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# Fixed-Income Pricing in a Non-Linear Interest-Rate Model

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**Abstract:** This paper introduces a novel kind of interest-rate model offering simple analytical pricing formulas for swaps, futures, swaptions, caps and floors. The model is based on an original use of regime-switching features that makes it consistent with the non-linear behavior of interest rates. In particular, it accommodates the fact that short-term rate fluctuations are mainly driven by discrete changes in the central-bank policy rates. An application on euro-area data shows how the model can be exploited to infer risk-neutral probabilities of central-bank rate decisions.

**JEL Codes:** E43, E47, G12, C53.

**Keywords:** yield curve, option pricing, regime switching, market expectations.

**Résumé:** Cet article présente un nouveau modèle de taux d'intérêt offrant des formules simples pour la valorisation de contrats d'échanges de taux (*swaps*), de contrats futures, d'options sur taux de swaps, de *caps* et de *floors*. L'approche est fondée sur une utilisation originale des changements de régimes. Cette spécificité rend le modèle cohérent avec le caractère non-linéaire de la dynamique des taux d'intérêt. En particulier, le modèle reproduit naturellement la façon dont les taux de court terme dépendent des taux directeurs fixés par la banque centrale (ces derniers étant à valeurs discrètes). Une application à la zone euro montre comment le modèle peut être utilisé afin d'inférer la distribution risque-neutre de valeurs futures des taux de politique monétaire.

**Codes JEL:** E43, E47, G12, C53.

**Mots-Clés:** courbe de taux d'intérêt, valorisation d'options, changement de régime, anticipations de marché.

# Non-technical summary

In spite of the pervasive relationship between central bank decisions and the yield curve, a very few term-structure models explicitly incorporate central-bank policy rates. Arguably, this stems from the difficulty of modeling the time series behavior of the latter by means of standard diffusion processes. In the present paper, by contrast, the step-like dynamics of policy rates occupies the central place. This is achieved thanks to an extensive and original use of Markov switching features. Though highly nonlinear, the model is surprisingly tractable, offering closed-form pricing formulas for various fixed-income instruments: swaps, futures, swaptions, caps and floors. These formulas involve only basic algebraic operations, ensuring simple model calibration.

In the basic form of the model, the state of the world is characterized by two discretized elements: a monetary-policy phase and a policy rate. Monetary policy can be either in an easing, a status-quo or a tightening phase. During status-quo phases, no changes in policy rates can take place. If the monetary-policy phase is of the easing (respectively tightening) kind, a cut (resp. a hike) in the policy rate can happen with a strictly positive probability. The model parameters include the probabilities of switching from one monetary-policy phase to another and the conditional probabilities of hikes and cuts in the policy rate. The model can easily be extended by the introduction of additional regimes, the only restriction being that the number of regimes remains finite. In particular, it is shown how the model can be extended in order to accommodate the existence of different monetary-policy corridor functioning regimes.

The facts that (a) the model explicitly incorporates policy rates and that (b) it can be calibrated on various financial market data make it an ideal laboratory to infer implicit risk-neutral probabilities attached to future central-bank rate decisions. This is illustrated by an empirical exercise based on euro-area data covering the last decade. For each date, the model is calibrated so as to fit overnight indexed swaps (OIS) and interest-rate options prices. This exercise highlights the flexibility of the model, which proves able to satisfyingly fit market data in various contexts. It is eventually shown how the model can be exploited to compute risk-neutral probabilities attached to future policy rate outcomes. Compared to existing approaches aimed at recovering market expectations of policy-rate decisions, the model results in a richer description of future states of the world. More precisely, in addition to providing the risk-neutral probabilities associated with different policy rates, the approach also yields probabilities of being in the different monetary-policy phases at any future horizon.

# 1 Introduction

Medium to long-term rates depend on expected future short-term rates, which are themselves driven by the central bank. Hence, monetary-policy decisions are central to account for the fluctuations of the whole yield curve, from short to long maturities. This mechanism, known as the expectation channel of monetary policy, contributes to the strong influence of central bank actions and communication on asset prices.[1]

In spite of the pervasive relationship between central bank decisions and the yield curve, a very few term-structure models explicitly incorporate central-bank policy rates. Arguably, this stems from the difficulty of modeling the time series behavior of the latter by means of standard diffusion processes. In the present paper, by contrast, the step-like dynamics of policy rates occupies the central place. This is achieved thanks to an extensive and original use of Markov switching features. Though highly nonlinear, the model is surprisingly tractable, offering closed-form pricing formulas for swaps, futures, swaptions, caps and floors. These formulas involve only basic algebraic operations, ensuring simple model calibration.[2]

In the basic form of the model, the state of the world is characterized by two discretized elements: a monetary-policy phase and a policy rate. Monetary policy can be either in an easing, a status-quo or a tightening phase. The definition of these phases is consistent with observed central banks' rate-setting behavior and communication (see Bernanke (2006) or Smaghi (2009)). During status-quo phases, no changes in policy rates can take place. If the monetary-policy phase is of the easing (respectively tightening) kind, a cut (resp. a hike) in the policy rate can happen with a strictly positive probability. The model parameters include the probabilities of switching from one monetary-policy phase to another and the conditional

[1] See, amongst many others, Cook and Hahn (1989), Kuttner (2001), Rigobon and Sack (2004) Gurkaynak, Sack, and Swanson (2005), Cochrane and Piazzesi (2005) or Rosa and Verga (2008).

[2] The model introduced in the present paper is closely related to the one used by Renne (2012). Closed-form formulas for option prices are however not available in the latter, which focuses more on time-series aspects of yields fluctuations.

probabilities of hikes and cuts in the policy rate. The model can easily be extended by the introduction of additional regimes, the only restriction being that the number of regimes remains finite.[3] While the addition of regimes comes at the cost of additional parameters – in the form of additional probabilities of transition –, closed-form pricing formulas then remain available.

The facts that (a) the model explicitly incorporates policy rates and that (b) it can be calibrated on various financial market data make it an ideal laboratory to infer implicit risk-neutral probabilities attached to future central-bank rate decisions. This is illustrated by an empirical exercise based on euro-area data covering the last decade. For each date, the model is calibrated so as to fit overnight indexed swaps (OIS) and interest-rate options prices. This exercise highlights the flexibility of the model, which proves able to satisfyingly fit market data in various contexts. It is eventually shown how the model can be exploited to compute risk-neutral probabilities attached to future policy rate outcomes.

The rest of the paper is organized as follows. Section 2 briefly reviews the literature to which this paper relates. Section 3 presents stylized facts about the dynamics of policy rates and their influence on short-term market rates. Section 4 introduces the model and the pricing formulas. Section 5 aims at familiarizing the reader with the model specifications and functioning. An empirical exercise on euro-area data is presented in Section 6 and Section 7 concludes.

## 2 Related literature

There is a vast literature studying the interactions between monetary policy and interest rates. A specific strand of this literature examines the dynamics of policy rates and their

[3]In the previous case, with three monetary-policy phases and $N$ (say) possible values of the policy rate, the number of regimes is $3N$.

impact on money-market rates. Rudebusch (1995), Balduzzi, Bertola, and Foresi (1997) and Balduzzi, Bertola, Foresi, and Klapper (1998) propose policy-rate models and show that the expectation of future changes in the target rate is the main driving force of short-term interest rates. Hamilton and Jorda (2002) and Hu and Phillips (2004) use probit models to account for policy rate decisions. Contrary to the previous studies that focus on short-term rates, Piazzesi (2005) and Fontaine (2009) propose models that capture the dynamics of the whole term structure of interest rates, from short-term policy rates to long-term ones. These frameworks are however not consistent with the zero-lower bound of nominal yields, which is an important limitation in the current context of close-to-zero short-term yields.[4]

The second strand of literature to which this paper relates is concerned with the pricing of interest-rate options. At least from a practical point of view, interest-rate derivatives pricing is dominated by the family of *LIBOR market models*. These frameworks, originally due to Brace, Gatarek, and Musiela (1997) and Miltersen, Sandmann, and Sondermann (1997), have the great advantage of being consistent with Black (1976)'s formula, that is the standard formula employed in the interest-rate options market. Nevertheless, the tractability of these models is limited to the pricing of options written on a single underlying instrument. That is, once it is calibrated to price derivatives whose payoffs depend on a specific reference rate – a swap rate of a given maturity, say – a LIBOR market model does not possess closed-form formulas for other derivatives.[5] Affine term-structure models (ATSM) have also been used to investigate the dynamics of interest-rate option prices (see e.g. Singleton and Umantsev (2002), Collin-Dufresne and Goldstein (2002), Jagannathan, Kaplin, and

[4]This bound stems from the existence of cash as an alternative investment to bonds. Since cash yields a nominal return of zero at any maturity, bond yields-to-maturity cannot be below zero. Notwithstanding, slightly negative nominal interest rates are sometimes observed; market frictions and/or the difficulty to store cash (i.e. the storing cost of cash) may explain these episodes.

[5]See Rebonato, McKay, and White (2009), Rebonato and Pogudin (2011) or Brigo and Mercurio (2007), Chapter 6. The latter authors in particular highlight the theoretical incompatibility between LIBOR models calibrated on forward rates and those calibrated on swap rates (Subsection 6.8 in Brigo and Mercurio (2007)).

Sun (2003), Almeida, Graveline, and Joslin (2011), Trolle and Schwartz (2014)).[6] Pricing derivatives with ATSM however remains more demanding than with market models.

Insofar as it entails a recombining lattice for the policy rate, our model shares common features with tree-based option-pricing methods (Ho and Lee (1986), Hull and White (1994) and Hull and White (1996)). The lattice developed in the present paper enriches traditional trees with the taking into account of switching monetary-policy phases. In spite of this sophistication, pricing turns out to be as simple, if not more, as in tree frameworks.

To end with, this paper contributes to the literature on the extraction of market expectations from financial data. Several studies rely on Fed funds futures to recover market expectations of changes in the U.S. Fed policy rate (e.g. Krueger and Kuttner (1998), Sack (2004), Evans (1998), Gurkaynak, Sack, and Swanson (2007)). Other contributions, in the spirit of Breeden and Litzenberger (1978), use options prices to recover risk-neutral densities – sometimes called implied densities – of future rates. This is notably the case of Carlson, Craig, and Melick (2005) who exploit the fact that changes in the policy rate are discrete. This methodology is e.g. applied by Emmons, Lakdawala, and Neely (2006)) to compute implied probabilities of changes in the policy rate for the next two to three rate-decision meetings.[7]

[6] These models are also termed with equilibrium of short-rate models by Hull (2006).

[7] In this approach, the density functions associated with different future rate-decision meetings correspond to different risk-neutral measures. Indeed, the Breeden and Litzenberger (1978)'s formula is valid under the forward-$T$ measure, where $T$ is the expiry date of the options used in the calibration. Hence, if two future expiry dates are jointly involved in the calibration of these models (the expiry dates matching those of the monetary-policy meetings), the probability measure used for the first central-bank meeting (forward-$T_1$ neutral measure) is not the same as the one used for the second meeting (forward-$T_2$ neutral measure).

# 3 Policy rates and the short-end of the yield curve

The relationship between the effective short-term rate and the policy rates, which are the different rates set by the central bank, depends on the monetary-policy framework in place.[8] This section provides a brief overview of these mechanisms, on which is based the model specification.

The link between the short-end of the yield curve and the monetary-policy framework stems from the necessity, for banks, to hold reserves at the central bank. Reserves are the ultimate asset for settling payments and the most risk-free and liquid asset in the economy; they also help banks to manage their liquidity risks. Only central banks can alter the supply of reserves, which provides them with a mechanism for achieving their policy objectives. The monetary-policy framework is designed in such a way as to allow the central bank to make the short-term interest rates close to a target rate, which is the rate that the central bank deems consistent with its desired stance.

It is important to make the distinction between the target rate and policy rates. While the former may be not explicitly communicated by the central bank, the latter are publicly disclosed.[9] Typically, the policy rates are those rates characterizing the so-called interest-rate corridor system, which is now used by most central banks ( Whitesell (2006), Berentsen and Monnet (2008) and Kahn (2010)). The corridor system derives from the existence of central banks' standing facilities, which allow banks to borrow or to make deposits with the central bank. The lending rate is typically above, and the deposit rate typically below, the rate at which the central bank deals in the market through open-market operations (OMOs). Banks being unwilling to deal in the market on worse terms than those that are available at

[8]These frameworks notably consist of policies on access right to central bank facilities, collateral policies and an operating system. As had been illustrated by the recent financial crisis, these frameworks continually evolve over time.

[9]For instance, contrary to the U.S. Federal Reserve or the Bank of England, the ECB does not have an explicit interest-rate target.

the central bank, the interbank short-term market rate is unlikely to fall below the central bank's deposit rate or to rise above the lending facility rate. As a result, the interbank rate evolves within the corridor, whose bounds are defined by the rate of these standing facilities. The corridor is often symmetric, in the sense that OMOs rate is the mean of the facilities rates.

There are two regimes under which the corridor system can operate (see Clews, Salmon, and Weeken (2010) and Martin, McAndrews, and Skeie (2014)). Under the first ("normal") one, the central bank adjusts the aggregate amount of reserves by undertaking open-market operations in such a way that, on average, the interbank overnight rate almost coincides with the OMO rate. Under the second regime, called "floor system", the central bank deliberately supplies reserves in excess of the level banks would voluntarily target. Since the banking system, in aggregate, has to hold all the reserves the central bank creates, this reduces the number of banks that fall short of reserves, which pushes the interbank rate downwards, possibly close to the bottom of the corridor.[10]

# 4 Model

## 4.1 Dynamics of the short-term rate

This section presents the risk-neutral dynamics followed by the overnight interbank rate $r_t$, and its resulting implications in terms of pricing.

The short-term rate is split into two components:

$$r_t = \Delta' z_t + \xi_t, \tag{1}$$

[10] Bech and Klee (2011) exhibit periods during which the effective federal funds rate fluctuated below the deposit-facility rate; they argue this could be due to the fact that, in the US., not all participants in the federal funds market are eligible to receive interest on their reserve balances (Government-Sponsored Enterprises, GSEs, in particular).

where $z_t$ is a $K$-dimensional selection vector (i.e. it is full of zero except one entry that is equal to one) and where $\xi_t$ is a serially-uncorrelated random variable whose distribution may depend on $z_t$. The form of the latter distribution can be general, it just has to be such that the following expectation exists:

$$E(\exp(-\xi_t)|z_t) = \exp(-\delta)' z_t. \tag{2}$$

The fact that $\xi_t$ is serially uncorrelated implies that the short-term rate persistence only comes from the regime component $\overline{r}_t \equiv \Delta' z_t$. Vector $z_t$, which will be referred to as the regime vector in the following, follows a Markovian process. The matrix of transition probabilities, between dates $t$ and $t+1$, is denoted by $\Pi_{t+1}$. This matrix may depend on time, though in a deterministic way. Here, we exploit this possibility in order to distinguish between (a) those dates for which a monetary-committee meeting is scheduled (opening the possibility of having changes in the policy rates) and (b) those for which there is no monetary-policy meeting (i.e. with no possibility of changes in the policy rates). In the former case, $\Pi_t = \overline{\Pi}$ and in the latter case, $\Pi_t = \underline{\Pi}$ (say). We follow Piazzesi (2005) and, in our pricing formula, we take into account the exact number of days before the next monetary-policy meeting only, the subsequent meetings being assumed to take place every 30 days.[11] Formally, denoting by $\Pi(-n)$ the matrix of transition probabilities that prevails when the next monetary-policy meeting takes place in $n$ days, we have: $\Pi(30k) = \overline{\Pi}$ for $k \in \mathbb{Z}$, and $\Pi(s) = \underline{\Pi}$ when $s$ is not a multiple of 30.

While the pricing formulas that are developed in the next subsection are valid for any parameterization of matrices of transition probabilities, Sections 5 and 6 will propose specific examples.

[11] This periodicity can easily be modified to match the frequency of monetary-policy meetings in the area into consideration. Typically, in the U.S., the average period between two FOMC meetings is of 45 days, as will be the case in the euro area from 2015 onwards.

## 4.2 Pricing formulas

### 4.2.1 Generic pricing

The following proposition introduces a general pricing formula that will be used repeatedly in the following.

**Proposition 4.1** *The price, at date $t$, of a security providing the payoff $\omega' z_{t+h}$ at date $t+k$ (with $1 \le h \le k$) is given by:*

$$\begin{aligned} E_t\left[\exp(-r_t - \dots - r_{t+k-1})(\omega' z_{t+h})\right] &= \mathbf{1}' \left\{ \prod_{i=k}^{1} \gamma_i \Pi(-n_t + i) \right\} \gamma_0 z_t e^{\delta' z_t - \xi_t} \qquad (3) \\ &=: \pi(\omega, n_t, h, k)' z_t e^{\delta' z_t - \xi_t} \quad \textit{(say)}, \end{aligned}$$

*where $n_t$ is the number of days between $t$ and the next monetary-policy meeting, $\mathbf{1}$ is a $K$-dimensional vector of ones and $\gamma_i$ is a diagonal matrix whose diagonal is:*

$$(\mathbb{I}_{\{i=h\}} \omega + \mathbb{I}_{\{i \neq h\}} \mathbf{1}) \odot exp\{-\mathbb{I}_{\{i<k\}}(\Delta + \delta)\},$$

*where the symbol $\odot$ denotes the element-by-element product.*

**Proof:** The law of iterated expectations implies that the expectation appearing on the left-hand side of Equation (3) is equal to $E_t\left[E_t\{(\exp(-r_t - \dots - r_{t+k-1})(\omega' z_{t+h})|z_{t+1}, \dots, z_{t+k}\}\right]$, i.e. to $e^{\delta' z_t - \xi_t} E_t[(e^{-(\Delta+\delta)'} z_t) \times \dots \times (e^{-(\Delta+\delta)'} z_{t+h-1}) \times ([e^{-(\Delta+\delta)} \odot \omega]' z_{t+h}) \times (e^{-(\Delta+\delta)'} z_{t+h+1}) \times \dots \times (e^{-(\Delta+\delta)'} z_{t+k-1})]$ if $h < k$ and $e^{\delta' z_t - \xi_t} E_t[(e^{-(\Delta+\delta)'} z_t) \times \dots \times (e^{-(\Delta+\delta)'} z_{t+k-1}) \times (\omega' z_k)]$ otherwise. The results then derives from Lemma A.1 (see Appendix A). ■

Due to the daily frequency of the model, the effect of the term $e^{\delta' z_t - \xi_t}$ in the pricing formula (3) is extremely small and hence negligible.[12] Accordingly, the formula can be

[12] To illustrate this, let us consider a typically high deviation between $\delta' z_t$ and $\xi_t$, of 0.50 percentage point (expressed in annualized terms). This implies $\exp(\delta' z_t - \xi_t) = \exp(-0.005/365) = 1 - 1.4 \times 10^{-5}$.

simplified with virtually no pricing effect by replacing $e^{\delta' z_t - \xi_t}$ by its conditional expectation (given $z_t$), which equals 1 by definition of $\delta$. This simplification is systematically used in the sequel of this paper.

As a second remark, it should be stressed that regularities in the matrix product appearing in Equation (3) can be exploited in order to optimize the computing time of this pricing formula. Specifically, the sequential calculation of the $k$ matrix products can be avoided by regrouping terms and using matrix powers.[13]

### 4.2.2 Zero-coupon rates

**Proposition 4.2** *The price, at date $t$, of a zero-coupon bond of maturity $\tau$, where $\tau$ is expressed in years, is $\pi(\mathbf{1}, n_t, 1, 365\tau)' z_t$. Equivalently, the annualized continuously-compounded zero-coupon yield of maturity $\tau$, denoted by $r_{t,\tau}$, is given by:*

$$\begin{aligned} r_{t,\tau} &= -\frac{1}{\tau} \ln(\pi(\mathbf{1}, n_t, 1, 365\tau)' z_t) \\ &=: \pi_r(n_t, \tau)' z_t, \end{aligned} \tag{4}$$

*which defines function $\pi_r$.*

**Proof:** This result is obtained by using Proposition 4.1 with $\omega = \mathbf{1}$. In this case, all the $\gamma_i$s are equal, which implies that $h$ does not appear in Equation (4). ■

### 4.2.3 Overnight indexed swap (OIS) and LIBOR rates

An OIS is an interest-rate derivative that allows for exchanges between a fixed-interest-rate cash flow and a variable-rate cash flow. We consider here OIS contracts of maturities $\tau$ lower

[13]First, it can be seen that most of the sequences appearing in the total product are of the form $D^q$ where $D = (\gamma_0 \underline{\Pi})^{29} \gamma_0 \overline{\Pi}$. While a naive computation of this term would involve $30q$ matrix products, only $30 + q - 1$ products are required to compute $D^q$ (30 for D and $q - 1$ additional products to elevate $D$ at power $q$).

than one year; these OIS involve a single payoff.[14] At maturity, the payoff received by the fixed-rate receiver is the difference between (a) the notional (say 1) inflated with the date-$t$ OIS fixed rate, and (b) the same unit notional capitalized with the realized short-term rates.

**Proposition 4.3** *At date $t$, the continuously-compounded OIS swap rate of maturity $\tau$ is equal to the zero-coupon-bond yield $r_{t,\tau}$. Using the Actual/360 day-count convention, OIS rates are given by:*

$$\begin{aligned} OIS_{t,\tau} &= \frac{360}{365\tau}\left[1-\pi(\mathbf{1},n_t,1,365\tau)\right]' z_t. \\ &=: \pi_R(n_t,\tau)' z_t, \end{aligned} \tag{5}$$

*which defines function $\pi_R$.*

**Proof:** Let us denote by $ois_{t,\tau}$ the continuously-compounded OIS rate. At maturity, i.e. date $t+\tau 365$, the fixed-leg payoff will be $\exp(\tau ois_{t,\tau})$. This payoff is predetermined at date $t$; its date-$t$ value is therefore $\exp(\tau ois_{t,\tau})\exp(-\tau r_{t,\tau})$. At maturity, the floating-rate payoff will be $\exp\{r_t+\ldots+r_{t+365\tau-1}\}$. The date-$t$ value of the latter payoff is $E_t(\exp\{-r_t-\ldots-r_{t+365\tau-1}\}\times\exp\{r_t+\ldots+r_{t+365\tau-1}\})=1$. In an OIS contract, the values of both legs equate at its inception (date $t$). This implies that $\exp(\tau ois_{t,\tau})\exp(-\tau r_{t,\tau})=1$, i.e. $ois_{t,\tau}=r_{t,\tau}$. Equation (5) is obtained by converting the continuously-compounded rate $ois_{t,\tau}$ using the Actual/360 day-count convention ($OIS_{t,\tau}$ follows this convention.) ∎

We denote by $R_{t,\tau}$ the interbank rate of maturity $\tau$. For $\tau=0.25$, $R_{t,\tau}$ corresponds for instance to the 3-month LIBOR. These interbank rates are also equivalent to zero-coupon-bond yields and they follow the Actual/360 day-count convention. Accordingly, we have:

$$R_{t,\tau} = \pi_R(n_t,\tau)' z_t. \tag{6}$$

[14] See e.g. Barclays (2008). OIS with longer maturities involve several reset dates. This case is treated in 4.2.5.

Since LIBORs act as the underlying indices of various fixed-income derivatives, function $\pi_R$ will repeatedly appear in the following pricing formula.

Remark that OIS and LIBOR rates are equivalent in this model. This is due to the fact that the model does not take into account credit and/or liquidity pricing effects that explain why LIBORs are higher than OIS rates with matching maturities (see Taylor and Williams (2009), Filipovic and Trolle (2013) or Dubecq, Monfort, Renne, and Roussellet (2013)). Neglecting these effects is broadly consistent with the pre-crisis context, but it is at odds with the episodes of increases in the LIBOR-OIS spreads that have been observed over the last few years. In our framework, this could be handled by adding interbank credit/liquidity stress regimes that would account for changes in banks' default intensities. This is however beyond the scope of this paper, that focuses on pure interest-rate risk. In our empirical exercise (Section 6), we will simply address this by allowing for a deterministic spread between OIS and LIBOR rates.

#### 4.2.4 Futures

**Proposition 4.4** *At date $t$, the quoted rate of the future contract on the maturity-$\tau$ interbank rate for a delivery date of $t+h$ is given by:*

$$F_{t,h,\tau} = \pi_R(n_t,\tau)' \left\{ \prod_{i=h}^{1} \Pi(-n_t+i) \right\} z_t. \tag{7}$$

**Proof:** Futures contracts are usually marked-to-market on a daily basis. Margins are such that, once they have been paid, the value of the contract is zero. Therefore, at date $t$, we have $0 = E_t(\exp(-r_t)[F_{t+1,h-1,\tau} - F_{t,h,\tau}])$, where $F_{t+1,h-1,\tau} - F_{t,h,\tau}$ is the margin paid (or received) on date $t+1$. This implies that $F_{t,h,\tau} = E_t(F_{t+k,h-k,\tau})$ for any $k \in [0,h]$. Since $F_{t,0,\tau} \equiv R_{t,\tau}$, we have $F_{t,h,\tau} = E_t(R_{t+h,\tau})$ (setting $k=h$). It follows that $F_{t,h,\tau} = \pi_R(n_t,\tau)'E_t(z_{t+h})$, which leads to Equation (7). ■

### 4.2.5 Multi-payoffs (LIBOR-type) swap

Let us consider a maturity-$h$ swap involving several exchanges of fixed versus floating cash-flows, such as standard LIBOR-based swaps. For a unit notional, at every date $T_i$ in a pre-specified set of dates $T_1, \dots, T_{h/\tau}$ (the reset dates), the fixed leg pays out the amount $\tau s_{t,h,\tau}$, where $\tau$ is the year fraction corresponding to the time lengths $T_i - T_{i-1}$, where $T_0 = t$ is the inception date of the swap, $T_{h/\tau} = t + h$ is its maturity date and $s_{t,h,\tau}$ is the swap rate. At each reset date $T_i$ ($i > 0$), the floating leg pays $\tau R_{T_{i-1},\tau}$.[15]

Here, as in the following of this paper, we assume that $\tau$ is a multiple of the period between two monetary-policy rate-decision meetings. This assumption, which is de facto valid for standard swaps, implies that the number of days before the next meeting is $n_t$ at each reset date, which slightly simplifies the formulas.

**Proposition 4.5** *At date $t$, the swap rate of maturity $h$, indexed on the maturity-$\tau$ interbank rate $R_{t,\tau}$ is given by:*

$$s_{t,h,\tau} \quad = \quad \left( \frac{\sum_{i=1}^{h/\tau} \pi(\pi_R(n_t,\tau), n_t, T_{i-1} - t, T_i - t)}{\sum_{i=1}^{h/\tau} \pi(\mathbf{1}, n_t, 0, T_i - t)} \right)' z_t, \tag{8}$$

*where $T_i = t + i\tau$ and, by abuse of notation, the division appearing in Equation (8) is performed element-wise.*

[15] The bulk of plain-vanilla swaps involves this "in-arrear" feature (Singleton and Umantsev (2002)).

**Proof:** At its inception, the present values of the two legs are equal, which writes:

$$
\begin{aligned}
& E_t\left[\sum_{i=1}^{h/\tau} \exp(-r_t - \cdots - r_{t+T_i-1})\left(\tau\pi_R(n_t,\tau)' z_{T_{i-1}}\right)\right] \\
= \;& E_t\left[\sum_{i=1}^{h/\tau} \exp(-r_t - \cdots - r_{t+T_i-1})\right]\tau s_{t,h,\tau}.
\end{aligned}
$$

Equation (8) directly derives from the last equation, using Proposition 4.1 on both sides. ■

## 4.3 Swaption pricing

Let us turn to the pricing of European swaptions. This kind of option gives the right to its holder to enter some pre-specified underlying swap contract on a pre-specified expiry date. Plain-vanilla swaptions are written on swaps whose references are LIBORs (i.e. those swaps considered in 4.2.5). The maturity of the underlying swap rate is called the tenor. There are two kinds of swaptions: (a) the *receiver swaption*, which gives the buyer the right to receive the fixed leg of the swap and (b) the *payer swaption*, which gives her the right to receive the floating leg of the swap.

**Proposition 4.6** *The price, at date $t$, of a payer swaption of expiry date $t+q$, whose tenor is of maturity $h$ and whose strike price is $K$ is given by $\pi[S(K,n_t,\tau,h),n_t,q,q]'z_t$, where $S(K,n_t,\tau,h)$ is the vector defined by:*

$$
S(K,n_t,\tau,h) = \tau\left(\sum_{i=1}^{h/\tau}\pi[(\pi_R(n_t,\tau) - K\mathbf{1}),n_t,(i-1)\tau,i\tau]\right)_+'. \tag{9}
$$

**Proof:** At the expiry of the swaption (date $t+q$), the option payoff is:

$$\begin{aligned}
&\left(E_{t+q}\left\{\sum_{i=1}^{h/\tau}\exp(-r_{t+q}-\cdots-r_{t+q+i\tau-1})[\tau\pi_R(n_t,\tau)'z_{t+q+(i-1)\tau}-\tau K]\right\}\right)_+ \\
= \ &\tau\left(\sum_{i=1}^{h/\tau}E_{t+q}\left\{\exp(-r_{t+q}-\cdots-r_{t+q+i\tau-1})[\pi_R(n_t,\tau)'z_{t+q+(i-1)\tau}-K]\right\}\right)_+ .
\end{aligned}$$

Therefore, at date $t+q$, the payoff of the swaption is of the form $S(K,n_t,\tau,h)'z_{t+q}$, where $S(K,n_t,\tau,h)$ is the vector specified by Equation (9). Setting $\omega = S(K,n_t,\tau,h))$, an application of Proposition 4.1 leads to the results. ∎

## 4.4 Cap and floor pricing

A cap (respectively a floor) is a contract by which a seller of protection agrees to provide the buyer of protection with a positive payoff if a given reference rate exceeds (resp. falls below) a pre-specified level called the exercise rate on given future dates (the reset dates). The reference rate of plain-vanilla caps and floors are usually LIBORs, whose maturity (called tenor) is the difference between two reset dates.

**Proposition 4.7** *At date $t$, the price of a cap whose underlying is the LIBOR of maturity $\tau$, of exercise rate $C$, and of expiry date $t+h$ is $Cap(C,n_t,\tau,h)'z_t$ where the vector $Cap(C,n_t,\tau,h)$ is:*[16]

$$Cap(C,n_t,\tau,h) = \left(\sum_{i=2}^{h/\tau}\pi[(\pi_R(n_t,\tau)-C\mathbf{1})_+,n_t,(i-1)\tau,i\tau]\right). \tag{10}$$

**Proof:** The price of the Cap is the expected discounted value of the flows of future payoffs,

[16] Note that caps and floors are usually defined so that the initial LIBOR rate does not lead to a payoff on the first reset date, i.e. at date $t+\tau$; that is why the sums appearing in the definition of *Cap* begins with $i=2$.

i.e.:

$$E_t \left\{ \sum_{i=2}^{h/\tau} \exp(-r_t - \cdots - r_{t+i\tau-1})(\pi_R(n_t,\tau)' z_{t+(i-1)\tau} - C)_+ \right\}.$$

Setting $\omega = (\pi_R(n_t,\tau) - C\mathbf{1})_+$, an application of Proposition 4.1 leads to the result. ■

Obviously, floor prices are given by similar formulas, replacing vector $(\pi_R(n_t,\tau) - C\mathbf{1})_+$ by $(C\mathbf{1} - \pi_R(n_t,\tau))_+$ in Equation (10).

# 5 An illustrative example

This section provides an illustrative example aimed at familiarizing the reader with our framework. As mentioned in Section 4.1, the model is essentially specified through matrices $\underline{\Pi}$ and $\overline{\Pi}$, which depict the dynamics of the policy rate that, in turn, drives the overnight interbank rate $r_t$. The next subsection details possible specifications of these matrices. Subsection 5.2 comments simulations generated by this model.

## 5.1 Specifications

Recall that matrices $\underline{\Pi}$ and $\overline{\Pi}$ contain the probabilities of switching from one state (regime) of the economy to another. The value of the current policy rate is one element describing the state of the economy. If it were the only one, it would imply, in particular, that the probabilities of having cuts or hikes in the policy rate depend only on the last value of the latter. This would notably be at odds with the stylized fact according to which one policy-rate change is much more likely to be followed by another change in the same direction (e.g. Rudebusch (1995)). In the context of the present model, this can be accommodated by introducing different monetary-policy phases. Specifically, three phases are considered: easing (E), status-quo (S) and tightening (T). At date $t$, a policy-rate cut (respectively hike) occurs with a strictly positive probability only if monetary policy then is in an easing (resp. tightening) phase and if a rate-decision meeting is scheduled for date $t$. If there are $N$ possible policy rates, the regime vector $z_t$ is therefore of dimension $3N \times 1$ in such a model.

At that stage, two probabilities have been explicitly mentioned: the conditional probabilities of having an increase and a decrease in the policy rate. Note that these probabilities will be zero in $\underline{\Pi}$, which corresponds to those dates at which no meeting is scheduled. It remains to specify the probabilities of having some changes in monetary-policy phases. Positing that there can be no direct transition – i.e. between $t$ and $t+1$ – from E to T and from T to E, four additional probabilities are then required: $E \rightarrow S$, $S \rightarrow E$, $T \rightarrow S$, $S \rightarrow T$.

The calibration of such transition-probability matrix is illustrated by Figure 1.[17] Denoting by $r_{max}$ the maximum value for the policy rate $\overline{r}_t$, the vector $\Delta$ that corresponds to this specification of the transition-probability matrix is of the form: $[0, 0.25\%, 0.50\%, \dots, r_{max},$ $0, 0.25\%, 0.50\%, \dots, r_{max},\ 0, 0.25\%, 0.50\%, \dots, r_{max}]'$. It can be seen that the specification proposed in Figure 1 allows only for single-tick changes in the policy rate. (This can be relaxed at the cost of introducing additional parameters governing the break down of the probabilities of cuts and hikes.)

To complete the model specification, we need to define the distribution of $\xi_t$. As stated by Equation (1), this variable is the deviation between the effective interbank rate $r_t$ and the policy rate $\Delta' z_t$, which corresponds here to the middle of the corridor (see Section 3).[18] To be consistent with the fact that $r_t$ has to lie within the corridor, we select a distribution with a compact support. For a corridor width of 100bps, a possibility is for instance to take $\xi_t$ equal to $\eta_t \beta_t$ where $\eta_t$ is a zero-mean exogenous binomial variable valued in $\{-1\%, 1\%\}$ and $\beta_t$ follows a Beta distribution.

[17] $\underline{\Pi}$ is obtained by setting $p_h$ and $p_c$ to zero.

[18] As explained in Section 3, the overnight interbank rate is often close to the middle of the corridor but can also drop to levels close to the bottom of the corridor. In our illustrative example, we will not account for such effects, which will however be dealt with in our empirical section (Section 6).

Table 1: Illustrative model – Example of calibration

| $p_{ES}$ | $p_{SE}$ | $p_{ST}$ | $p_{TS}$ | $p_h$ | $p_c$ |
|---|---|---|---|---|---|
| 20% | 20% | 20% | 10% | 50% | 80% |

**Notes**: This table reports the model parameterization used to generate the outputs presented in Figures 2 to 4. E, S and T respectively stand for the Easing, Status-quo and Tightening regimes. The probability $p_{ij}$ ($i, j \in \{E, S, T\}$) corresponds to the probability to switch from regime $i$ to regime $j$. For a daily periodicity, these probabilities are small. Therefore, for sake of readability, these probabilities appear here in monthly terms; that is, if $\underline{p}$ is the daily transition probability, the table reports $p$, where $(1-p) = (1-\underline{p})^{30}$.

To end with, we posit a maximum policy rate of 8%, which is a level that has not been reached by the policy rates of the main monetary areas over the last 20 years.

## 5.2 Simulations

Table 1 reports the calibration of the model on which are based Figures 2 to 4. Similar figures can be generated by using an interface available on the internet.[19]

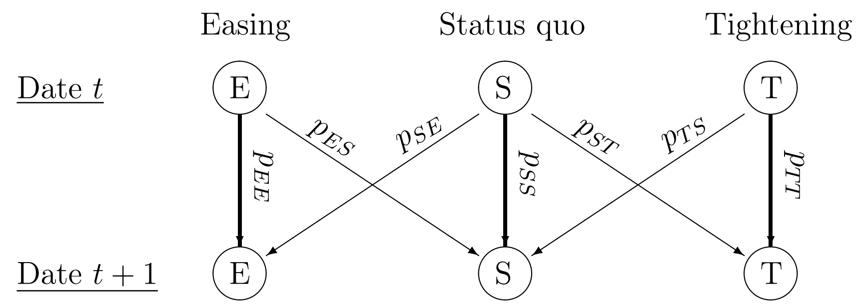
Figure 2 displays a simulated path for the effective interbank rate $r_t$, together with the policy rate $\Delta' z_t$. One can in particular see the influence of the monetary-policy phases – shown in the lower panel of the figure – on the interest-rate dynamics.

Figure 3 shows the term-structure of swap rates corresponding to different policy rates (0.50%, 3% and 5.5%) and different monetary-policy phases. Naturally, the swap curve is higher during tightening phases and vice versa for easing phases. This figure also illustrates that the model can generate the usual yield-curve shapes: flat, upward sloping, hump shaped and inverse-hump shaped.

Model-implied term-structures of swaption prices are displayed on Figure 4. These prices correspond to different expirations and tenors. While option premiums appear on the left-

[19]This interface, available at: https://jrenne.shinyapps.io/essai_shiny, allows the users to investigate the influence of the model calibration on model outputs, i.e. to generate Figures 2 to 4 with alternative model calibrations.

Figure 1: Specifying matrix Π: an example




$$
\Pi = \left[\begin{array}{ccccc|ccccc|ccccc}
p_{EE} & p_{EE}p_c & 0 & \cdots & 0 & p_{SE} & p_{SE}p_c & 0 & \cdots & 0 & 0 & \cdots & & \cdots & 0 \\
0 & p_{EE}p_{nc} & & & \vdots & 0 & p_{SE}p_{nc} & & & \vdots & \vdots & & & & \vdots \\
\vdots & & \ddots & & 0 & \vdots & & \ddots & & \vdots & & & & & \\
\vdots & & & p_{EE}p_{nc} & p_{EE}p_c & \vdots & & & p_{SE}p_{nc} & p_{SE}p_c & & & & & \\
0 & \cdots & & 0 & p_{EE}p_{nc} & 0 & \cdots & & 0 & p_{SE}p_{nc} & 0 & \cdots & & \cdots & 0 \\
\hline
p_{ES} & 0 & \cdots & & 0 & p_{SS} & 0 & \cdots & & 0 & p_{TS} & 0 & \cdots & & 0 \\
0 & p_{ES} & & & \vdots & 0 & p_{SS} & & & \vdots & 0 & p_{TS} & & & \vdots \\
 & & \ddots & & & & & \ddots & & & & & \ddots & & \\
\vdots & & & p_{ES} & 0 & \vdots & & & p_{SS} & 0 & \vdots & & & p_{TS} & 0 \\
0 & \cdots & & 0 & p_{ES} & 0 & \cdots & & 0 & p_{SS} & 0 & \cdots & & 0 & p_{TS} \\
\hline
0 & \cdots & & \cdots & 0 & p_{ST}p_{nr} & 0 & & \cdots & 0 & p_{TT}p_{nr} & 0 & & \cdots & 0 \\
\vdots & & & & \vdots & p_{ST}p_r & p_{ST}p_{nr} & & & \vdots & p_{TT}p_r & p_{TT}p_{nr} & & & \vdots \\
 & & & & & 0 & \ddots & & & 0 & 0 & \ddots & & & \\
 & & & & & \vdots & & & p_{ST}p_{nr} & 0 & \vdots & & & p_{TT}p_{nr} & 0 \\
0 & \cdots & & \cdots & 0 & 0 & \cdots & 0 & p_{ST}p_r & p_{ST} & 0 & \cdots & 0 & p_{TT}p_r & p_{TT}
\end{array}\right]
$$

hand panel in this figure, Black volatilities are plotted on the right-hand panel.[20] This plot demonstrates the ability of the model to generate humped-shaped term-structures of Black volatilities, a phenomenon frequently observed on these markets (Amin and Morton (1994), Mercurio and Moraleda (2000) and Ritchken and Chuang (1999)).

# 6 Empirical exercise

## 6.1 Overview of the exercise

In this last section, we show how the model can be used to infer risk-neutral expectations of future policy rates. As is formally illustrated by Propositions 4.2 to 4.7, the prices of fixed-income instruments incorporate market expectations of future policy rates. It is a common approach, among practitioners, central bankers and academics, to use fixed-income instruments to back out such implied expectations.

It can be remarked that certain untransformed market data can readily be interpreted as market expectations. This is typically the case of Fed funds futures rates or overnight indexed swaps (OIS), that are often used as proxies for risk-neutral expectations of future central-bank target rates (Gurkaynak, Sack, and Swanson (2007) or Christensen and Kwan (2014)). However, while these rates may be informative to reveal the mean path expected by market participants, they do not provide information about market views regarding possible deviations from this path. For that, these rates have to be complemented with option prices and a model is then required to convert all these market data into implied probabilities. By offering simple formulas to price various fixed-income instruments, our model offers a convenient way to recover such probabilities.[21]

[20]The common practice in the financial markets is to price interest-rate options using Black (1976)'s model. To price a swaption, this model assumes a log-normal process for the forward rate, see e.g. Hull (2006).

[21]These probabilities are risk-neutral in the sense that they are associated to the pricing (risk-neutral) measure. They may deviate from physical probabilities due to the existence of risk premia, see e.g. Piazzesi

## 6.2 Data

We use euro-area data covering the period from January 23, 2004 to September 12, 2014 at the weekly frequency (end-of-week data; 556 dates). The data are extracted from Bloomberg; option prices are obtained from the ICAP Bloomberg page. The model is fitted on both interest rates and interest-rate derivatives. Interest rates are OIS rates of maturities 1, 3 and 6 months, 1, 2, 3 and 5 years. Nine at-the-money swaption prices are used: these prices are for three different times-to-expiry (6 months, 1 year and 2 years) and for three tenors (1, 2 and 5 years). We use prices of caps and floors of three maturities (3, 4 and 5 years) and we consider four exercise rates (0.50%, 1.00%, 1.50% and 2.00%). Cap/floor prices are available from March 2012 onwards; except for floors with exercise rates of 1.50%, which are available from September 2009 onwards. In order to cope with temporary periods of illiquidity of cap/floor prices, we remove price observations that remain unchanged for 10 consecutive trading days. Eventually, between 16 and 31 prices are fitted for each date.

In spite of the growing importance of OIS yields (BIS (2013)), plain vanilla interest-rate options, such as those used here, are indexed on EURIBORs. As mentioned in Subsection 4.2.3, the present form of the model is not consistent with the existence of credit and liquidity premia in LIBOR-type rates. As a result, adjustments have to be made so as to make the approach compatible with the use of EURIBOR-based options (see Appendix B).

## 6.3 The extended model

The model presented in Section 5 is extended in order to better fit market data for all the considered dates. Note that while this extension results in an increase in the number of parameters to be calibrated – from 6 to 7 or 9 depending on the dates –, it does not imply a modification of the pricing formulas.

and Swanson (2008).

First, we allow the conditional probability of policy-rate hikes to depend on the level of the current policy rate. Formally, instead of assuming that the probability of having an increase in the policy rate between $t$ and $t+1$ is a constant parameter $p_h$ (conditional on being in the tightening regime), it is allowed to be a function of the policy rate $\overline{r} = \Delta' z$. In order to ensure that these probabilities lie in the $[0,1]$ interval, we assume that they are given by the logit function: $p_h(\overline{r}) = e^{\zeta_1+\zeta_2\overline{r}}/(1+e^{\zeta_1+\zeta_2\overline{r}})$. For instance, hikes in the policy rate can be made less frequent when policy rates are high by setting $\zeta_2 < 0$.[22]

Second, the basic model is specified in such a way as to accommodate the shift in the position of the overnight interbank rate within the monetary-policy corridor that was experienced by the Eurosystem in the fall of 2008. (See the last paragraph of Section 3 for precisions regarding the relationships between the corridor and interbank rates.) Before then, the overnight interbank rate was, on average, close to the rate of the main refining operations (MRO) conducted by the ECB, the latter rate corresponding to the middle of the corridor (see top panel in Figure 5). We consider here that $\overline{r}_t$ is the MRO rate. In October 2008, following the announcement and implementation of non-standard monetary-policy measures that were taken in response to the financial crisis, overnight interbank rates fell to levels close to the bottom of the corridor.[23] In the model, we account for this by making the distribution of $\xi_t$ depend on the MRO rate $\overline{r}_t$. Then, since $r_t = \overline{r}_t + \xi_t$ (Equation 1), $\xi_t$ is the spread between the overnight interbank rate and the MRO rate. As mentioned above, this spread is, on average, close to zero under a normal functioning of the corridor system (regime C) and negative under the floor system (regime F). Compared to the model presented in Section 3,

[22] In terms of coding, this leads to a change in the specification of matrix $\overline{\Pi}$, where the $p_h$ probabilities have to be replaced accordingly by the $p_h(\overline{r})$ ones (see Figure 1).

[23] These unconventional monetary policy measures notably included the move from variable rate tender procedures in liquidity providing operations to fixed-rate full-allotment procedures (announced on October 8, 2008), whereby the ECB accommodates any demand for liquidity its bank counterparties might express at the policy rate – against eligible collateral – in unlimited amounts (see Beirne (2012) and Fahr, Motto, Rostagno, Smets, and Tristani (2010)).

we therefore introduce additional regimes to account for the fact that $\delta$ can take two values. (Recall that $\delta$ is the log-Laplace transform of $\xi_t$ conditional on $z_t$, which is close to the mean of $\xi_t$ conditional on $z_t$ because the order of magnitude of $\xi_t$ is 1%.) More precisely, all those regimes for which the MRO rate is below 2% are duplicated: the first group of regimes (state C) is such that $\delta_C = 0$ and the second group (state F) is such $\delta_F < 0$. Consistently with the fact that the switch from state C to state F was not anticipated before October 2008, we assume that the probability to exit state C is zero. This leaves us with two additional parameters to estimate only for those dates after October 2008: the probability $p_{FC}$ to exit the floor system and $\delta_F$.[24]

To sum up, the model is calibrated by adjusting 7 parameters for each date until October 2008: $p_{ES}$, $p_{SE}$, $p_{TS}$, $p_{ST}$, $p_c$, $\zeta_1$ and $\zeta_2$ and 9 for subsequent dates (the previous seven ones plus $p_{FC}$ and $\delta_F$).

## 6.4 Regime detection

To calculate model-implied prices for a given date $t$, one requires to know the state $z_t$ of the economy. Let us recall that a regime is defined by (i) the policy rate prevailing at date $t$, i.e. $\overline{r}_t$, (ii) the monetary-policy phase (E,S or T) and (iii) the functioning regime of the corridor system (C or F, as defined in the previous subsection). The policy rate, that we take equal to the MRO rate, is directly observable. As for point (iii), we consider that the floor system has been operating since October 8, 2008, that is the date on which the ECB announced important changes in its monetary-policy operational framework (see panels (a) and (b) in Figure 5).

The identification of monetary phases is based on slope of the short-end of the yield curve, proxied by the spread between the 6-month and the 1-month OIS rates. As can be

[24]That is, we implicitly assume that state C is absorbing.

seen by looking at both panels (a) and (b) in Figure 5, this spread is high in tightening phase and low during easing phases. Monetary policy phase is assumed to be in a tightening (respectively easing) phase when this spread is above (resp. below) 10 basis points.[25] This leads to the identification of monetary-policy phases displayed in panel (c) of Figure 5.

## 6.5 Results

Let us denote by $\theta$ the vector of parameters required to specify the model. The model calibration is obtained by minimizing, for each date, the loss function $\mathcal{L}$ defined by:

$$\mathcal{L}(\theta) = \sum_{s \in \Omega} \omega_s (v_s^m - v_s^o)^2, \tag{11}$$

where $s$ denotes one of the fixed-income instruments used to calibrate the model, whose set is denoted by $\Omega$; $\omega_s$ is a weight associated with instrument $s$; $v_s^m$ and $v_s^o$ are, respectively, the model-implied and observed yield (or price for options) associated with instrument $s$.

Tables 2 and 3 show the results for four dates. The former reports the model calibrations and the latter presents the resulting fit as well as, in its first column, the weights $\omega_s$ that are used in the loss function.[26]

An interesting output of the model is displayed in Figure 6, which plots model-implied 6-month-ahead distributions of the policy rate $\overline{r}_t$. The black vertical line indicates the position of the then-prevailing policy rate. Importantly, these distributions do not only show the probabilities associated with the different possible levels of the policy rate; they also incorporate the probabilities of being in the different monetary-policy phases in the future. Let us consider for instance the top-left panel, that corresponds to March 16, 2007.

[25] As an alternative to this identification procedure, one could select, for each considered date, the type of monetary-policy phase that provides the best fit of observed prices (for instance). The results however proved to be already satisfying with the simpler approach proposed here.

[26] These weights have been chosen arbitrarily in order to yield a fit deemed satisfying across the different market data. Naturally, a user that would focus on option pricing (say) would increase option-related weights.

On this date, the central bank is engaged in a tightening phase and the rate of the main refinancing operations is of 3.75%. If we sum the probabilities associated with all black triangles, we get the risk-neutral probability of being in an easing phase 6-month from then. This probability is equal to 5%. On October 31, 2008 (top-right panel), the MRO rate is the same but this probability is of 29% because the monetary-policy phase is "easing" then. The two bottom plots correspond to more recent situations, with very low policy rates.

Over the whole sample, the root mean squared pricing errors (RMSPE) associated with OIS rates is of 3 basis points. A closer inspection suggests that the size of the pricing errors tends to depend on the level of the yield curves. Indeed, while the RMSPE is of 4 bps when the policy rate is equal or above 2%, it falls to 2 bps when it is lower than 2%. Regarding swaption prices, the RSMPE are of 8 bps. On average – across time, maturities and strikes – the absolute pricing errors correspond to 14% of the swaption prices. Turning to caps and floors, which are only available over the most recent period, absolute pricing errors amount to 11% of the option prices.

To end with, Figure 7 shows time series of model outputs. The second panel displays probabilities of changes in the policy rate. More precisely, for each date $t$, the chart shows the model-implied risk-neutral probabilities that, 6-month later, the policy rate stands at least 1, 2, 3 or 4 ticks above or below its date-$t$ level. Unsurprisingly, these probabilities closely relate to the monetary-policy phases prevailing at each date. The lowest panel compares the model-implied expectations of 6-month changes in the MRO rate $\overline{r}_t$ and those reported in the survey of professional forecasters published by *Consensus Economics*.[27] This last chart suggests that both measures of policy-rate expectations are closely linked.

[27] These forecasts are the mean values of the respondents' forecasts. The Consensus Forecast survey is released around the middle of the month. These survey data are available only since July 2009.

Table 2: Model calibrations for selected dates

| Dates | $p_{ES}$ | $p_{SE}$ | $p_{ST}$ | $p_{TS}$ | $p_c$ | $\zeta_1$ | $\zeta_2$ | $p_{FC}$ | $\delta_F$ |
|---|---|---|---|---|---|---|---|---|---|
| 16/03/2007 | 0.180 | 0.017 | 0.039 | 0.292 | 0.904 | 0.007 | -0.006 | – | – |
| 31/10/2008 | 0.179 | 0.014 | 0.336 | 0.029 | 0.953 | 1.648 | -0.172 | 0.550 | -0.982 |
| 31/08/2012 | 0.182 | 0.349 | 0.139 | 0.029 | 0.276 | -0.54 | 0.000 | 0.056 | -0.923 |
| 16/08/2013 | 0.182 | 0.121 | 0.094 | 0.035 | 0.119 | 1.243 | -0.156 | 0.053 | -0.860 |

**Notes**: This table reports the model calibration obtained for four selected dates (the same dates are used in Figure 6). E, S and T respectively stand for the Easing, Status-quo and Tightening regimes. Letters F and C refer to the two functioning regimes of the corridor system (C corresponds to the "normal" functioning of the corridor system and F to the floor system, see Section 3). The probability $p_{ij}$ ($i, j \in \{E, S, T, C, F\}$) corresponds to the probability to switch from regime $i$ to regime $j$. For a daily periodicity, these probabilities are small. Therefore, for sake of readability, these probabilities appear here in monthly terms; that is, if $\underline{p}$ is the daily transition probability, the table reports $p$, where $(1-p) = (1-\underline{p})^{30}$. For the first date, $p_{FC}$ and $\delta_F$ are not estimated because the floor system was not operating then (see end of Subsection 6.3 and Subsection 6.4).

# 7 Conclusion

This paper proposes a novel kind of term-structure model offering closed-form formulas for various interest-rate derivatives. In this model, the short-term rate follows an innovative process based on an extensive use of regime-switching features. Consistently with empirical observation, the fluctuations of the short-term rate mainly stem from those of the central-bank policy rates. The framework adequately captures the non-linear step-like path of policy rates. An empirical exercise is conducted on euro-area fixed-income data covering the last decade. This exercise illustrates the flexibility of the model, by showing that it can match fixed-income prices in various contexts. The calibrated models are eventually exploited to infer market-data-implied probabilities of future changes in the policy rate.

Table 3: Model fit for selected dates

| Instrument | weight $\omega$ | 16/03/2007 mod. | 16/03/2007 obs. | 31/10/2008 mod. | 31/10/2008 obs. | 31/08/2012 mod. | 31/08/2012 obs. | 16/08/2013 mod. | 16/08/2013 obs. |
|---|---|---|---|---|---|---|---|---|---|
| Yields | | | | | | | | | |
| 1m OIS | 0.50 | 3.80 | 3.83 | 3.15 | 3.12 | 0.07 | 0.10 | 0.08 | 0.09 |
| 3m OIS | 0.50 | 3.88 | 3.84 | 2.97 | 2.9 | 0.08 | 0.07 | 0.10 | 0.10 |
| 6m OIS | 0.50 | 3.95 | 3.94 | 2.79 | 2.76 | 0.08 | 0.06 | 0.12 | 0.12 |
| 1y OIS | 2.00 | 4.01 | 4.05 | 2.63 | 2.60 | 0.06 | 0.06 | 0.17 | 0.18 |
| 2y OIS | 2.00 | 4.03 | 4.02 | 2.73 | 2.73 | 0.08 | 0.09 | 0.32 | 0.31 |
| 3y OIS | 2.00 | 4.02 | 3.99 | 2.98 | 3.02 | 0.19 | 0.19 | 0.52 | 0.51 |
| 5y OIS | 2.00 | 3.97 | 3.98 | 3.47 | 3.47 | 0.54 | 0.54 | 1.00 | 1.01 |
| Swaptions | | | | | | | | | |
| 6m - 1y | 0.05 | 0.16 | 0.11 | 0.37 | 0.16 | 0.15 | 0.10 | 0.13 | 0.11 |
| 6m - 2y | 0.05 | 0.35 | 0.24 | 0.70 | 0.52 | 0.31 | 0.24 | 0.33 | 0.31 |
| 6m - 5y | 0.05 | 0.71 | 0.63 | 1.50 | 1.29 | 0.91 | 0.94 | 0.99 | 1.05 |
| 1y - 1y | 0.05 | 0.24 | 0.17 | 0.36 | 0.20 | 0.22 | 0.16 | 0.24 | 0.19 |
| 1y - 2y | 0.05 | 0.47 | 0.36 | 0.70 | 0.60 | 0.45 | 0.36 | 0.53 | 0.50 |
| 1y - 5y | 0.05 | 0.88 | 0.89 | 1.52 | 1.45 | 1.29 | 1.35 | 1.50 | 1.51 |
| 2y - 1y | 0.05 | 0.33 | 0.26 | 0.31 | 0.25 | 0.34 | 0.31 | 0.44 | 0.38 |
| 2y - 2y | 0.05 | 0.59 | 0.51 | 0.64 | 0.67 | 0.75 | 0.67 | 0.93 | 0.84 |
| 2y - 5y | 0.05 | 1.10 | 1.22 | 1.49 | 1.61 | 2.12 | 2.09 | 2.26 | 2.21 |
| Caps | | | | | | | | | |
| 0.50% - 3y | 0.05 | 8.41 | – | 7.55 | – | 0.77 | – | 1.26 | – |
| 0.50% - 4y | 0.05 | 11.47 | – | 11.29 | – | 1.60 | – | 2.59 | – |
| 0.50% - 5y | 0.05 | 14.36 | – | 15.22 | – | 2.84 | – | 4.41 | – |
| 1.00% - 3y | 0.05 | 7.27 | – | 6.40 | – | 0.43 | 0.35 | 0.78 | 0.72 |
| 1.00% - 4y | 0.05 | 9.90 | – | 9.7 | – | 1.02 | 0.95 | 1.79 | – |
| 1.00% - 5y | 0.05 | 12.4 | – | 13.21 | – | 1.98 | 1.94 | 3.26 | – |
| 1.50% - 3y | 0.05 | 6.13 | – | 5.27 | – | 0.25 | 0.21 | 0.49 | 0.46 |
| 1.50% - 4y | 0.05 | 8.35 | – | 8.13 | – | 0.65 | 0.63 | 1.24 | 1.20 |
| 1.50% - 5y | 0.05 | 10.45 | – | 11.22 | – | 1.36 | 1.37 | 2.37 | 2.31 |
| 2.00% - 3y | 0.05 | 4.99 | – | 4.17 | – | 0.13 | 0.13 | 0.28 | 0.32 |
| 2.00% - 4y | 0.05 | 6.81 | – | 6.59 | – | 0.40 | 0.44 | 0.79 | 0.86 |
| 2.00% - 5y | 0.05 | 8.53 | – | 9.25 | – | 0.91 | 1.00 | 1.63 | 1.72 |
| Floors | | | | | | | | | |
| 0.50% - 3y | 0.05 | 0.00 | – | 0.01 | – | 0.50 | 0.44 | 0.20 | 0.24 |
| 0.50% - 4y | 0.05 | 0.01 | – | 0.01 | – | 0.60 | 0.58 | 0.26 | 0.32 |
| 0.50% - 5y | 0.05 | 0.01 | – | 0.01 | – | 0.66 | 0.71 | 0.30 | 0.37 |
| 1.00% - 3y | 0.05 | 0.01 | – | 0.04 | – | 1.40 | – | 0.96 | – |
| 1.00% - 4y | 0.05 | 0.02 | – | 0.04 | – | 1.77 | – | 1.19 | 1.16 |
| 1.00% - 5y | 0.05 | 0.05 | – | 0.04 | – | 2.03 | 1.98 | 1.35 | 1.31 |
| 1.50% - 3y | 0.05 | 0.02 | – | 0.10 | – | 2.47 | – | 1.91 | – |
| 1.50% - 4y | 0.05 | 0.05 | – | 0.10 | – | 3.14 | – | 2.36 | – |
| 1.50% - 5y | 0.05 | 0.1 | – | 0.10 | – | 3.65 | – | 2.68 | – |
| 2.00% - 3y | 0.05 | 0.04 | – | 0.17 | – | 3.60 | – | 2.94 | – |
| 2.00% - 4y | 0.05 | 0.10 | – | 0.18 | 0.23 | 4.63 | – | 3.65 | – |
| 2.00% - 5y | 0.05 | 0.18 | – | 0.18 | 0.30 | 5.42 | – | 4.14 | – |

**Notes**: This table compares observed market data with their model-implied counterparts for four selected dates. The calibrations of the corresponding models are reported in Table 2. Column $\omega$ reports the weights used in the loss function (see Equation 11). Details about the instruments used are provided in Subsection 6.2. All data are expressed in percentage points.

# A Proof of Proposition 4.1

This appendix introduces a lemma from which Proposition 4.1 derives. We consider here a Markovian process denoted by $z_t$ and valued in $\{e_1, \dots, e_n\}$, where $e_i$ denotes the $i^{th}$ column of the identity matrix of dimension $n \times n$. The matrix of transition probabilities between date $t$ and date $t+1$ is deterministic and denoted by $\Pi_{t+1}$ (the columns of $\Pi_t$ sum to one).

**Lemma A.1** *For all $h \in \mathbb{N}$, we have:*

$$E_t[(\alpha_1' z_{t+1}) \times \cdots \times (\alpha_h' z_{t+h})] = \mathbf{1}' \left\{ \prod_{i=h}^{1} D(\alpha_i) \Pi_{t+i} \right\} z_t, \tag{12}$$

*where $\prod_{i=h}^{1}$ is a backward product operator (i.e. $\prod_{i=2}^{1} M_i = M_2 \times M_1$) and where $D$ is a matrix operator that is such that $D(\alpha)$ is a diagonal matrix with vector $\alpha$ on its diagonal.*

**Proof:** To begin with, let us show that Equation (12) holds for $h = 1$. We have

$$\begin{aligned} E_t(\alpha_1' z_{t+1}) &= \sum_{i=1}^{n} \alpha_{1,i} e_i' \Pi_{t+1} z_t = \left( \sum_{i=1}^{M} \alpha_{1,i} e_i' \right) \Pi_{t+1} z_t \\ &= \mathbf{1}' D(\alpha_1) \Pi_{t+1} z_t. \end{aligned} \tag{13}$$

Now, let us consider the case $h = 2$. The law of iterated expectations leads to:

$$\begin{aligned} E_t((\alpha_1' z_{t+1}) \times (\alpha_2' z_{t+2})) &= E_t(E_t \left[ (\alpha_1' z_{t+1}) \times (\alpha_2' z_{t+2}) | \, z_{t+1} \right]) \\ &= E_t \left( (\alpha_1' z_{t+1}) E_t \left[ \alpha_2' z_{t+2} | \, z_{t+1} \right] \right). \end{aligned}$$

Then, using Equation (13) leads to:

$$\begin{aligned} E_t((\alpha_1' z_{t+1}) \times (\alpha_2' z_{t+2})) &= E_t \left( (\alpha_1' z_{t+1}) \mathbf{1}' D(\alpha_2) \Pi_{t+2} z_{t+1} \right) \\ &= E_t \left( \mathbf{1}' D(\alpha_2) \Pi_{t+2} z_{t+1} (\alpha_1' z_{t+1}) \right) \\ &= E_t \left( \mathbf{1}' D(\alpha_2) \Pi_{t+2} z_{t+1} (z_{t+1}' D(\alpha_1) \mathbf{1}) \right). \end{aligned}$$

Using the facts that $z_{t+1} z_{t+1}'$ commutes with any matrix and that $z_{t+1} z_{t+1}' \mathbf{1} = z_{t+1}$, we get:

$$\begin{aligned} E_t((\alpha_1' z_{t+1}) \times (\alpha_2' z_{t+2})) &= E_t \left( \mathbf{1}' D(\alpha_2) \Pi_{t+2} D(\alpha_1) z_{t+1} \right) \\ &= \mathbf{1}' \left\{ D(\alpha_2) \Pi_{t+2} \right\} \left\{ D(\alpha_1) \Pi_{t+1} \right\} z_t. \end{aligned}$$

It is straightforward to generalize to $h > 2$. ■

# B EURIBOR-related data adjustment

The model does not account for potential credit-risk and liquidity-risk effects in LIBOR-type rates. Therefore, if one calibrates the model on option prices whose underlying are LIBORs, adjustments have to be made not to bias the estimation of the model parameters.

The calibration exercise presented in Section 6 is based on OIS rates as well as interest-rate options (swaptions, caps and floors). Since the reference rate of these plain-vanilla options is the 6-month EURIBOR – and not the 6-month OIS rate – adjustments are required. For sake of simplicity, we propose adjustments based on the hypothesis of a deterministic spread between EURIBORs and OIS rates.

The swaptions used in our study are at-the-money (ATM), meaning that their strike rates are forward rates deriving from the EURIBOR swap curve. Our assumption that the spread between EURIBORs and OIS rates is deterministic implies that the price of a (synthetic) ATM OIS-based swaption is the same as the price of the (observed) EURIBOR-based one. That is, the only adjustment that we need to make here is to compute the strike rate on the basis of the OIS curve (and not on the EURIBOR-swap curve).

Turning to cap and floor prices, we subtract from the exercise rate the spread between EURIBOR swaps and OIS rates of the same maturity. To be sure, assume for instance that the spread between the 3-year EURIBOR swap and the OIS with same maturity is of 30 basis points. This would mean that market expects the EURIBOR rate to be approximately 30 basis points higher than the 6-month OIS rate over the next three years. Hence, the payoffs associated with a (EURIBOR-indexed) cap of exercise rate $c$ are the same as those associated with a (OIS-indexed) based swap of exercise rate $c - 30bps$.

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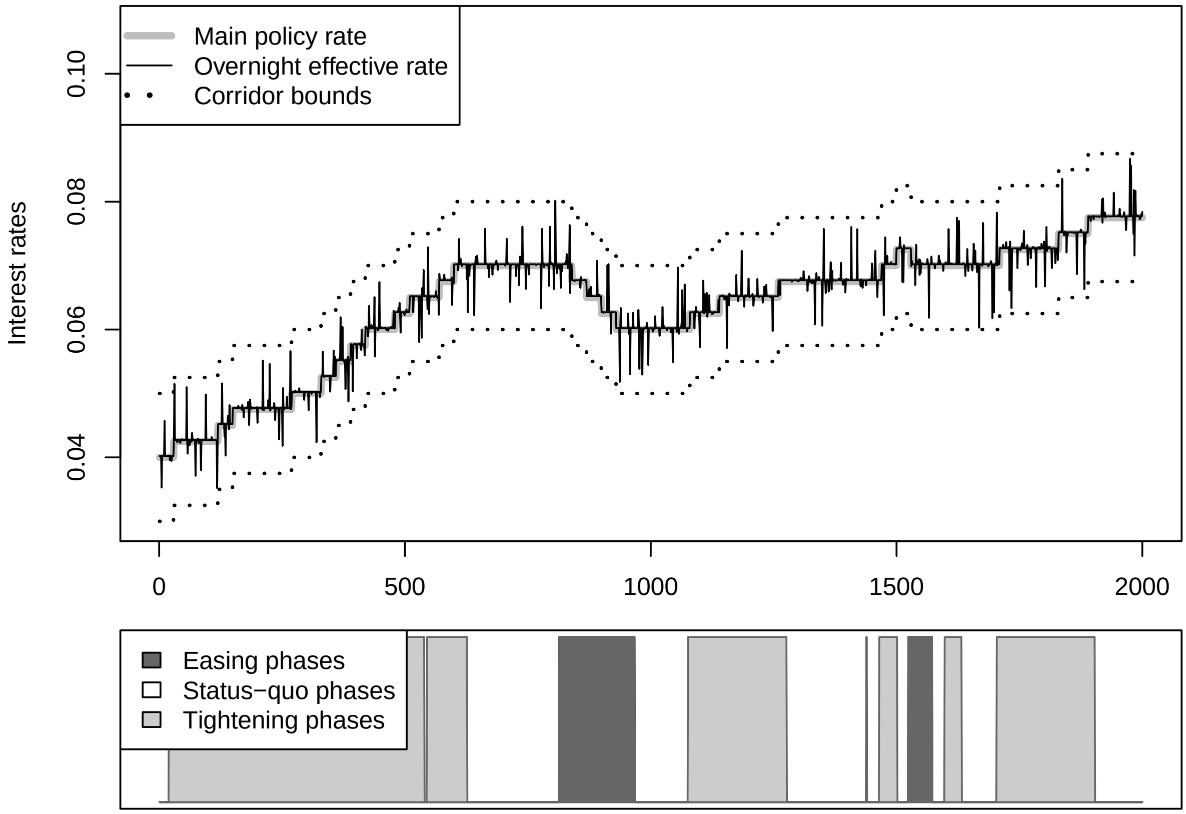
Figure 2: Simulated overnight rate




**Notes**: This figure displays simulated paths for the overnight and policy rates over a period of 2000 days. The dotted lines delineate the monetary-policy corridor. The upper (lower) bound of this corridor is the rate of the lending facility (deposit facility), which allows banks to obtain overnight liquidity from the central bank (to make overnight deposits with the central bank). The main policy rate (solid gray thick line) is the rate targeted by the central bank at its open-market operations. The model used to simulate these series is the one presented in Section 5; the calibration of the model is reported in Table 1.

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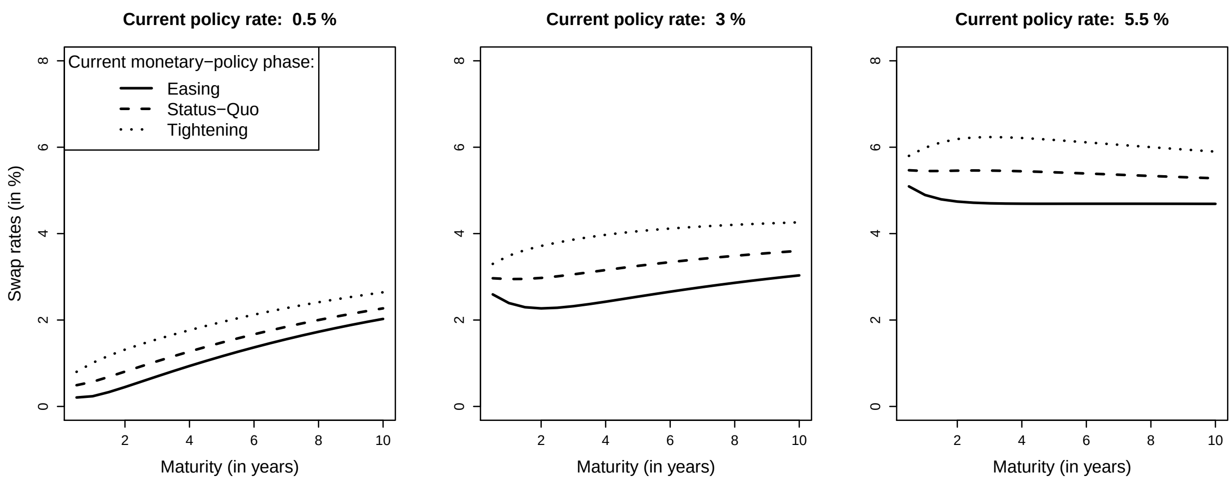
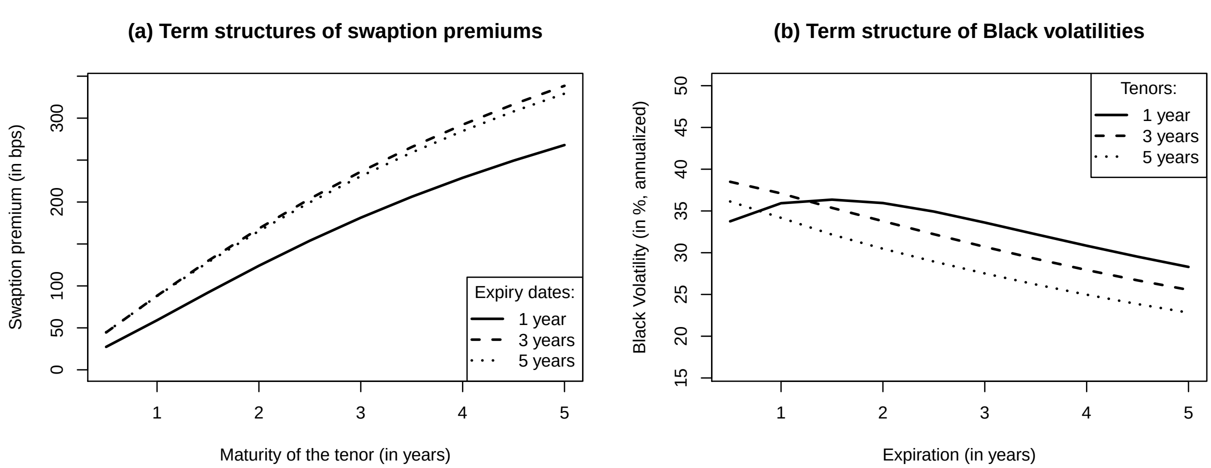
Figure 3: Swap curves



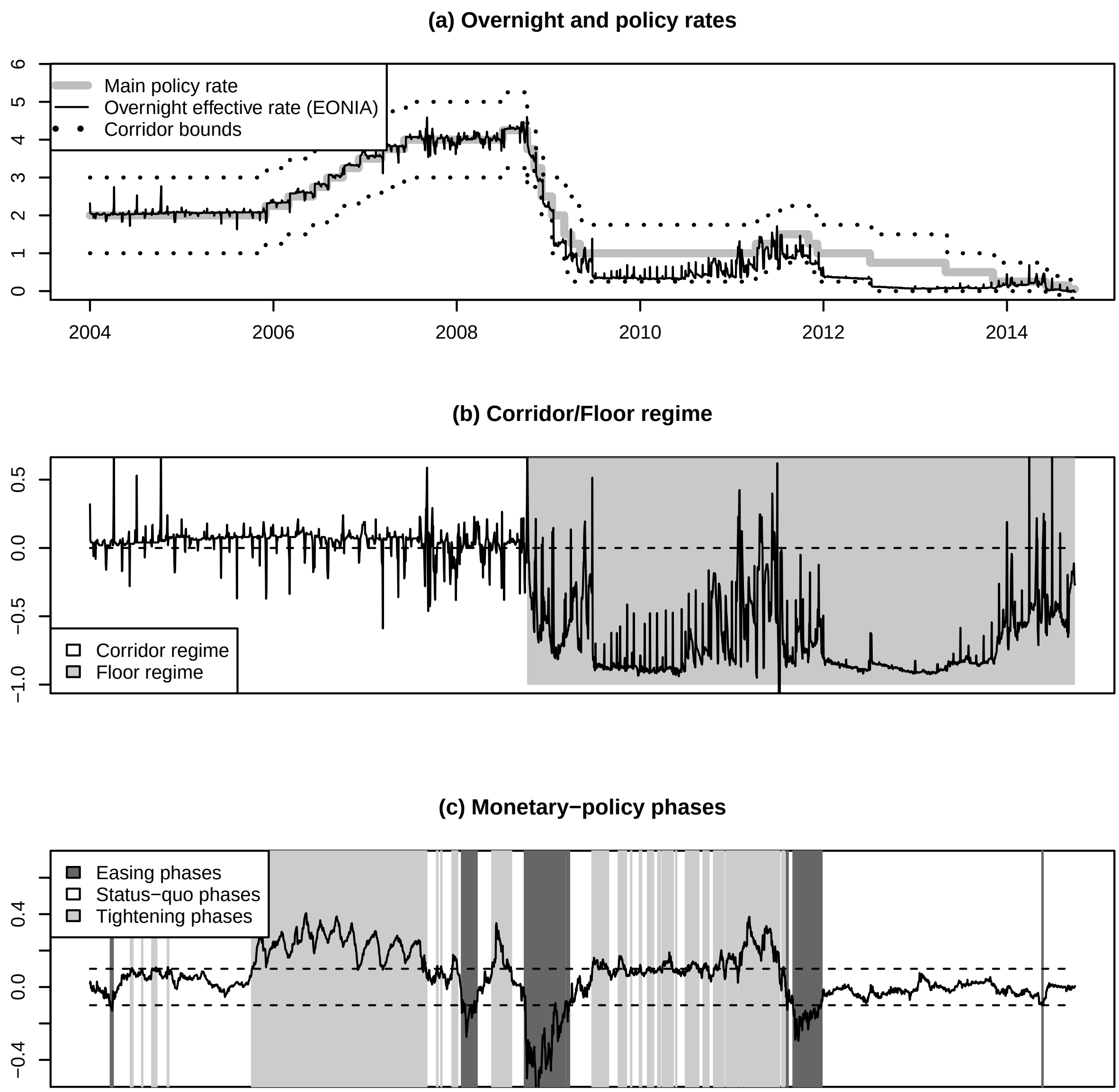

**Notes**: This figure displays swap curves that derive from the illustrative model presented in Section 5, the model parameters being those reported in Table 1. The three panels correspond to three possible values of the current policy rate (0.50%, 3.00% and 5.50%). Each plot shows the yield curves that prevail under each of three possible monetary-policy phases: Easing (solid lines), Status-quo (dashed lines) or Tightening (dotted lines).

Figure 4: Swaption pricing



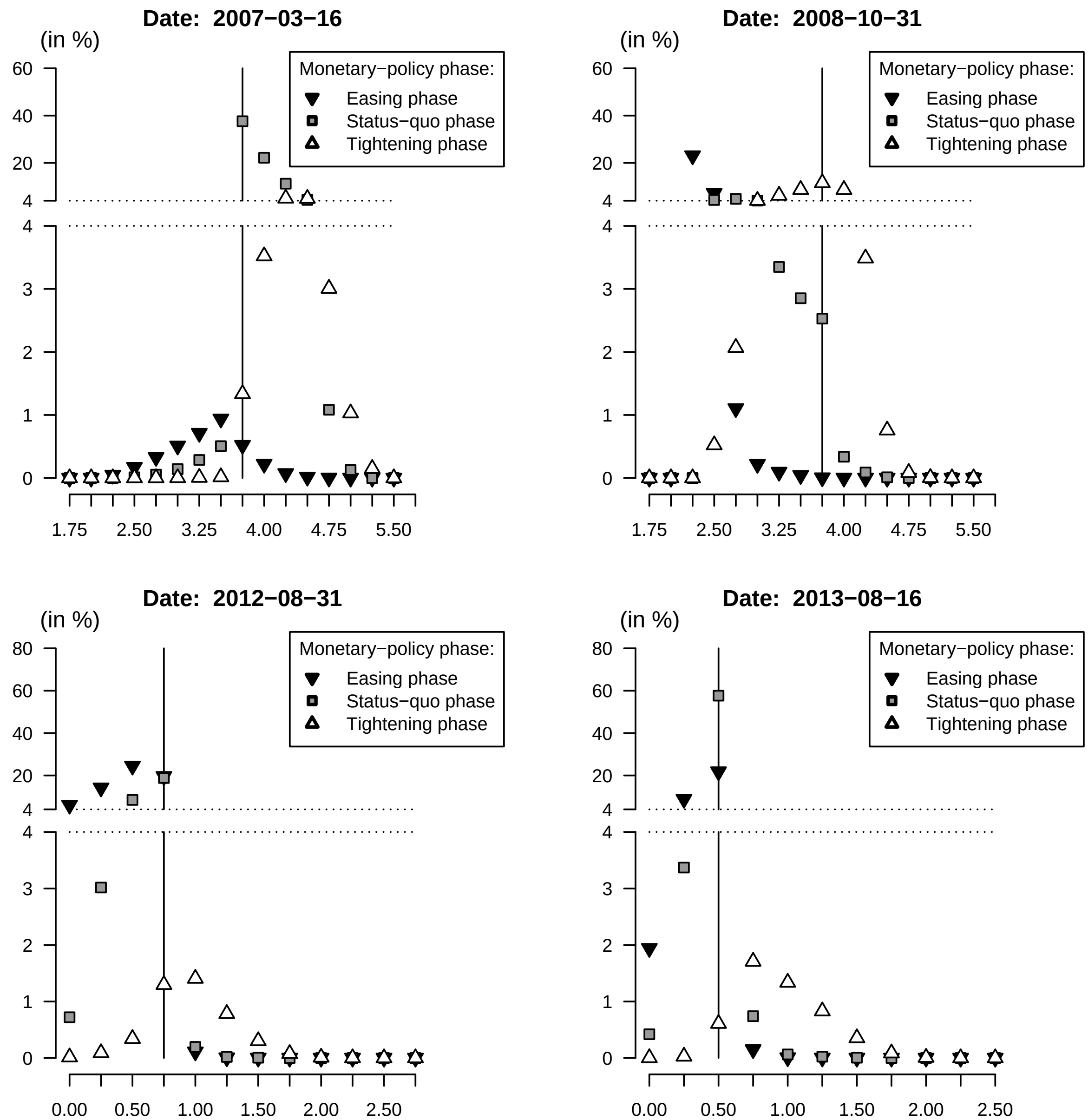

**Notes**: This figure shows swaption prices derived from the illustrative model presented in Section 5 (the model parameters are those reported in Table 1). The left-hand panel shows swaption prices expressed in basis points (0.01%) of the notional amounts. The right-hand panel plots Black volatilities, which are commonly used in the financial markets is to quote interest-rate options.

Figure 5: Identification of monetary-policy phases and corridor regimes



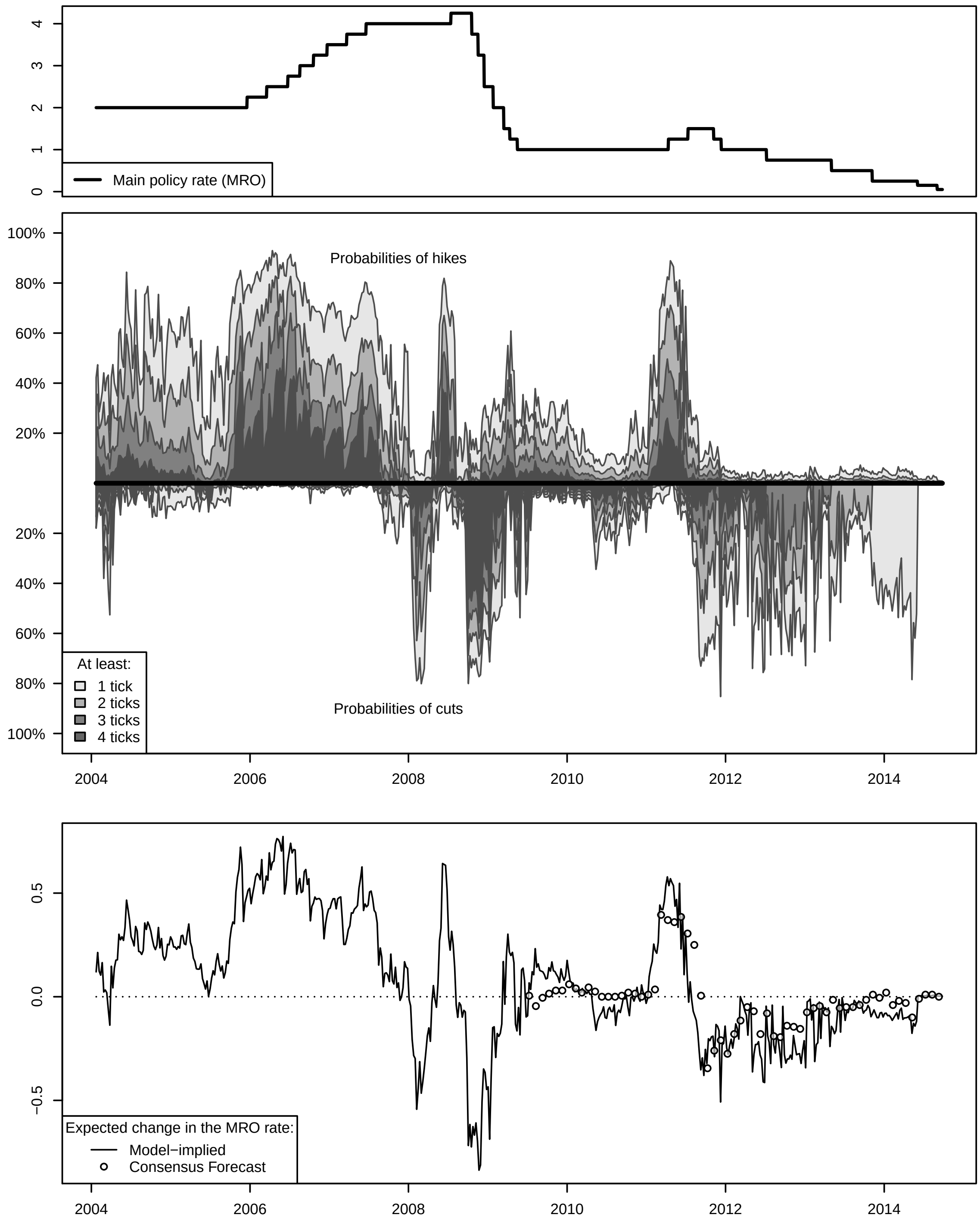

**Notes**: This figure illustrates the identification schemes of monetary-policy phases (Easing, Status-quo or Tightening, see Panel b) and of the corridor functioning regimes (normal or floor system, see Panel c). The black solid line in Panel (b) is the spread between the 6-month and the 1-month OIS spread; monetary policy is said to be (i) in a tightening phase if this spread is above 10 bps, (ii) in an easing phase if this spread is below 10 bps and (iii) in a status-quo phase otherwise. The black solid line in Panel (c) is the spread between the EONIA (overnight interbank rate in the euro area) and the lower bound of the corridor (that is the rate of the EBC deposit facility). When the 5-day moving average of this spread is below −0.20%, the floor system prevails (otherwise it is assumed that the corridor system operates under "normal" conditions).

Figure 6: Distribution of future policy rate (6-month horizon)




**Notes**: This figure displays distributions associated with 6-month-ahead forecasts of the policy rate ($\Delta' z_t$ of Equation 1). For each date, the model is calibrated on a set of observed market data (see Subsection 6.2). The distributions represented on these four plots are based on these calibrated models for four selected dates. (The calibrations are reported in Table 2). For each of these four plots, the plotted probabilities sum to one. This implies for instance that the market-implied probability of being in the tightening regime (6-month ahead) is given by the sum of the probabilities associated with the white triangles. The vertical bar indicates the location of the policy rate at the date where the market data are collected.

Figure 7: Recovering risk-neutral probabilities of hikes/cuts in the policy rate




**Notes**: This middle panel displays model-implied probabilities of possible hikes and cuts in the policy rate over a 6-month horizon. For instance, for date $t$, the darkest area that is above the zero line represents the probability of having a policy rate, at date $t+6$ mths, that is four ticks above the current policy rate (a tick corresponds to 25 basis points). The lower panel compares model-implied expectations of changes in the policy rate $\overline{r}_t$ (over the next 6 months) with those (mean values of the respondents' forecasts) reported in the survey of professional forecasters published by *Consensus Economics*. These survey data are available only since July 2009.

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