DOCUMENT
DE TRAVAIL
N° 527

AN ARBITRAGE-FREE NELSON-SIEGEL TERM STRUCTURE MODEL WITH STOCHASTIC VOLATILITY FOR THE DETERMINATION OF CURRENCY RISK PREMIA

Sarah Mouabbi

December 2014

BANQUE DE FRANCE
EUROSYSTÈME

DIRECTION GÉNÉRALE DES ÉTUDES ET DES RELATIONS INTERNATIONALES

DIRECTION GÉNÉRALE DES ÉTUDES ET DES RELATIONS INTERNATIONALES

AN ARBITRAGE-FREE NELSON-SIEGEL TERM STRUCTURE MODEL WITH STOCHASTIC VOLATILITY FOR THE DETERMINATION OF CURRENCY RISK PREMIA

Sarah Mouabbi

December 2014

An arbitrage-free Nelson-Siegel term structure model with stochastic volatility for the determination of currency risk premia

Sarah Mouabbi[‡]
DGEI-DEMFI Banque de France

Abstract

Abstract: This paper uses a risk-averse formulation of the uncovered interest rate parity to determine exchange rates through interest rate differentials, and ultimately extract currency risk premia. The method proposed consists of developing an affine Arbitrage-Free class of dynamic Nelson-Siegel term structure models with stochastic volatility to obtain the domestic and foreign discount rate variations, which in turn are used to derive a representation of exchange rate depreciations. No-arbitrage restrictions allow the endogenous capturing of currency risk premia. Empirical findings suggest that estimated currency risk premia are able to account for the forward premium puzzle and their properties are examined.

JEL classification numbers: E43, F31, G15
Keywords: term structure of interest rates; affine; exchange rates; risk premia

Résumé: Cet article se sert de la parité des taux d'intérêt non couverte à aversion au risque pour déterminer les taux de change à travers des spread de taux d'intérêt et pour extraire des primes de risque de change. La méthode proposée consiste à la modélisation affine de la courbe des taux d'intérêt par non-arbitrage, incorporant la méthode de Nelson-Siegel et de la volatilité stochastique, afin d'obtenir les facteurs d'escompte stochastique domestique et étranger, qui par leur tour sont utilisés pour déduire une représentation du taux de variation du cours de change. L'hypothèse d'absence d'opportunités d'arbitrage permet l'obtention endogène de primes de risque de change. Les résultats empiriques suggèrent que les primes de risque du taux de change estimées sont capables d'expliquer les dv́iations de la parité des taux non couverte. Leurs propriétés sont examinées.

Classification JEL: E43, F31, G15
Mots-clefs: structure par terme; affine; taux de change; primes de risque

---

I am grateful to Andrea Carriero for his valuable comments and suggestions and to Marcelo Fernandes and Erik Hjalmarsson for helpful discussions and constructive feedback. I also thank conference participants at the CSDA International Conference on Computational and Financial Econometrics and the Postgraduate Meeting of the Royal Economic Society, as well as seminar participants at the Banque de France and Queen Mary, University of London, particularly Richard Baillie, Ana Galvao, Emmanuel Guerre, George Kapetanios, Jean-Stephane Mesonnier, Jean-Paul Renne and Martin Weale.

This paper expresses the views of the author only; they do not necessarily reflect those of the Banque de France.

‡Corresponding author.
E-mail: sarah.mouabbi@banque-france.fr. Phone: +33 (0)1 42 92 49 18.

Non-technical summary

Exchange rate fluctuations have substantial implications for the pricing and allocation of assets. Characterized by seemingly weak links to fundamentals and by a volatile nature, exchange rates still remain at the forefront of a multitude of papers. These stylized facts, better known as the exchange rate determination and excess volatility puzzles, render the modeling of exchange rate movements and the caption of their volatility increasingly intricate.

A significant strand of the exchange rate literature has long been devoted to tying exchange rates to interest rates through the so called covered and uncovered interest rate parities. Under the validity of perfect asset substitutability and capital mobility, the principle of these two parities revolves around the premise of no-arbitrage, whereby low interest rate countries ought to be compensated by an appreciated currency in order to maintain the indifference of the global investor. Despite the highly intuitive nature of these theoretical equilibrium relations, severe deviations from postulated equilibrium levels have, on multiple occasions, been recorded through empirical tests. The observed divergences are expressed by the susceptibility of low interest rate countries to currency depreciations and are typically known as the forward premium puzzle.

A plethora of studies has been dedicated to justifying these deviations. What seems to be the most convincing interpretation so far is the one proposed by Fama (1984), advocating the presence of a time-varying risk premium. The latter represents the compensation to the investor for being exposed to exchange rate risk.

Though unobserved, currency risk premia have the potential to enhance asset allocation and risk management decisions. This explains why attempts to estimate currency risk premia are persistently found in the literature.

The purpose of this study is to examine whether a newly established framework for the term structure of interest rates, the Arbitrage Free Nelson Siegel term structure model (AFNS) with stochastic volatility, introduced by Christensen, Lopez, and Rudebusch (2010a), can be further extended to jointly price both the term structure of interest rates of two countries and exchange rate depreciations. Once the exchange rate depreciation is estimated through the Bilateral Arbitrage-Free Nelson-Siegel model with stochastic volatility (BAFNS), no-arbitrage conditions allow for the endogenous extraction of the risk premium.

I. Introduction

Exchange rate fluctuations have substantial implications for the pricing and allocation of assets. Characterized by seemingly weak links to fundamentals and by a volatile nature, exchange rates still remain at the forefront of a multitude of papers. These stylized facts, better known as the exchange rate determination and excess volatility puzzles, render the modeling of exchange rate movements and the caption of their volatility increasingly intricate.

A significant strand of the exchange rate literature has long been devoted to tying exchange rates to interest rates through the so called covered and uncovered interest rate parities. Under the validity of perfect asset substitutability and capital mobility, the principle of these two parities revolves around the premise of no-arbitrage, whereby low interest rate countries ought to be compensated by an appreciated currency in order to maintain the indifference of the global investor. Despite the highly intuitive nature of these theoretical equilibrium relations, severe deviations from postulated equilibrium levels have, on multiple occasions, been recorded through empirical tests. The observed divergences are expressed by the susceptibility of low interest rate countries to currency depreciations and are typically known as the forward premium puzzle.

A plethora of studies has been dedicated to justifying these deviations. What seems to be the most convincing interpretation so far is the one proposed by Fama (1984), advocating the presence of a time-varying risk premium. The latter represents the compensation to the investor for being exposed to exchange rate risk. Fama (1984) stipulates that currency risk premia ought to have a greater variance than expected exchange rate variations and that both variables need to be negatively correlated in order to explain the puzzle. Following this noteworthy account, many papers have attempted to model a currency risk premium using statistical methods and conventional asset pricing methods, including consumption based asset pricing theory, equilibrium models, but with arguably limited success (see, for example, Frankel and Engel (1984), Domowitz and Hakkio (1985), Mark (1988), Bekaert (1996) and Lustig and Verdelhan (2011)).

Though unobserved, currency risk premia have the potential to enhance asset allocation and risk management decisions. This explains why attempts to estimate currency risk premia are persistently found in the literature. The purpose of this study is to examine whether a newly established framework for the term structure of interest rates, the Arbitrage Free Nelson Siegel term structure model (AFNS) with stochastic volatility, introduced by Christensen, Lopez, and Rudebusch (2010a), can be further extended to jointly price both the term structure of interest rates of two countries and exchange rate depreciations. Once the exchange rate depreciation is estimated through the Bilateral Arbitrage-Free Nelson-Siegel model with stochastic volatility (BAFNS), no-arbitrage conditions allow for the endogenous extraction of the risk premium. The above-mentioned approach of exploiting existing affine term structure models in order to derive risk premia has previously been employed in several different contexts. In an influential study by Backus, Foresi, and Telmer (2001), the issue of whether the popular affine term structure model by Duffie and Kan (1996) is capable of capturing the forward premium anomaly is considered. Similarly, Sarno, Schneider, and Wagner (2012) derive a multi-currency term structure model that gives rise to the foreign exchange risk premium, the properties of which are examined. Graveline (2006), examines the forward premium anomaly using an arbitrage-free model, including options prices. Similar methods and applications can be found in Brandt and Santa-Clara (2001), where excess volatility in an incomplete market setting is examined, in Ahn (2004), Inci and Lu (2004) and Anderson, Hammond, and Ramezani (2010), who compare the different implications of local and global factors, and Brennan and Xia (2006), where the volatility of pricing kernels is tied to exchange rate volatility and risk premia. More recently, term structure models have been used to obtain equity premia (see Brennan, Wang, and Xia (2004) and Lettau and Wachter (2011)) and underpin inflation expectations and risk premia (see Christensen, Lopez, and Rudebusch (2010b) and Chernov and Mueller (2012)).

Although Sarno, Schneider, and Wagner (2012)'s analysis appears to be the most complete and well-rounded piece of work to date, it suffers from a cumbersome Bayesian estimation procedure. Moreover, an additional step is further required stemming from the necessity to use rotations in order to interpret the latent factors. In this paper, attention is drawn towards employing the AFNS model due to the favorable properties it agglomerates. In particular, this model encompasses sound theoretical grounds through no-arbitrage restrictions, whilst also preserving robust estimation procedures with the imposition of the Dynamic Nelson Siegel (DNS) structure. Specifically, the imposition of the DNS structure provides a level, slope and curvature interpretation to the latent factors without performing any rotation. Additionally, the flexibility of the AFNS model allows to extend its use beyond simple estimation and makes it appealing for forecasting exercises. Furthermore, the AFNS is found to be successful not only in the blunt determination of the term structure of interest rates but also in more synthesized problems such as the estimation of inflation expectations; hence motivating the use of this specific model to estimating currency risk premia. This paper further shifts its focus towards analyzing the impact of the different assumptions set on the diffusion of the process (ie. Gaussian or with stochastic volatility) on the properties acquired by the estimates of the model, namely, the yields, exchange rate variations and currency risk premia.

A six-factor AFNS model with stochastic volatility is estimated to jointly underpin the term structure of two countries, whilst exchange rate depreciations and risk premia are derived endogenously. For robustness purposes, a Gaussian multilateral AFNS model with twenty one factors (three factors for each country included) is examined in Appendix B of this paper. Results suggest that the Gaussian AFNS model provides a better fit for interest rates and allows for a joint multi-currency estimation rather than restricting the model to a bilateral estimation. On the other hand, the volatility of exchange rate differentials is better captured using the AFNS model with stochastic volatility rather than the Gaussian version of the model. Additionally, the risk premium generated from the bilateral AFNS model with stochastic volatility respects the above mentioned Fama conditions, hence offering a legitimate explanation for the forward bias puzzle without resorting to departures from rational expectations. The main drivers of exchange rate depreciations and risk premia are found to be the two curvature factors whilst currency risk premia display a countercyclical nature. Finally, Graveline (2006) argues that the use of options helps in fitting the volatility of exchange rates. In this regard, this paper shows that it is possible to reasonably capture the volatility of exchange rate depreciations and risk premia without the inclusion of options in the model. More specifically, this result extends to first and second conditional moments.

The remainder of the paper is structured as follows. Section II consists of a selective overview of the uncovered interest rate parity, the existing AFNS model with stochastic volatility, and pricing kernels as the connecting link of interest rates to exchange rates. In section III, the BAFNS model is derived with the aim of extrapolating both exchange rate depreciations and risk premia. Section IV comprises of an empirical study of the performance of the BAFNS model in determining exchange rate changes and extracting risk premia. This section also specifies the estimation procedure followed and its substantial benefits. Section V provides conclusive remarks.

II. Exchange rates and interest rates at a glance

This segment aims to motivate the sections that follow by building a review of the link between interest rates and exchange rates as well as the affine term structures model that is utilized to derive exchange rate variations.

A. The uncovered interest rate parity

Let $y^D(t,T)$ and $y^F(t,T)$ denote the zero coupon bond yields with maturity T at time t, of the domestic and foreign countries, and s_t and $f_{t,T}$ denote the logarithm of the spot and T-forward exchange rate, respectively. For the remainder of the paper, the United States is considered as the domestic country. The United Kingdom represents the foreign country in the main analysis of the paper, whilst additional foreign countries, including Australia, Canada, Switzerland, Japan and Sweden are examined in Appendix B. All exchange rates are denominated in U.S. dollars, and hence represent the price of one unit of foreign currency in US dollars.

The covered interest rate parity stipulates that, under rational expectations and risk-neutrality, the expected exchange rate depreciation equals the difference between the forward and spot exchange rates. By the same token, the uncovered interest rate parity builds an exact relationship between the expected exchange rate depreciation and the domestic and foreign interest rate differential. The two relationships are shown in the equations below,

$$\mathbb{E}^{\mathbb{P}}\left[\Delta s_{t,T}|\mathcal{F}_t\right] = f_{t,T} - s_t \tag{1}$$

$$\mathbb{E}^{\mathbb{P}}\left[\Delta s_{t,T}|\mathcal{F}_t\right] = y^D(t,T) - y^F(t,T) \tag{2}$$

where $\mathbb{E}^{\mathbb{P}}$ is the expectation under the data generating probability measure, $\mathcal{F}_t$ is the filtration and $\Delta s_{t,T} = s_T - s_t$. Drawing from equation (1), the forward exchange rate ought to be an unbiased predictor of the future spot exchange rate. Using the traditional Fama regressions given below, the validity of the forward rate unbiasedness hypothesis is confirmed if $\alpha_i = 0$, $\beta_i = 1$ and $\xi_{i;t,T}$ displays no serial correlation, for $i = 1, 2$.

$$\Delta s_{t,T} = \alpha_1 + \beta_1 (f_{t,T} - s_t) + \xi_{1;t,T} \tag{3}$$

$$\Delta s_{t,T} = \alpha_2 + \beta_2 \left[y^D(t,T) - y^F(t,T)) \right] + \xi_{2;t,T} \tag{4}$$

The preponderance of empirical results have, however, disputed the claim of the hypothesis, hence raising theories for the existence of a time-varying risk premium, amongst others. Conceptually, the existence of a risk premium signifies a departure from risk-neutrality given it represents a compensation, to the investor, for being exposed to currency risk as well as interest rate risk. A risk-averse interpretation of the uncovered interest rate parity is given below,

$$\Delta s_{t,T} = \left[y^D(t,T) - y^F(t,T) \right] - \rho_{t,T} + \zeta_{t,T} \tag{5}$$

with $\rho_{t,T}$ representing the risk premium, which varies with time t and maturity T and $\zeta_{t,T}$ being the regression residual. The risk premium component bears a negative sign due to the fact that exchange rates are denominated in US dollars (ie. the domestic currency). A negative exchange rate depreciation signals an appreciated US currency, hence implying a higher purchasing power and risk premium. Fama (1984) stipulates that there are two necessary conditions the risk premium needs to feature in order to ensure its ability to explain the departures from the levels dictated by the uncovered interest rate parity. These conditions are stated below,

$$\mathbb{V}^{\mathbb{P}} \left[\rho_{t,T} \right] > \mathbb{V}^{\mathbb{P}} \left[\mathbb{E}^{\mathbb{P}}{}_t \left(\Delta s_{t,T} \right) \right] \tag{6}$$

$$\mathbb{C}ov^{\mathbb{P}} \left[\rho_{t,T}, \mathbb{E}^{\mathbb{P}}{}_t \left(\Delta s_{t,T} \right) \right] < 0 \tag{7}$$

where $\mathbb{V}^{\mathbb{P}}$ and $\mathbb{C}ov^{\mathbb{P}}$ represent the variance and covariance under the physical measure, respectively.

Specifically, omitting the risk premium typically generates a negative slope of the Fama regression in equation (4). Fama (1984) shows that if the risk premium admits these two conditions then the negative bias of the slope is corrected, advocating, hence, in favor of the risk premium hypothesis as a reasonable correction to the risk neutral uncovered interest rate parity.

B. THE ARBITRAGE-FREE NELSON-SIEGEL MODEL WITH STOCHASTIC VOLATILITY

In this segment, the model, used to fit the term structure of interest rates of the domestic and foreign countries, is presented in its simplest, unilateral form.

One of the most prominent models, empirically, for the term structure of interest rates is the one developed by Nelson and Siegel (1987). The popularity of this model mainly stems from its stable estimation and its flexibility in fitting both the cross section and time series properties of interest rates. Diebold and Li (2006) have extended it to a dynamic factor model where latent factors bear the level, slope and curvature interpretation, whilst, Koopman, Mallee, and Van der Wel (2010) have allowed for time-varying parameters and a non-Gaussian setting. Although empirically these models have been highly praised for their performance, they have sustained some criticism for their lack of theoretical grounding.

Conversely, affine term structure models imposing no-arbitrage restrictions, such as the canonical model by Duffie and Kan (1996), have been found challenging in their estimation due to the difficulty in pinning down the global optimum, (see Joslin, Singleton, and Zhu (2011) and Duffee and Stanton (2012)), as well as in their empirical success (see Duffee (2002). Christensen, Diebold, and Rudebusch (2011) develop an affine Arbitrage-Free class of dynamic Nelson-Siegel term structure models which combine the benefits of the two strands of models above whilst simultaneously alleviating their disadvantages. However, due to the Gaussian nature of the model, it is highly unlikely to be able to capture the volatility displayed by exchange rates. A stochastic version of the AFNS model is hence adopted following Christensen, Lopez, and Rudebusch (2010a).

The details of the three-factor AFNS model with stochastic volatility generated by all three factors ($AFNS_3$) are provided below. Let $X_t = (L_t, S_t, C_t)'$ denote the latent state variables, which can be interpreted as level, slope and curvature factors. In addition, assume the state vector X_t follows a Cox-Ingersoll-Ross process under the risk neutral $\mathbb{Q}$ measure. $\kappa^{\mathbb{Q}}$ is the mean-reversion matrix, $\theta^{\mathbb{Q}}$ the unconditional mean vector and $W_t^{X,\mathbb{Q}}$ denotes a three dimensional Wiener process.

$$dX_t = \kappa^{\mathbb{Q}}\left[\theta^{\mathbb{Q}} - X_t\right]dt + \Sigma diag[\sqrt{X_t}]dW_t^{X,\mathbb{Q}} \tag{8}$$

Christensen, Diebold, and Rudebusch (2011) show that with no loss of generality, $\theta^{\mathbb{Q}}$ can be set to zero. The system of stochastic differential equations, under the risk neutral probability measure, is hence re-written as follows,

$$\begin{pmatrix} dL_t \\ dS_t \\ dC_t \end{pmatrix} = -\begin{pmatrix} \epsilon & 0 & 0 \\ 0 & \lambda & -\lambda \\ 0 & 0 & \lambda \end{pmatrix}\begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix}dt + \begin{pmatrix} \sigma_{11} & 0 & 0 \\ 0 & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{pmatrix}\begin{pmatrix} \sqrt{L_t} & 0 & 0 \\ 0 & \sqrt{S_t} & 0 \\ 0 & 0 & \sqrt{C_t} \end{pmatrix}\begin{pmatrix} dW_t^{L,\mathbb{Q}} \\ dW_t^{S,\mathbb{Q}} \\ dW_t^{C,\mathbb{Q}} \end{pmatrix} \tag{9}$$

where λ is the mean-reversion parameter and $\epsilon = 10^{-6}$ to have a near unit root behavior for the level factor. In particular, the level factor typically displays a unit root, implying that the first element of the mean-reversion matrix ought to be equal to zero. However, the breach of Gaussianity would prevent the use of the Kalman filter. Setting this element equal to ϵ, a very small yet non-zero number, allows to preserve a near unit root feature whilst still allowing the use of the Kalman filter.

As demonstrated by Ang and Piazzesi (2003), nominal zero-coupon bond prices are exponentially affine functions of the state variables,

$$P(t,T) = E_t^{\mathbb{Q}}\left[exp\left(-\int_t^T r_u du\right)\right] = exp\left(A(t,T) + B(t,T)' X_t\right) \tag{10}$$

where r_t denotes the instantaneous risk-free rate and $(A(t,T))$ and $(B(t,T))$ are , respectively, the intercept and slope of the affine expression.

Consequently, the representation of zero-coupon yields with maturity T at time t is given by an affine function of the state variables, as shown below,

$$y(t,T) = -\frac{1}{T-t} log P(t,T) = -\frac{A(t,T)}{T-t} - \frac{B(t,T)'}{T-t} X_t \tag{11}$$

with $A(t,T)$ and $B(t,T)$ being the unique solutions to a system of Riccati equations. $A(t,T)$ is known as the adjustment term, which is added to maintain no-arbitrage conditions, whilst the factor loadings $B(t,T)$, retain the interpretation of level, slope and curvature, although they no longer match the exact form of the Nelson-Siegel factor loadings. The Riccati differential equations are listed below.

$$\begin{cases} \frac{B_1(t,T)}{dt}(t,T) = 1 + \epsilon B_1(t,T) - \frac{1}{2}\sigma_{11}^2 B_1^2(t,T) \\ \frac{B_2(t,T)}{dt}(t,T) = 1 + \lambda B_2(t,T) - \frac{1}{2}\sigma_{22}^2 B_2^2(t,T) \\ \frac{B_3(t,T)}{dt}(t,T) = -\lambda B_2(t,T) + \lambda B_3(t,T) - \frac{1}{2}\sigma_{33}^2 B_3^2(t,T) \\ \frac{A(t,T)}{dt}(t,T) \ = -B(t,T)'\kappa^{\mathbb{Q}}\theta^{\mathbb{Q}} \end{cases} \tag{12}$$

The instantaneous risk-free rate is an affine function of the state variables given by the sum of the level and slope factors, as stated in equation (13). This representation is justified by the fact that the level factor affects yields of all maturities, including the short rate, while the slope factor typically influences yields of short maturities. The curvature factor is unnecessary in the spectrum of the short rate since it typically influences yields of medium horizons.

$$r_t = L_t + S_t \tag{13}$$

The AFNS model with stochastic volatility is a continuous-time model and Girsanov's theorem ensures the change from the data generated process measure, also known as the physical measure, to the risk-neutral measure as such, $dW_t^{\mathbb{Q}} = dW_t^{\mathbb{P}} + \Gamma_t dt$, where Γ_t is the market price of risk and under the extended affine risk premium specification defined in Cheridito, Filipovic, and Kimmel (2007), it takes the form below:

$$\begin{aligned} \Gamma_t = & \begin{pmatrix} \sqrt{L_t} & 0 & 0 \\ 0 & \sqrt{S_t} & 0 \\ 0 & 0 & \sqrt{C_t} \end{pmatrix} \begin{pmatrix} \gamma_{1,1} \\ \gamma_{1,2} \\ \gamma_{1,3} \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{S_t}} & 0 \\ 0 & 0 & \frac{1}{\sqrt{C_t}} \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ \gamma_{2,21} & 0 & \gamma_{2,23} \\ \gamma_{2,31} & \gamma_{2,32} & 0 \end{pmatrix} \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} \\ & + \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{S_t}} & 0 \\ 0 & 0 & \frac{1}{\sqrt{C_t}} \end{pmatrix} \begin{pmatrix} 0 \\ \gamma_{3,2} \\ \gamma_{3,3} \end{pmatrix} \end{aligned} \tag{14}$$

The extended specification for the market price of risk encompasses the essentially affine risk premium specification provided by Duffee (2002), which itself is a generalization of the completely affine formulation of the canonical model by Dai and Singleton (2000). Subtracting $\Sigma diag[\sqrt{X_t}]\Gamma_t dt$ from the risk-neutral dynamics and substituting the Brownian motion under the risk-neutral measure with its physical counterpart allows the extraction of the latent state variables $X_t = (L_t, S_t, C_t)'$ under the physical measure. The dynamics are given by the following stochastic differential equation.

$$\begin{pmatrix} dL_t \\ dS_t \\ dC_t \end{pmatrix} = \begin{pmatrix} \kappa_{11}^{\mathbb{P}} & 0 & 0 \\ \kappa_{21}^{\mathbb{P}} & \kappa_{22}^{\mathbb{P}} & \kappa_{23}^{\mathbb{P}} \\ \kappa_{31}^{\mathbb{P}} & \kappa_{32}^{\mathbb{P}} & \kappa_{33}^{\mathbb{P}} \end{pmatrix} \left[\begin{pmatrix} \theta_t^{L,\mathbb{P}} \\ \theta_t^{S,\mathbb{P}} \\ \theta_t^{C,\mathbb{P}} \end{pmatrix} - \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} \right] dt + \begin{pmatrix} \sigma_{11} & 0 & 0 \\ 0 & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} \sqrt{L_t} & 0 & 0 \\ 0 & \sqrt{S_t} & 0 \\ 0 & 0 & \sqrt{C_t} \end{pmatrix} \begin{pmatrix} dW_t^{L,\mathbb{P}} \\ dW_t^{S,\mathbb{P}} \\ dW_t^{C,\mathbb{P}} \end{pmatrix} \tag{15}$$

It is important to note that Feller conditions need to be satisfied in order to prevent states from hitting the zero-bound, as it would induce the states to remain at zero. These conditions are:

$$\begin{cases} \kappa_{21}^{\mathbb{P}}\theta_1^{\mathbb{P}} + \kappa_{22}^{\mathbb{P}}\theta_2^{\mathbb{P}} + \kappa_{23}^{\mathbb{P}}\theta_3^{\mathbb{P}} > \frac{1}{2}\sigma_{22}^2 \\ \lambda\theta_2^{\mathbb{Q}} - \lambda\theta_3^{\mathbb{Q}} > \frac{1}{2}\sigma_{22}^2 \\ \kappa_{31}^{\mathbb{P}}\theta_1^{\mathbb{P}} + \kappa_{32}^{\mathbb{P}}\theta_2^{\mathbb{P}} + \kappa_{33}^{\mathbb{P}}\theta_3^{\mathbb{P}} > \frac{1}{2}\sigma_{33}^2 \\ \lambda\theta_3^{\mathbb{Q}} > \frac{1}{2}\sigma_{33}^2 \end{cases} \tag{16}$$

There are additional admissibility restrictions that also need to be respected in order to ensure that the Nelson-Siegel factor loadings are being as feasibly approximated as possible, as well as for the model to remain free from arbitrage opportunities. These are:

$$\begin{cases} \epsilon\theta_1^{\mathbb{Q}} = \kappa_{11}^{\mathbb{P}}\theta_1^{\mathbb{P}} \\ \epsilon\theta_1^{\mathbb{Q}} > 0, \kappa_{11}^{\mathbb{P}}\theta_1^{\mathbb{P}} > 0 \\ \kappa_{21}^{\mathbb{P}} \leq 0, \kappa_{23}^{\mathbb{P}} \leq 0, \kappa_{31}^{\mathbb{P}} \leq 0, \kappa_{32}^{\mathbb{P}} \leq 0 \\ \theta_3^{\mathbb{Q}} = \frac{\lambda\theta_2^{\mathbb{Q}} - \frac{1}{2}\sigma_{22}^2}{\lambda} - \epsilon \end{cases} \tag{17}$$

Further, to ensure stationarity, the eigenvalues of $\kappa^{\mathbb{P}}$ have to be strictly positive. Finally, the latent factor L_t is interpreted as a level factor, which theoretically has a unit root. However, a unit root in the diffusion process induces complications in the estimation procedure. An adequate compromise is to settle for a near unit root behavior. Hence, in order to prevent the latent factor from displaying a unit root, additional restrictions are imposed on the relevant parameters. More specifically, $\kappa_{11}^{\mathbb{P}}$ and $\theta_1^{\mathbb{P}}$ are set to be strictly positive and $\kappa_{11}^{\mathbb{Q}} = \epsilon = 10^{-6}$, thus ensuring a near unit root behavior.

C. Stochastic discount factors

Let P_t^D and P_t^F denote the domestic and foreign price at time t of a future payment P_T^D and P_T^F, respectively,

$$P_t^D = \mathbb{E}^{\mathbb{P}}\left[\frac{M_T^D}{M_t^D} P_T^D\right] \tag{18}$$

$$P_t^F = \mathbb{E}^{\mathbb{P}}\left[\frac{M_T^F}{M_t^F} P_T^F\right] \tag{19}$$

where M^D and M^F are the domestic and foreign stochastic discount factors. Stochastic discount factors, also known as pricing kernels, establish the existence of a risk neutral probability measure and dictate the price of state-dependent claims. According to Graveline (2006), there exists a unique minimum variance stochastic discount factor with the following dynamics,

$$\frac{dM_t^D}{M_t^D} = -r_t^D dt - \Gamma_t^{D'} dW_t^{\mathbb{P}} \tag{20}$$

$$\frac{dM_t^F}{M_t^F} = -r_t^F dt - \Gamma_t^{F'} dW_t^{\mathbb{P}} \tag{21}$$

r_t^D and r_t^F denote the instantaneous domestic and foreign risk-free rate, respectively, and $W_t^{\mathbb{P}}$ represents a Wiener process. The diffusions of the pricing kernels, Γ_t^D and Γ_t^F, are the domestic and foreign prices of risk. The benefits of adopting a no-arbitrage setting come into play by enforcing a relationship between domestic and foreign bond prices and more importantly by setting a direct link relating interest rates to exchange rates, as shown below.

$$\frac{M_T^F}{M_t^F} \equiv \frac{S_T}{S_t}\frac{M_T^D}{M_t^D} \tag{22}$$

The above relationship states that one of the three random variables can be replicated using the remaining two variables. Hence, one of the stochastic processes can be determined endogenously, assuming that the remaining two dynamics are known. As in Backus, Foresi, and Telmer (2001), the two pricing kernels are used to endogenously extract the exchange rate dynamics. This strategy allows the preservation of symmetry between the theoretical frameworks of the two countries. In particular, this paper aims to extract information from the term structures of interest rates in order to explain exchange rate movements. Thus, the two term structures are modeled using exactly the same theoretical model for consistency purposes.

III. Theoretical framework: a dynamic bilateral asset pricing model

This section builds a bilateral extension for the AFNS model with stochastic volatility generated by all factors included in the model. The endogenous representations of the exchange rate depreciation, expected exchange rate return and currency risk premia are then derived.

A. The bilateral arbitrage-free Nelson-Siegel model with stochastic volatility

Extrapolating from the $AFNS_3$ to encompass two countries requires six factors. Let $X_t^J = \left(L_t^D, S_t^D, C_t^D, L_t^F, S_t^F, C_t^F\right)'$ denote the state vector for the joint model, including the level, slope and curvature factors for the domestic and foreign countries. An advantage in using an extension of the AFNS stems from the fact that no additional rotation is necessary to interpret the latent factors. Under the risk-neutral measure, the state variable $X_t^J = \left(X_t^D, X_t^F\right)'$ solves the following stochastic differential equation.

$$dX_t^J = -\begin{pmatrix} \kappa^{D,\mathbb{Q}} & 0 \\ 0 & \kappa^{F,\mathbb{Q}} \end{pmatrix} \begin{pmatrix} X_t^D \\ X_t^F \end{pmatrix} dt + \begin{pmatrix} \Sigma^D & 0 \\ 0 & \Sigma^F \end{pmatrix} \begin{pmatrix} diag\sqrt{X_t^D} & 0 \\ 0 & diag\sqrt{X_t^F} \end{pmatrix} \begin{pmatrix} dW_t^{D,\mathbb{Q}} \\ dW_t^{F,\mathbb{Q}} \end{pmatrix} \tag{23}$$

where $W_t^{D,\mathbb{Q}}$ and $W_t^{F,\mathbb{Q}}$ are three dimensional Brownian motions and $\kappa^{D,\mathbb{Q}}$, $\kappa^{F,\mathbb{Q}}$, Σ^D and Σ^F are defined as follows.

$$\kappa^{D,\mathbb{Q}} = \begin{pmatrix} \epsilon & 0 & 0 \\ 0 & \lambda^D & -\lambda^D \\ 0 & 0 & \lambda^D \end{pmatrix}; \; \kappa^{F,\mathbb{Q}} = \begin{pmatrix} \epsilon & 0 & 0 \\ 0 & \lambda^F & -\lambda^F \\ 0 & 0 & \lambda^F \end{pmatrix}; \; \Sigma^D = diag \begin{pmatrix} \sigma_{11}^D \\ \sigma_{22}^D \\ \sigma_{33}^D \end{pmatrix}; \; \Sigma^F = diag \begin{pmatrix} \sigma_{44}^F \\ \sigma_{55}^F \\ \sigma_{66}^F \end{pmatrix} \tag{24}$$

It is important to note that the off-diagonal elements of the mean-reversion matrix, in equation (23), are set to zero in order to preserve an independence between the latent factors in the domestic and foreign economy. Specifically, using the pairwise approach for the analysis of more than two countries, say $n+1$ countries including the domestic economy, induces the domestic economy to have n sets of estimates, one for each pair of currencies; generating hence a consistency problem. Keeping domestic and foreign latent factors independent alleviates this issue and preserves the consistency of the model in a bilateral setting. However, in a multilateral setting, consistency can be achieved in two ways, either by using a joint pricing for the $n+1$ term structures of interest rates, or by conducting the estimation for each country on an individual basis.

The instantaneous risk-free rates for the domestic and foreign countries are affine functions of the state variables and are given below.

$$r_t^D = L_t^D + S_t^D \tag{25}$$

$$r_t^F = L_t^F + S_t^F \tag{26}$$

Additionally, let $y(t,T)$ be the column vector of dimension 2Nx1, composed of the concatenation of N-maturities of domestic and foreign yields. The representations of domestic and foreign zero-coupon yields with maturity T at time t are given by an affine function of the state variables, as shown below,

$$y(t,T) = \begin{bmatrix} y^D(t,T) \\ y^F(t,T) \end{bmatrix} = -\begin{pmatrix} \frac{A^D(t,T)}{T-t} \\ \frac{A^F(t,T)}{T-t} \end{pmatrix} - \begin{pmatrix} \frac{B^D(t,T)'}{T-t} & 0 \\ 0 & \frac{B^F(t,T)'}{T-t} \end{pmatrix} X_t^J \tag{27}$$

where $A^D(t,T)$, $A^F(t,T)$, $B^D(t,T)$ and $B^F(t,T)$ are the unique solutions to a system of Riccati equations which are a natural extension to the system in equation (12). The intercept terms are the no-arbitrage adjustment terms and the factor loadings capture the level, slope and curvature interpretations.

Suppose a diffusion process of the form $dx_t = \mu(x_t)dt + \sigma(x_t)dW_t$ with $\mu^{\mathbb{P}}(x_t)$ and $\mu^{\mathbb{Q}}(x_t)$ denoting the drift terms of the state diffusion process under the physical and risk neutral probability measures, respectively. The price of risk is defined as follows.

$$\Gamma_t(x_t) = (\sigma(x_t))^{-1} \left[\mu^{\mathbb{P}}(x_t) - \mu^{\mathbb{Q}}(x_t)\right] \tag{28}$$

The dynamics of the state vector X_t^J, under the physical probability measure $\mathbb{P}$, are consequently drawn and given by the following stochastic differential equation,

$$dX_t^J = \kappa^{J,\mathbb{P}} \left[\theta^{J,\mathbb{P}} - X_t^J\right] dt + \Sigma^J \, diag[\sqrt{X_t^J}] dW_t^{J,\mathbb{P}} \tag{29}$$

with $\kappa^{J,\mathbb{P}}$ being set to a diagonal matrix for simplicity[1] and $W_t^{J,\mathbb{P}}$ being a six dimensional Brownian motion. The square matrix $\kappa^{J,\mathbb{P}}$ and vectors $\theta^{J,\mathbb{P}}$ and X_t^J are all six-dimensional.

B. Deriving exchange rate depreciations

In order to derive the exchange rate differences, a formulation for the domestic and foreign pricing kernels is necessary. Denote by M^D and M^F the domestic and foreign stochastic discount factors with the following dynamics,

$$\frac{dM_t^D}{M_t^D} = -r_t^D dt - \Gamma_t^D (X_t^J)' dW_t^{\mathbb{P}} \tag{30}$$

$$\frac{dM_t^F}{M_t^F} = -r_t^F dt - \Gamma_t^F (X_t^J)' dW_t^{\mathbb{P}} \tag{31}$$

$$= -r_t^F dt - \left(\Gamma_t^D (X_t^J)' - \gamma^* \Sigma^J \sqrt{X_t^J}\right) dW_t^{\mathbb{P}} \tag{32}$$

with $\gamma^* = (0, 0, 0, 1, 1, 1)$ and $W_t^{\mathbb{P}}$ being a six dimensional Wiener process. It is interesting to note that the foreign stochastic discount factor has two representations given by equations (31) and (32). The latter is the one used in the extraction of the depreciation of exchange rates due to its ability to create correlations amongst the domestic and foreign economies (see Brennan and Xia (2006) and Sarno, Schneider, and Wagner (2012)). In a more general setting, with n currency pairs, the domestic risk factors act as global risk factors for the international economy.

[1]An alternative is to consider a lower-triangular $\kappa^{J,\mathbb{P}}$ matrix which implies that the foreign economy relies not only on foreign factors but also on domestic ones (see Jotikasthira, Le, and Lundblad (2015)). However, a general-to-specific method shows that all off-diagonal elements are insignificant; reinforcing the idea of independent factors.

Using equation (22), the dynamics of the exchange rate S_t are derived. Moreover, using Ito's lemma, the dynamics of the logarithm of the exchange rate, denoted by s_t are also retrieved. It is interesting to note that the dynamics of the exchange rate are no longer affine in the state variable.

$$\frac{dS_t}{S_t} = \left(r_t^D - r_t^F + \gamma^* \Sigma^J \sqrt{X_t^J} \Gamma_t^D(X_t^J) \right) dt + \gamma^* \Sigma^J \sqrt{X_t^J} dW_t^{\mathbb{P}} \tag{33}$$

$$ds_t = \left(r_t^D - r_t^F + \gamma^* \Sigma^J \sqrt{X_t^J} \Gamma_t^D(X_t^J) - \frac{1}{2} \gamma^* \Sigma^J X_t^J \Sigma^{J'} \gamma^{*'} \right) dt + \gamma^* \Sigma^J \sqrt{X_t^J} dW_t^{\mathbb{P}} \tag{34}$$

A clear parallelism is derived between the two equations above and equation (5), keeping in mind that $r_t^D - r_t^F$ is the short rate differential and $\gamma^* \Sigma^J \sqrt{X_t^J} dW_t^{\mathbb{P}}$ is the disturbance term.

C. Extracting currency risk premia

Having established the endogenous relationship of the variation in the logarithm of exchange rates implied by the model, the extraction of the risk premium is fairly straight-forward. Using equation (33), the drift is now composed of two components, the interest rate differentials and a second component, which englobes the risk premium, as shown below.

$$rp_t = -\gamma^* \Sigma^J \sqrt{X_t^J} \Gamma_t^D(X_t^J) \tag{35}$$

The risk premium is hence obtained by differencing the expectations of the exchange rate depreciation under the risk-neutral and physical probability measures. An equivalent representation can be derived using the dynamics of the logarithm of the exchange rate.

It is further possible to obtain a representation of the continuously compounded expected return of exchange rates by taking the expectation, under the physical measure, of equation (33).

$$\mathbb{E}^{\mathbb{P}} \left[S_t^{ret} | \mathcal{F}_t \right] = r_t^D - r_t^F + \gamma^* \Sigma^J \sqrt{X_t^J} \Gamma_t^D(X_t^J) \tag{36}$$

The expected return of exchange rates assumes rational expectations and sets the expectations of $\gamma^* \Sigma^J \sqrt{X_t^J} dW_t^{\mathbb{P}}$, under the data generating process measure, equal to zero.

IV. Empirical analysis

This section is devoted to the empirical estimation of the bilateral AFNS with stochastic volatility on domestic and foreign zero-coupon yields. In a first instance, the characteristics of the data set are studied, sequentially, the estimation method is described and finally, all empirical results are presented.

A. Data description

The data set consists of monthly nominal yields for the United Kingdom and the United States, spanning from September 1989 to October 2008 and includes a set of nine maturities for each country, namely 3, 6, 12, 18, 24, 30, 36, 42 and 48 months. The time period includes the abandonment of the European Exchange Rate Mechanism in September 2002 by the UK as well as the beginning of the recent financial crisis caused by the burst of the housing bubble in the US market. The data set's timespan is specifically selected to coincide with the timespan of the data set included in Sarno, Schneider, and Wagner (2012), given it is the most recent paper in this strand of the literature, thus facilitating comparison of results. However, the use of short and medium term maturities is perfectly warranted as most violations of the uncovered interest rate parity are reported to occur in the short run, whilst empirical evidence supports claims of the parity holding in the long run. It is important to note that the sample conveniently excludes the period of the zero lower bound, as its inclusion would induce estimation problems. More particularly, during the zero lower bound, term structure models need to account for non-negative nominal rates that can stay at zero for a considerable amount of time, without zero being reflecting or absorbing. Such models include shadow-rate models as well as affine models à la Monfort, Pegoraro, Renne, and Roussellet (2014).

The data set is kindly made available by Jonathan Wright and can be found on the following link -http://econ.jhu.edu/directory/jonathan-wright-.

Additionally, the monthly GBP/USD spot exchange rate is obtained through Datastream, and is denominated in US dollars; the same timespan applies, commencing in September 1989 and ending in October 2008.

Table 1 displays the descriptive statistics, namely the mean, standard deviation, skewness, kurtosis and first lag autocorrelation, of the level of interest rates for the US and the UK as well as the level of the exchange rate and logarithm of the exchange rate. The UK yields are characterized by a positive skew and excess kurtosis, especially at short and medium term maturities. All variables have a high first autocorrelation, close to unity, indicating highly persistent behaviors.

[Table 1]

Throughout the paper, differentials of variables are used. Panel A of Table 2 presents the descriptive statistics for the variables' differentials. Those are defined as the difference between domestic and foreign rates for yields at all maturities, and a first lag difference for exchange rates and the logarithm of exchange rates. Both exchange rate differentials display strong excess kurtosis. The results for the Fama regression in equation (4) are reported in Panel B of Table 2. The findings confirm the empirical results found in the majority of the literature, whereby the intercept of the regressions is statistically insignificant, while the slope coefficient rejects the null hypothesis of unity at all conventional significance levels. Additionally, the R squared coefficient displays a very weak goodness of fit. These results motivate the methodology of incorporating a time-varying risk premium.

[Table 2]

It is common practice to use three factors to fit the term structure of interest rates of a single country. Additionally, following convention, the level factor affects yields at all maturities, the slope factor influences short-term yields, whilst the curvature factor is of importance for medium-term maturities. The maturities used in this empirical section span from 3 to 48 months, hence justifying the use of three factors per economy. However, before proceeding to the estimation procedure, a preliminary study is conducted to best specify the model. A principal component analysis (PCA) is used to determine how many pricing factors are required to explain the cross-sectional variation of domestic and foreign yields. The loadings for the six first principal components for the entire set of maturities are reported on Table 3. The PCA results validate our use of 6 latent factors given the first six components explain 99.98% of the cross-sectional yield variation.

[Table 3]

B. Estimation procedure: Kalman filtering

The model, so far presented, naturally adopts a state space representation, with equations (37) and (38) below being the transition and measurement equations, respectively. The state-space representation is given below, in its discretized form, with $X_t^J = \left(X_t^D, X_t^F\right)'$ and $y(t,T) = \left(y(t,T)^D, y(t,T)^F\right)'$,

$$X_T^J = \left[I - exp(-\kappa^{\mathbb{P}}(T-t))\right]\theta^{\mathbb{P}} + exp(-\kappa^{\mathbb{P}}(T-t))X_t^J + \eta_t \tag{37}$$

$$y(t,T) = -\frac{A(t,T)}{T-t} - \frac{B(t,T)'}{T-t}X_t^J + \epsilon_t \tag{38}$$

where the measurement errors η_t and ϵ_t are assumed to be orthogonal and ϵ_t is i.i.d.

The bilateral AFNS model with spanned volatility is estimated using a quasi-maximum likelihood method, in the same spirit as in Fisher and Gilles (1996), Jacobs and Karoui (2009) and Christensen, Lopez, and Rudebusch (2010a). A quasi-maximum likelihood procedure is straight-forward to implement and requires solely the first and second conditional moments. The moments conditions are displayed below.

$$\mathbb{E}^{\mathbb{P}}\left[X_T^J|\mathcal{F}_t\right] = \left[I - exp(-\kappa^{\mathbb{P}}(T-t))\right]\theta^{\mathbb{P}} + exp(-\kappa^{\mathbb{P}}(T-t))X_t^J \tag{39}$$

$$\mathbb{V}^{\mathbb{P}}\left[X_T^J|\mathcal{F}_t\right] = \int_t^T exp(-\kappa^{\mathbb{P}}(T-s))\Sigma\sqrt{\mathbb{E}^{\mathbb{P}}\left[X_s^J|\mathcal{F}_t\right]}\sqrt{\mathbb{E}^{\mathbb{P}}\left[X_s^J|\mathcal{F}_t\right]}'\Sigma' exp(-\kappa^{\mathbb{P}'}(T-s))ds \tag{40}$$

The initial conditions for the Kalman filter are set to the unconditional mean and covariance matrix, given in equation (41) and (42).

$$\hat{X}_0^J = \theta^{\mathbb{P}} \tag{41}$$

$$\hat{\Sigma}_0 = \int_0^\infty exp(-\kappa^{\mathbb{P}}s)\Sigma\sqrt{\theta^{\mathbb{P}}}\sqrt{\theta^{\mathbb{P}}}'\Sigma' exp(-\kappa^{\mathbb{P}'}s)ds \tag{42}$$

The conditional and unconditional covariance matrix in equation (42) are estimated using the analytical solutions provided in Jacobs and Karoui (2009).

Finally, to estimate the logarithmic exchange depreciation implied by the model, a discretization of equation (34) is used,

$$\Delta s_{t+\omega} = \left[r_t^D - r_t^F + \gamma^* \Sigma^J \sqrt{X_t^J} \Gamma_t^D (X_t^J) - \frac{1}{2} \gamma^* \Sigma^J X_t^J \Sigma^{J\prime} \gamma^{*\prime} \right] \omega + \gamma^* \Sigma^J \sqrt{X_t^J} \Delta W_{t+\omega}^{\mathbb{P}} \quad (43)$$

where $\Delta W_{t+\omega}^{\mathbb{P}}$ is approximated by the following expression.

$$\Delta W_{t+\omega}^{\mathbb{P}} \approx \left[\Sigma^J \sqrt{X_t^J} \right]^{-1} \left[\Delta X_{t+\omega}^J - \left(\kappa^{\mathbb{P}} \left(\theta^{\mathbb{P}} - X_t^J \right) \right) \omega \right] \quad (44)$$

The above expression is derived by re-arranging a discretized version of the state dynamics.

C. Empirical findings

The estimates for the six factor bilateral AFNS model with stochastic volatility are provided in Table 4. These results are found using solely the US and UK nominal yields with maturities 3, 6, 12, 18, 24, 30, 36, 42 and 48 months. The specification for the mean reversion matrix $\kappa^{\mathbb{P}}$ is set to a diagonal matrix. The results indicate that the first and fourth factor do display near unit root behaviors. This result is clearer when the discretized states are considered. The estimates for the unconditional mean $\theta^{\mathbb{P}}$ and diffusion matrix Σ are also displayed. The two mean reversion parameters, under the risk neutral probability measure, λ^D and λ^F are comparable to the ones found in the literature.

[Table 4]

Table 5 elaborates on the fit of the six factor bilateral AFNS model. Both the mean and root mean squared error (RMSE) are provided. It is clearly visible that the short maturities are extremely hard to fit. The shorter the maturity of the first yield in the sample, the higher the ability of extracting the appropriate cross-section of the yields. The fact that the shortest maturity used is 3 months, could explain the difficulty in fitting the short yields appropriately. Using swap and libor rates to bootstrap short rates has the potential to improve significantly the fit of short term yields, however, this exercise is left for future research. On the other hand, the fit of yields is successful especially in medium term maturities. Attention is drawn to Appendix B which contains a robustness check using a multilateral Gaussian AFNS model. The results for the US, using the bilateral AFNS model with stochastic volatility, are comparable to those found in the multilateral Gaussian AFNS, while the fit for the UK is visibly poorer. This can be explained by the particularity of the UK term structure of interest rates which has succumbed an inversion of the yield curve.

[Table 5]

Table 6 allows to compare the findings of the model's implied logarithmic exchange rate depreciations with the actual variation in log exchange rates. The means of the two variables are significantly similar, while the standard deviation of the model implied depreciation is lower, which is a major improvement to the findings under the Gaussian AFNS model. The mean and standard deviation of the implied risk premium and expected exchange rate return are also reported. The risk premium is comparable to the ones found in similar studies.

[Table 6]

Moreover, Table 7 indicates that the correlation found between the actual and estimated exchange rate depreciations is equal to 16.03%. This finding might be misinterpreted as a poor fit, however it is important to note that comparable bilateral studies have found correlations well below 10% and on some occasions correlations slightly below 0%, thus indicating an improvement in the fit (see Sarno, Schneider, and Wagner (2012)). Additionally, the implied risk premium does validate the two Fama conditions, hence providing empirical support to Fama (1984)'s claim and indicating that the model does offer a correction to the uncovered interest rate parity by incorporating a time-dependent risk premium.

[Table 7]

In addition, Figure 1 displays the actual and estimated exchange rate depreciations' time series. It is noticeable that the mean is successfully captured, and the variance is closely matched. It is also clear that interest rate differentials are not the only drivers of exchange rate changes. The consideration of unspanned volatility and a macro-finance approach to the model are two interesting extensions of the current study which are left for future investigation.

[Figure 1]

Figure 2 plots the estimated expected exchange rate return and exchange rate risk premia. In recent literature, claims increasingly stipulate that currency risk premia are countercyclical. This graph also supports theories of countercyclicality, the risk premium thus tracking expected returns. Assuming the foreign country has a lower interest rate than the domestic country, the risk premium tends to be positive given an appreciation of the domestic currency is denoted by a decrease of the exchange rate. Vice-versa, a foreign country with historically higher interest rates than the domestic country will mostly display a negative currency risk premium in order to reflect the appreciation of the foreign currency which is coupled with an increase of the exchange rate. The more the domestic country is considered risky vis-a-vis the foreign country, the larger the magnitude of the risk premium. Hence, the higher the liquidity constraints and economic uncertainty, the more likely the risk premium is to increase, thus reinforcing arguments of flight-to-liquidity and flight-to-quality. Moreover, the expected return on the pound fluctuates between -1.07% and 8.75%; whilst its mean and standard deviation are equal to 3.06% and 2.08%, respectively. The estimation provides similar results with Graveline (2006)'s findings using options prices. In particular, Graveline (2006) did a comparative study between two models, with and without options. He concluded that models that do not use option prices usually display a lot of variability in currency expected returns. The findings in this paper show that option prices are not necessary to retrieve expected return on currencies that have a low variance.

[Figure 2]

Figure 3 provides a graphical representation of the contribution of each of the six risk factors to the risk premium. Interestingly, this figure corroborates Graveline (2006)'s results by displaying the same low variances in risk premium contributions for risk factors that have a greater impact on exchange rates. Hence, the domestic and foreign curvature factors appear to be the key drivers of both exchange rate depreciations and currency risk premia, whilst they also appear to be the most persistent factors.

[Figures 3 and 4]

Finally, Figure 4 plots the contribution of a carry trade risk factor to the currency risk premium. The carry trade factor, in this case, is represented by the short interest rate differential. Using equation (25) and (26), the carry trade risk factor is easily derived by summing the fourth and fifth risk factors (ie. level and slope of the foreign economy, which in this case is the UK) and subtracting the first and second risk factors (ie. level and slope of the domestic economy, in this case the US). The contribution of the carry trade factor to currency risk premia, on average, is equal to -1.60%, whilst the integrity of the currency risk premium is on average equal to -5.66%. It is clear that the carry trade factor is a driver of currency risk premia, as demonstrated by Lustig, Roussanov, and Verdelhan (2010). However, the carry trade factor is found not to contain all the information of currency risk premia in its integrity, hence rendering the two curvature factors particularly important. Moreover, the carry trade risk factor's contribution to currency risk premia mainly contains exchange rate risk in short maturities, whilst it is contaminated by an additional component for interest rate risk in long maturities. A recent study by Lustig, Stathopoulos, and Verdelhan (2013) indicates that carry trade risk premia are indicative of temporary shocks and hence their term structure tends to be downward sloping. This finding is confirmed by the persistence of curvature factors which do not feature within carry trade factors. On the other hand, currency risk premia at long horizons seem to be driven by the permanent component of stochastic discount factors.

V. Conclusion

In conclusion, in this paper a bilateral AFNS model with stochastic volatility for the joint pricing of the term structure of interest rates for both the domestic and foreign countries that is further able to derive exchange rate variations is developed. The model proposed benefits from the Nelson-Siegel factor loadings yielding a robust and tractable estimation procedure. The no-arbitrage restrictions enhance the theoretical grounds whilst simultaneously allowing the extraction of currency risk premia.

This paper compares the effect of the different assumptions set on the diffusion of the process (ie. Gaussian or with stochastic volatility) on the properties adopted by the estimates of the yields, exchange rate variations and currency risk premia. To summarize, the use of a stochastic volatility version rather than a Gaussian take of the AFNS model comes with the detriment of having an inferior fit for the yields. However, the very inclusion of stochastic volatility endows the model with the capacity to capture to some extent the volatility of exchange rate depreciations and successfully derive a risk premium that respects the two Fama conditions. The model's implied risk premium provides, thus, an adaptation of the uncovered interest rate parity that alleviates the recorded puzzle in the literature whilst solely assuming a departure from risk neutrality. On the other hand, a Gaussian AFNS model allows a better fit for the yields, whilst the variance of exchange rate fluctuations is not fully captured. It is interesting to note that the Gaussian AFNS is easily extended to a multi-currency model which not only benefits from an elegant estimation procedure, but also takes advantage of the fact that currency portfolios tend to be more predictable than individual exchange rates.

Finally, the extension of the stochastic volatility AFNS model to a multi-currency framework is left for future research.

Appendix A: Bilateral AFNS model with stochastic volatility

Table 1: Descriptive statistics of the level of interest rates and exchange rates

Panel A: Domestic Country -United States-					
Maturity	Mean	Standard Deviation	Skewness	Kurtosis	Autocorrelation
3 months	0.0405	0.0178	-0.1129	2.4204	0.9716
6 months	0.0434	0.0184	-0.1388	2.3578	0.9761
12 months	0.0449	0.0183	-0.1729	2.3694	0.9735
18 months	0.0464	0.0179	-0.1604	2.4065	0.9710
24 months	0.0476	0.0174	-0.1286	2.4320	0.9689
30 months	0.0488	0.0170	-0.0874	2.4381	0.9675
36 months	0.0498	0.0165	-0.0415	2.4270	0.9667
42 months	0.0508	0.0162	0.0061	2.4036	0.9664
48 months	0.0517	0.0158	0.0533	2.3731	0.9666
Panel B: Foreign Country -United Kingdom-					
Maturity	Mean	Standard Deviation	Skewness	Kurtosis	Autocorrelation
3 months	0.0666	0.0289	1.7187	5.1836	0.9790
6 months	0.0632	0.0270	1.7349	5.3003	0.9759
12 months	0.0624	0.0244	1.5979	5.0414	0.9711
18 months	0.0626	0.0233	1.4388	4.5814	0.9708
24 months	0.0630	0.0226	1.2987	4.1433	0.9717
30 months	0.0634	0.0222	1.1902	3.7899	0.9730
36 months	0.0637	0.0219	1.1094	3.5196	0.9745
42 months	0.0640	0.0217	1.0496	3.3161	0.9758
48 months	0.0642	0.0216	1.0046	3.1617	0.9771
Panel C: Exchange rate and logarithm of the exchange rate					
Maturity	Mean	Standard Deviation	Skewness	Kurtosis	Autocorrelation
S_t	1.6778	0.1685	0.5156	2.2601	0.9602
s_t	0.5126	0.0987	0.3673	2.1500	0.9608

NOTE: The descriptive statistics for the level of domestic and foreign yields at all the maturity set and the exchange rate and logarithmic exchange rate are given. The data comprises of monthly nominal zero coupon bond yields for the US and the UK and the GBP/USD exchange rate denominated in US dollars, from September 1989 to October 2008.

Table 2: Stylized facts of interest rates and exchange rates differentials

Panel A: Descriptive Statistics on variable differentials

Variable	Mean	Standard Deviation	Skewness	Kurtosis	Autocorrelation
$y_3^D - y_3^F$	-0.0260	0.0214	-1.0106	3.0320	0.9747
$y_6^D - y_6^F$	-0.0198	0.0194	-0.9846	3.1179	0.9796
$y_{12}^D - y_{12}^F$	-0.0175	0.0165	-0.8513	2.9694	0.9726
$y_{18}^D - y_{18}^F$	-0.0163	0.0148	-0.8185	3.0088	0.9695
$y_{24}^D - y_{24}^F$	-0.0154	0.0135	-0.7860	3.0647	0.9668
$y_{30}^D - y_{30}^F$	-0.0146	0.0126	-0.7493	3.1037	0.9644
$y_{36}^D - y_{36}^F$	-0.0139	0.0118	-0.7096	3.1137	0.9623
$y_{42}^D - y_{42}^F$	-0.0132	0.0113	-0.6698	3.0963	0.9606
$y_{48}^D - y_{48}^F$	-0.0134	0.0113	-0.5359	2.9016	0.9627
$S_t - S_{t-1}$	0.0009	0.0466	-1.3608	8.5506	0.1267
$s_t - s_{t-1}$	0.0005	0.0270	-1.1966	7.6103	0.1219

Panel B: Fama Regression

Variable		α	β	$t[\beta = 1]$	R^2
		-0.0003	-0.0313	-12.2090	0.0006
		(0.0028)	(0.0845)		

NOTE: The descriptive statistics for the differentials of domestic and foreign yields at all the maturity set and the exchange rate and logarithmic exchange rate are given in Panel A. The results of the Fama regression are provided in Panel B. The numbers in parenthesis are the standard errors of the estimates. The data comprises of monthly nominal zero coupon bond yields for the US and the UK and the GBP/USD exchange rate denominated in US dollars, from September 1989 to October 2008.

Table 3: First three principal components in nominal yields

Maturity	First PC	Second PC	Third PC	Fourth PC	Fifth PC	Sixth PC
y_3^D	0.1801	0.3175	0.2843	0.4628	0.0661	0.6147
y_6^D	0.1862	0.3433	0.2584	0.3217	0.1315	-0.2112
y_{12}^D	0.1895	0.3340	0.1353	0.1071	0.0349	-0.3551
y_{18}^D	0.1895	0.3101	0.0381	-0.0449	-0.0320	-0.2756
y_{24}^D	0.1880	0.2817	-0.0379	-0.1512	-0.0599	-0.1437
y_{30}^D	0.1858	0.2527	-0.0979	-0.2266	-0.0584	-0.0110
y_{36}^D	0.1833	0.2249	-0.1458	-0.2810	-0.0379	0.1066
y_{42}^D	0.1806	0.1988	-0.1844	-0.3212	-0.0059	0.2053
y_{48}^D	0.1778	0.1747	-0.2159	-0.3511	0.0323	0.2860
y_3^F	0.3251	-0.3049	0.4696	-0.3411	0.5749	-0.1615
y_6^F	0.3106	-0.2494	0.3475	-0.1346	-0.1926	0.3827
y_{12}^F	0.2863	-0.1885	0.1653	0.0071	-0.4419	-0.0901
y_{18}^F	0.2760	-0.1594	0.0107	0.0729	-0.3792	-0.1258
y_{24}^F	0.2679	-0.1442	-0.1083	0.1183	-0.2355	-0.1100
y_{30}^F	0.2617	-0.1370	-0.1977	0.1507	-0.0735	-0.0743
y_{36}^F	0.2569	-0.1342	-0.2656	0.1750	0.0854	-0.0297
y_{42}^F	0.2533	-0.1339	-0.3178	0.1943	0.2339	0.0175
y_{48}^F	0.2504	-0.1350	-0.3588	0.2104	0.3702	0.0637
% explained	87.83	97.46	99.61	99.83	99.94	99.98

NOTE: The loadings of the yields of the set of maturities on the first six principal components are given. The percentage of all bond yields' cross-sectional variation accounted for by each component is displayed on the final row. The data comprises of monthly zero coupon bonds from September 1989 to October 2008 for the United States and the United Kingdom.

Table 4: 6 factor BAFNS estimates for domestic and foreign rates

$\kappa^{\mathbb{P}}_{i,i}$	θ^{P}	$\Sigma_{i,i}$
0.1000	0.0099	0.0163
(0.000032)	(0.000034)	(0.000120)
0.1996	0.0243	0.0583
(0.000032)	(0.000058)	(0.001229)
0.4997	0.0328	0.0325
(0.000032)	(0.000077)	(0.000216)
0.0991	0.0100	0.0319
(0.000048)	(0.000068)	(0.008248)
0.1985	0.0396	0.0841
(0.000077)	(0.000901)	(0.030020)
0.4995	0.0264	0.0442
(0.000041)	(0.000625)	(0.024991)

NOTE: The estimated parameters of the $\kappa^{\mathbb{P}}$ matrix, $\theta^{\mathbb{P}}$ vector, and diagonal diffusion matrix $\Sigma_{i,i}$ are given for the six-factor bilateral AFNS model for domestic and foreign yields. The estimated value of λ^{D} is 0.4974 with standard deviation of 0.000045 and λ^{F} is 0.4965 with standard deviation of 0.000156. The numbers in parentheses are the standard deviations of the estimated parameters. The log likelihood is equal to 10290.1208.

Table 5: Measures of fit for the bilateral AFNS model

Maturity in months	Mean(in bp)	RMSE(in bp)
y_3^D	17.1459	36.9274
y_6^D	-1.9856	16.1301
y_{12}^D	-0.7299	5.5918
y_{18}^D	-0.2130	1.6230
y_{24}^D	0.0060	0.0382
y_{30}^D	-0.0479	1.0435
y_{36}^D	-0.4106	2.3444
y_{42}^D	-1.0902	3.9866
y_{48}^D	-2.0643	5.8888
y_3^F	-14.7128	47.0974
y_6^F	9.6541	18.0715
y_{12}^F	6.5452	8.6486
y_{18}^F	0.6465	4.6115
y_{24}^F	-1.1010	1.3787
y_{30}^F	0.9950	3.3367
y_{36}^F	6.0909	6.8855
y_{42}^F	13.3451	9.7477
y_{48}^F	22.0650	16.1391

NOTE: The mean and RMSE of fitted errors of the six-factor bilateral AFNS model with stochastic volatility for domestic and foreign yields are given. All values are measured in basis points. The nominal yields span from September 1989 to October 2008.

Table 6: Model implied findings

Variable	Δs_{t+1}	$\Delta \hat{s_{t+1}}$	$\hat{rp}_t$	$\mathbb{E}^{\mathbb{P}}\left[\hat{S_t^{ret}} \vert \mathcal{F}_t\right]$
Mean	0.0005	0.0006	-0.0566	0.0306
Standard deviation	0.0270	0.0172	0.0224	0.0203

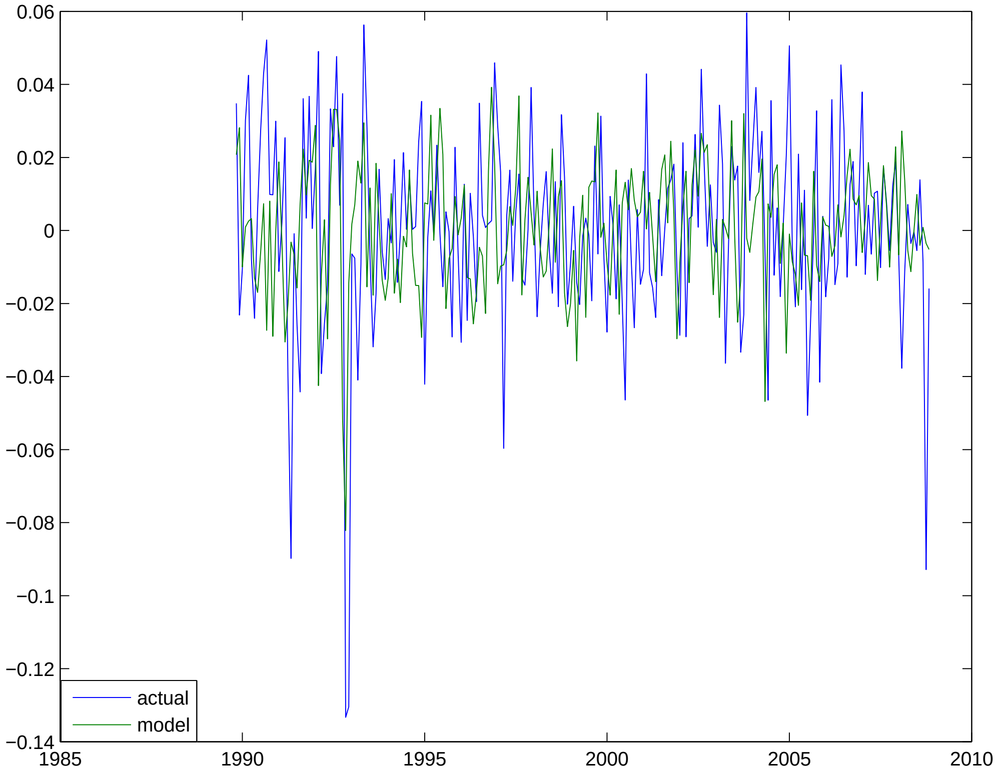
NOTE: The mean and standard deviation of the implied exchange rate depreciation, risk premium and exchange rate expected return are provided. The actual depreciation exchange rate mean and standard deviation are also included to facilitate the comparison with the estimates. The exchange rates span from September 1989 to October 2008.

Table 7: Analysis of the model implied exchange rate depreciation and risk premium

Panel A: model implied exchange rate depreciation	
$corr(\Delta s_{t+1}, \Delta \hat{s_{t+1}})$	0.1603
Panel B: Fama conditions	
$VR = \frac{r\hat{p}_t}{\Delta \hat{s_{t+1}}}$	1.7017
$corr(\Delta \hat{s_{t+1}}, r\hat{p}_t)$	-0.0718

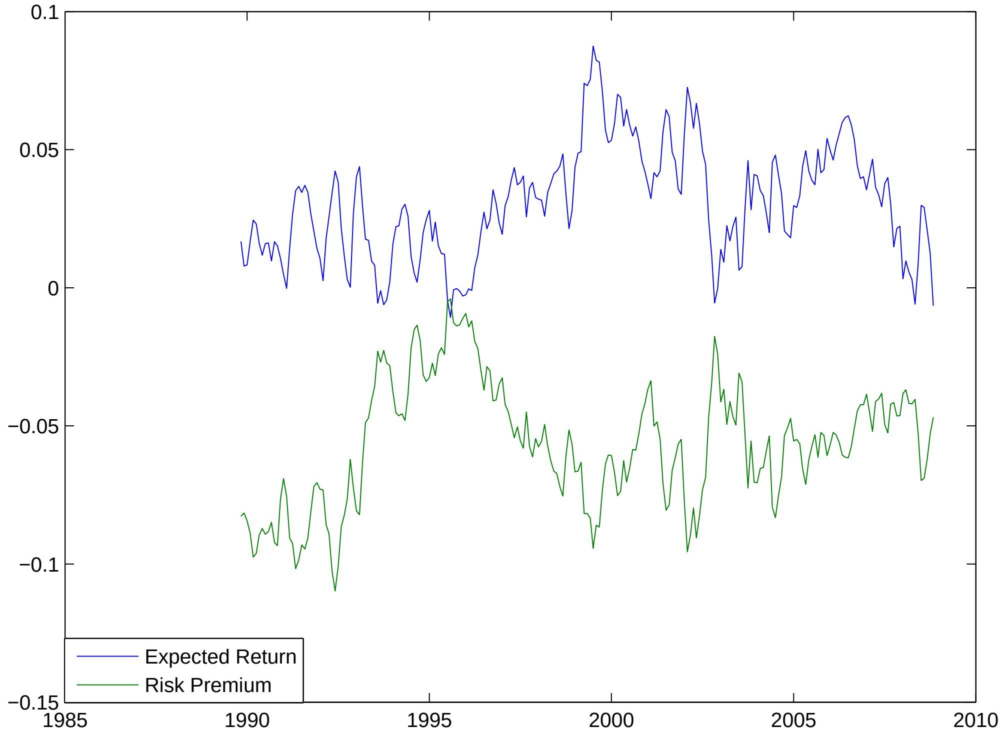
NOTE: Panel A displays the correlation between the actual and model implied exchange rate depreciations. In panel B, the variance ratio of the implied risk premium and actual exchange rate depreciations are provided. The correlation of the implied risk premium and actual exchange rate depreciations are also displayed. If the variance ratio figure is above 1 and the correlation is below 0 then the Fama conditions are verified. The exchange rates span from September 1989 to October 2008.

Figure 1: Actual and model implied log exchange rate depreciations



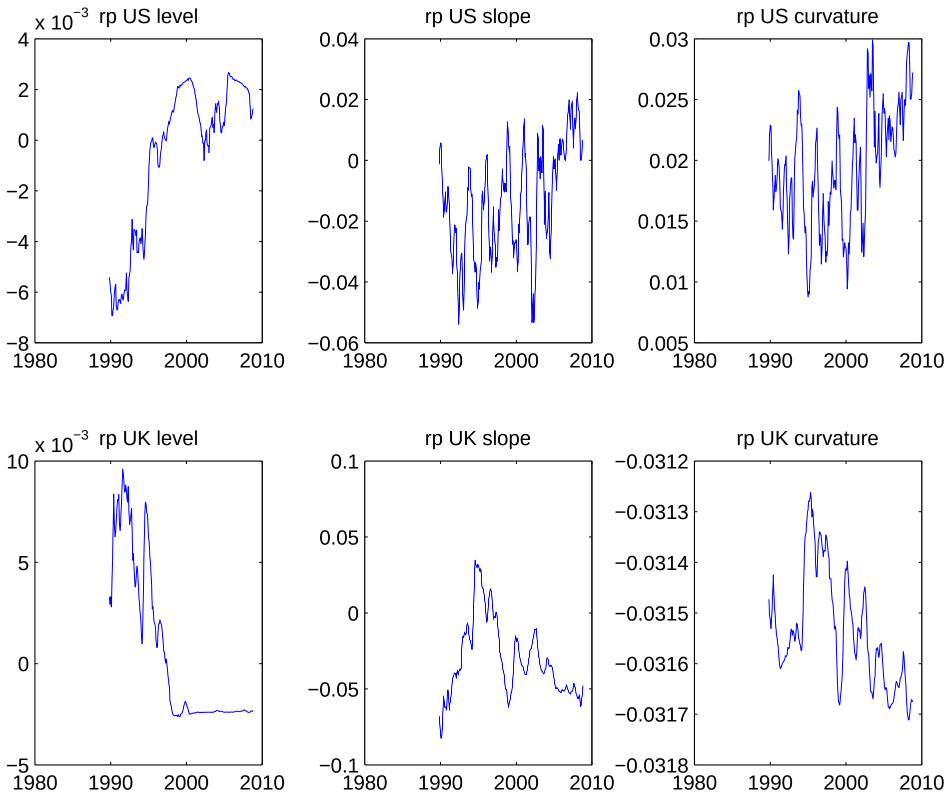
NOTE: Comparison of the actual and model implied log GBP/USD exchange depreciations across time. The exchange rates span from September 1989 to October 2008.

Figure 2: Expected exchange rate return and exchange rate risk premium



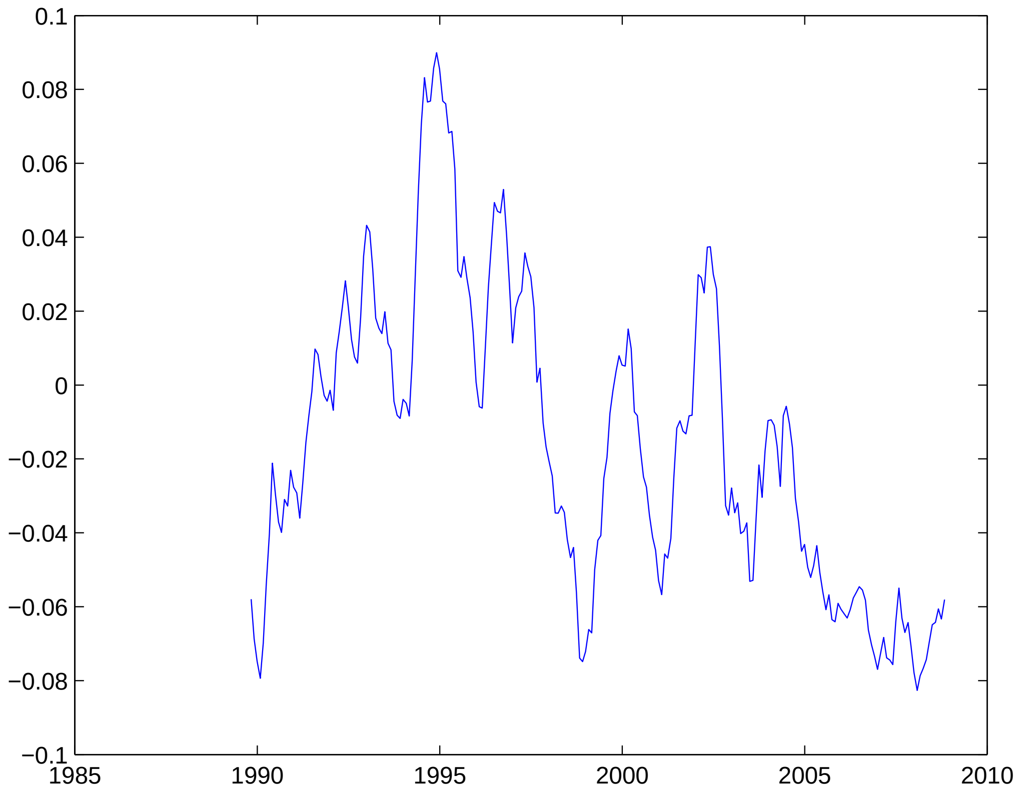
NOTE: Comparison of the expected exchange rate return and exchange rate risk premium across time, with exchange rates spanning from September 1989 to October 2008.

Figure 3: Contribution of risk factors to risk premium



NOTE: Comparison of the contribution of each risk factor to the risk premium. The six risk factors considered are namely the domestic and foreign level, slope and curvature factors.

Figure 4: Contribution of the carry trade factor to risk premium



NOTE: Contribution of a carry trade risk factor to the risk premium. The carry trade factor is computed by summing the foreign-UK level and slope factors and deducting the domestic-US level and slope factors.

Appendix B: Multilateral Gaussian AFNS model

This appendix segment is dedicated to conducting a robustness check with a different specification for the model. The empirical exercise, for this section, consists of an analysis of the Gaussian AFNS model extended to a multi-currency setting. Specifically, the United States is preserved as the domestic country and six more countries, including the United Kingdom, are treated as foreign countries. The model investigated includes twenty one latent factors; three factors for each country in the sample.

The data set consists of monthly nominal yields for the United States, the United Kingdom, Australia, Canada, Switzerland, Japan and Sweden spanning from January 1995 to May 2009 and includes a set of six maturities for each country, namely 3, 6, 12, 24, 36 and 48 months. The yields are available in Jonathan Wright's homepage.

Moreover, the monthly GBP/USD, AUD/USD, CAD/USD, CHF/USD, JPY/USD and SEK/USD spot exchange rates are obtained through Datastream, using a denomination in US dollars. The same timespan applies, commencing in January 1995 and ending in May 2009. The data set is comprised of a balanced panel and is truncated vis a vis to the empirical analysis' data set due to unavailability of data.

It is important to note that the model is Gaussian, which allows the uncontested use of the Kalman filter to obtain the maximum likelihood estimates.

Table 8 reports the fit of the yields for all seven countries across the entire set of maturities. The mean and Root Mean Squared Error (RMSE) indicate that with the exception of the three month yield for the US and the UK, all remaining yields are strikingly well captured.

[Table 8]

Figures 5 to 10 display the comparison between the actual and the model implied logarithmic exchange rate depreciations for all the six pairs of currencies. The mean of the exchange rate depreciations seems to be appropriately captured, however their variance is clearly underestimated. The correlation between the two time series above mentioned tend to be significantly lower than in the setting of the bilateral AFNS model.

As a final note, the fit of the yields is superior under the Gaussian multilateral AFNS rather than the bilateral AFNS with stochastic volatility. However, there seems to be an obvious trade-off between fitting yields and capturing the exchange rate depreciation properties. As Sarno, Schneider, and Wagner (2012) suggest, selecting between two extensions of a given model, in this case between the bilateral AFNS with stochastic volatility and the multilateral Gaussian AFNS model, will depend entirely on the purpose of the exercise, hence by whether the objective of the analysis is to fit yields or exchange rates.

[Figures 5 to 10]

Table 8: Measures of fit for the multilateral AFNS model

Maturity in months	Mean(in bp)	RMSE(in bp)
Panel A: Fit for domestic yields - US		
y_3^D	23.2560	35.2626
y_6^D	0.5516	7.3573
y_{12}^D	-0.0003	0.0086
y_{24}^D	-0.3060	1.1311
y_{36}^D	0.0000	0.0000
y_{48}^D	-0.9072	2.1180
Panel B: Fit for foreign yields - UK		
y_3^F	-30.9604	46.0448
y_6^F	-3.2131	8.6486
y_{12}^F	0.1757	0.6775
y_{24}^F	0.0587	0.3522
y_{36}^F	-0.0001	0.0003
y_{48}^F	0.3937	0.9682
Panel C: Fit for foreign yields - Australia		
y_3^F	-0.1564	5.3435
y_6^F	0.0000	0.0000
y_{12}^F	-0.0984	2.2626
y_{24}^F	0.0000	0.0000
y_{36}^F	0.0000	0.0000
y_{48}^F	-0.5561	2.2206
Panel D: Fit for foreign yields - Canada		
y_3^F	-1.2376	7.6264
y_6^F	0.0000	0.0000
y_{12}^F	0.0033	3.1487
y_{24}^F	-0.0643	0.1456
y_{36}^F	0.5218	0.8229
y_{48}^F	-1.2093	2.1356

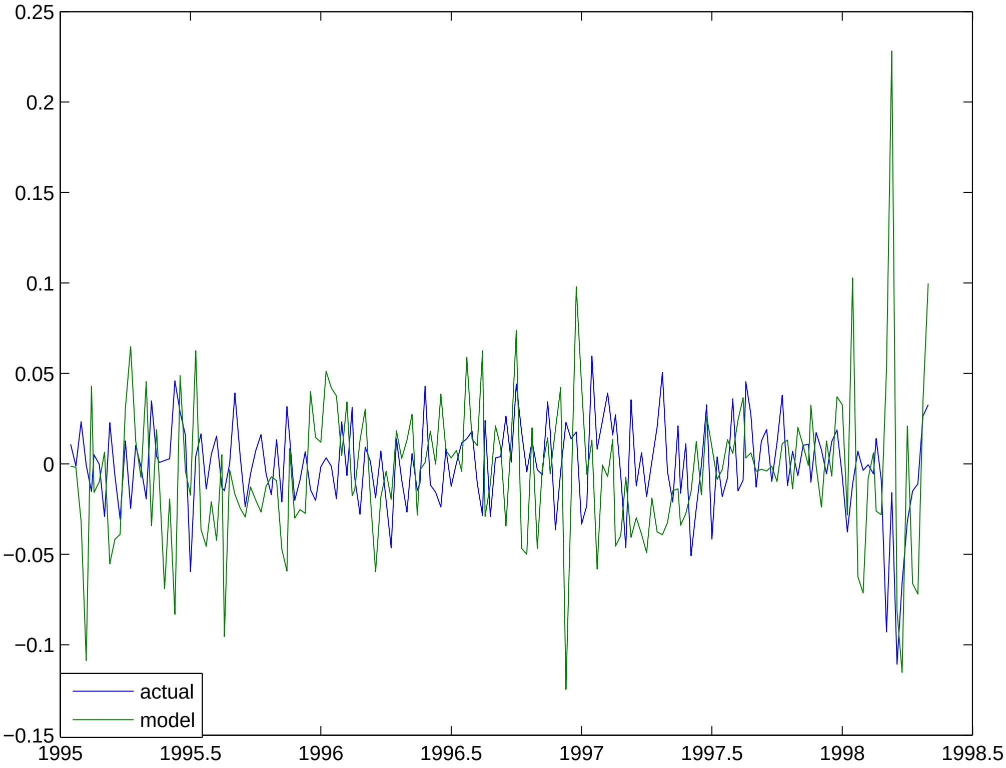
NOTE: The mean and RMSE of fitted errors of the multilateral Gaussian AFNS model for domestic and foreign yields are given. Panel A displays the fit for the US (domestic) yields, panel B for the UK (foreign), panel C for Australia (foreign), panel D for Canada (foreign), panel E for Switzerland (foreign), panel F for Japan (foreign) and panel G for Sweden (foreign). All values are measured in basis points. The nominal yields span from from September 1989 to October 2008.

Table 8 Continued: Measures of fit for the multilateral AFNS model

Maturity in months	Mean(in bp)	RMSE(in bp)
Panel E: Fit for foreign yields - Switzerland		
y_3^F	6.6244	13.1451
y_6^F	0.0531	0.6131
y_{12}^F	-2.8266	5.4623
y_{24}^F	-0.7319	2.5425
y_{36}^F	0.1603	0.2882
y_{48}^F	-0.8433	1.9154
Panel F: Fit for foreign yields - Japan		
y_3^F	0.2589	0.5446
y_6^F	0.0000	0.0000
y_{12}^F	0.0000	0.0001
y_{24}^F	0.4940	0.8120
y_{36}^F	0.0000	0.0000
y_{48}^F	-2.1471	3.0299
Panel G: Fit for foreign yields - Sweden		
y_3^F	-0.9590	5.9960
y_6^F	0.0003	0.0006
y_{12}^F	-0.4138	2.1575
y_{24}^F	-0.0026	0.0082
y_{36}^F	0.2150	0.4063
y_{48}^F	-0.9196	1.8548

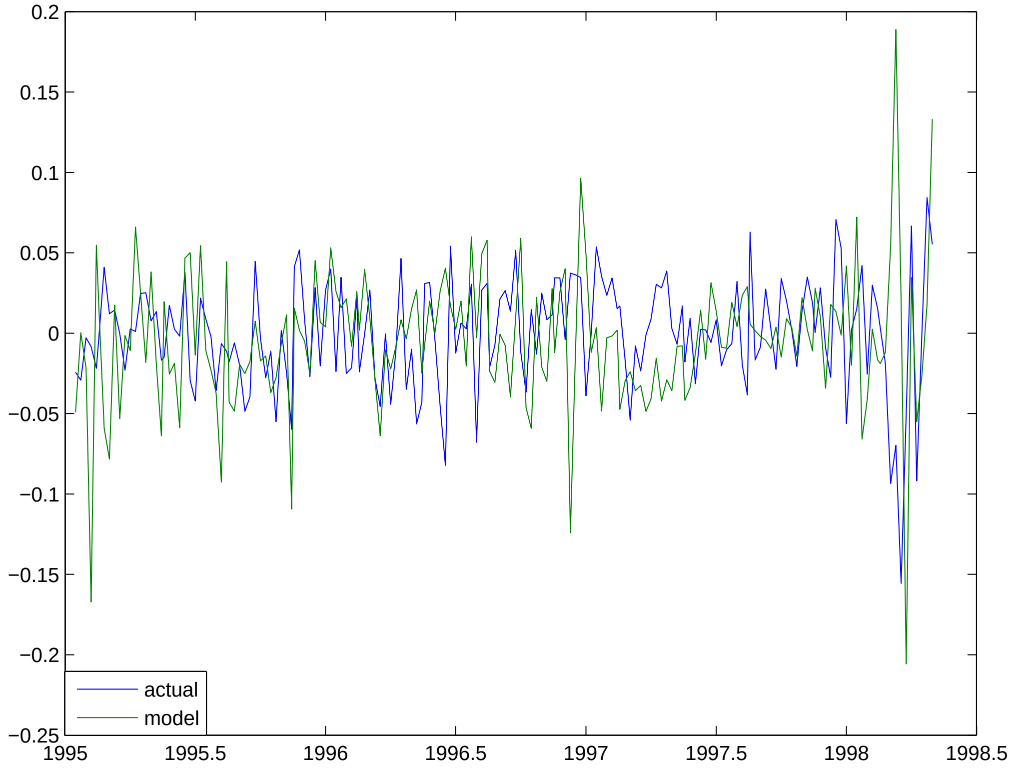
NOTE: The mean and RMSE of fitted errors of the multilateral Gaussian AFNS model for domestic and foreign yields are given. Panel A displays the fit for the US (domestic) yields, panel B for the UK (foreign), panel C for Australia (foreign), panel D for Canada (foreign), panel E for Switzerland (foreign), panel F for Japan (foreign) and panel G for Sweden (foreign). All values are measured in basis points. The nominal yields span from from September 1989 to October 2008.

Figure 5: Actual and model implied log exchange rate depreciations for the GBP/USD



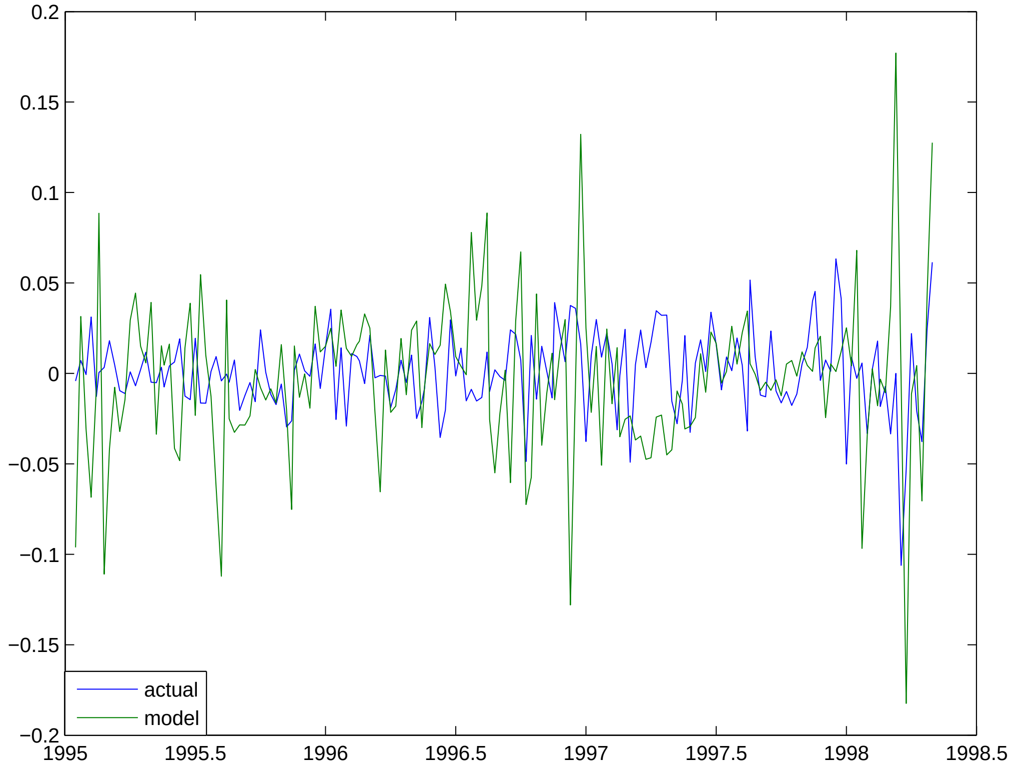
NOTE: Comparison of the actual and model implied log GBP/USD exchange depreciations across time.

Figure 6: Actual and model implied log exchange rate depreciations for the AUD/USD



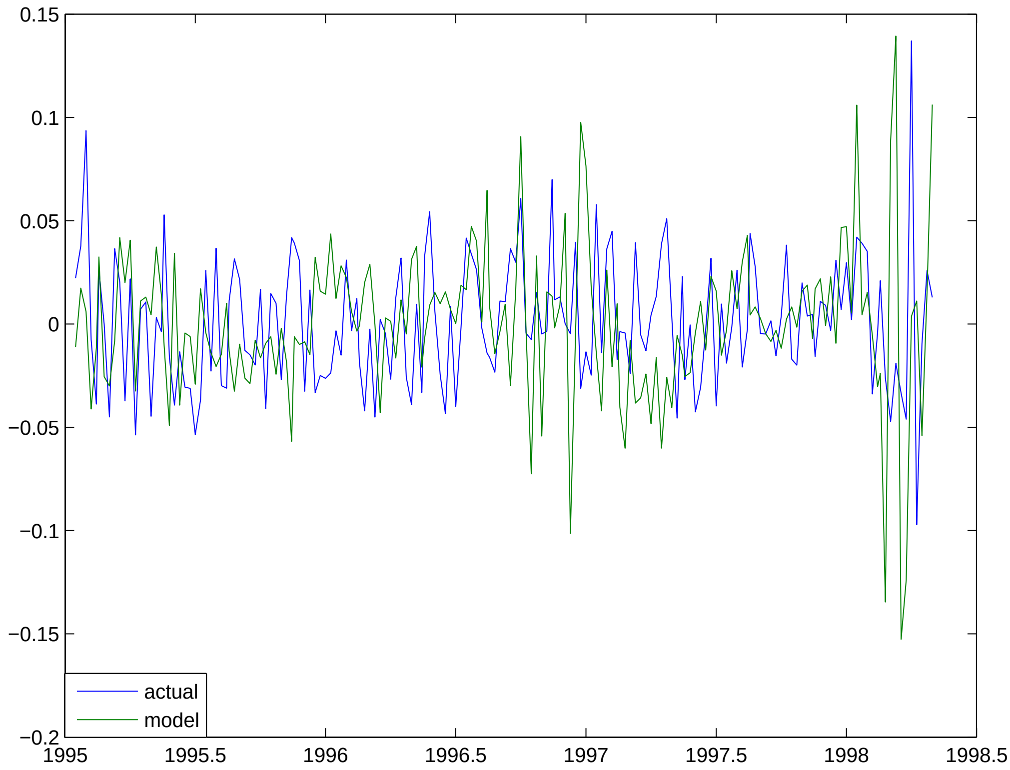
NOTE: Comparison of the actual and model implied log AUD/USD exchange depreciations across time.

Figure 7: Actual and model implied log exchange rate depreciations for the CAD/USD



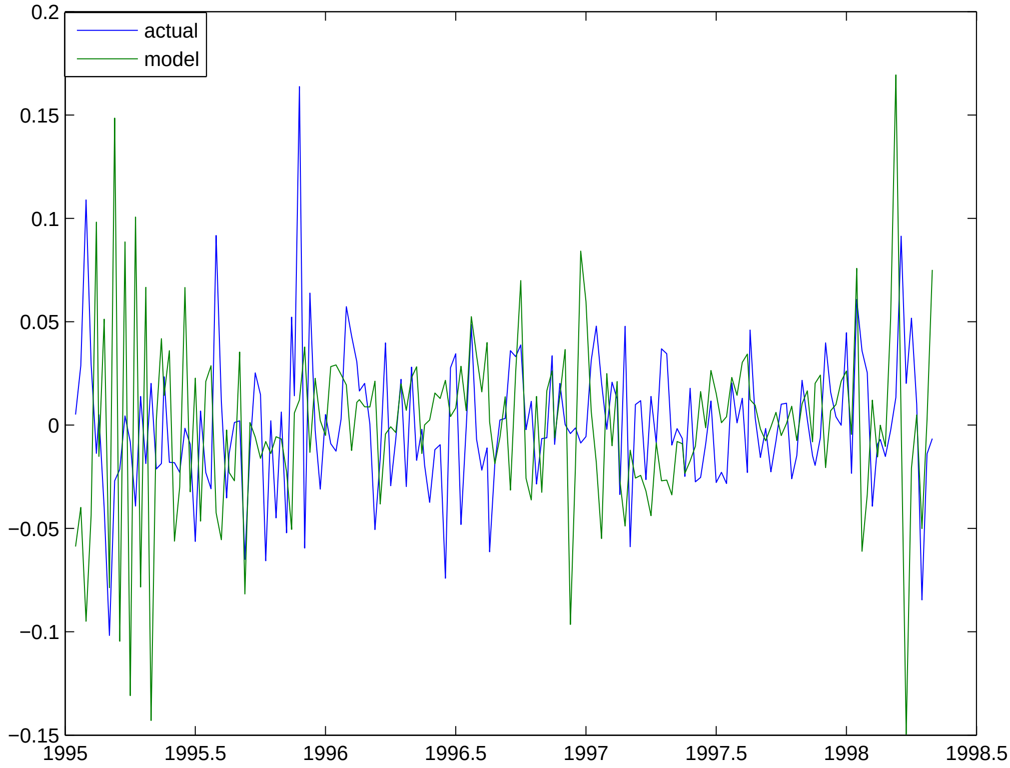
NOTE: Comparison of the actual and model implied log CAD/USD exchange depreciations across time.

Figure 8: Actual and model implied log exchange rate depreciations for the CHF/USD



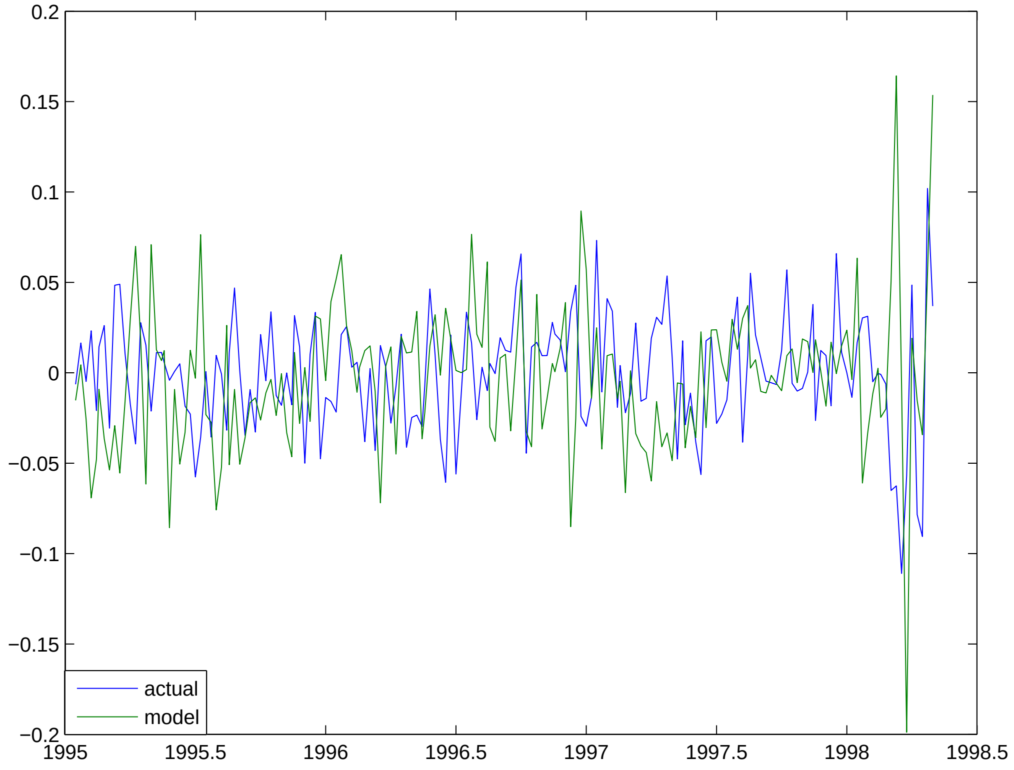
NOTE: Comparison of the actual and model implied log CHF/USD exchange depreciations across time.

Figure 9: Actual and model implied log exchange rate depreciations for the JPY/USD



NOTE: Comparison of the actual and model implied log JPY/USD exchange depreciations across time.

Figure 10: Actual and model implied log exchange rate depreciations for the SEK/USD



NOTE: Comparison of the actual and model implied log SEK/USD exchange depreciations across time.

References

Ahn, D.-H. (2004): "Common Factors and Local Factors: Implications for Term Structures and Exchange Rates," *Journal of Financial and Quantitative Analysis*, 39(01), 69–102.

Anderson, B., P. J. Hammond, and C. A. Ramezani (2010): "Affine Models of the Joint Dynamics of Exchange Rates and Interest Rates," *Journal of Financial and Quantitative Analysis*, 45(05), 1341–1365.

Ang, A., and M. Piazzesi (2003): "A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables," *Journal of Monetary Economics*, 50(4), 745–787.

Backus, D. K., S. Foresi, and C. I. Telmer (2001): "Affine Term Structure Models and the Forward Premium Anomaly," *The Journal of Finance*, 56(1), 279–304.

Bekaert, G. (1996): "The Time Variation of Risk and Return in Foreign Exchange Markets: A General Equilibrium Perspective," *Review of Financial Studies*, 9(2), 427–70.

Brandt, M. W., and P. Santa-Clara (2001): "Simulated Likelihood Estimation of Diffusions with an Application to Exchange Rate Dynamics in Incomplete Markets," NBER Technical Working Papers 0274, National Bureau of Economic Research, Inc.

Brennan, M. J., A. W. Wang, and Y. Xia (2004): "Estimation and Test of a Simple Model of Intertemporal Capital Asset Pricing," *The Journal of Finance*, 59(4), 1743–1776.

Brennan, M. J., and Y. Xia (2006): "International Capital Markets and Foreign Exchange Risk," *The Review of Financial Studies*, 19(3), 753–795.

Cheridito, P., D. Filipovic, and R. L. Kimmel (2007): "Market price of risk specifications for affine models: Theory and evidence," *Journal of Financial Economics*, 83, 123–170.

Chernov, M., and P. Mueller (2012): "The term structure of inflation expectations," *Journal of Financial Economics*, 106(2), 367–394.

Christensen, J. H., F. X. Diebold, and G. D. Rudebusch (2011): "The affine arbitrage-free class of Nelson-Siegel term structure models," *Journal of Econometrics*, 164(1), 4–20.

Christensen, J. H. E., J. A. Lopez, and G. D. Rudebusch (2010a): "Can Spanned Term Structure Factors Drive Stochastic Volatility?," unpublished work, Federal Reserve Bank of San Francisco.

——— (2010b): "Inflation Expectations and Risk Premiums in an Arbitrage-Free Model of Nominal and Real Bond Yields," *Journal of Money, Credit and Banking*, 42(s1), 143–178.

Dai, Q., and K. J. Singleton (2000): "Specification Analysis of Affine Term Structure Models," *The Journal of Finance*, 55(5), 1943–1978.

Diebold, F. X., and C. Li (2006): "Forecasting the term structure of government bond yields," *Journal of Econometrics*, 130(2), 337–364.

Domowitz, I., and C. S. Hakkio (1985): "Conditional variance and the risk premium in the foreign exchange market," *Journal of International Economics*, 19(1-2), 47–66.

Duffee, G. R. (2002): "Term Premia and Interest Rate Forecasts in Affine Models," *Journal of Finance*, 57(1), 405–443.

Duffee, G. R., and R. Stanton (2012): "Estimation of dynamic term structure models," *Quarterly Journal of Finance*, 2, page numbers undetermined.

Duffie, D., and R. Kan (1996): "A yield-factor model of interest rates," *Mathematical Finance*, 6(4), 379–406.

Fama, E. F. (1984): "Forward and spot exchange rates," *Journal of Monetary Economics*, 14(3), 319–338.

Fisher, M., and C. Gilles (1996): "Term Premia in Exponential Affine Models of the Term Structure," unpublished manuscript, Board of Governors of the Federal Reserve System.

FRANKEL, J., AND C. M. ENGEL (1984): "Do asset-demand functions optimize over the mean and variance of real returns? A six-currency test," *Journal of International Economics*, 17(3-4), 309–323.

GRAVELINE, J. J. (2006): "Exchange Rate Volatility and the Forward Premium Anomaly," unpublished manuscript.

INCI, A. C., AND B. LU (2004): "Exchange rates and interest rates: can term structure models explain currency movements?," *Journal of Economic Dynamics and Control*, 28(8), 1595–1624.

JACOBS, K., AND L. KAROUI (2009): "Conditional volatility in affine term-structure models: Evidence from Treasury and swap markets," *Journal of Financial Economics*, 91(3), 288–318.

JOSLIN, S., K. J. SINGLETON, AND H. ZHU (2011): "A New Perspective on Gaussian Dynamic Term Structure Models," *Review of Financial Studies*, 24(3), 926–970.

JOTIKASTHIRA, C., A. LE, AND C. LUNDBLAD (2015): "Why do term structures in different currencies co-move?," *Journal of Financial Economics*, 115(1), 58 – 83.

KOOPMAN, S. J., M. I. P. MALLEE, AND M. VAN DER WEL (2010): "Analyzing the Term Structure of Interest Rates Using the Dynamic NelsonSiegel Model With Time-Varying Parameters," *Journal of Business & Economic Statistics*, 28(3), 329–343.

LETTAU, M., AND J. A. WACHTER (2011): "The term structures of equity and interest rates," *Journal of Financial Economics*, 101(1), 90–113.

LUSTIG, H., N. ROUSSANOV, AND A. VERDELHAN (2010): "Countercyclical Currency Risk Premia," NBER Working Papers 16427, National Bureau of Economic Research, Inc.

LUSTIG, H., A. STATHOPOULOS, AND A. VERDELHAN (2013): "The Term Structure of Currency Carry Trade Risk Premia," NBER Working Papers 19623, National Bureau of Economic Research, Inc.

LUSTIG, H., AND A. VERDELHAN (2011): "The Cross-Section of Foreign Currency Risk Premia and Consumption Growth Risk: Reply," *American Economic Review*, 101(7), 3477–3500.

MARK, N. C. (1988): "Time-varying betas and risk premia in the pricing of forward foreign exchange contracts," *Journal of Financial Economics*, 22(2), 335–354.

MONFORT, A., F. PEGORARO, J.-P. RENNE, AND G. ROUSSELLET (2014): "Staying at Zero with Affine Processes: A New Dynamic Term Structure Model," Discussion paper, Available at SSRN: http://ssrn.com/abstract=2408495 or http://dx.doi.org/10.2139/ssrn.2408495.

NELSON, C. R., AND A. F. SIEGEL (1987): "Parsimonious Modeling of Yield Curves," *The Journal of Business*, 60(4), 473–89.

SARNO, L., P. SCHNEIDER, AND C. WAGNER (2012): "Properties of foreign exchange risk premiums," *Journal of Financial Economics*, 105(2), 279–310.

Documents de Travail

510. G. Bazot, M. D. Bordo and E. Monnet, "The Price of Stability. The balance sheet policy of the Banque de France and the Gold Standard (1880-1914) ," October 2014

511. K. Istrefi and A. Piloiu, "Economic Policy Uncertainty and Inflation Expectations," October 2014

512. C. Jardet and A. Monks, "Euro Area monetary policy shocks: impact on financial asset prices during the crisis?," October 2014

513. J. Dugast and T. Foucault, "False News, Informational Efficiency, and Price Reversals," October 2014

514. G. Cette, J. Lopez and J. Mairesse, "Product and Labor Market Regulations, Production Prices, Wages and Productivity," October 2014

515. L. Ferrara and C. Marsilli, "Nowcasting global economic growth: A factor-augmented mixed-frequency approach," October 2014

516. F. Langot and M. Lemoine, "Strategic fiscal revaluation or devaluation: why does the labor wedge matter? ," October 2014

517. J.-P. Renne, "Fixed-Income Pricing in a Non-Linear Interest-Rate Model," October 2014

518. J.-P. Renne, "Options Embedded in ECB Targeted Refinancing Operations," October 2014

519. A. Penalver, "Pre-crisis credit standards: monetary policy or the savings glut?," November 2014

520. C. Marsilli, "Variable Selection in Predictive MIDAS Models," November 2014

521. P. Lopez, "The Term Structure of the Welfare Cost of Uncertainty," November 2014

522. F. Ferroni and B. Klaus, "Euro Area business cycles in turbulent times: convergence or decoupling? ," November 2014

523. F. Bec and A. De Gaye, "How do oil price forecast errors impact inflation forecast errors? An empirical analysis from French and US inflation forecasts," November 2014

524. P. Andrade, R. Crump, S. Eusepi and E. Moench, "Fundamental Disagreement," November 2014

525. P. Antipa, "How Fiscal Policy Affects the Price Level: Britain's First Experience with Paper Money," November 2014

526. A. Berthou and H. Ehrhart, "Trade networks and colonial trade spillovers," December 2014

527. S. Mouabbi, "An arbitrage-free Nelson-Siegel term structure model with stochastic volatility for the determination of currency risk premia," December 2014

Pour accéder à la liste complète des Documents de Travail publiés par la Banque de France veuillez consulter le site : www.banque-france.fr

For a complete list of Working Papers published by the Banque de France, please visit the website: www.banque-france.fr

Pour tous commentaires ou demandes sur les Documents de Travail, contacter la bibliothèque de la Direction Générale des Études et des Relations Internationales à l'adresse suivante :

For any comment or enquiries on the Working Papers, contact the library of the Directorate General Economics and International Relations at the following address :

BANQUE DE FRANCE
49- 1404 Labolog
75049 Paris Cedex 01
tél : 0033 (0)1 42 97 77 24 ou 01 42 92 63 40 ou 48 90 ou 69 81
email : 1404-ut@banque-france.fr