}})$ is updated with a draw from the conditional posterior distribution $p\left(p_2 | y^{1..T}, K_2^{1..T}\right) \sim Be(\overline{b_{12}}, \overline{b_{22}})$ with $\overline{b_{12}} = \underline{b_{12}} + \sum_{t=1}^{T} K_{2t}$ and $\overline{b_{22}} = \underline{b_{22}} + T - \sum_{t=1}^{T} K_{2t}$.

- <u>Step 8</u>: Simulation of $p\left(S | y^{1..T}, \alpha^{1..T}, K_2^{1..T}\right)$

S is a diagonal matrix: $S = \begin{pmatrix} s_1 & 0 & ... & 0 \\ 0 & s_2 & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & s_5 \end{pmatrix}$.

For each i, the prior $s_i \sim IG\left(\underline{s_i}, \underline{\nu_s}\right)$ is updated with draw from the conditional posterior distribution of $s_i | y^{1..T}, \alpha_i^{1..T} \sim IW\left(\overline{s_i}, \overline{\nu_s}\right)$ with $\overline{s_i} = \underline{s_i} + \frac{1}{2} \cdot \sum \left(\alpha_{i,t+1} - \alpha_{i,t}\right)^2$ and $\overline{\nu_s} = \underline{\nu_s} + \frac{1}{2} \cdot \sum_{t=1}^{T} K_{2t}$.

- <u>Step 9</u>: Simulation of $p\left(s_t | y_{i,t}^{***}, h^{1..T}\right)$

A new state $s_{i,t}$ is drawn for each $i \in \{1,..,3\}$ and each $t \in \{1,..,T\}$ using the probability: $Pr\left(s_{i,t} = j | y_{i,t}^{***}\right) \propto q_j \cdot f_N\left(y_{i,t}^{***} | h_{i,t} + m_j - 1.2704, v_j^2\right)$.

- Step 10: Simulation of $p\left(h^{1..T} | y^{1..T}, A^{1..T}, \beta^{1..T}, s^{1..T}, W, K^{1..T}\right)$

Introducing the notation $y_t^{**} \equiv A_t \cdot A_{idtf} \cdot (y_t - Z_t \cdot \beta_t)$, $y_t^{***} \equiv \log\left({y_t^{**}}^2\right)$ and $\varepsilon_t^{***} = \log\left({\varepsilon_t}^2\right)$, the model may be rewritten in the state-space form:

$$\begin{cases} y_t^{***} = h_t + \varepsilon_t^{***} \\ h_t = h_{t-1} + K_{3t} \cdot \eta_t \end{cases}$$

Following Kim et al. (1998), the distribution of $\varepsilon_{i,t}^{***}$ for $i \in \{1,..,3\}$ is approximated by a mixture of 7 normals: $p\left(\varepsilon_{i,t}^{***}\right) \approx \sum q_j \cdot f_N\left(\varepsilon_{i,t}^{***} | m_j - 1.2704, v_j^2\right)$ where the parameters q_j, m_j and v_j are given by Kim, Shephard and Chib (1998). Concretely, we introduce 7 states for each component of the vector y_t^{***}: $s_{i,t} \in \{1,..,7\}$. Conditional on being in one of these states, $\varepsilon_{i,t}^{***}$ follows a normal law: $\varepsilon_{i,t}^{***} |\ s_{i,t} = j \ \sim \ N\left(m_j - 1.2704, v_j^2\right)$.

Given priors on h_0, the Carter and Kohn (1994) algorithm allows us to draw vectors $h^{1..T}$ from the conditional posterior distribution $p\left(h^{1..T} | y^{1..T}, A^{1..T}, \beta^{1..T}, s^{1..T}, W, K^{1..T}\right)$.

Eventually, the draw on $h^{1..T}$ is transformed into a draw on $\sigma^{1..T}$ using the formula $\sigma_{i,t} \equiv \exp\left(\frac{h_{i,t}}{2}\right)$.

- Step 11: Simulation of $p\left(K_{3t} | y^{1..T}, Q, \alpha^{1..T}, S, h^{1..T}, W, K_3^{1..T\backslash\{t\}}, K_1^{1..T}, K_2^{1..T}\right)$

Given a prior on p_3 ($p_3 \sim Be(b_{13}, b_{23})$), we draw the state variables K_{3t} from their posterior distribution using the algorithm of Gerlach et al. (2000). The same method is implemented in Koop et al. (2009). This step is repeated for each date t sequentially, allowing to progressively update K_3, but some computations are common to all dates, allowing gains in efficiency (see Gerlach et al. (2000) for details).

- Step 12: Simulation of $p\left(p_3 | y^{1..T}, K_3^{1..T}\right)$

The prior $p_3 \sim Be(\underline{b_{13}}, \underline{b_{23}})$ is updated with a draw from the conditional posterior distribution $p\left(p_3 | y^{1..T}, K_3^{1..T}\right) \sim Be(\overline{b_{13}}, \overline{b_{23}})$ with $\overline{b_{13}} = \underline{b_{13}} + \sum\limits_{t=1}^{T} K_{3t}$ and $\overline{b_{23}} = \underline{b_{23}} + T - \sum\limits_{t=1}^{T} K_{3t}$.

- Step 13: Simulation of $p\left(W | y^{1..T}, h^{1..T}, K_3^{1..T}\right)$

W is a diagonal matrix: $W = \begin{pmatrix} w_1 & 0 & ... & 0 \\ 0 & w_2 & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & w_5 \end{pmatrix}$.

For each i, the prior $w_i \sim IG\left(\underline{w_i}, \underline{\nu_w}\right)$ is updated with draw from the conditional posterior distribution of $w_i | y^{1..T}, h_i^{1..T} \sim IW\left(\overline{w_i}, \overline{\nu_w}\right)$ with $\overline{w_i} = \underline{w_i} + \frac{1}{2} \cdot \sum \left(h_{i,t+1} - h_{i,t}\right)^2$ and

$\overline{\nu_w} = \underline{\nu_w} + \frac{1}{2} \cdot \sum_{t=1}^{T} K_{3t}$.

D.3 Specification of the priors

Following Canova and Ciccarelli (2009), priors on α_0, β_0, h_0, Q, S and W are specified using OLS estimates on the sample where the TV-SVAR model is also estimated[40].

$$
\begin{aligned}
\beta_0 &\sim N\left(\hat{\beta}_{OLS}, 4 \cdot \hat{V}\left(\hat{\beta}_{OLS}\right)\right) \\
\alpha_0 &\sim N\left(\hat{\alpha}_{OLS}, 4 \cdot \hat{V}\left(\hat{\alpha}_{OLS}\right)\right) \\
h_0 &\sim N\left(2 \cdot \log \hat{\sigma}_{OLS}, 4 \cdot \hat{V}\left(\hat{\sigma}_{OLS}\right)\right)
\end{aligned}
$$

Q, S and W are diagonal matrices with diagonal elements q_i, s_i and w_i

$$
\begin{aligned}
q_i &\sim IG\left(k_q^2 \cdot \nu_q \cdot \hat{V}_{i,i}\left(\hat{\beta}_{OLS}\right), \nu_q\right) \\
s_i &\sim IG\left(k_s^2 \cdot \nu_s \cdot \hat{V}_{i,i}\left(\hat{\alpha}_{OLS}\right), \nu_s\right) \\
w_i &\sim IG\left(k_w^2 \cdot \nu_w \cdot \hat{V}_{i,i}\left(\hat{\sigma}_{OLS}\right), \nu_w\right)
\end{aligned}
$$

$\hat{V}_{i,i}\left(\hat{\beta}_{OLS}\right)$ and $\hat{V}_{i,i}\left(\hat{\alpha}_{OLS}\right)$ correspond, respectively, to the diagonal elements of matrices $\hat{V}\left(\hat{\beta}_{OLS}\right)$ and $\hat{V}\left(\hat{\alpha}_{OLS}\right)$.

[40] More precisely, we use OLS estimates on the 1980Q1-2007Q2 sample, ending just before the Great Recession.

E Bayesian model selection criteria

Following Koop et al. (2009) who estimate similar models, we compute two model selection criteria: the marginal likelihood and the expected log-likelihood. In fact, the marginal likelihood of a model is often difficult to compute and may be more sensitive to prior information than to posterior information, especially in highly parameterized models such as TV-VARs. The computation of the expected log-likelihood is advocated by Carlin and Louis (2000) and can be considered as a robustness check.

E.1 Marginal likelihood

In order to compute the marginal likelihood, we use the modified harmonic mean estimator of Gelfand and Dey (1994). This estimator derives from the equality:

$$m\left(y\right)=\left[\int\frac{1}{L\left(y|\theta\right)}\frac{f\left(\theta\right)}{\pi\left(\theta\right)}p\left(\theta|y\right)d\theta\right]^{-1}$$

where $m\left(y\right)$ is the marginal likelihood, $L\left(y|\theta\right)$ the (conditional) data likelihood, $\pi\left(\theta\right)$ the prior density, $p\left(\theta|y\right)$ the posterior density and f any function such that $\int f(\theta)d\theta=1$. It takes the following form:

$$\hat{m}\left(y\right)\equiv\left[\frac{1}{N}\sum_{i=1}^{N}\frac{1}{p\left(y|\theta^{(i)}\right)}\frac{f\left(\theta^{(i)}\right)}{\pi\left(\theta^{(i)}\right)}\right]^{-1}$$

where $\theta^{(i)}$ is a draw from the posterior distribution $p\left(\theta|y\right)$ simulated by the Gibbs sampler.

The weighting factor $\frac{f(\theta)}{\pi(\theta)}$ must be chosen so that the random variable $\frac{1}{L(y|\theta)}\frac{f(\theta)}{\pi(\theta)}$ has a finite variance, ensuring that the estimator $\hat{m}\left(y\right)$ is stable and follows a central limit theorem[41]. In practice, following Geweke (1998), f is chosen to be a truncated normal variable with mean and variance equal to the mean and variance of the posterior draws $\theta^{(i)}$.

θ is the vector of model parameters that have to be integrated out. As a first way to reduce the dimension of θ, we use an output of the Gibbs sampling algorithm (step 2, see Appendix D above) where all autoregressive parameters $\beta^{1..T}$ have been analytically integrated out. More precisely, we rely on a by-product of the algorithm of Gerlach et al. (2000): $L\left(y^{1..T}|Q,\alpha^{1..T},S,h^{1..T},W,K_1^{1..T},K_2^{1..T},K_3^{1..T}\right)$.

[41] A non-modified harmonic mean estimator could be defined following the equality:

$$m\left(y\right)=\left[\int\frac{1}{L\left(y|\theta\right)}p\left(\theta|y\right)d\theta\right]^{-1}$$

However, the (conditional) data likelihood may take very low values on some regions of the posterior density. Hence, the assumption of a finite variance for the random variable $\frac{1}{L(y|\theta)}$ may be violated.

Following Justiniano and Primiceri (2008), (1) we choose

$$f(Q,\alpha^{1..T},S,h^{1..T},W,K_1^{1..T},K_2^{1..T},K_3^{1..T}) = f(Q)f(S)f(W)f(\alpha^{1..T},h^{1..T})f(K_1^{1..T},K_2^{1..T},K_3^{1..T})$$

and (2), for computational convenience, we set

$$f(\alpha^{1..T},h^{1..T}) = \pi(\alpha^{1..T},h^{1..T})$$

This choice is only motivated by the fact that the dimension of matrices $\alpha^{1..T}$ and $h^{1..T}$ is large.

Moreover, $K_1^{1..T}$, $K_2^{1..T}$ and $K_3^{1..T}$ are discrete variables, with values 0 or 1. Hence it seems unlikely that a severe instability in the computation of the marginal likelihood can originate from these K parameters. That is why we set

$$f(K_1^{1..T},K_2^{1..T},K_3^{1..T}) = \pi(K_1^{1..T},K_2^{1..T},K_3^{1..T})$$

E.2 Expected log-likelihood

Carlin and Louis (2000) (p.220) recommend to use the expected log-likelihood as an overall measure of model fit to be compared across models. In practice, this function of interest is computed using posterior samples:

$$\hat{l}\,(y) \equiv \frac{1}{N}\sum_{i=1}^{N} \log L\left(y|\theta^{(i)}\right)$$

We do not consider penalizing the expected log-likelihood for the number of parameters because all the models that we consider are of the same size. They only differ in the specification of the prior distributions.

F Convergence of the Gibbs sampling algorithm

This appendix addresses the convergence of the Gibbs sampler in the benchmark specification. Results are essentially the same in the alternative specifications. We apply two types of convergence checks to the three classes of model parameters (autoregressive parameters, parameters of the impact matrix A_t and variances of the orthogonalized shocks).

We first compute the 20-th-order autocorrelations of the parameters (figure 22). They all remain below 0.2 (with only one exception, still below 0.3), which is usually considered as satisfactory.

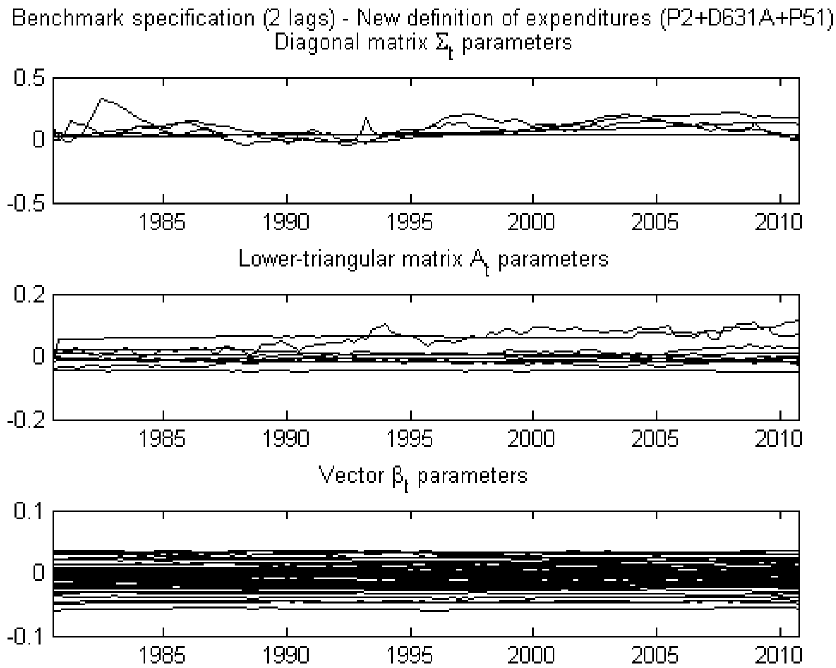

Figure 22: 20-th-order autocorrelations of the simulated parameters.

Our second convergence diagnostic consists in computing inefficiency factors (IFs, figure 23). The IF is the inverse of the relative numerical efficiency measure of Geweke (1992), i.e. $IF = 1 + 2\sum_{k=1}^{+\infty} \rho_k$, where ρ_k is the k-th autocorrelation of the chain. An IF close to 1 suggests that draws of the Gibbs sampler are almost independent. Values of the IFs below or around 20 are regarded as satisfactory, according to Primiceri (2005). In practice, IFs are estimated using a Bartlett kernel and an automatic bandwidth selection procedure (Andrews 1991). The values of the IFs are all beneath 20, indicating a good convergence of the Gibbs sampler.

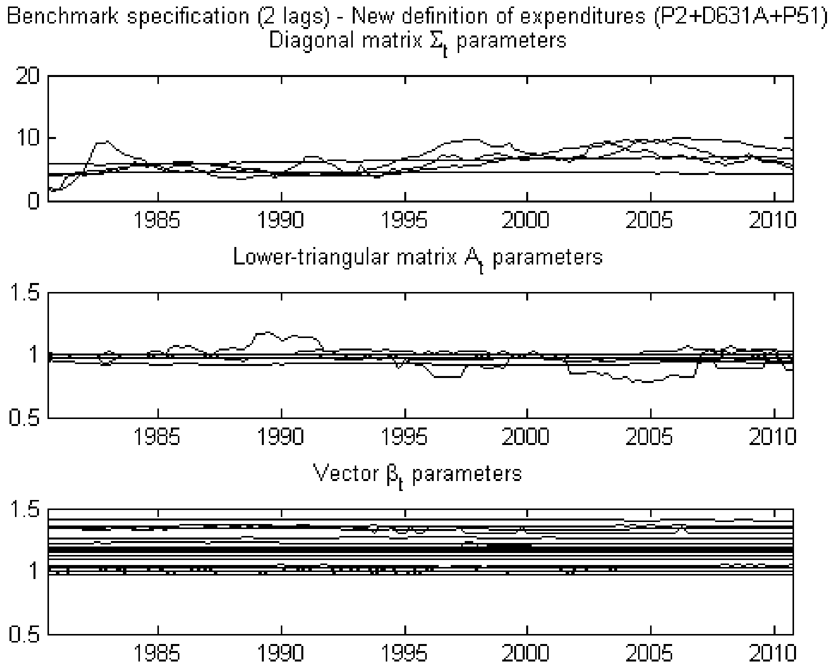

Figure 23: Inefficiency factors of the simulated parameters.

G Results from a SVAR model without inflation or interest rates as endogenous variables

The long run spending multiplier remains significantly positive, with a point-estimate above 1, in a 3-variable VAR (excluding prices and interest rates) whereas it reverts to 0 with a 5-variable VAR (figures 24 and 25). Multipliers computed with the two models may be different for two reasons: either the coefficients on the first 3 variables are biased when the last 2 variables are omitted from the model (omitted variable bias), or there are relevant feedback variables excluded from the 3-variable VAR. When the lagged coefficients on prices and interest rates are put to zero in the GDP equation of the 5-variable VAR, the long run spending multiplier remains significantly positive (figure 26), with a point-estimate close to the one found in the 3-variable VAR (around 1.5)[42]. Thus, differences come from the inclusion of a supplementary feedback loop.

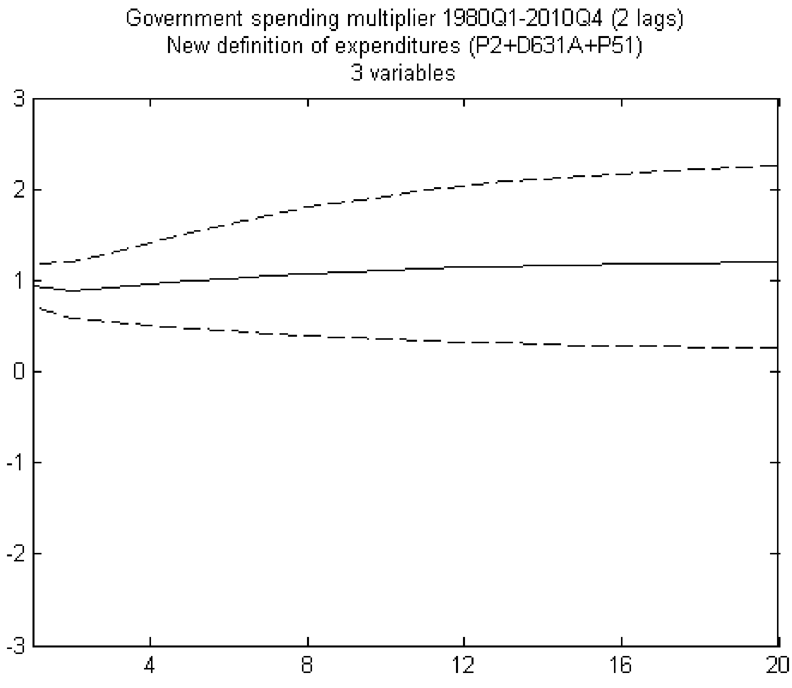

Figure 24: Government spending multiplier of a 3-variable VAR.

[42]In all cases, multipliers and 68% confidence intervals are computed using Kilian's (1998) bootstrap-after-bootstrap.

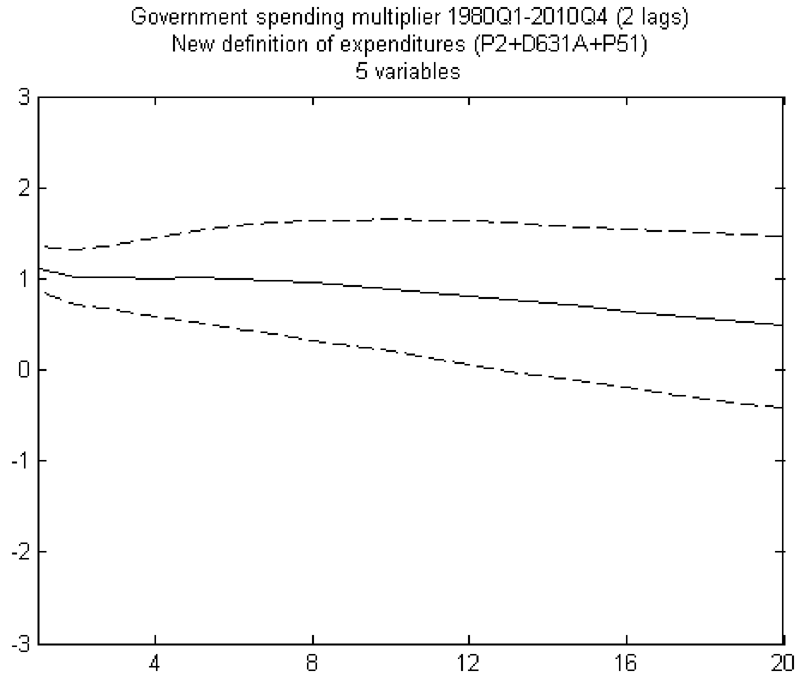

Figure 25: Government spending multiplier of a 5-variable VAR.

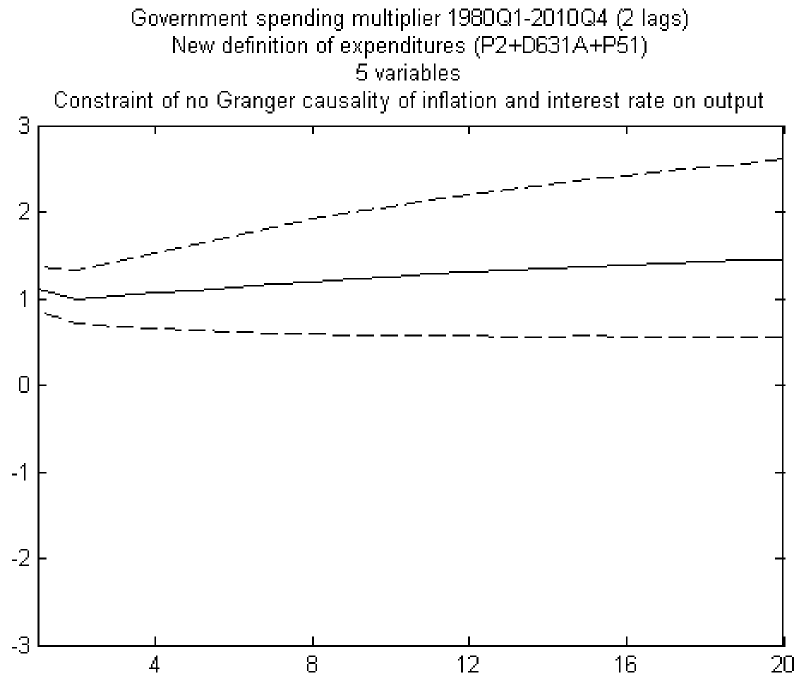

Figure 26: Government spending multiplier of a 5-variable VAR without feedback effects from inflation and interest rate.

Documents de Travail

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