DOCUMENT DE TRAVAIL

N° 528

ASSESSING THE MACROECONOMIC EFFECTS OF LTROS

Christophe CAHN, Julien MATHERON and Jean-Guillaume SAHUC

December 2014

BANQUE DE FRANCE
EUROSYSTÈME

DIRECTION GÉNÉRALE DES ÉTUDES ET DES RELATIONS INTERNATIONALES

DIRECTION GÉNÉRALE DES ÉTUDES ET DES RELATIONS INTERNATIONALES

ASSESSING THE MACROECONOMIC EFFECTS OF LTROS

Christophe CAHN, Julien MATHERON and Jean-Guillaume SAHUC

December 2014

Les Documents de travail reflètent les idées personnelles de leurs auteurs et n'expriment pas nécessairement la position de la Banque de France. Ce document est disponible sur le site internet de la Banque de France « www.banque-france.fr ».
Working Papers reflect the opinions of the authors and do not necessarily express the views of the Banque de France. This document is available on the Banque de France Website "www.banque-france.fr".

ASSESSING THE MACROECONOMIC EFFECTS OF LTROS

CHRISTOPHE CAHN, JULIEN MATHERON, AND JEAN-GUILLAUME SAHUC

Date: December 12, 2014.

Cahn: Banque de France, Observatoire des Entreprises (44-1356), 31 rue croix des petits champs, 75049, Paris Cedex 01 (e-mail: christophe.cahn@banque-france.fr); Matheron: Banque de France, Service d'Etude sur la Politique Monétaire (41-1422), 31 rue croix des petits champs, 75049, Paris Cedex 01 (e-mail: julien.matheron@banque-france.fr); Sahuc: Banque de France, Service d'Etude sur la Politique Monétaire (41-1422), 31 rue croix des petits champs, 75049, Paris Cedex 01 (e-mail: jean-guillaume.sahuc@banque-france.fr). We wish to thank Jordi Galí, Benoît Mojon, and participants at several seminars for their helpful feedback. We are also very grateful to Michel Juillard for his help with the Dynare code. The views expressed herein are those of the authors and should under no circumstances be interpreted as reflecting those of the Banque de France or the Eurosystem.

ABSTRACT: In response to the 2008-2009 crisis, faced with distressed financial intermediaries, the ECB embarked in long-term refinancing operations (LTROs). Using an estimated DSGE model with a frictional banking sector, we find that such liquidity injections can have large macroeconomic effects, with multipliers up to 0.5. However, the latter depend in an important way on how standard monetary policy is adjusted in conjunction with these non-standard measures. We find that the effects are larger when the separation principle is breached, that is to say when we force monetary policy not to react to the stimulative effects of LTROs.

KEYWORDS: Financial frictions, unconventional monetary policy, long-term refinancing operations, DSGE model.

JEL CLASS.: E32, E58

RÉSUMÉ: Suite à la crise de 2008-2009, confrontée à un système financier sous stress, la BCE a mis en place des opérations de refinancement à long terme (LTRO). En nous appuyant sur un modèle DSGE avec frictions bancaires, nous trouvons que ces injections de liquidité peuvent avoir d'importants effets macroéconomiques, avec des multiplicateurs pouvant atteindre 0.5. Cependant, ces effets dépendent crucialement de la manière dont la politique monétaire standard est menée conjointement aux mesures non conventionnelles. Nous obtenons que ces effets sont plus importants si le principe de séparation est abandonné.

MOTS-CLÉS: Frictions financières, politique monétaire non-conventionnelle, opération de refinancement à long terme, modèle DSGE.

CLASSIFICATION JEL: E32, E58

NON TECHNICAL SUMMARY

In response to the 2008-2009 crisis, central banks in most advanced countries embarked in large-scale asset purchase programs. In the euro area, in which the financing of investment relies predominantly on the banking system, such programs played only a minor role. Instead, the bulk of non-standard interventions took the form of long-term refinancing operations (LTROs), with maturities of three, six, and finally twelve months in July 2009. Through these operations, the ECB aimed at increasing the average maturity of outstanding liquidity, from approximately 20 days before the crisis to more than 200 days in the second half of 2009. This policy was addressing funding concerns in the banking sector, in an attempt to allow banks to keep lending in spite of an acute confidence crisis. While LTROs were crucial to combat the crisis in the euro area, they have not received much attention in the literature.

In this paper, we assess the macroeconomic impact of LTROs in a dynamic stochastic general equilibrium (DSGE) model *à la* Smets and Wouters (2007), augmented with a frictional banking sector. In doing so, the paper makes two contributions. First, we extend the discount window setup developed by Gertler and Kiyotaki (2010, section 3) to allow for maturities longer than just one quarter. Importantly, we find that lengthening the maturity of LTROs helps relax the bankers incentive constraint *above and beyond* the direct effect of short-term liquidity injections. Second, we estimate the model on pre-crisis data via Bayesian techniques, using macroeconomic and financial variables. Armed with the posterior distributions of the model's structural parameters, we assess quantitatively the macroeconomic effects of LTROs by allowing the central bank in our model to open alternative longer-term refinancing facilities.

As in Gertler and Karadi (2011), we assume that bankers face an incentive constraint stemming from their ability to divert assets financed via households deposits. As a consequence, in equilibrium, bankers charge a higher interest rate on loans than the interest rate they pay on deposits. This credit spread turns out to be counter-cyclical, thus proving an important amplification and propagation device. This mechanism accords well with standard descriptions of the bank lending channel and rationalizes how disruptions in the latter can threaten the proper transmission of monetary policy in times of crisis. In this context, an injection of central bank liquidity has the power of alleviating the aggregate effects of the bankers incentive constraint. However, the details of how standard monetary policy is conducted in conjunction with LTROs turn out to be crucial. Our baseline estimate is obtained under the separation principle. According to the latter, standard monetary policy actions are conducted irrespective of LTROs. In our DSGE model, we interpret the separation principle as follows: Insofar as the LTROs are expansionary, the separation principle has it that the nominal interest rate should be raised, to the extent warranted by the monetary policy reaction function. Alternatively, we consider liquidity injections when the separation principle is breached as long as the LTRO facility is in place. This is achieved by forcing the systematic component of monetary policy not to react to the stimulative effects of LTROs, while reacting normally to all the other structural shocks.

To assess the impact of the maturity lengthening of LTROs per se, we first consider the case of purely exogenous liquidity injections, with maturities of one, two, or four quarters. It turns out that LTROs are modestly expansionary, with longer maturities resulting in larger effects. In response to a four-quarter LTRO amounting to 2 percent of annual output, quarterly output increases by about 0.30 percent after three quarters. The multiplier after one year is only 0.12. Importantly, the separation principle calls for a counterbalancing increase in the nominal interest rate. The effects of a two-quarter exogenous LTRO are approximately half as small.

We find that breaching the separation principle magnifies substantially the macroeconomic effects of one-year LTROs. The mean response of quarterly output to a one-year LTRO of the same size as above is now an increase by about 1.2 percent after three quarters. The associated multiplier after one year amounts to slightly more than 0.5. Thus, combining LTROs with an accommodative conventional monetary policy results in much larger effects. This finding provides quantitative support to the statement by several Eurosystem officials that LTROs and standard monetary policy are complementary.

In a second set of simulations, we pay particular attention to an essential characteristic of the ECB's non conventional refinancing operations. Since the fourth quarter of 2008, all the refinancing operations of the ECB, including LTROs, have been conducted under the fixed rate full allotment (FRFA) tender procedure. Under the latter, the ECB sets the interest rate (fixed rate) and stands ready to offer whatever amount of liquidity demanded by the banking sector (full allotment).The key feature of the FRFA tender procedure is that, under the latter, the central bank balance sheet expansion is endogenous, mirroring the endogenous demand for central bank funds by financial intermediaries.

While our model is too crude to offer a detailed account of the demand for liquidity, we capture the endogeneity of the balance sheet expansion by postulating rule-based liquidity injections. Having this rule depend on the credit spread allows us to approximate *(i)* the demand for liquidity emanating from the banking sector and *(ii)* the policy intent of flooding the system with excess liquidity. We then feed in the model shocks triggering a 2008-like recession. In response, the central bank opens alternative rule-based LTRO facilities, with increasing maturities. We judge the efficacy of these facilities by comparing the resulting pathes of macroeconomic variables with those obtained absent LTROs.

Under the separation principle, we find that when a four-quarter LTRO facility is in place for three quarters, as in the 2008 and 2009 facilities, output increases by about 0.80 percent in response to an endogenous liquidity injection amounting to slightly more than 4 percent of annual GDP. The multiplier after one year is approximately 0.12, very similar to what obtained in the exogenous LTRO scenario. When the separation principle is breached, once again, we obtain larger effects. The multiplier on output is about 0.5 after one year. More importantly, the amount borrowed at the LTRO facility is now close to 2 percent of annual GDP, much smaller than under the separation principle. This confirms our finding of a complementarity between LTROs and more conventional monetary policy actions.

1. Introduction

In response to the 2008-2009 crisis, central banks in most advanced countries embarked in large-scale asset purchase programs. In the euro area, in which the financing of investment relies predominantly on the banking system, such programs played only a minor role.[1] Instead, the bulk of non-standard interventions took the form of long-term refinancing operations (LTROs), with maturities of three, six, and finally twelve months in July 2009. Through these operations, the ECB aimed at increasing the average maturity of outstanding liquidity, from approximately 20 days before the crisis to more than 200 days in the second half of 2009. This policy was addressing funding concerns in the banking sector, in an attempt to allow banks to keep lending in spite of an acute confidence crisis. While LTROs were crucial to combat the crisis in the euro area, they have not received much attention in the literature.

In this paper, we assess the macroeconomic impact of LTROs in a dynamic stochastic general equilibrium (DSGE) model *à la* Smets and Wouters (2007), augmented with a frictional banking sector. In doing so, the paper makes two contributions. First, we extend the discount window setup developed by Gertler and Kiyotaki (2010, section 3) to allow for maturities longer than just one quarter. Importantly, we find that lengthening the maturity of LTROs helps relax the bankers incentive constraint *above and beyond* the direct effect of short-term liquidity injections. Second, we estimate the model on pre-crisis data via Bayesian techniques, using macroeconomic and financial variables.[2] Armed with the posterior distributions of the model's structural parameters, we assess quantitatively the macroeconomic effects of LTROs by allowing the central bank in our model to open alternative longer-term refinancing facilities.

As in Gertler and Karadi (2011), we assume that bankers face an incentive constraint stemming from their ability to divert assets financed via households deposits. As a consequence, in equilibrium, bankers charge a higher interest rate on loans than the interest rate they pay on deposits. This credit spread turns out to be counter-cyclical, thus proving an important amplification and propagation device. This mechanism accords well with standard descriptions of the bank lending channel and rationalizes how disruptions in the latter can threaten the proper transmission of monetary policy in times of crisis. In this context, an injection of central bank liquidity has the power of alleviating the aggregate effects of the bankers incentive constraint. However, the details of how standard monetary policy is conducted in conjunction with LTROs turn out to be crucial. Our baseline estimate is obtained under the separation principle. According to the latter, standard monetary policy actions are conducted irrespective of LTROs. In our DSGE model, we interpret the separation principle as follows: Insofar as the LTROs are expansionary, the separation principle has it that the nominal interest rate should be raised, to the extent warranted by the monetary policy reaction function. Alternatively, we consider liquidity injections when the separation principle is breached as long as the LTRO facility is in place. This is achieved by forcing the systematic component of monetary policy not to react to the stimulative effects of LTROs, while reacting normally to all the other structural shocks.

[1]The ECB conducted two separate asset purchase programs (the Covered Bond Purchase Program and the Securities Market Program), with much more limited size than US- or UK-style Quantitative Easing programs. See European Central Bank (2011) for more details about those programs and the other non conventional policies undertaken in the euro area.

[2]As in previous studies, the focus on pre-crisis data is justified by our concern about a possible bias in estimation if we were to extend the sample to cover the crisis period.

To assess the impact of the maturity lengthening of LTROs per se, we first consider the case of purely exogenous liquidity injections, with maturities of one, two, or four quarters. It turns out that LTROs are modestly expansionary, with longer maturities resulting in larger effects. In response to a four-quarter LTRO amounting to 2 percent of annual output, quarterly output increases by about 0.30 percent after three quarters. The multiplier after one year is only 0.12. Importantly, the separation principle calls for a counterbalancing increase in the nominal interest rate. The effects of a two-quarter exogenous LTRO are approximately half as small.

We find that breaching the separation principle magnifies substantially the macroeconomic effects of one-year LTROs. The mean response of quarterly output to a one-year LTRO of the same size as above is now an increase by about 1.2 percent after three quarters. The associated multiplier after one year amounts to slightly more than 0.5. Thus, combining LTROs with an accommodative conventional monetary policy results in much larger effects.[3] This finding provides quantitative support to the statement by several Eurosystem officials that LTROs and standard monetary policy are complementary.

In a second set of simulations, we pay particular attention to an essential characteristic of the ECB's non conventional refinancing operations. Since the fourth quarter of 2008, all the refinancing operations of the ECB, including LTROs, have been conducted under the fixed rate full allotment (FRFA) tender procedure. Under the latter, the ECB sets the interest rate (fixed rate) and stands ready to offer whatever amount of liquidity demanded by the banking sector (full allotment).[4] The key feature of the FRFA tender procedure is that, under the latter, the central bank balance sheet expansion is endogenous, mirroring the endogenous demand for central bank funds by financial intermediaries.

While our model is too crude to offer a detailed account of the demand for liquidity, we capture the endogeneity of the balance sheet expansion by postulating rule-based liquidity injections. Having this rule depend on the credit spread allows us to approximate *(i)* the demand for liquidity emanating from the banking sector and *(ii)* the policy intent of flooding the system with excess liquidity. We then feed in the model shocks triggering a 2008-like recession. In response, the central bank opens alternative rule-based LTRO facilities, with increasing maturities. We judge the efficacy of these facilities by comparing the resulting pathes of macroeconomic variables with those obtained absent LTROs.

Under the separation principle, we find that when a four-quarter LTRO facility is in place for three quarters, as in the 2008 and 2009 facilities, output increases by about 0.80 percent in response to an endogenous liquidity injection amounting to slightly more than 4 percent of annual GDP. The multiplier after one year is approximately 0.12, very similar to what obtained in the exogenous LTRO scenario. When the separation principle is breached, once again, we obtain larger effects. The multiplier on output is about 0.5 after one year. More importantly, the amount borrowed at the LTRO facility is now close to 2 percent of annual GDP, much smaller than under the separation principle. This confirms our finding of a complementarity between LTROs and more conventional monetary policy actions.

As a caveat, we emphasize that this paper does not consider the case of the two three-year LTROs conducted in December 2012 and February 2013. These were implemented in the particular context of

[3]This echoes findings by Gertler and Karadi (2011) in the context of large-scale asset purchases programs.

[4]This official terminology is somewhat confusing since, in practice, the interest rate of LTROs is not fixed. The ECB sets the latter to the average rate of the weekly main refinancing operations over the life of the LTRO.

the European sovereign debt crisis. As a result of these liquidity injections, a number of banks in the euro area embarked in carry-trade operations, which resulted in a sharp decline in sovereign spreads. Addressing this would require allowing for sovereign default risk in the model, an endeavor that we leave for future research.

In spite of their quantitative relevance, LTROs have received surprisingly little attention in the literature. Using a structural VAR with sign restrictions, Peersman (2011) concludes that a 10 percent increase in the monetary base (i.e., an increase close to that corresponding to the July 2009 LTRO) results in a 2 percent increase in output after about 18 months and a commensurate increase in the supply of private loans. Giannone, Lenza, Pill and Reichlin (2012) focus on the intermediation role that the ECB took after the crisis. Using a large Bayesian VAR, they conduct several counterfactual scenarios which show that monetary policy actions taken in 2009 resulted in a cumulated increase in industrial production by slightly more than one percent. However, their analysis does not allow to insulate the exact contribution of one-year LTROs. By contrast, Fahr, Motto, Rostagno, Smets and Tristani (2013) focus on the lengthening of refinancing operations maturities, in the context of an estimated DSGE model à la Christiano, Motto and Rostagno (2010). Through a set of indirect counterfactuals, they find that these liquidity injections together with FRFA allotments substantially reduced the intensity of the crisis. Our paper complements this literature by providing an empirical analysis of LTROs through the lenses of the canonical banking friction model by Gertler and Karadi (2011).[5] This allows us to incorporate explicitly the bank lending channel initially targeted by the ECB through its unconventional monetary policy actions.

The rest of the paper is as follows. In section 2, we expound the model and describe how we incorporate LTROs in this setup. In section 3, we describe our empirical approach and comment on the results, insisting on the posterior distribution of parameters pertaining to the frictional banking sector. We then report our estimate of macroeconomic effects LTROs. The last section briefly concludes.

2. THE DSGE MODEL

In this section we describe the structural model we will use in the subsequent quantitative analysis. To begin with, we augment the frictional banking sector *à la* Gertler and Karadi (2011) to allow for LTROs of maturities longer than just one quarter. This frictional banking sector is then plundeged into an augmented version of the standard DSGE model, along the lines of Christiano, Eichenbaum and Evans (2005), Justiniano, Primiceri and Tambalotti (2010), and Smets and Wouters (2007). After a brief overview of the model, the bankers problem is expounded in details. A full description of the model is relegated in appendix A.

2.1. Model Overview.

The economy is inhabited by a unit-mass continuum of identical households, each composed of a continuum of members. Household members can be either workers, in proportion $1 - f$, or bankers, in proportion f. Workers supply labor and earn wage income while bankers manage financial intermediaries and accumulate net worth. All these sources of income are pooled at the household level. There is perfect consumption insurance within the household, so that workers and bankers

[5] See also Del Negro, Eggertsson, Ferrero and Kiyotaki (2011) for a thorough study of the discount window policy in the US.

share the same consumption level. In each period, a banker has probability $1-\sigma$ of becoming a worker next period. By the law of large numbers, $(1-\sigma)f$ bankers exit every period, in which case they bring back their retained earnings as dividends to their respective household. The bankers who exit become workers and are replaced by a similar number of workers randomly becoming bankers. The household also provides its new bankers with a small amount of start up funds.

Households maximize a separable utility function in consumption and the labor effort of workers over an infinite life horizon. Consumption appears in the utility function relative to a time-varying internal habit that depends on past consumption. Workers provides differentiated labor inputs. Monopoly power in the labor market results in an explicit wage equation and allows for the introduction of sticky nominal wages as in the Calvo model (households are allowed to reset their wage each period with an exogenous probability). Households save by lending funds to financial intermediaries. Though the household effectively owns the intermediaries that its bankers manage, it deposits funds in intermediaries that it does not own.

There are four distinct agents on the production side of the model. An intermediate good is produced by a unit-mass continuum of perfectly competitive firms. They decide on labor and capital inputs and the utilization rate of capital. The introduction of variable capital utilization implies that as the rental price of capital changes, the capital stock can be used more or less intensively according to some cost schedule. At the end of each period, intermediate goods firms finance capital purchases to be used in the next period by raising funds from financial intermediaries. At the beginning of the next period, a capital quality shock $\varphi_{k,t}$ is realized, through which capital can turn out to be more or less productive than expected. Monopolistic retailers combine intermediate goods and material goods, as in Basu (1995) and Rotemberg and Woodford (1995), to produce differentiated goods.[6] They set prices according to the Calvo model. Competitive firms then pack the retail goods into an aggregate final good, which is then split into three different usages: material goods, consumption goods, and investment goods. Finally, capital producers build new capital units using the final good as the sole input in production, subject to dynamic adjustment costs.

The model is closed by postulating a Taylor-like monetary policy rule with partial adjustment. Here, the desired nominal interest rate is set in function of the year-on-year inflation rate and the year-year growth rate of output, each in deviation from their respective steady-state values.

2.2. **Bankers.** In this section, we briefly expound the Gertler and Karadi (2011) frictional banking sector. In this sector, banks issue short-term debt (deposits), which they combine with their net worth to finance purchases of claims on physical capital issued by intermediate goods producers. For concreteness, consider banker j. At the end of t, he has deposits $D_t(j)$, net worth $N_t(j)$, and purchases $k_t(j)$ claims on capital at price $P_{k,t}$. Banker j's balance sheet is accordingly

$$P_{k,t}k_t(j) = N_t(j) + D_t(j). \tag{1}$$

[6]As argued by Woodford (2003), this formulation of the production function helps insulate inflation from the real marginal cost, resulting in a lower degree of nominal price rigidities given an estimated slope for the New Keynesian Phillips curve.

Letting R_t denote the non-contingent interest rate payable on deposits and $R_{k,t+1}$ the return on capital, banker j's net worth evolves according to

$$N_{t+1}(j) = (R_{k,t+1} - R_t)P_{k,t}k_t(j) + (1 + R_t)N_t(j), \tag{2}$$

where we used (1) to eliminate $D_t(j)$.

Banker's j objective is then to maximize its franchise value, i.e. the expected and discounted value of funds she brings back to the household upon exit

$$V_t(j) = \mathsf{E}_t \left\{ \sum_{s=1}^{\infty} \beta^s \frac{\Lambda_{t+s}}{\Lambda_t}(1-\sigma)\sigma^{s-1}N_{t+s}(j) \right\}, \tag{3}$$

where $\beta^s \Lambda_{t+s}/\Lambda_t$ is the nominal stochastic discount factor between dates $t+s$ and t.

At the end of each period t (after deposits have been collected), we assume that bankers can divert a fraction α_t of their asset side $P_{k,t}k_t(j)$. If the banker decides to divert assets, depositors have the ability to force him into bankruptcy. However, it is prohibitively costly for them to recover the diverted assets. Knowing this in advance, depositors are not willing to lend to bankers unless the franchise value of a banker is higher than the gain from asset diversion. Hence, banker j faces the incentive constraint

$$V_t(j) \geq \alpha_t P_{k,t}k_t(j). \tag{4}$$

The fraction of divertible assets α_t evolves over time according to

$$\alpha_t = \frac{1}{1 + \mathrm{e}^{-\bar{\alpha} - \varphi_{\alpha,t}}},$$

where $\bar{\alpha}$ pins down the steady-state value of α and $\varphi_{\alpha,t}$ is an exogenous shock.[7]

We guess the form of banker j's value function

$$V_t(j) = \gamma_{k,t}P_{k,t}k_t(j) + \gamma_t N_t(j). \tag{5}$$

Here, $\gamma_{k,t}$ can be interpreted as the net (nominal) value of an extra unit of private assets while γ_t is the net value of an extra unit of net worth. Assuming that condition (4) binds (as is the case in the steady state considered here), straightforward manipulations of (4) and (5) yield

$$\phi_t N_t(j) = P_{k,t}k_t(j),$$

where the endogenous leverage ratio ϕ_t obeys

$$\phi_t = \frac{\gamma_t}{\alpha_t - \gamma_{k,t}}.$$

Plugging this back into (3), using (1) and (2), and differentiating term by term, we obtain

$$\gamma_{k,t} = \beta \mathsf{E}_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \Omega_{t+1}(R_{k,t+1} - R_t) \right\}, \tag{6}$$

[7] The setup just described implicitly features an interbank market. To see this, imagine that on top of net worth and deposits, banker j can also borrow from other banks, in an amount $D_t^{\mathrm{ib}}(j)$ and at rate R_t^{ib}. Provided these funds are divertible to the same extent as deposits, it must be the case that in equilibrium $R_t^{\mathrm{ib}} = R_t$. Since in aggregate we also have $\int_j D_t^{\mathrm{ib}}(j)\mathrm{d}j = 0$, interbank loans will cancel out when aggregating the different bankers balance sheets. With this alternative setup in mind, a shock to α_t should be broadly interpreted as a drop in confidence striking the interbank market, a shock allegedly responsible for part of the crisis in the euro area.

$$\gamma_t = \beta \mathsf{E}_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \Omega_{t+1}(1+R_t) \right\}, \tag{7}$$

where we defined

$$\Omega_t = 1 - \sigma + \sigma \alpha_t \phi_t.$$

2.3. **Introducing LTROs.** In this section, we introduce LTROs in the model. This is done by extending the discount window setup devised by Gertler and Kiyotaki (2010) to allow for maturities longer than just one quarter. Formally, we now assume that in a given period $t_0 > 0$, the Central Bank announces (credibly) that a new refinancing facility is available over the following T quarters. During this lapse of time, banks have access in each quarter to LTROs with a maturity of $q > 0$ quarters. The non-contingent interest rate payable in each quarter of the LTROs is $R_{m,t}$ and is a priori susceptible to changes. Starting from period $t_0 + T$, the facility is closed and there are no more funds borrowed from the central bank. Letting $M_{t,t-i}(j)$ denote the period t outstanding amount of ECB's liquidity borrowed from the LTRO facility at date $t-i$ by banker j, it must be the case that

$$M_{t+i|t}(j) = M_{t|t}(j) \text{ for } 0 \leq i \leq q-1.$$

Also, since the LTROs have a q-quarter maturity, we have $M_{t+i|t}(j) = 0$ for $i \geq q$. So, in each quarter, banker j has total amount of liquidity $M_{f,t}(j)$

$$M_{f,t}(j) = \sum_{i=0}^{q-1} M_{t|t-i}(j) = \sum_{i=0}^{q-1} M_{t-i|t-i}(j).$$

The balance sheet of banker j now rewrites

$$P_{k,t}k_t(j) = N_t(j) + D_t(j) + M_{f,t}(j). \tag{8}$$

The net worth of bank j, while active, accumulates according to

$$N_{t+1}(j) = (R_{k,t+1} - R_t)P_{k,t}k_t(j) + (1+R_t)N_t(j) - (R_{m,t} - R_t)M_{f,t}(j). \tag{9}$$

As before, bankers have the ability to divert a fraction α_t of their asset side, but this time this is net the fraction $\varsigma \in [0,1]$ of funds $M_{f,t}(j)$ borrowed from the Central Bank (for example because the loan has been extra collateralized). In our simulations, we will impose $\varsigma = 1$. Thus, banker j faces the reformulated incentive constraint

$$V_t(j) \geq \alpha_t(P_{k,t}k_t(j) - \varsigma M_{f,t}(j)). \tag{10}$$

We guess the new form of banker j's value function

$$V_t(j) = \gamma_{k,t}P_{k,t}k_t(j) + \gamma_t N_t(j) - \gamma_{m,t}M_{f,t}(j), \tag{11}$$

where $\gamma_{k,t}$ and γ_t are the same as before, and $\gamma_{m,t}$ can be interpreted as the net cost of an extra unit of LTRO and obeys

$$\gamma_{m,t} = \mathsf{E}_t \left\{ \beta \frac{\Lambda_{t+1}}{\Lambda_t} \Omega_{t+1}(R_{m,t} - R_t) \right\}. \tag{12}$$

The optimal choice for $M_{t|t}(j)$ implies the arbitrage condition

$$\gamma_{m,t} = \varsigma \gamma_{k,t}. \tag{13}$$

Importantly, equations (12) and (13) tell us that financial intermediaries are willing to pay a premium to have access to the LTRO facility. This premium is the price for the ability to expand credit with a less stringent incentive constraint. The Central Bank should thus set $R_{m,t}$ so as to make financial intermediaries indifferent at the margin between LTROs and private credit.

Assuming that condition (10) binds (as is the case in the steady state considered here), straightforward manipulations of (10), (11), and (13) yield

$$\phi_t N_t(j) = P_{k,t}k_t(j) - \varsigma M_{f,t}(j),$$

where the endogenous leverage ratio ϕ_t is still defined as above. This reformulation of the incentive constraint tells us that LTROs help banker j to extend more loans than otherwise permitted by the endogenous leverage ratio ϕ_t.

To conclude, the LTRO facility helps relax the incentive constraint. This happens because in this setup, the Central Bank has two key advantages. First, as long as $\varsigma > 0$, assets financed with funds borrowed from the Central Banks are less divertible than those financed via deposits. This mechanically renders the incentive constraint less stringent. Second, because it never defaults, the Central Bank itself is not balance-sheet constrained. Taking these two special advantages into account, the Central Bank can effectively take up an intermediation role in times of crisis by issuing short-term debt to households and lending those funds to financial intermediaries through an LTRO facility.

2.4. **Aggregation.** Let N_t and $M_{f,t}$ denote the aggregate counterparts of $N_t(j)$ and $M_{f,t}(j)$, respectively, and let Ξ_t denote the nominal start up funds that households transfer to new bankers. We assume that exiting bankers reimburse the outstanding amount of funds that they borrowed from the Central Bank. The aggregate law of motion of net worth is thus

$$N_{t+1} = \sigma[(R_{k,t+1} - R_t)P_{k,t}k_t - (R_{m,t} - R_t)M_{f,t} + (1 + R_t)N_t] + \Xi_{t+1}.$$

In the quantitative part of the paper, the real transfer from the households Ξ_t/P_t is assumed to be proportional to the beginning of period real net worth n_{t-1}, where ω is the proportionality coefficient and where P_t denotes the aggregate price level. Similarly, the the aggregate incentive constraint rewrites

$$\phi_t N_t = P_{k,t}k_t - \varsigma M_{f,t}.$$

Finally, the Central Bank finances its liquidity facility by issuing short-term debt B_t^{st}, bought by the households. We assume that these assets are perfectly substitutable with deposits. Accordingly, the Central Bank's balance sheet is $B_t^{\mathrm{st}} = M_{f,t}$. Finally, we assume that any profit the Central Bank makes through its liquidity facility is rebated to the government (in the form of dividends). These profits stem from the difference between the rate at which the Central Bank loans funds to banks ($R_{m,t}$) and the interest rate it pays on short-term debt (R_t).

As argued above, the LTRO facility helps relax the incentive constraint. Assuming a certain pattern for the liquidity injections, the following proposition states this more formally.

Proposition 1. *Suppose the central bank sets up a facility of q-quarter LTROs for T consecutive periods, starting from period t_0. Let M_t denote the aggregate amount of funds borrowed from the LTRO facility at date t and assume* *that* $M_t = \tilde{\psi}_t P_{k,t} k_t$, *with*

$$\tilde{\psi}_t = \begin{cases} \psi_t & \text{for } t_0 \leq t \leq t_0 + T - 1 \\ 0 & \text{for } t \geq t_0 + T, \end{cases}$$

where $\psi_t \in (0,1)$ *follows a process known to the agents. The aggregate incentive constraint is then*

$$\underbrace{\frac{\phi_t}{1 - \varsigma\tilde{\psi}_t} N_t}_{\text{Direct Effect}} + \underbrace{\frac{\varsigma}{1 - \varsigma\tilde{\psi}_t} \sum_{i=1}^{q-1} \sigma^i M_{t-i}}_{\text{Maturity Lengthening Effect}} = P_{k,t} k_t.$$

Proof. See appendix B. ■

Proposition 1 shows that the liquidity injections has two effects on the aggregate incentive constraint. The direct effect on the left-hand side shows that, given a level of aggregate net worth and a leverage ratio, having the LTRO facility in place helps banks extend more loans than they would have done absent the liquidity injections. Basically, the LTRO facility acts as a multiplier on the endogenous leverage ratio. This is similar to the effect documented by Gertler and Karadi (2011) in the case of asset purchase programs. The maturity lengthening effect stems from the funds inherited from past LTROs. They too contribute to a less stringent aggregate incentive constraint. These effects are all the more pronounced as he LTRO facility is in place. Yet, past LTROs continue to relax the incentive constraint even after the LTRO facility is closed. Indeed, since the last LTRO takes place at period $t_0 + T - 1$, by construction, it takes an extra q quarters to drive $M_{f,t}$ to zero. This second term accounts for the extra stimulative impact yielded by a maturity lengthening.

3. Quantitative Analysis

In this section, we describe how the above model is solved and estimated. We discuss our choice of priors and comment on the posterior distribution of the structural parameters related to the frictional banking sector. We then use the estimated model to assess the macroeconomic effects of a series of LTRO facilities.

3.1. Estimation.

Using pre-crisis data, we estimate a version of the model without LTRO facilities. Before, taking the model to the data, we first induce stationarity by getting rid of the stochastic trend component z_t and we log-linearize the resulting system in the neighborhood of the deterministic steady state.[8] In the process of inducing stationarity, it is convenient to define the following auxiliary parameters

$$\hat{\omega} = \omega e^{-\gamma_z}, \quad \omega_k = \beta(1 - \hat{\omega}) - \sigma.$$

The composite parameter $\hat{\omega}$ is simply a growth-adjusted version of the share of net worth redirected to new bankers ω. In contrast, ω_k is a more sophisticated object linked to the steady-state value of the equilibrium spread between R_k and R. It turns out to be more convenient to estimate ω_k rather than the probability of surviving for bankers σ. This is so because by imposing a prior with positive support, we make sure that the incentive constraint binds in steady state.[9]

[8] See the technical appendix for further details on the procedure used to induce stationarity.

[9] The steady-state multiplier on the incentive constraint is proportional to ω_k. See the technical appendix for further details.

The vector of structural parameters $\boldsymbol{\vartheta}$ is split in two sub-vectors $\boldsymbol{\vartheta}_1$ and $\boldsymbol{\vartheta}_2$. The first one, $\boldsymbol{\vartheta}_1$ contains parameters calibrated prior to estimation. Typically, these are parameters difficult to estimate in our framework. They are described in appendix C. The remaining parameters are estimated. We follow the Bayesian approach to estimate the log-linearized model, as advocated by An and Schorfheide (2007). After having cast the dynamic system in a state-space representation for the set of observable variables, we use the Kalman filter (i) to measure the likelihood of the observed variables and (ii) to form the posterior distribution of the structural parameters by combining the likelihood function with a joint density characterizing some prior beliefs. Given the specification of the model, the posterior distribution cannot be recovered analytically but may be computed numerically, using a Monte-Carlo Markov Chain (MCMC) sampling approach. More specifically, we rely on the Metropolis-Hastings algorithm to obtain a random draw of size 1,000,000 from the posterior distribution of the parameters.

We estimate the model using eight series of euro area quarterly data: GDP growth, consumption growth, investment growth, the growth rate of real hourly compensation, the growth rate of hours worked, the inflation rate, measured as the growth rate of the GDP deflator, the short-term nominal interest rate, measured as the three-month Euribor rate, and a measure of non-financial corporate spreads for the euro area proposed by Gilchrist and Mojon (2014). The latter develop euro-area credit risk indicators, applying the methodology devised by Gilchrist and Zakrajsek (2012).

Real variables are divided by the civilian population, age 16 and over. The population series is only available at yearly frequency. It is converted to quarterly frequency by resorting to a simple interpolation procedure. Except for the credit risk indicators, all series are from Eurostat. The sample runs from 1999:1 to 2007:4. All the data are multiplied by 100 so that the reported standard errors of structural shocks are all expressed in percent.

3.1.1. *Prior Distributions.* Our choice of priors is standard. We impose Beta distributions for all the parameters the theoretical support of which is the compact $[0, 1]$. We use Gamma distributions for positive parameters. Finally, we use Inverse Gamma distributions for the standard errors of shocks. Table 1 reports the priors of parameters related to the banking sector and the financial shocks $\varphi_{\alpha,t}$ and $\varphi_{k,t}$. The other choices of prior distributions are discussed in Appendix D.

We impose a prior mean of 0.5 for α, implying that bankers have the ability to divert a large fraction of funds borrowed from the households. However, the standard error for the prior distribution is 0.1, thus allowing for a rich set of values for α. The growth-adjusted share of aggregate net worth redirected to new bankers is comprised between 0 and 1 and should be small if one follows Gertler and Karadi (2011). To impose this, we select a Beta prior for $100\hat{\omega}$, with mean 0.2 and standard error 0.1. The composite parameter ω_k is expected to be small, since it is defined as the difference between two quantities close to but below 1. However, ω_k must be greater than zero for the equilibrium around which the model is log-linearized to be well defined. We thus impose a Gamma distribution for $100\omega_k$, with a prior mean of 1 and an associated standard error of 0.5.

TABLE 1. Banking Sector Parameters

Parameter	Prior shape	Prior Mean	Prior s.d.	Post. Mean	Post. s.d.	Low	High
$100\hat{\omega}$	Beta	0.20	0.10	0.23	0.12	0.06	0.40
$100\omega_k$	Gamma	1.00	0.50	1.27	0.44	0.55	1.99
α	Beta	0.50	0.10	0.48	0.07	0.35	0.60
ρ_α	Beta	0.80	0.15	0.70	0.12	0.49	0.93
ρ_k	Beta	0.80	0.15	0.70	0.10	0.50	0.90
σ_α	Inverse Gamma	0.25	2.00	0.23	0.06	0.08	0.37
σ_k	Inverse Gamma	0.25	2.00	0.16	0.04	0.06	0.25

Note: 'low' and 'high' denote the bounds of the 90% probability interval for the posterior distribution.

3.1.2. *Posterior Distributions.* The last four columns of Table 1 report information on the posterior distribution of parameters related to the banking sector and financial shocks: the mean, the standard deviation, and the bounds of the associated 90 percent confidence region. The rest of the parameters are discussed in Appendix D.

The first result that stands out is the relatively high bankers' ability to divert funds. The posterior mean for α is 0.48 with a posterior 90 percent interval (0.35,0.60). The data thus seem to favor a substantial level of asset diversion to match the spread over the risk-free rate.

Another key parameter of interest is the growth-adjusted fraction of net worth directed to new bankers. The posterior mean for this parameter is small and estimated to 0.23 percent. Notice however that the posterior distribution is hardly different from the prior, suggesting that aggregate data are close to silent on this parameter. This can be seen from the fact that the posterior mean and the posterior standard deviation for this parameter do not differ much from their prior counterpart.

Finally, the third parameter related to the banking sector is the composite parameter ω_k, directly linked to the bankers' survival probability, σ. With a mean for $100\omega_k$ estimated to 1.27 (with associated 90 percent confidence interval (0.55,1.99)), the implied probability of survival for bankers is 0.9867, a value close to that used in Gertler and Karadi (2011). This implies a life expectancy close to 19 years for bankers.

Finally, the asset diversion shock φ_α and the capital quality shock φ_k, exhibit some persistence with autoregressive parameters and associated 90 percent confidence bands $\rho_\alpha = 0.70$ (0.49,0.93) and $\rho_k = 0.70$ (0.50,0.90). However, their standard deviations, $\sigma_\alpha = 0.19$ and $\sigma_k = 0.16$, are more than four times as small as the standard deviation of productivity shocks ($\sigma_z = 0.75$).

3.2. **Quantifying the Macroeconomic Effects of LTROs.** We now quantify the macroeconomic effects of LTROs. We begin with exogenous liquidity injections. This allows us to control for the size of the LTRO and thus assess the effects of the maturity lenghtening per se. We then consider endogenous LTROs calibrated to replicate the actual size of their 2009 actual counterparts.

3.2.1. *Exogenous Liquidity Injections.* To assess the impact of the maturity lengthening of LTROs per se, we first consider the case of purely exogenous liquidity injections, with increasing maturities. This approach enables us to control for the size of the liquidity injection. Formally, we set the liquidity injection at date $t = 0$ equal to $\psi_0 P_{k,0} k_0$. We set ψ_0 so that the total amount borrowed at the different LTROs considered corresponds to 2 percent of annual GDP. This is approximately the net amount borrowed at the series of three-month and six-month LTROs launched in October 2008. We consider one-time exogenous liquidity injections with maturities of one, two, and four quarters, even though only three-month and six-month LTROs actually took place at the end of 2008. In turn, the central bank sets $R_{m,t}$ according to (12). The effects of these shocks depend on how the central bank adapts its standard monetary policy to accompany the liquidity injection. To investigate this, we also consider two alternative scenarios.

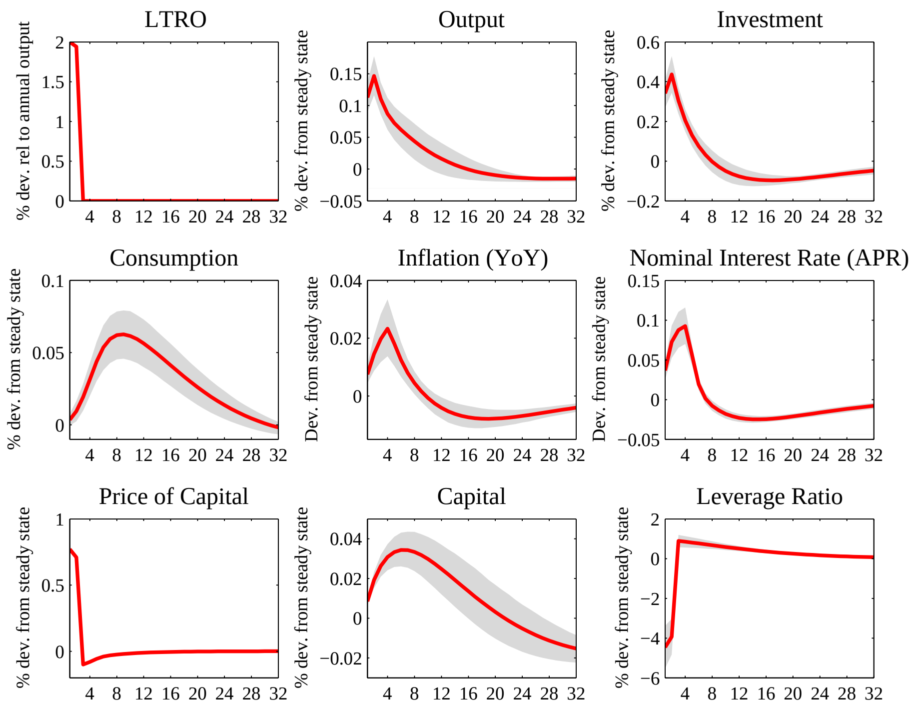
Our baseline scenario is that these liquidity injections take place under the separation principle. According to the latter, liquidity support policies *(i)* can be and *are* conducted irrespective of standard monetary policy actions and *(ii)* pursue different objectives (essentially that of restoring the proper functioning of money markets). Our reading of the separation principle in the context of our DSGE model is the following: Insofar as liquidity-support operations are expansionary (generating higher output and higher inflation via their positive impact on the credit supply), the separation principle has it that the nominal interest rate should be raised to the extent warranted by the conventional monetary policy reaction function.

Figures 1(A) and 1(B) report the outcome of this simulation for two-quarter and four-quarter LTROs. For the sake of brevity, we do not report the impulse responses to a one-quarter exogenous LTRO. The latter are just a scaled-down version of the responses to a two-quarter LTRO. Both figures show the dynamic responses of output, investment, consumption, capital, the price of capital, and the leverage ratio, all in percentage deviation. The figures also report year-on-year inflation and the annualized nominal interest rate, all in deviation from steady state. Finally, they report the amount of central bank funds borrowed by financial intermediaries, under the heading LTRO. The latter is reported in deviation relative to annual output. The grey area corresponds the 68 percent confidence region obtained by drawing 1000 times from the posterior distribution of $\boldsymbol{\vartheta}_2$. The red line corresponds to the mean response.

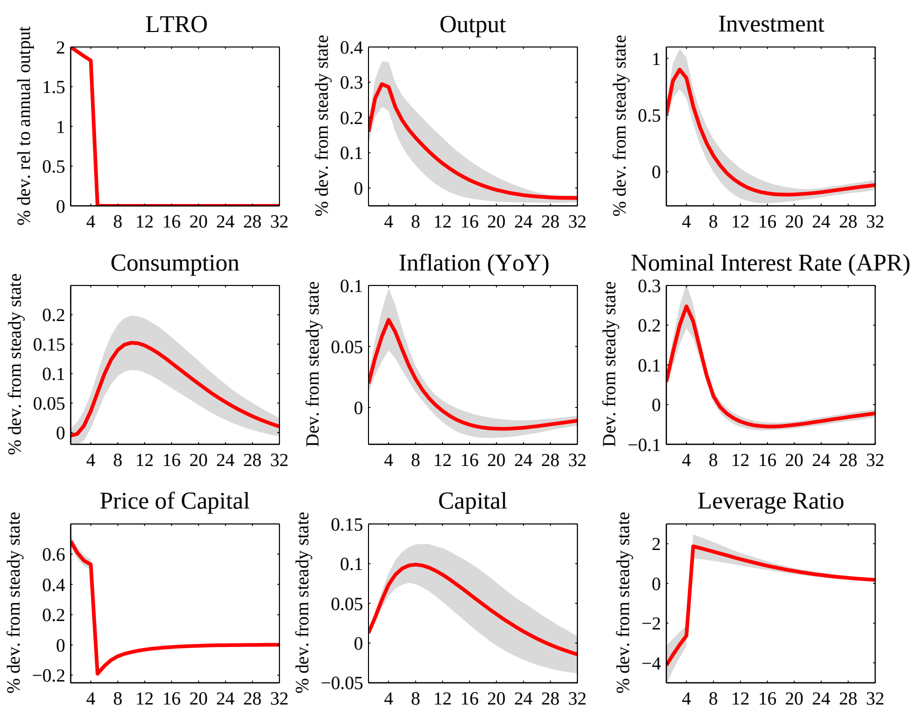
The LTRO shock triggers modest hump-shaped responses of output, with a 0.1 percent increase in output on impact which builds up to culminate at an increase by about 0.15 percent after two quarters for the two-quarter LTRO, and approximately twice as much for the four-quarter LTRO. Another way of summarizing the effects of the LTRO shocks is to compute the associated multipliers. The latter is simply the cumulated increase in output divided by the exogenous liquidity injection. For the two-quarter LTRO, this multiplier is slightly less than 0.06 while it reaches 0.12 with a four-quarter LTRO.

The increase in output is essentially driven by an increase in investment. In effect, consumption hardly reacts over the initial LTRO phase. This is so because the separation principle dictates a series of increases in the short-term nominal interest rate, triggered by the gradual rise in inflation and output. Due to the Taylor principle, the real interest rate is set on an increasing path over the LTRO horizon, which in turn mutes the response of consumption.

In contrast, the peak response of investment is more than twice as large as that of output. In turn, the increase in investment is rendered possible by a less stringent incentive constraint. The inflow of fresh



(A) Two-Quarter LTROs



(B) Four-Quarter LTROs

FIGURE 1. Exogenous LTROs - Separation Principle

Note: The grey area is the 68 percent confidence region. The plain line is the mean response. Output, investment, consumption, the price of capital, capital, and the leverage ratio are in percent deviation from steady state. LTRO are reported in percent relative to annual output. The year-on-year inflation rate and the annualized short-term interest rate are in deviation from steady state.

funds into the banking sector enables financial intermediaries to extend more loans. However, as the dynamics of capital suggest, this process is very gradual. This in turn bids up the price of capital. The increase in the price of capital strengthens the aggregate net worth of the banking sector. This can be seen from the dynamics of the leverage ratio ϕ_t, which declines by approximately 5 percent on impact. Notice that this decline is all the more protracted as the LTRO maturity is long. When the operation expires, the price of capital declines. This corresponds to the moment when the banking sector has to reimburse the funds borrowed from the central bank. At this time, banks start to reduce the scale of their operations, thus triggering a drop in the real price of capital. In turn, after the LTRO expiry, the leverage ratio sharply increases. It then gradually returns to its steady-state value from above. Notice however that there is a fair amount of uncertainty surrounding the mean responses of investment, the price of capital, and the leverage ratio.

Importantly, the inflationary effects of the LTRO shock are somewhat limited. In the two-quarter LTRO, year-on-year inflation gradually rises by slightly more than 2 basis points above target after two quarters and then declines. In the four-quarter LTRO, inflation increases by slightly less than 8 basis points. Thus, despite a massive injection of funds in the banking sector, inflation developments remain relatively subdued.

To sum up, an exogenous LTRO successfully reduces the leverage ratio, as initially targeted by the ECB. Nevertheless, it has a very limited effect on output or inflation. As the maturity of the operation lengthens, the effects are bigger. However, even with a four-quarter maturity, the effects of an exogenous LTRO remain quite small.

The central bank's decision to increase the short-term interest rate while injecting funds in the banking sector seems remotely linked to what the ECB did in practice. As a matter of fact, the ECB accompanied its liquidity injections at the end of 2008 with a series of interest rate cuts. In our second scenario, we force the central bank to breach the separation principle. In doing so, we have to take a stand on how the ECB could do this in practice.

We assume that the central bank now adapts its monetary policy by neutralizing the endogenous response of the short-term nominal interest rate for as many periods as the maturity of the LTRO. We further assume that this policy move is announced at the same time as the LTRO shocks. While inconsistent with the ECB's reluctance to announce an interest rate path, we still view this exercise as providing a useful illustration of the ECB's policy. In practice, the truth probably lies in between our two alternative scenarios (separation principle versus a breach of the latter). The ECB did not announce that it would keep interest rates unchanged over the LTRO horizon. At the same time, the private sector was not expecting any interest rate hike over this horizon, viewing such hikes as inconsistent with the policy intention. Thus, our experiments are best viewed as providing a lower and an upper bounds of the LTRO policy.

To operationalize this alternative scenario, we append q shocks $\epsilon^i_{R,t-i}$ for $i = 0, \dots, q-1$ to the monetary policy rule, thus allowing for the possibility of pre-announcing deviations from the standard monetary policy over the life of the q-quarter LTRO. In our alternative scenario, the LTRO shock is thus accompanied by the surprise announcement at date $t = 0$ of q shocks to the $\epsilon^0_{R,0}, \dots, \epsilon^{q-1}_{R,0}$. We compute these shocks so as to neutralize the systematic part of the Taylor rule, resulting in a nominal rate stuck at its steady-state value for the first q quarters of the simulation.[10]

The outcome of our alternative scenario is reported on Figures 2(A) and 2(B). As before, the figure reports the mean response obtained by drawing 1000 times from the posterior distribution of $\boldsymbol{\vartheta}_2$. This corresponds to the red line. The grey area is the 68 percent confidence band. For ease of comparison, the figure also reports the mean response obtained under the baseline scenario (dashed red line). Once again, we do not report the responses to a one-quarter LTRO, since the latter are just a muted version of the two-quarter LTRO.

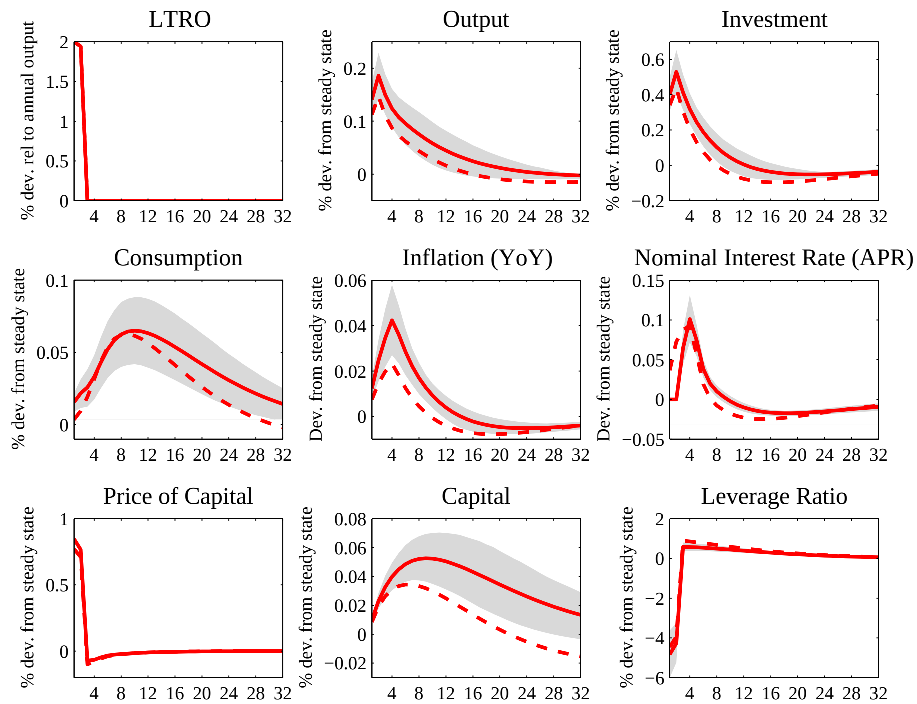
The figure makes clear that under this alternative scenario, the dynamics of the aggregate variables reported have broadly the same general shape as before. The main difference is that now, the amplitude and persistence of the dynamic responses are larger. While the amplification is somewhat limited in the case of two-quarter LTROs, it is more pronounced in the case of four-quarter LTROs. The extra monetary policy accommodation delivered by the surprise shocks has essentially two effects. First, the surprise shocks magnify the response of consumption. Indeed, the lower path of R_t brings about a moderate increase in consumption which contributes to the reinforced response of output. Notice that this positive effect on consumption extends well after the expiry of the LTRO.

Second, the initial surprise cut in R_t reduces the expected required return on capital. This triggers a larger inflow of investment projects, with a stronger response of capital, particularly apparent in the case of four-quarter LTROs. In turn, through a form of credit channel accelerator, the increase in the real price of capital and the associated decline in the leverage ratio call for a further increase in investment. Importantly, the leverage ratio now converges back to its steady-state value from below. All in all, this translates into a higher and more persistent boom in output. In particular, the mean peak response of output is now close to 1.2 percent with a the four-quarter LTRO. Notice finally that the uncertainty surrounding these responses is larger than in the benchmark scenario. Yet, the mean responses obtained when the separation principle holds are almost always outside these probability bands. This is particularly evident in the case of four-quarter LTROs.

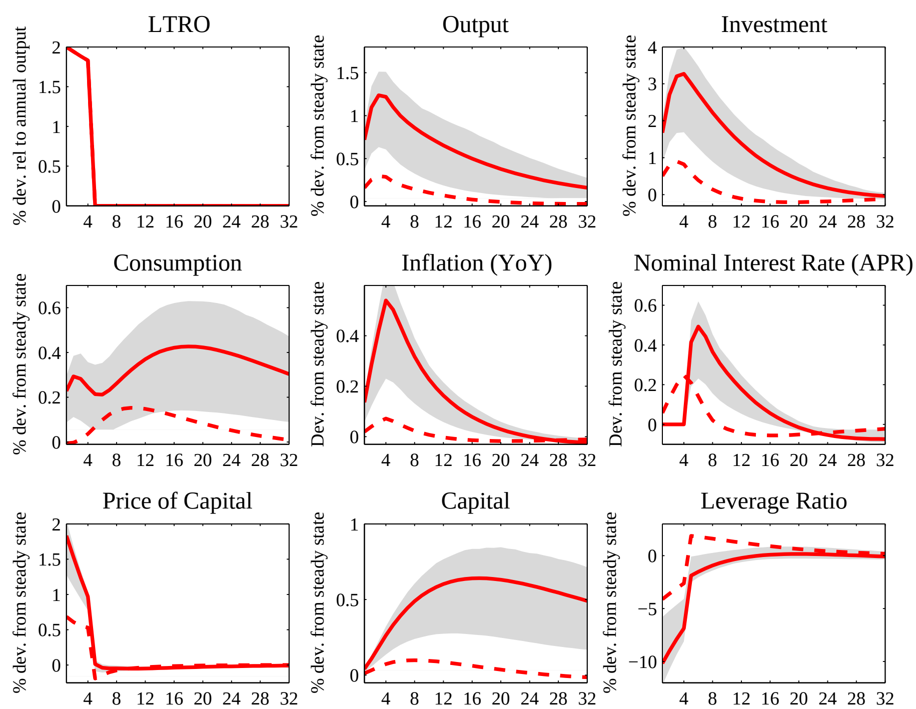
To sum up, breaching the separation principle magnifies the effects of the exogenous liquidity injection. This magnification effect is all the pronounced as the LTRO maturity is long. That being said, we still find a small macroeconomic effect for two-quarter LTROs. The multiplier associated with a two-quarter LTRO is about 0.075, as opposed to 0.530 for a four-quarter LTRO.

Based on this, one could be tempted to argue that the ECB did not react appropriately to the 2008-2009 crisis by adopting unconventional measures with muted effects. We claim below that such a negative assessment is not warranted for two reasons. First, the LTROs implemented at the end of 2008 were not one-time liquidity injections but rather took the form of a facility set to stay in place for almost three quarters, with a preset calendar known to market participants. Second, the mode in which liquidity was allotted at that time does not resemble exogenous injections but rather reflected the endogenous demand

[10]Del Negro, Giannoni and Patterson (2012) use a similar method to simulate forward guidance shocks. Here, we do not consider any serial correlation in the deviation from the plain Taylor rule once the central banks decides to re-adhere strictly to its policy rule after q quarters. This allows us to consider policy shocks the effects of which immediately vanish when the central bank decides to revert back to its normal policy rule.



(A) Two-Quarter LTROs



(B) Four-Quarter LTROs

FIGURE 2. Exogenous LTROs - No Separation Principle

Note: The grey area is the 68 percent confidence region. The plain line is the mean response; the dashed line is the mean response under the separation principle. Output, investment, consumption, the price of capital, capital, and the leverage ratio are in percent deviation from steady state. LTRO are reported in percent relative to annual output. The year-on-year inflation rate and the annualized short-term interest rate are in deviation from steady state.

for liquidity emanating from the banking sector. In the following simulations, we address explicitly these characteristics.

3.2.2. *Endogenous Liquidity Injections.* As the first act of its Enhanced Credit Support policy, at the beginning of the fourth quarter of 2008, the ECB moved to the so-called Fixed Rate Full Allotment (FRFA) tender procedure for its refinancing operations, including the LTROs conducted in 2008 and 2009. Under this allotment procedure, in stark contrast with its usual variable tender procedure, the ECB stands ready to offer whatever amount demanded by the banking sector at the rate of the main refinancing operations over the life of the operations, bet it a regular operation or a supplementary LTRO.

The key feature of the FRFA tender procedure is that, under the latter, the central bank balance sheet expansion is endogenous, mirroring the endogenous demand for central bank funds by financial intermediaries. While our model is too crude to offer a detailed account of the demand for liquidity, we capture the endogeneity of the balance sheet expansion by postulating rule-based liquidity injections. Having this rule depend on the credit spread $\text{Spr}_t \equiv \mathsf{E}_t\{R_{k,t+1} - R_t\}$ allows us to approximate *(i)* the demand for liquidity emanating from the banking sector and *(ii)* the policy intent of flooding the system with excess liquidity. Formally, we now assume that, for the number T of periods in which the LTRO facility is in place, $M_t = \psi_t P_{k,t} k_t$ with

$$\psi_t = \max\{0, \psi(\text{Spr}_t - \text{Spr})\},$$

where $\psi > 0$ and Spr is the steady-state value of Spr_t. The max operator is included to make sure that whenever the credit spread is lower than its steady-state value, banks cannot borrow negative amounts to the central bank. We calibrate ψ so that the amount borrowed at the two-quarter LTRO facility is 2 percent of annual GDP, approximately the amount borrowed actually at the end of 2008. In turn, the central bank sets $R_{m,t}$ according to (12).

The number T of periods in which the LTRO facility is in place remains to be calibrated. In the 15th of October 2008, the ECB announced a particular calendar for standard three-month LTROs, a series of special-term refinancing operations, and a number of supplementary LTROs with maturities of three-month and six-month. The calendar indicated that two three-month LTROs and one six-month LTRO were to be conducted each month until and including March 2009. As a matter of fact, the last six-month LTRO of this series took place mid-March 2009. Taking this observation into account, we calibrate $T = 3$, so that the facility stays in place for 3 consecutive quarters. We use the same T for two-quarter and four-quarter LTROs, even though four-quarter LTROs did not happen before June 2009. In each case, the facility is opened at t_0 and closed at $t_0 + T$.

In the model with endogenous LTROs, absent any disturbance, bankers would never borrow from the LTRO facility if the later were in place. To motivate such a recourse to central bank funds, we feed in the model surprise shocks allowing us to replicate the crisis that culminated in the fourth quarter of 2008. To retrieve these shocks, we use the posterior mean of the structural parameters and run the Kalman smoother on a sample extended up to the fourth quarter of 2008. The shocks thus obtained are: $\epsilon_c = -3.7$, $\epsilon_h = 1.9$, $\epsilon_k = -3.7$, $\epsilon_i = 1.7$, $\epsilon_p = -1.3$, $\epsilon_g = 2.1$, $\epsilon_\alpha = 1.9$, $\epsilon_R = 1.9$, and $\epsilon_z = -2.1$.

There are two striking findings. First, the trough reached in the fourth quarter of 2008 seems essentially driven by large negative shocks to ϵ_c and ϵ_k. However, the crisis is also the result of a sudden tightening of the incentive constraint, via a shock to ϵ_α, and a negative shock to technology ϵ_z. Finally, to a lesser extent, the crisis also originates from a large contractionary monetary policy shock. As argued before, there are two interpretations for such shocks. They can capture *(i)* the apparent reluctance of the ECB to cut interest rates with the same vigor as the Federal Reserve did at the same time and/or *(ii)* the adverse liquidity situation in the interbank market.

Subjecting the model to these shocks yields a benchmark path for the aggregate variables against which we can compare the dynamics obtained when LTROs are in place. To get such a comparison, we simulate the model with the exact same shocks and allow the central bank to open an LTRO facility in the last quarter of 2008. The efficacy of the LTROs is judged through the extent to which they allow for a milder downturn than in the benchmark crisis path. Below, we thus report the difference between the impulse response to the series of shocks that drive the economy into a 2008-like recession, with and without an LTRO facility. We interpret the resulting path as the impulse response to an LTRO shock. We also assess the uncertainty surrounding the counterfactual response by drawing in the posterior distribution of the structural parameters and repeating the exercise described above for each draw, thus yielding a posterior distribution for our counterfactual impulse response functions.

An important caveat in this analysis is that three-month and six-month LTROs actually took place in the fourth quarter of 2008, as discussed above. Given the discrete nature of these policy actions, it is difficult to take them into account in our smoothing exercise. Here, we deliberately ignore this problem, so that the shocks that we back out in the fourth quarter of 2008 are probably smaller than the shocks that actually hit the economy at this time, insofar as three-month and six-month LTROs were effective at mitigating the effects of the crisis. We argued before that, for three-month LTROs, this need not be a major problem for our analysis given that they hardly affect the dynamics of aggregate variables. However, we do find below that six-month LTROs have a non trivial impact on aggregate dynamics. Strictly speaking, thus, the exercise undertaken below should not be interpreted as a counterfactual analysis but rather as an analysis of the likely effects of an LTRO shock under economic circumstances resembling those that affected the euro area in the last quarter of 2008.

As before, we start by assessing the macroeconomic effects of the two-quarter and four-quarter LTRO facilities under the separation principle. The outcome of this simulation is reported on Figures 3(A) and 3(B). The figure reports the mean response obtained by drawing 1000 times from the posterior distribution of $\boldsymbol{\vartheta}_2$. This corresponds to the red line. The grey area is the 68 percent confidence band.

The LTRO is accompanied by a large amount of funds borrowed from the central bank. By construction, the mean response on impact is about 2 percent of annual output for the two-quarter LTRO facility, close to the actual amount borrowed at the end of 2008. With the four-quarter LTRO facility, the amount borrowed is larger, culminating at more than 4 percent of annual GDP after three quarters. The shock also triggers hump-shaped responses of output, investment, consumption, capital, year-on-year inflation, and the annualized nominal short-term interest rate. In the two-quarter LTRO facility, the peak response of output is close to 0.3 percent, while it is higher than 0.7 percent with the four-quarter LTRO facility. Not surprisingly, having an LTRO facility in place is a more powerful tool than implementing one-time exogenous liquidity injections. Yet, if we define the multiplier as the cumulated deviation of output divided by the overall amount of liquidity borrowed, we obtain 0.063 for the two-quarter LTRO and 0.124 for the four-quarter LTRO after one year. These are figures close to what obtained with the exogenous LTROs. Once again, judged through this metric, the effects of the LTRO are modest.

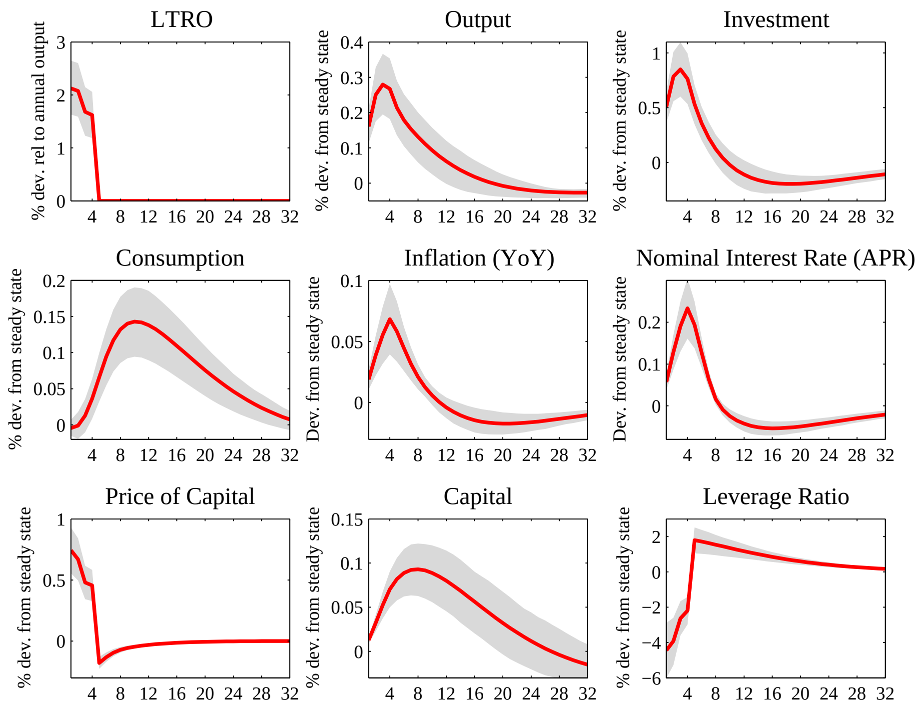
As before, the separation principle is partly responsible for these relatively modest effects. In the two-quarter LTRO facility, the peak response of the annualized interest rate is about 25 basis points, while it is close to 50 basis points with the four-quarter LTRO facility. Importantly, the inflationary effects of the LTRO shock are somewhat limited. With the two-quarter LTRO facility, year-on-year inflation gradually rises by less than 10 basis points above target after four quarters and then declines. With the four-quarter LTRO facility, it gradually rises by approximately 20 basis points above target after five quarters and then declines. Thus, despite a massive injection of funds in the banking sector, inflation developments remain once again relatively subdued.

Using the same motivation as before, we now redo the previous simulations assuming that the separation principle is breached. In the context of these simulations, there is an additional rationale for abandoning the separation principle. Since under the FRFA tender procedure the banking system is in a regime of excess liquidity, the three-month EURIBOR is driven to the floor of the channel system. Insofar as LTROs participate to maintain this regime of excess liquidity, one should expect that they contribute to exerting a downward pressure on the money market rates.

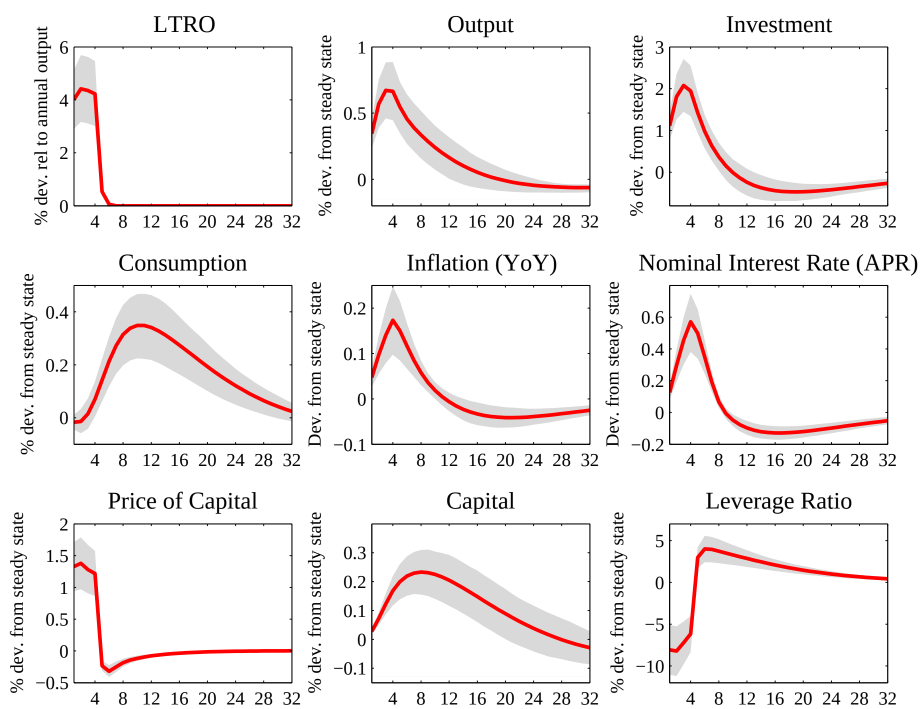
In practice, breaching the separation principle under endogenous LTROs is slightly more subtle than with exogenous liquidity injections. Under the latter, it was enough to unplug the Taylor-like rule during the q quarters of the LTRO. Now, we want to make sure that the Taylor rule reacts normally to the shocks hitting the economy and yet does not react to the LTROs. To this end, we consider two economies subject to the same shocks as before: In the first economy, there are no LTROs; in the second one, we consider alternatively two-quarter and four-quarter LTROs. In addition, we force the standard monetary policy in the second economy to equal that of the first economy over a pre-specified horizon. In the case of either two-quarter or four-quarter LTROs, this happens for 4 consecutive quarters, corresponding to the number of periods before liquidity starts to resorb. Thus, in the second economy, we make sure that the systematic component of the Taylor-like rule does not react to that portion of the dynamics of output and inflation triggered by the LTRO facilities. Importantly, this restriction is imposed ex ante, so that agents in the model economy are aware of it and form expectations by taking it into account.

The outcome of our alternative scenario is reported on Figures 4(A) and 4(B). As before, the figure reports the mean response obtained by drawing 1000 times from the posterior distribution of $\boldsymbol{\vartheta}_2$. This corresponds to the red line. The grey area is the 68 percent confidence band. For ease of comparison, the figure also reports the mean response obtained under the baseline scenario (dashed red line). Once again, we do not report the responses to a one-quarter LTRO, since the latter is just a muted version of the two-quarter LTRO.

The first striking result is that the amount of funds borrowed at the LTRO facility is now smaller. For the two-quarter LTRO facility, the maximal outstanding amount borrowed is slightly below 2 percent of annual GDP. With the four-quarter LTRO, it is close to 2 percent instead of 4 percent under the separation principle. This confirms our previous finding of a form of complementarity between standard monetary



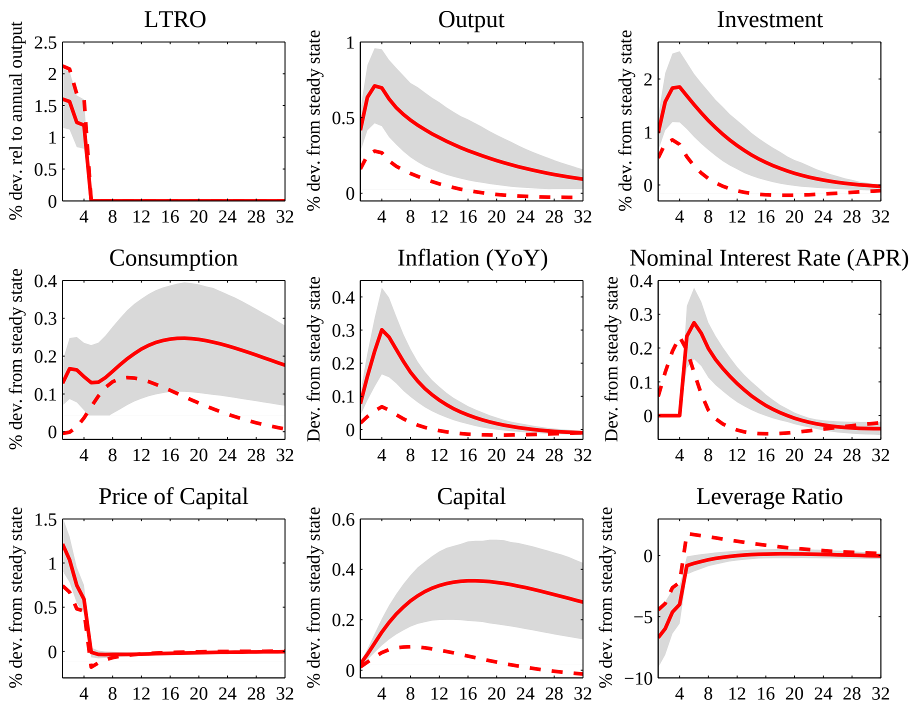
(A) Two-Quarter LTROs



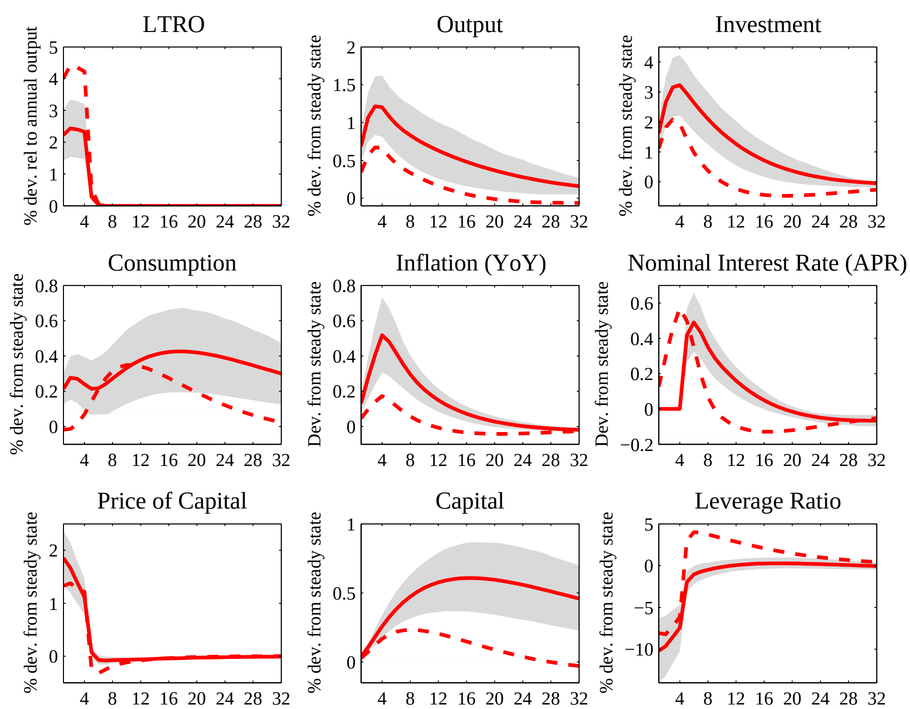
(B) Four-Quarter LTROs

FIGURE 3. Endogenous LTROs - Separation Principle

Note: The grey area is the 68 percent confidence region. The plain line is the mean response. Output, investment, consumption, the price of capital, capital, and the leverage ratio are in percent deviation from steady state. LTRO are reported in percent relative to annual output. The year-on-year inflation rate and the annualized short-term interest rate are in deviation from steady state.



(A) Two-Quarter LTROs



(B) Four-Quarter LTROs

FIGURE 4. Endogenous LTROs - No Separation Principle

Note: The grey area is the 68 percent confidence region. The plain line is the mean response; the dashed line is the mean response under the separation principle. Output, investment, consumption, the price of capital, capital, and the leverage ratio are in percent deviation from steady state. LTRO are reported in percent relative to annual output. The year-on-year inflation rate and the annualized short-term interest rate are in deviation from steady state.

policy and the non-standard refinancing facility: With the nominal interest rate unresponsive to the stimulus provided by LTROs, the latter can have much larger effects with substantially smaller amounts.

The second striking result pertains to the dynamics of the nominal interest rate. In the case of exogenous LTROs when the separation principle was breached, we found that the peak response of nominal interest rate after the LTRO expired was higher than that obtained under the separation principle. With endogenous LTROs, this difference is much less marked. In particular, with four-quarter LTROs, the nominal interest rate raises by a higher amount under the separation principle than when the latter is breached.

Overall, the multipliers are about 0.27 and 0.56 for the two-quarter and the four-quarter LTRO facilities, respectively. For the four-quarter LTRO facility, this is approximately what we obtained in the case of exogenous liquidity injections. For the two-quarter LTRO facility, the exogenous liquidity injection resulted in a substantially smaller multiplier. The difference stems from the fact that in the endogenous LTRO, the systematic component of monetary policy (in response to the LTRO) is frozen for two extra quarters compared with the exogenous injection.

3.2.3. *LTROs versus Interest Rate Cuts.* In the previous simulations, we saw that LTROs and interest-rate cuts are complementary. Indeed, when the central bank announces its intention to neutralize the systematic reaction of the standard Taylor rule to the LTRO shock, the amount borrowed at the facility turns out to be smaller and yet produces larger macroeconomic effects. These observations just reflect that both policies work their way to the economy through similar channels.

Against this backdrop, a legitimate question arises: How does an LTRO shock compare to a standard monetary policy shock? Indeed, at the end of 2008 and in 2009, there was plenty of room for further standard policy accommodation. In the context of our model, the reluctance of the Governing Council to cut interest rates was interpreted as a positive monetary policy shock ($\epsilon_R = 1.9$) counteracting the decrease in R_t that the systematic part of the Taylor rule would have otherwise induced. In this context, the above question can be refined as: How does a standard monetary policy shock of reverse sign ($\epsilon_R = -1.9$) compare to an LTRO?

To answer this question, we simply compute the impulse response function to an exogenous decrease in the nominal interest rate, scaled appropriately. Figure 5 reports the impulse response functions to a exogenous interest rate cut of output, investment, consumption, capital, the price of capital, and the leverage ratio, all in percentage deviation. It also reports year-on-year inflation and the annualized nominal interest rate, all in deviation from steady state. The grey area corresponds the 68 percent confidence region obtained by drawing from the posterior distribution of $\boldsymbol{\vartheta}_2$. The red line corresponds to the mean response.

The shock considered corresponds to an annualized interest rate cut of close to 50 basis points. While large compared to pre-crisis record, an interest rate cut of such size is not completely off mark when compared to the series of cuts undertaken during the crisis. The shock triggers a hump-shaped increase in output culminating to close to 0.4 percent after three quarters. The corresponding increase in investment is about 1.2 percent. The surge in investment is reflected in the gradual dynamics of capital, culminating

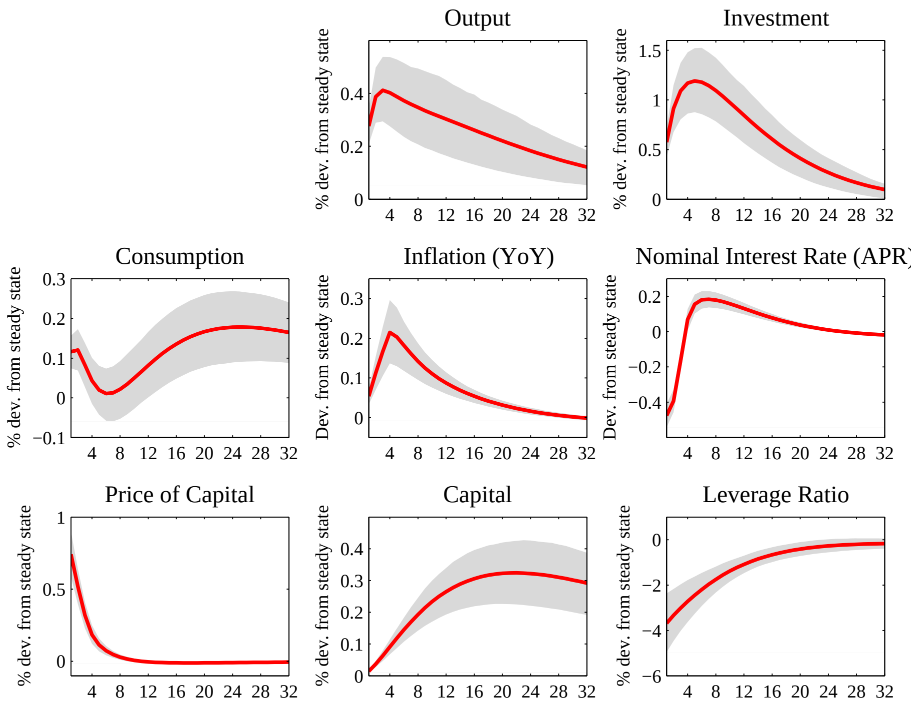


FIGURE 5. Aggregate Effects of an Interest Rate Cut

Note: The grey area is the 68 percent confidence region. The plain line is the mean response. Output, investment, consumption, the price of capital, capital, and the leverage ratio are in percent deviation from steady state. The year-on-year inflation rate and the annualized short-term interest rate are in deviation from steady state.

at 0.3 percent after about 20 quarters. In turn, the price of capital jumps by 75 basis points. The monetary policy shock also results in a decline in the leverage ratio by about 4 percent. Consumption first increases by a modest 0.1 percent, then declines, and then increases again before converging back to its steady-state level from above. This mirrors the dynamics of the interest rate itself which first declines due to the shock and then increases as the shock resorbs and the systematic part of policy comes into play. Far after the initial shock (beyond the reported simulation horizon), the interest rate converges back to it from below, which governs the consumption pick-up. Finally, year-on-year inflation increases by slightly less than 25 basis points.

The dynamics triggered by an exogenous interest rate cut share resemblance with those originating from an LTRO shock. Overall, the relatively large interest rate cut considered here generates aggregate effects on output, consumption, and investment that are slightly smaller than those of an endogenous two-quarter LTRO facility set to stay in place for three quarters, without the separation principle. Thus, even though the ECB did not cut its policy rate to the same extent the Fed did after the crisis, and even stood above the Taylor prescription, our results suggest that its policy actions were in fact accommodative and stimulative.

4. Conclusion

In this paper, our objective was to assess quantitatively the macroeconomic effects of LTROs. To this end, we first formulated and estimated a DSGE model featuring a frictional banking sector *à la* Gertler and Karadi (2011). We then augmented this setup to allow for LTROs. To this end, we extended the the model to allow for refinancing operations with maturities longer than just one quarter. In this augmented setup, we found that lengthening the maturity of LTROs helps relax the bankers incentive constraint above and beyond the direct discount window effect documented by Gertler and Kiyotaki (2010).

Our quantitative results confirmed this, under particular conditions. We found that liquidity injections that replicate the actual LTROs undertaken by the ECB after the crisis, can have large macroeconomic effects. However, the latter depend in an important way on how standard monetary policy is adjusted in conjunction with the non-standard measures. If the separation principle holds, that is if standard monetary policy reacts to the stimulative effects of LTROs, the nominal interest rate hikes tend to mute the benefits of LTROs. In contrast, when we force the systematic component of monetary policy not to react to LTROs, the effects are significantly larger.

We also showed that a two-quarter LTRO facility set to stay in place for three quarters, without the separation principle, has effects slightly larger than a shock to monetary policy resulting in a 50 basis points decline in the nominal interest rate. Thus, in the face of economic circumstances resembling the 2008-2009 crisis, a two-quarter LTRO facility appears a powerful tool to mitigate the severity of the downturn. Under similar circumstances, a four-quarter LTRO is even more powerful.

References

An, Sungbae and Frank Schorfheide, "Bayesian Analysis of DSGE Models," *Econometric Reviews*, 2007, *26* (2-4), 113–172.

Basu, Susanto, "Intermediate Goods and Business Cycle: Implications for Productivity and Welfare," *American Economic Review*, June 1995, *85* (3), 512–531.

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans, "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, February 2005, *113* (1), 1–45.

Christiano, Lawrence, Roberto Motto, and Massimo Rostagno, "Financial Factors in Economic Fluctuations," Working Paper 1192, European Central Bank May 2010.

Del Negro, Marco, Gauti Eggertsson, Andrea Ferrero, and Nobuhiro Kiyotaki, "The Great Escape? A Quantitative Evaluation of the Fed's Liquidity Facilities," Staff Reports 520, Federal Reserve Bank of New York 2011.

———, **Marc Giannoni, and Christina Patterson**, "The Forward Guidance Puzzle," Staff Reports 574, Federal Reserve Bank of New York 2012.

European Central Bank, "The ECB's Non Standard Measures: Impact and Phasing Out," *Monthly Bulletin*, July 2011, pp. 55–70.

Everaert, Luc and Werner Schule, "Why It Pays to Synchronize Structural Reforms in the Euro Area Across Markets and Countries," *IMF Staff Paper*, 2008, *55* (2), 356–366.

Fahr, Stephan, Roberto Motto, Massimo Rostagno, Frank Smets, and Oreste Tristani, "A Monetary Policy Strategy in Good and Bad Times: Lessons from the Recent Past," *Economic Policy*, April 2013, *28* (74), 243–288.

Gertler, Mark and Nobuhiro Kiyotaki, "Financial Intermediation and Credit Policy in Business Cycle Analysis," in Benjamin M. Friedman and Michael Woodford, eds., *Handbook of Monetary Economics*, Vol. 3, Elsevier, 2010, chapter 11, pp. 547–599.

——— **and Peter Karadi**, "A Model of Unconventional Monetary Policy," *Journal of Monetary Economics*, January 2011, *58* (1), 17–34.

Giannone, Domenico, Michele Lenza, Huw Pill, and Lucrezia Reichlin, "The ECB and the Interbank Market," *Economic Journal*, November 2012, *122* (564), F467–F486.

Gilchrist, Simon and Benoît Mojon, "Credit Risk in the Euro Area," Working Paper 482, Banque de France 2014.

——— **and Egon Zakrajsek**, "Credit Spreads and Business Cycle Fluctuations," *American Economic Review*, June 2012, *102* (4), 1692–1720.

Justiniano, Alejandro, Giorgio E. Primiceri, and Andrea Tambalotti, "Investment Shocks and Business Cycles," *Journal of Monetary Economics*, March 2010, *57* (2), 132–145.

McAdam, Peter and Alpo Willman, "Medium Run Redux," *Macroeconomic Dynamics*, June 2013, *17* (4), 695–727.

Peersman, Gert, "Macroeconomic Effects of Unconventional Monetary Policy in the Euro Area," Discussion Papers 8348, Center for Economic Policy Research April 2011.

Rotemberg, Julio and Michael Woodford, "Dynamic General Equilibrium Models with Imperfectly

Competitive Product Markets," in Thomas Cooley, ed., *Frontiers of Business Cycle Research*, Princeton, NJ: Princeton University Press, 1995, chapter 9, pp. 243–293.

Smets, Frank and Rafael Wouters, "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, June 2007, *97* (3), 586–606.

Woodford, Michael, *Interest and Prices*, Princeton, NJ: Princeton University Press, 2003.

Appendix A. Rest of the Model

In this appendix, we expound the remaining parts of the model.

A.1. **Households.** There is perfect consumption insurance within the household, so that workers and bankers share the same consumption level. Households save by lending funds to financial intermediaries. A typical household has the utility function

$$\mathsf{E}_t \sum_{s=0}^{\infty} \beta^s \mathrm{e}^{\varphi_{c,t+s}} \left\{ \log(c_{t+s} - \eta c_{t+s-1}) - \mathrm{e}^{\varphi_{h,t+s}} \frac{\chi}{1+\zeta} \int_0^{1-f} h_{t+s}(i)^{1+\zeta} \mathrm{d}i \right\} \tag{14}$$

where c_t is private consumption. The flow of consumption services is subject to habit formation, with degree $\eta \in [0,1)$. Each member experiences disutility from work $h_t(i)$. Labor disutility is described by two parameters: $\chi > 0$ is a scale parameter and $\zeta \geq 0$ governs the elasticity of labor supply. The overall impact on the representative household is simply the sum of individual disutilities. Finally, $\varphi_{c,t}$ and $\varphi_{h,t}$ are preference shocks.

The representative household maximizes (14) subject to the sequence of budget constraints

$$P_t c_t + D_t + B_t^{\mathrm{st}} = \int_0^{1-f} W_t(i) h_t(i) \mathrm{d}i + (1 + R_{t-1})(D_{t-1} + B_{t-1}^{\mathrm{st}}) + \mathrm{Div}_t + T_t \tag{15}$$

where P_t is the aggregate price level, $W_t(i)$ is the nominal wage rate paid to labor of type i, and D_t denotes deposits, paying the gross nominal interest rate R_t. The representative household also receives nominal transfers T_t (pays lumps sum taxes if negative) from the government. Finally Div_t denotes aggregate nominal profits redistributed by monopolistic firms, capital producers, and bankers (net of startup funds) to the households.

A.2. **Wage Setting.** Unions sell specialized labor units to competitive labor intermediaries. In turn, these intermediaries combine the differentiated labor inputs into an aggregate labor index h_t. This is done through the technology

$$h_t = \left(\frac{1}{1-f} \right)^{1/(\theta_w - 1)} \left(\int_0^{1-f} h_t(i)^{(\theta_w - 1)/\theta_w} \mathrm{d}i \right)^{\theta_w/(\theta_w - 1)},$$

where $\theta_w > 1$ is the elasticity of substitution between any two labor types. Labor intermediaries take $\{W_t(i), i \in [0,1]\}$ as given. Let W_t denote the nominal wage paid to aggregate labor, which is defined as

$$W_t = \left(\frac{1}{1-f} \int_0^{1-f} W_t(i)^{1-\theta_w} \mathrm{d}i \right)^{1/(1-\theta_w)}.$$

Within each household, workers of type $i \in [0, 1-f]$ are represented by a continuum $[0, 1-f]$ of unions, each specialized in a specific labor type. The union does not care about the worker history (i.e., it does not care whether the worker it represents was a worker or a banker last period). It simply sets the wage rate paid to labor.

Each labor union sets wages on a staggered basis, taking as given the demand for their specific labor input. It is assumed that at each point in time only a fraction $1 - \xi_w$ of the labor unions can set a new wage, which will remain fixed until the next time period the union is drawn to reset its wage. The remaining unions simply revise their wages according to the rule

$$W_{t+1}(i) = [\mathrm{e}^{\gamma_z}(1+\pi)]^{1-\iota_w}[(1+\pi_t)\mathrm{e}^{\gamma_z+\varphi_{z,t}}]^{\iota_w} W_t(i).$$

where $\iota_w \in [0,1]$ measures the degree of indexation to the most recently available inflation measure, $\pi_t \equiv P_t/P_{t-1} - 1$ is the inflation rate, and π is the inflation target set by the central bank. Also, when wages are not re-optimized, they are indexed to an average of steady-state total factor productivity (TFP) growth, e^{γ_z}, and to last period's realized TFP growth, $\mathrm{e}^{\gamma_z+\varphi_{z,t}}$. It is useful to define

$$1+\delta^w_{t,t+s} = \prod_{j=t}^{t+s-1} [\mathrm{e}^{\gamma_z}(1+\pi)]^{1-\iota_w}[(1+\pi_j)\mathrm{e}^{\gamma_z+\varphi_{z,j}}]^{\iota_w}$$

Let us now turn to the wage setting decision and define $h^\star_{t,t+s}(i)$ the supply of hours at $t+s$ by union i if it last re-optimized its wage at t. In period t, if drawn to re-optimize, union i chooses his wage rate $W^\star_t(i)$ so as to solve

$$\max_{W^\star_t(i)} \mathrm{E}_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \left\{ \Lambda_{t+s}(1+\delta^w_{t,t+s}) W^\star_t(i) h^\star_{t,t+s}(i) - \mathrm{e}^{\varphi_{h,t+s}+\varphi_{c,t+s}} \frac{\chi}{1+\zeta} \left(h^\star_{t,t+s}(i)\right)^{1+\zeta} \right\}$$

subject to

$$h^\star_{t,t+s}(i) = \frac{1}{1-f} \left(\frac{(1+\delta^w_{t,t+s}) W^\star_t(i)}{W_{t+s}} \right)^{-\theta_w} h_{t+s},$$

where Λ_t denotes the Lagrange multiplier on the household's nominal budget constraint.

A.3. **Intermediate Goods and Capital Producers.** At the end of period $t-1$, a unit-mass continuum of intermediate goods firms finance capital purchases to be used in the next period by issuing k_{t-1} claims at price $P_{k,t-1}$. At the beginning of period t the quality of capital is revealed to the firms through the realization of a shock $\varphi_{k,t}$, so that efficient capital is $\bar{k}_t = \mathrm{e}^{\varphi_{k,t}} k_{t-1}$.

In period t, these firms have access to the constant-return technology

$$y_{m,t} = (v_t \bar{k}_t)^\theta (\mathrm{e}^{z_t} h_t)^{1-\theta},$$

where $\theta \in (0,1)$ is the elasticity of production with respect to efficient capital (and will turn out to be the capital share in output in the steady-state equilibrium), h_t is the input of aggregate labor, $\bar{k}_t$ is the input of efficient capital (i.e., capital after the capital quality has been revealed), and v_t is the capital utilization rate, entailing a cost $a(v_t)\bar{k}_t$ (measured in final good units). For the purpose of estimation, it is useful to define $\tilde{\nu}_a \equiv a''(1)/a'(1)$ and $\nu_a \equiv \tilde{\nu}_a/(1+\tilde{\nu}_a)$, which proves more convenient to estimate than $\tilde{\nu}_a$. The permanent TFP shock z_t is assumed to evolve according to the process $z_t = \gamma_z + z_{t-1} + \varphi_{z,t}$, where $\varphi_{z,t}$ is an exogenous shock to the growth of z_t.

Let $P_{m,t}$ denote the nominal price of intermediate goods. The per unit cash flow accruing to effective capital $\bar{k}_t$ is

$$Z_{k,t} = \theta P_{m,t} \frac{y_{m,t}}{\bar{k}_t} - P_t a(v_t)$$

In equilibrium, the return on capital obeys

$$1+R_{k,t} = \frac{Z_{k,t} + (1-\delta) P_{k,t}}{P_{k,t-1}} \mathrm{e}^{\varphi_{k,t}}.$$

This return is entirely rebated to banks to pay for the date $t-1$ loan.

Capital is produced by a continuum of perfectly competitive capital producers. These producers buy back $\tilde{k}_t$ units of old efficient capital units, add new capital units using the input of final output (subject to adjustment costs) and sell the new capital to firms at the price $P_{k,t}$. Given that households own capital producers, the objective of a capital producer is to choose a contingent plan for investment i_t so as to maximize

$$\mathsf{E}_t \sum_{s=0}^{\infty} \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \left\{ P_{k,t+s} \left[\tilde{k}_{t+s} + i_{t+s} \left(1 - S\left(\frac{i_{t+s}}{i_{t+s-1}} \right) \right) \mathrm{e}^{\varphi_{i,t+s}} \right] - P_{t+s} i_{t+s} - P_{k,t+s} \tilde{k}_{t+s} \right\},$$

where $S(\cdot)$ is an adjustment cost function such that $S(\mathrm{e}^{\gamma_z}) = S'(\mathrm{e}^{\gamma_z}) = 0$ and $S''(\mathrm{e}^{\gamma_z})\mathrm{e}^{2\gamma_z} = \nu_i$, and $\varphi_{i,t}$ is a shock to the investment technology. Notice that the first order condition on $\tilde{k}_t$ implies that any value of $\tilde{k}_t$ is consistent with profit maximization. It follows that $\tilde{k}_t$ is pinned down by the equilibrium on the market for used capital, yielding

$$k_t = (1-\delta)\, \mathrm{e}^{\varphi_{k,t}} k_{t-1} + i_t \left(1 - S\left(\frac{i_t}{i_{t-1}} \right) \right) \mathrm{e}^{\varphi_{i,t}}.$$

A.4. **Price Setting.** There is a unique aggregate good, q_t, which can be used either for consumption and investment purposes, y_t, or used as an input in production, x_t. Thus, $q_t = y_t + x_t$. The aggregate good is produced by competitive firms according to the constant elasticity of substitution technology

$$q_t = \left(\int_0^1 q_t(i)^{(\theta_{p,t}-1)/\theta_{p,t}} \mathrm{d}i \right)^{\theta_{p,t}/(\theta_{p,t}-1)},$$

where $q_t(i)$ is the input of retail good i, with $i \in [0,1]$ and $\theta_{p,t}$ is the time-varying elasticity of substitution between any two retail goods. The aggregate nominal price is defined as

$$P_t = \left(\int_0^1 P_t(i)^{1-\theta_{p,t}} \mathrm{d}i \right)^{1/(1-\theta_{p,t})},$$

where $P_t(i)$ is the nominal price of retail good i. Finally, the resource constraint is

$$c_t + g_t + i_t + a(v_t)\mathrm{e}^{\varphi_{k,t}} k_{t-1} = y_t, \tag{16}$$

where g_t denotes government consumption (to be described below).

Each retail good $i \in [0,1]$ is produced by a monopolistic firm with the same index. Firm i has the technology

$$q_t(i) = \min\left\{ \frac{y_{m,t}(i) - \kappa \mathrm{e}^{z_t}}{1 - s_x}, \frac{x_t(i)}{s_x} \right\},$$

where $y_{m,t}(i)$ is the input of the intermediate good, $x_t(i)$ is the input of material good, and $\kappa\mathrm{e}^{z_t}$ is a fixed production cost. In this specification, material goods and intermediate goods are complements. The share of material goods in production is $s_x \in [0,1)$. This formulation of the production function has been advocated by Rotemberg and Woodford (1995).[11] The assumption that the fixed cost grows at the same rate as exogenous technical progress z_t ensures existence of a well–defined deterministic balanced growth path. In addition, we assume that the fixed cost κ is such that aggregate monopolistic profits are zero in the deterministic steady state.

[11] See also Basu (1995).

We assume that in each period of time, a monopolistic firm can re-optimize its price with probability $1-\xi_p$, irrespective of the elapsed time since it last revised its price. If the firm cannot re-optimize its price, the latter is re-scaled according to the simple revision rule

$$P_{t+1}(i) = (1+\pi)^{1-\iota_p}(1+\pi_t)^{\iota_p} P_t(i),$$

where $\iota_p \in [0,1]$ measures the degree of indexation to the most recently available inflation measure π_t. Again, it is useful to define

$$1+\delta^p_{t,t+s} = \prod_{j=t}^{t+s-1} (1+\pi)^{1-\iota_p}(1+\pi_j)^{\iota_p}$$

Let $q^\star_{t,t+s}(i)$ denote the production of firm i at $t+s$ if it last revised its price in period t. Let $P^\star_t(i)$ denote the price which was chosen at t. According to the demand function for intermediate goods, $q^\star_{t,t+s}(i)$ obeys the relationship

$$q^\star_{t,t+s}(i) = \left(\frac{(1+\delta^p_{t,t+s})P^\star_t}{P_{t+s}}\right)^{-\theta_{p,t+s}} q_{t+s}. \tag{17}$$

Thus, if drawn to re-optimize in period t, firm i selects $P^\star_t(i)$ so as to maximize the present discounted sum of profit streams

$$\mathsf{E}_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\Lambda_{t+s}}{\Lambda_t} \left\{(1+\delta^p_{t,t+s})P^\star_t(i) q^\star_{t,t+s}(i) - S_{t+s} q^\star_{t,t+s}(i)\right\},$$

subject to (17), where $S_t = (1-s_x)P_{m,t} + s_x P_t$ is the nominal marginal cost. We can then define the desired markup $\mu_{p,t}$ as

$$\mu_{p,t} = \frac{\theta_{p,t}}{\theta_{p,t}-1} = \mu_p \mathrm{e}^{\varphi_{p,t}},$$

where $\varphi_{p,t}$ is a price markup shock.

A.5. **Government Policy and Shocks.** The Central Bank sets R_t according to the Taylor-like rule

$$\hat{R}_t = \rho \hat{R}_{t-1} + (1-\rho)\left[\frac{a_\pi}{4}\hat{\pi}^a_t + \frac{a_y}{4}\hat{\gamma}_{y,t}\right] + \varphi_{R,t}$$

with $\hat{R}_t = R_t - R$, $\hat{\pi}^a_t = \pi^a_t - \pi^a$, $\hat{\gamma}_{y,t} = \gamma_{y,t} - \gamma_y$, where $1+\pi^a_t = (1+\pi_t)(1+\pi_{t-1})(1+\pi_{t-2})(1+\pi_{t-3})$, $\gamma_{y,t} = \log(y_t/y_{t-4})$, and a letter without subscript refers to the steady-state value of the associated variable. Here, monetary policy reacts to (i) the year-on-year growth rate of the aggregate price level P_t, in deviation from its target level π^a, with responsiveness a_π, and (ii) the year-on-year growth rate of output, in deviation from its average level, with responsiveness a_y. Notice that, since R_t is a quarterly rate, we include the term $1/4$ in the systematic part of the rule for consistency. The monetary policy rule also features inertia, with degree ρ. Finally, monetary policy is perturbed by a stochastic shock $\varphi_{R,t}$ which accounts for policy errors and exogenous variations in the liquidity situation of the economy.

Government expenditures g_t are assumed to be exogenous and evolve according to

$$\frac{g_t}{y_t} = s_g \mathrm{e}^{\varphi_{g,t}},$$

where s_g is the steady-state share of government expenditures and $\varphi_{g,t}$ is an exogenous process.

The government adjusts transfers T_t so as to balance its budget period by period. Thus

$$P_t g_t + T_t = 0.$$

Finally, the shocks $\varphi_{x,t}$, for $x \in \{c, h, k, i, \alpha, p, g, R, z\}$ evolve according to

$$\varphi_{x,t+1} = \rho_x \varphi_{x,t} + \sigma_x \epsilon_{x,t+1}, \quad \epsilon_{x,t} \sim N(0,1),$$

where $0 \leq \rho_x < 1$ and $\sigma_x > 0$.

Appendix B. Proof of Proposition 1

We now proceed to derive the proof of proposition 1. To begin with, recall that we defined M_t as the aggregate amount of funds borrowed at the date t LTRO. Similarly, $M_{f,t}$ is the aggregate amount of outstanding funds borrowed at t and at previous periods. We initialize the problem by noticing that prior to t_0, it must be the case that $M_t = M_{f,t} = 0$. At t_0, we have

$$M_{f,t_0} = M_{t_0}.$$

At the beginning of $t_0 + 1$, a fraction $1 - \sigma$ of bankers exit. Recall that we assumed that they reimburse the outstanding amount of funds borrowed from the Central Bank. After exit, new bankers arrive and are allowed to participate to the new LTRO that takes place at $t_0 + 1$. We thus obtain

$$M_{f,t_0+1} = \sigma M_{t_0} + M_{t_0+1}$$

Continuing up until $t_0 + q - 1$, we obtain

$$M_{f,t_0+q-1} = \sigma^{q-1} M_{t_0} + \sigma^{q-2} M_{t_0+1} + \cdots + \sigma M_{t_0+q-2} + M_{t_0+q-1}.$$

More compactly, for $i = 0, \ldots, q-1$

$$M_{f,t_0+i} = \sum_{s=0}^{i} \sigma^{i-s} M_{t_0+s}.$$

Now, at $t_0 + q$, the liquidity borrowed at the t_0 LTRO is entirely reimbursed. We thus have

$$M_{f,t_0+q} = \sigma \sum_{i=1}^{q-1} \sigma^{q-1-i} M_{t_0+i} + M_{t_0+q}.$$

Using the change of index $t = t_0 + q$, we obtain

$$M_{f,t} = \sum_{i=1}^{q-1} \sigma^{q-i} M_{t-q+i} + M_t = \sum_{i=1}^{q} \sigma^{q-i} M_{t-(q-i)} = \sum_{i=0}^{q-1} \sigma^{i} M_{t-i}.$$

Given our initialization, this formula holds for any t. For example, taking $t = t_0 + 2$ yields

$$M_{f,t_0+2} = \sum_{i=0}^{q-1} \sigma^{i} M_{t_0+2-i} = M_{t_0+2} + \sigma M_{t_0+1} + \sigma^2 M_{t_0},$$

where we used the fact that $M_t = 0$ for $t < t_0$. This formula allows us to keep track of the aggregate amount of Central Bank funds present in the system at each time. It follows that the aggregate incentive constraint rewrites

$$\phi_t N_t = P_{k,t} k_t - \varsigma \sum_{i=0}^{q-1} \sigma^{i} M_{t-i}.$$

Now, recall that we assumed that

$$M_t = \tilde{\psi}_t P_{k,t} k_t.$$

The aggregate incentive constraint thus rewrites

$$\frac{\phi_t}{1-\varsigma\tilde{\psi}_t} N_t + \frac{\varsigma}{1-\varsigma\tilde{\psi}_t} \sum_{i=1}^{q-1} \sigma^i M_{t-i} = P_{k,t} k_t.$$

APPENDIX C. CALIBRATION

In this section, we describe the parameters calibrated prior to estimation. They are collected in the vector $\boldsymbol{\vartheta}_1 = (\beta, \delta, \theta, \gamma_z, s_g, \pi, \theta_p, \theta_w, , s_x, A)$, The subjective discount factor, β, is set to 0.9986, yielding an annual nominal interest rate of 4 percent. The depreciation rate, δ, is set to 0.025. We set $\theta = 0.30$, so that the labor income share in output is 70 percent. This is consistent with estimates for the euro area reported in McAdam and Willman (2013). Parameters γ_z and s_g are set to 0.005 and 0.2, so as to reproduce the mean economic growth rate and the average ratio of government expenditures to output. Steady-state inflation, π, is set to 0.005 which corresponds to an annual inflation rate of 2 percent, consistent with the ECB's quantitative definition of price stability. The elasticity of substitution between intermediate goods θ_p is set to 6, implying a price mark-up of 20 percent, implying $\mu_p = 1.20$. The elasticity of substitution between labor types θ_w is set to 3.8571, which corresponds to a wage mark-up of 35 percent, yielding $\mu_w = 1.35$, as in Everaert and Schule (2008). The share of material goods in value added s_x, is set to 0.5, consistent with Eurostat's input-output tables.[12] Table 2 summarizes the calibrated values.[13]

TABLE 2. Calibrated parameters

Parameter	Description	Value
β	Subjective discount factor	0.9986
δ	Depreciation rate	0.0250
θ	Share of Capital in value added	0.3000
γ_z	Quarterly productivity growth rate	0.0050
s_g	Steady-state share of gov. expenditure	0.2000
π	Quarterly inflation target	0.0050
θ_p	Product elasticity of substitution	6.0000
θ_w	Labor elasticity of substitution	3.8570
s_x	Share of material goods in production	0.5000

APPENDIX D. PRIOR AND POSTERIOR DISTRIBUTIONS

Table 3 reports a complete summary of our estimation results. All the ρ_x for $x \in \{c, h, \alpha, z, k, i, R, g\}$ have prior means 0.8, except ρ_z for which the prior mean is 0.3, reflecting our prior of a much lower degree of serial correlation for the growth rate of permanent technology shocks. All these priors have

[12] See Eurostat's website, http://epp.eurostat.ec.europa.eu/portal/page/portal/esa95_supply_use_input_tables.

[13] The parameter χ does not appear in the log-linear system, so that we do not need to calibrate it.

a standard errors of 0.15, except ρ_z for which the standard deviation of the prior is set to 0.10. Most of the σ_x for $x \in \{c, h, \alpha, z, k, i, R, g\}$ have prior means 0.25 percent, except σ_g, σ_i, and σ_z which have prior means equal to 1 percent. All these priors have a standard deviation of 2.

The inverse of the Frisch elasticity of labor supply ζ has a prior mean equal to 2, as is now standard, following the work by Smets and Wouters (2007). The standard error of the prior distribution is 0.5, allowing for a wide range of values. The growth-adjusted degree of habit $\hat{\eta} \equiv \eta e^{-\gamma_z}$ has a prior mean of 0.75, with standard error equal to 0.1. Consistent with previous results, the prior mean for ν_i is equal to 4, a large value. The associated standard error is 1. The prior mean for ν_a is 0.5 (implying a unit value for $\tilde{\nu}_a$), with standard error 0.1.

The parameters pertaining to price and wage nominal rigidities share the same priors. The Calvo probabilities ξ_p and ξ_w have prior means equal to 0.75, with standard error 0.1, and the indexation parameters ι_p and ι_w have prior leans equal to 0.5, with standard error 0.2. The degree of responsiveness to inflation in the Taylor-like rule, a_π, has prior mean equal to 2, with standard error equal to 0.3, while the degree of responsiveness to output growth has a much slower prior mean of 0.5, with standard error equal to 0.1.

We now discuss our posterior results. As regards the nominal rigidity parameters, the posterior means imply an average duration of price and wage adjustment in the euro area of about 9 months ($\xi_p = 0.66$) and slightly more than 12 months ($\xi_w = 0.77$), respectively. The means of the wage and price indexation parameters are $\iota_w = 0.16$ and $\iota_p = 0.17$, respectively. This is consistent with the now standard result that the euro area data do not require too high a degree of price or wage indexation to match the persistence in the data. The posterior moments for the monetary policy rule imply that the monetary authority reacts slightly more aggressively to inflation and output than the standard Taylor prescription ($a_\pi = 1.67$ and $a_y = 0.73$), and shows a modest degree of interest rate smoothing ($\rho = 0.61$).

Concerning household's preferences, the habit persistence parameter, $\hat{\eta}$, is estimated at 0.55 at the posterior mean, with a 90 percent confidence interval (0.43,0.68). The posterior mean for the inverse of the Frisch elasticity of labor supply, ζ, is 2.14 with a 90 percent confidence interval of (1.32,2.95). Again, these values are consistent with several contributions in the medium-scale DSGE literature. Notice however that the posterior distribution of ζ is almost identical to the prior, suggesting that our aggregate data are close to silent on this parameter too. Once again, this can be seen from the fact that the posterior mean and the posterior standard deviation for this parameter do not differ much from their prior counterpart. Turning to real frictions in production, estimates for the curvatures of investment adjustment costs and capacity utilization costs are $\nu_i = 3.31$ and $\nu_a = 0.29$ at the posterior mean. In turn, the estimated value of ν_a implies $\tilde{\nu}_a = 0.41$, yielding a small degree of curvature for the utilization cost function.

TABLE 3. Complete Estimation Results

Parameter	Prior shape	Prior Mean	Prior s.d.	Post. Mean	Post. s.d.	Low	High
$100\hat{\omega}$	Beta	0.20	0.10	0.23	0.12	0.06	0.40
$100\omega_k$	Gamma	1.00	0.50	1.27	0.44	0.55	1.99
α	Beta	0.50	0.10	0.48	0.07	0.35	0.60
$\hat{\eta}$	Beta	0.75	0.10	0.55	0.07	0.43	0.68
a_π	Gamma	2.00	0.30	1.67	0.19	1.36	1.97
a_y	Gamma	0.50	0.10	0.73	0.10	0.56	0.90
ρ	Beta	0.75	0.15	0.61	0.07	0.49	0.74
ξ_p	Beta	0.75	0.10	0.66	0.06	0.56	0.77
ξ_w	Beta	0.75	0.10	0.77	0.09	0.62	0.93
ι_p	Beta	0.50	0.20	0.17	0.05	0.09	0.26
ι_w	Beta	0.50	0.20	0.16	0.07	0.02	0.29
ν_i	Gamma	4.00	1.00	3.31	0.77	2.03	4.55
ν_a	Beta	0.50	0.10	0.29	0.07	0.18	0.40
ζ	Gamma	2.00	0.50	2.14	0.49	1.32	2.95
ρ_α	Beta	0.80	0.15	0.70	0.12	0.49	0.93
ρ_c	Beta	0.80	0.15	0.49	0.14	0.26	0.71
ρ_h	Beta	0.80	0.15	0.52	0.19	0.21	0.83
ρ_k	Beta	0.80	0.15	0.70	0.10	0.50	0.90
ρ_i	Beta	0.80	0.15	0.90	0.04	0.81	1.00
ρ_p	Beta	0.80	0.15	0.54	0.13	0.32	0.76
ρ_g	Beta	0.80	0.15	0.99	0.00	0.97	1.00
ρ_R	Beta	0.80	0.15	0.48	0.12	0.29	0.67
ρ_z	Beta	0.30	0.10	0.16	0.05	0.06	0.25
σ_α	Inverse Gamma	0.25	2.00	0.23	0.06	0.08	0.37
σ_c	Inverse Gamma	0.25	2.00	0.64	0.11	0.39	0.89
σ_h	Inverse Gamma	0.25	2.00	0.17	0.04	0.11	0.23
σ_k	Inverse Gamma	0.25	2.00	0.16	0.04	0.06	0.25
σ_i	Inverse Gamma	1.00	2.00	1.68	0.33	1.04	2.29
σ_p	Inverse Gamma	0.25	2.00	0.13	0.02	0.09	0.17
σ_g	Inverse Gamma	1.00	2.00	1.25	0.14	1.00	1.48
σ_R	Inverse Gamma	0.25	2.00	0.08	0.01	0.06	0.09
σ_z	Inverse Gamma	1.00	2.00	0.75	0.08	0.60	0.90

Note: 'low' and 'high' denote the bounds of the 90% probability interval for the posterior distribution.

Documents de Travail

510. G. Bazot, M. D. Bordo and E. Monnet, “The Price of Stability. The balance sheet policy of the Banque de France and the Gold Standard (1880-1914) ,” October 2014

511. K. Istrefi and A. Piloiu, “Economic Policy Uncertainty and Inflation Expectations,” October 2014

512. C. Jardet and A. Monks, “Euro Area monetary policy shocks: impact on financial asset prices during the crisis?,” October 2014

513. J. Dugast and T. Foucault, “False News, Informational Efficiency, and Price Reversals,” October 2014

514. G. Cette, J. Lopez and J. Mairesse, “Product and Labor Market Regulations, Production Prices, Wages and Productivity,” October 2014

515. L. Ferrara and C. Marsilli, “Nowcasting global economic growth: A factor-augmented mixed-frequency approach,” October 2014

516. F. Langot and M. Lemoine, “Strategic fiscal revaluation or devaluation: why does the labor wedge matter? ,” October 2014

517. J.-P. Renne, “Fixed-Income Pricing in a Non-Linear Interest-Rate Model,” October 2014

518. J.-P. Renne, “Options Embedded in ECB Targeted Refinancing Operations,” October 2014

519. A. Penalver, “Pre-crisis credit standards: monetary policy or the savings glut?,” November 2014

520. C. Marsilli, “Variable Selection in Predictive MIDAS Models,” November 2014

521. P. Lopez, “The Term Structure of the Welfare Cost of Uncertainty,” November 2014

522. F. Ferroni and B. Klaus, “Euro Area business cycles in turbulent times: convergence or decoupling? ,” November 2014

523. F. Bec and A. De Gaye, “How do oil price forecast errors impact inflation forecast errors? An empirical analysis from French and US inflation forecasts,” November 2014

524. P. Andrade, R. Crump, S. Eusepi and E. Moench, “Fundamental Disagreement,” November 2014

525. P. Antipa, “How Fiscal Policy Affects the Price Level: Britain’s First Experience with Paper Money,” November 2014

526. A. Berthou and H. Ehrhart, “Trade networks and colonial trade spillovers,” December 2014

527. S. Mouabbi, “An arbitrage-free Nelson-Siegel term structure model with stochastic volatility for the determination of currency risk premia,” December 2014

528. C. Cahn, J. Matheron and J-G. Sahuc, “Assessing the macroeconomic effects of LTROS,” December 2014

Pour accéder à la liste complète des Documents de Travail publiés par la Banque de France veuillez consulter le site : www.banque-france.fr

For a complete list of Working Papers published by the Banque de France, please visit the website: www.banque-france.fr

Pour tous commentaires ou demandes sur les Documents de Travail, contacter la bibliothèque de la Direction Générale des Études et des Relations Internationales à l'adresse suivante :

For any comment or enquiries on the Working Papers, contact the library of the Directorate General Economics and International Relations at the following address :

BANQUE DE FRANCE
49- 1404 Labolog
75049 Paris Cedex 01
tél : 0033 (0)1 42 97 77 24 ou 01 42 92 63 40 ou 48 90 ou 69 81
email : 1404-ut@banque-france.fr