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Honoring Sovereign Debt or Bailing Out Domestic Residents? A Theory of Internal Cost of Default.

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Résumé

Le coût interne du défaut, un des facteurs importants conduisant les pays à repayer leur dette souveraine, s'accroissent avec le bais domestique des portefeuilles des résidents. En conséquence, lorsqu'il souhaite accentuer ce biais en mettant en place des contrôles de capitaux, un pays met en regard les bénéfices en termes de crédibilité avec les pertes liées à une moindre diversification des portefeuilles. Cependant, pourquoi le souverain n'évite-t-il pas ce coût interne du défaut, par exemple en sauvant ses résidents avec des transferts ? Pourquoi, par ailleurs, ne met-il pas lui-même en place la diversification des portefeuilles domestiques, par des swaps ou par n'importe quelle autre stratégie de couverture ? Répondre à ces deux questions permet de mieux saisir la nature du coût interne du défaut. Ce papier étudie la soutenabilité de la dette souveraine dans un modèle où les investisseurs domestiques et étrangers choisissent leurs portefeuilles et où le souverain détermine son niveau de dette, ses décisions de défaut et de sauvetage de ses résidents. Il détermine sous quelles conditions les plans de sauvetage n'empêchent pas le paiement souverain et établit quand, malgré leur gains en termes de crédibilité, les contrôles de capitaux ne sont pas désirables.

Mots-clés: dette souveraine, coût interne au défaut, plans de sauvetage, contrôles de capitaux.

Classification JEL: F34, G15, G18.

Abstract

The internal cost of default, an important driver of sovereign debt repayment, increases with domestic portfolios' home bias. And so, when using capital controls or other instruments to steer these portfolios, a country faces a trade-off between commitment to repay and diversification. But why does a borrowing country not eschew the internal cost of default through domestic sector bailouts? And why does their sovereign not intermediate the diversification through swaps and other hedging devices? Answering these two questions is key to fathom the nature of internal costs of default. This paper investigates sovereign debt sustainability in a model where domestic and foreign investors optimally select their portfolios and the sovereign optimizes over its debt, default and bailout policies. It derives conditions under which internal bailouts do not preclude sovereign borrowing and establishes when, despite their disciplining benefits, capital controls are undesirable.

Keywords: sovereign debt, internal cost of default, bailouts, capital controls.

JEL numbers: F34, G15, G18.

Executive summary

This paper studies sovereign debt sustainability when governments only care about the collateral damage of a default on their domestic residents. To this purposes, it builds a model where: 1) residents use public debt as private liquidity, 2) the government is willing to default on its debt to expropriate foreign debt holders, 3) the government cannot discriminate among debt holders but it can implement transfers to domestic residents (i.e. domestic bailouts). I

The paper's main result is to identify a necessary and sufficient friction that leads governments to honor their debts: the inability to precisely observe individual domestic exposures to domestic debt. In other words, sovereign repayment derives from the fear of the messy consequences of a default and from the government's inability to compensate for those consequences. More precisely, the paper shows default costs increase both the level of domestic exposures and the heterogeneity of those exposures.

In addition, this paper explores the desirability of capital controls. Those controls allow to increase the home bias of domestic portfolios and, hence, boost the country's ability to borrow, but this comes at the cost of a lack of diversification of portfolios. Here, the paper main contribution is to show that the government is not necessarily able to intermediate this diversification, e.g. using swaps, as it cannot necessarily commit to honor its debt in good states of nature, when receiving revenues. This result sheds light, for example, on the possibility of contingent sovereign bonds.

In the end, the paper's main point is to relate government's debt creditworthiness with financial opacity. Financial opacity, by making domestic portfolios more difficult to assess, prevents government to costlessly default by perfectly reallocating resources. Empirically, this suggests that debt repayment is related to the dispersion, the opaqueness and complexity of domestic exposures. Sovereign credibility would be therefore enhanced by opaque domestic interbank or OTC derivatives markets. According to the model, "financial fragility" (e.g. potential domestic contagion of losses), if costly *ex post*, is desirable *ex ante* for sovereign credibility.

1 Introduction

Reputation concerns in explaining country repayment in a world of limited sanctions have been a major focus of scholarly work on sovereign debt (e.g. Eaton and Gersovitz, 1981; Bulow and Rogoff, 1989; Hellwig and Lorenzoni, 2009). This paper analyses an often-discussed alternative motive for honoring one's sovereign liabilities: the internal cost inflicted by a default on the country's private sector. Sovereign default does not only expropriate foreign investors, it also affects domestic residents and companies holding government securities[1].

The internal-cost-of-default theory of sovereign credibility rests on the premise that a country cannot default selectively on foreign-owned debt. A sovereign can, however, replicate such a selective default by defaulting wholesale and by compensating domestic residents and companies by an amount equal to their losses[2]. Yet, this ability to compensate domestic residents may be hindered by information scarcity as a perfect bailout would involve tracking not only domestic *holdings* but also all domestic *exposures* to domestic debt, both direct and indirect (CDS and other derivative instruments, private sector exposures to foreign institutions or foreign subsidiaries that might be jeopardized by the default, etc.).

Furthermore, the internal-cost-of-default theory advocates for capital controls[3], as the resulting home bias in domestic portfolios boosts the country's ability to borrow. This neglects the potential cost of a lack of diversification in domestic portfolios. However, the diversification may be intermediated by the sovereign, using swaps or other hedging devices, and the desirability of capital controls hinges on the government's ability to perform such an intermediation.

This paper investigates sovereign debt credibility in a model where domestic and foreign investors optimally select their portfolios and where the sovereign optimizes over default and internal bailout policies with only limited information on domestic exposures. It derives conditions under which the country can borrow abroad and determines when capital controls are desirable for boosting country's borrowing.

Section 2 builds a model of internal cost of default. At date 0, the government needs to borrow money and issues bonds to this purpose; the price fetched by these bonds is endogenous and depends on repayment expectations. The country's private sector, described as a set of firms needing cash at date 1 to finance a decreasing-returns-to-scale investment, and foreign investors decide whether to buy these bonds or to go for a safe alternative abroad. When

[1] Cf. Guembel and Sussman (2009) or Broner, Martin, and Ventura (2010) among others.

[2] An example of a - imperfect - bailout after a default is the so-called "corralito", the banks' deposits convertibility suspension decided in Argentina in december 2001.

[3] Cf. Broner and Ventura (2011) among others.

indifferent between these two options, domestic entrepreneurs may randomize their positions, allowing for *ex post* portfolio heterogeneity. At date 1, the government makes two decisions: whether to engage in (non-selective) default and, in case of default, whether to bail out the domestic sector. The efficiency of a bailout is limited by the government's available information about individual portfolio positions. Portfolio choices depend on the expectation of repayment and, for domestic residents, on the prospect of a bailout in case of default (section 3). When being indifferent between portfolios, investors may randomize, and so, *ex ante* homogeneous domestic entrepreneurs may differ in their *ex post* positions. Conversely, the internal cost of default, and therefore the country's incentive to default and bail out depends on past foreign and domestic portfolio allocations (section 4). This results in both a feedback loop between portfolio allocations and policies and a coordination problem among domestic and foreign investors.

Our first insight is that domestic bailouts do not preclude sovereign repayment (section 5) as long as government's information on domestic exposures remains imperfect[4]. Bailouts only provide a limited insurance against a default, and the resulting misallocation of resources due to domestic holdings incentivizes the country to honor its debt, making it attractive for foreign investors. Thus heterogeneity in domestic portfolios exacerbates the inability to target transfers accurately, and so, improves further the country's credibility[5]. Domestic bailouts provide residents with some liquidity insurance as well, making up for risk aversion, and thereby bolster domestic holdings of risky government securities. This risk-management effect may actually boost the country's access to international borrowing.

Our second insight is that capital controls (section 6), while making the government more accountable, come at the cost of a lack of international diversification of domestic portfolios. A country-level hedging policy cannot substitute for individual residents' desirable portfolio diversification, as hedging repayment adds to outstanding debt: when this debt is already large or when domestic shocks are likely, the sovereign is prone to renege on its commitment.

In the end, this paper argues that sovereign credibility relies on misallocation of resources resulting from a default. This is consistent with recent findings that internal costs of default of sovereign defaults do not necessarily derive from factor employment but from falls in total factor productivity (cf. Sandleris and Wright, 2014; Wright, 2014). Furthermore, the misallocation of resources stems, in this paper, from imperfect financial markets associated with the government's inability to reallocate resources through transfers, due to portfolio non-

[4]With perfect information, the government always defaults and perfectly bails out domestic residents.

[5]Unobservable but homogeneous portfolios would allow the government to perfectly bail out domestic residents, as it would have sufficiently accurate information.

observability. A further testable implication of the theory would be that sovereign repayment is positively correlated with the dispersion, the opaqueness[6] and complexity of domestic exposures. Sovereign credibility would be therefore enhanced by domestic interbank or OTC derivatives markets associated with a large dispersion of bond holdings and with the country's degree of financial integration. According to the model, "financial fragility" (e.g. potential domestic contagion of losses), if costly *ex post*, is desirable *ex ante*[7].

Related literature This paper's main concern is the connection between bailouts and international capital flows as, for example, Schneider and Tornell (2004). Their emphasis is, however, on the effects of external bailouts, that is, bailouts of foreign investors, while this paper emphasizes the role of domestic bailouts on domestic sovereign borrowing.

Foreign borrowing has been the object of a large literature (e.g. Eaton and Gersovitz, 1981; Bulow and Rogoff, 1989; Hellwig and Lorenzoni, 2009). The impact of reneging on domestic agents goes back at least to Cole and Kehoe (1998), where a sovereign default sends a negative signal to domestic agents, who in turn adapt their behavior, making the option of default less worthy. Recent contributions include Guembel and Sussman (2009) who consider the political economy cost of internal redistribution, or Brutti (2011) who introduce an internal cost based on domestic liquidity needs[8]. My approach differs from theirs as the government's repayment incentives do not derive from factor employment (e.g. investment) but from misallocation of resources as the government is unable to efficiently compensate domestic losses. Finally, Mengus (2013a) establishes a general connection between foreign debt sustainability and domestic fiscal policies, thus generalizing this paper's insights. Finally, this paper rules out selective defaults on foreign-owned sovereign debt because of asymmetric information on portfolios in contrast with Broner et al. (2010) who emphasize the role of secondary markets (Cf. Section 7.2 for a more detailed discussion).

My approach shares with Farhi and Tirole (2012), Philippon and Skreta (2012) and Tirole (2012) the idea that bailouts are costly because of asymmetric information, and applies it to investigate its implications for sovereign debt repayment.

The use of public debt for macroeconomic liquidity needs may stem from several causes, such as, here, an non-synchronicity between endowment and investment opportunities *à la*] Woodford (1990) or Holmstrom and Tirole (1998)[9].

[6]Of course, OTC/opaque markets have costs in terms of prudential monitoring of banks; but it is interesting to note that transparency has this unintended negative consequence.

[7]This contrasts with Broner et al. (2010) who emphasize the need for liquid secondary markets.

[8]Cf. also Basu (2009), Bolton and Jeanne (2011), Simon (2012) or Gennaioli et al. (2011).

[9]For empirical evidence, cf. Krishnamurthy and Vissing-Jorgensen (2012).

The problem of coordination among investors for sovereign debt was highlighted in Wright (2005)'s analysis for punishment schemes. Here punishments are endogenously chosen as a function of the final asset allocation. A by-product of this coordination problem is multiple equilibria as in Kareken and Wallace (1981), Calvo (1988) or Chang and Velasco (2000).

Finally, usually studied for their prudential role against potential over-borrowing, capital controls allow to increase the government's borrowing capacity as in Broner and Ventura (2011)[10].

2 The environment

Consider a two-period model of a small economy. This economy is populated by a government and a continuum of domestic entrepreneurs. The rest of the world consists of foreign investors. There is a single tradable and non-storable good. I denote by $t = 0, 1$ the two dates.

2.1 The agents

Domestic entrepreneurs The domestic entrepreneurs[11] are risk-neutral and make decisions so as to maximize utility $u(c_0, c_1) = c_0 + c_1$ where c_t is their consumption at date t.

Each entrepreneur receives an endowment of 1 unit of good in period 0. He has access either in period 0 or in period 1 to a concave production technology: $F(I) = \rho_1 \min(1, I) + \rho_2 \max(0, I - 1)$ where I denotes what he has invested. I assume that $\rho_1 > \rho_2 > 1$, i.e. F is concave. The concavity of the production function introduces entrepreneurial risk aversion. The piecewise linear form of the production function is assumed for tractability and involves no loss of generality (cf. appendix). Entrepreneurs are privately informed in period 0 whether they can produce in period 0 or 1. In the end, a mass 1 of entrepreneurs receives their investment opportunity in period 1 and a mass $\nu \geq 0$ receives the investment in period 0.

I assume that this production technology's income can be fully concealable. As a result, entrepreneurs cannot borrow and cannot be taxed by the government after having produced.

To transfer wealth from period 0 to period 1, entrepreneurs may purchase either domestic public bonds or foreign risk-free bonds. Entrepreneur i's position in foreign bonds is denoted

[10] See also Kremer and Mehta (2000), Tirole (2003) or Wright (2006).

[11] Without loss of generality, this can also be banks with strictly positive pledgeable income that provide loans to firms or simply risk-averse households.

by $x^i \in [0,1]$ and so he invests $1 - x^i$ in domestic public bonds. As the price of public bonds is p, investing $1 - x^i$ domestically allows to hold $z^i = (1 - x^i)/p$ domestic public bonds.

Aggregation I define, respectively, aggregate investment I, aggregate portfolio decision x and aggregate entrepreneurs' domestic bond holdings Z^e as:

$$I = \int_0^1 I^i di, \quad x = \int_0^1 x^i di, \quad Z^e = \int_0^1 z^i di \tag{1}$$

H denotes the equilibrium cumulative distribution function of the x^is and h is its density.

Finally, I assume that the weight assigned by the government to each entrepreneur is identical across entrepreneurs, so we only need to consider the aggregate welfare of entrepreneurs:

$$W^e = \int_0^1 F(I^i) di.$$

Foreign investors Foreign investors are risk-neutral agents endowed in period 0 with an infinite wealth. Their utility function is: $u(c_0^*, c_1^*) = c_0^* + c_1^*$ where c_t^* is date-t consumption. Throughout the paper, the superscript * refers to foreign investors.

Foreign investors can also purchase domestic public bonds and/or foreign bonds to transfer wealth from period 0 to period 1. I denote by Z^* the external debt, namely the nominal repayment foreign investors are entitled to because of their holdings of domestic public bonds.

Asset markets As mentioned above, domestic entrepreneurs and foreign investors trade only two assets in period 0: foreign and domestic public bonds.

Foreign bonds are risk-free bonds in unlimited supply. Each yields one unit of good in period 1 for one unit of good invested in period 0.

Domestic public bonds are traded in period 0 at the endogenous price p. Each of them promises a nominal repayment of 1 in period 1. Yet, actual repayment depends on the government's date-1 decisions. Agents form expectations by anticipating the government's best response and trade domestic bonds at a price consistent with their beliefs. As they share the same information, domestic entrepreneurs and foreign investors also share the same beliefs.

Government The government has access to a deterministic production technology: by investing $G \leq \overline{G}$. $\overline{G}$ is sufficiently large so that this constraint will not be binding in equilibrium. in period 0, it produces RG units of goods in period 1, with $R > 1$. The government has no resources in period 0 and must therefore borrow from domestic entrepreneurs and/or foreign investors.

This government's investment opportunity summarizes the rest of the economy. More specifically, it may be interpreted as the supply of a public good and the return R then includes the possible cost of taxes.

However the government cannot commit to repay its debt and contracts are not enforceable.

In period 0, the government issues Z bonds. Z is divided between issuances to domestic and foreign investors, so that $Z = Z^e + Z^*$.

In period 1, the government chooses its policies so as to maximize *ex post* domestic welfare W. Domestic welfare includes both the welfare of entrepreneurs and the production of the government, net of repayments. This can be summarized by:

$$W = RG - P + \beta W^e \tag{2}$$

where P are the total payments to both foreign investors and domestic entrepreneurs and $\beta > 0$ denotes the government's weight assigned to domestic entrepreneurs.

In period 1, the government can renege on its debt. Without loss of generality[12], I only consider complete default and π denotes the endogenous government's repayment probability. In addition, when defaulting, the government may choose to bail out domestic entrepreneurs by implementing transfers. B^i denotes the transfer to domestic entrepreneur i from the government.

Non-observability of portfolios I assume that the government cannot observe promised repayment to an individual bondholder (formally the $1 - x^i$'s) but only the aggregate distribution of promised repayment to domestic investors (formally H)[13]. Furthermore, I assume that the government cannot observe entrepreneurs' production opportunity.

As a result, the government is unable to discriminate between domestic and foreign lenders when repaying, precluding selective default on the fraction of debt held abroad. Also, when bailing out, the government has to implement a uniform transfer $B^i = B$ that is not contingent to domestic entrepreneurs portfolio (x^i) (cf. Proposition 5).

To summarize, the government's payment P equals Z when honoring its debt and, when defaulting and bailing out domestic entrepreneurs, this payment equals $(1+\nu)B$, where $1+\nu$ is the total mass of entrepreneurs.

[12] Section 7 extends the analysis to partial default.

[13] Section 7 provides further foundations and discussion for this assumption. The analysis can also be easily extended to the case where the government chooses its policies using beliefs on the aggregate distribution. In equilibrium, the set of equilibria of this extended game is exactly the set of equilibria of the game considered here.

Finally, I make the following assumption throughout this paper:

Assumption 1. $\beta\rho_2 < 1$ and $\beta\rho_1 > 1 + \nu \geq 1$.

This makes suboptimal to finance (under symmetric information) domestic entrepreneurs who have enough cash to fund their most profitable activities (i.e. with return ρ_1), and optimal (under both symmetric and asymmetric information) to finance them when they do not have enough cash.

In addition, I assume that R, the slope of the government's production function, is sufficiently large to allow for repayment or bailout. Without any further mention, this means that I consider only strategic default. Section 6 relaxes this assumption by considering stochastic government's production functions.

2.2 Description of the game and equilibrium definition

Timing The timing of the game is summarized by Figure 1.

Strategies and equilibrium At date 0, domestic entrepreneurs and foreign investors compare their beliefs on government's future moves (π, B) and the price of domestic bonds on the market (p) to choose portfolios (the x^i's). Section 3 describes this portfolio allocation.

Based on the portfolio allocation, the government chooses in period 1 whether to default (π) and how much it transfers (B). Section 4 characterizes these choices.

Section 5 determines the equilibrium outcome of the no-commitment game between investors and the government.

3 Portfolio allocations

This section characterizes domestic and foreign investors' optimal portfolio allocation.

Domestic Entrepreneurs Each entrepreneur i infers beliefs about future policies $\{\pi, B\}$ from the portfolios' distribution (h). Depending whether they have an investment opportunity in period 0 or in period 1, entrepreneurs are willing to save to be able to invest.

Date-0 investment opportunity Entrepreneurs with an investment opportunity in date-0 invest their endowment in period 0 and consume.

Date-1 investment opportunity Given these beliefs, the program is:

$$\begin{aligned} \max E_0 u(c_0^i, c_1^i) &= c_0^i + E_0 c_1^i, \\ \text{s.t. } c_0^i + x^i + pz^i &= 1, \\ \text{if no default: } c_1^i + I^i &= F(I^i) + x^i + z^i, \\ \text{if default with bailout } B\text{: } c_1^i + I^i &= F(I^i) + x^i + B. \end{aligned}$$

As a result, entrepreneurs invest everything in period 0 ($c_0^i = 0$) and the demand function for domestic bonds $1 - x^i$ is:

$$\begin{aligned} &1 - x^i = 0 \text{ if } \; p > \pi \; ; \; 1 - x^i = [0, B] \text{ if } \; p = \pi \; ; \\ &1 - x^i = B \text{ if } p \in [\overline{p}, \pi] \; ; \; 1 - x^i = [B, 1] \text{ if } p = \overline{p} \text{ and } 1 - x^i = 1 \text{ if } p < \overline{p}, \end{aligned}$$

with

$$\overline{p} = \frac{\pi \rho_2}{\pi \rho_2 + (1 - \pi)\rho_1} \leq \pi. \tag{3}$$

When choosing their portfolios, domestic entrepreneurs take into account two factors: a speculative motive as they purchase domestic bonds depending on their expectations of default (π) and a liquidity motive as they try to guarantee a minimum of resources in every state of nature. This latter motive shows up when government's promised repayment exceeds the bailout B. In this case, a default corresponds to less liquidity for them. To determine their holdings above B, they compare the expected marginal profit of holding x^i (or conversely the cost to hold $1 - x^i$) without default: $\pi \rho_2 (1 - 1/p)$, with the expected marginal profit in case of default: $(1 - \pi)\rho_1$.

Figure 2 plots this demand function given arbitrary values for π and B. When the price is sufficiently low, domestic entrepreneurs purchase only domestic bonds. This corresponds to the *speculative demand*. When the price increases, domestic entrepreneurs purchase domestic bonds insofar they are insured against the default by the bailout. This corresponds to the *liquidity demand*.

Interestingly, by providing insurance, bailouts limit the flight to quality towards safer foreign bonds. This role of bailouts arises as long as domestic agents have a demand for liquidity, either because they are risk-neutral entrepreneurs with a concave production function as in this paper's framework or because they are risk-averse households.

Finally, when $p = \pi$ or when $p = \overline{p}$, entrepreneurs are indifferent between a large range of portfolios. This indifference allows entrepreneurs to randomize their domestic debt position leading possibly to *ex post* heterogeneity, even though entrepreneurs are *ex ante* homogeneous.

Foreign investors Foreign investors also compare foreign assets' yields (1 invested yields 1) with domestic public bonds' yields (1 invested yields π in expectation). Thus, foreign investors purchase domestic bonds if and only if $p \leq \pi$.

Aggregation As $\overline{p} \leq \pi$ with equality if and only if $\pi = 1$, foreign investors always accept to pay a higher price than domestic entrepreneurs to purchase domestic bonds.

Because of randomization, the portfolio distribution h may be non-degenerate and, more generally, can be any distribution on $[0, 1]$.

4 Government's optimal response

In period 1, the government faces nominal repayment Z to both domestic and foreign lenders.

If the government were able to discriminate between domestic and foreign bondholders, it would default selectively on foreign-owned bonds and repay its domestic bondholders as there are no *ex post* incentives to repay[14].

However, even when unable to discriminate bondholders, if the government were able to perfectly monitor portfolios, the government can replicate the selective default by defaulting wholesale and perfectly compensate every domestic entrepreneurs exactly up to the losses implied by the default, i.e. by transferring $B^i = 1 - x^i$ to entrepreneur i. In this case, a default followed by a bailout would be equivalent to a selective default, as they both leave domestic entrepreneurs unaffected. In equilibrium, investors would anticipate the default and would not lend the government as well.

The inability to observe portfolios introduces an internal cost which may allow external debt. Under no-discrimination, the values obtained from default and repayment may differ. These values can be written as follows, in case of no-default:

$$W_1 = RG - Z + \beta \int_0^1 F(z^i + x^i) di, \tag{4}$$

in case of default:

$$W_0 = RG - (1 + \nu(1 - \beta))B + \beta \int_0^1 F(B + x^i) di. \tag{5}$$

The *internal cost of default* corresponds to the difference between these two options.

[14]This conclusion can be extended to infinite horizon environments using Bulow and Rogoff (1989)'s argument, i.e. when the small open economy has to option to save abroad.

Government's problem The government chooses π and B so as to maximize:

$$\pi W_1 + (1-\pi) W_0(B)$$

I characterize first the optimal bailout in case of default and then the decision to default itself.

4.1 Optimal bailouts

In case of default, the government has the option to bail out domestic firms. The transfer chosen by the government maximizes W_0 with respect to B. The following proposition describes the outcome of this maximization:

Proposition 1. *The optimal bailout B is* $\max\{b, H(1-b) \geq 1-\hat{x}\}$ *with*
$\hat{x} = \frac{\beta\rho_1 - 1 - \nu(1-\beta)}{\beta(\rho_1-\rho_2)} \in (0,1)$.
As a consequence, $B \leq 1$.

Proof. See appendix. □

The government chooses a bailout $B > 0$ only if there are enough domestic entrepreneurs who invested enough domestically. The two conditions matter: the government chooses not to implement large bailouts ($B << 1$) either if a lot of entrepreneurs invested only a little or if only a few invested a lot in domestic bonds.

Because of the concavity of the production function and due to assumption 1, the government does not want to transfer wealth to entrepreneurs who are already able to invest at least 1 in their projects. Recall that the marginal welfare for an entrepreneur who invested x^i in foreign assets is $\beta\rho_1 - 1 - \nu(1-\beta) > 0$ if $x^i + B \leq 1$ but $\beta\rho_2 - 1 - \nu(1-\beta) < 0$ if $x^i + B > 1$. As a consequence, when the government increases the transfer in case of bailout B, welfare rises only if there is more entrepreneurs in the former case than in the latter. $H(1-b)$ measures the number of entrepreneurs with holdings x^i less than $1-b$. Then, as long as, $H(1-b)$ is large enough (i.e. compared with $1-\hat{x}$), the government can increase b. Formally, this condition can be written as follows:

$$H(1-b)\left[\beta\rho_1 - 1 - \nu(1-\beta)\right] + (1 - H(1-b))\left[\beta\rho_2 - 1 - \nu(1-\beta)\right] \geq 0 \quad (6)$$

Finally, the bailout B is the greatest value of b such that $H(1-b)$ is still large enough to obtain welfare gains.

In terms of economic interpretation, the ability to bail out hinges on the government's available information, and so, on the heterogeneity of domestic portfolios, as measured by the distribution H.

Example (No diversification). When domestic entrepreneurs hold either only domestic bonds or only foreign bonds, the average holding of foreign bonds denoted by x corresponds to the fraction of entrepreneurs who hold foreign bonds only. Because $H(1-b) = 1-x$ for $b \in [0,1)$, we obtain that $B = 1$ for $x \leq \hat{x}$ and $B = 0$ otherwise.

Example (Symmetric holdings). When domestic entrepreneurs' holdings are symmetric, the average holding x corresponds to the fraction of each entrepreneur's portfolio invested in foreign bonds. We obtain easily that $B = 1 - x$.

Symmetry implies that the government has perfect information on domestic entrepreneurs' exposure. It can then perfectly bail them out. Some resources are, however, still diverted by entrepreneurs whose investment opportunity occurs at date-0, involving a proportional cost $\nu(1-\beta)B$ for transfers.

4.2 Optimal default

Given an optimal bailout B, the government chooses to default by comparing W_0 and W_1. More precisely it selects a repayment probability π so as to maximize $\pi W_1 + (1-\pi)W_0$. This amounts to comparing the internal cost of default to repayment Z:

$$W_0 \geq W_1 \Leftrightarrow (1+\nu(1-\beta))B + \beta \int_0^1 \left[F(z^i + x^i) - F(B + x^i)\right] di \geq Z \tag{7}$$

Equation (7) clarifies the cost and the benefit of a default. The benefit consists of the absence of repayment Z. The cost is the detrimental effect of the default on domestic entrepreneurs: $\beta \int_0^1 [F(z^i + x^i) - F(B + x^i)]\, di$ combined with the direct cost of the bailout $(1+\nu(1-\beta))B$. Thus, the *internal cost of default* corresponds to the difference for each agent between nominal repayments of domestic public bonds z^i and the transfer received in case of bailout B.

As the option to bail out gives the ability to raise the value of default W_0, the effect of limited information propagates from bailouts to the default decision: the more opaque the domestic economy, the less the government defaults.

Finally, notice that the government chooses $\pi \in (0,1)$ if and only if it is indifferent between repaying and defaulting ($W_1 = W_0$), and so, it may then randomize its decision.

Example (No diversification). Depending on the value of x, inequality (7) becomes:

- $(1+\nu(1-\beta)) + \beta\rho_2(Z^e - 1 - x) \geq Z$ if $x \leq \hat{x}$.
- $\beta\left[\rho_2(Z^e - 1) + \rho_1(1-x)\right] \geq Z$ if $x \geq \hat{x}$.

Notice that when $x \leq \overline{x}$, the bailout makes the internal cost of default lower.

Example (Symmetric holdings). As $B = 1 - x$, inequality (7) can be rewritten as follows:

$$W_0 \geq W_1 \Leftrightarrow (1 + \nu(1 - \beta))(1 - x) + (Z^e - (1 - x))\beta\rho_2 \geq Z$$

If repayment to domestic entrepreneurs Z^e equal $1 - x$, which holds if agents anticipate that the government will repay for sure, this expression boils down to $(1 + \nu(1 - \beta))(1 - x) \geq Z$. In that case, W_0 can be lower than W_1 only when the cost of bailing out entrepreneurs with early investment opportunities $\nu(1 - \beta)$ is strictly positive.

5 Equilibrium

5.1 Commitment

If the government were able to commit, it would repay for sure ($\pi = 1$) and would not bail out ($B = 0$). The price of public bonds would be equal to 1 and domestic public bonds would be risk-free. In turn, domestic entrepreneurs would invest their entire endowments indifferently between the two assets, invest 1 in period 1 and produce ρ_1 in period 2. Foreign investors would be indifferent between domestic and foreign assets. Finally, the government would borrow $\overline{G}$ produce $R\overline{G}$ in period 1. In the end, the government would freely borrow and the allocation of domestic entrepreneurs' portfolios would have no effect.

5.2 No commitment

From now on, the government cannot commit at date 0 to honor its debt at date 1. Investors form expectations $\{\pi, B\}$ on these decisions and select their investment as described in section 3. Let $\Gamma(\pi, B)$ denote the set of probabilities of repayment and bailouts that are consistent with $\{\pi, B\}$: the expectation of $\{\pi, B\}$ leads to some optimal reactions in the form of a distribution of assets h, which in turn generates a correspondence of equilibrium probabilities of repayment and bailouts $\Gamma(\pi, B)$. An equilibrium is then a fixed point of this correspondence. Formally, equilibria of the no-commitment problem consist of beliefs on repayment probability π and expected bailout B such that:

$$\{\pi, B\} \in \Gamma(\{\pi, B\}) \tag{8}$$

Three kinds of equilibria may exist: no-default equilibria where $\pi = 1$, the default equilibrium where $\pi = 0$ and, between the two, a continuum of random default equilibria where $\pi \in (0, 1)$. The following theorem describes the set of equilibria using this classification:

Theorem 1. *For a given repayment Z, there may exist multiple equilibria:*

(i) *No-default equilibria exist ($\pi = 1$) if and only if repayment is sufficiently small $Z \leq 1 + \nu(1-\beta)$.*

(ii) *Random default equilibria $\{\pi, B\}$ with $\pi \in (0,1)$ exist if and only if agents anticipate to be bailed out, $B > 0$, and repayment is sufficiently small: $Z < \overline{Z}_\pi$ (with $\overline{Z}_\pi \to \infty$ when $\pi \to 0$ and $\overline{Z}_\pi \to 1 + \nu(1-\beta)$ when $\pi \to 1$).*

(iii) *Default $\{\pi = 0, B = 0\}$ is always an equilibrium.*

Proof. Cf. Appendix. □

No-default equilibria When the government is expected to repay for sure ($\pi = 1$), domestic bonds' price equals 1 ($p = 1$), and so, are perfect substitutes for foreign bonds. External debt is sustained without reputation arguments as long as repayment is sufficiently small ($Z \leq 1 + \nu(1-\beta)$). The upper bound depends on the ability to bail out which restricts repayment to be less than $1 + \nu(1-\beta)$. When $Z = 1 + \nu(1-\beta)$, there exists only one no-default equilibrium, where domestic entrepreneurs own only domestic bonds. The external debt equals $Z^* = \nu(1-\beta)$. For smaller repayment values, other no-default equilibria exist, which feature richer portfolio distributions.

Finally, to the extent that the allocation is at a corner and repayment is strictly preferred ($W_1 > W_0$), no-default equilibria are robust to small changes in portfolios.

Random default equilibria A continuum of equilibria where repayment is random, i.e. $\pi \in (0,1)$ stretch between default and repayment for sure. In these equilibria, the price of domestic bonds p equals π.

Random equilibria tend not to exist when repayment (Z) is too large or when the political weight of domestic entrepreneurs (β) is too small (Cf. appendix for a more detailed discussion of this point). Furthermore, a single repayment probability may correspond to multiple equilibria, as the value of the bailout may differ, and, hence, the physical outcome[15].

[15]For example, if one distribution features no-diversification and the other symmetric portfolios, B differs for these two distributions. In the symmetric case, the government will bail out entrepreneurs so that they will produce ρ_1 in aggregate in case of default and $\rho_1 + \rho_2(1/\pi - 1)$ when there is no default. In the no-diversification case, entrepreneurs produce $\rho_1 + \rho_2(1/\pi - 1)$ when there is no default and, when there is a default, $x\rho_1 + (1-x)(\rho_1 + \rho_2)$ if $B = 1$ and $(1-x)\rho_1$ if $B = 0$. When two portfolio distributions lead to the same repayment probability and to the same bailout, the physical outcome would be the same on aggregate, but not at entrepreneurs' level. For example, with no-diversification, the outcome for an entrepreneur when a default occurs depends on whether he invested in foreign or in domestic bonds.

In random-default equilibria, default is not related to fundamentals. This gives a rationale for Tomz and Wright (2007)'s results, who document that default and output are only weakly negatively correlated[16], even though the conventional view is that countries default in bad times: a low level of outputs pushes a country to default.

Remark. As risky domestic debt makes domestic holdings costly, random default equilibrium outcome resembles Fudenberg and Tirole (1990)'s result on renegociation and moral hazard. Indeed, by interpreting domestic holdings of debt as effort, the government offers a menu of compensation schemes that is safe for low-effort entrepreneurs (small holdings of domestic debt) and risky for high-effort ones (large holdings).

Default equilibrium Whatever the value of parameters, an equilibrium always exists in which the government defaults for sure ($\pi = 0$) and where the domestic public debt is worthless ($p = 0$). In this equilibrium, the portfolio distribution h is degenerate and peaks at $x = 1$. Domestic entrepreneurs transfer their endowment solely using foreign bonds. In period 1, they receive 1 from their investment and produce ρ_1. Foreign investors may hold all the domestic debt, clearing the domestic bond market[17]. The government is unable to borrow since $p = 0$.

Notice that this equilibrium is not robust to arbitrarily small deviations in domestic agents' portfolios, as a small but strictly positive measure of domestic holdings would make default undesirable.

The TFP effects of defaults A default does not result only in less resources (e.g. less investment) invested by entrepreneurs but in a misallocation of resources. This misallocation translates into a lower total factor productivity.

When repaying, the TFP is: ρ_1. Resources are perfectly allocated. When defaulting, the TFP is at least lower than ρ_1. Indeed, the country's TFP can be written as:

$$TFP = \rho_1 + (\rho_2 - \rho_1) \left(\int_{1-B}^{1} (x + B)h(x)dx + (1 - H(1 - B)) \right).$$

As a result, as soon as the portfolio distribution is not symmetric, the total factor productivity is lower than ρ_1: among entrepreneurs having investment opportunities in period 1 invest too much while other invest too little.

As argued by Wright (2014), sovereign crises lead to major falls in total factor productivities, while leading to minor changes in factor employment. In the case of Greece, Wright

[16] Their result is computed over a sample of 175 sovereigns with HP-filtered annual data from 1820 to 2005 (cf. also Tomz and Wright, 2013, for a detailed discussion.).

[17] Worthless domestic debt can also be held by the domestic private sector. However, because $\beta\rho_2 < 1$, the government does not repay.

documents that in the fall of -22.07% of Greek GDP between 2007 and 2012, -20.48% were due to total factor productivity. At the same time, capital services contributed positively to GDP (+5.79%).

In this paper, the misallocation of resources takes place across firms, as documented by Sandleris and Wright (2014) in the case of Argentina. Resources' misallocation across sectors[18] may also occur as soon as the government is unable to redistribute resources across these sectors using transfers.

Complementarity between domestically and foreign held debt The government repays with a strictly positive probability only when there are some domestic holdings of debt, making complements foreign and domestically held debt.

However, when domestic savings are sufficiently invested domestically, the dispersion of portfolios decreases, making bailouts less costly. The option to default becomes more attractive, limiting the complementarity between the two stocks of debt. Formally, when the cost of bailouts c increases, the government honors its debt with probability 1 for a larger range of promised repayment Z. The set of no-default equilibria is enriched with equilibria featuring combination of higher external debt (Z^*) and higher domestically held debt (Z^e)[19].

Remark. When foreign debt is sufficiently large compared with domestic savings capacity, government repayment is necessarily stochastic and so is domestic entrepreneurs production. Interpreting lower private sector production as a financial crisis, this is reminiscent of Reinhart and Rogoff (2011)'s observation that a high level of external debt leads to a banking crisis (understood here as losses suffered by the domestic private sector).

The role of bailouts When debt is risky, the country's incentive to honor its commitment relies crucially on its possibility to bail out domestic residents. The presence of costs for implementing transfers (either because of heterogeneity of portfolios - cf. Proposition 1 - or due to opportunistic agents, $\nu(1-\beta)$, allows for debt repayment.

As a result the bailout option acts as an off-equilibrium condition for no-default equilibria. This is measured by the dependence on $\nu(1-\beta)$: costly bailouts make repayment desirable. $\nu(1-\beta)$ represents the relative cost of bailouts to debt repayment. This cost may be interpreted as domestic financial opacity or complexity, as it may derive from various sources such as opportunistic parasite entrepreneurs, political reputation or costs to plan and implement

[18]Cf. Sandleris and Wright (2014) or Benjamin and Meza (2009). Cf. also Wright (2014) for a discussion.

[19]Let $c_1 > c_2$, the set of no-default equilibria resulting from c_1 includes the one resulting from c_2. It contains, furthermore, equilibria with higher repayment that correspond either to higher foreign repayment (Z^*) or to higher domestic repayment (Z^*).

the bailout. Empirically, this suggests that measures of financial opacity or complexity are negatively correlated with the frequency of default.

Paradoxically, bailouts also allow to make the government more accountable as they make domestic debt less risky and, thus, attractive for domestic residents. In equilibrium, these transfers help investors to coordinate on equilibria featuring domestic holdings and, hence, public debt repayment.

This mechanism plays a crucial role for random default equilibria. In that equilibria, the government should bail out domestic entrepreneurs. Indeed, suppose that $B = 0$. Then domestic entrepreneurs' willingness to pay is $\overline{p} = \pi\rho_2/\left(\pi\rho_2 + (1-\pi)\rho_1\right)$. This expression equals π, foreign investors' willingness to pay, if and only if $\pi = 1$ or $\pi = 0$. This corresponds exactly to the argument raised in section 3. Without bailouts, domestic entrepreneurs prefer to invest abroad when the repayment probability is strictly less than 1 as the price of domestic bonds is too high for them when considering the default risk. The insurance that a bailout should provide is:

$$B \geq \left(1 - \frac{\rho_2}{\rho_1}\right)(1 - x^i).$$

In other words, bailouts should insure domestic agents against their losses weighted by their degree of risk-aversion, as measured by the curvature of the production function $1 - \frac{\rho_2}{\rho_1}$.

Risk aversion Risk aversion is also a key factor shaping the role of bailouts.

Without risk aversion (i.e., here, with a linear production function, $\rho_2 = \rho_1$), all probabilities $\pi \in [0, 1]$ are always equilibria. Domestic and foreign agents value the default risk in the same way and bailouts do not play any insurance role anymore. Furthermore, when, contrary to the benchmark model, domestic agents are less risk averse than foreign investors, the formers are more willing to hold domestic debt than foreigners, making the insurance role of bailouts useless as well. As a consequence, difficulties to keep domestic savings home and the corresponding role of bailouts arise as long as domestic agents are more risk averse than foreign investors.

The difference in the concavity of production functions, or, equivalently, in risk aversions may stem from different diversification abilities or from asymmetric business cycle fluctuations: in a domestic recession, with CRRA preferences, $1/c^{\sigma}$ is relatively larger. A country is, thus, more subject to sovereign default risk when its risk aversion is relatively higher than the rest of world. In particular, when the country's current economic situation is relatively worser or when its domestic financial sector is relatively less developed, the country's degree of risk-aversion can be higher, making debt repayment less likely.

Thus, this sheds light on the reasons why, even in recessions, risk averse private agents in less reliable countries purchase their sovereign's bonds rather than going for safer assets abroad: they still anticipate a government's bailout in case of default.

Remark. To fix ideas, one can interpret foreign investors as the US economy, where the relative development of the financial sector allows for a better diversification (cf. Gourinchas, Rey, and Govillot, 2010). In addition, notice that this relative development also makes US treasuries safer as the internal cost of default is large. This is in line with my assumption of a safe foreign asset.

Remark (Domestic savings capacity)*.* As the domestic savings capacity is normalized to 1, all the results are in relative terms with this savings capacity. In particular, note that no-default equilibria always exist when repayment is lower than this capacity ($Z \leq 1$).

Heterogeneity of portfolios By affecting the cost of bailouts (cf. Proposition 1), domestic portfolios' heterogeneity determines the government's willingness to honor its debt. Heterogeneity results, here, from equilibrium portfolio choices, even though domestic residents are *ex ante* homogeneous, and appears endogenously, making domestic portfolios opaque. This precludes both perfect bailouts and selective default[20].

Sensitivity of debt repayment hinges on the degree of portfolio heterogeneity as well. Indeed, heterogenous portfolios help small changes in these portfolios to translate into large variations in policy responses[21] and sufficiently dispersed portfolios may even create a discontinuity in the repayment probability. For example, a very little portfolio rearrangement may shift the bailout's transfer from $B > 0$ to 0, making random default equilibria not sustainable anymore. As a result a jump in the repayment rate arises (cf. appendix for the formal description of this class of discontinuous equilibria).

In terms of assessment of sovereign debt risk, this makes portfolio allocations a sensitive factor for sovereign debt repayment, not only in terms of allocation between domestic and foreign agents, but also in terms of debt's dispersion among domestic agents. This completes rating agencies' methodology, as they solely focus on aggregate criteria[22].

[20]This insight carries over to financial complexity: allowing domestic entrepreneurs for trading indirect exposures, that are costly to determine, leads to an endogenous financial complexity that makes portfolio observability even more difficult (cf. appendix).

[21]It is a well-known result that the properties of equilibria are not continuous functions of parameters (cf. Kreps and Wilson (1982) among others). This gives, however, no indication on where discontinuity may appear.

[22]Standard and Poor's considers five broad criteria: institutional effectiveness and political risks, economic structure and growth prospects, external liquidity and international investment position, flexibility and fiscal performance combined with debt burden and monetary flexibility.

In addition to endogenous heterogeneity, exogenous heterogeneity, such as differences in endowments, productivity or even in preferences, would further reduce the government's ability to bail out. Yet, such exogenous heterogeneity is not necessary and only adds to the endogenous heterogeneity.

Remark. Notice that these key conclusions do not depend on the specific setting of this paper's model and they can be extended to any model that features some form of internal cost of default, some domestic risk aversion and where portfolios may be heterogenous. For example, it is sufficient to include a foreign store of value and concavity in utility function in Guembel and Sussman (2009) (or in other models like Broner and Ventura (2011)) to obtain similar results.

5.3 Government bond issuance

So far repayment Z has been taken as given. This subsection endogenizes it by considering the government's *ex ante* problem of bond issuance. To begin with, I need to characterize domestic portfolios maximizing country's borrowing Z. In the following, I assume that investors coordinate on the equilibrium that, for a given Z, maximizes government's welfare.

Optimal portfolio The distribution allowing for the largest repayment Z maximizes the difference between the options to repay and to default $W_1 - W_0$. As only W_0 depends on the portfolio distribution, finding optimal portfolios amounts to minimizing W_0, or equivalently, to increasing the cost of bailout.

As a first step, notice that, for some average domestic holdings x, non-diversified portfolios maximizes the cost of bailouts. Indeed, when $x \geq \hat{x}$, such portfolios completely preclude bailouts and, when $x \leq \hat{x}$, they require inefficiently large bailouts ($B = 1$).

As a second step, maximizing over the average holdings x, optimal portfolios are invested only in domestic public bonds.

Bonds issuance This allows to write the government's problem as:

$$\max_{Z} \max_{\pi,B} \pi W_1 + (1-\pi) W_0$$
$$\text{s.t. } W_1 = RpZ - Z + \beta \int_0^1 F(z^i + x^i) di$$
$$W_0 = RpZ - (1 + \nu(1-\beta))B + \beta \int_0^1 F(B + x^i) di$$

and where the price p and the distribution of portfolios follow from private agents' decisions.

Rewriting $\pi W_1 + (1-\pi)W_0$ as $W_1 + (1-\pi)(W_0 - W_1)$, this problem amounts to comparing the gains from borrowing RpZ with the expected cost of default, that is:

$$(1-\pi^{opt})\left((1+\nu(1-\beta))B^{opt} - Z + \beta\int_0^1 F(B^{opt}+x^i) - F(z^i+x^i)di\right) \leq 0$$

where π^{opt} and B^{opt} are date-1 government's policies.

The borrowing capacity pZ is a concave function of Z as what the government gains by increasing the number of circulating bonds Z, it may lose as the price p declines. This capacity is maximized when $Z = 1+\nu(1-\beta)$. In that case, the government repays with probability 1 in period 1 and the corresponding cost of default is minimized and equals 0. Hence, maximizing the borrowing capacity is equivalent to maximizing *ex ante* welfare. This leads to the following proposition:

Proposition 2. *The optimal amount of bonds issued by the government is:* $Z^{max} = 1+\nu(1-\beta)$.

As a consequence, the optimal external debt is $Z^* = \nu(1-\beta)$. The government maximizes domestic holdings of debt ($Z^e = 1$) and borrows as long as its debt remains risk-less. Risky debt is not desirable *ex ante* as, on the one hand, the gain from issuing more debt is exactly compensated by the lower price of issuance induced by the additional sovereign risk, and, on the other hand, default risk reduces domestic entrepreneurs' welfare.

5.4 Two examples

This subsection illustrates the general results with two subclasses of equilibria: non-diversified and symmetric portfolios.

Non-diversified portfolios The relevant variable is, here, the average domestic holdings x which corresponds here to the fraction of domestic entrepreneurs holding foreign assets. Figure 3 plots equilibria as a function of x and repayment Z.

A discontinuity appears in the relation between the ability to borrow and holdings x. In the neighborhood of the threshold $\hat{x}$, small variations of x may trigger two major changes depending on the value of repayments Z. When considering changes in portfolios around $\hat{x}$, two cases arise as plotted in Figure 3 by the two segments:

Case 1: a change in portfolios along the barre may shift repayment from $\pi = 1$ (repayment for sure) to a lower but strictly positive probability.

Case 2: a change in portfolios may shift government's decision from a probability strictly positive to a complete default $\pi = 0$ (case 2).

Symmetric portfolios Similarly, the relevant variable is also x, which now corresponds to the fraction of foreign assets in each domestic entrepreneur's portfolio. Because of symmetry, the government knows exactly how much to transfer to each domestic resident. The cost of bailing out boils down to $(1 + \nu(1 - \beta))B$. Figure 4 plots this subclass of equilibria.

6 The pros and cons of capital controls

In the benchmark model, domestic agents may underinvest in domestic bonds and, thus, reduce inefficiently the government's borrowing capacity. In response, the latter can use capital controls to increase its commitment to repay.

However capital controls prevent diversification of domestic portfolios. In most cases, I show that this lack of diversification cannot be solved by the government intermediating an insurance contract with the rest of the world. Indeed, such a contract's implied payment would add to the country's promised repayment and thereby compromise credibility.

Turning to implementation, I allow domestic entrepreneurs to skirt the controls, by investing abroad using non-controlled, yet riskier, assets. This advocates for large-scale rather than targeted controls[23].

6.1 Adding macroeconomic shock

In the baseline model, domestic entrepreneurs have no diversification motive. This subsection adds such a motive and considers its interaction with capital controls.

Suppose that the government's production equals R with probability γ and 0 otherwise. In the latter case, the government is not able to repay nor to bail out. This constitutes a macroeconomic shock for domestic debt repayment against which domestic entrepreneurs are eager to insure.

In the absence of capital controls and when $\gamma < 1$, the only equilibrium is such that domestic agents prefer to hold only foreign debt and, consequently, the government never repays ($\pi = 0$). The macroeconomic shock reinforces the flight-to-quality towards the safest asset, the foreign asset, and bailouts are unable to prevent this preference.

Remark. Interpreting foreign liquidity as US treasuries, this sheds light on the "savings glut" experienced by the US in the 2000s: domestic sovereign risk and country-specific macroeconomic risks prevent domestic holdings of public debt. This constitutes an alternative view

[23]This is consistent with the fact that a complete and long-lasting closing of the financial account and strong limitations on currency exchanges are much more efficient than temporary and targeted measures.

to liquidity explanations of global imbalances pioneered by Caballero, Farhi, and Gourinchas (2008), who emphasize the ability to generate financial assets from real investment.

Could the government insure against this shock? In response to the macroeconomic shock, the government can try to self-insure by contracting with foreigners. Thus, it may receive resources in the bad state (i.e. the return is 0) against a payment in the good state (i.e. the return is R). As in the baseline model, the participation of foreigners to this contract depends on the observability of the promised payments:

(i) When the repayment associated with the insurance contract is observable, such insurance contract cannot exist. The government does not honor its contract, as it has no incentives to repay foreigners.

(ii) When insurance contract repayment cannot be distinguished from other repayment, this fungibility leads to a similar commitment problem as in section 5.

Let me consider this new commitment problem.

To cover repayments in the bad state, the transfer should be greater than or equal to Z. Foreigners' break-even condition pins down payment by the government in the good state:

$$\gamma \pi P^* = (1 - \gamma) Z$$

with P^* denoting the insurance contract payment and π the repayment probability in the good state. This allows to write total repayment to foreigners in the good state as

$$\left[\frac{1 - \gamma}{\pi \gamma} + 1 \right] Z.$$

Total repayment includes repayment associated with the insurance contract and repayment of domestic public bonds Z. Plugging this new value into the baseline model, we obtain the following result:

Proposition 3. *When Z is high enough or γ low enough, the government defaults with probability* 1 *in the good state.*

Proof. See appendix. □

Too large repayment in the good state prevents foreigners to insure the government against domestic macroeconomic shocks. Here, large repayment corresponds either to already large domestic public debt (Z large), or to very likely macroeconomic risks (γ low), implying large insurance contract's payment in the good state.

Remark. This result is reminiscent of the contingent sovereign debt literature (e.g. Grossman and Van Huyck, 1988; Borensztein and Mauro, 2004; Chamon et al., 2005). Using such contingent instruments rather than non-contingent debt allows to achieve better risk-sharing and to avoid pro-cyclical policies in bad times. Here, contingent bonds allow for lower observability of repayment compared with explicit international insurance contract, but Proposition 3 limits this possibility of contingent bonds, as the option of default restricts promised repayment in good times.

Optimal portfolios Without macroeconomic shocks, optimal portfolios are invested only in domestic bonds, so as to maximize the government's ability to borrow (cf. section 5). Macroeconomic shocks introduce a trade-off for portfolios because of the additional need of diversification. The gain of domestically-invested portfolios is the greater government's ability to borrow $\gamma RG = \gamma RpZ$. The diversification cost results from reduced domestic entrepreneurs' investment in bad times: $\beta(1-\gamma)\int_0^1 (F(z^i + x^i) - F(x^i))$.

As long as R or γ are large enough, the government strictly prefers to increase its borrowing capacity, even at the cost of a lack of diversification. Otherwise, capital controls are not desirable.

6.2 Introducing a tax on flows of funds

When capital controls are desirable, domestic residents can circumvent these controls, for example, by replicating asset positions using uncontrolled markets or financial instruments, or by directly using subsidiaries abroad. When, for example, Brazil introduced capital controls on stock- and bond markets in the late 2000s, intra-companies loans skyrocketed in response, presumably, to circumvent the controls.

In this subsection, I introduce a tax on capital flows and I consider the possibility of skirting tools. As in the benchmark model, portfolios are not observable and government can only intervene by taxing flows of funds received by investors. Let $\kappa \in (0, 1)$ be the tax rate on flows of funds.

If the government were able to observe who initiated a transfer of funds, it would tax domestic investment abroad and let foreign investors purchase domestic bonds. I assume, however, that the government does not have this ability so that it has to tax uniformly all flows of funds. As a result, capital controls affect purchases of foreign bonds by domestic agents along with investment by foreigners in domestic bonds.

Without loss of generality, I consider a tax on outflows in period 0 and 1[24].

[24]Notice that, because of the timing of the model, taxing outflows in period 0, or inflows in period 1, would

6.2.1 General outcome

Foreign investors purchase domestic bonds if and only if $p \leq \gamma\pi(1-\kappa)$ and domestic entrepreneurs' payoff as a function of foreign bonds' holding is:

$$P(x^i) = \gamma\left(\pi F\left(x^i(1-\kappa) + \frac{1-x^i}{p}\right) + (1-\pi)F(x^i(1-\kappa)+B)\right) + (1-\gamma)F(x^i(1-\kappa)) \tag{9}$$

I let for the appendix the description of the corresponding demand function.

Turning to equilibria, with capital controls $\kappa > 0$, no-default equilibria exist, and the price of domestic bonds is $\gamma(1-\kappa)$. Default $\{\pi = 0, B = 0\}$ is always an equilibrium.

The introduction of a tax on financial flows prevents domestic agents from purchasing foreign assets rather than domestic debt and, in turn, this gives incentive to the government to repay. The tax, however, reduces the price at which foreign investors accept to purchase domestic bonds.

An example without macroeconomic shock. This corresponds to $\gamma = 1$. Each entrepreneur invests his endowment only in domestic bonds ($x^i = 0$, i.e. a symmetric equilibrium where $x = 0$). The price of domestic bonds is $1 - \kappa$.

The tax on external flows delivers $\nu(1-\beta)\kappa$ to the government. This implies a trade-off between government financing and the risk of multiple equilibria. Overall the government obtains $(1-\kappa)(1+\nu(1-\beta)) + \kappa\nu(1-\beta) = 1 + \nu(1-\beta) - \kappa$ from the issuance of bonds and capital controls. Without capital controls, the government faces multiple equilibria but it can obtain as much as $1 + c$ in some equilibria.

Remark (Ex ante problem of bonds issuance). When the government issues only a mass 1 of bonds rather than $1 + c$, they obtain 1 of resource, which has to be compared with $(1-\kappa)(1+\nu(1-\beta)) + \kappa\nu(1-\beta)$. As long as $\nu(1-\beta) \geq \kappa$, the government gains to issue more bonds than a mass of 1.

6.2.2 Opening the financial account

Adding another investment opportunity abroad may allow domestic entrepreneurs to avoid the controls. Here this alternative investment is a foreign risky asset that yields $1/\zeta$ with probability ζ, with $\zeta \in [0, 1]$ and 0 otherwise, so that the expected return equals 1.

The government can impose capital controls (κ) on sovereign debt markets, or also on risky foreign asset markets. Depending on the government's choice, we obtain:

be sufficient to preclude domestic entrepreneurs' investment abroad. This is not robust to extensions such as overlapping generation.

Proposition 4. *In absence of capital controls, domestic entrepreneurs purchase directly only foreign bonds: they do not save through banks (and banks do not purchase risky foreign debt) and do not purchase domestic bonds.*

With capital controls restricted to sovereign debt markets, there exists a threshold $\overline{\zeta} \in [\gamma, 1]$ *such that, for any* $\zeta > \overline{\zeta}$*, domestic entrepreneurs save their whole endowment through banks in risky foreign assets.*

With capital controls on both markets, domestic entrepreneurs do not save through banks (banks do not purchase risky foreign debt): the outcome follows the previous subsection.

Incomplete capital controls combined with sufficiently rewarding skirting structures (ζ is not too low), prevents the government from keeping domestic savings at home. The degree of completeness of controls is here inversely proportional to the degree of openness as measured by indexes *de jure* as defined by Chinn and Ito (2006) or de facto by Milesi-Ferretti and Lane (2005).

7 Further discussions and extensions

7.1 Incentive compatible bailouts

In the benchmark model, transfers are restricted to be uniform across agents. In this subsection, I allow for a more general class of bailouts, potentially allowing for portfolio revelation.

The government offers a schedule $\{T_1(\hat{x}^i), T_2(\hat{x}^i)\}$, where T_1 is the transfer taking place *before* entrepreneurs invest and T_2 the one taking place *after* entrepreneurs invested and $\hat{x}^i$ is the portfolio announced by an entrepreneur. The delayed component T_2 could help achieve incentive compatibility[25].

The key ingredient here is the hiding constraint of domestic entrepreneurs. Intuitively, incentive compatible bailouts would have been implemented by loans with interest rates high enough such that only entrepreneurs with sufficiently high marginal returns (ρ_1 here) accept to participate. The hiding constraint rules out these kinds of loans and also rules out any government's replication of these loans by implementing transfers before investment and by expropriating date-1 incomes, as the government cannot tax entrepreneurs.

The following proposition summarizes the results of the mechanism design problem:

[25]Furthermore, I allow T_1 to be random as in Maskin and Riley (1984) and Stiglitz (1987). In contrast, without loss of generality, T_2 can be assumed to be deterministic, as agents are risk-neutral after investing.

Proposition 5 (Incentive-compatible bailouts)**.** *Incentive-compatible bailouts are such that:*

(i) *T_1 is deterministic and is such that $T_1 = 1 - x^i$ and $T_2 = \rho_2 x^i$.*

(ii) *When $\beta\rho_1 \leq \rho_2(1 + \nu(\rho_2 - 1)/\rho_2$, there is no loss of generality involved in considering only uniform bailouts.*

Proof. See appendix. □

An incentive-compatible mechanism implies an additional transfer (T_2) to domestic entrepreneurs after they invested. This transfer is a strictly increasing function of entrepreneurs' announcement $\hat{x}^i$ in order to give an incentive to entrepreneurs with large net positions in foreign assets to reveal their type. A lower bound on this subsidy is obtained by compensating the production that an entrepreneur would have obtained if he had announced holding only domestic bonds. Then, to choose the form of its bailout, the government compares the costs associated with the two transfers T_1 and T_2 with the loss implied by a single and uniform bailout:

$$\max_B \left\{ \beta \int_0^1 \left[f(x^i + B) \right] di - B(1 + \nu(1 - \beta)) \right\}$$
$$- \int_0^1 \left[\beta\rho_1 - (1 + \nu)(1 - x^i) + (\beta - (1 + \nu(1 - \beta)))Rx^i \right] di \geq 0 \tag{10}$$

To consider uniform bailouts is relevant when $\beta\rho_1 \leq \rho_2(1 + \nu(\rho_2 - 1)/\rho_2$, that is, when the cost of not sufficiently bailing out some entrepreneurs (ρ_2) is lower than the cost of implementing two transfers.

Extensions This paragraph's conclusions can be extended by continuity to imperfectly concealable date-1 income and, similarly, to imperfectly pledgeable income. In particular, this would ensure that sufficiently exposed financial institutions would borrow from the government. This would be consistent with evidence on discount window borrowing during the 2008 financial crisis (cf. Armantier et al., 2011).

Yet, to this extent that income is sufficiently concealable, such loans (or their equivalent in terms of transfers and *ex post* expropriation) are not sufficient for optimally bailing out domestic residents.

7.2 Non-observability of portfolios

This subsection discusses this paper's central assumption: the government's inability to observe domestic portfolios[26].

Verifiability of domestic holdings A simple argument against non-observability is verifiability: the government can assess each domestic agent's exposure by requiring from him to disclose his bondholdings. In the absence of secondary markets, domestic agents would announce exactly what they hold and, hence, the government would be able to default and to perfectly bail out domestic residents, or, equivalently, to default selectively on external debt.

However, verifiability becomes more difficult and costlier in complex financial systems. Domestic residents may be exposed to debt indirectly through loans (collateralized or non-collateralized) or equity participations in foreign financial institutions that, in turn, own domestic public debt. Conversely, domestic residents may have domestic debt on their balance sheet, but, at the same time, they could have transferred the corresponding default risk to foreign agents by purchasing credit default swaps[27]. In other words, domestic *holdings* may differ from domestic *exposures*. As soon as information acquisition is costly, keeping track of all the domestic exposures becomes inefficient[28].

Hints from data? Even though domestic residents may make up for exposures using derivatives or other instruments (Proposition 8), looking at data may provide hints to the quantitative relevance of unobservable and dispersed portfolios.

First most datasets at the disaggregated level concern only banks (the European Banking Authority stress-tests in 2010 and 2011 for example). Usually not collected on a regular basis, these data concern only a fraction of domestic bondholders (from 23% for Finland or 29% for Greece to 81% for Switzerland, comparing the EBA stress tests with the IMF aggregate data from Arslanalp and Tsuda (2012)). As far as I know, more detailed datasets on a wider range of ultimate-risk exposures do not exist and remain barely known.

Indirect exposures can be of the same order of magnitude as direct holdings. For example, the EBA have assessed the banks' exposures through loans to the private sector (so excluding exposures to the country's public bonds) exposed to one country's default. For the main Italian banks, these defaulted exposures range between 52% (Intesa San Paolo) of the bank's

[26] I leave for the appendix the description of further theoretical foundations of that inability (e.g. endogenous financial complexity or asymmetry of information between the government and taxpayers).

[27] Similarly, domestic agents' exposure hinge on credit derivatives and not through direct holdings of domestic public debt.

[28] Cf. Caballero and Simsek (2013) for a theoretical understanding of information acquisition and complexity.

direct holdings of Italian debt to 103% (Banco Popolare), and for the main Greek banks, these range between only 6% for the Hellenic PostBank to 74% for AlphaBank. Notice, however, that indirect exposures contribute to the inability to bail out to the extend that they allow for more opacity.

Finally, bank's holdings, themselves, are very dispersed among banks. The fraction of domestic bank's holdings of domestic bonds concentrated in the largest banks is usually small compared with the concentration of banking capital or assets: the 4 largest spanish banks concentrate only 27.8% of banks' holdings of spanish debt but 67% of spanish banking capital and 70% of bank-owned assets[29].

Finally, empirical evidence also emphasizes the fact that financial institutions may adapt immediately to bailouts. He et al. (2010) provide evidence that, in the 2008 financial crisis, hedge funds and broker-dealers sold assets to commercial banks in anticipation of a bailout of the latter[30]. Turning to *ex post* verifications of bailout's efficiency, money's fungibility makes the use of transfers to banks hard to track (cf. COP, 2011) and public authorities can only rely on broad proxies such as overall lending activities for measuring the effect of a bailout.

Secondary markets Broner, Martin, and Ventura (2010) argue that the presence of secondary markets, allowing foreign investors to resell their domestic country's bonds to domestic residents, prompts the government to honor its debt. Is there conclusion robust to the introduction of bailouts?

In this paper's framework, secondary markets are redundant with primary markets: no additional shock occurs between period 0 and 1, and thus, the introduction of such markets does not modify Section 5's results. If domestic entrepreneurs buy back the all the debt on secondary markets in period 1, the government repays. This is, however, exactly equivalent to a no-default equilibrium. In particular, the amount of debt that the government can credibly repay is bounded above by the cost of a bailout. Thus embedding secondary markets in this paper's framework would not alter the set of equilibria.

Notice, however, when portfolios are fully observable, domestic agents cannot purchase more domestic bonds than their savings, i.e. 1, and so, equilibria with external debt higher than 1 cannot be sustained: total government borrowing . As a result, in this paper's context, secondary markets *per se* are unable to explain why the government can borrow more than

[29]Similarly, this fraction is 32% for the 4 largest french banks, 49.7% for the 12 largest german banks, 32.6% for the 4 largest Italian banks and 27.8% for the 4 largest spanish banks.

[30]Such as monetary bailouts: Federal Reserve's lending facilities, or government's guarantees such as the Trouble Asset Relief Program I and II.

in autarky.

The main reason is that secondary markets do not prevent the government from replicating a selective default by defaulting wholesale and compensating domestic residents, if secondary markets are associated with perfect information[31]. Yet, by contributing to the opacity of domestic exposures, and hence, to portfolio non-observability, secondary markets may still improve sovereign credibility.

Intuitively, sovereign repayment does not require secondary markets. Suppose that a domestic bank claims to be exposed to a foreign bank holding domestic debt through, for example, the interbank market. The government may be better off by honoring its commitment in order to avoid the foreign bank's default on the loan granted by the domestic bank, even though domestic residents are unable to directly repurchase bonds held by foreigners.

Conversely, domestic bailouts suggest an alternative role for secondary markets for sovereign repayment: agents that are more likely bailed out (e.g. banks) repurchase the debt from agents less likely to be bailed out (e.g. hedge funds) making transfers more costly, and so, repayment more attractive. Notice that this emphasizes the role of domestic secondary markets, in contrast with Broner, Martin, and Ventura (2010) who emphasize international secondary markets. Yet, to the extend that information on portfolios is imperfect, the possibility of bailouts may also preempt secondary markets. When anticipating a default followed by a bailout, as transfers do not compensate exactly domestic agents' losses, these agents do not have individually the incentive to deviate and purchase domestic bonds in secondary markets, unless the corresponding change in the domestic portfolio distribution modifies the country's willingness to bailout[32].

7.3 Internal costs of default and bailouts

The quantitative relance of internal costs of default through the banking sector has been documented by Rajan and Zingales (1998), who show that the probability of banking crises increases significatively after a domestic default. Brutti (2011) provides further evidence on the association of sovereign debt crises and domestic private sector's liquidity crises, and

[31]Under perfection information, this conclusion carries over when including an arbitrarily small external cost of default as in Broner, Martin, and Ventura (2010): as the government can perfectly compensate domestic entrepreneurs against a potential default, this additional external cost ends up being the only cost resulting from the default.

[32]In the case of perfect information, domestic residents, anticipating perfect bailouts are indifferent to domestic debt repurchase on secondary markets, and so, anything can be an equilibrium. When information is imperfect, this does not hold anymore as, for any given level of non-targeted transfers, domestic investors would be strictly worse off buying defaulted domestic bonds, unless this affects the level of transfers.

Gennaioli, Martin, and Rossi (2011) highlight the quantitative role of banks[33].

Bailouts in the baseline model can be implemented in several ways, such as direct transfers, loans at discounted interest rates to banks or financial institutions or any other instruments aiming to redistribute resources and sustain economic activity.

For example, in the 2001 Argentinian default, banks' deposits convertibility was suspended (the so-called "corralito" decided the 3rd of december 2001). Despite criticisms of the efficiency of such a suspension, other suggested solutions would have in fact be bailouts as well. This includes, among others, central banks interventions as lender of last resort suggested. Similar banks' liabilities suspensions were decided by Russian authorities in August 1998 after having defaulted on their debt. More generally, monetary policy, through devaluation or inflation, may also redistribute resources after a default. Yet, these tools remain imperfect as the transfers they may implement do not exactly the effects of a default.

Bailouts and reputation costs A possible objection against the possibility of bailouts after a default can be the reputation cost it would involve: implementing transfers to domestic residents is a clear signal to foreigners that the country has defaulted despite having the resources for reimbursing. Yet, this would correspond to reputation costs similar to those implied by the default itself. And so, if these reputation costs are unable, both quantitatively and theoretically following Bulow and Rogoff (1989), to incentivize the country to repay, they are also unable to prevent the country from bailing out its residents.

Nevertheless, even small, reputation costs may encourage the country to make the presence of domestic bailouts difficult to assess, as this can decrease the stigma associated with an non-excusable default. To the extend that non-targeted transfers are less identifiable than direct transfers (e.g. through monetary policy), reputation costs may reduce as well the country's ability to implement perfect bailouts.

Are bailout expectations desirable? Ex ante, the anticipation of a bailout can make the government more accountable as it increases the internal demand of bonds. Conversely, the possibility of a bailout decreases the government's ability to borrow, as, then, the government can make up for internal costs. As a result, the government faces a trade-off, if it is willing to manage bailout expectations (e.g. through past actions).

[33] Cf. also Arteta and Hale (2008) or Borensztein and Panizza (2009) and Panizza, Sturzenegger, and Zettelmeyer (2009) for recent surveys. For the role of banks, further quantitative as well as theoretical research could yield interesting insights from the mutual feedback between countries' debt and banks' fragility using, for example, Diamond and Rajan (2001)'s theory of banking.

Nevertheless, the role of bailouts for fostering domestic holdings of bonds arises when these bonds appear to be risky to investors. This suggests that, when managing expectations, the government should favor expectations of bailouts when its debt appears to be risky to investors, while it should favor no-bailout expectations when the debt is perceived as safe.

7.4 Partial defaults

Countries usually do not renege on their whole stock of debt, preferring partial defaults. For example, Sturzenegger and Zettelmeyer (2007) document that debt restructuring in the 1998-2005 period implied losses between 13% and 75% for creditors. This subsection shows that partial defaults replicate, to some extent, complete defaults followed by bailouts, as they leave some wealth to domestic agents. The key difference is that partial defaults do not involve bailouts' costs, but they still imply some repayment to foreigners. When the latter exceeds the former, I show that there is no loss of generality not to consider partial defaults.

When defaulting partially, a government can either repay only a fraction of each bond it has issued or it can repay fully only some bondholders, as in a lottery. I assume in the following that agents prefer to perfectly diversify their portfolios and that domestic bonds are sufficiently divisible. Using the law of large numbers, the two implementations are equivalent, so that I consider only default on a fraction of bonds.

7.4.1 Partial default equilibria

I denote by λ the fraction of the debt that the government repays. From lenders' perspective, because of its inability to observe portfolios, the government has to randomize repayment across the lenders. Thus λ also represents the probability for a lender to be repaid.

Foreign investors compare p with λ and so do domestic entrepreneurs. The government's problem is to maximize the following value function:

$$W = RG - \lambda Z - (1 + \nu(1 - \beta))B + \beta \int_0^1 F(\lambda z^i + B + x^i)di \tag{11}$$

The maximization of W with respect to the fraction λ also requires having determined optimal bailout and its dependence with respect to the fraction λ. Then, we obtain:

Proposition 6 (Optimal partial default)**.** *Optimal fraction of default and optimal bailout are such that $\lambda = \lambda^e$ and $B = 0$.*

Proof. See appendix. □

Equilibria Every $\lambda \in [0,1]$ is an equilibrium and partial default has the comparative advantage not to require bailouts. Notice, however, that the production function introduces a bias towards partial defaults, as domestic entrepreneurs are risk-neutral against such defaults.

7.4.2 Welfare comparison

To choose *ex post* between partial default and complete default followed by a bailout, the government compares the welfare outcome of these two options. Given a distribution of portfolios h compatible with λ^e, the expected defaulted fraction, the government prefers to default and bail out if and only if:

$$-\lambda^e Z + \beta\rho_1 \leq \max_B \beta \int_0^1 F(x+B)h(x)dx - (1+\nu(1-\beta))B$$

The right hand term corresponds to the welfare outcome of partial default and the left hand term to the outcome of a complete default with a bailout. A sufficient condition for this inequality to hold is that repayment Z is sufficiently large. With large repayment, partial default requires to repay foreigners considerably as well, making bailouts' costs comparatively more affordable.

8 Conclusion

This paper investigates sovereign debt sustainability in a model in which domestic and foreign investors optimally select their portfolios and the sovereign optimizes over its debt, default and bailout policies. It derives conditions under which the country can borrow abroad and shows that internal bailouts do not preclude sovereign borrowing. It determines when capital controls are undesirable in spite of their disciplining benefits. Its main contribution is to connect sovereign credibility with the ability to bail out.

Turning to future research, this paper only considers the case of a small open economy. A richer framework would include multiple countries that would compete to make their debt attractive to their residents, and in turn, to the other country's residents. This would result in coordination problems in borrowing capacities between the two countries, that may prevent an efficient allocation of capital (cf. Mengus, 2013b).

Moreover, this paper emphasizes opacity and complexity as relevant drivers for a country to honor its liabilities. A major amplification channel is, thus, the potential contagion of domestic losses to the whole domestic economy. Future empirical research on internal cost of default should assess quantitatively the link between the government's ability to prevent contagion and financial crisis and its ability to repay sovereign debt.

Finally, the model in this paper is in real terms and does not allow for nominal adjustments and monetary policy. The framework could be enriched to include nominal variables in order to study the interactions between default and monetary policy's standard objectives such as inflation targeting. Indeed, buybacks of public bonds by central banks give rise to another endogenous cost of default through inflation.

I leave these questions for future research.

A Appendix

A.1 Portfolio non-observability and capture

So far, the main friction was the government's inability to observe domestic bondholders' exposures. Here I provide another form of asymmetry preventing perfect bailouts arising between taxpayers and the government. When taxpayers have imperfect information on domestic bondholders' exposures and when the government may be captured by these bondholders[34], taxpayers restrict in response the set of possible transfers that the government implements.

Formally, the government has perfect information on entrepreneurs' wealth allocation (x^i), while taxpayers only have information on the aggregate distribution h. Importantly, they are able to observe the transfers granted by the government to domestic entrepreneurs.

Without taxpayers' limited information, portfolios would be fully observable to everyone and optimal bailouts would be $1 - x^i$ and production $F(1)$.

However, each domestic entrepreneur can try to bribe the government by offering a share $\xi_i > 0$ of additional benefits linked to excessive bailout: $F(B^i + x^i) - F(1)$. Symmetry and the production function monotonicity imply that $x^i = 1$ and $B^i = B$, which diverges to ∞. In response, taxpayers limit the transfer by imposing a upper bound $\overline{B}$ using their information on h.

Proposition 7. *With a captured government, $\overline{B}$ is:* $\max\{b|H(1-b) \geq 1 - \hat{x}\}$ *with* $\hat{x} = \frac{\beta\rho_1 - 1 - \nu(1-\beta)}{\beta(\rho_1 - \rho_2)} \in (0,1)$ *and* $B = \overline{B}$.

$\overline{B}$ is obtained in a similar way as in proposition 1. Bailouts are restricted by the least informed agents: the taxpayers, although the government is better informed than in the baseline case.

Notice that here verifiability would be costly as well as the government should provide taxpayers hard information on bondholders' exposures. This would be both costly and lengthy and, thus, prevents the government from finely tailored bailouts.

In addition, any form of political cost would require similar asymmetries of information. Otherwise, the government would be able to compensate residents for the default, precluding internal costs of default.

A.2 Endogenous financial complexity

So far, indirect exposures through interbank or derivatives markets have not been explicitly modeled. Hereafter, I allow for such exposures modeled as credit derivatives (CDS for instance) correlated with the country's decisions. The government is endowed with a costly auditing technology for assessing these exposures. When auditing is sufficiently costly, in equilibrium, domestic agents may engage in financial complexity, i.e. numerous indirect exposures. As a consequence, the government strictly prefers to honor its debt rather than auditing portfolios.

Formally, suppose that there are N "derivative" assets $\{a_j\}_{1 \leq i \leq N}$. Each is in net zero supply and a_j's payoffs are contingent to government's default. I denote by $P_0(a_j)$ the payoff when the government defaults and by $P_1(a_j)$ the payoff when the government repays.

[34] Cf. Laffont and Tirole (1991) for a model of captured regulation.

The government has an auditing technology that allows to assess the payoffs of assets and an agent's exposure by paying a fixed cost κ. This can be generalized to richer distribution of costs. Without loss of generality, when the entrepreneur has lied on his exposure, I assume that he receives no transfer.

The problem of the government amounts to comparing perfect bailouts implemented at the cost of auditing portfolios with uniform bailouts. The following proposition establishes a condition under which uniform bailouts are preferred:

Proposition 8. *Given a number of assets N, for sufficiently high cost κ, there exist at least two equilibria:*

(i) *Domestic entrepreneurs issue and purchase "derivatives". The government strictly prefers not to audit domestic portfolios, chooses to implement uniform bailouts, and repays with probability π.*

(ii) *Domestic entrepreneurs do not issue and purchase "derivatives", nor they purchase domestic debt. In response, the government defaults for sure.*

The proof uses continuity and derives easily from applying the intermediate value theorem on κ. Derivatives need not be traded by both foreign and domestic agents, as it is sufficient for sovereign repayment that precise individual exposures are difficult to assess. Yet, when a country is internationally financially integrated, not only individual exposures but also aggregate exposures become uncertain, preventing further perfect bailouts. So, empirically, sovereign credibility has to be correlated with the development of domestic interbank or OTC derivatives markets but also with the country's degree of financial integration.

This result can be extended to random monitoring. In that case, the government has to audit a sufficient number of domestic entrepreneurs to force them to disclose their portfolios (the probability to be audited has to be strictly positive and sufficiently high). Otherwise, they have no incentives to disclose as emphasized by Proposition 5. Furthermore, auditing a fraction of the portfolio combined with disclosure is not sufficient to assess the precise exposure of one particular portfolio as this exposure may result from the unaudited fraction - either because the remaining of the portfolio hedges against the default or because it exposes to the default.

Notice that the proposition can be restated in terms of number of the assets, taking as given the cost κ and that an alternative formulation would include complex indirect exposures where domestic agents may be exposed to domestic debt through a sequence of intermediaries. The government would have to audit all these intermediaries to identify the precise domestic residents exposures.

In addition to uncertainty about exposures and the corresponding cost of auditing, another source of cost is the time required to acquire information and, thereafter, to implement finely tailored bailouts. This delay of implementation may lead to confusion about the government's ability to bail out and, hence, contribute to additional costly disturbances.

A.3 Comparative statics with random default equilibria

Contrary to no-default equilibria, comparative statics cannot be performed easily with random equilibria. On the one hand, the set of equilibria S_π is not upper- nor lower- hemi-continuous with respect to changes in parameters (e.g. β, Z or ν). Indeed, after a change of parameter, S_π includes only equilibria with distributions that do not correspond to equilibria before the change. Each distribution that corresponds to an equilibrium before a change in parameters does not correspond to an equilibrium thereafter. On the other hand, there always exists some distribution h such that $\{\pi, B, h\}$ is an equilibrium with the new set of parameters (These two properties are formalized by lemma 10 in appendix.). However, the existence of these new equilibria is based on a non-constructive proof. This limits the comparative statics to asymptotic properties of S_π: when repayment goes to infinity, random equilibria tend not to exist when repayment (Z) is too large or when the political weight of domestic entrepreneurs (β) is too small. As a general picture, for a distribution of portfolios h, the higher is repayment Z, the lower is the probability of repayment π: the probability of repayment equals one for a range of repayment values and then decreases continuously to converge to 0 as repayment goes up.

A.4 A class of discontinuous repayment equilibria

I provide here an example of equilibria where policies are not continuous.

Let H be a distribution associated to a bailout B (as derived as in proposition 1) such that: H is flat on $[B, 1)$ and H equals $1 - \hat{x}$ on this segment. Any change of portfolio leading to a distribution H' such that $H'(x) < H(x)$ for any $x \in [0, 1)$ implies that the bailout associated to the distribution H' is $B' = 0$.

As a consequence, if H sustains a mixed equilibrium with a repayment probability π, $H + dH$ does not sustain any equilibrium.

Indeed, consider H such that $H(1) = 1$. Given $\epsilon > 0$, let dH a small deviation such that $(H + dH)(1) = 1$, such that $\|dH\|_\infty < \epsilon$.

$$H(1^-) + dH(1) < 1 - \hat{x} \text{ and } H(1 - B) > 1 - \hat{x}$$
$$\text{implies that } 0 < H(1^-) - H(1 - B) < -dH(1)$$

H is flat for $1 - B$ and 1. When there is a discontinuity of B with respect to dH, there is H, for Z high enough, mixed equilibria exist but not for $H + dH$.

This is only an example of a class of distributions which features potentially some discontinuities. However it is easy to see that the two ingredients defining this class of equilibria are almost necessary conditions to obtain these discontinuities.

A.5 Verification of net positions

The verification of domestic entrepreneurs' net positions is another way to enforce capital controls in my setting. As an example, Italy and South Korea implemented such controls on portfolio choices in the past.

When the government is able to observe these net positions, the form of capital controls are easy to choose and to implement: the government forces domestic entrepreneurs to invest only in domestic assets. However, domestic agents may use strategies to avoid capital controls[35], making this ability only imperfect. In parallel to the coordination problem in the baseline model, choices of evading strategies has to be coordinated as there is complementarity between evading strategies of entrepreneurs. This coordination problem delivers similar equilibria, where bailouts and the repayment probability of the government depend endogenously on portfolios. Such a form of capital controls turns out to be ineffective.

In between, the government may identify a fraction $\mu \in [0, 1]$ of investors and force them to hold domestic bonds. In case of default, it bails out completely these identified investors ($B = 1$). For the other investors, it proceeds as in the baseline case.

[35] These strategies include, for example, the issuance of liabilities abroad. See Forbes (2007) for a survey.

Then the government compares the value of its two options:

$$W_1(\mu) = -Z + \beta\mu\rho_1 + \beta(1-\mu)\int_0^1 F(z^i + x^i)di >$$
$$W_0(\mu) = -\mu - (1-\mu)(1+\nu(1-\beta))B + \beta\mu\rho_1 + \beta(1-\mu)\int_0^1 F(B + x^i)di$$

This comparison should be independent from the distribution of holdings among the other domestic entrepreneurs:

$$(1-\mu)\left(W_1(0) - W_0(0)\right) + \mu(1-Z) > -(1-\mu)Z + \mu(1-Z) > 0$$

If μ is sufficiently high, this is sufficient to tie the hands of the government and force the coordination towards domestic bonds:

Proposition 9. *For $Z < 1$, the government does not default for sure as long as the fraction μ of investors identified by the government satisfies:*

$$\mu > \mu^* = \frac{1}{1 + \frac{(1-Z)}{Z}} \tag{12}$$

whatever the distribution of holdings among the unidentified domestic entrepreneurs.

For $Z > 1$, $\mu = \mu^* = 1$.

An easier access to information on some portfolios may be achieved in several ways: 1) with legal disclosure of information for prudential policies, 2) through the structure of the bond market (identified buyers, development of secondary markets). In terms of institution design, this suggests that the agency in charge of the allocation of bailouts should be also in charge of banking regulation, triggering potentially more risks of collusion with banks (domestic entrepreneurs in this model). Presumably, a potential capture of the government causes these gains from additional information to vanish.

Remark. Z is compared to 1 and not to $1 + c$ as bailouts of identified investors are costless. The identification of investors represents both gains and costs: it facilitates debt sustainability but in a limited amount.

A.6 Demand for domestic bonds under capital controls

The maximization of domestic entrepreneurs payoff's leads to Figure 5 which plots domestic entrepreneurs' demand function (see appendix for the derivation and the details of the computation. p_1, p_2, p_3 and p_4 are thresholds derived in the appendix.), for arbitrary values of π and B.

Red dotted lines indicate that entrepreneurs have to compare their payoffs with the two remaining portfolios: either $P((1-B)/(1-\kappa))$ and $P(1)$ or $P(0)$ and $P(1)$. Arrows denote the sign of the derivate with respect to x^i in the region delimited by small and black dashed lines.

As in the baseline model, $\pi = 0$ is an equilibrium. When investors anticipate that $\pi = 0$, they will systematically pay the tax to invest abroad, whatever its value ($\kappa \in (0,1]$): a tax does not allow to avoid default equilibria.

$\pi = 1$ can be sustained as an equilibrium. A necessary and sufficient condition is that $p_1^S(\pi = 1) \geq \gamma(1-\kappa)$. Such a condition can be fulfilled as long as κ is such that:

$$p_1^S(\pi = 1) \geq \gamma(1-\kappa) \Leftrightarrow (1-\kappa)\left((1-\gamma)\frac{\rho_1}{\rho_2}(1-\kappa) + \gamma\left(1 - \kappa\frac{\rho_1}{\rho_2}\right)\right) \geq 1 \tag{13}$$

This last inequality has a solution for $\kappa < (1-\gamma) + \gamma\frac{\rho_2}{\rho_1} < 1$ (there is also a solution for $\kappa > 1$).

Thus, there exists capital controls such that $\pi = 1$ can be sustained as an equilibrium. Now, the price of bonds is $\gamma(1-\kappa)$. Additionally, mixed equilibria exist as well.

A.7 More general production functions

Here I consider a more general domestic entrepreneurs' production function: f is concave and satisfies Inada conditions. In addition, f is twice differentiable a.s.

Assumption 1 writes: for $I \geq 1$, $\beta f'(I) < 1 + c$. Domestic entrepreneurs also compare p with π and with $\overline{p}$, which now depends on x^i. Indeed, $\overline{p}$ is such that domestic entrepreneur i is indifferent between investing $1 - x^i$ in domestic assets and investing everything abroad, i.e.:

$$\pi \left[f'(x^i) + (1/p - 1) f'(x^i + 1/p(1 - x^i)) \right] = f'(1)$$

However, this does not change qualitatively the equilibria, as the equilibrium price is pegged by foreigners. Finally, there exists $B \leq 1$ such that W_0 is maximized by B, because of the equivalent of assumption 1. Indeed, $W_0(b) \leq W_0(1)$ for $b \geq 1$ and then, continuity yields immediately the result.

A.8 Symmetric and non-diversified portfolios

Equilibria with non-diversified portfolios

Strategies of agents Strategies of agents both in period 0 and in period 1 are special cases of sections 3 and 5.

Domestic agents and the demand for bonds First, recall that without diversification, the demand function for any entrepreneur is either:

(if $B = 1$): $1 - x^i = 0$ if $p > \pi$; $= \{0, 1\}$ if $p = \pi$; $= 1$ if $p < \pi$

(if $B = 0$): $1 - x^i = 0$ if $p > \overline{p}$; $= \{0, 1\}$ if $p = \overline{p}$; $= 1$ if $p < \overline{p}$

As a consequence $z^i = 1/p$ or $z^i = 0$ are two only possible repayments. The distribution of repayment has two peaks at 0 and 1/p. As in the general case, for every beliefs $\{\pi, B\}$ foreign investors accept to pay a higher price (π) than domestic investors for domestic bonds.

Government policies For this subclass of equilibria, optimal bailouts are such that: if $x > \hat{x}$, $B = 0$ and if $x < \hat{x}$, $B = 1$. The government does not default as long as:

$$(1 + \nu(1-\beta)) + \beta\rho_2(Z - Z^f - 1) \geq Z \text{ if } x < \hat{x} \text{ and } \beta\left[\rho_2(Z - Z^f - 1) + \rho_1(1 - x)\right] \geq Z \text{ if } x > \hat{x}.$$

Equilibria As in the general case, equilibria are divided into three subspecies: when the government either defaults or does not default for sure, and probabilistic default (or mixed strategies). Gathering equilibria per fraction x is more relevant here.

Thus, mixed equilibria exist as long as $x \leq \hat{x}$. Indeed, with no diversification of portfolios, $x \leq \hat{x}$ is equivalent to $B > 0$.

Notice that $\{\pi = 0, B = 0\}$ is still an equilibrium and the outcome properties are the same as before ($p = 0$).

Equilibria where $x \geq \hat{x}$ In this case, only no-default equilibria may exist. The condition for no-default equilibria to exist is $Z \leq \beta\rho_1 (1-x)$.

This completes the results found in the general case. For every $x \geq \hat{x}$, the set $S_x \cap S_{\pi=1}$ is expanding with β and with ρ_1. Besides, it is expanding with $1-x$.

Remark. Recall that $x \geq \hat{x}$ then, an upper bound here for Z is $Z \leq \rho_1/(\rho_1-\rho_2)(1-\beta\rho_2)$.

Equilibria where $x \leq \hat{x}$ In this case both no-default and mixed equilibria may exist. The function $Z_x(\pi) = 1 + c - \beta\rho_2 \frac{x-(1-\pi)}{\pi}$ is such that:

- For every $Z \leq Z_x(1)$, $\{\pi = 1, B = 1\}$ is an equilibrium.
- For every Z such that there exists $\pi \in (0,1)$ which satisfies $Z_x(\pi) = Z$, $\{\pi, B = 1\}$ is an equilibrium.

Remark. The set of functions $\{Z_x(\pi), x \in [0,1]\}$ satisfies two properties:

1. The function $Z_x(\pi)$ is decreasing and takes values between $Z_x(1) = 1 + \nu(1-\beta) - \beta\rho_2 x$ and $\lim_{\pi\to 0} Z_x(\pi) = \infty$. Morevover, this is a continuous function.
2. For every $x \in [0,1]$, $Z_x(1) = 1 + c - \beta\rho_2 x \geq 1 + \nu(1-\beta) - \beta\rho_2 > 0$ using assumption 1.

The set of equilibria - Properties of equilibria as a function of Z I describe here how equilibria of the game between domestic entrepreneurs and foreign investors depend on the amount of bonds issued by the government. The upper bounds for Z such that no default occurs are essential here and more precisely the way they are ordered. Upper bounds are ordered as follows:

$$Z_{\hat{x}}(1) \leq \beta\rho_1 (1-\hat{x}) \leq Z_1(1) = 1 + \nu(1-\beta) \tag{14}$$

Indeed, the first inequality is equivalent to:

$$1/\rho_1(1+\nu(1-\beta)) + \frac{\beta\rho_1 - 1 - \nu(1-\beta)}{\rho_1} \geq \beta$$

which is true by definition. The second inequality is equivalent to:

$$(1+\nu(1-\beta))(1-\rho_2/\rho_1) \geq (1-\beta\rho_2) \tag{15}$$

Symmetric portfolios

Given the information structure, symmetric portfolios is a degenerate case: in this case the government knows exactly what each domestic entrepreneurs holds. This feature simplifies a lot this subclass: bailouts are designed to match exactly the needs of domestic entrepreneurs and the government always bails out when defaulting, except when entrepreneurs hold no domestic bonds.

Strategies of agents The demand function of each agent i is:

$$x^i = 1 \text{ if } p > \pi \;;\; = [x,1] \text{ if } p = \pi \;;\; = x \text{ if } p \in [\overline{p}, \pi] \;;\; = [0,x] \text{ if } p = \overline{p} \;;\; = 0 \text{ if } p < \overline{p}$$

Aggregation implies that $x^i = x$.

Government policies Given that the government knows perfectly how many public bonds each domestic entrepreneur holds, optimal bailout is $B = 1 - x$. The government default condition becomes:

$$W_0 \geq W_1 \Leftrightarrow (1 + \nu(1-\beta))(1-x) + \left(\frac{1-x}{p} - (1-x)\right)\beta\rho_2 \geq Z$$

Equilibria First, recall that $\{\pi = 0\}$ is an equilibrium which belongs to this subclass of equilibria. Besides, the complete description of the subclass follows lemma 10: for each distribution $x \in [0,1)$, there exists a function $Z_x(\pi)$ which is such that:

1. Its expression is: $Z_x(\pi) = (1 + \nu(1-\beta))(1-x) + (\frac{1-x}{\pi} - (1-x))\beta\rho_2$.
2. For $Z \leq Z_x(1) = (1+c)(1-x)$, $\{\pi = 1, B = 1-x\}$ is an equilibrium.
3. For Z such that there exists $\pi \in (0,1)$ such that : $Z = Z_x(\pi)$, $\{\pi, B = 1-x\}$ is an equilibrium.

B Proofs

B.1 Proof of proposition 1

Government chooses B in order to maximize its objective function. Then the derivative of this objective function with respect to B is:

$$\begin{aligned}
\frac{\partial W_1}{\partial B} &= -1 - \nu(1-\beta) + \beta\frac{\partial}{\partial B}\int_0^1 F(B + x^i)di = -1 - \nu(1-\beta) + \beta\frac{\partial}{\partial B}\int_0^1 F(B+x)h(x)dx \\
&= -1 - \nu(1-\beta) + \beta\frac{\partial}{\partial B}\left[\int_0^{1-B}\rho_1(B+x)h(x)dx + \rho_1(1 - H(1-B)) + \int_{1-B}^1 \rho_2(B+x-1)h(x)dx\right] \\
&= -1 - \nu(1-\beta) + \beta\left[\int_0^{1-B}\rho_1 h(x)dx + \int_{1-B}^1 \rho_2 h(x)dx\right] = -1 - \nu(1-\beta) + \beta\left[(\rho_1 - \rho_2)(H(1-B)) + \rho_2\right]
\end{aligned}$$

Even though the production function is not differentiable everywhere, W_1 is differentiable. This derivative is positive as long as:

$$-1 - \nu(1-\beta) + \beta\left[(\rho_1 - \rho_2)(H(1-B)) + \rho_2\right] \geq 0 \Leftrightarrow H(1-B) \geq \frac{1 + \nu(1-\beta) - \beta\rho_2}{\beta(\rho_1 - \rho_2)} \tag{16}$$

Using $\hat{x} = \frac{\beta\rho_1 - 1 - \nu(1-\beta)}{\beta(\rho_1 - \rho_2)}$, I have: $H(1-B) \geq 1 - \hat{x}$.

B.2 Proof of proposition 5

Each entrepreneur i has a private type x^i which is its own wealth. The government implements transfers $T_1(x^i)$ and $T_2(x^i)$. Using the revelation principle, the government's problem can be written as follows:

$$\begin{aligned}
&\max \int_0^1 \beta\left[f\left(x^i + T_1(x^i)\right) + T_2(x^i)\right] - \left[T_1(x^i) + T_2(x^i)\right] di + \int_0^\nu T_1(x^i) + T_2(x^i) di \\
&\forall x^i, \forall \tilde{x}^i,\ f\left(x^i + T_1(x^i)\right) + T_2(x^i) \geq f\left((x^i + T_1(\tilde{x}^i)\right) + T_2(\tilde{x}^i) \\
&\qquad T_1(0) + T_2(0) \geq T_1(\tilde{x}^i) + T_2(\tilde{x}^i) \\
&\qquad T^2(x^i) \geq 0
\end{aligned}$$

Assuming an interior solution, derivates are such that:

$$\frac{\partial T_1}{\partial \tilde{x}^i}\rho_1 + \frac{\partial T_2}{\partial \tilde{x}^i} \leq 0 \text{ when } x^i + T_1 \leq 1 \text{ and } \frac{\partial T_1}{\partial \tilde{x}^i}\rho_2 + \frac{\partial T_2}{\partial \tilde{x}^i} \geq 0 \text{ when } x^i + T_1 \geq 1$$

One can check that: $T_1(\tilde{x}^i) = 1 - \tilde{x}^i$ and hence:

$$\frac{\partial T_2}{\partial \tilde{x}^i} \geq \rho_2 \ \forall \tilde{x}^i \leq x^i \ ; \ \frac{\partial T_2}{\partial \tilde{x}^i} \leq \rho_1 \ \forall \tilde{x}^i \geq x^i$$

In particular: $T_2(\tilde{x}^i) = -R(1-\tilde{x}^i)+T_2(1)$ satisfies the IC constraints when $R \in (\rho_2, \rho_1)$. The hiding constraint requires that: $T_2(0) = 0$. Maximization yields $R = \rho_2$.

Then we have to compare as in equation 10

$$\max_B \left\{\beta \int_0^1 \left[f(x^i + B)\right] di - B(1 + \nu(1-\beta))\right\} - \int_0^1 \left[\beta\rho_1 - (1-x^i) - (1+\nu(1-\beta) - \beta)\rho_2 x^i\right] di$$

In particular, one can take $B = \int(1 - x^i)di$:

$$\left\{\beta \int_0^1 \left[f(x^i + B)\right] di - B\nu\right\} - \int_0^1 \left[\beta\rho_1 - (1+\nu)\rho_2 x^i\right] di \geq 0$$

Finally, a sufficient condition for this inequality to hold is that $\beta\rho_1 \leq \rho_2 + \nu(\rho_2 - 1)$.

Randomization with T_1 and T_2 The government can try to decrease the cost of bailouts by randomizing T_1. First remark that the support of T_1 is a subset of $[0, 1-\tilde{x}^i]$. Indeed, as $\tilde{x}^i \leq x^i$, over $1 - \tilde{x}^i$ the marginal return for the government is negative almost surely.

Without loss of generality, I can consider only distributions where the government $T_1 = 1 - x^i$ with some probability and 0 otherwise, using the piecewise linearity of the production function. Consequently, $T_2(x^i) = p\rho_2 x^i$.

$$\int_0^1 \beta\left[p\rho_1 + (1-p)\rho_1 x^i + p\rho_2 x^i\right] - (1+\nu)\left[p(1-x^i) + p\rho_2 x^i\right] di$$

The derivative with respect to p is positive as long as:

$$\int_0^1 \beta\left[\rho_1(1-x^i) + \rho_2 x^i\right] - (1+\nu)\left[(1-x^i) + \rho_2 x^i\right] di \geq 0$$

$$\frac{\beta\rho_1 - (1+\nu)}{\beta(\rho_1 - \rho_2) - (1+\nu)(1-\rho_2)} \geq \int_0^1 x^i di$$

As a consequence, for $\frac{\beta\rho_1 - (1+\nu)}{\beta(\rho_1-\rho_2)-(1+\nu)(1-\rho_2)} \geq \int_0^1 x^i di$, the probability is such that $p = 1$. In that case, the results of the last paragraph hold. Otherwise, $p = 0$: there are no transfers at all.

B.3 Proof of Theorem 1

The proof relies on the following lemma:

Lemma 10. *Let h be a distribution of portfolios, which is not degenerate at $x = 1$ and B the corresponding optimal bailout as derived in proposition 1. For every $\pi \in [0, 1]$, there exists a function $Z_h(\pi)$, such that:*

- *If $B > 0$, $Z_h(\pi)$ is a continuous and decreasing function of π. For $Z \leq Z_h(1)$, $\{\pi = 1, B, h\}$ is an equilibrium and for $Z = Z_h(\pi)$, $\{\pi, B, h\}$ is an equilibrium.*
- *If $B = 0$, $Z_h(\pi)$ is constant and for $Z \leq Z_h(1)$, $\{\pi = 1, B, h\}$ is an equilibrium.*

Consider then an equilibrium i where $\pi = 1$. There exists then a distribution of repayment to domestic entrepreneurs, but this repayment do not exceed 1 as $p = 1$. Then for $Z > 1$, W_1 is a decreasing function of Z on the one hand and W_0 does not depend on Z. There exists then a value Z_i such that $W_0 > W_1$ for $Z > Z_i$ but $W_0 \leq W_1$ for $Z \leq Z_i$. Defining $\overline{Z} = sup_{\{i\}} Z_i$, we have that for $Z \leq \overline{Z}$, there exists an equilibrium where $\pi = 1$. Notice furthermore that:

- β: $\frac{\partial(W_1 - W_0)}{\partial \beta} = \int_0^1 F(z^i + x^i) - F(B + x^i) di$: the sign is ambiguous and depends on the distribution.
- ν: $\frac{\partial(W_1 - W_0)}{\partial \nu} = B \geq 0$.

B.4 Proof of proposition 3

For any distribution of portfolios h, π is a fixed point:

$$\pi^* = \arg\max_{\pi} \max_{B} \pi W_1 \left(Z \left[1 + \frac{1-\gamma}{\pi^* \gamma} \right] \right) + (1-\pi) W_0(B)$$

Let $Z_h(\pi)$ be a function as in lemma 10. If $Z \left[1 + \frac{1-\gamma}{\gamma} \right] \leq Z_h(1)$, then $\pi = 1$ is still an equilibrium. Otherwise, there exists π such that $Z \left[1 + \frac{1-\gamma}{\gamma} \right] = Z_h(\pi)$. We have to change the value of repayment and then π is the solution of the following equation:

$$Z \left[1 + \frac{1-\gamma}{\pi \gamma} \right] = Z_h(\pi)$$

As $Z_h(\pi)$ is of the form: $Z_h(\pi) = \frac{\zeta_1}{\pi} + \zeta_2$ and $\zeta_2 > 0$, the only solution of the equation is $\pi = 0$.

B.5 Proof of proposition 6

Optimal bailouts in case of partial default are such that:

$$B = \max\{b, H(1 - \frac{b}{1 - \frac{\lambda}{\lambda^e}}) \geq 1 - \hat{x}\} \text{ for } \lambda < \lambda^e \text{ and } B = 0 \text{ for } \lambda \geq \lambda^e$$

Indeed,

$$\frac{\partial W(\lambda)}{\partial B} = -1 - \nu(1-\beta) + \beta \frac{\partial}{\partial B} \int_0^1 F\left(\lambda \frac{1-x^i}{\lambda^e} + B + x^i \right) di$$

Remark that:

$$\lambda \frac{1-x^i}{\lambda^e} + B + x^i \leq 1 \Leftrightarrow \left(\frac{\lambda}{\lambda^e} - 1 \right)(1-x^i) \leq -B \Leftrightarrow$$

$$\text{either for } \lambda > \lambda^e,\ (1-x^i) \leq -\frac{B}{\frac{\lambda}{\lambda^e} - 1} \text{ or for } \lambda < \lambda^e,\ (1-x^i) \geq -\frac{B}{\frac{\lambda}{\lambda^e} - 1}$$

The first inequality is violated and thus $B = 0$ in that case. In the other case, using the same computation as in proposition 1. Furthermore, notice that B is a continuous function of λ. The derivative of welfare with respect to the fraction of default is:

$$\frac{\partial W}{\partial \lambda} = -Z + \beta \frac{\partial}{\partial \lambda} \int_0^1 F(\lambda \frac{1-x}{\lambda^e} + B + x) h(x) dx,$$

and

$$\lambda \frac{1-x}{\lambda^e} + B + x > 1 \Leftrightarrow B > \left(1 - \frac{\lambda}{\lambda^e} \right)(1-x).$$

This latter inequality is always satisfied when $\lambda^e \leq \lambda$. Then:

$$\frac{\partial W}{\partial \lambda} = -Z + \beta \frac{\partial}{\partial \lambda} \int_0^1 \left[\rho_2 \left(\lambda \frac{1-x}{\lambda^e} + x - 1 \right) + \rho_1 \right] h(x)dx = -Z + \beta \int_0^1 \left[\rho_2 \frac{1-x}{\lambda^e} \right] h(x)dx$$

and for $\lambda^e \geq \lambda$,

$$\frac{\partial W}{\partial \lambda} = -Z + \beta \frac{\partial B}{\partial \lambda} + \beta \int_0^{\min(1-\frac{B}{1-\frac{\lambda}{\lambda^e}},1)} \rho_1 \frac{1-x}{\lambda^e} h(x)dx + \beta \int_{\min(1-\frac{B}{1-\frac{\lambda}{\lambda^e}},1)}^1 \rho_2 \frac{1-x}{\lambda^e} h(x)dx$$

As B is such that:

$$B = \max\{b, H\left(1 - \frac{B}{1-\frac{\lambda}{\pi^e}}\right) \geq 1 - \hat{x}\} \text{ and then: } \frac{\partial B}{\partial \lambda} \geq 0 \text{ and } \frac{\partial^2 B}{\partial \lambda^2} \leq 0.$$

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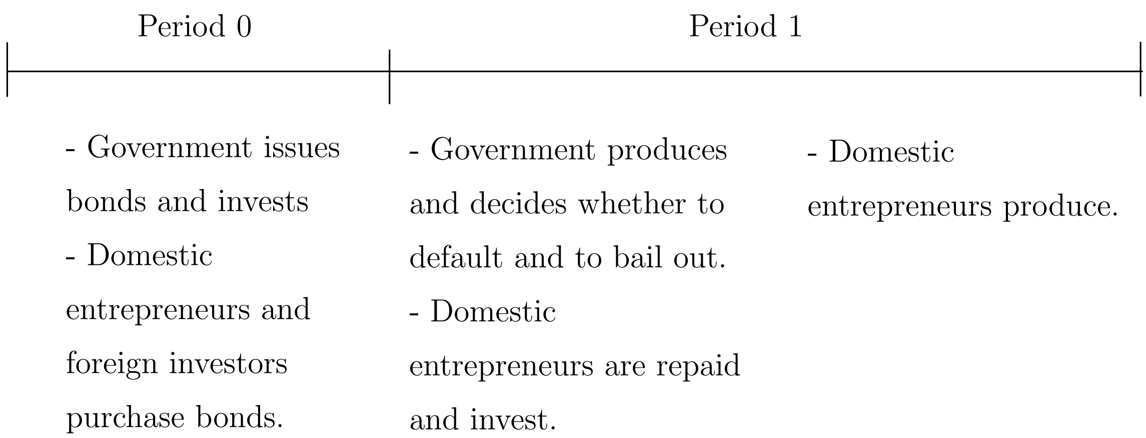

Figure 1 – Timing

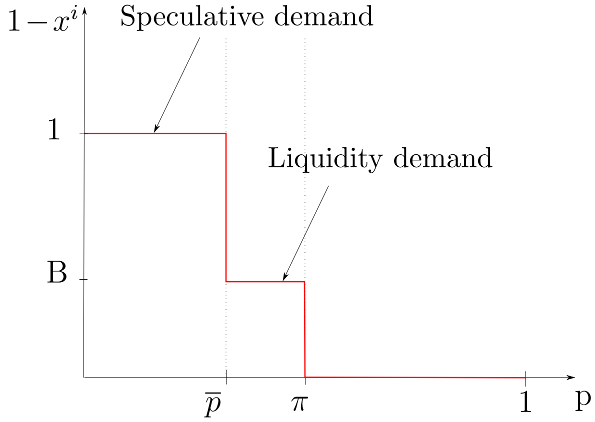

Figure 2 – Entrepreneurs' demand for domestic bonds

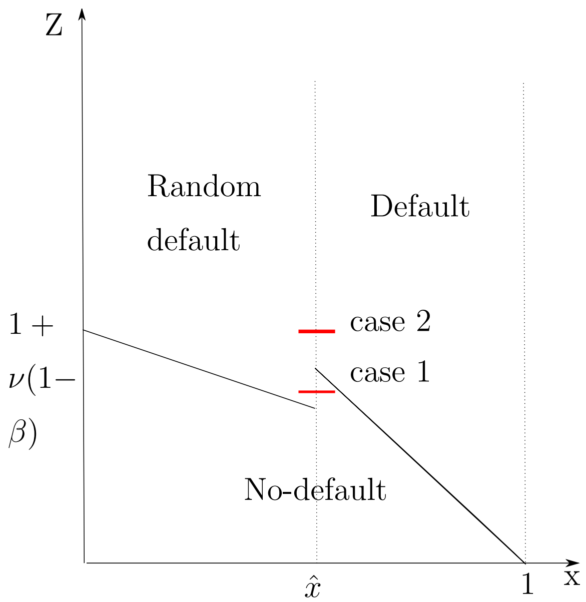

Figure 3 – Equilibria as functions of domestic holdings (x) and repayments (Z) for undiversified portfolios

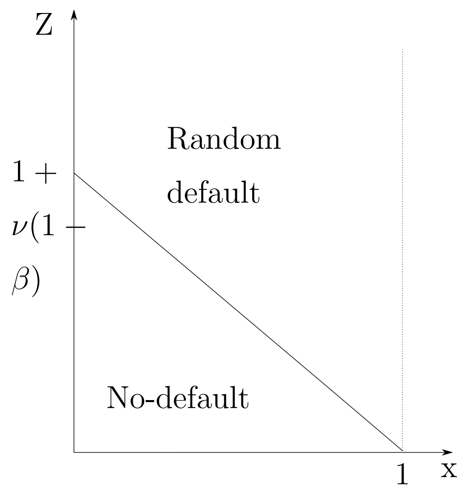

Figure 4 – Equilibria as functions of domestic holdings (x) and repayments (Z)

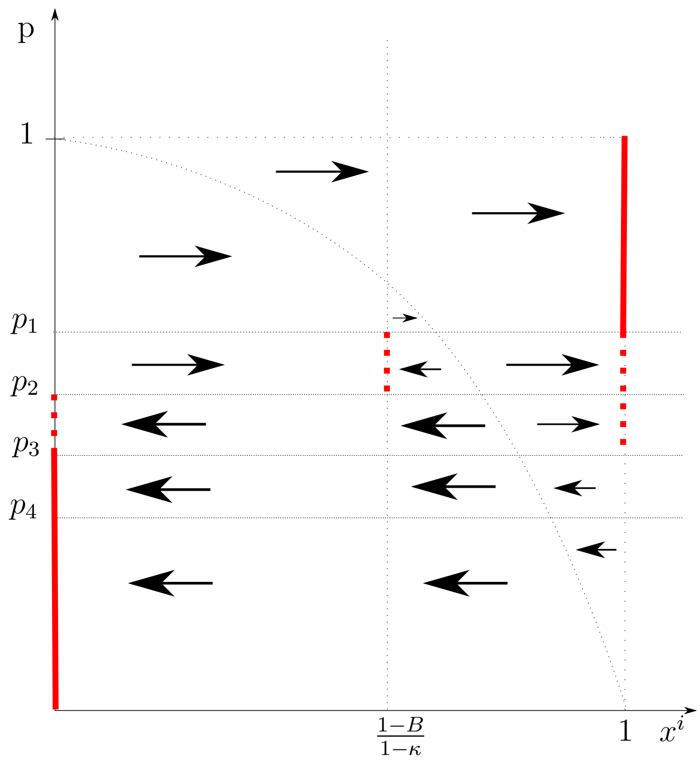

Figure 5 – Domestic entrepreneurs' demand for foreign bonds with capital controls

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