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Résumé

Lorsque la monnaie était d'or et d'argent, les individus cherchaient à en déterminer la valeur dans l'échange. Cet article s'intéresse à une solution bien connue à ce problème: l'évaluation des espèces. Nous construisons un modèle de prospection et de rencontre dans lequel les agents échangent des pièces dont la qualité est a priori difficile à déterminer. Ils peuvent payer pour utiliser une technologie d'évaluation des monnaies qui révèle avec certitude les caractéristiques intrinsèques (poids et alliage). Nous étudions deux sources d'information imparfaite : la contrefacon et le rognage des monnaies. Dans le cas de la contrefacon, nous montrons que la technologie d'évaluation permet de réduire les inefficiences créées par la difficulté à reconnaître la qualité des espèces, augmentant les quantités échangées et la fréquence des échan ges. Cependant cette transparence sur la qualité des actifs n'augmente pas toujours le bien être car la technologie d'évaluation réduit le pouvoir d'achat des espèces contrefaites qui sont alors acceptées au rabais. Dans le cas du rognage des pièces, nous montrons que les agents rognent pour deux raisons. Tout d'abord dans l'espoir de faire passer une pièce de mauvais aloi pour une pièce de bon aloi. Ensuite pour réduire le pouvoir d'achat des espèces ayant une trop forte valeur. Alors que les technologies d'évaluation de pièce aident à supprimer le premier motif au rognage, il n'a aucun effet sur le second type. Bien qu'illustré par le cas des monnaies marchandises, cette analyse s'applique également aux échanges sur des marchés décentralisés d'actifs liquides dont la qualité est incertaine.

Mots-clés: Monnaie marchandise, information asymétrique, rognage des monnaies, évaluation des actifs.
Code JEL: D82, E42, N23

Abstract

When money was made of gold and silver, individuals faced the problem of determining the intrinsic content of coins in many exchange situations. In this paper we look at a well-documented solution to this problem, and a key institution of the commodity money system: coin assaying. To that goal we build a model of search and matching in which agents trade using coins that are imperfectly recognizable, but have access to a coin inspection technology that reveals the intrinsic content of coins for a fee. We consider two sources of imperfect information: counterfeit coins and clipping. With counterfeits we show that coin assaying reduced the extent of inefficiencies associated with imperfect coins recognizability (namely lower traded quantities and lower trading frequencies). Yet it did not necessarily increase welfare because it unmasked counterfeits which then traded at a discount, reducing total output. With clipping, we show that agents clip for two reasons: in the hope of passing an inferior coin for a superior one, and to reduce the purchasing power of coins that are too valuable. While coin assaying could remove the first type of clipping, it had no effect on the second. While framed in the context of the commodity money system, our analysis relates to the more general issue of asset trading under imperfect information.

Keywords: Commodity Money, Asymmetric Information, Coin Assaying, Clipping.
JEL: D82, E42, N23.

Non technical summary

How does transparency on asset quality impact on agents' welfare? As first shown by Hirshleifer (1971), while traders may have an incentive to acquire information, more transparency does not necessarily improve welfare. In this paper, we study whether and when more transparency on assets can increase welfare. The distinctive features of our model are 1) that assets with positive yield have an explicit role in trade and 2) that when the yield is sometimes unknown to buyers, sellers can use a costly state verification (CSV) technology to signal the quality of their assets to prospective buyers. We embed these features in a model in which the assets have an explicit liquidity component and study the trade-offs between transparency and liquidity from an optimality standpoint and the consequences of these trade-offs on the redistribution of gains from trades.

Our model has two types of assets that may differ in their intrinsic value (the revenues accruing to the owner) and trading value (assets can be used to pay for the purchase of non-durable goods). The trading of the assets on the market is complicated by the buyers' difficulty to distinguish low-yield assets from high-yield assets. This creates an adverse selection problem whereby some mutually beneficial trades either do not occur (good assets are hoarded when not recognized) or do occur but at a discounted price (good assets trade at a discount when not recognized). We characterize the equilibria in which those assets are traded and in which the agents choose (or not) to use the technology of signaling the quality of their assets. We show that even though transparency on assets is an optimal choice by individual sellers, it does not necessarily improve welfare.

Our study is framed into the context of the commodity money system. Indeed this system is a natural setting to discuss the impact of asset transparency on welfare. Commodity money shares many features with the trading of heterogeneous assets on over-the-counter markets. Gold or silver coins were indeed assets with positive yields that were used to purchase other goods or assets in decentralized trades. As is the case with some recent financial products, the quality of their alloy was difficult to tell and verification technologies were used to ascertain and certify their quality and help in their pricing. This task of assessing and certifying was performed by specialists, known as money changers or goldsmiths.

Historical evidence show that the availability and affordability of the assaying technology did not suppress all informational problems on coins: Agents continue to trade using 'clipped' coins —i.e. coins to which part of the metal was removed— or counterfeited coins —i.e. coins with a lower alloy than believed. The fact that clipped coins continued to be used in trade despite the pervasiveness and affordability of the coin assaying technology suggests that the absence of transparency on assets was not always the main impediment to trades. The tradeoff was not simply between the cost of acquiring information and the benefits of trading at the full informational value. Rather our model suggests richer trade-offs involving redistribution of the gains from trade between buyers and sellers triggered by the decision of private individuals to acquire information on the circulating assets.

We characterize the optimal trading behavior of agents with regards to the decision both to sell the asset and to acquire the CSV technology. We compute the welfare impact of the use of the coin assaying technology in two cases. First, we study situations in which the proportion of the high yield relative to low yield assets is given. Second, we allow agents to endogenously choose this proportion. In both cases we find that increasing transparency is not always welfare improving. In the first case, the reason is that the decision by high-quality coin holders to signal their coin unmasks holders of low-quality coins who were trading it at a premium. Because of decreasing marginal utility to consumption, the utility loss of unmasked holders of low yield assets is greater than the utility gains earned by holders of high-yield assets that now trade their asset at its full information value. When agents choose the relative proportion of assets, transparency does not improve welfar e because agents are better off using lower yield coins to pay for the goods.

1 Introduction

What are agents' incentives to reveal information? What is the effect of information revelation on welfare? In this paper we study these questions within the context of the commodity money system.

In the commodity money system money was made of precious metal coins (gold or silver usually) whose exchange value was influenced by their intrinsic content (weight and fineness). Because assessing the intrinsic content of coins was not straightforward, good (high quality) coins could be hard to tell apart from bad (low quality) coins. As shown in Velde, Weber and Wright (1999), henceforth VWW, the quality difference created an adverse selection problem whereby some mutually beneficial trades either did not occur (good coins were hoarded when not recognized) or occurred at a discounted price (good coins traded at a discount when not recognized).

In addition to hoarding and discounting, agents typically responded in two additional ways to this information problem. They could alter good coins by removing some of their intrinsic content, and then try to pass them on as unaltered good coins. This operation was known as *clipping*. Or they could certify the intrinsic content of their coin via an expert, an operation known as *coin assaying*. Both clipping and assaying are amply documented by economic historians (e.g. Sargent and Velde, 2002). A central contribution of this paper is to study within the framework developed by VWW the impact of clipping and coin assaying on trade, output and welfare. Despite their importance, to our knowledge there does not exist any study of their impact.

While our study is framed within the historical context of commodity money, the questions it raises apply to the broader context of asset trading under imperfect information. There are good and bad assets that are hard to distinguish, and opportunities to produce bad assets (as with clipping) or certify their value (as with coin assaying). Given that those assets are monetary and therefore improve welfare by expanding the set of feasible trades, what happens to welfare when these two opportunities are thrown in? Similar questions are asked in the context of the recent financial crisis (Gorton and Ordoñez, 2012). It turns out sometimes more information does not necessarily improve welfare (Andolfatto and Martin, 2012; Andolfarro, Berentsen and Waller 2014).

When the proportion of high and low quality coins is exogenous, we show that transparency in the quality of coins increases welfare by suppressing the adverse selection problem impeding on the number of trades. The coin assaying technology increases the extensive margin (i.e. the number of trades) because coin sellers obtain better terms of trade, thereby increasing their willingness to sell. This intuition, however, does not carry through when the adverse selection problem translates into lower quantities traded rather than less trade. The reason is simply that the decision by high-quality coin holders to certify their coin unmasks holders of low-quality coins who were trading it at a premium. The screening technology substitutes a lottery (a low quantity with probability p or a high quantity with probability $1 - p$) to an average quantity with probability 1. Risk aversion ensures that agents prefer the sure payment to the lottery, i.e. imperfect information to the certification of the high quality coin.

We then study whether greater transparency increases welfare when sellers are allowed to choose the quality of their coin. We do so by allowing agents to clip their good coin, consume the difference and trade with the resulting bad coin. In an environment where coin assaying was available, one would expect clipping to survive because of the high cost of operating the certification technology. But evidence suggests the opposite: despite availability and affordability of coin assaying, clipping was pervasive until the invention of edges. In this section we show that the trade off behind the decision to clip is richer than simply comparing costs and benefits of the technology. We show in particular that there were two forces behind clipping. First, because coins were hard to tell apart, agents clipped their coin in the hope of passing off a inferior coin for a good one and pocketing the proceed. But there also existed a second motive behind clipping: agents clip coins that are too heavy (i.e. their purchasing power is too high). By reducing their intrinsic content, hence their purchasing power, this increases the gains from trade by raising the marginal utility and reducing the marginal cost of the consumed output. While coin assaying can remove clipping caused by imperfect asset recognizability, it cannot remove clipping motivated by too 'heavy' coins. This finding may explain why clipping was pervasive during the commodity money system despite the well-documented availability and affordability of coin assaying (more on this in the text).

From a modeling point of view, we fit our story within the search and matching theory of money, in the vein of Trejos and Wright (1995) and Velde, Weber and Wright (1999). These papers have indeed several comparative advantages for the problem at hand. First, this class of models is explicit about the frictions that allow some assets to be used as medium of exchange (Williamson and Wright, 2010). Second, they naturally nest decentralized trading, a feature that fits the commodity money system well. Third, the liquid asset is indivisible in our model. Although this assumption can be relaxed building on Shi (1997) and Lagos and Wright (2005), we view indivisibility as descriptive of the economy we want to model. The history of the commodity money, and to some extent of the financial system, is also about the technological difficulties of achieving a system with portable, divisible and recognizable assets that would make them as liquid as fiat money. Indivisibility, in the form of the lack of small change was indeed a major impediment to trade in the commodity money system and has been abundantly documented (Munro 1988, Glassman and Redish 1988, Redish 2000, Sargent and Velde 2002). While we focus on the answer to the information problem, we also want to maintain the other defining characteristics of the commodity system, especially indivisibility.[1] We will nonetheless discuss the relaxation of our key assumptions along the way and show how the results are affected.

Our paper is closest to the research that investigates the role of money when agents have private information about the goods or assets they trade. In those economies money works as a substitute for information acquisition on goods quality (Brunner and Meltzer, 1971; King and Plosser, 1986) and alleviates the moral hazard problem by reducing the incentive to produce 'lemons' (Williamson and Wright, 1994, Trejos 1999, Berentsen and Rocheteau, 2004). Here we ask: What if money itself can be a lemon? Imperfect recognizability of money is also central to models of counterfeiting in the fiat money system, in particular Kullti (1996), Green and Weber (1996), Williamson (2002), Nosal and Wallace (2007) and Quercioli and Smith (2009). To our knowledge, however, this paper is the first to study the impact of both clipping and coin assaying. Though framed in the context of the commodity monety system, our work also

[1] A similar approach is adopted in Wallace and Zhou (1998).

relates to the literature on the role played by information on markets transparency. See for instance Morris and Shin (2002) and Gorton and Odonez (2012) where more information does not necessarily lead to higher welfare (Hirshleifer 1971).

2 The Environment

The background economy is essentially VWW. The economy is populated by a $[0, 1]$ continuum of infinitely lived agents indexed by k and there are $I \geqslant 3$ types of non storable goods. A type $k \in I$ agent consumes good k and produces good $k + 1$, ruling out barter trade. Agents meet bilaterally according to an anonymous random matching Poisson process with arrival rate α. They discount the future at rate $\frac{1}{1+r} > 0$.

To trade, agents use precious metal coins and each agent can hold at most one coin. Coins are of two types, light (L) and heavy (H). We will first study an economy where money stocks are exogenous (the proportion of light and heavy coins is fixed). In this environment light coins will be called counterfeit coins and correspond to full weight low fineness coins. Later we endogenize the composition of the money supply by letting agents choose whether to keep their heavy coin, or turn it into a light coin by clipping it. Light coins will then correspond to clipped coins (low weight full fineness).

We let M_i be the measure of agents holding a coin of type $i = \{L, H\}$ with $M = M_H + M_L$ so that $1 - M$ represents the fraction of sellers. Because commodity money always has an alternative usage (such as a consumption good), each coin yields to its owner a flow of utility proportional to its intrinsic content: γ_L for a light coin and $\gamma_H > \gamma_L$ for a heavy coin. Note that if agents decide to hoard heavy coins for instance, those coins are still part of the money supply since they can be used in trade any time.

We note $\beta = \frac{\alpha}{I}(1 - M)$ the probability per unit of time of a single-coincidence-of-wants meeting, i.e. a buyer meets a seller who produces his consumption good. In such meeting, it is assumed that terms of trade are formed via bargaining where (for simplicity) the buyer has all the bargaining power. That is, when a buyer chooses to make an offer, his offer leaves the seller indifferent between accepting and refusing. If the buyer decides to trade, agents swap their inventories so that the buyer becomes a seller and vice versa. Consuming q units of their consumption good yields agents $u(q)$ with $u(0) = 0$, $u'(q) > 0$ and $u''(q) < 0$. Producing q units of their production good costs agents $c(q) = q$. Further, there is a unique $\hat{q} > 0$ such that $u(\hat{q}) = \hat{q}$.

The information problem on coins is captured as follows: while buyers always know the type of their coin (i.e. buyers can evaluate the quality of their coin at no cost), sellers cannot always tell the true intrinsic content of the coin that the buyer offers to pay with. Specifically, we assume that when presented with a coin, a seller learns its true quality via a common knowledge signal that is informative with probability $\theta \in (0, 1)$ and uninformative with probability $1 - \theta$. When the signal is uninformative, a seller cannot tell what type of coin the buyer is offering. An interpretation is that buyers have full knowledge of their coin because they have more time to inspect it.

Based on historical records (e.g. Spufford 1986, Bompaire 1987, Gandall and Sussman 1997, Bompaire 2007), we allow buyers to rent a coin assaying device for a fee δ. Such technology achieved two operations: the weighting of the coin and the testing of its fineness. Weight was determined using precise scales, and fineness was evaluated by rubbing the coin on a special stone (the touchstone) and comparing the color of the trace left with that of needles of known fineness (Gandall and Sussman, 1997). A more precise assay involved melting down a sample of coins to weigh the quantity of pure metal. For obvious reasons, this last test was limited to large payments involving many coins.[2] Coin assaying was often intermediated via agents specialized in monetary affairs such as moneychangers or goldsmiths (Spufford 1986; Bompaire 2007). There is ample evidence of their activity as coin assayers in Medieval Europe (see e.g. De Roover, 1948; Favreau, 1964; Bonnet, 1973; Chevalier, 1973; De La Roncière, 1973; Bompaire, 1987) but also in Ancient Greece and in the Roman Empire (Lothian, 2003), in Bizance (Kaplanis 2003) and the Islamic world (Udovitch, 1975). Coin assaying was then a central feature of the commodity money system.

Thanks to coin assaying, a buyer is able to certify the quality (weight and fineness) of his coin in front of the seller and produce a certificate of quality, which unambiguously reveals the quality of the coin, light or heavy. It should be noted that sellers have no incentive to rent the technology because of the buyer-takes-all assumption. As will be clear shortly, buyers holding coins of lower quality have no incentive to rent the technology either since they actually benefit from the information problem by trading their coin above its full information value in some circumstances. Therefore, only buyers holding good coins may want to pay for the technology in equilibrium.

3 Counterfeits and Coin Assaying

We start with an economy in which money stocks are given: there is a fraction M_L of buyers holding counterfeit coins and there is a fraction M_H of agents holding genuine coins. Counterfeiting was widespread in the commodity money system (Ashley, 1888, p. 172). Munro (2000), for instance, provides a detailed account of "the war of the gold nobles" between England and Flanders in the late 14th century: On October 1, 1388, the Flemish count, Duke Philip the Bold of Burgundy, began striking counterfeit imitations of the English gold nobles struck in London's Tower Mint and the Calais Stable mint (Calais was a recently conquered enclave on the French side of the English Channel). While identical in weight and alloy to its British counterpart in the first issues, Flemish authorities soon started minting nobles of lower fineness, making it difficult for agents to tell genuine nobles from fake ones. Similar examples can be found in Bompaire (2007).

The sequence of events is as follows: At the beginning of the period, each buyer decides whether to rent the coin-testing technology or not. He then searches for a seller. If he has the technology and finds a seller producing his consumption good, he uses the technology to show the quality of his coin and then makes an offer. If the parties agree to trade, the seller produces the agreed quantity and they swap inventories. The technology is returned at the end of the

[2] While assaying determined fineness and scales determined weight, for sake of simplicity we will use 'coin assaying' for both fineness and weight testing.

period by the former buyer.[3,4]

To conduct the study we proceed as follows. First, we characterize the equilibria in which agents chose not to use the coin assaying technology. Then we characterize equilibria where agents choose to use the coin assaying technology. We will see that both equilibria coexists for some parameter values (i.e. there is multiplicity), and that there also exists a mixed-strategy equilibrium in which agents randomize over the use of coin assaying.

Let us note λ_{ij} the probability (endogenously determined) that a buyer with a coin of type $i \in \{L, H\}$ wants to trade with a seller of type $j \in \{K, U\}$ where K means that the quality of the coin is known to the seller (the signal is informative), and U means that the quality of the coin is unknown (the signal is uninformative). The (steady-state) Bellman equation for a buyer with a light coin is

$$V_L = \frac{1}{1+r}\left\{\begin{array}{c} \gamma_L + \beta\theta \max_{\lambda_{LK}} \left[\lambda_{LK}\left[u(q_L)+V_0\right]+\left(1-\lambda_{LK}\right)V_L\right] \\ +\beta\left(1-\theta\right)\max_{\lambda_{LU}}\left[\lambda_{LU}\left[u(\bar{q})+V_0\right]+\left(1-\lambda_{LU}\right)V_L\right] \\ +\left(1-\beta\right)V_L \end{array}\right\}. \tag{1}$$

Multiplying by $(1+r)$ and rearranging yields the flow version of the Bellman equation,

$$rV_L = \gamma_L + \beta\theta \max_{\lambda_{LK}} \lambda_{LK}\left[u(q_L)+V_0-V_L\right] + \beta\left(1-\theta\right)\max_{\lambda_{LU}}\lambda_{LU}\left[u(\bar{q})+V_0-V_L\right]. \tag{2}$$

Equation (2) gives the flow return to a buyer holding a light coin, made of three components. The first part gives the periodic return on holding the coin, γ_L. The second part corresponds to the probability that he meets a producer and there is a single coincidence of wants, β, multiplied by the probability that the seller recognizes the light coin, θ, times the net gain from trading the light coin against q_L, which is equal to consuming q_L and switching from buyer with a light coin to producer, that is $u(q_L) + V_0 - V_L$, times the probability that he decides to trade with him, λ_{LK}. The last part has a similar interpretation. The difference is that because the coin is not recognized, it is not traded for q_L but for an average quantity $\bar{q}$ defined in equation (8) below.

Similarly, the flow Bellman equation for a buyer holding a heavy coin is given by

$$rV_H = \gamma_H + \beta\theta \max_{\lambda_{HK}} \lambda_{HK}\left[u(q_H)+V_0-V_H\right] + \beta\left(1-\theta\right)\max_{\lambda_{HU}}\lambda_{HU}\left[u(\bar{q})+V_0-V_H\right]. \tag{3}$$

From the take-it-or-leave-it bargaining protocol, the informed seller is indifferent between not trading or producing q_i for the buyer and becoming a buyer with a coin of type i. Therefore the offers made by buyers satisfy

$$V_0 = -q_i + V_i \text{ for } i \in \{L, H\}. \tag{4}$$

[3]Renting the technology takes the form of a side payment of real output that can be consumed after the transaction by the coin assayer for an implied utility δ. The advantage of this approach is that it keeps the total number of coins constant.

[4]The timing of assaying decisions by buyers is important. In our model buyers decide whether to rent the technology before they know if the seller has recognized the coin or not. Assuming otherwise would make the model more realistic, but less tractable as there would always be a mix of certified and non-certified good coins in addition to bad coins. Moreover, since the assaying decision would no longer be based on the probability of an informative signal but on the realization of the signal itself, it would no longer be possible to partition the (r, θ) space.

Similarly the uninformed seller is indifferent between not producing and producing and trading $\bar{q}$ against the unknown coin so that

$$V_0 = -\bar{q} + \pi V_H + (1-\pi)\, V_L \tag{5}$$

where π is the probability that the buyer has a heavy coin given that he wants to trade,

$$\pi = \frac{\lambda_{HU} M_H}{\lambda_{HU} M_H + \lambda_{LU} M_L}.$$

Because sellers never get any utility from trade, we have $V_0 = 0$ and then $V_H = q_H$ and $V_L = q_L$. Once we insert these values into (2) and (3) we obtain

$$rq_L = \gamma_L + \beta\theta \max_{\lambda_{LK}} \lambda_{LK} \left[u(q_L) - q_L\right] + \beta\,(1-\theta) \max_{\lambda_{LU}} \lambda_{LU} \left[u(\bar{q}) - q_L\right] \tag{6}$$

$$rq_H = \gamma_H + \beta\theta \max_{\lambda_{HK}} \lambda_{HK} \left[u(q_H) - q_H\right] + \beta\,(1-\theta) \max_{\lambda_{HU}} \lambda_{HU} \left[u(\bar{q}) - q_H\right] \tag{7}$$

with

$$\bar{q} = \pi q_H + (1-\pi)\, q_L. \tag{8}$$

Finally, the λ_{ij} satisfy the following incentive conditions: for $i \in \{L, H\}$,

$$\lambda_{iK} = \begin{cases} 1 & \text{if } u(q_i) - q_i \geq 0 \\ 0 & \text{otherwise,} \end{cases} \tag{9}$$

$$\lambda_{iU} = \begin{cases} 1 & \text{if } u(\bar{q}) - q_i \geq 0 \\ 0 & \text{otherwise.} \end{cases} \tag{10}$$

As shown by Velde Weber and Wright (1999), there exist three types of pure-strategy monetary equilibria in this commodity money economy where coin assaying is not available: (i) both coins circulate *by weight* (heavy coins are traded only when recognized, i.e. $\lambda_{LK} = \lambda_{LU} = \lambda_{HK} = 1$ and $\lambda_{HU} = 0$), (ii) both coins circulate *by tale* (unrecognized coins trade at the same price, hence a premium on light coins and a discount on heavy ones, i.e. $\lambda_{LK} = \lambda_{LU} = \lambda_{HK} = \lambda_{HU} = 1$), and ($iii$) only light coins circulate (i.e. single currency equilibrium with $\lambda_{LK} = \lambda_{LU} = 1$ and $\lambda_{HK} = \lambda_{HU} = 0$).

To show how the coin-testing technology impacts on circulation and welfare, we will start by considering the first two of these equilibria[5] and offer an agent the opportunity to deviate from his strategy and rent the assaying technology. This will add a non-deviating condition for each equilibrium. For instance, a by-weight equilibrium will now be an equilibrium in which light coins always circulate, heavy coins circulate only when recognized *and* no buyer holding a heavy coin deviates by renting the technology to certify his coin. Note that all this happens at time 0 where agents have a one-time chance to deviate from the equilibrium we characterize. Possible mixed-strategy equilibria where agents randomize between hoarding the heavy coin, trading it at a discount or certifying it will also be characterized.

[5] Coin assaying is of no use in a single currency equilibrium.

3.1 By-weight equilibrium

With circulation by weight, a heavy coin trades only if it is recognized by the seller. It follows that light coins always circulate whether recognized or not (and at the same price q_L) since unrecognized circulating coins can only be light coins. Therefore $\lambda_{LK} = \lambda_{LU} = 1$. From (9) the light coin circulates in informed meetings if $\lambda_{LK} = 1$ equivalent to $u(q_L) \geq q_L$, and the heavy coin circulates in informed meetings if $\lambda_{HK} = 1$ equivalent to $u(q_H) \geq q_H$. These two conditions are met if $r > \gamma_H$.[6] Finally, heavy coins do not circulate when not recognized if $\lambda_{HU} = 0$, which from (10) is equivalent to $u(\bar{q}) = u(q_L) \leq q_H$ since $\bar{q} = q_L$ when unrecognized heavy coins are hoarded.

Assume first that coin assaying is not available. Inserting above values for λ and q into (6) and (7), a by-weight equilibrium is a list (q_L, q_H) given by[7]

$$rq_L = \gamma_L + \beta\left[u(q_L) - q_L\right] \tag{11}$$

$$rq_H = \gamma_H + \beta\theta\left[u(q_H) - q_H\right] \tag{12}$$

that satisfy

$$r \geq \gamma_H, \tag{13}$$

$$q_H \geq u(q_L). \tag{14}$$

Set to equality, equation (14) together with (11)-(12) define the by-weight frontier (BWF), as in VWW, which is the set of points in (r, θ) space such that the pair $q_L = q_L(r, \theta)$ and $q_H = q_H(r, \theta)$ that solves (11)-(12) satisfies (14) with equality. A by-weight equilibrium exists for all points in the parameter space (r, θ) to the right of $r = \gamma_H$ and to the left of the BWF (see Figure 1).[8]

Assume now that coin assaying is available. Suppose that every buyer plays the by-weight equilibrium, and one buyer contemplates deviating and certifying his coin. If he does not deviate, he receives $rV_H = rq_H$ given by (12). If he does deviate, he pays δ to rent the technology, certifies the coin in front of the seller and makes a take-it-or-leave-it (deviant) offer $\tilde{q}_C$ to the seller such that

$$-\tilde{q}_C + V_H = V_0. \tag{15}$$

With this offer the seller is indifferent between producing a quantity $\tilde{q}_C$ and becoming a holder of an uncertified heavy coin, V_H, or staying as a producer, V_0. Note that the deviant buyer believes that future buyers will not deviate and then not use the assaying technology, hence

[6] To see why $u(q_H) \geq q_H$ requires $r \geq \gamma_H$, insert $\lambda_H = 0$ into (7) which shows that the return to keeping home the coin is $rq_H = \gamma_H$ so that $q_H = \gamma_H / r$. There is then an incentive to deviate and trade the coin if $u(q_H) - q_H \geq 0$, which is equivalent to $q_H \leq \hat{q}$, or $r\hat{q} \geq \gamma_H$. Given that $u(q_H) = q_H^\eta$ and recalling that $c(q_H) = q_H$, we have $\hat{q} = 1$ so that $u(q_H) - q_H \geq 0$ requires $r \geq \gamma_H$.

[7] In general q_L and q_H are different across equilibria. In this paper, unless specified otherwise, q_L and q_H will implicitly refer to the q_L and q_H of the equilibrium we are considering.

[8] Frontiers are derived numerically using the following functional forms and parameter values, none of which are critical to our results: $u(q) = \sqrt{q}$, $c(q) = q$, $\gamma_H = 0.04$, $\gamma_L = 0.02$ and $\delta = 0.005$. The algorithm is as follows. Step 1: take one $\tilde{r} \geq \gamma_H$ and find the equilibrium q_H and q_L given by (11)-(12) for all $\theta \in (0, 1)$. Then pick the $\tilde{\theta}$ such that (14) holds with equality. Step 2: repeat Step 1 for all $\tilde{r} \geq \gamma_H$. All the couples $\left(\tilde{r}, \tilde{\theta}\right)$ that satisfy (14) with equality constitute the frontier. All frontiers in the rest of the article are built using in the same algorithm.

the V_H in equation (15).[9] Since $V_0 = 0$ from the the take-it-or-leave-it protocol, equation (15) implies

$$\tilde{q}_C = V_H = q_H. \tag{16}$$

That is, the deviant buyer certifying his coin actually asks the seller for exactly the same quantity as if he was not deviating, $\tilde{q}_C = q_H$.

Noting $\tilde{\lambda}_C$ the deviant buyer's strategy whether to trade the certified heavy coin or not, the Bellman equation for the deviator who certifies is

$$\tilde{V}_C = \frac{1}{1+r}\left\{-\delta + \gamma_H + \beta \max_{\tilde{\lambda}_C}\left[\tilde{\lambda}_C\left\{u(\tilde{q}_C) + V_0\right\} + \left(1 - \tilde{\lambda}_C\right)V_H\right] + (1-\beta)\,V_H\right\}. \tag{17}$$

Note that with probability $\beta\left(1 - \tilde{\lambda}_H\right) + (1-\beta)$ he does not trade, returns the technology and moves back to holding an uncertified heavy coin. Using $\tilde{q}_C = q_H$ and $V_0 = 0$, the flow version (assuming he wants to trade the certified heavy coin) is

$$r\tilde{V}_C = \gamma_H - \delta + \beta\left[u(q_H) - q_H\right] + V_H - \tilde{V}_C. \tag{18}$$

Equation (18) says that the net gain from deviating and renting the technology is equal to the periodic return on the heavy coin minus the rent for the technology, plus the net gains from trading the heavy coin at its full information value in all single coincidence of wants meetings, plus the net gain from swapping from deviator back to holding an uncertified heavy coin that trades by weight.

In the end there is no incentive to deviate if the payoff to trading the heavy coin by weight is larger than the payoff to deviating and shopping with a certified heavy coin, that is

$$V_H > \tilde{V}_C \tag{19}$$

which, using (12) and (18), yields

$$\delta > \beta\,(1-\theta)\left[u\left(q_H\right) - q_H\right] + V_H - \tilde{V}_C. \tag{20}$$

Inequality (20) says that a buyer will *not* deviate by certifying his heavy coin if the cost of expertise is larger than the benefit.

To characterize the frontier between circulation by weight and certification (labelled $CF1a$), we insert the indifference condition between deviating or not, i.e. $V_H = \tilde{V}_C$, into (20) and set it to equality. This gives

$$\delta = \beta\,(1-\theta)\left[u\left(q_H\right) - q_H\right], \tag{21}$$

Although buyers holding genuine coins can have them certified now, a by-weight equilibrium still exists to the right of $r = \gamma_H$ and to the left of BWF and $CF1a$. See Figure 1. We postpone comments to section 4.3.

[9] A deviation has then no permanent effect on the coin. As we will see later the same cannot be said when a deviant buyer clips his coin since the coin is then permanently damaged.

3.2 By-tale equilibrium

With circulation by tale, both light and heavy coins trade at the same price $\bar{q}$ when not recognized so that $\lambda_{LK} = \lambda_{LU} = \lambda_{HK} = \lambda_{HU} = 1$ (cf. VWW). As an example of circulation by tale Grierson (1988) notes that in Egypt in the late Middle Ages the circulation of many counterfeits of the Venetian ducat triggered an undervaluation of the real coin. From (9), the two coins circulate in informed meetings if $u(q_L) \geq q_L$ and $u(q_H) \geq q_H$, which again require $r > \gamma_H$. Finally, heavy coins circulate at a discount when not recognized if $\lambda_{HU} = 1$, which from (10) is equivalent to $u(\bar{q}) \geq q_H$.

Assume first that coin assaying is not available. Inserting those values into (6) and (7), a by-tale equilibrium is a list (q_L, q_H) given by

$$rq_L = \gamma_L + \beta\theta\left[u(q_L) - q_L\right] + \beta\left(1-\theta\right)\left[u\left(\bar{q}\right) - q_L\right] \quad (22)$$
$$rq_H = \gamma_H + \beta\theta\left[u(q_H) - q_H\right] + \beta\left(1-\theta\right)\left[u\left(\bar{q}\right) - q_H\right] \quad (23)$$

satisfying the two conditions

$$r \geq \gamma_H \quad (24)$$
$$u(\bar{q}) \geq q_H. \quad (25)$$

Set to equality, equation (25) together with (22)-(23) define the by-tale frontier (BTF). A by-tale equilibrium exists for all points in the parameter space (r, θ) to the right of the BTF (see Figure 1).

Assume now coin assaying is available. Suppose that every buyer plays the by-tale equilibrium and one contemplates deviating and renting the technology. If he does not deviate, he receives $rV_H = rq_H$ given by (23). If he deviates, he pays δ to rent the equipment and makes a deviating offer $\tilde{q}_C$ to the seller that also satisfies (15) so that $\tilde{q}_C = V_H = q_H$. The continuation payoff to the deviating buyer with a heavy coin $\tilde{V}_C$ is again given by (18). Therefore a buyer does not deviate if $V_H > \tilde{V}_C$, which using (18) and (23) transforms into

$$\delta > \beta\left(1-\theta\right)\left[u\left(q_H\right) - u(\bar{q})\right] + V_H - \tilde{V}_C. \quad (26)$$

As per equation (26), the heavy coin will not be certified if the cost of certification is greater than the increase in gains from trade due to the full recognizability of heavy coins $\beta\left(1-\theta\right)\left[u\left(q_H\right) - u(\bar{q})\right]$, plus the net gain from shifting from deviator back to playing by tale, that is $V_H - \tilde{V}_C$. Inserting the indifference condition $V_H = \tilde{V}_C$ into (26) and setting it to equality yields

$$\delta = \beta\left(1-\theta\right)\left[u\left(q_H\right) - u(\bar{q})\right]. \quad (27)$$

We label this frontier $CF2a$. On Figure 1, to the right of BTF and $CF2a$ a circulation-by-tale equilibrium exists even though coin assaying is available.

3.3 Assaying Equilibrium

We now conduct a mirror exercise, i.e. characterize the region where agents certify their heavy coin and have no incentive to deviate and trade it uncertified. Since the deviator can trade an uncertified heavy coin by weight or by tale (as seen above) we will have to consider both deviations.

Let q_C be the quantity traded against a certified heavy coin and V_C be the Bellman equation for the buyer with a certified heavy coin. Because all unrecognized coins can only be light coins in a full-certification equilibrium, light coins circulate at full information value ($\lambda_{LK} = \lambda_{LU} = 1$) with associated payoff

$$rq_L = \gamma_L + \beta \left[u(q_L) - q_L\right]. \tag{28}$$

For a holder of a certified heavy coin we have

$$rq_C = \gamma_H - \delta + \beta \left[u(q_C) - q_C\right]. \tag{29}$$

Equations (28) and (29) describe a full information economy with a periodic return on light coins equal to γ_L, and a periodic return on heavy coins equal to $\gamma_H - \delta$. From (9) the circulation of light coins requires $r \geq \gamma_L$ while that of certified heavy coins requires $r > \gamma_H - \delta$ although we will see that it is dominated by another constraint.

Assume now a buyer contemplates deviating by not certifying his heavy coin. Let us note $\tilde{\lambda}_{HK}$ the probability for the deviant buyer of trading the uncertified heavy coin when it is recognized, and let $\tilde{q}_H$ be the corresponding quantity purchased. This quantity is determined by the take-it-or-leave-it offer $-\tilde{q}_H + V_C = V_0$. Since $V_0 = 0$ we have $\tilde{q}_H = q_C$, i.e. the deviant buyer offers to buy the same quantity as if the coin was certified. Similarly, let $\tilde{\lambda}_{HU}$ be the probability for the deviant buyer of trading the heavy coin when it is not recognized, in which case the coin is inferred to be light since all unrecognized coins have to be light in an equilibrium with full certification. The Bellman equation for the deviator is then

$$\tilde{V}_H = \frac{1}{1+r}\left\{\begin{array}{c} \gamma_H + \beta\theta \max\limits_{\tilde{\lambda}_{HK}} \left[\tilde{\lambda}_{HK}\left\{u(\tilde{q}_H) + V_0\right\} + \left(1 - \tilde{\lambda}_{HK}\right)V_C\right] \\ +\beta\left(1-\theta\right) \max\limits_{\tilde{\lambda}_{HU}} \left[\tilde{\lambda}_{HU}\left\{u(q_L) + V_0\right\} + \left(1 - \tilde{\lambda}_{HU}\right)V_C\right] \\ +(1-\beta)V_C \end{array}\right\}. \tag{30}$$

Using $\tilde{q}_H = q_C$ and $V_0 = 0$ it simplifies into

$$r\tilde{V}_H = \gamma_H + \beta\theta \max_{\tilde{\lambda}_{HK}} \tilde{\lambda}_{HK}\left[u(q_C) - q_C\right] + \beta\left(1-\theta\right) \max_{\tilde{\lambda}_{HU}} \tilde{\lambda}_{HU}\left[u(q_L) - q_C\right] + V_C - \tilde{V}_H. \tag{31}$$

In the end, there is no incentive to deviate from certification and play either by-weight or by-tale if

$$V_C > \tilde{V}_H. \tag{32}$$

3.3.1 Deviation to by-weight

Assume first that a buyer with a heavy coin deviates by trading the uncertified coin only when it is recognized, i.e. by-weight ($\tilde{\lambda}_{HK} = 1$ and $\tilde{\lambda}_{HU} = 0$). Inserting these values into (31) his payoff is

$$r\tilde{V}_H = \gamma_H + \beta\theta\left[u(q_C) - q_C\right] + V_C - \tilde{V}_H. \tag{33}$$

Proceeding as in the previous section, the indifference condition is given by $V_C = \tilde{V}_H$ so that the frontier between certification and circulation by weight, noted $CF1b$, is characterized by

$$\delta = \beta\left(1-\theta\right)\left[u\left(q_C\right) - q_C\right], \tag{34}$$

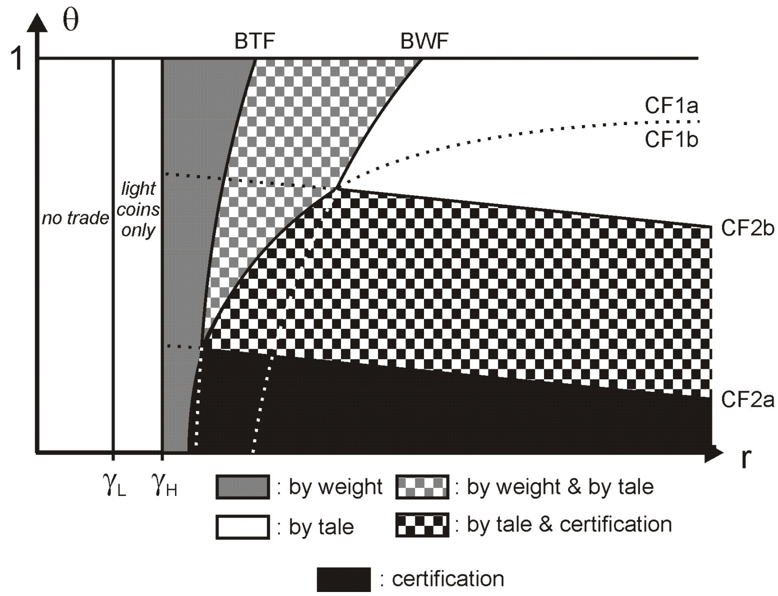

Figure 1: Equilibrium region in the (r, θ) space.

and represented on Figure 1. Note that it is identical to $CF1a$. An equilibrium with certification exists to the right of $CF1a \equiv CF1b$ and to the left of BWF.[10]

3.3.2 Deviation to by-tale

Assume now that the buyer deviates by trading the uncertified heavy coin at a discount when not recognized, that is by-tale ($\tilde{\lambda}_{HK} = \tilde{\lambda}_{HU} = 1$). Inserting these values into (31) his payoff is

$$r\tilde{V}_H = \gamma_H + \beta\theta \left[u(q_C) - q_C\right] + \beta\left(1-\theta\right)\left[u(q_L) - q_C\right] + V_C - \tilde{V}_H, \tag{35}$$

so that the frontier, noted $CF2b$, is given by

$$\delta = \beta\left(1-\theta\right)\left[u\left(q_C\right) - u(q_L)\right]. \tag{36}$$

By contrast to $CF1a \equiv CF1b$, $CF2a$ and $CF2b$ are different. An equilibrium with certification exists to the right of $CF1a \equiv CF1b$ and to the left of (or below) $CF2b$. It follows that circulation by tale and certification coexist as equilibria.[11]

Before summarizing our results, note from (21), (27), (34) and (36) that since $u(q)-q > 0$, if $\delta = 0$ (coin assaying is free, meaning the economy is now one of full information on coins), then all coins would be certified and would circulate at a price that reflect their intrinsic content.

[10] To see why $CF1a \equiv CF1b$ let us note q_i^{bw}, q_i^{bt} and q_i^c the quantities traded for a coin of type i in a by-weight, by-tale and certification equilibrium respectively. From (21), on $CF1a$ we have $\beta\theta\left[u\left(q_H^{bw}\right) - q_H^{bw}\right] = \beta\left[u\left(q_H^{bw}\right) - q_H^{bw}\right] - \delta$. Inserting this into (12) gives $rq_H^{bw} = \gamma_H + \beta\left[u(q_H^{bw}) - q_H^{bw}\right] - \delta$ identical to (29). Then $q_H^{bw} = q_H^c$ on $CF1a$ and we can substitute q_H^{bw} for q_H^c into (21) which yields (34) characterizing $CF1b$.

[11] We show in Appendix A.1. that, in the region where circulation by tale and certification coexist, there also exists a unique mixed-strategy equilibrium in which some heavy coins are certified and some are not, called mixed-strategy certification equilibrium.

By contrast, if $\delta > \bar{\delta}$, agents do not assay their coin and the economy reverts to VWW. The above results can be summarized in a proposition.

Proposition 1 *If $\delta = 0$ all coins circulate at their full information value. For $0 < \delta < \bar{\delta}$ the possible equilibria, which exist in the regions shown in Figure 1, are as follows:*
(i) A no trade equilibrium;
(ii) A single-currency equilibrium in which only light coins circulate;
(iii) A pure-strategy by-weight equilibrium in which light coins always circulate while heavy coins circulate only when recognized;
(iv) A pure-strategy by-tale equilibrium in which both types of coins always circulate and light and heavy coins trade at the same price when not recognized;
(v) A pure-strategy certification equilibrium in which light coins circulate and heavy coins are certified and circulate;
(vi) A mixed-strategy certification equilibrium in which light coins circulate and some heavy coins are certified while the rest is not and trade at the same price as light coins when not recognized.

First note that, as δ decreases, $CF1a \equiv CF1b$ shifts to the left and becomes steeper while both $CF2a$ and $CF2b$ shift up and get closer to each other. Eventually, as noted in Proposition 1, as δ reaches 0 all heavy coins are certified and both coins circulate at their full information value.

Assuming now that $\delta > 0$, certification of heavy coins is an equilibrium when information on coins is low and the discount rate not too high since this is where gains from trade that are large enough to compensate for the cost of assaying.[12] Importantly, there can be multiple equilibria as the circulation-by-tale and certification regions overlap. In order to trade the coin by tale (deviating from certification), buyers understand that their unrecognized heavy coin will be treated as a light coin by sellers since all other unrecognized coins are necessarily light ones in an equilibrium with certification. But when considering deviating from by-tale to certification, buyers understand that their unrecognized heavy coin will be treated as a weighted average of the two coins. Because unrecognized coins are valued more in a by-tale equilibrium than in a certification equilibrium, the two symmetric deviations do not yield the same payoff. As a result, the two equilibrium regions overlap. Finally, note that Velde Weber and Wright (1999)'s set of equilibria is fully partitioned (compare Figure 1. p. 302 in their paper and Figure 1 in this paper).[13]

Given estimations provided by historians on the low cost of coin assaying (between 0.3% and 1% of the transaction), this suggests that unless restrictions on coin assaying applied, the extent of the low-quantity (by tale) and low-frequency (by weight) inefficiencies must have been quite limited. For instance, if we use γ_H and γ_L as a proxy for the intrinsic content of the coins

[12] The result that buyers certify more when both the cost of certification is lower and the probability of a meeting with an uninformed seller is higher is intuitive. Related conclusions can be found in Lester et al. (2011, forthcoming), but in a very different environment.

[13] The no-trade and single-curreny equilibria are artefacts of the indivisibility assumption. Since Berentsen and Rocheteau (2002) it is well known that the indivisibility of money generates inefficient terms of trade. We have a clear illustration here as divisible coins or lotteries would allow both types of coins to circulate at low discount rates, at least in some trades.

and set $\delta = 0.01 * \gamma_H$ then all by-weight equilibria disappear and only a narrow band of by-tale equilibria survives in the region where θ is close to 1.

3.4 Welfare

Proposition 2 *Assume* $0 < \delta < \bar{\delta}$. (i) : *When coin assaying triggers a shift from a by-weight equilibrium to certification, welfare increases;* (ii) : *When coin assaying triggers a shift from the pure strategy by-tale equilibrium to a certification equilibrium, welfare decreases.*

The proof of Part (i) is straightforward. When coins trade by weight, unrecognized circulating coins can only be light coins, therefore light coins always circulate at their full information value. It follows that if holders of heavy coins opt for certification, it does not impact on buyers holding light coins, but it increases their own payoff. These observations imply that welfare is higher with certification than with circulation-by-weight.

Things are different for circulation-by-tale (a formal proof can be found in Appendix A.2.). With circulation by tale, when her counterfeit coin is not recognized, a buyer can still purchase the average quantity $\bar{q}$. But when holders of genuine coins certify, all coins become fully recognizable since uncertified coins can only be counterfeits now. This means that certification substitutes a lottery (q_L with probability M_L and q_C with probability M_H) to a sure payment $\bar{q}$ in former uninformed meetings. Concavity of the utility function implies that agents prefer the sure payment to the lottery, or $u\left[E(q)\right] > E\left[u(q)\right]$, i.e. circulation by-tale to certification. It also implies that, assuming the economy settles on an equilibrium with certification that overlaps with a by-tale equilibrium, which we have shown to be possible, agents would collectively be better off by dropping the coin inspection technology, but have no incentive individually to do that. To that extent historical restrictions on the use of coin assaying may actually have increased welfare.[14]

Although some form of indivisibility is desirable when modelling the commodity money system, an alternative to fully divisible money would be to introduce lotteries on the delivery of the coin in the vein of Berentsen Molico and Wright (2001). There is little evidence, however, that such mechanism was used during the commodity money era despite the lack of small change (we discuss some ingenious ways to circumvent the lack of small change such as cutting coins in half or quarters at the end of section 5). But we should be aware that the use of lotteries is likely to eliminate the pooling type equilibria that we have if one appeals to the Cho-Kreps refinement. Randomization would indeed give the buyer holding a heavy coin another way (on top of coin assaying) to signal that his coin is of high intrinsic.

4 Clipping

We now assume that there is just one type of coin in circulation but that buyers can clip it. Clipping consisted in removing some of the metal of a coin by cutting or shaving the edges using tin snips or shears. We assume that clipping is costless and takes the form of the buyer permanently removing a fixed portion of the metal of which the coin is made (we discuss below

[14]See, e.g., Bigwood (1921) and Favreau (1964) for accounts of the restrictions applying to coin assaying in Medieval Belgium and France, respectively.

what happens if we let buyers choose this portion). Consistent with historical evidence, we assume that the proceed of clipping is sold to bullion dealers who export the metal. Latimer (2001), for instance, reports that clipping in England in 1180-1220 aimed at exporting the extracted metal to Amsterdam. Quinn (1996) documents similar types of bullion exports in the later part of the 17th century, also in England. An alternative would be to melt the proceed of clipping into additional coins, as was also frequently observed. One advantage of assuming that the proceed is exported is that it does not alter the stock of money, and hence matching probabilities in $\beta = \frac{\alpha}{I}(1 - M)$.

To facilitate comparison with the previous section we call the unclipped coin heavy (H) and the clipped coin light (L). Algebraically, we assume that the price paid by bullion dealers exactly compensates for the difference in periodic rates of return between the clipped coin and the unclipped coin. That is, while non-clipped coins yield γ_H per period, clipped coins yield $\gamma_L < \gamma_H$ so that if a buyer decides to clip his coin, he sells the proceed in exchange for a lump-sum (utility equivalent) payment of $\frac{\gamma_H - \gamma_L}{r}$. He then tries to trade the clipped coin. As in the previous section, buyers know whether their coin is clipped or not, but a seller is informed about the type of the coin offered in payment with probability θ. In this section we will characterize the equilibria with and without clipping assuming no coin assaying technology is available. In the next section we will study the impact of coin assaying in this economy, in particular on clipping activities.

This section shares some elements with section III on debasements in Velde Weber and Wright's (1999) where agents are offered a one-off chance by the mint to have their heavy coin swapped for a light coin plus some compensating payment. This should not be surprising since a debasement was nothing but legalized clipping by the authorities. There are two differences, however. First, the clipper keeps the proceed whereas in their paper the proceed of a debasement is shared between the mint and the coin holder. Second, and more fundamentally, we propose a different partition of the set of parameters. In particular we highlight the role that both imperfect information and trading frictions play in clipping decisions. This prepares the ground for the next section where we study the impact of coin assaying on clipping activities, which is our main goal.

4.1 No coin is clipped

First, consider the equilibrium in which no coin is clipped, that is $M = M_H$. Since unrecognized coins can only be full-bodied (i.e. not clipped), the flow payoff to holding a full-bodied coin is

$$rV_H = rq_H = \gamma_H + \beta \left[u(q_H) - q_H \right]. \tag{37}$$

For such an equilibrium to exist, we need to check that no coin holder has an incentive to clip his coin, given that no one else clips.

Assume then that a buyer deviates and clips his coin. Let us note $\tilde{\lambda}_{LK}$ the probability with which the deviant buyer trades the clipped coin if it is recognized, in which case he makes a take-it-or-leave-it offer $\tilde{q}_L$ such that

$$-\tilde{q}_L + \tilde{V}_L = V_0, \tag{38}$$

so that $\tilde{q}_L = \tilde{V}_L$. Note that by contrast to certification, clipping permanently alters the coin, hence $\tilde{V}_L$ in (38) instead of V_L. Now let $\tilde{\lambda}_{LU}$ denote the probability with which the deviant buyer trades the clipped coin if it is not recognized, in which case it trades as if it was not clipped, that is q_H, since all unrecognized coins are inferred to be full-bodied. The Bellman equation for a deviator is then

$$\tilde{V}_L = \frac{1}{1+r} \left\{ \begin{array}{c} \gamma_L + \beta\theta \max\limits_{\tilde{\lambda}_{LK}} \left[\tilde{\lambda}_{LK} \left\{ u(\tilde{q}_L) + V_0 \right\} + \left(1 - \tilde{\lambda}_{LK} \right) \tilde{V}_L \right] \\ +\beta \left(1-\theta \right) \max\limits_{\tilde{\lambda}_{LU}} \left[\tilde{\lambda}_{LU} \left\{ u(q_H) + V_0 \right\} + \left(1 - \tilde{\lambda}_{LU} \right) \tilde{V}_L \right] \\ +(1-\beta)\tilde{V}_L \end{array} \right\} \tag{39}$$

Multiplying both sides by $1+r$ and simplifying yields the flow payment to deviating by holding a clipped coin:

$$r\tilde{q}_L = \gamma_L + \beta\theta \left[u(\tilde{q}_L) - \tilde{q}_L \right] + \beta \left(1-\theta \right) \left[u(q_H) - \tilde{q}_L \right]. \tag{40}$$

In this equation it is assumed that the deviant trades the clipped coin when recognized, $u(\tilde{q}_L) - \tilde{q}_L > 0$, which implies that it is traded when not recognized since $u(q_H) - \tilde{q}_L > u(\tilde{q}_L) - \tilde{q}_L$.

A buyer will not clip his coin if

$$V_H > \frac{\gamma_H - \gamma_L}{r} + \tilde{V}_L \tag{41}$$

which using (37) and (40) yields

$$u(q_H) - q_H > \theta \left[u(\tilde{q}_L) - \tilde{q}_L \right] + (1-\theta) \left[u(q_H) - \tilde{q}_L \right]. \tag{42}$$

Figure 2 represents the area in the (r, θ) space where buyers do not clip their coin. Such an equilibrium exists above the *non-clipping frontier* (NCL), given by (42) set to equality, that is

$$\theta \left[u(q_H) - u(\tilde{q}_L) \right] = q_H - \tilde{q}_L. \tag{43}$$

Equation (43) gives the set of points in (r, θ) space such that the pair $q_H = q_H(r, \theta)$ and $q_L = q_L(r, \theta)$ that solves (37) and (40) satisfies (42) with equality. We delay all explanations to section 4.4 together with the welfare results.

4.2 All coins are clipped

Now consider the case in which all coins are clipped, that is $M = M_L$. Since unrecognized coins can only be clipped, the flow payoff to holding a clipped coin is:

$$rV_L = rq_L = \gamma_L + \beta \left[u(q_L) - q_L \right]. \tag{44}$$

For such an equilibrium to exist, we need to check that no coin holder keeps his coin unclipped, given that everyone else clips.

Assume that a buyer deviates, keeps his coin unclipped and tries to trade it. Let us note $\tilde{\lambda}_{HK}$ the probability with which the deviant buyer trades the unclipped coin if it is recognized, in which case he makes a take-it-or-leave-it offer $\tilde{q}_H$ such that

$$-\tilde{q}_H + \frac{\gamma_H - \gamma_L}{r} + V_L = V_0, \tag{45}$$

so that $\tilde{q}_H = \frac{\gamma_H - \gamma_L}{r} + q_L$. With this offer a seller is indifferent between rejecting the offer and staying as a producer with steady-state payoff V_0, or producing $\tilde{q}_H$, receiving the heavy coin, gaining $\frac{\gamma_H - \gamma_L}{r}$ by clipping it and becoming a holder of a clipped coin with steady-state payoff V_L. Note here that by contrast to (38) the deviation has no permanent effect on the coin as the deviant must believe that the recipient of the coin will clip it.

Let us note $\tilde{\lambda}_{HU}$ the probability with which the deviant buyer trades the unclipped coin if it is not recognized, in which case he has two options: he can pass it as a clipped coin, which is what sellers will infer the coin to be in this equilibrium, or he can hoard it and wait for a seller to recognize it and produce more goods accordingly. By analogy to the previous section we will describe the former as circulation by tale and the later as circulation by weight and will consider both as possible deviations.[15]

The Bellman equation for a deviator is then

$$\tilde{V}_H = \frac{1}{1+r}\left\{\begin{array}{c} \gamma_H + \beta\theta \max\limits_{\tilde{\lambda}_{HK}} \left[\tilde{\lambda}_{HK}\left\{u(\tilde{q}_H) + V_0\right\} + \left(1 - \tilde{\lambda}_{HK}\right)\left(\frac{\gamma_H - \gamma_L}{r} + V_L\right)\right] \\ +\beta\left(1-\theta\right) \max\limits_{\tilde{\lambda}_{HU}} \left[\tilde{\lambda}_{HU}\left\{u(q_L) + V_0\right\} + \left(1 - \tilde{\lambda}_{HU}\right)\left(\frac{\gamma_H - \gamma_L}{r} + V_L\right)\right] \\ +(1-\beta)\left(\frac{\gamma_H - \gamma_L}{r} + V_L\right) \end{array}\right\}. \tag{46}$$

Multiplying both sides by $1+r$, using $\tilde{q}_H = \frac{\gamma_H - \gamma_L}{r} + V_L$ and simplifying yields the flow payment to deviating by holding an unclipped coin:

$$r\tilde{V}_H = \gamma_H + \beta\theta \max_{\tilde{\lambda}_{HK}} \tilde{\lambda}_{HK}\left[u(\tilde{q}_H) - \tilde{q}_H\right] + \beta\left(1-\theta\right) \max_{\tilde{\lambda}_{HU}} \tilde{\lambda}_{HU}\left[u(q_L) - \tilde{q}_H\right] + \frac{\gamma_H - \gamma_L}{r} + V_L - \tilde{V}_H. \tag{47}$$

4.2.1 No deviation to by-weight: $\tilde{\lambda}_{HK} = 1$ and $\tilde{\lambda}_{HU} = 0$

First, for the unclipped coin to circulate by weight we must have $u(q_L) < \tilde{q}_H$ which using (45) and $V_L = q_L$ is equivalent to

$$u(q_L) < \frac{\gamma_H - \gamma_L}{r} + q_L. \tag{48}$$

When set to equality this equation represents the *by-weight frontier* demarcating the combination of r and θ to the left of which any unclipped coin that is not recognized is hoarded. We label it BW on Figure 2.

In addition, a buyer prefers to clip his coin rather than trade an unclipped coin by weight if

$$\frac{\gamma_H - \gamma_L}{r} + V_L > \tilde{V}_H. \tag{49}$$

Using (44), (47) and setting (49) to equality the *clipping-versus-by-weight* equation for the frontier, labelled CL/BW, is given by

$$u(q_L) - q_L = \theta\left[u(\tilde{q}_H) - \tilde{q}_H\right] \tag{50}$$

with $\tilde{q}_H = \frac{\gamma_H - \gamma_L}{r} + q_L$. This frontier demarcates the combinations of r and θ below which all coins are clipped and no one has an incentive to keep his coin full-bodied and trade it by weight. On Figure 2 such an equilibrium exists to the right of $r = \gamma_L$ and to the left of CL/BW and BW. Since BW stands on the left of CL/BW the latter is redundant.

[15] Note that no such dilemma exists in a no-clipping equilibrium since holders of clipped coins are always happy to pass on a clipped coin for a full-bodied one.

4.2.2 No deviation to by-tale: $\tilde{\lambda}_{HK} = \tilde{\lambda}_{HU} = 1$

First, for the unclipped coin to circulate by tale we must have $u(q_L) > \tilde{q}_H$ or

$$u(q_L) > \frac{\gamma_H - \gamma_L}{r} + q_L. \tag{51}$$

When set to equality the resulting equation characterizes the frontier that demarcates the combinations of r and θ to the right of which any unclipped coin that is not recognized trades at a discount. We label it BT on Figure 2 and note that $BT \equiv BW$.

In addition, a buyer prefers to clip his coin rather than trade an unclipped coin at a discount if

$$\frac{\gamma_H - \gamma_L}{r} + V_L > \tilde{V}_H. \tag{52}$$

Using (44), (47) and setting (52) to equality the *clipping-versus-by-tale* frontier, labelled CL/BT, is given by

$$u(q_L) - q_L = \theta\left[u(\tilde{q}_H) - \tilde{q}_H\right] + (1-\theta)\left[u(q_L) - \tilde{q}_H\right] \tag{53}$$

or

$$\theta\left[u(\tilde{q}_H) - u(q_L)\right] = \tilde{q}_H - q_L \tag{54}$$

with $\tilde{q}_H = \frac{\gamma_H - \gamma_L}{r} + q_L$. This frontier demarcates the combinations of r and θ to the left of which all coins are clipped and no one has an incentive to keep his coin full-bodied and trade it at a discount. On Figure 2 such an equilibrium exists to the right of $BT \equiv BW$ and to the left of CL/BT.

4.3 Some coins are clipped

In this equilibrium the payoff to clipping or sticking to one's full-bodied coin is the same, $\frac{\gamma_H - \gamma_L}{r} + V_L = V_H$. As a result buyers randomize so that the money stock is a mix of clipped and unclipped coins. It is easy to show that if such an equilibrium exists then unclipped coins must circulate by tale.[16]

Assume then that non-clipped coins circulate by tale. Given a pair (r, θ) in between CL/BT and NCL, an equilibrium is a triple (q_H, q_L, π) given by

$$rq_H = \gamma_H + \beta\theta\left[u(q_H) - q_H\right] + \beta(1-\theta)\left[u(\bar{q}) - q_H\right] \tag{55}$$

$$rq_L = \gamma_L + \beta\theta\left[u(q_L) - q_L\right] + \beta(1-\theta)\left[u(\bar{q}) - q_L\right] \tag{56}$$

$$\frac{\gamma_H - \gamma_L}{r} + q_L = q_H, \tag{57}$$

with $\bar{q} = \pi q_H + (1-\pi) q_L$. Using (55)-(56), (57) becomes

$$\theta\left[u(q_H) - u(q_L)\right] = q_H - q_L. \tag{58}$$

For instance, if $\pi = 1$ (no coin is clipped) then $\bar{q} = q_H$ in (55)-(56) and (58) transforms into equation (43) for NCL. And if $\pi = 0$ (all coins are clipped) then $\bar{q} = q_L$ and (58) transforms

[16] Assume that some coins are clipped and that non-clipped coins circulate by weight instead of by tale, i.e. are hoarded when not recognized. This implies $\bar{q} = q_L$ in (55) and (56) so that (55) transforms into $rq_H = \gamma_H + \beta\theta\left[u(q_H) - q_H\right] + \beta(1-\theta)\left[u(q_L) - q_H\right]$, (56) transforms into $rq_L = \gamma_L + \beta\left[u(q_L) - q_L\right]$, and (57) transforms into $q_L = q_H$, which is impossible.

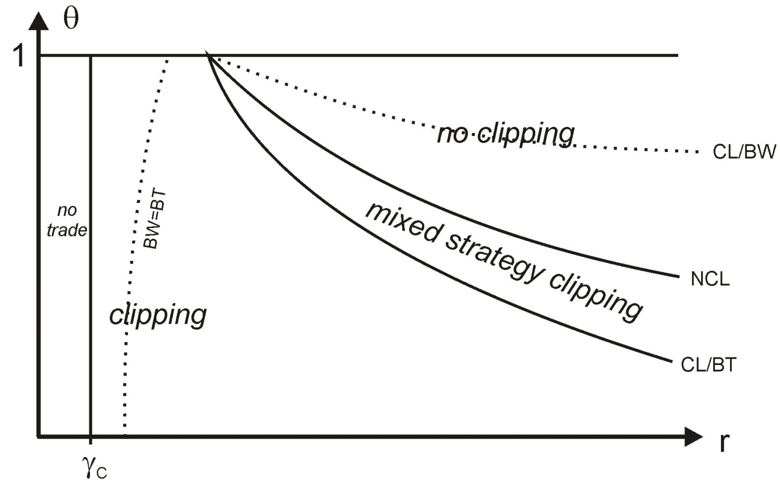

Figure 2: Equilibria with clipping.

into equation (54) for CL/BT. That is, given $\theta < 1$, for any r between CL/BT and NCL, there exists a unique triple $[q_H(r,\theta), q_L(r,\theta), \pi(r,\theta)]$ that satisfies (55)-(57). Intuitively, as r increases away from CL/BT, agents clip less on average, which means that the fraction of unclipped coins π increases from 0 on CL/BT to reach 1 on NCL.

4.4 Welfare and Comments

Welfare is simply M times the buyer's payoff,

$$W_H = Mq_H, \tag{59}$$

$$W_L = M\left\{\frac{\gamma_H - \gamma_L}{r} + q_L\right\}, \tag{60}$$

so that welfare is higher with clipping whenever agents chose to clip and vice versa.

When the discount rate is high, that is when trading frictions are severe, quantities traded are small, marginal utility of consuming goods is high so that gains from trade are larger with full-bodied coins. As a result agents prefer to keep their coin unclipped. If, in addition, information on coins is good (high θ), unclipped coins are frequently recognized making it even more profitable to keep the coin full-bodied. But if the discount rate is low and/or information on coins is poor, then agents are better off clipping their coin. In the next section we investigate how the availability of coin assaying impacts on clipping activities and welfare.

4.5 The optimal amount of clipping

First note that the value to holding a clipped coin, V_L, and the amount of good that can be purchased with a clipped coin, q_L, are both functions of its intrinsic content γ_L, so that $V_L = V_L(\gamma_L)$ and $q_L = q_L(\gamma_L)$. The payoff to clipping, denoted V, is given by

$$V = \frac{\gamma_H - \gamma_L}{r} + V_L(\gamma_L).$$

Substituting for V_L, multiplying by r and taking the derivative of the resulting expression with respect to γ_L yields

$$\frac{\partial q_L}{\partial \gamma_L}\left[\theta u'(q_L) - 1\right] = 0$$

such that

$$u'(q_L) = \frac{1}{\theta}.$$

In particular if agents are fully informed ($\theta = 1$) this simplifies to $u'(q_L) = 1$ which simply says that the intrinsic content of the clipped coins must be such that gains from trade are maximized when it is traded (more on this in the next section). By contrast if information on coins is very poor, coins will be heavily clipped.

5 Clipping and Coin Assaying

In this section we study the impact of a coin assaying technology in an economy with clipping. The new sequence of events is as follows: At time 0, each buyer holding a non-clipped coin decides whether to clip it, certify it, or leave it full-bodied. Then buyers and sellers search for each other. If a buyer decided to rent the technology and finds a seller producing his consumption good, he uses the technology to prove the quality of his coin to the seller and then makes an offer. If the parties agree to trade, they swap inventories so that the seller becomes a buyer and the buyer a seller. Finally, the former buyer (and new seller) returns the technology.

There can be three pure-strategy equilibria now (in addition to possible mixed-strategy equilibria): all coins are clipped, all coins are certified, and all coins are left full-bodied. A pure-strategy clipping equilibrium, for example, is one in which agents prefer to clip their coin over any of the two alternatives, i.e. not clipping the coin or certifying it. In order to characterize those pure-strategy equilibria, we will use what we have done in the previous section and see how robust the clipping and no-clipping equilibria are to coin assaying. Then we will characterize equilibria in which all coins are certified.

Before we start, note that coins are not clipped in the upper right corner of Figure 2. Since all coins a full-bodied in this region agents have no incentive to certify their unclipped coin. Therefore, if an equilibrium with certification exists it must be in the region where coins are clipped and be such that agents prefer certification to clipping. In such an equilibrium, if a non certified coin is spotted yet its intrinsic content cannot be identified, it is inferred to be a clipped coin.

5.1 All coins are clipped

For an equilibrium with full clipping to exist when coin assaying is available, agents must prefer clipping to assaying their coin. The payoff to a non-deviant buyer is

$$rq_L = \gamma_L + \beta\left[u(q_L) - q_L\right]. \tag{61}$$

If he deviates he pays δ to certify the coin and offers a quantity $\tilde{q}_C$ such that

$$-\tilde{q}_C + \frac{\gamma_H - \gamma_L}{r} + V_L = V_0, \tag{62}$$

which implies $\tilde{q}_C = \frac{\gamma_H - \gamma_L}{r} + q_L$. The Bellman equation for such a deviator is given by

$$\tilde{V}_C = \frac{1}{1+r} \left\{ \begin{array}{c} \gamma_H - \delta + \beta \max_{\tilde{\lambda}_C} \left[\tilde{\lambda}_C \left\{ u(\tilde{q}_C) + V_0 \right\} + \left(1 - \tilde{\lambda}_C\right) \left(\frac{\gamma_H - \gamma_L}{r} + V_L\right) \right] \\ +(1-\beta)(\frac{\gamma_H - \gamma_L}{r} + V_L) \end{array} \right\}. \tag{63}$$

This equation shows in particular that the buyer considers a one-time deviation and that the recipient of the certified coin would immediately return to the equilibrium by clipping it. Assuming the deviator trades the certified coin his flow payoff is

$$r\tilde{V}_C = \gamma_H - \delta + \beta \left[u(\tilde{q}_C) - \tilde{q}_C \right] + \frac{\gamma_H - \gamma_L}{r} + V_L - \tilde{V}_C. \tag{64}$$

Therefore a buyer prefers clipping to certification if $\frac{\gamma_H - \gamma_L}{r} + V_L > \tilde{V}_C$ with corresponding frontier given by

$$\beta \left[u(q_L) - q_L \right] = -\delta + \beta \left[u(\tilde{q}_C) - \tilde{q}_C \right]. \tag{65}$$

Because agents compare two full-information options, the level of information θ plays no role in this decision, only the discount rate does. We label this *clipping-versus-certification* frontier CL/CF on Figure 3. To the left of that frontier agents clip their coin despite the availability of certification.

5.2 All coins are certified

Assume now that all coins are certified with associated payoff

$$rq_C = \gamma_H - \delta + \beta \left[u(q_C) - q_C \right]. \tag{66}$$

A deviator who clips offers to buy a quantity $\tilde{q}_L$ such that

$$-\tilde{q}_L + \tilde{V}_L = V_0 \tag{67}$$

implying that $\tilde{V}_L = \tilde{q}_L$. Since unrecognized coins are inferred to be clipped, the Bellman equation for the deviator is

$$\tilde{V}_L = \frac{1}{1+r} \left\{ \begin{array}{c} \gamma_L + \beta \max_{\tilde{\lambda}_L} \left[\tilde{\lambda}_L \left\{ u(\tilde{q}_L) + V_0 \right\} + \left(1 - \tilde{\lambda}_L\right) \tilde{V}_L \right] \\ +(1-\beta)(\frac{\gamma_H - \gamma_L}{r} + \tilde{V}_L) \end{array} \right\} \tag{68}$$

so that

$$r\tilde{q}_L = \gamma_L + \beta \left[u(\tilde{q}_L) - \tilde{q}_L \right]. \tag{69}$$

It follows that the indifference condition between certification and clipping, $\frac{\gamma_H - \gamma_L}{r} + \tilde{V}_L = V_C$, yields the same cutoff r (or CL/CF) as above in equation (65). To the right of that frontier agents certify their coin instead of clipping it. See Figure 3.

Finally, let us look at what happens when only some coins are clipped, i.e. in the mixed-strategy clipping region of Figure 2. In such an equilibrium, buyers are indifferent between clipping or not with associated payoffs $q_H = \frac{\gamma_H - \gamma_L}{r} + q_L$ given by (55)-(56). If one buyer deviates, he pays δ, certifies the coin, makes the same offer as in (62) with payoff given by (63).

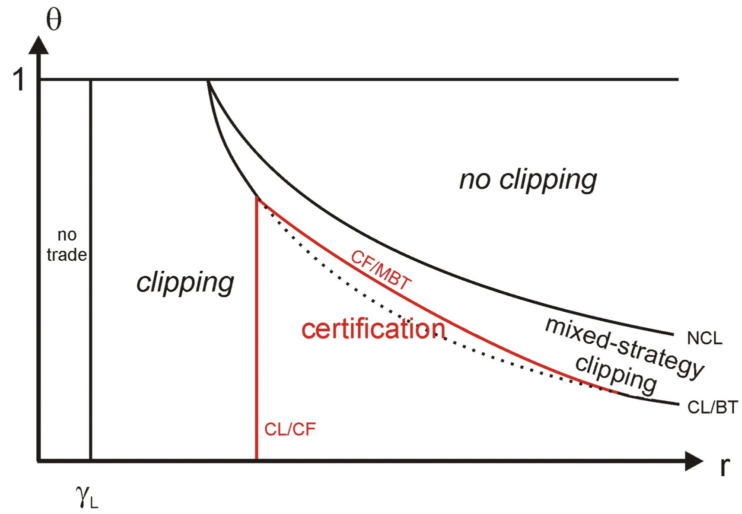

Figure 3: Equilibria with clipping and coin assaying.

A buyer would then prefer a clipped coin (or an unclipped coin since he is indifferent between the two) to a certified one if

$$\frac{\gamma_H - \gamma_L}{r} + V_L = V_H > \tilde{V}_C, \tag{70}$$

which once set to equality yields the frontier equation

$$\delta = \beta \left[u\left(q_C\right) - q_C \right] - \beta \left\{ \theta \left[u(q_H) - q_H \right] + (1-\theta) \left[u\left(\bar{q}\right) - q_H \right] \right\}. \tag{71}$$

Here θ enters the equation for the frontier because of the mix of clipped and unclipped coins. We label this frontier CF/MBT on Figure 3. As can be seen, CF/MBT never touches NCL, indicating that there must be a minimum percentage of clipped coins (for a given δ) for agents to certify their unclipped coin rather than randomize between clipping or leaving the coin unaltered. On CF/MBT agents are indifferent between certification, clipping, or trading the unclipped coin by tale.

When coin assaying is free ($\delta = 0$), the CL/CF frontier shifts to the left and stops where CL/BT and NCL intersect on $\theta = 1$, leaving space between γ_L and CL/CF where agents still clip despite free and full information on coins. There, clipping is not motivated by the hope to pass an inferior coin for a superior one. Clipping is due to the coin being too heavy and clipping it enables to bring its intrinsic content closer to the optimal intrinsic content from the buyer's point of view. There is nothing coin assaying can do against this type of clipping.[17]

Note that one could as well construct an example in which coins are too small with regards to the buyer's need. Clipping would then bring the intrinsic content of coins further away from the optimum, in which case agents would rather keep the coin full-bodied than reduce its size further. Even if we let buyers choose how much metal to remove from the coin, they would still not touch it.

[17]This second motive is similar to endogenous money creation. Smaller coins may reduce inefficiencies on the intensive and extensive margins, which can lead to a positive effect when money is indivisible. Related papers on divisibility and money creation are Berentsen and Rocheteau (2002), Camera (2005) and Deviatov (2006).

One may wonder to what extent this type of clipping is an artefact of the strong indivisibility assumption. If the menu of coins available to buyers makes it possible for them to carry their optimal purchasing power in the form of coins, then that sort of clipping would not exist. Strong evidence on the lack of small change and other curious practises suggest that this sort of clipping was present. For instance, it was once accepted practice to cut coins in half (or quarters) to produce two coins of half (or four coins of one fourth) the value of the original coin. Ashley (1888) reports for instance that the first *round* halfpennies were first introduced in England only in 1220. This can easily be interpreted within our model as the second form of clipping, that motivated by too heavy coins in the face of an urgent need for change.

In the end, as can be seen by comparing Figure 2 and Figure 3, the existence of assaying does impact on the incentive to clip. In particular, when coins are difficult to tell apart (θ is small), assaying induces agents to certify their coin instead of clipping them. But again, assaying can do nothing against clipping motivated by too heavy coins.

5.3 Welfare

Proposition 3 *Assume* $0 < \delta < \bar{\delta}$. (i) : *When coin assaying triggers a shift from clipping to certification, welfare increases;* (ii) : *When coin assaying triggers a shift from a mixed-strategy clipping equilibrium to certification, welfare decreases.*

Part (i) is straightforward. Since welfare is M times the buyer's payoff, if agents chose certification over clipping then welfare is higher with certification. As for part (ii), similar to Proposition 2, welfare is lower in an economy with certification than in an economy with a mixed of clipped and unclipped coins because certification unmasks clipped coins.

6 Conclusion

Because money was made of precious metal, trade in the commodity money system suffered from clipping and the tedious task of evaluating the intrinsic content of the coins offered in payment. The goal of this paper was to evaluate the effect of a well-documented solution to the problem: coin assaying. Coin assaying, usually intermediated by moneychangers, was a central feature of the commodity money system. To our knowledge, however, no theoretical assessment of its impact on circulation, output and welfare is available. The problem is somehow similar in some of today's financial markets where assets with monetary value are traded, raising the question of transparency and its effect on assets' liquidity and welfare.

Three main conclusions were reached. First, coin assaying did not necessarily increase welfare. Second, coin assaying could not get rid of all forms of clipping. Third, because coin assaying seems to have been relatively affordable, there must have been restrictions to its use given the prevalence of information problems on coins.

Although some form of indivisibility should be part of a model of commodity money, it takes a convenient yet extreme form in our model: buyers can hold at most one coin. Our main results are unlikely to be challenged by introducing some divisibility, yet more work on the source of indivisibility and the lack of small change in the commodity money system is probably warranted.

Appendix

A.1 Existence of a mixed-strategy equilibrium in which some heavy coins are certified and some are not.

Using superscript m for mixed-strategy, if such an equilibrium exists, it is characterized by q_L^m, q_H^m, $\bar{q}^m, q_C$, and Σ given by

$$rq_L^m = \gamma_L + \beta\theta \left[u(q_L^m) - q_L^m\right] + \beta(1-\theta)\left[u(\bar{q}^m) - q_L^m\right] \tag{72}$$

$$rq_H^m = \gamma_H + \beta\theta \left[u(q_H^m) - q_H^m\right] + \beta(1-\theta)\left[u(\bar{q}^m) - q_H^m\right] \tag{73}$$

$$rq_C^m = \gamma_H - \delta + \beta\left[u(q_C^m) - q_C^m\right] \tag{74}$$

$$\bar{q}^m = \frac{(1-\Sigma) M_H}{(1-\Sigma) M_H + M_L} q_H^m + \frac{M_L}{(1-\Sigma) M_H + M_L} q_L^m \tag{75}$$

$$\delta = \beta\left[u(q_C^m) - q_C^m\right] - \beta\left\{\theta\left[u(q_H^m) - q_H^m\right] + (1-\theta)\left[u(\bar{q}^m) - q_H^m\right]\right\} \tag{76}$$

where Σ is the proportion of heavy coins that are certified.

To see this note that if $\Sigma = 0$ then $\bar{q}^m = \bar{q}^{bt}$, $q_H^m = q_H^{bt}$ and $q_L^m = q_L^{bt}$ where superscript bt goes for by-tale. Then, except on *CF2a*, we have $\delta > \beta\left[u(q_C^m) - q_C^m\right] - \beta\left\{\theta\left[u(q_H^{bt}) - q_H^{bt}\right] + (1-\theta)\left[u\left(\bar{q}^{bt}\right) - q_H^{bt}\right]\right\}$ $= \beta(1-\theta)\left[u(q_H^{bt}) - u(\bar{q}^{bt})\right]$ and buyers are better-off not certifying. If $\Sigma = 1$ then $\bar{q}^m = q_L^c$ given by (28) and $q_H^m = q_C$ given by (29) where superscript c goes for certified. Then, except on *CF2b*, we have $\delta < \beta\left[u(q_C^m) - q_C^m\right] - \beta\left\{\theta\left[u(q_H^c) - q_H^c\right] + (1-\theta)\left[u(q_L^c) - q_H^c\right]\right\} = \beta(1-\theta)\left[u(q_C^m) - u(\bar{q}^{bt})\right]$ and buyers are better-off certifying. Hence there is a unique $\Sigma \in (0,1)$ that satisfies (76) where q_L^m, q_H^m, q_C, and $\bar{q}^m$ are given by (72)-(75).

A.2 Proof of Proposition 2, part (ii)

In each equilibrium, welfare is given by

$$\begin{aligned} rW_{bt} = {} & M_L\left\{\gamma_L + \beta\theta\left[u(q_L^{bt}) - q_L^{bt}\right] + \beta(1-\theta)\left[u(\bar{q}^{bt}) - q_L^{bt}\right]\right\} \\ & + M_H\left\{\gamma_H + \beta\theta\left[u(q_H^{bt}) - q_H^{bt}\right] + \beta(1-\theta)\left[u(\bar{q}^{bt}) - q_H^{bt}\right]\right\} \end{aligned} \tag{77}$$

and

$$rW_c = M_L\left\{\gamma_L + \beta\left[u(q_L^c) - q_L^c\right]\right\} + M_H\left\{\gamma_H - \delta + \beta\left[u(q_C) - q_C\right]\right\} \tag{78}$$

where subscripts bt and c refer to by-tale and certification, respectively. Grouping all the terms in θ and $(1-\theta)$ in (77), we obtain

$$\begin{aligned} rW_{bt} = {} & M_L\gamma_L + M_H\gamma_H + \theta\beta\left\{M_L\left[u(q_L^{bt}) - q_L^{bt}\right] + M_H\left[u(q_H^{bt}) - q_H^{bt}\right]\right\} \\ & + (1-\theta)\beta\left\{(M_L + M_H)u(\bar{q}^{bt}) - \left(M_L q_L^{bt} + M_H q_H^{bt}\right)\right\}. \end{aligned} \tag{79}$$

From the concavity of u, we have $u(\bar{q}^{bt}) = u\left[\pi q_H^{bt} + (1-\pi) q_L^{bt}\right] > \pi u(q_H^{bt}) + (1-\pi) u\left(q_L^{bt}\right)$. Using the definition of π this inequality can be rewritten $(M_L + M_H) u(\bar{q}^{bt}) > M_L u\left(q_L^{bt}\right) + M_H u\left(q_H^{bt}\right)$. It follows that for all θ we have $rW_{bt} > rW(\theta)$ with

$$\begin{aligned} rW(\theta) = {} & M_L\gamma_L + M_H\gamma_H + \theta\left\{M_L\beta\left[u(q_L^{bt}) - q_L^{bt}\right] + M_H\beta\left[u(q_H^{bt}) - q_H^{bt}\right]\right\} \\ & + (1-\theta)\beta\left\{M_L u\left(q_L^{bt}\right) + M_H u\left(q_H^{bt}\right) - \left(M_L q_L^{bt} + M_H q_H^{bt}\right)\right\} \end{aligned} \tag{80}$$

which simplifies into

$$rW(\theta) = M_L \left\{ \gamma_L + \beta \left[u(q_L^{bt}) - q_L^{bt} \right] \right\} + M_H \left\{ \gamma_H + \beta \left[u(q_H^{bt}) - q_H^{bt} \right] \right\}. \tag{81}$$

Note now that, when $\theta = 1$, from (22)-(23) and (28) we have $q_L^{bt} = q_L^c$. Similarly, when $\theta = 1$, from (22)-(23) and (29) we have $\gamma_H + \beta \left[u(q_H^{bt}) - q_H^{bt} \right] > \gamma_H - \delta + \beta \left[u(q_C) - q_C \right]$ so that $rW(\theta = 1) > rW_c$. Since $rW_{bt} > rW\ (\theta = 1)$, we conclude $rW_{bt} > rW_c$.

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