NOTES D'ÉTUDES

ET DE RECHERCHE

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June 1998

NER # 53

BANQUE DE FRANCE

DIRECTION GÉNÉRALE DES ÉTUDES

DIRECTION GÉNÉRALE DES ÉTUDES ET DES RELATIONS INTERNATIONALES
DIRECTION DES ÉTUDES ÉCONOMIQUES ET DE LA RECHERCHE

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October 1998
(Revised draft)

Abstract

In this paper we give a precise definition of long-run causality in a multivariate non-stationary, possibly cointegrated, framework. A variable is said to be causal for another in the long run if knowledge of the past of the former improves long-run predictions of the latter. In a VAR framework, we show that long-run non-causality can be easily tested with a Wald statistics, conditionnally on the cointegration rank. The methodology is used to study long-run causal links between US, German, and French long-term interest rates from January 1990 to June 1997.

Key words: Causality, Prediction Improvement, Cointegration.

JEL classification: C12, C32.

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The authors aknowledge helpful comments of Dick van Dijk, Clive W.J. Granger, Soren Johansen, Michael Rockinger, Pierre Sicsic, Anindya Banerjee (the Associate Editor), and an anonymous referee. They are also grateful for the helpful comments of participants at the seminar at the Banque de France, at the 1997 Econometric Society European Meeting and at the Maastricht University.

1 Introduction

One of the applications of estimating dynamic systems is to test for causality between subsets of time series. Indeed, causality plays a key role in studying the predictive properties of multivariate time series models and, more specifically, in testing exogeneity properties (Engle *et alii*, 1983). Causality is then defined in the usual Granger (1969) sense.

In a stationary framework, tests for causality are straightforward. In integrated systems, however, such tests are much more complex, essentially due to the uncertain number of unit roots in the dynamics. These empirical difficulties have been extensively studied in the literature (Sims *et alii*, 1990, Mosconi and Giannini, 1992, Toda and Phillips, 1993, Toda and Yamamoto, 1995) and practical recommendations are available to implement causality tests in possibly cointegrated systems. It is worth emphasizing that, in this strand of literature, causality is then defined as a one-step ahead prediction improvement property.

Other authors (Stock and Watson, 1989, Lütkepohl, 1990, Lütkepohl and Reimers, 1992) prefer to measure causal effects by estimating confidence bands on dynamic multipliers. Apart from the well known debate about the orthogonalization of the impulses, this approach has the advantage of providing a specific measure of persistency of shocks through the "long-run" dynamic multipliers. Moreover, the estimation of the long-run dynamic multipliers is easy to perform, whatever the cointegration properties of the dynamics are, provided that the orthogonalization is achieved by using a Choleski decomposition (Lütkepohl and Reimers, 1992).

Careful scrutiny of the above mentioned literature reveals two kinds of difficulties. First, the usual Granger characterization seems to be insufficient to measure persistent causal links, since the associated prediction improvement properties only deal with one step-ahead predictions. It should be noted that the error-correction effects are often associated with long-run causal links in cointegrated system, along the lines suggested by Granger (1988) and Granger and Lin (1995). However, these effects do not seem to be related to prediction improvement properties for the levels of the variables. In the same spirit Toda and Phillips (1993) do not explicitly propose a distinction between short-run and long-run causalities, even if they focus on sufficient conditions for non-causality that involve, separately, "short-run" and "long-run" parameters as they are usually denoted in the Error Correction Model (ECM) framework.

Second, as pointed out by Bruneau and Nicolaï (1995), the non-nullity of long-run dynamic multipliers cannot generally be interpreted in terms of prediction improvement. Actually, an impulse response analysis is not explicitly aimed at measuring causal links between time series but rather at identifying "structural shocks" that may account for the fluctuations of the system. The problem is that certain identification schemes may exclude any association between the shocks and the initial series (Blanchard and Quah, 1989). Therefore, the effect of these shocks cannot be seen as a causal link between the series of interest.

In this paper, we give a precise definition of long-run non-causality in a multivariate context, which can be interpreted in terms of prediction improvement for long-term predictions: *a variable X is not causal for another variable Y in the long-run if and only if the knowledge of the past of X does not improve the long-run prediction of Y*. In a VAR framework, we prove that long-run non-causality is equivalent to a bilinear constraint on long-run dynamic multipliers and parameters of the VAR in levels. We show that this constraint can be easily tested in this framework, conditionally on the long-run properties of the dynamics, that is, on the cointegration rank.

The remainder of the paper is organized as follows. In Section 2 we define the long-run causality property and we give a statistical characterization of this property in a non-stationary VAR framework. Section 3 details the test procedure. Section 4 presents an illustration of the method, which is used to study the long-run causal links between US, German, and French long-term interest rates. Section 5 concludes.

2 Unidirectional Long-Run Causality

Granger causality is characterized as a property of prediction improvement. Here, we are interested in long-run causality from X_j to X_k. Accordingly, we have to focus on prediction improvement properties for long-term predictions, and we therefore compare, for any horizon h, which is "large enough", the best linear predictions $EL\left(X_{k,t+h}\left|\left\{\underline{X_{i,t}}; i \neq k\right\}\right.\right)$ and $EL\left(X_{k,t+h}\left|\left\{\underline{X_{i,t}}; 1 \leq i \leq N\right\}\right.\right)$, where $\underline{X_{i,t}}$ denotes, for any i, $1 \leq i \leq N$, the set of past variables $X_{i,t-k}$, $k \geq 0$. More precisely, we adopt the following definition:

Definition 1 *For the N-dimensional process $X = \begin{pmatrix} X_1 & \cdots & X_N \end{pmatrix}'$, integrated of order one, X_j is said to be not unidirectional prior causal for X_k in the long-run if and only if, at any date t, the knowledge of the lagged variables $X_{j,t-h}$, $h \geq 0$, does not improve the best linear long-run prediction of X_k, according to, $\forall t \geq 1$:*

$$\lim_{h \to +\infty} EL\left(X_{k,t+h}\left|\left\{\underline{X_{i,t}}; 1 \leq i \leq N\right\}\right.\right) = \lim_{h \to +\infty} EL\left(X_{k,t+h}\left|\left\{\underline{X_{i,t}}; 1 \leq i \leq N, i \neq j\right\}\right.\right).$$

We therefore define causality along the lines suggested by Granger (1969), since we focus on the prediction performance of the whole past of the assumed causal variable. It differs from the usual definition of causality, since it deals with infinite horizon predictions.

Now, we focus on a testable condition of long-run non-causality, as defined above. Suppose that the dynamics of the N-dimensional process X_t obeys a non-stationary VAR model of finite order p:

$$\Phi(L) X_t = \rho D_t + \varepsilon_t, \ \forall t \geq 1 \tag{1}$$

where $\Phi(L) = I_N - \sum_{i=1}^{p} \Phi_i L^i$, $\Phi(0) = I_N$, and $\{\varepsilon_t\}_{t \geq 1}$ denotes a white noise process with a regular covariance matrix equal to Σ. D_t is a d-dimensional vector of zero-one dummy variables. Moreover, suppose that $\det \Phi(x)$ has all its roots outside or at $z = 1$. Accordingly, the white noise process $\{\varepsilon_t\}_{t \geq 1}$ is the process of the canonical innovations of the process X. It is useful to introduce the $(N, N(p+1))$ matrix $\Phi = \begin{bmatrix} I_N & -\Phi_1 & \cdots & -\Phi_p \end{bmatrix}$.

Then, long-run non-causality is characterized according to the following proposition:

Proposition 1 *Consider the N-dimensional process $X = \begin{pmatrix} X_1 & \cdots & X_N \end{pmatrix}'$, integrated of order one, whose dynamics obeys the VAR model (1). Then, X_j is not unidirectional prior causal for X_k in the long-run, if and only if the condition:*

$$\left\{\sum_i C_{ki}(1)\, \Phi_{ij}(L) = 0\right\}$$

or, equivalently,

$$\left\{ C_{kj}(1) = 0 \text{ and } \sum_{i \neq j} C_{ki}(1)\, \Phi_{ij}(L) = 0 \right\}$$

is satisfied, where $C(L)$ denotes the canonical moving-average operator of the Wold decomposition (MA) written for the first differences:

$$\Delta X_t = C(L)(\varepsilon_t + \rho D_t). \tag{2}$$

Proof: See Appendix.

It is worth emphasizing that the previous definition holds, whatever the cointegration rank is. Moreover the characterization of long-run non-causality is not affected by the introduction of dummy variables. Nevertheless it is obvious that the estimates of the VAR and MA parameters take into account the effects of these dummies on the dynamics.

Let us note that the "neutrality" condition, $\{C_{kj}(1) = 0\}$, as defined by Stock and Watson (1989), in an integrated, not cointegrated, bivariate system, appears as a necessary condition for excluding long-run causality, as shown by the non-causality condition of Proposition 1.

Moreover, in the bivariate case, the long-run non-causality condition reduces to the non-causality condition in the usual Granger (1969) sense:

$$\underbrace{\{C_{kj}(1) = 0 \text{ and } C_{kk}(1)\, \Phi_{kj}(L) = 0\}}_{\text{long-run non-causality}} \Leftrightarrow \underbrace{\{\Phi_{kj}(L) = 0\}}_{\text{Granger non-causality}}.$$

Indeed the following equivalences are true:

$$\{C_{kj}(1) = 0 \text{ and } C_{kk}(1)\, \Phi_{kj}(L) = 0\} \Leftrightarrow \{C_{kj}(1) = 0 \text{ and } \Phi_{kj}(L) = 0\}$$

and

$$\{C_{kj}(1) = 0 \text{ and } \Phi_{kj}(L) = 0\} \Leftrightarrow \{\Phi_{kj}(L) = 0\}.$$

The first equivalence results from the non-stationarity of the variables, which excludes the joint nullity of the long-run multipliers $C_{kj}(1)$ and $C_{kk}(1)$. The second equivalence is obtained by inverting the VAR model: in that case, $(1-L)\,X_t = \frac{\operatorname{adj}(\Phi(L))}{\varphi(L)}\varepsilon_t$ where $\det(\Phi(L)) = \varphi(L)(1-L)$ and $\operatorname{adj}(\Phi(L))$ denotes the adjoint of matrix $\Phi(L)$, and one obtains $C_{kj}(L) = -\frac{\Phi_{kj}(L)}{\varphi(L)}$.

By contrast, for higher dimensional systems, the variable X_j may be not unidirectionally prior causal for X_k in the long-run, but prior causal in the usual Granger sense $\{\Phi_{kj}(L) \neq 0\}$. Indeed, the following conditions may jointly hold:

$$\{\Phi_{kj}(L) \neq 0 \text{ and } \Phi_{ij}(L) \neq 0\} \qquad \text{(Granger causality)}$$

and

$$\{C_{kj}(1) = 0 \text{ and } (C_{kk}(1)\, \Phi_{kj}(L) + C_{ki}(1)\, \Phi_{ij}(L)) = 0\} \qquad \text{(long-run non-causality)}$$

As in the bivariate case, neutrality remains a necessary but not sufficient condition for long-run non-causality. More generally, the neutrality restriction appears to be an interesting restriction to test for long-run non-causality, although it is not sufficient to be interpreted as a prediction improvement property for long-run predictions.

It is also interesting to relate the long-run non-causality property to the significance of the error-correction terms in the ECM representation of the dynamics. Indeed, Granger

(1988) suggests to analyze "long-run" causality in a bivariate cointegrated system by focusing on the significance of the error-correction terms. Let us briefly recall the assumptions of the ECM representation which will be used later: this representation can be written as

$$\Gamma(L)\,\Delta X_t = \Pi X_{t-1} + \rho D_t + \varepsilon_t, \tag{3}$$

where $\Gamma(L) = I_N - \sum_{i=1}^{p-1} \Gamma_i L^i$, $\Gamma_i = -\sum_{j=i+1}^{p} \Phi_j$, $i = 1, ..., p-1$, and $\Pi = -\left(I_N - \sum_{i=1}^{p} \Phi_i\right)$. We define the $(N, N(p-1))$ matrix $\Gamma = \begin{bmatrix} \Gamma_1 & \cdots & \Gamma_{p-1} \end{bmatrix}$. If there are r $(0 < r \leq N)$ cointegration relationships, we have $\Pi = \alpha\beta'$, where α and β are (N, r) matrices, with β defining the cointegration vectors. Since ΔX_t is stationary, ΠX_{t-1} must be stationary too and it must be a linear function of $Z_{t-1} = \beta' X_{t-1}$. Equation (3) can be rewritten as:

$$\Gamma(L)\,\Delta X_t = \alpha Z_{t-1} + \rho D_t + \varepsilon_t. \tag{4}$$

In the bivariate cointegrated case, equation (4) reduces to, omitting dummy variables:

$$\begin{aligned} \Delta X_{1t} &= \gamma_{11}(L)\,\Delta X_{1t-1} + \gamma_{12}(L)\,\Delta X_{2t-1} + \alpha_1 Z_{t-1} + \varepsilon_{1t} \\ \Delta X_{2t} &= \gamma_{21}(L)\,\Delta X_{1t-1} + \gamma(L)\,\Delta X_{2t-1} + \alpha_2 Z_{t-1} + \varepsilon_{2t} \end{aligned}$$

where $Z_t = \beta_1 X_{1t} + \beta_2 X_{2t}$ and it can be proved that $(\alpha_1, \alpha_2) \neq (0, 0)$ (Engle and Granger, 1987). Thus, if the lagged error-correction term Z_{t-1} influences only ΔX_{2t} $(\alpha_1 = 0)$, then X_2 should not be prior causal for X_1 in the long run (Granger, 1988, Granger and Lin, 1995). Note that, in Granger and Lin (1995), the condition $\{\alpha_1 = 0\}$ is shown to be equivalent to the nullity of Hosoya's causality measure computed for frequencies ω near (but different) from zero, indicating something like non-causality in the long run. We note that condition $\{\alpha_1 = 0\}$ appears as a necessary condition of long-run non-causality as characterized in Definition 1. Indeed the condition $\{\alpha_1 = 0\}$ is equivalent to $C_{12}(1)$ being equal to zero, since $C(1)\,\alpha = 0$. Note however, that, like in the non-cointegrated case, in bivariate systems, long-run causality and causality in the usual Granger sense are the same, since condition $\{\Phi_{12}(L) = 0\}$ implies $\{\Phi_{12}(L) = 0 \text{ and } C_{12}(1) = 0\}$.[1]

Let us consider the trivariate case to illustrate the role plaid by an auxiliary variable in transmitting the long-run causal links. Suppose that we focus on long-run causality from X_2 to X_1 in the system $\{X_1, X_2, X_3\}$ whose dynamics has two cointegration relationships. In that case, $C(1)$ is of rank one. Suppose that $C_{11}(1) = 0$. Then $C(1)$ reduces to

$$C(1) = \begin{bmatrix} 0 & 0 & C_{13}(1) \\ 0 & 0 & C_{23}(1) \\ 0 & 0 & C_{33}(1) \end{bmatrix}.$$

Accordingly, the following non-causality condition holds:

$$\{C_{12}(1) = 0 \text{ and } A_{32}(L) = 0\}$$

because $C_{13}(1)$ cannot be null, according to the non-stationarity of X_1.

First, we notice that, as in the bivariate case, the neutrality is necessary but not sufficient to exclude long-run causality.

[1] $\text{adj}(C(L)) = (1-L)^{r-1}\Phi(L)$ (see Engle and Granger, 1987), and, for $r = 1$, $\text{adj}(C(L)) = \Phi(L)$.

Second the vector of error-correction terms

$$\begin{bmatrix} \alpha_{11} \\ \alpha_{21} \\ \alpha_{31} \end{bmatrix} \text{ and } \begin{bmatrix} \alpha_{12} \\ \alpha_{22} \\ \alpha_{32} \end{bmatrix}$$

must satisfy:

$$\{\alpha_{31} = \alpha_{32} = 0\}$$

because they obey:

$$C(1) \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \\ \alpha_{31} & \alpha_{32} \end{bmatrix} = 0.$$

The nullity of the error-correction terms appears one more time necessary but not sufficient to exclude long-run causality. More generally, the error-correction effects are better related to weak exogeneity properties as pointed out by Johansen (1992) and Urbain (1992).

Third the non-causality condition highlights the crucial role plaid by X_3. Indeed long-run non-causality implies the nullity of $A_{32}(L)$.

Finally it is worth emphasizing that this condition cannot be expressed as a linear constraint on the parameters of the VAR in levels or the VECM models.

3 Statistical Analysis of Long-Run Causality

To implement the test for long-run non-causality condition, we have to estimate a function of the long-run dynamic multipliers $C(1)$. As $C(1)$ cannot be directly estimated, we have to estimate an auxiliary VAR representation, which is inverted in a second step. In the cointegrated case, a useful alternative representation of VAR in levels and ECM is the restricted VAR representation (RVAR) with p lags (Campbell and Shiller, 1988, Mellander *et alii*, 1990, Warne, 1993):

$$B(L) Y_t = \tilde{\rho} D_t + \eta_t, \tag{5}$$

where:

$Y_t = D_{\perp}(L) M X_t$, $M = \begin{bmatrix} S_{N-r} \\ \beta' \end{bmatrix}$, $\eta_t = M\varepsilon_t$, $\tilde{\rho} = M\rho$,

$D(L) = \begin{bmatrix} I_{N-r} & 0 \\ 0 & (1-L) I_r \end{bmatrix}$, $D_{\perp}(L) = \begin{bmatrix} (1-L) I_{N-r} & 0 \\ 0 & I_r \end{bmatrix}$.

The $(N-r, N)$ matrix S_{N-r} can be chosen such that: $S_{N-r} = \begin{bmatrix} I_{N-r} & 0_{(N-r,r)} \end{bmatrix}$.

It is easy to see that if $\{X_t\}$ is cointegrated of order (1,1), with r cointegration relationships, then the following relation holds between the parameters of the ECM and the RVAR models:

$$B(L) = M\left[\Gamma(L) M^{-1} D(L) - \alpha^* L\right] \tag{6}$$

where $\underset{(N,N)}{\alpha^*} = \begin{bmatrix} 0_{(N,N-r)} & \alpha \end{bmatrix}$.

Given the RVAR representation, long-run dynamic multipliers are deduced by simply inverting the $B(L)$ matrix of polynomials as:

$$C(L) = M^{-1} D(L) B(L)^{-1} M \tag{7}$$

in which dynamic multipliers C_h (with $C(L) = \sum_{h=0}^{\infty} C_h L^h$) are directly introduced (Mellander *et alii*, 1990).

3.1 Estimation of the Long-Term Dynamic Multipliers and Test for Neutrality

The estimation of the long-term dynamic multipliers is based on the RVAR representation. This allows us to estimate directly these multipliers and not their approximation $\sum_{h=0}^{H} C_h$ obtained for a H large enough, as in Lütkepohl and Reimers (1992) who used the VAR in levels, estimated from the ECM. Once the matrix M (containing the ECM long-run parameters), and $B(L)$ (the parameters of the RVAR representation) are estimated, one can directly deduce the estimators of long-term dynamic multipliers and their asymptotic distribution. Note that M can be considered as known. This result comes from the rapid convergence of the long-run parameters (Stock, 1987, and Lütkepohl and Reimers, 1992).

We assume that N time series $X = \begin{pmatrix} X_1 & \cdots & X_N \end{pmatrix}'$ of length T and p presample values are available. We define $\Delta X = \begin{pmatrix} \Delta X_1' & \cdots & \Delta X_T' \end{pmatrix}'$. The parameters of interest from the ECM are $\Theta = \begin{bmatrix} -\Gamma & -\alpha \end{bmatrix}$, with $\theta = \text{vec}(\Theta)$, but one also has to deal with $\Psi = \begin{bmatrix} I_N & -\Gamma & -\alpha \end{bmatrix} = \begin{bmatrix} I_N & \theta \end{bmatrix}$ in the following, with $\psi = \text{vec}(\Psi)$. We also define the $(N(Np+r), N(N(p-1)+r))$ matrix $\zeta = \frac{\partial \psi}{\partial \theta'} = \begin{bmatrix} 0_{(N^2, N(p-1)+r)} \\ I_{(N(p-1)+r)} \end{bmatrix}$. "*vec*" denotes the column stacking operator.

At a first step, one has to test for the cointegration rank r and to estimate the cointegration vectors β. For this purpose, one can use the Johansen's (1988) maximum-likelihood procedure to estimate the parameters of the ECM representation (3) and their asymptotic distribution. The asymptotic distribution of θ is given by (Johansen, 1988), by denoting $X_{-1} = \begin{pmatrix} X_{1-p}' & \cdots & X_{T-p}' \end{pmatrix}'$ and $\tilde{X} = \begin{pmatrix} \tilde{X}_1' & \cdots & \tilde{X}_T' \end{pmatrix}'$ where $\tilde{X}_t = \begin{pmatrix} \Delta X_{t-1} & \cdots & \Delta X_{t-p} \end{pmatrix}$:

$$\sqrt{T}\left(\hat{\theta} - \theta\right) \xrightarrow{d} N\left(0, \Sigma_\theta\right)$$

with

$$\Sigma_\theta = \begin{bmatrix} I_{N(p-1)} & 0 \\ 0 & \beta' \end{bmatrix} \Omega^{-1} \begin{bmatrix} I_{N(p-1)} & 0 \\ 0 & \beta' \end{bmatrix}' \otimes \Sigma$$

where

$$\Omega = \text{plim} \frac{1}{T} \begin{bmatrix} \tilde{X}\tilde{X}' & \tilde{X}X_{-1}'\beta' \\ \beta X_{-1}\tilde{X}' & \beta X_{-1}X_{-1}'\beta' \end{bmatrix}$$

and Σ is the covariance matrix of ε.

At a second step, the estimators of the RVAR parameters and their asymptotic covariance matrix are derived from the estimators of the ECM parameters. Accordingly, one can easily obtain the $B(1)$ parameters from (6). In order to obtain the asymptotic covariance matrix of $B(1)$, one uses the assumption that long-run parameters, contained in M, can be treated as known, as pointed out before.

Then from (7), the long-run dynamic multipliers $C(1)$ are estimated by:

$$C(1) = M^{-1}D(1)B(1)^{-1}M = M^{-1}D(1)\left[M\Gamma(1)M^{-1}D(1) - M\alpha^*\right]^{-1}M$$

that is:

$$C(1) = M^{-1}D(1)\left[M\Psi G\right]^{-1}M \tag{8}$$

where $\underset{(Np+r,N)}{G} = \begin{bmatrix} \Lambda M^{-1}J & 0_{(Np,r)} \\ 0_{(r,N-r)} & I_r \end{bmatrix}$, $\underset{(Np,N)}{\Lambda} = \frac{\partial vec(\Gamma(1))}{\partial vec(\Gamma)'} = e_p \otimes I_N$,
and $\underset{(N,N-r)}{J} = \begin{bmatrix} I_{N-r} \\ 0_{(r,N-r)} \end{bmatrix}$. e_p is the $(p,1)$ vector of ones.

Finally, the asymptotic distribution of the long-run dynamic multipliers is given by the following proposition, by denoting $c = \text{vec}(C(1))$:

Proposition 2 *If X_t is a Gaussian process as in (1), the asymptotic distribution of long-term dynamic multipliers is given by:*

$$\sqrt{T}(\hat{c} - c) \xrightarrow{d} N(0, \Sigma_c) \tag{9}$$

with

$$\Sigma_c = \lambda \Sigma_\theta \lambda'$$

and

$$\lambda = \left(M' \otimes M^{-1}D(1)\right)\left(\left((M\Psi G)^{-1}\right)' \otimes (M\Psi G)^{-1}\right)(G' \otimes M)\zeta.$$

Proof: The asymptotic covariance matrix of $\hat{c}$ is given by:

$$\Sigma_c = \frac{\partial c}{\partial \theta'}\Sigma_\theta \frac{\partial c'}{\partial \theta}$$

with

$$\frac{\partial c}{\partial \theta'} = \frac{\partial c}{\partial \text{vec}\left[M\Psi G\right]^{-1\prime}} \frac{\partial \text{vec}\left[M\Psi G\right]^{-1}}{\partial \text{vec}\left[M\Psi G\right]'} \frac{\partial \text{vec}\left[M\Psi G\right]}{\partial \psi'} \frac{\partial \psi}{\partial \theta'}$$

such that

$$\frac{\partial c}{\partial \theta'} = \left(M' \otimes M^{-1}D(1)\right)\left(\left((M\Psi G)^{-1}\right)' \otimes (M\Psi G)^{-1}\right)(G' \otimes M)\zeta. \blacksquare$$

Note the asymptotic distribution of $\hat{c}$ is also given in Johansen (1995) or in Paruolo (1997). From this proposition, it is easy to perform tests for neutrality. One has just to estimate the ECM, derive the RVAR parameters, and apply Proposition 2.

3.2 Testing for Long-Run Non-Causality

Now, if one wants to test for the null hypothesis that X_j is not prior causal for X_k in the long-run, one has to estimate the expression: $\sum_{i=1}^{N} C_{ki}(1)\Phi_{ij}(L)$ for j and k, which can be written as $u_k' C(1)\Phi v_j$. The $(N, N(p+1))$ matrix Φ contains the parameters of the VAR in levels. It can be estimated as:

$$\Phi = \begin{bmatrix} I_N & -\Phi_1 & \ldots & -\Phi_p \end{bmatrix} = \begin{bmatrix} I_N & -\hat{\Gamma} & -\hat{\alpha} \end{bmatrix} D = \Psi D$$

where:

$$\underset{(Np+r,N(p+1))}{D} = \begin{bmatrix} I_N & -I_N & 0 & \cdots & 0 & 0 \\ 0 & I_N & -I_N & \ddots & \vdots & \vdots \\ 0 & 0 & I_N & \ddots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & -I_N & 0 \\ 0 & 0 & \cdots & 0 & I_N & -I_N \\ 0 & \beta' & 0 & \cdots & 0 & 0 \end{bmatrix}.$$

The $(N,1)$ selection vector u_k has unity in the kth element and zero elsewhere, and v_j is the $(N(p+1),p)$ matrix, the lth column of which has unity in the $(N(l-1)+j)$th row, for $l=1,...,p$. Note that v_j is a $(N(p+1),p)$ (not $(N(p+1),p+1)$) matrix, because one constraint is redundant among the p last (N,N) matrices of $C(1)\Phi$. This can be easily seen, since one has:

$$C(1)\Phi = \left[\begin{array}{cccccc} C(1) & C(1)(-I_N-\Gamma_1) & C(1)(\Gamma_1-\Gamma_2) & \cdots & C(1)(\Gamma_{p-2}-\Gamma_{p-1}) & C(1)\Gamma_{p-1}\end{array}\right],$$

where the sum of the $(p+1)$ matrices is null. This result is due to the fact that the second matrix, which should be $C(1)(-I_N-\Gamma_1-\alpha\beta')$, reduces to $C(1)(-I_N-\Gamma_1)$ (because cointegration implies $C(1)\alpha=0$). So only the p first non-redundant constraints are tested. Then, we define the estimator of the function:

$$g_{kj}(\theta) = vec\left(u_k' C(1)\Phi v_j\right)$$

which can be rewritten as:

$$g_{kj}(\theta) = \text{vec}\left(u_k'\left[M^{-1}D(1)\left(M\Psi G\right)^{-1}\Psi D\right]v_j\right) = \Lambda_{kj}\text{vec}\left[\left(M\Psi G\right)^{-1}\Psi D\right] \tag{10}$$

where: $\Lambda_{kj} = v_j' \otimes u_k' M^{-1}D(1)$.

Proposition 3 *If X_t is a Gaussian process as in (1), the asymptotic distribution of $g_{kj}(\theta)$ is:*

$$\sqrt{T}\left(g_{kj}\left(\hat{\theta}\right)-g_{kj}(\theta)\right)\xrightarrow{d}N\left(0,\Sigma_{g_{kj}}\right),\quad \forall k,j=1,...,N,k\neq j, \tag{11}$$

where:

$$\Sigma_{g_{kj}} = \frac{\partial g_{kj}}{\partial\theta'}\Sigma_\theta\frac{\partial g_{kj}'}{\partial\theta}$$

and

$$\frac{\partial g_{kj}}{\partial\theta'} = \frac{\partial g_{kj}}{\partial\psi'}\zeta \tag{12}$$

with

$$\begin{aligned}\frac{\partial g_{kj}}{\partial\psi'} &= \Lambda_{kj}\frac{\partial}{\partial\psi'}vec\left[\left(M\Psi G\right)^{-1}\Psi D\right]\\ &= -\Lambda_{kj}\left((\Psi D)'\otimes I_N\right)\left(\left(M\Psi G\right)^{-1\prime}\otimes\left(M\Psi G\right)^{-1}\right)\left(G'\otimes M\right)\\ &\quad+\Lambda_{kj}\left(I_{N(p+1)}\otimes\left(M\Psi G\right)^{-1}\right)\left(D'\otimes I_N\right).\end{aligned}$$

Proof: This result is obtained from equation (10), which implies that:

$$\begin{aligned}\frac{\partial g_{kj}}{\partial\psi'} &= \Lambda_{kj}\left[\frac{\partial}{\partial\psi'}\text{vec}\left(\left(M\Psi G\right)^{-1}\Psi D\right)\right]\\ &= \Lambda_{kj}\left[\left((\Psi D)'\otimes I_N\right)\frac{\partial}{\partial\psi'}\text{vec}\left(M\Psi G\right)^{-1}\right.\\ &\quad\left.+\left(I_{N(p+1)}\otimes\left(M\Psi G\right)^{-1}\right)\frac{\partial}{\partial\psi'}\text{vec}\left(\Psi D\right)\right].\ \blacksquare\end{aligned}$$

It is now possible to propose a test statistics of long-run non-causality:

Proposition 4 *If X_t is a Gaussian process as in (1), the test of H_0: $g_{kj}(\theta) = 0$ is based on the statistics:*

$$\xi_{kj} = T g_{kj}\left(\hat{\theta}\right)' \left(\hat{\Sigma}_{g_{kj}}\right)^{-1} g_{kj}\left(\hat{\theta}\right), \quad \forall k, j = 1, ..., N, k \neq j, \tag{13}$$

which is distributed as a χ^2 with p degrees of freedom.

Proof: The asymptotic distribution of ξ_{kj} is directly deduced from equation (10). The degree of freedom (p) comes from the fact that p non-redundant constraints are tested. ■

In derivating our test statistic, we use the VAR in levels and the MA representations. This reference to the MA representation is intended to clarify the presentation, since the long-run non-causality is defined as $C(1)\Phi(L) = 0$. Besides, the neutrality condition, which appears as a necessary condition to exclude long-run causality, is directly defined as $C_{kj}(1) = 0$. But it is worth noting that the expression of $g_{kj}(\theta)$ in equation (10) essentially depends on the ECM parameters (θ). Thus the implementation of our test, using equation (13)), can be directly derived from the ECM parameters only.

4 Illustration

By way of illustration, we use the previous definition of causality to analyze international links between long-term interest rates. Many studies have focused on causal links between interest rates. For example, Arshanapalli and Doukas (1994) explored common trends in systems of Eurorates. More specifically, some authors questioned the asymmetric functioning of the EMS, by testing the existence of causality from German rates toward other EMS member countries rates (Germany being seen as the leading country of the EMS). See, e.g., von Hagen and Fratianni (1990), Kirchgässner and Wolters (1993), and Henry and Weidmann (1995).

Many authors studied international linkages of long-term interest rates, often from the US-European connection point of view (for example, Cumby and Mishkin, 1986, Kirchgässner and Wolters, 1987, Johnson, 1993, Hansen, 1996). Generally these studies did not take into account the possible long-term relationships between interest rates (with the exception of Hansen, 1996). To shed some light on international long-run causal links between long-term interest rates, we analyze a system of US, German, and French long-term interest rates. The dataset consists of 10-year benchmark interest rates for the Deutschmark (DEM), the French franc (FRF), and the US dollar (USD). The sample covers weekly data from January 5, 1990 to June 27, 1997 (Graph 1).[2] We aim at comparing the results obtained from the various approaches commonly used to assess causal links "in the long run" —that is error-correction terms and neutrality analysis— and our long-run causality approach.

4.1 Cointegration Analysis

As a first step, augmented Dickey-Fuller tests indicate that the null hypothesis of a unit root cannot be rejected for each of the interest rate series, but that it is rejected for each

[2]Data come from Datastream. All the computations were done using GAUSS. Our GAUSS code of the programm is available upon request by e-mail.

of the changes in interest rates (the results are reported in Table 1). These results are confirmed by the multivariate tests for stationarity (based on the estimates of the ECM using maximum likelihood methodology). We then conclude that 10-year interest rates are integrated of order one.

The ECM for the joint dynamics of US, German and French rates is estimated using Johansen's (1988) maximum likelihood methodology. Tests for the deterministic components of the system lead us to allow a constant term in the cointegration relationships. The optimal lag length, selected by the HQ (Hannan and Quinn, 1979) information criterion[3], is $p = 4$, whatever the cointegration rank. Lastly, based on the usual maximal eigenvalue λ_{max} and trace statistics λ_{trace}, proposed by Johansen (1988) and Johansen and Juselius (1990), at the significance level of 5%, we choose one cointegration vector ($r = 1$) (Table 2). The corresponding estimated cointegration vector can be written, when normalized with the USD coefficient:

$$USD_t - 4.677DEM_t + 3.062FRF_t + 3.166 = z_t$$

which is depicted in Graph 2.

Exclusion tests (not reported here) indicate that none of the variables can be excluded from the long-term relationship. The ECM estimates are reported in Table 3. Some dummy variables have been included in the model to take account of large shocks on German and French interest rates.[4] Multivariate and univariate misspecification test statistics are reported in Table 4. Residuals are not serially correlated; they do not present any ARCH effects, and they satisfy the normality assumption. Graph 3 shows the residuals.

More information regarding the links between the variables of the system can be obtained from the ECM estimates. First the error-correction terms are significant in the three equations, although at a 8 percent significance level only for German rates. Therefore all rates are causal vis-à-vis each other, and, accordingly, one cannot identify any leading rate in the long-run, according to the definition proposed by Granger (1988) and Granger and Lin (1995).[5]

We then turn to the so-called short-run causality, which involves the first differences of the variables. This term is not quite appropriate, since it has something to do with the long-run properties of the series, as indicated by the long-run non-causality condition (proposition 2). But this condition involves AR parameters and, accordingly, the parameters of the "short-run dynamics" of the ECM. We notice that the US rate have a large and significant effect on the German rate; conversely the German rate has a smaller impact on the US rate. In the short run, the German rate also influences the French rate, whose lagged changes are never significant.

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[3]The HQ information criterion for a model i is defined as: $HQ_i = \ln(\det(\Sigma_i)) + 2p_i \ln(\ln(T))/T$, where Σ_i is the covariance matrix of the residuals of model i, p_i is the number of estimated parameters in model i, T is the number of observations.

[4]The dummy variables are for 92:09:04, 94:06:17, 94:07:29, 94:09:30 and 97:01:17. The first one is intended to take into account the strong decrease in European rates; we introduce the dummy variables of 94 to explain the large movements at the end of the period of rising rates; lastly the goal of the 97 dummy variable is to account for the large decrease in French interest rate. It is worth noting that these dummy variables help to obtain normality of residuals.

[5]Note that the analysis of the so-called matrix of long-run responses, Π, provides the same interpretation, since we just have one cointegration vector.

To summarize, the analysis of the ECM estimates does not provide us with a clear identification of the leading rates in the long run: the German or/and US rate? This is the reason why we turn to neutrality and long-run causality analysis.

4.2 Neutrality and Long-Run Non-Causality Results

Since there is only one cointegration vector, two independent stochastic trends drive the long-run dynamics of the system. Table 5 reports first the estimates of the orthogonal complement to α, which indicates the contribution of each normalized residuals to each of the common trends, as pointed out by Juselius (1996); second the factor loadings, which describe how each variable is affected by the common trends; lastly the t-statistics for the neutrality test.

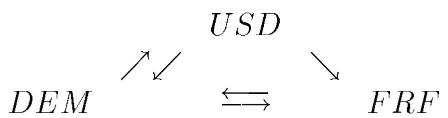
The estimates of the $\alpha_{\perp}$ suggest that both common trends are determined by the innovations of each of the rates. Moreover, as indicated by the estimates of the factor loadings, both trends appear to have an impact on the dynamics of each rate. However, the results are difficult to interpret because of the well-known problem of the identification of the common trends.

Now, if we perform neutrality tests, at the significance level of 5%, we obtain the main following results: the US rate is only influenced by the German rate, and conversely the German rate only depends on the innovations of the US rate. Lastly, the French rate is influenced by the German innovations, but not by the US innovations. Accordingly, we partly confirm the results obtained from the causal analysis performed in the ECM framework, since the US rate and the German rate appear jointly as causal variables in the long run. However, the French rate has lost any causal role in the long run, contrary to the indications given by the analysis of the ECM.

So, the two types of analysis do not give us a clear interpretation of the causal links in the long run. Actually, as pointed out before, neither the analysis of the error-correction effects nor the neutrality tests allow to measure the long-run causal links, according to our definition.

Lastly, we perform the test of long-run non-causality (Table 6). For example, the test statistic for the long-run causality of the US rate toward the German rate is $\xi_{21} = 15.818$. Since the test statistic is asymptotically distributed as $\chi^2(p)$, with $p = 4$, we conclude that the US rate plays a long-run causal role toward the German rate at a 1 percent significance level. More generally there is a bidirectional long-run causality between US and German rates, on the one hand, and, between German and French rates, on the other hand. However the causal link from the French rate toward the German rate is quite low. It is worth noting that we identify a small direct long run causal link from the US rate to the French rate, which is significant at a 10 percent level (Figure 1).

Figure 1: long-term causal links between 10-year interest rates



We verify that neutrality is necessary but not sufficient to exclude long-run causality. First, neutrality tests indicate that the innovations in the French rate do not influence the German rate. On the contrary, according to long-run non-causality tests, we cannot reject long-run causality from the French rate toward the German rate. Second, we also obtain a long-run causality from the US rate toward the French rate, whereas we do not reject the neutrality condition.

Our illustration shows that interpreting separately error-correction terms as well as neutrality properties can be misleading in identifying long-run causal links. When they are jointly investigated, they can even lead to opposite conclusions regarding these links. By contrast, tests for long-run non-causality shed light on causal links between the long-term interest rates under study, which are clearly defined, well identified and, lastly, plausible.

5 Conclusion

In this paper, we give a definition for long-run non-causality in terms of long-run prediction improvement for long-term predictions. We prove that non-causality in that sense can be easily characterized for integrated and possibly cointegrated VAR dynamics. Indeed, the non-causality condition can be expressed as the nullity of a function of long-run dynamic multipliers and the parameters of the VAR in levels. This result shows that the properties which are usually related to long-run causality (error-correction effects, neutrality) are not sufficient to be interpreted as long-run prediction properties.

From an empirical point of view, the test procedure can be easily implemented once the long-run features (stationarity, cointegration) of the dynamics have been identified: non-causality can be tested by a standard Wald test, distributed as a chi-square.

By way of illustration, we performed an analysis of long-run causal links between US, German, and French long-term interest rates and presented evidence that there exists reciprocal long-run causality between the US rate and the German rate as well as between the German rate and the French rate. Beyond the application, the concepts and statistical tools developed in this paper should be useful to characterize more precisely the long-run properties of multivariate systems.

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Table 1: Test for non-stationarity of interest rates

currency	ADF tests level	ADF tests first difference	Multivariate tests $r = 1$	Multivariate tests $r = 2$
USD	-1.98	-8.46 [a]	18.67 [a]	10.04 [a]
DEM	-0.62	-18.97 [a]	18.29 [a]	9.59 [a]
FRF	-1.29	-5.56 [a]	17.86 [a]	9.44 [a]

Note: The ADF statistics are based on the following regression: $\Delta x_t = a + bx_{t-1} + \sum_{i=1}^{l} c_i \Delta x_{t-i} + u_t$, where x_t is the interest rate, u_t is the error term. The order of the autoregressive process, l, is selected in order to whiten the residuals. The critical values are from Fuller (1976). The multivariate tests are based on the Johansen's methodology. They are based on a constant term in the cointegration relations, $p = 4$ lags and conditional to the cointegration rank r. The test statistics have a chi-squared distribution with $p - r$ degrees of freedom.

For all tables, the sample goes from January 5, 1990 to June 27, 1997. [a], [b] and [c] indicate that the statistics are significant at a 1%, 5% and 10% significance level.

Table 2: Tests for the cointegration rank

$N - r$	λ	λ_{max}	λ_{trace}
1	0.008	3.290	3.290
2	0.030	11.888	15.178
3	0.058	23.325 [b]	38.503 [b]

Note: The estimation is performed with a constant term in the cointegration relations and $p = 4$. N is the number of variables, r is the cointegration rank. λ contains the eigenvalues of the system. λ_{max} and λ_{trace} are the cointegration test statistics proposed by Johansen (1988). The critical values for λ_{max} and λ_{trace} are from Osterwald-Lenum (1992, Table 2*).

Table 3: Estimates of the ECM

	USD eq.	DEM eq.	FRF eq.
$\boldsymbol{\alpha}$	-0.014	0.010	-0.020
(error-corr. term)	(1.964)	(1.735)	(4.122)
$\boldsymbol{\Gamma}_1$			
USD_t	-0.094	0.099	-0.014
	(1.850)	(2.512)	(0.400)
DEM_t	0.043	0.050	0.022
	(0.672)	(1.001)	(0.511)
FRF_t	0.078	0.057	-0.100
	(1.091)	(1.038)	(2.056)
$\boldsymbol{\Gamma}_2$			
USD_t	0.021	0.096	0.055
	(0.409)	(2.402)	(1.554)
DEM_t	0.147	-0.032	0.682
	(2.313)	(0.656)	(15.832)
FRF_t	-0.017	-0.049	0.001
	(0.314)	(1.157)	(0.018)
$\boldsymbol{\Gamma}_3$			
USD_t	0.037	0.064	-0.033
	(0.723)	(1.634)	(0.942)
DEM_t	0.082	0.023	0.173
	(1.034)	(0.383)	(3.233)
FRF_t	-0.005	-0.067	-0.002
	(0.098)	(1.565)	(0.064)
$\boldsymbol{\rho}_1$	-0.249	-0.331	-0.098
(92:09:04)	(1.986)	(3.425)	(1.147)
$\boldsymbol{\rho}_2$	0.107	0.066	0.292
(94:06:17)	(0.828)	(0.670)	(3.346)
$\boldsymbol{\rho}_3$	-0.171	0.446	-0.053
(94:07:29)	(1.353)	(4.575)	(0.621)
$\boldsymbol{\rho}_4$	0.018	-0.388	-0.088
(94:09:30)	(0.145)	(3.979)	(1.025)
$\boldsymbol{\rho}_5$	-0.066	-0.150	-0.354
(97:01:17)	(0.519)	(1.547)	(4.139)

Note: α denotes the error-correction vector; Γ_i, $i = 1, 2, 3$, denotes the matrix containing the estimates of the parameters associated to lag i. ρ_i, $i = 1, ..., 5$, denotes the matrix containing the estimates of the parameters associated to dummy variable i. Numbers in parentheses indicate t-statistics.

Table 4: Misspecification tests

Multivariate tests			
Residual autocorrelation	statistic	p-value	
LB: $\chi^2\,(36)$	337.832	(0.15)	
LM_1: $\chi^2\,(9)$	4.607	(0.87)	
JB: $\chi^2\,(6)$	12.818	(0.05)	
Univariate tests	ARCH(4)	Norm(2)	R^2
USD	3.928	4.550	0.083
DEM	8.468^c	3.665	0.157
FRF	3.419	4.602	0.517

Note: LB is the Ljung-Box statistic for residual autocorrelation up to order 36. LM_1 is the Lagrange Multiplier statistic for first order residual autocorrelation. JB is the Jarque-Bera statistic testing for normality. ARCH is the Lagrange Multiplier statistic for squared residual autocorrelation up to order 4. Norm is the univariate Jarque-Bera statistic testing for normality.

Table 5: Analysis of common trends and test for neutrality

Panel A:	Analysis of common trends			
	$\alpha_{\perp}$		factor loadings	
USD	0.546	−0.411	0.186	0.794
DEM	−0.083	−0.894	0.246	0.064
FRF	−0.834	−0.180	0.315	−0.161

Panel B:	Test statistics for $C_{kj}(1) = 0$		
from:	to: USD	DEM	FRF
USD	0.980 [a]	0.310 [a]	0.154
DEM	0.632 [a]	0.897 [a]	1.164 [a]
FRF	-0.387	0.211	0.449 [c]

Note: $\alpha_{\perp}$ denotes the orthogonal complement to α. The second part of the Table reports test statistics for $C_{kj}(1) = 0$, which are asymptotically normally distributed. Numbers in parentheses indicate t-statistics.

Table 6: Tests for long-run non-causality

	Test statistics for $(C(1)\Phi(L))_{kj} = 0$		
from:	to: USD	DEM	FRF
USD	–	15.810 [a]	9.423 [c]
DEM	11.725 [b]	–	83.787 [a]
FRF	4.090	8.101 [c]	–

Note: The test statistics for $(C(1)\Phi(L))_{kj} = 0$ are asymptotically χ^2 distributed with p degrees of freedom under the null hypothesis of long-run non-causality. 15.810 is the test statistics for the null of long-run non-causality from the US rate to the German rate.

Appendix: Proof of Proposition 1.

Let us suppose that X_j is not unidirectionally prior causal for X_k in the long run. Then, we have, for any date $t \geq 1$, the equality:

$$\lim_{H\to+\infty} EL\left(X_{k,t+H}\left|\left\{\underline{X_{i,t}}; 1 \leq i \leq N\right\}\right.\right) = \lim_{H\to+\infty} EL\left(X_{k,t+H}\left|\left\{\underline{X_{i,t}}; 1 \leq i \leq N, i \neq j\right\}\right.\right).$$

In other words, the variables $X_{j,1}$, ..., $X_{j,t}$ must not enter the limit:

$${}_t\hat{X}_{k,\infty} = \lim_{H\to+\infty} EL\left(X_{k,t+H}\left|\left\{\underline{X_{i,t}}; 1 \leq i \leq N\right\}\right.\right).$$

So one has to make sure that the relevant coefficients disappear.

i) Let us first focus on the simple case where the order of the VAR is equal to one ($p = 1$). Then we find the equations:

$$\Delta X_t = \alpha\beta' X_{t-1} + \varepsilon_t,$$

and the results:

$$EL\left(X_{k,t+H}\left|\left\{\underline{X_{i,t}}; 1 \leq i \leq N\right\}\right.\right) = \left(I_N + \alpha\beta'\right)^H X_t.$$

Accordingly, the best linear prediction:

$$\beta' EL\left(X_{k,t+H}\left|\left\{\underline{X_{i,t}}; 1 \leq i \leq N\right\}\right.\right) = \beta' \left(I_N + \alpha\beta'\right)^H X_t$$

has to tend to 0, when H tends to infinity.

Moreover, we must have:

$$\begin{aligned}\alpha'_\perp EL\left(X_{k,t+H}\left|\left\{\underline{X_{i,t}}; 1 \leq i \leq N\right\}\right.\right) &= \alpha'_\perp \left(I_N + \alpha\beta'\right)^H X_t \\ &= \alpha'_\perp X_t.\end{aligned}$$

Finally, the best linear prediction ${}_t\hat{X}_{t+H} = EL\left(X_{k,t+H}\left|\left\{\underline{X_{i,t}}; 1 \leq i \leq N\right\}\right.\right)$, which can be written as:

$${}_t\hat{X}_{t+H} = \beta_\perp \left(\alpha'_\perp\beta_\perp\right)^{-1} \alpha'_{\perp t}\hat{X}_{t+H} + \alpha\left(\beta'\alpha\right)^{-1} \beta'_t\hat{X}_{t+H}$$

has to tend to CX_t with C defined as:

$$C = \beta_\perp(\alpha'_\perp\beta_\perp)^{-1}\alpha'_\perp.$$

In this case, if one does not want the limit expression for ${}_t\hat{X}_{k,\infty}$ to depend on $X_{j,t}$, one clearly needs: $C_{kj} = 0$.

ii) In the general case, where $p > 1$, we just have to consider the equation:

$$\Delta X_t = \alpha\beta' X_{t-1} + \sum_{h=1}^{p-1} \Gamma_h \Delta X_{t-h} + \varepsilon_t,$$

in companion form:

$$\Delta\tilde{X}_t = \tilde{\alpha}\tilde{\beta}'\tilde{X}_{t-1} + \tilde{\varepsilon}_t$$

with $\tilde{X}_t = \begin{pmatrix} X'_t & \cdots & X'_{t-p+1} \end{pmatrix}'$.

So, we find that:

$$\begin{aligned}
\tilde{\beta}_\perp &= \begin{pmatrix} \beta'_\perp & \cdots & \beta'_\perp \end{pmatrix}' \\
\tilde{\alpha}'_\perp &= \begin{pmatrix} \alpha'_\perp & -\alpha'_\perp \Gamma_1 & \cdots & -\alpha'_\perp \Gamma_{p-1} \end{pmatrix} \\
\tilde{\alpha}'_\perp \tilde{\beta}_\perp &= \tilde{\alpha}'_\perp \left(I_N - \sum_{h=1}^{p-1} \Gamma_h \right) \tilde{\beta}_\perp \\
\tilde{C} &= \tilde{\beta}_\perp \left(\tilde{\alpha}'_\perp \tilde{\beta}_\perp \right)^{-1} \tilde{\alpha}'_\perp .
\end{aligned}$$

Hence, the best linear limit prediction ${}_t\hat{X}_\infty$ can be written as:

$${}_t\hat{X}_\infty = C \left(X_t - \sum_{h=1}^{p-1} \Gamma_h \Delta X_{t-h} \right)$$

so that, if one wants ${}_t\hat{X}_{k,\infty}$ to be independent of $X_{j,1}, ..., X_{j,t}$, one needs the conditions:

$$(C\Gamma_h)_{kj} = 0, \text{ for any } h$$

or, equivalently:

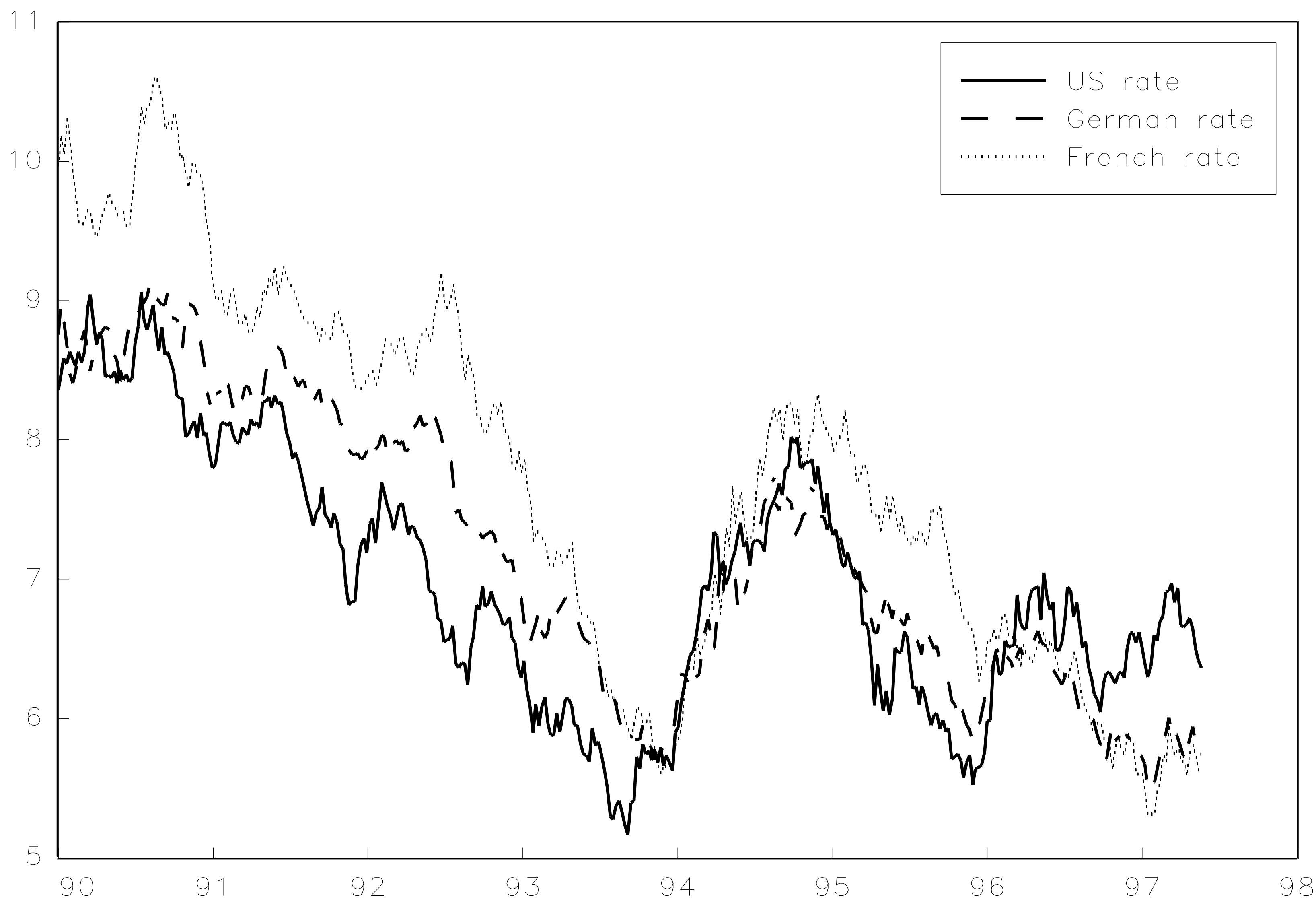
$$(C\Phi(L))_{kj} = 0$$

that is, exactly, the condition given in the proposition:

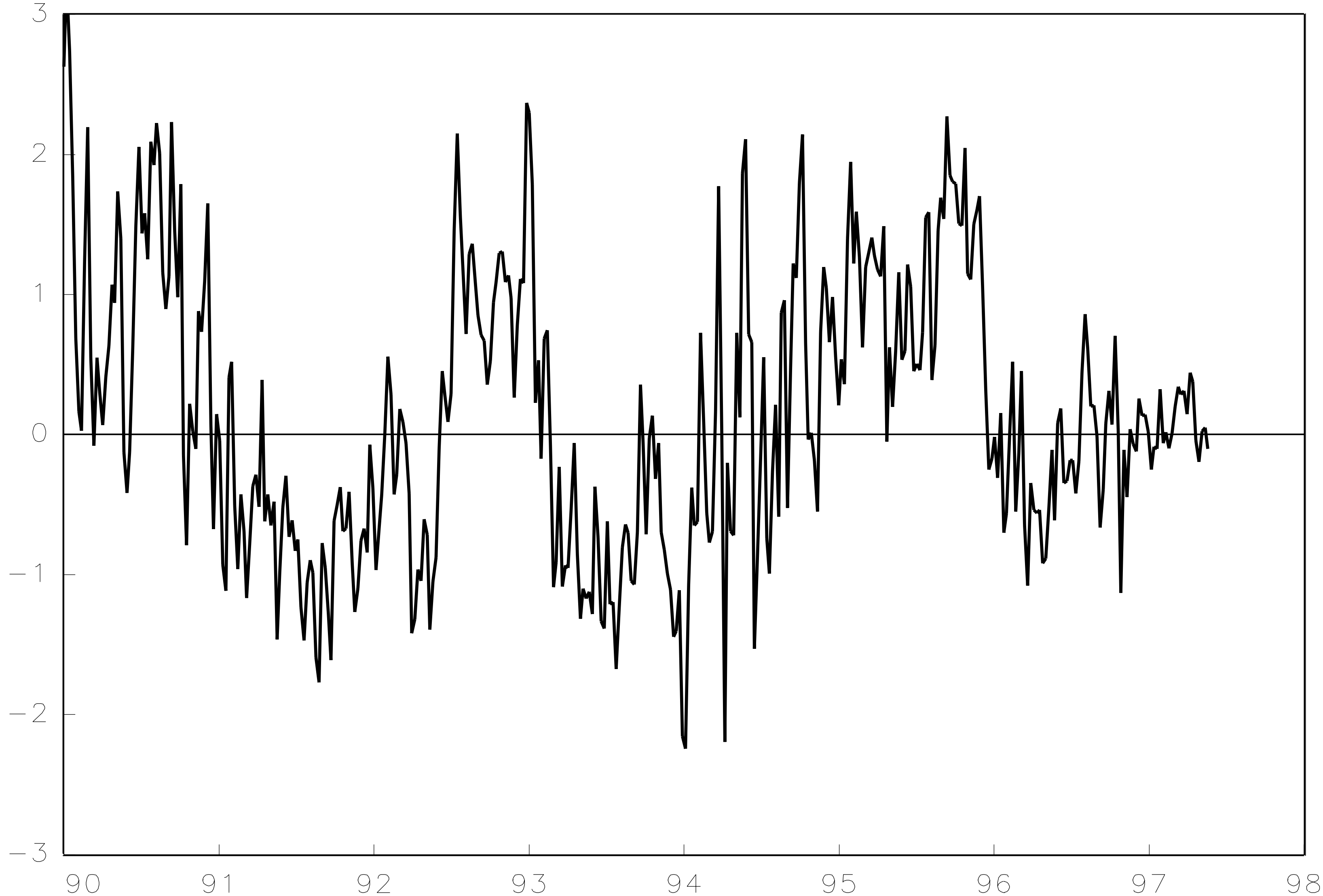
$$\sum_{i=1}^{N} C_{ki}(1) \sum_{l=1}^{N} \Phi_{ij}(L) = 0.$$

■

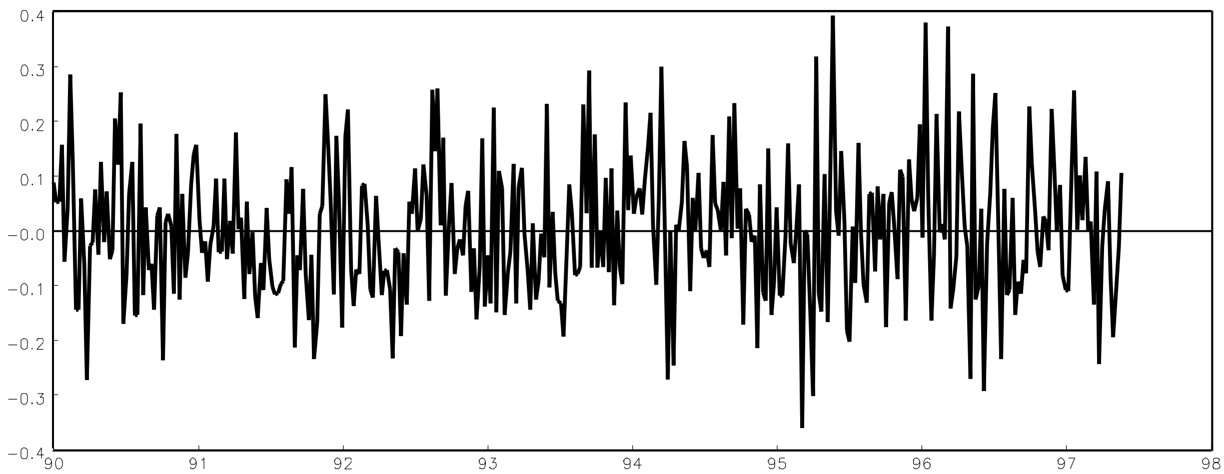
Graph 1: Long-Term Interest Rates



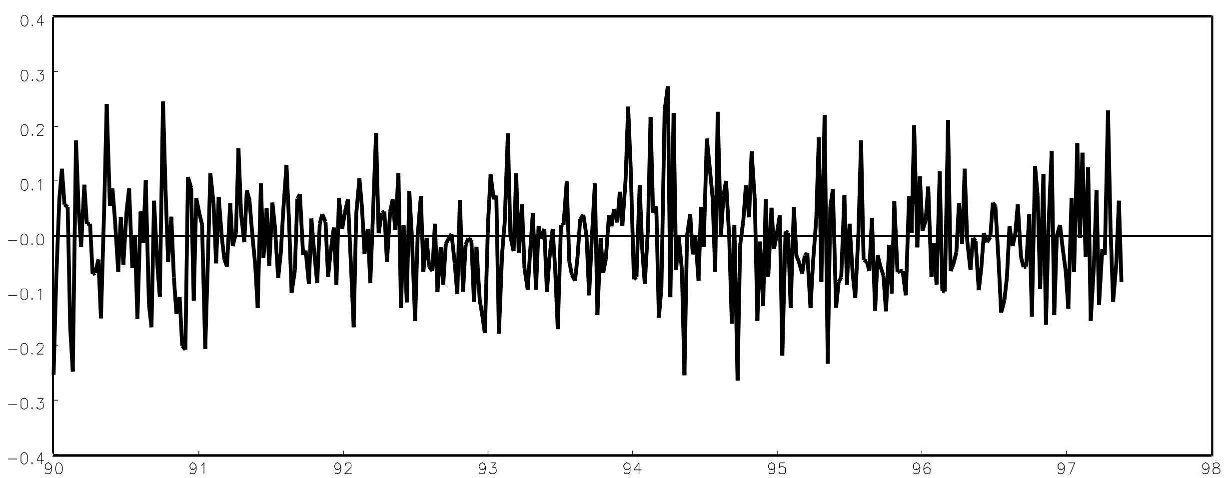
Graph 2: Error-Correction Term



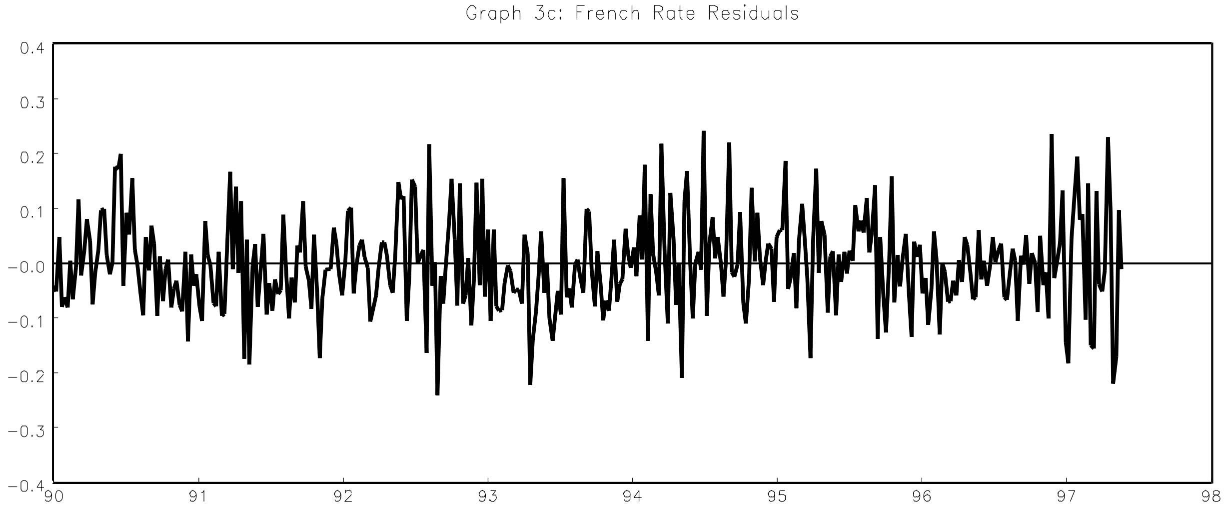
Graph 3a: US Rate Residuals



Graph 3b: German Rate Residuals



Graph 3c: French Rate Residuals



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