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AND IMPERFECT CREDIT MARKET INTEGRATION

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Monetary Union with A Single Currency and Imperfect Credit Market Integration*

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Résumé

Afin d'analyser les effets de l'union bancaire en zone Euro, nous montrons que l'union monétaire peut avoir une incidence sur le volume de crédit distribué par le canal des incitations au défaut. Nous construisons un modèle à deux pays dans lequel les paiements sont faits en monnaie et à crédit. L'imperfection de l'intégration du marché du crédit se traduit par un coût plus élevé du paiement à crédit pour les achats à l'étranger. Pour des niveaux suffisamment forts de ce coût, l'union monétaire amplifie les incitations au défaut, ce qui conduit un rationnement du crédit et à un bien-être plus faible que dans un régime de monnaies séparées. L'intégration des marchés du crédit restore l'optimalité de l'union monétaire.

Mots-clés: union monétaire, banques, crédit, défaut.
Codes JEL: E42, E50, F3, G21

Abstract

With the Euro Area context in mind, we show that currency arrangements impact on credit available through default incentives. To this end we build a symmetric two-country model with money and imperfect credit market integration. Differences in credit market integration are captured by variations in the cost for banks to grant credit for cross-border purchases. We show that for high enough levels of this cost, currency integration may magnify default incentives, leading to more stringent credit rationing and lower welfare than in a regime of two currencies. The integration of credit markets restores the optimality of the currency union.

Keywords: banks, currency union, monetary union, credit, default.
JEL codes: E42, E50, F3, G21.

Non technical summary

The conditions on the feasibility and optimality of a monetary union are key to the ongoing policy debate in Europe, as policymakers and scholars discuss the features of a banking union to complement the European monetary union. Discussions focus mainly on the creation of a single supervisory mechanism and on the need for a federal fiscal backstop for banking resolution.

In this paper we stress the complementarity between the single *currency*—high powered money—and the degree of integration among the various bank-intermediated *credit markets*. Although public authorities can easily unify currency conditions—suppress the currency risk—they have no direct command over the degree of credit market integration, since the determination of loan conditions depends on private banks. A regime of monetary union may therefore be characterized by a perfect integration of the currency—no cost to pay with cash—but imperfect integration of credit markets—hence a higher cost for cross-border purchases paid with credit.

With the Eurozone context in mind, we define imperfect credit market integration as a situation in which residents face more stringent credit conditions (higher cost) when financing purchases abroad compared to borrowing for domestic purchases. We compare two currency arrangements, depending on whether the currency risk is nil or positive and two types of credit market integration, depending on whether the cost of granting cross-border credit is nil or positive.

The transaction costs of paying with cash and credit are embedded in a symmetric two-country model of fiat money and bank credit. We show that a regime of monetary union is always optimal when credit markets are sufficiently integrated. But for high enough level of the cross border credit premium, the welfare gains of a single currency is wiped out. This analysis provides a normative argument for the integration of bank credit markets of a currency zone in order to reap the benefits expected from the unification of the currency.

As long as the authorities have some command on the level of the cross border credit premium, the policy implications for a monetary union are threefold. First, a banking union that aims at fostering credit market integration is a decisive addition to a currency union. Second, regulators and supervisors should avoid taking actions that create differences in the cost of managing credit across jurisdictions, notably ring fencing banks to domestic activities. Third, the unification of the credit markets will usefully be completed by guaranteeing an equal treatment of cross-border claims in national bankruptcy procedures and by ensuring an equal access to the information on borrowers' creditworthiness.

1 Introduction

Cross-border financing of non-financial corporations and households represents a small fraction of total financing to non-bank entities within the euro area. For example, the share of cross-border bank lending to non-bank entities across member states has varied between 3% and 6% since the creation of the euro. Recent policy discussions on the sustainability of the European monetary union have revealed that there is no consensus on whether more integration of credit and financial markets such as an increase of cross-border lending would be beneficial to the performance of euro area economies. Federal institutions, namely the ECB and the European commission, have supported policies fostering integration of those markets, including retail finance, in order to complete monetary unification. A contrasting standpoint in the policy debate defends instead greater credit market segmentation across member states with the view that currency arrangements and financial market structures are to a large extent two independent matters.

This paper constructs a model to determine whether the desirability of currency unification depends on the degree of integration of retail credit markets.[1] We take seriously the idea that one can have perfect integration with respect to the currency dimension—common rules governing the legal tender and currency issuance—but varying degrees of integration of retail credit markets. We capture a lower level of integration across credit markets by a higher cost for agents to obtain credit for cross-border purchases.[2] We show that for relatively low levels of the inflation rate credit market integration is a prerequisite to fully reap the gains of monetary unification. By contrast, a regime of a unique currency coupled with segmented

[1] We refer to retail credit as any type of debt contracted by households and small and medium enterprises for which the lender factors in the incentives to default.

[2] The level of cross-border credit to households and small and medium enterprises actually depends on several factors, such as the knowledge of specifics of local markets, the role of relationship-based information in the assessment of creditworthiness of small businesses and households, the availability of information on non-resident borrowers, the degree of harmonization of state legal systems, and the automaticity of the enforcement of cross-border repayment and foreclosure procedures.

credit markets across member states is sustainable only when the rate of inflation is sufficiently high.

In order to study the interplay between currency arrangements and credit market integration, we develop a symmetric two-country model in which currency (fiat money) and bank credit are used in equilibrium by residents of each country to purchase domestic or foreign goods. The underlying setup is a limited-commitment economy in which both currency and credit have a welfare-improving role. As in Rocheteau and Wright (2005), in each period agents are subject to individual consumption and production shocks that cannot be efficiently insured by their cash holdings. Banks provide insurance against those shocks. As in Berentsen, Camera, and Waller (2007), agents with no current need for cash (producers) can deposit their currency holdings rather than keeping them as idle balances, while those with a current need for cash (buyers) can obtain credit from banks to finance additional purchases. By lending out the cash received in deposits, banks effectively redistribute the money stock according to agents' current transaction needs.

Lending is potentially limited by the fact that agents cannot commit to repay their debt. Banks resort to borrowing constraints and to the exclusion of defaulters from future access to their services in order to ensure debt repayment, and agents disclose their identity and their financial history to banks in order to obtain credit—i.e., they must provide evidence on their creditworthiness. We model the difference in the conditions to obtain local credit *vis-à-vis* cross-border credit by the differential in the disclosure cost that residents of a country must incur whenever they want to obtain credit to fund operations abroad. We refer to this differential as the cross-border credit premium.

To evaluate the gains generated by a currency union, we compare two monetary arrangements: a single currency regime, and a 'one country-one currency' regime with positive conversion costs between the two currencies. The only difference between the two regimes lies in the conversion costs, and we ask whether the case with strictly positive conversion costs is dominated in terms of welfare by a currency union—which is equivalent to costless conversion.

Our analysis delivers two sets of results.

The first set of results concerns the conditions for the optimality of a currency union. We show that with perfect credit market integration, a unique currency is always the optimal arrangement. When credit market integration is imperfect, a unique currency is optimal if the borrowing constraint is not binding. This occurs when agents are patient enough and inflation is sufficiently high. Conversely, a regime of separate currencies with positive conversion costs may be preferred when credit market integration is sufficiently low and the borrowing constraint is binding, which occurs if agents are impatient and inflation is sufficiently low. The reason is that in this case the volume of credit is higher in a regime of separate currencies than in a currency union.

The intuition for this result is as follows. When credit market integration is imperfect, the reduction in conversion costs associated with a single currency may *worsen* default incentives on bank loans. Given the cross-border credit premium, the wedge between the cost of financing foreign versus domestic purchases induces borrowers—agents with no record of default—to be biased towards domestic goods compared to their preferred basket of goods in the absence of the cross-border credit premium. An endogenous home bias arises. Instead agents who have defaulted and lost access to credit—something that does not happen on the equilibrium path—are not impacted by the cross-border credit premium. Unlike agents with access to credit, agents who have defaulted are not home biased since they make their purchase decisions solely based on their preferences. Therefore, positive conversion costs between currencies can make default less attractive, as this cost affects defaulters more severely than non-defaulters, thereby relaxing borrowing constraints and allowing for a higher amount of credit in equilibrium. By contrast, when financing conditions are the same for domestic and cross-border purchases, there is no home bias, and a conversion cost between currencies does not attenuate default incentives.

The second set of results concerns how credit varies with the cross-border premium when there is credit rationing in equilibrium. We first show that for both monetary regimes, the volume of credit is monotonically decreasing in the cross-border credit premium. The logic is that a higher cost of cross-border credit leads to a reduction in agents' welfare. This reduces the value of maintaining future access to bank credit which negatively impacts repayment incentives, and results in a lower volume of credit in equilibrium. We then investigate how this impact of the cross-border credit premium varies across monetary regimes. We show that credit crunches—defined as a reduction in the quantity of credit caused by a substantial increase in the cross-border credit premium—are sharper in a currency union than in a regime of separate currencies. This follows from two effects. First, as agents consumption is lower in a regime of separate currencies when the cost of cross-border credit is low, an increase in this cost leads to a lesser reduction in the value of credit, thereby attenuating the negative impact on repayment incentives. Second, an increase in the cross-border premium can trigger an increase in the home bias sufficiently strong for the positive effect of conversion costs on repayment incentives identified above to outweigh the negative impact of an increase in conversion costs on trade.

These results have implications for the current policy debate regarding the architecture of the euro area. Our focus on stationary equilibrium highlights the long-term (structural) ingredients needed for a sustainable currency union, and independently of the design of the tools tailored to deal with financial crises. The policy agenda of the European Commission aims at deepening credit market integration, and is negotiated under the headings "banking union" for banking matters and "capital market union" for direct finance matters, see *inter alia* Beck (2012), Nieto and White (2013). The model suggests that in a world of low inflation (or in which the central bank does not have a complete command of the inflation rate), deeper banking and capital market integration across member states improves the efficiency of the currency union by reducing the incentives to default on credit and thereby supporting both a higher level of welfare and credit.

Our paper extends previous research on the use of money and credit in equilibrium to a two-country framework with potentially imperfect credit market integration. It contributes to the macroeconomic literature on the benefits and costs of monetary unions by showing that their sustainability requires integrated cross-border credit markets, independently of risk-sharing mechanisms or perfect capital and labor mobility. In addition, we contribute to the literature on monetary theory by suggesting a new rationale for the optimality of multiple currencies *vis-à-vis* a unique currency. In our setup, a regime of separate currencies mitigates the incentives to default on credit and, hence, may be socially preferred even though it entails higher transaction costs in cross-border trades.

The rest of the paper is organized as follows. The environment is laid out in Section 2. The conditions for the existence of equilibria are presented in Section 3. Section 4 presents the results pertaining to the welfare implications of a regime of unique versus multiple currencies for different degrees of credit market integration. Section 5 describes the causes of limited credit market integration in the euro area since its inception. Section 6 discusses our contribution to the literature. Section 7 concludes. Proofs are relegated to the Appendix.

2 Environment

Time is discrete and continues forever. There are two identical countries, the home country and the foreign country, each populated by a continuum of infinitely-lived agents of unit mass. There are two perfectly divisible non-storable country-specific goods: a home good, denoted as q_h, and a foreign good, denoted as q_f. Agents discount across periods with factor β. A period is divided in two subperiods. In each period, two competitive markets open sequentially in each country. Before the first market opens, agents receive an idiosyncratic shock that determines whether they are sellers for the current period and gain no utility from consumption (with probability $(1-b)$), or buyers who want to consume but cannot produce (with probability b). In the second market, all agents can produce and consume a quantity of a generic good denoted as x, and utility from consumption (or disutility from working) is linear in the quantity of good.

Buyers' preferences in the first subperiod are

$$\max\left[u\left(q_f\right)+\eta q_f, u\left(q_h\right)\right] \tag{1}$$

where η is a preference shock which can take values $\eta = 0, \eta_1, \eta_2$ with probabilities π_0, π_1 and π_2, and $0 < \eta_1 < \eta_2$.[3] The function u satisfies $u'(q), -u''(q) > 0$, $u'(0) = \infty$ and $u'(\infty) = 0$. In addition we assume that $-u''(q)\,q \leq u'(q)$. Preferences in (1) are such that in equilibrium buyers will consume the home good in periods in which their preference shock η is low and consume the foreign good in periods in which η is high. In the former case they trade in the first market of the home country. In the latter case they travel costlessly to trade in the foreign country and come back to the home country to participate in the second market. For sellers, producing a quantity $q$in the first subperiod represents a disutility equal to $c(q) = q$.

There are two storable, perfectly divisible and intrinsically useless currencies, the home currency and the foreign currency. For simplicity, the quantity of each currency at the beginning of period t is denoted as M. The money supply in each country grows at the gross rate $\gamma = M_{+1}/M$ where the subscript $+1$ indicates the

[3] We assume three realizations of the preference shock η to allow for domestic and foreign consumption on the equilibrium path while creating a wedge between the consumption patterns of agents who borrow and those who do not.

following period. Agents receive monetary lump-sum transfers from the central bank equal to $T = (\gamma - 1) M_{-1}$ during the second market in period t. Consistently with the current euro area situation, we assume that the central bank has no power to tax agents, such that $\gamma \geq 1$.[4] In order to motivate a role for a medium of exchange, traders are assumed to be anonymous so that sellers require immediate compensation when they produce. This assumption rules out bilateral credit but not banking credit.

In each country there are competitive banks which take deposits and use them to grant loans as in Berentsen, Camera, and Waller (2007).[5] Banking activities take place before the first market opens. Loans and deposits are paid back during the second subperiod with the corresponding interest. Hence credit is intra-period.[6]

Banks have no enforcement power. However banks are able to exclude agents who have defaulted in the past from banking activities—loans and deposits—for the rest of their lifetimes. Defaulters are also excluded from monetary transfers. Home (foreign) banks can recognize agents from the home (foreign) country.

Agents must incur a disclosure cost in order to reveal their identity and their financial history to banks. We assume that the disclosure cost is dependent on the distance between the agent and the contracting bank. For simplicity, the disclosure cost for a home agent who is located in the home country and contacts a home bank is set to zero. The disclosure cost for a home agent who is located in the foreign country is proportional to the real amount borrowed with factor $c \geq 0$.[7] Throughout the paper we refer to parameter c as the cross-border credit

[4] This restriction implies that the Friedman rule is not a feasible policy, so that it is optimal for agents to insure against idiosyncratic shocks using both (costly) cash holdings and banks. This assumption could be relaxed, for instance by assuming that the government can use lump-sum taxes but that agents can evade taxation by not participating in the market - see Hu, Kennan, and Wallace (2009) and Andolfatto (2010).

[5] Given our emphasis on credit markets integration, an important aspect of banks in our setup is that they extend loans in addition to taking deposits. See Bencivenga and Camera (2011) for a model in which banks provide liquidity insurance on the liability side but cannot extend credit.

[6] As pointed out by Berentsen et al. (2007) quasi-linear preferences in the second market imply that one-period debt contracts are optimal.

[7] We assume that home agents can only be recognized by home banks. However our setup

premium. When $c = 0$, taking out a loan to consume in the home country or in the foreign country is equivalent; i.e. credit market integration is perfect. When $c > 0$, financing consumption abroad is more costly than financing consumption in the home country; i.e., there is imperfect credit market integration.[8]

We assume that agents can only hold the currency of their country of residence across subperiods, since our purpose is to assess the effect of forming a currency union when national currencies circulate only in their issuing country.[9] Currencies can be exchanged before the first market opens. As is usual in the literature (see references in footnote 9), exchanging currencies represents a disutility cost ε proportional to the real amount of money exchanged. Notice that our modelling choice is very conservative since the welfare-improving effect of conversion costs would be much easier to obtain if the conversion cost was modelled as a monetary cost that is transferred back to agents as lump-sum, since defaulters can be excluded from monetary transfers. Given that the money growth rate is assumed to be the same for the two countries, the case in which the cost ε is equal to zero is equivalent to a currency union.

The sequence of trades within a period is depicted in figure 1. At the beginning of the period, the preference shocks are realized. Then, buyers decide whether to stay in their home country or to travel to the foreign country. Next, banking

is equivalent to assuming that home agents can contract with foreign banks by incurring the disclosure cost c.

[8]In our model, the difference in the conditions to obtain local credit vs. cross-border credit is captured by the disclosure cost c which reflects the information friction caused by spatial separation and lack of information sharing across borders. In reality, many features render cross-border credit or domestic credit for cross-border purchases more costly or more difficult to obtain. For details see Section 5.

[9]If agents could costlessly hold the foreign currency, the desirability of a currency union between symmetric countries would presumably not be an issue. Indeed it would amount to removing conversion costs that would only partially affect the holding of diversified portfolios. The 'one-country one-currency' setup allows for greater tractability while being a reasonable assumption in a model intended to formalize the retail credit market. See King, Wallace, and Weber (1992) for a paper which considers different types of agents, those forced by law to hold the currency of their country of residence, and those free to hold any currency. See Engineer (2000), Head and Shi (2003), Camera, Craig, and Waller (2004), Zhang (2014) and Geromichalos and Simonovska (2014) for models on the choice of an asset portfolio.

activities (loans and deposits) take place. Then the first market and the second market open sequentially. Agents who have travelled in the current period come back to their home country before the opening of the second market.

[Figure 1 about here.]

3 Symmetric equilibrium

We focus on stationary equilibria in which end-of-period real money balances are constant and positive, so that

$$\gamma = M/M_{-1} = \phi_{-1}/\phi \tag{2}$$

where ϕ is the price of money in real terms during the second market. Let $V(m)$ denote the value function of an agent who holds an amount m of money at the beginning of a period, before learning the realization of the preference shock. $W_k(m, \ell_k)$ is the expected value from entering the second market with m units of money balances and an amount ℓ_k of loans (a negative value of ℓ_k denotes deposits). The index $k = h, f, s$ denotes whether the agent has taken out a loan for home consumption, taken out a loan for foreign consumption or deposited at the beginning of the period. In what follows, we analyze a representative period t and solve the model backwards from the second market to the first market. Since countries are perfectly symmetric, we only present the optimal choices by agents from the home country.

3.1 The second market

In the second market, agents consume or produce, reimburse loans or redeem deposits, and adjust their money balances. Denote as i_h (i_f) the interest rate on loans for home (foreign) consumption and i_s the interest rate on deposits. If an agent has borrowed an amount ℓ_h (ℓ_f), he must repay an amount equal to $\ell_h (1 + i_h)(\ell_f (1 + i_f))$. If an agent has deposited an amount ℓ_s, he obtains $\ell_s (1 + i_s)$. The representative agent chooses his next period monetary holdings, m_{+1}, and his consumption (production) of the generic good, x, in order to maximize $W_k (m, \ell_k)$ subject to the budget constraint:

$$\max_{x,m_{+1}} W_k (m, \ell_k) = x + \beta V (m_{+1})$$
$$\text{s.t. } x + \phi \ell_k (1 + i_k) + \phi m_{+1} = \phi m + \phi T$$

where $T = (\gamma - 1) M_{-1}$ is a lump-sum transfer from the central bank. The budget constraint states that the sum of an agent's current consumption, loan repayment (or deposit's redemption if $\ell_k = \ell_s < 0$) and next-period money holdings equals his current money holdings plus the monetary transfer from the central bank.

Inserting the budget constraint into the objective function, the above program simplifies to

$$\max_{m_{+1}} \left[-\phi m_{+1} + \phi m - \phi \ell_k (1 + i_k) + \phi T + \beta V (m_{+1}) \right].$$

The first-order condition on m_{+1} is

$$\beta V' (m_{+1}) = \phi \tag{3}$$

where $V' (m_{+1})$ is the marginal value of an additional unit of money taken into period $t + 1$. Notice that m_{+1} is the same for all agents, regardless of their initial money holdings m. The envelope conditions are

$$
\begin{aligned}
W_{k,m} &= \phi \\
W_{k,\ell_k} &= -\phi\left(1+i_k\right).
\end{aligned}
\tag{4}
$$

3.2 The first market

3.2.1 Sellers

Since sellers do not derive utility from consumption, they choose to deposit their currency holdings at the bank. Let p denote the price of first-market goods. In the first market the representative seller chooses how much to produce q_s and the amount of his deposits ℓ_s, subject to the deposit constraint by which the seller's deposits are limited by his currency holdings. The program for a seller in the first market is

$$
\max_{q_s,\ell_s}\left[-q_s + W_s\left(m_{-1}+\ell_s+pq_s,\ell_s\right)\right]
$$

$$
\text{s.t.} \quad -\ell_s \leq m_{-1}
$$

where m_{-1} are currency holdings taken from the previous period. The first-order condition on q_s is

$$
W_{s,m}p = 1.
$$

Using (4), it becomes

$$
\phi p = 1. \tag{5}
$$

Condition (5) states that prices ϕ and p are such that sellers are indifferent between producing in the first market and producing in the second market.

The first-order condition on ℓ_s can be written as

$$\phi i_s = \mu_s \tag{6}$$

where μ_s is the multiplier associated with the deposit constraint. According to condition (6), if the interest rate i_s is positive, the deposit constraint is always binding and sellers deposit their entire currency holdings.

3.2.2 Buyers

At the beginning of each period, buyers learn the realized value of the preference shock η that determines the utility of consuming the foreign good. Then, buyers decide to consume in their home country or abroad during the first market. Given preferences (1), it is straightforward to see that buyers' travel decisions follow a simple cutoff rule: Buyers consume the home good when $\eta \leq \eta^*$ and consume the foreign good when $\eta > \eta^*$, where the threshold η^* is endogenously determined (see Section 3.5).

Denote as q_h^η (q_f^η) the quantity of home (foreign) goods consumed by a buyer with preference shock η. Denote as ℓ_h^η (ℓ_f^η) the loan taken out by a buyer with preference shock η who consumes the home (foreign) good. Since banks can distinguish domestic from foreign transactions, they can potentially set different borrowing limits. Let $\bar{\ell}_f$ indicate the maximal amount that an agent traveling abroad can borrow. Similarly $\bar{\ell}_h$ indicates the borrowing limit for an agent who consumes the home good.

Since optimal quantities may differ for buyers who stay in the home country and those who travel abroad, we distinguish two cases. Consider first a buyer who consumes the home good (that is with shock $\eta \leq \eta^*$). This buyer maximizes the utility from consuming q_h^η subject to two constraints:

$$\max_{q_h^\eta, \ell_h^\eta} u\left(q_h^\eta\right) + W_h\left(m_{-1} + \ell_h^\eta - pq_h^\eta, \ell_h^\eta\right)$$

$$\text{s.t. } pq_h^\eta \leq m_{-1} + \ell_h^\eta \tag{7}$$

$$\ell_h^\eta \leq \bar{\ell}_h. \tag{8}$$

The first constraint is the cash constraint by which the buyer cannot spend more than his initial money holdings plus his loan. The second constraint is the borrowing constraint set by banks to ensure loan repayment (see Section 3.6).

Using (4) and (5), the first-order condition on q_h^η is

$$u'\left(q_h^\eta\right) = 1 + \mu_h^\eta/\phi \tag{9}$$

where μ_h^η is the multiplier associated with the cash constraint (7). The first-order condition on ℓ_h^η for this buyer can be written as

$$\mu_h^\eta - \phi i_h = \lambda_h^\eta, \tag{10}$$

where λ_h^η is the multiplier associated with the borrowing constraint (8). Using (9) to substitute for μ_h^η, condition (10) can be rewritten as

$$u'\left(q_h^\eta\right) = 1 + i_h + \lambda_h^\eta/\phi. \tag{11}$$

Consider next the program for a buyer who consumes abroad (with shock $\eta > \eta^*$). His consumption quantity solves:

$$\max_{q_f^\eta, \ell_f^\eta} u\left(q_f^\eta\right) + (\eta - \varepsilon)\, q_f^\eta - c\ell_f^\eta/p + W_f\left(m_{-1} + \ell_f^\eta - pq_f^\eta, \ell_f^\eta\right)$$

$$\text{s.t. } pq_f^\eta \leq m_{-1} + \ell_f^\eta, \tag{12}$$

$$\ell_f^\eta \leq \bar{\ell}_f. \tag{13}$$

Compared to the buyer who consumes the home good, the buyer who consumes the foreign good bears the cross-border credit premium to disclose his identity and his financial history to banks ($c\ell_f^\eta/p$) and incurs conversion costs on his purchase (εq_f^η).

Using (4) and (5), the first-order condition on q_f^η is

$$u'\left(q_f^\eta\right) + \eta = 1 + \varepsilon + \mu_f^\eta/\phi \tag{14}$$

where μ_f^η is the multiplier associated with the cash constraint (12). The first-order condition on ℓ_f^η can be written as

$$u'\left(q_f^\eta\right) + \eta - \varepsilon - c = 1 + i_f + \lambda_f^\eta/\phi \tag{15}$$

where λ_f^η is the multiplier associated with the borrowing constraint (13).

3.3 Market clearing

Market clearing in the loan market yields

$$(1-b)\,\ell_s + b\sum_{\eta\leq\eta^*}\pi_\eta \ell_h^\eta + b\sum_{\eta>\eta^*}\pi_\eta \ell_f^\eta = 0. \tag{16}$$

The sum of the deposits made by sellers and the loans taken out by all buyers—i.e., those who consume the home good and those who consume the foreign good—is equal to zero. For sellers, it is optimal to deposit their entire money holdings for any $\gamma \geq 1$. Thus $m_{-1} = -\ell_s$, and (16) becomes

$$(1-b)\, m_{-1} = b \sum_{\eta \leq \eta^*} \pi_\eta \ell_h^\eta + b \sum_{\eta > \eta^*} \pi_\eta \ell_f^\eta. \tag{17}$$

Since countries are symmetric, market clearing in the first market for goods yields

$$b \sum_{\eta \leq \eta^*} \pi_\eta q_h^\eta + b \sum_{\eta > \eta^*} \pi_\eta q_f^\eta = (1-b)\, q_s. \tag{18}$$

3.4 Marginal value of money

The expected utility for an agent who starts a period with m units of money is:

$$\begin{aligned} V(m) = {} & b \sum_{\eta \leq \eta^*} \pi_\eta \left[u\left(q_h^\eta\right) + W_h\left(m + \ell_h^\eta - p q_h^\eta, \ell_h^\eta\right) \right] \\ & + b \sum_{\eta > \eta^*} \pi_\eta \left[u\left(q_f^\eta\right) + (\eta - \varepsilon)\, q_f^\eta - \phi \ell_f^\eta c + W_f\left(m + \ell_f^\eta - p q_f^\eta, \ell_f^\eta\right) \right] \\ & + (1-b)\left[-q_s + W_s\left(m + \ell_s + p q_s, \ell_s\right) \right]. \end{aligned}$$

Given (5) cash constraints (7) and (12) imply that

$$\begin{aligned} q_h^\eta &\leq \phi\left(m_{-1} + \ell_h^\eta\right) \\ q_f^\eta &\leq \phi\left(m_{-1} + \ell_f^\eta\right). \end{aligned} \tag{19}$$

Using (4), (5), (6), (9) and (14), the marginal value of money is

$$\partial V / \partial m = b\phi \sum_{\eta \leq \eta^*} \pi_\eta u'\left(q_h^\eta\right) + b\phi \sum_{\eta > \eta^*} \pi_\eta \left[u'\left(q_f^\eta\right) + \eta - \varepsilon \right] + (1-b)\, \phi\, (1 + i_s).$$

Using (2) and (3), this condition becomes

$$\gamma/\beta = b\sum_{\eta\leq\eta^*}\pi_\eta u'\left(q_h^\eta\right) + b\sum_{\eta>\eta^*}\pi_\eta\left[u'\left(q_f^\eta\right)+\eta-\varepsilon\right] + (1-b)\,(1+i_s)\,. \tag{20}$$

The left-hand side of this equation represents the marginal cost of acquiring an additional unit of money while the right-hand side represents its marginal benefit: With probability b the agent consumes the home good (for $\eta \leq \eta^*$) or the foreign good (for $\eta > \eta^*$), and with probability $(1-b)$ the agent is a seller and earns interest on his deposits.

3.5 Travel decision

As discussed above, buyers' travel equilibrium decisions can be represented by a threshold η^* such that buyers with shock $\eta \leq \eta^*$ consume at home while buyers with $\eta > \eta^*$ consume abroad. This threshold corresponds to the virtual value of the preference parameter η such that the value of staying in the home country is equal to the value of traveling to the foreign country. The threshold η^* is defined by

$$u\left(q_h^{\eta^*}\right) - \phi\ell_h^{\eta^*}\,(1+i_h) = u\left(q_f^{\eta^*}\right) + q_f^{\eta^*}\,(\eta^*-\varepsilon) - \phi\ell_f^{\eta^*}\,(1+i_f+c)\,. \tag{21}$$

On the left-hand side of (21), the value of purchasing $q_h^{\eta^*}$ is equal to the utility from consumption minus the cost of reimbursing the loan for home-good consumption. On the right-hand side of (21), the value of purchasing $q_f^{\eta^*}$ is equal to the utility from consumption minus the cross-border credit premium, the cost of reimbursing the loan for foreign-good consumption and the conversion costs.

3.6 Borrowing constraint

Banks have no enforcement power. Therefore they must set a borrowing constraint that ensures voluntary debt repayment: They choose the amount of loans $\bar{\ell}_h$ and $\bar{\ell}_f$ such that the payoff to an agent who repays his debt is at least equal to the payoff to a defaulter.

Denote as $\hat{q}_h^\eta$ ($\hat{q}_f^\eta$) the quantity of the home (foreign) good consumed by an agent with preference shock η who has defaulted in the past. The term $\hat{m}_{-1}$ denotes money holdings brought by a defaulter from the previous period.

Since utility from consuming the foreign good is higher than utility from consuming the home good for $\eta > 0$ given (1), defaulters could be cash-constrained for $\eta = \eta_1, \eta_2$ and not cash-constrained for $\eta = 0$. Lemma 1 states that defaulters are cash-constrained for all realizations of the preference shock η if agents are sufficiently impatient.

Lemma 1 *Let $\tilde{\beta} = [1 + b(\pi_1\eta_1 + \pi_2\eta_2)]^{-1}$. Defaulters are cash-constrained for all realizations of η for $\beta > \tilde{\beta}/\gamma$. In particular, if $\beta > \tilde{\beta}$ defaulters are cash-constrained for all realizations of η for all $\gamma \geq 1$.*[10]

The condition for Lemma 1 will be satisfied in all the equilibria that we consider. We can thus without loss in generality focus on situations such that defaulters are cash-constrained for all realizations of η, and set $\hat{q}_h^\eta = \hat{q}_f^\eta = \hat{q}$ and $\hat{m}_{-1} = p\hat{q}$ for all η (since defaulters do not have access to the banking system). Let $\hat{\eta}^*$ denote the threshold describing the optimal travel decision for an agent who has defaulted in the past. In periods in which $\eta \leq \hat{\eta}^*$ the defaulter consumes the home good, whereas in periods in which $\eta > \hat{\eta}^*$ the defaulter consumes the foreign good. The threshold $\hat{\eta}^*$ is given by

$$u(\hat{q}) = u(\hat{q}) + (\hat{\eta}^* - \varepsilon)\hat{q}.$$

[10]Notice that, in this model, without preference shocks on the utility provided by foreign goods relative to home goods ($\pi_0 = 1, \pi_1, \pi_2 = 0$) and given that $\gamma \geq 1$, the condition in Lemma 1 is simply $\beta \leq 1$.

According to this condition $\hat{\eta}^*$ is determined such that the utility derived from consuming the home good is equal to the utility from consuming the foreign good minus the conversion cost. Hence,

$$\hat{\eta}^* = \varepsilon. \tag{22}$$

Let $\hat{V}(\hat{m})$ indicate the expected utility for a defaulter who starts a period with $\hat{m}$ units of money and $\hat{W}(\hat{m})$ indicate the expected utility for a defaulter with $\hat{m}$ units of money at the beginning of the second market. $\hat{V}(\hat{m})$ is

$$\begin{aligned}\hat{V}(\hat{m}) = {} & b\sum_{\eta\leq\hat{\eta}^*}\pi_\eta\left[u(\hat{q})+\hat{W}(0)\right]+b\sum_{\eta>\hat{\eta}^*}\left[u(\hat{q})+(\eta-\varepsilon)\hat{q}+\hat{W}(0)\right]\\ & +(1-b)\left(-q_s+\hat{W}(\hat{m}+pq_s)\right)\end{aligned}$$

where $\hat{q}$ is determined by the optimal condition on the money holdings of the defaulter:

$$\gamma/\beta = bu'(\hat{q})+b\sum_{\eta>\hat{\eta}^*}\pi_\eta(\eta-\varepsilon)+1-b. \tag{23}$$

Lemma 2 *Assume that the condition stated in Lemma 1 holds. Interest rates satisfy $i_h = i_f = i_s = i$. An agent who borrows debt ℓ has an incentive to repay his debt if, and only if,*

$$-\phi\ell(1+i)-\phi m_{+1}+\phi T+\beta V(m_{+1}) \geq -\phi\hat{m}_{+1}+\beta\hat{V}(\hat{m}_{+1}). \tag{24}$$

Equation (24) can be expressed equivalently as

$$
\begin{aligned}
&- \phi\left[\ell\left(1+i\right)+m_{+1}-T\right] \\
&+ \frac{\beta b}{1-\beta}\left\{\sum_{\eta\leq\eta^*}\pi_\eta\left[u\left(q_h^\eta\right)-q_h^\eta\right]+\sum_{\eta>\eta^*}\pi_\eta\left[u\left(q_f^\eta\right)+\left(\eta-1-\varepsilon\right)q_f^\eta-\phi\ell_f^\eta c\right]\right\} \\
&\geq \frac{\beta b}{1-\beta}\left[u\left(\hat{q}\right)-\hat{q}+\sum_{\eta>\hat{\eta}^*}\pi_\eta\left(\eta-\varepsilon\right)\hat{q}\right]-\frac{\left(\gamma-\beta\right)\hat{q}}{1-\beta}.
\end{aligned}
\tag{25}
$$

Thus banks set identical limits $\bar{\ell}_h = \bar{\ell}_f = \bar{\ell}$ for home-goods consumption loans and foreign-goods consumption loans.

The left-hand side of the borrowing constraint in equation (25) represents the pay-off to an agent who does not default. In period t, this agent works to pay his loan with the corresponding interest and to recover his money holdings. From $t+1$ onwards, his expected utility is determined by the net utility he obtains from consuming the home good each time he turns out to be a buyer with $\eta \leq \eta^*$, or by the net utility he obtains from consuming the foreign good (minus conversion costs and the cross-border credit premium) each time he turns out to be a buyer with $\eta > \eta^*$.

The right-hand side of the borrowing constraint represents the pay-off to a defaulter. If an agent defaults, he does not work to repay the loan taken out at the beginning of t, nor does he pay the interest on it. His expected lifetime utility is given by the net utility from consuming $\hat{q}$ as a buyer from $t+1$ onwards, minus the cost of adjusting money holdings from t onwards, equal to $\left(\gamma-\beta\right)\hat{q}/\left(1-\beta\right)$.

According to Lemma 2, the equilibrium interest rates on loans for home and foreign consumption are equal ($i_h = i_f$) and banks set the borrowing limits $\bar{\ell}_h$ and $\bar{\ell}_f$ at the same value $\bar{\ell}$. The reason is that the cost c is born at the moment at which the loan is granted, and hence it does not affect the continuation value to an agent at the repayment stage. In addition, since banks make zero profit in equilibrium, it must be that $i_h = i_f = i_s = i$.

3.7 Unconstrained and fully-constrained equilibria

In this section we provide conditions on parameter values for the existence of symmetric and stationary equilibria. We consider unconstrained equilibria, in which buyers are not credit constrained regardless of the value of their preference shock η, and fully constrained equilibria in which all buyers are credit constrained.

Definition 1 *An equilibrium is a vector of consumption quantities* $\left\{q_h^\eta, q_f^\eta, \hat{q}\right\}$*, traveling thresholds* $\{\eta^*, \hat{\eta}^*\}$*, interest rate* i*, price of money* ϕ*, money holdings* m_{-1}*, loans* $\left\{\ell_h^\eta, \ell_f^\eta\right\}$*, borrowing limit* $\bar{\ell}$ *and multipliers associated with the borrowing constraint* $\left\{\lambda_h^\eta, \lambda_f^\eta\right\}$ *for* $\eta \in \{0, \eta_1, \eta_2\}$ *which satisfy* $m_{-1} = M_{-1}$*, (11), (15), (17), (19)-(23) and (25). An equilibrium is unconstrained if the borrowing constraint (25) is slack for all values of* η*. An equilibrium is fully constrained if the borrowing constraint (25) binds for all values of* η *(*$\ell_h^\eta = \ell_f^\eta = \bar{\ell}$ *for all* η*).*

The following propositions refer to the existence of the unconstrained equilibrium in which no buyer is credit constrained (Proposition 1) and the fully constrained equilibrium in which all buyers are credit constrained (Proposition 2).

Proposition 1 *If* β *is sufficiently high there is* $\tilde{\gamma}$ *such that if* $\gamma \geq \tilde{\gamma} \geq 1$*, a unique unconstrained equilibrium exists.*

Proposition 1 states that if the rate of money growth γ is high enough, then an unconstrained equilibrium exists and this equilibrium is unique. For all realizations of η, agents are able to borrow as much as they desire at the prevailing interest rate. This result is usual in monetary models with limited commitment, see Aiyagari and Williamson (2000), Corbae and Ritter (2004). It extends the result of Proposition 4 in Berentsen *et alii* (2007) to a two-country framework with potentially imperfect credit market integration.

This result comes from the impact of inflation on consumption and thus on expected utility. Agents choose the consumption quantity bought in the first market by equating the marginal utility of consumption in this market to the marginal cost of carrying money from the second market in t to the first market in $t+1$. If the rate of money growth γ is higher than the discount factor β, carrying money throughout periods is costly, because agents need to acquire their money holdings before purchasing goods. The higher γ is, the higher the cost of carrying money is, and therefore the higher the marginal utility of consumption in the first market—or the lower the level of consumption. The cost of carrying money is mitigated for non-defaulters by the interest that they earn on their idle cash balances when they turn out to be sellers and by the monetary transfers. Therefore, the mere existence of banks allows agents with access to the banking system to enjoy a higher level of consumption in the first market. On the contrary, defaulters are unable to deposit their cash balances and hence do not earn any interest on them. Consequently, they bear a higher cost of carrying money and enjoy a lower level of consumption. When inflation reaches a certain point, defaulters' consumption is so low that agents are unwilling to default. Thus the borrowing constraint is not binding. As a result, there is a level of inflation above which agents borrow their desired amount of money at equilibrium interest rates.

Proposition 2 *If β, η_1 and η_2 are sufficiently low, there is $\left\{\gamma^1, \gamma^2\right\}$ with $1 \leq \gamma^1 < \gamma^2 < \tilde{\gamma}$ such that if $\gamma \in \left[\gamma^1, \gamma^2\right]$ a fully constrained equilibrium exists. In this fully constrained equilibrium the threshold η^* satisfies*

$$\eta^* = \varepsilon + (1-b)\,c. \tag{26}$$

If $\eta_1 > \eta^*$, buyers consume the home good with probability π_0. If $\eta^* \geq \eta_1 > \varepsilon$ buyers consume the home good with probability $(\pi_0 + \pi_1)$.

In a fully constrained equilibrium, all buyers would like to borrow more money than the banks are willing to provide at the prevailing equilibrium interest rate. Proposition 2 states that a fully constrained equilibrium exists when the inflation rate is positive and low enough, provided that the discount factor β and the values of the preference shock η_1 and η_2 are low enough.[11] When inflation is low, the marginal cost of carrying money is low, and defaulters obtain a relatively high level of consumption. Incentives to default are high and the borrowing constraint is binding: Only a limited amount of credit can be sustained in equilibrium because the threat of being excluded from the banking system imposes too mild a cost of default.

Next, we discuss how the travel decision is determined in this equilibrium. Buyers are credit-constrained for all realizations of the preference shock η, thus they borrow the same amount of credit and consume the same quantity of goods regardless of the country in which they consume. Therefore equation (21) can be reduced to equation (26). In (26), the threshold η^* depends on the extra cost of purchasing foreign goods which consists of the cross-border credit premium and the conversion cost. The conversion cost is paid on the total amount purchased whereas the cross-border credit premium c is proportional to the share of consumption financed with a bank loan, equal to $(1-b)$.[12]

To decide on the country in which he wishes to trade in the first market, the buyer compares the utility derived from the consumption of the foreign good, that depends on the realization of η and the extra cost of financing it, with the utility derived from the consumption of the home good. Given the realized preference

[11] In the main text we do not present the case in which c is so high that buyers never consume the foreign good. The Appendix presents this case with the corresponding proofs.

[12] In this equilibrium the share of consumption financed with credit l/pq is equal to $(1-b)$ whereas the share of consumption financed with cash holdings is b, see equation (42) in the proof of Proposition 2 in the Appendix. Intuitively, cash holdings depend positively on the probability b of becoming a buyer since agents are more inclined to accumulate costly money holdings when they have a greater opportunity to spend them. Credit is used to finance the difference between desired consumption and cash holdings.

shock, there is a level of the financing cost above which an agent switches from consumption of the foreign good to consumption of the home good even for a positive value of η. As stated in Proposition 2, if $\eta_1 > \eta_1^* = \varepsilon + (1-b)\,c$, the cross-border credit premium is low, so buyers consume the home good only when η is zero—with probability π_0—and there is no home-bias. If $\eta_2 > \eta_1^* = \varepsilon + (1-b)\,c \geq \eta_1 > \varepsilon$ the cross-border credit premium is high and buyers consume the home good when η is equal to zero or to η_1—i.e., with probability $(\pi_0 + \pi_1)$. This defines a home bias in consumption which is triggered by a sufficiently high cross-border credit premium and (or) conversion cost. When the cost of converting one currency into the other is negligible, an agent's bias towards home consumption will be due to imperfect credit market integration.

4 Currency conversion costs, credit and welfare

This section presents the main results of the paper. We analyze the effect of making currency exchange costly on both credit and welfare; i.e., the expected lifetime utility of the representative agent. Given (1), (5) and (18), welfare is defined as

$$\mathcal{W} = \frac{b}{1-\beta}\left\{\sum_{\eta\leq\eta^*}\pi_\eta\left[u\left(q_h^\eta\right) - q_h^\eta\right] + \sum_{\eta>\eta^*}\pi_\eta\left[u\left(q_f^\eta\right) + (\eta - 1 - \varepsilon)\,q_f^\eta - \phi\ell_f^\eta c\right]\right\}. \tag{27}$$

We ask when a monetary union is optimal; i.e., for which values of the parameter space welfare is maximal when $\varepsilon = 0$. We derive conditions on c and γ such that agents prefer a regime of separate currencies ($\varepsilon > 0$) instead of a unified currency.[13] We then provide a comparative statics result on how credit and welfare depend on c. Finally, we construct an example in which a regime of separate

[13]We focus on the comparison of steady state welfare levels and abstract from any cost of entry or exit from a currency union. This comparison can be extended to a setup in which the cost of exit is fixed, as suggested by the empirical discussion in Eichengreen (2007).

currencies is optimal, even if inflation is optimally chosen.

4.1 When is a monetary union optimal?

In this section, we show that in economies with money and credit agents prefer a monetary union if, for exogenous reasons, the inflation rate γ is high enough or if the credit market integration between countries is deep enough; i.e., the level of the cross-border credit premium c is low enough. The next proposition assesses the effect of implementing conversion costs between the two currencies when agents are not credit-constrained.

Proposition 3 *In an unconstrained equilibrium, imposing a conversion cost $\varepsilon > 0$ leaves the consumption of the home good (q_h^η) unchanged, decreases the consumption of the foreign good (q_f^η) for all η, increases the real quantity of credit financing home-good consumption ($\phi\ell_h^\eta$) and decreases the real quantity of credit financing foreign-good consumption ($\phi\ell_f^\eta$). The overall effect is welfare worsening.*

Proposition 3 states that imposing positive conversion costs is unambiguously detrimental to welfare if agents are not credit constrained. There are redistributive effects across types of agents depending on the good—home or foreign—that they consume. A positive conversion cost increases the marginal cost of purchasing the foreign good. Hence buyers decrease their expected consumption of the foreign good so that its marginal utility matches its marginal cost. In addition, conversion costs do not affect the individually optimal consumption quantity of the home good. Consequently, agents choose to carry a lower amount of costly monetary holdings across periods, as they are anyway able to borrow as much as they want. Thus buyers who stay in the home country take out greater loans following an increase in conversion costs. Conversely, agents who consume abroad need to borrow less because the decrease in money holdings is lower than the decrease in their desired consumption of the foreign good. Since the consumption of the foreign good decreases and the consumption of the home good is unaffected with conversion costs, it follows that the overall effect on utility is negative.

Next, we analyze the effect of conversion costs when agents are credit-constrained. The following proposition refers to the case in which agents are credit-constrained and credit market integration among countries is sufficiently deep; i.e., $c < \eta_1/(1-b)$.

Proposition 4 *Let $c < \eta_1/(1-b)$. In a fully constrained equilibrium, the imposition of a conversion cost $\varepsilon > 0$ triggers a reduction in the consumption of both goods (q_h^η, q_f^η) and in the real quantity of credit $(\phi\ell_h^\eta,\ \phi\ell_f^\eta)$ and worsens welfare.*

According to Proposition 4, imposing positive conversion costs is welfare-worsening when agents are credit constrained and the credit markets of the two countries are relatively well integrated; i.e., if the cross-border credit premium c multiplied by the share $(1-b)$ of consumption financed with credit is smaller than the intermediate value of the preference shock for the foreign good, η_1. In the fully constrained equilibrium, agents are constrained for all realizations of η. Thus, they all borrow the same amount, equal to the borrowing limit, regardless of the value of their preference shock. In addition agents reduce their money holdings when conversion costs increase, since the marginal value of money decreases with conversion costs, see equation (20). As a result, an increase in conversion costs entails a reduction in the consumption of both the home good and the foreign good. As in the case in which agents are not constrained, when agents are credit-constrained and credit market integration is deep enough, the imposition of conversion costs makes agents reduce their consumption and so it is unambiguously detrimental to welfare.

We can conclude that a monetary union is always optimal when no agent is credit constrained and when all agents are credit-constrained and the cross-border credit premium is low.

4.2 Monetary and non-monetary causes for monetary disunion

In this section we explain why the previous result on the optimality of a monetary union may be reversed. We depart from existing models that study the conditions for the optimality of monetary union by explicitly considering the possibility of imperfect credit market integration among countries; i.e., when the premium on cross-border credit c is high. We start from a situation of a monetary union between countries—agents do not pay any currency conversion cost ε—and imperfect credit market integration. We ask whether agents' welfare may be improved by imposing a positive conversion cost between currencies.

Proposition 5 *Let $c > \eta_1/(1-b)$. There are $\hat{\pi}_2 > 0$ and $\hat{\gamma}^2$ with $\gamma^1 < \hat{\gamma}^2 \leq \gamma^2$, such that for $\pi_2 \leq \hat{\pi}_2$ and $\gamma \in \left[\gamma^1, \hat{\gamma}^2\right]$ in a fully constrained equilibrium the imposition of a conversion cost $\varepsilon > 0$ increases the consumption of both goods (q_h^η, q_f^η) and the quantity of credit $(\phi\ell_h^\eta,\ \phi\ell_f^\eta)$, and improves welfare.*

Proposition 5 states that imposing positive conversion costs is welfare improving if agents are credit-constrained, the cross-border credit premium c is sufficiently high, and the probability π_2 of having a strong preference for the foreign good is sufficiently low. A positive conversion cost has a differential impact on the lifetime utility of a defaulter on loan repayment, compared to a non-defaulter. The reason is that defaulters consume more often abroad than non-defaulters and hence pay the conversion cost more frequently. A positive conversion cost therefore reduces the *ex ante* incentives to default, which relaxes the borrowing constraint. To understand why defaulters are not home-biased while non-defaulters are, let us compare their respective travel and consumption choices. A high level of c reduces the willingness of a non-defaulter to consume the foreign good. When the cost of using credit to finance purchases abroad $(1-b)\,c$ is greater than η_1, buyers choose to consume the foreign good only when the realized value of η is η_2, and choose to consume the home good when $\eta = 0, \eta_1$. The foreign good is consumed with probability π_2. By contrast, a defaulter cannot borrow and hence his decision $\hat{\eta}^*$ is independent of c (see equation 22). When $\varepsilon = 0$, he consumes the foreign good for any η higher than 0 (for $\eta = \eta_1, \eta_2$); i.e., with probability $(\pi_1 + \pi_2)$.

Since defaulters pay the conversion cost more often than home-biased non-defaulters, a positive conversion cost makes default less attractive. In equilibrium a higher level of credit can be sustained, thereby allowing higher consumption. However, conversion costs increase the marginal cost of purchasing goods for non-defaulters as well. Therefore, for conversion costs to be welfare improving, it must be that the probability π_2 is sufficiently small so that the negative effect of conversion costs on the consumption of the foreign good is more than compensated by the effect of conversion costs on incentives to default. The condition that π_2 is lower than the threshold value $\hat{\pi}_2$ in Proposition 5 states that the probability π_2 that non-defaulters pay the conversion cost must be relatively low.[14]

The reason why the positive effect of conversion costs on welfare does not hold when the cross-border credit premium is low and agents are credit-constrained, is that the consumption pattern is the same for defaulters and non-defaulters. For $c < \eta_1 (1 - b)$ and $\varepsilon = 0$, non-defaulters travel if their preference shock η is η_1 or η_2, since (26) implies that $\eta_1 > \eta^*$. As a result, $\eta^*, \hat{\eta}^* \leq \eta_1$; i.e., non-defaulters consume the foreign good and therefore pay the conversion costs as often as defaulters.

Next we discuss two potential causes for monetary disunion: first a monetary cause—a variation of the level γ of monetary injections—and then a non-monetary cause—an increase in the cross-border credit premium c.

[14] If c is high enough to lead buyers to consume the home good for all realizations of the preference shock η, conversion costs are only born by defaulters and hence their unique effect is to relax the borrowing constraint. Therefore an increase in conversion costs unambiguously improves welfare regardless of the probabilities associated with the different values of the preference shock. The Appendix contains the proof of this result.

Monetary cause for currency disunion. We now ask whether a currency disunion may be optimal following a variation in the growth rate of the money supply and hence in the rate of inflation. Proposition 1 states that agents are unconstrained for sufficiently high values of γ, in which case they always prefer trading in a monetary union according to Proposition 3, regardless of the level of the cross-border credit premium. Proposition 2 states that agents may be credit-constrained for values of γ below a certain threshold γ^2. Propositions 4 and 5 refer to the case in which agents are credit-constrained. They state that if the cross-border credit premium c is low enough, welfare is higher in a regime with no conversion costs between currencies than in a regime with positive conversion costs (Proposition 4), whereas the opposite is true when the cross-border credit premium is sufficiently high if the probability π_2 is sufficiently low (Proposition 5). Therefore comparison of propositions 1 and 3 with propositions 2 and 5 suggests the following interpretation: For any sufficiently high level of the cross-border credit premium and a sufficiently low level of π_2, a reduction in the level of monetary injections below γ^2 makes agents switch from a preference for the monetary union to a preference for separate monies. The following corollary sums up this discussion.

Corollary 6 *A comparison of Propositions 1 and 3 with Propositions 2 and 4 shows that if $c < \eta_1/(1-b)$, the currency union is optimal regardless of the level of γ. Comparison of Propositions 1 and 3 with Propositions 2 and 5 shows that if $c \geq \eta_1/(1-b)$ and π_2 is relatively low, the level of γ matters for the optimality of the currency union. In particular, a decrease in the rate of inflation from a high enough level of inflation ($\gamma > \widetilde{\gamma}$) to low levels ($\gamma < \gamma^2$) can lead to a shift from a situation in which a currency union is optimal to one in which separate currencies are preferred.*

Non-monetary cause for monetary disunion. We now look at a potential non-monetary cause for the sub-optimality of a monetary union. We follow a traditional interpretation of financial crises that sees their origin in an increase in the real cost associated with the extension of bank credit, see for example Bernanke (1983), Gertler and Kiyotaki (2007). In our model, the non-monetary factor is a variation of the real cost c for agents to obtain cross-border loans. This interpretation is consistent with recent empirical evidence which has shown that the Japanese and the subprime crises had an asymmetric impact on bank lending to the economy: Credit granted by foreign banks decreased more than credit granted by domestic banks, something that may be interpreted as a differential cost of getting different types of credit.[15]

Following this view, our model suggests that the sustainability of a monetary union is directly impacted by the non-monetary factor given by an increase in the cost c when the inflation rate is low enough. The next corollary summarizes the effect of an increase in c in a situation of low inflation.

Corollary 7 *Comparison of Propositions 4 and 5 shows that for low levels of inflation, an increase in the cross-border credit premium from a low level ($c < \eta_1/(1-b)$) to a high level ($c > \eta_1/(1-b)$) may lead to a shift from a situation in which a currency union is optimal to one in which separate currencies are preferred.*

Credit crunch compared across monetary regimes. We define a credit crunch as a decrease in the real amount of credit triggered by an exogenous increase in c that is sufficiently high to induce a home bias in consumption. Before comparing the size of a credit crunch across currency arrangements, Proposition 8 establishes that any increase in the cross-border credit premium c reduces the quantity of credit when agents are credit-constrained.

Proposition 8 *Let $0 < c_0 < \eta_1/(1-b) < c_1$. If a fully constrained equilibrium exists for all $c \in [c_0, c_1]$, an increase in c from c_0 to $c \leq c_1$ leads to a decrease in the real amount of total credit and worsens welfare.*

[15] See Peek and Rosengren (1997), De Haas and van Lelyveld (2010), Popov and Udell (2012).

Proposition 8 shows that an increase in c reduces the amount of credit both when it impacts the travel decision and when it does not. The reason is that a greater value of c reduces agents' expected lifetime utility and hence their repayment incentives. The dashed curve in Figure 2 plots the volume of credit as a function of c in a fully constrained equilibrium under a regime of currency union.[16] For low levels of c, credit is continuously decreasing in c. When c reaches the threshold value $\eta_1/(1-b)$, credit shrinks sharply—the credit crunch—because the agents' decision on how often to consume the foreign good gets distorted, with a resulting fall in their lifetime utility. Agents who previously consumed the foreign good with probability $(\pi_1+\pi_2)$ now opt for consuming it with probability π_2. For values of c greater than $\eta_1/(1-b)$, the effect of c on credit is monotonously negative.

Corollary 9 *Let $0 < c_0 < \eta_1/(1-b) < c_1$. A comparison of Propositions 4 and 5 shows that if c increases from c_0 to c_1 and π_2 is sufficiently low there is a range of values of γ such that the decrease in credit is greater if $\varepsilon = 0$ than if $\varepsilon > 0$.*

Corollary 9 deals with the case in which the increase in c is sufficiently high to generate a home bias in consumption when agents are credit constrained. Such an increase in c generates a sharper decrease in the quantity of credit in a regime of currency union—when $\varepsilon = 0$—than in a regime of separate currencies—i.e. when $\varepsilon > 0$. The solid line in Figure 2 represents the evolution of credit in a regime of separate currencies. Comparison with the dashed line shows that a currency union is the regime that provides the highest volume of credit and consumption when $c < \eta_1/(1-b)$. However the credit crunch triggered by an increase in c above the threshold $\eta_1/(1-b)$ is less acute in a regime of separate currencies than in a currency union.

[16] Figure 2 is drawn assuming that $u(q) = (q^{\alpha})/\alpha$ and parameter values $\alpha = 0.2$, $\beta = 0.9$, $b = 0.3$, $\eta_1 = 0.02$, $\eta_2 = 0.05$, $\pi_1 = 0.2$, $\pi_2 = 0.02$, $\gamma = 1.01$ and $\varepsilon = 0.001$ for the regime of separate currencies. The software program Mathematica was used to check that the conditions for the existence of the fully constrained equilibrium are satisfied.

[Figure 2 about here.]

The simulation reported in Figure 3 shows that parameter values exist for which welfare is higher with positive conversion costs than with an optimal positive rate of inflation in a currency union. The dashed line corresponds to welfare as a function of the inflation rate when there are no conversion costs. Welfare is maximized at an inflation rate equal to 2.6% ($\gamma = 1.026$). The solid line represents the welfare attained in a regime of separate currencies. In this example, conversion costs between currencies improve welfare in the fully constrained and partially constrained equilibria even when inflation is chosen optimally. Consistent with Proposition 3, in a unconstrained equilibrium conversion costs worsen welfare.

5 Credit market integration in the euro area

In this section we review the evidence on existing barriers to cross-border credit in the euro area. Financial market integration in the euro area increased considerably with the introduction of the euro in 1999 and the various regulatory initiatives aimed at creating a single European financial market (Hartmann, Maddaloni, and Manganelli, 2003, ECB, 2007, 2012). The money market and the government bonds market became fully integrated, whereas the degree of integration of corporate bonds and equity markets across member states also increased (De Haan, Oosterloo, and Schoenmaker, 2009).[17]

Nevertheless, the financing of households and small and medium enterprises, highly reliant on bank credit, has remained mostly segmented across member states since the creation of the euro (Sørensen and Gutiérrez, 2006, Kleimeier and Sander, 2007, ECB, 2008, Gropp and Kashyap, 2009). The share of cross-border loans in total loans granted by monetary financial institutions to non-financial entities within the euro area increased from 3% by the end of the 1990s to slightly less

[17]A dramatic contraction in cross-border banking activity in the aftermath of the subprime crisis has been documented (Milesi-Ferreti and Tille, 2011, Manna, 2011).

than 6% in 2008 to further decrease to about 5%.[18] As the European Central Bank asserts, "*cross-border banking through branches or subsidiaries has remained limited*"(ECB, 2012, p.90-91). ECB President M. Draghi stated that "*integration [in the euro area] was largely based on short term interbank debt rather than on equity or direct cross-border lending to firms and households*" (Draghi, 2014a).

Informational asymmetries across the borders are key in explaining the low-level of cross-border credit to non-financial entities within the euro area. Creditors' access to information on non-resident borrowers remains limited despite regulatory measures taken by the European Commission to ensure non-discriminatory access to credit data.[19] While data on debtors is reported at the state level to credit registers operated by central banks and to private credit bureaus, cross-border information sharing occurs only among a subset of public credit registers and mainly on legal persons. In addition, the lack of harmonization among states both on the type of information reported and on the standards for processing it hampers the use of credit information by foreign creditors (Jentzsch, 2007). As a result, for borrowers it is difficult to obtain credit in a member state in which they have no credit history. The informational disadvantage of foreign creditors within the euro area has also negatively affected their entry through branches into other member states (Giannetti, Jentzsch, and Spagnolo, 2009).

In our model, the difference in the conditions to obtain cross-border credit *vis-à-vis* local credit is captured by the extra cost c that borrowers must incur to disclose their identity and their financial history whenever they want to obtain credit across the borders. Aside from informational frictions, however, several institutional features make it ultimately more difficult for borrowers to obtain cross-border credit in

[18] Calculations based on data available at http://sdw.ecb.europa.eu/.

[19] See for instance European Commission (2014b). Jentzsch and San José Rientra (2003) report that EU banks have less access to cross-border information on their EU customers than U.S. banks have on their customers across U.S. states. Jentzsch (2007) points out to the existence of discriminatory rules on cross-border data exchange adopted by EU countries to limit competition within their jurisdictions.

the euro area. Differences in debt recovery and foreclosure procedures with no automatic judicial cooperation among states hinder cross-border credit. The diversity in standards for property valuation, tax systems and even languages across member states also limits the provision of credit across the borders Allen, Beck, Carletti, Lane, Schoenmaker, and Wagner (2011), European Commission (2014b,a). Finally, the extension of cross-border credit has been constrained by supervisory and regulatory policies at the state level.[20] Although the creation of the euro was accompanied by EU initiatives to reduce barriers to the inter-state exchange of financial services, the timing of the transposition of the EU directives reflected each state's preference towards cross-border financial integration (Kalemli-Ozcan, Papaioannou, and Peydró, 2010). While between 1999 and 2014 banks' supervision remained a state-level prerogative in the euro area, during the financial crisis some policies of supervisors could have encouraged the fragmentation of local credit markets (Gros, 2012). The existence of country-specific financial safety nets is found to act as a barrier to cross-border banking (Bertay, Demirguç-Kunt, and Huizinga, 2011). In the words of the ECB President M. Draghi, the insufficient credit market integration in the euro area is related to *"hidden barriers to cross-border activity linked to national preferences"* (Draghi, 2014b).[21]

6 Relation to the literature

Our work is related to four streams of literature. The paper contributes to the broad macroeconomic literature analyzing the costs and benefits of monetary unions.[22]

[20] See Aglietta and Scialom (2003) for a discussion related to the euro area supervisory authorities and Houston, Lin, and Ma (2012) for an empirical investigation showing that banks activity is influenced by the regulatory environment.

[21] See also Constâncio (2014). The trend towards the ring-fencing of banking activities at the state level may be reversed by the devolution of the supervision of banks to the ECB in November 2014. In this respect, a recently stated objective of the ECB is that *"a Spanish firm should be able to borrow from a Spanish bank at the same price at which it would borrow from a Dutch bank"* (Draghi, 2013).

[22] See Mongelli (2002) and Beetsma and Giuliodori (2010) for surveys.

To underscore our contribution, our stylized framework deliberately leaves aside several dimensions already analyzed in that literature. Our work is also relevant to the literature using monetary search models to assess the effects of multiple currencies.

Asymmetric shocks. We abstract from any source of heterogeneity or asymmetric shocks, so that the type of tradeoffs emphasized in the literature on optimal currency areas do not arise in our setup (Mundell, 1961, Benigno, 2004). The main focus of these investigations is on macroeconomic stabilization, and on the cost associated with the loss of the ability to use monetary policy to react to country-specific shocks. It has been argued that this cost to monetary unification is lower when alternative stabilization tools are available at the national level (Cooper and Kempf, 2004, Gali and Monacelli, 2008) or in the presence of supra-national risk-sharing arrangements; e.g., through financial integration (Mundell, 1973) or fiscal transfers (Kenen, 1969). By contrast our paper offers a case for credit market integration in a currency union independent of any stabilization or risk-sharing considerations.

Frictions of monetary policy. Several papers have argued that a currency union can mitigate the inflation bias that results from the time inconsistency of monetary policy identified by Barro and Gordon (1983). In Alesina and Barro (2002), countries lacking internal discipline can commit to monetary stability by joining a currency union with a low-inflation anchor country. Cooley and Quadrini (2003) demonstrate that monetary unification allows countries to benefit from lower inflation by internalizing a negative externality arising between independent monetary authorities under no commitment. Such inflation-generating externalities and the resulting gains from monetary policy coordination can stem from individual countries incentives to manipulate terms of trade, or from an attempt to tax the domestic currency holding of foreigners by means of inflation (Cooper and Kempf, 2003, Liu and Shi, 2010). Our work differs from these studies by considering monetary authorities that are fully committed to a given (exogenous) inflation rate. Importantly, we compare the currency union and the separate currencies regimes for the same monetary policy.[23] This allows us to derive new insights into the link between inflation, credit integration and the desirability (or lack thereof) of monetary unions. In particular, we show in Section 4.2 that when credit integration is low a unique currency regime may be optimal only for sufficiently high levels of inflation. This suggests that high-inflation monetary unions are sustainable, and that countries experiencing high inflation may choose to form a monetary union for reasons unrelated to a reduction in the level of the inflation. By contrast, papers analyzing monetary unions as a way to commit to low inflation suggest that one should mainly observe monetary unions with low inflation levels.

Fiscal and monetary policies interactions. Our results are not driven by fiscal considerations and we have no role for public spending and borrowing. This distinguishes our work from the numerous studies that have analyzed the interactions between monetary and fiscal policies in a monetary union. Motivated by the debate on the European Monetary Union, several papers have discussed the need for fiscal constraints to contain the risk of applying monetary financing of the fiscal deficits to (sub)national governments. Chari and Kehoe (2007) argue that the time-inconsistency problem of the single monetary authority creates a free-rider problem in fiscal policies, leading to excessive debts and inflation in the absence of debt constraints. Beetsma and Uhlig (1999) show that currency integration exacerbates pre-existing moral hazard problems in government borrowing arising from political distortions, thus explaining why countries have incentives to commit to a balance budget *after* joining a monetary union.[24] These studies focus

[23]However, our results do not hinge on exogenous inflation, and the effect of conversion costs that we identify does not disappear when inflation is chosen optimally (see Section 7.1).

[24]One important assumption underlying these results is that the central bank inflates away some of the real value of public debt. In the terminology of Sargent and Wallace (1981), this corresponds

primarily on the issue of the monetisation of fiscal deficits,[25] ruling out default on government debt. Other papers suggests that the possibility of default on public debt may impact the sustainability of monetary unions. It has been argued that a currency union may be unsustainable because it forbids over-indebted governments to reduce their real debt-burden through inflation and currency devaluation (Goodhart, 2011, De Grauwe, 2013, Sims, 2013). In Aguiar, Amador, Farhi, and Gopinath (2014), the level of welfare and vulnerability to rollover debt crises of countries in a monetary union are shown to depend on the distribution of government debts in the union. These studies focus on default on government debt and are silent about credit market integration. By contrast, we focus on the default incentives of private borrowers and show how credit market integration affects the sustainability of the currency union *from the perspective of private agents*.

Costs and benefits of multiple currencies. Our work is also related to a few papers analyzing the potential benefits of multiple monies when there is a commitment issue on the side of private agents rather than public authorities. Early search-theoretic models of monetary exchange which investigate the issue of multiple currencies found that one currency is always optimal (see e.g. Matsuyama, Kiyotaki, and Matsui (1993), Wright and Trejos (2001)). Building on Ravikumar and Wallace (2001), Kiyotaki and Moore (2003) show that multiple currencies may be preferred to a single currency when it allows agents to enjoy the benefits of a greater degree of specialization in the production of goods. Kocherlakota and Krueger (1999) provides a setup where multiple monies can be optimal, because they allow agents to credibly signal private information concerning the type of goods (home vs. foreign) that they prefer. In a related vein, Kocherlakota (2002)

to a regime of 'fiscal dominance'. These problems need not arise, and debt constraints may be sub-optimal, in a monetary-dominance regime where the central bank credibly commits to not accommodate fiscal authority's profligacy (Dornbusch, 1997, Chari and Kehoe, 2007).

[25] See Dixit and Lambertini (2001), Trejos (2004) and Cooper, Kempf, and Peled (2010) for related studies. Sargent (2012) uses U.S. history to draw lessons on the coordination of fiscal and monetary policies in a currency union.

demonstrates that two monies can be useful by allowing agents to signal their (unobservable) money holdings. We emphasize a distinct tradeoff, making the point that when credit integration is imperfect, a unique currency can *increase* the outside option associated with default, and exacerbate agents' incentives to default on their bank loans.

7 Conclusion

With the euro area context in mind, we study a situation in which there is perfect integration with respect to the currency dimension but potentially imperfect integration of credit markets across different jurisdictions. The model presented shows that the ability of a currency union to improve agents' welfare depends on the degree of credit market integration. We capture a high level of credit market integration by a low level of the premium that agents bear in order to have access to credit to trade in another jurisdiction. We show that when this premium is sufficiently low agents always prefer using a unique currency. However, if countries are unable or unwilling to sufficiently reduce the cross-border credit premium, welfare may be impaired by the adoption of a unique currency. The reason is that a currency union may be a cause of credit rationing when the supply of bank credit adapts to borrowers default incentives. This issue may be especially acute in times of crisis when impediments to cross-border credit, condensed in our model by the cross-border credit premium, increase.

The unification of banking markets has remained an overlooked issue of monetary union arrangements. In the two prominent examples of monetary unions formed during the last two centuries—the United States and the euro area—the original design endowed a single organization with the right to issue currency, while credit markets remained segmented across states mainly owing to decentralized bank regulation and supervision at the state level.[26] Both ended up by seeking

[26]The U.S. constitution is the founding act of the currency union in the United States. The

greater credit market integration across states and devolving part of banking regulation and supervision to the federal authorities.[27] This paper suggests that this did not occur by chance, but instead that with low inflation credit market integration is a requisite to reap the gains of a unique currency.

This paper provides a stylized model with both spatial separation between borrowers and lenders and the need for lenders to identify the borrowers' credit history in order to rationalize the imperfection in credit market integration. Thus, our analysis remains silent on other specific obstacles to credit market integration. At the state level, the limited capacity that banks have in seizing collateral or revenue across jurisdictions and the absence of automatic inter-state judicial cooperation constitute barriers to cross-border credit. In addition, state banking supervisory or regulatory authorities may impose limits to cross-border credit. We leave the analysis of the welfare impact of these underlying factors of the degree of credit market integration for future research.

Appendix

Proof of Lemma 1. If defaulters are cash-constrained for all realizations of η, it must be that $u'(\hat{q}), u'(\hat{q}) + \eta_1 - \varepsilon, u'(\hat{q}) + \eta_2 - \varepsilon > 1$. Since we only consider parameter values such that $\eta_1, \eta_2 > \varepsilon$, it is sufficient to show that in the conjectured equilibria $u'(\hat{q}) > 1$ holds to ensure that defaulters are cash-constrained for all

U.S. regime of banknote issuance varied during the 19th century but the Mint was always the authority in charge of issuing the dollar in specie, see Rolnick, Smith, and Weber (2003). The Maastricht treaty creates the euro and endows the ECB with the right to issue the currency.

[27] The U.S. experience exemplifies the distinction between a currency union and a fully-fledged monetary union. During the 19th century periodic systemic banking crises triggered discussions on the redesign of the regime of currency issuance (Rousseau, 2013). Differences in regulatory frameworks during the National Banking Act period caused distortions on the credit market that "stimulated the public to press for currency and banking reform" (White, 1982).

realizations of η. From (22) and (23) we get

$$u'(\hat{q}) - 1 = (\gamma/\beta - 1)/b - [\pi_1(\eta_1 - \varepsilon) + \pi_2(\eta_2 - \varepsilon)]. \tag{28}$$

Thus if $\beta > \{[1 + b(\pi_1\eta_1 + \pi_2\eta_2)]\gamma\}^{-1}$ it follows that $u'(\hat{q}) > 1$ always holds for $\varepsilon \geq 0$ whereas if $\beta \leq \tilde{\beta}$ it follows that $u'(\hat{q}) > 1$ always holds for $\gamma \geq 1$ and $\varepsilon \geq 0$.

Proof of Lemma 2. We first show that any agent repays iff (24) holds. Conjecture that $\mathbf{i}_h = \mathbf{i}_f = \mathbf{i}.$ First, observe that (24) corresponds to the incentive constraint for any agent at the repayment date (in the second market), and as such must hold for any η. To show that condition (24) is also sufficient, it suffices to show that no type η has an incentive to deviate at the borrowing stage. Let $\Gamma = \beta\left(V(m_{+1}) - \hat{V}(\hat{m}_{+1})\right) - \phi(m_{+1} - T - \hat{m}_{+1})$, and rewrite (24) as

$$\phi\ell(1+i) \leq \Gamma. \tag{29}$$

This defines a first debt limit $\bar{\ell}^1 \equiv \frac{\Gamma}{\phi(1+i)}$ for all η. Now, consider an agent with preference shock η and debt $\ell^\eta \leq \bar{\ell}$ (with $\bar{\ell} \geq 0$ arbitrary). With no loss of generality, consider the case of local consumption. For this agent not to deviate at the borrowing stage, it must be the case that

$$u\left(\frac{m+\ell^\eta}{p}\right) - \phi\ell^\eta(1+i) - \phi m_{+1} + \phi T + \beta V(m_{+1}) \geq u\left(\frac{m+\bar{\ell}}{p}\right) - \phi\hat{m}_{+1} + \beta\hat{V}(\hat{m}_{+1}), \tag{30}$$

since an agent that will default borrows up to the limit $\bar{\ell}$. Notice that because the right-hand side is increasing in $\bar{\ell}$, (30) defines a second debt limit $\bar{\ell}^2$ to be imposed on type η. To show that (30) is redundant, we show that $\bar{\ell}^1 \leq \bar{\ell}^2$. Assume the contrary, that is $\bar{\ell}^1 > \bar{\ell}^2$. Using (30),

$$u\left(\frac{m+\bar{\ell}^2}{p}\right) = u\left(\frac{m+\ell^\eta}{p}\right) - \phi\ell^\eta(1+i) + \Gamma, \tag{31}$$

where ℓ^η is the equilibrium borrowing for type η. Since ℓ^η is chosen optimally (and $\bar{\ell}^2$ can be chosen) we have

$$u\left(\frac{m+\ell^\eta}{p}\right) - \phi\ell^\eta(1+i) \geq u\left(\frac{m+\bar{\ell}^2}{p}\right) - \phi\bar{\ell}^2(1+i). \tag{32}$$

From (31) and (32),

$$u\left(\frac{m+\bar{\ell}^2}{p}\right) \geq u\left(\frac{m+\bar{\ell}^2}{p}\right) - \phi\bar{\ell}^2(1+i) + \Gamma,$$

which gives $\bar{\ell}^2 \geq \frac{\Gamma}{\phi(1+i)} = \bar{\ell}^1$, a contradiction. Hence, $\bar{\ell}^1 \leq \bar{\ell}^2$ and (24) is both sufficient and necessary for repayment incentives.

Since $\bar{\ell}^1$ does not depend on η, it also follows that $\bar{\ell}_h = \bar{\ell}_f = \bar{\ell}$. Furthermore, since agents' continuation value at the settlement stage is equal for all buyers regardless of the good that they have consumed in the first market, the interest rate on loans for home consumption and the interest rate on loans for foreign consumption must also be equal in equilibrium; i.e. $i_h = i_f$. In addition, since banks make no profits it must be that $i_h = i_f = i_s = i$. Otherwise a bank could attract all borrowers by offering a lower interest rate and/or all depositors by offering a higher interest rate.

To conclude, we check that (25) is equivalent to (24). Denote as x_j^η and x_s the amount of consumption by the buyer with preference shock η who consumes good $j = (h, f)$ and the amount of consumption by the seller, respectively, in the second market. When the settlement stage arrives, the pay-off to a buyer with preference given by η who repays his debt for consumption of good $j = (h, f)$ is:

$$x_j^\eta + \frac{\beta b}{1-\beta}\left\{\sum_{\eta\leq\eta^*}\pi_\eta\left[u\left(q_h^\eta\right)+x_h^\eta\right]+\sum_{\eta>\eta^*}\pi_\eta\left[u\left(q_f^\eta\right)+(\eta-\varepsilon)\,q_f^\eta-\phi\ell_f^\eta c+x_f^\eta\right]\right\}$$
$$-\frac{\beta(1-b)}{1-\beta}\left(q_s-x_s\right).$$

The pay-off to a defaulter with preference shock η who consumes good $j=(h,f)$ is

$$\bar{x}_j^\eta + \frac{\beta b}{1-\beta}\left\{u(\hat{q})+\sum_{\eta\leq\hat{\eta}^*}\pi_\eta\hat{x}_h^\eta+\sum_{\eta>\hat{\eta}^*}\pi_\eta\left[(\eta-\varepsilon)\,\hat{q}+\hat{x}_f^\eta\right]\right\}-\frac{\beta(1-b)}{1-\beta}\left(q_s-\hat{x}_s\right)$$

where $\bar{x}_j^\eta$ is consumption by the agent in the period in which he defaults and $\hat{x}_h^\eta$, $\hat{x}_f^\eta$ and $\hat{x}_s$ are net consumption by the defaulter in subsequent periods in case he is a buyer with preference shock $\eta\leq\hat{\eta}^*$, a buyer with preference shock $\eta>\hat{\eta}^*$, or a seller.

Consumption quantities x_j^η and x_s are

$$\begin{aligned} x_j^\eta &= -\phi\ell_j^\eta(1+i)-\phi m_{+1}+\phi T \\ x_s &= -\phi\ell_s(1+i)+\phi pq_s-\phi m_{+1}+\phi T. \end{aligned} \tag{33}$$

where $T=(\gamma-1)\,M_{-1}$. In a symmetric equilibrium, $m_{-1}=M_{-1}$. In addition, $m_{-1}=-\ell_s$. Using (5), (16)-(19) and (33), we verify the market clearing condition in the second market:

$$b\sum_{\eta\leq\eta^*}\pi_\eta x_h^\eta+b\sum_{\eta>\eta^*}\pi_\eta x_f^\eta+(1-b)\,x_s=0.$$

Consumption quantities by the defaulter $\bar{x}_j^\eta$, $\hat{x}_h^\eta$, $\hat{x}_f^\eta$ and $\hat{x}_s$ are

$$\bar{x}_j^\eta = \hat{x}_h^\eta = \hat{x}_f^\eta = -\phi \hat{m}_{+1} = -\gamma \hat{q}$$
$$\hat{x}_s = \hat{x}_j^\eta + \phi \hat{m}_{-1} + q_s = -(\gamma - 1)\,\hat{q} + q_s \tag{34}$$

since $\phi \hat{m}_{-1} = \hat{q}$ and $\hat{m}_{+1}/\hat{m}_{-1} = \gamma$. Using (33) and (34), the borrowing constraint can be rewritten as in (25).

Proof of Proposition 1. We first rewrite the equilibrium equations that correspond to an unconstrained equilibrium and then show that the borrowing constraint is effectively slack for γ sufficiently high.

Conjecture an unconstrained equilibrium by setting $\lambda_h^\eta = 0$ and $\lambda_f^\eta = 0$ for all η. From (1), note that the consumption quantity of home goods does not depend on η so in what follows we set $q_h^\eta = q_h$ and $\ell_h^\eta = \ell_h$. (11) and (15) become

$$u'(q_h) = 1 + i$$
$$u'\left(q_f^\eta\right) + \eta - \varepsilon - c = 1 + i. \tag{35}$$

Hence (20) can be rewritten as

$$\gamma/\beta - b \sum_{\eta > \eta^*} \pi_\eta c = 1 + i. \tag{36}$$

Thus, q_h and q_f^η are immediately pinned down for a given value of γ.

From (17) and (19), we get

$$\phi \ell_h = (1 - b)\, q_h - b \sum_{\eta > \eta^*} \pi_\eta \left(q_f^\eta - q_h\right) \tag{37}$$

and

$$q_h - \phi \ell_h = q_f^\eta - \phi \ell_f^\eta \tag{38}$$

for all η.

From (28), a defaulter effectively consumes $\hat{q}$ for γ sufficiently high regardless of the value of the preference shock. From (23), (35), and (36), it follows that $\hat{q} < q_h, q_f^\eta$ for γ sufficiently high. Hence, by the mean value theorem $u(q_h) - u(\hat{q}) > u'(q_h)(q_h - \hat{q})$. Similarly, $u\left(q_f^\eta\right) - u(\hat{q}) > u'\left(q_f^\eta\right)\left(q_f^\eta - \hat{q}\right)$. Therefore, a sufficient condition for the borrowing constraint (25) to be non-binding is

$$\begin{aligned}
&-\phi\left[\ell(1+i) + m_{-1}\right] + \frac{\beta b}{1-\beta}\left\{\sum_{\eta>\eta^*}\pi_\eta\left[(\eta-\varepsilon) q_f^\eta - \phi \ell_f^\eta c\right] - \sum_{\eta>\hat{\eta}^*}\pi_\eta(\eta-\varepsilon)\hat{q}\right\} \\
&+ \frac{\beta b}{1-\beta}\left\{\sum_{\eta\le\eta^*}\pi_\eta\left[u'(q_h)-1\right](q_h-\hat{q}) + \sum_{\eta>\eta^*}\pi_\eta\left[u'\left(q_f^\eta\right)-1\right]\left(q_f^\eta-\hat{q}\right)\right\} \\
&\geq -\frac{(\gamma-\beta)\hat{q}}{1-\beta}.
\end{aligned}$$

Given (35), (37) and (38), this condition can be rewritten as

$$\begin{aligned}
&-\phi\left[\ell(1+i) + m_{-1}\right] + \frac{\beta b^2 c}{1-\beta}\sum_{\eta>\eta^*}\pi_\eta\left[q_h + \sum_{\eta>\eta^*}\pi_\eta\left(q_f^\eta - q_h\right)\right] \\
&+ \frac{\beta b i}{1-\beta}\left(\sum_{\eta\le\eta^*}\pi_\eta q_h + \sum_{\eta>\eta^*}\pi_\eta q_f^\eta\right) + \frac{\beta b}{1-\beta}\left[\sum_{\eta>\eta^*}\pi_\eta(\eta-\varepsilon) - \sum_{\eta>\hat{\eta}^*}\pi_\eta(\eta-\varepsilon)\right]\hat{q} \\
&\geq -\frac{(\gamma-\beta)\hat{q}}{1-\beta} + \frac{\beta b}{1-\beta}\left(i + c\sum_{\eta>\eta^*}\pi_\eta\right)\hat{q}.
\end{aligned} \tag{39}$$

We consider two different cases. First, consider the case of an agent who has consumed the home good in the current period. Using (19), (36) and (37), (39) becomes

$$
\begin{aligned}
&-(1-b)\, q_h i + b \sum_{\eta>\eta^*} \pi_\eta \left(q_f^\eta - q_h\right) i - q_h + \frac{\beta b^2 c}{1-\beta} \sum_{\eta>\eta^*} \pi_\eta \left[q_h + \sum_{\eta>\eta^*} \pi_\eta \left(q_f^\eta - q_h\right) \right] \\
&+ \frac{\beta b i}{1-\beta} \left(\sum_{\eta\leq\eta^*} \pi_\eta q_h + \sum_{\eta>\eta^*} \pi_\eta q_f^\eta \right) \\
&\geq -\beta i \frac{1-b}{1-\beta} \hat{q} - \frac{\beta b}{1-\beta} \left[\sum_{\eta>\eta^*} \pi_\eta (\eta-\varepsilon) - \sum_{\eta>\hat{\eta}^*} \pi_\eta (\eta-\varepsilon) \right] \hat{q}.
\end{aligned}
$$

Since all terms with q_f^η in the above inequality are positive, one way to show that this inequality holds for γ sufficiently high is to consider the following sufficient condition

$$
\begin{aligned}
&- q_h i - q_h + \frac{\beta b^2 c q_h}{1-\beta} \sum_{\eta>\eta^*} \pi_\eta \sum_{\eta\leq\eta^*} \pi_\eta + q_h i \frac{b}{1-\beta} \sum_{\eta\leq\eta^*} \pi_\eta \\
&\geq -\frac{\beta i (1-b)}{1-\beta} \hat{q} - \frac{\beta b}{1-\beta} \left[\sum_{\eta>\eta^*} \pi_\eta (\eta-\varepsilon) - \sum_{\eta>\hat{\eta}^*} \pi_\eta (\eta-\varepsilon) \right] \hat{q}.
\end{aligned}
$$

Since $\eta^* \geq \hat{\eta}^*$, note that if γ is high enough the right-hand side in the above inequality is unambiguously negative given (36). Therefore, the right-hand side can be dismissed and it is sufficient for this inequality to hold that

$$
\left(-1 + \frac{b}{1-\beta} \sum_{\eta\leq\eta^*} \pi_\eta \right) i \geq 1 - \frac{\beta b^2 c}{1-\beta} \sum_{\eta>\eta^*} \pi_\eta \sum_{\eta\leq\eta^*} \pi_\eta. \tag{40}
$$

From (36), the left-hand side in the above inequality is increasing in γ, provided that β is sufficiently high (it is sufficient that $\beta > 1 - b\pi_0$ since $\sum_{\eta\leq\eta^*} \pi_\eta \geq \pi_0$).

Second, consider the case of an agent who has consumed the foreign good in the current period. Using (19), (36), (37) and (38), (39) can be written as

$$\left(-q_f^\eta + b\sum_{\eta>\eta^*}\pi_\eta q_f^\eta + b\sum_{\eta\leq\eta^*}\pi_\eta q_h\right) i - q_f^\eta + \frac{\beta b^2 c}{1-\beta}\sum_{\eta>\eta^*}\pi_\eta\left(\sum_{\eta\leq\eta^*}\pi_\eta q_h + \sum_{\eta>\eta^*}\pi_\eta q_f^\eta\right)$$
$$+\frac{\beta b i}{1-\beta}\left(\sum_{\eta\leq\eta^*}\pi_\eta q_h + \sum_{\eta>\eta^*}\pi_\eta q_f^\eta\right)$$
$$\geq -\beta i\frac{1-b}{1-\beta}\hat{q} - \frac{\beta b}{1-\beta}\left[\sum_{\eta>\eta^*}\pi_\eta\left(\eta-\varepsilon\right) - \sum_{\eta>\hat{\eta}^*}\pi_\eta\left(\eta-\varepsilon\right)\right]\hat{q}.$$

In the above inequality, all terms with q_h are positive and the right-hand side is negative if γ is high enough (as stated in the case of the borrowing constraint for consumption of home goods). Thus, one way to show that this inequality holds for γ sufficiently high is to consider the following sufficient condition

$$\left(-q_f^\eta + \frac{b}{1-\beta}\sum_{\eta>\eta^*}\pi_\eta q_f^\eta\right) i - q_f^\eta + \frac{\beta b^2 c}{1-\beta}\sum_{\eta>\eta^*}\pi_\eta\sum_{\eta>\eta^*}\pi_\eta q_f^\eta \geq 0.$$

Since in an equilibrium with positive consumption of foreign goods $q_f^\eta \leq q_f^{\eta_2}$ and $\sum_{\eta>\eta^*}\pi_\eta q_f^\eta \geq \pi_2 q_f^{\eta_2}$, it is sufficient that

$$\left(-q_f^{\eta_2} + \frac{b}{1-\beta}\pi_2 q_f^{\eta_2}\right) i - q_f^{\eta_2} + \frac{\beta b^2 c\pi_2 q_f^{\eta_2}}{1-\beta}\sum_{\eta>\eta^*}\pi_\eta \geq 0.$$

If $\beta > 1 - b\pi_2$, then a sufficient condition is

$$\left(-1 + \frac{b\pi_2}{1-\beta}\right) i \geq 1 - \frac{\beta b^2 c\pi_2}{1-\beta}\sum_{\eta>\eta^*}\pi_\eta. \tag{41}$$

From (36), the left-hand side in the above inequality is increasing in γ, provided that β is sufficiently high.

To sum up, (40) and (41) hold if γ is sufficiently high. In addition from (36) a high value of γ ensures $i \geq 0$. Hence an unconstrained equilibrium exists. Since

(36) pins down a unique value of i and (35) pins down unique values of q_h and q_f^η for all η this equilibrium is unique.

Proof of Proposition 2. First, we derive the threshold η^* in a conjectured fully constrained equilibrium. In this equilibrium all buyers are credit-constrained. Since the multiplier associated to the borrowing constraint is positive for all realizations of η, it follows from (10), (14) and (15) that the multiplier associated to the cash constraint is also positive for all values of η and so all buyers are cash-constrained. From Lemma 2, $\bar{\ell}_h = \bar{\ell}_f$. Thus we can write $q_h = q_f^\eta = q$ and $\ell_h = \ell_f^\eta = \ell$ for all η. Combining (17) and (19) yields

$$\begin{aligned} \phi\ell &= (1-b)\,q \\ \phi m_{-1} &= bq. \end{aligned} \tag{42}$$

Therefore, from (21) the threshold η^* is equal to $\varepsilon + (1-b)\,c$.

Second, we prove the existence of a fully constrained equilibrium. We distinguish three cases depending on the value of c: $\eta_1 > \varepsilon + (1-b)\,c$, $\varepsilon < \eta_1 \leq \varepsilon + (1-b)\,c < \eta_2$ and $\varepsilon + (1-b)\,c > \eta_2$. We show for the three cases that a fully constrained equilibrium exists for $\gamma \in \left[\gamma^1, \gamma^2\right]$ where γ^1 and γ^2 depend on the value of c. The proof proceeds as follows. First, we rewrite equilibrium equations by conjecturing a fully constrained equilibrium and show that $i \geq 0$ for $\gamma \geq \gamma^1$. Then we show that there is an interval $\left[\gamma^1, \gamma^2\right]$ such that the borrowing constraint binds for all buyers; i.e., for any value of η.

For the cases $\eta_1 \leq \varepsilon + (1-b)\,c < \eta_2$ and $\eta_2 \leq \varepsilon + (1-b)\,c$, we show that sufficiently low values of η_1 and η_2 ensure that an agent with preference shock η_1 or η_2 always prefers borrowing in order to consume the home good instead of consuming the foreign good by using only his money holdings.

Case $\eta_1 > \varepsilon + (1-b)\,c$.

Using the solutions for η^* and $\hat{\eta}^*$ stated in (22) and (26), $\eta^*, \hat{\eta}^* < \eta_1$. (20) and (23) can be rewritten as

$$\gamma/\beta - 1 = b\left[u'(q) + \pi_1(\eta_1 - \varepsilon) + \pi_2(\eta_2 - \varepsilon) - 1\right] + (1-b)\,i \tag{43}$$

and

$$\gamma/\beta - 1 = b\left[u'(\hat{q}) + \pi_1(\eta_1 - \varepsilon) + \pi_2(\eta_2 - \varepsilon) - 1\right]. \tag{44}$$

For a constrained equilibrium to exist, it must be that $i \geq 0$, which requires $q \geq \hat{q}$ given (43) and (44). Denote as $\gamma^{1\prime}$ the value of γ such that $\hat{q} = q$ and as γ^1 the value of γ such that $i = 0$ in a fully constrained equilibrium. From (43) and (44), $\gamma^1 = \gamma^{1\prime}$. Rewrite the borrowing constraint (25) by conjecturing a fully constrained equilibrium for the case $\eta^*, \hat{\eta}^* < \eta_1$. Using (42) equation (25) becomes

$$\begin{aligned} & -i(1-b)\,q - q \\ & + \frac{\beta b}{1-\beta}\left[u(q) - q + \pi_1(\eta_1 - \varepsilon)\,q + \pi_2(\eta_2 - \varepsilon)\,q - (\pi_1 + \pi_2)(1-b)\,cq\right] \\ & = \frac{\beta b}{1-\beta}\left[u(\hat{q}) - \hat{q} + \pi_1(\eta_1 - \varepsilon)\,\hat{q} + \pi_2(\eta_2 - \varepsilon)\,\hat{q}\right] - \frac{(\gamma - \beta)\,\hat{q}}{1-\beta}. \end{aligned} \tag{45}$$

From (45) it follows that

$$\gamma^{1\prime} = 1 + \beta b\,(\pi_1 + \pi_2)(1-b)\,c.$$

Next we must ensure that $\partial i/\partial\gamma \geq 0$ for $\gamma \geq \gamma^{1\prime} = \gamma^1$. Differentiate (45) with respect to γ to get

$$-\frac{\partial i}{\partial \gamma}(1-b)q-[i(1-b)+1]\frac{\partial q}{\partial \gamma} \tag{46}$$
$$+\frac{\beta b}{1-\beta}\left\{u'(q)-1+\pi_1(\eta_1-\varepsilon)+\pi_2(\eta_2-\varepsilon)-(\pi_1+\pi_2)(1-b)c\right\}\frac{\partial q}{\partial \gamma}$$
$$=\frac{\beta b}{1-\beta}\left\{u'(\hat{q})-1+\pi_1(\eta_1-\varepsilon)+\pi_2(\eta_2-\varepsilon)\right\}\frac{\partial \hat{q}}{\partial \gamma}-\frac{\gamma-\beta}{1-\beta}\frac{\partial \hat{q}}{\partial \gamma}-\frac{\hat{q}}{1-\beta}.$$

From (43),

$$(1-b)\frac{\partial i}{\partial \gamma}=1/\beta-bu''(q)\frac{\partial q}{\partial \gamma}. \tag{47}$$

Use (43), (44), (46) and (47) to get

$$\frac{\partial q}{\partial \gamma}=\frac{(1-\beta)q/\beta-\hat{q}}{(1-\beta)bu''(q)q+\gamma-1-i(1-b)-\beta b(\pi_1+\pi_2)(1-b)c}$$

and

$$(1-b)\beta\frac{\partial i}{\partial \gamma}=\frac{\gamma-1-i(1-b)-\beta b(\pi_1+\pi_2)(1-b)c+\beta bu''(q)\hat{q}}{(1-\beta)bu''(q)q+\gamma-1-i(1-b)-\beta b(\pi_1+\pi_2)(1-b)c}. \tag{48}$$

From (45), we get

$$\gamma-i(1-b)-1-\beta b(\pi_1+\pi_2)(1-b)c \tag{49}$$
$$=\gamma-\beta b(\pi_1+\pi_2)(1-b)c+\frac{\beta b}{1-\beta}\frac{u(\hat{q})+[-1+\pi_1(\eta_1-\varepsilon)+\pi_2(\eta_2-\varepsilon)]\hat{q}}{q}$$
$$-\frac{(\gamma-\beta)\hat{q}/q}{1-\beta}-\frac{\beta b}{1-\beta}\frac{u(q)-q+\pi_1(\eta_1-\varepsilon)q+\pi_2(\eta_2-\varepsilon)q-(\pi_1+\pi_2)(1-b)cq}{q}.$$

By the mean value theorem, $u(q)-u(\hat{q})>u'(q)(q-\hat{q})$ for $q>\hat{q}$. Therefore, for $q>\hat{q}$ (or $i>0$) we verify from (49) that

$$\gamma-i(1-b)-1-\beta b(\pi_1+\pi_2)(1-b)c<-\beta(1-b)i\frac{\hat{q}}{q}$$

so $\gamma - i(1-b) - 1 - \beta b(\pi_1 + \pi_2)(1-b)c$ is unambiguously negative for $i > 0$ and given γ^1 it is equal to zero for $i = 0$. Therefore from (48) it follows that $\partial i/\partial\gamma > 0$ for $i \geq 0$ provided that the borrowing constraint binds. Since $\partial i/\partial\gamma > 0$ at $\gamma = \gamma^1$, $i > 0$ at γ slightly higher than γ^1. In turn, this implies that $\partial i/\partial\gamma > 0$ for γ slightly higher than γ^1. Therefore, $i > 0$ for a higher value of γ. Thus there is an interval of values of $\gamma \geq \gamma^1$ for which $i \geq 0$.

To conclude, we must ensure that the representative agent is credit-constrained for all values of η as we conjectured at the beginning of the proof. We show that he is credit-constrained for a range of values of γ. Since $\eta_1 > (1-b)c + \varepsilon$, two subcases may exist: $\eta_1 - c - \varepsilon > 0$ and $\eta_1 - c - \varepsilon \leq 0$.

Subcase $\eta_1 - c - \varepsilon > 0$: For the agent who consumes the home good, given (11) the multiplier of the borrowing constraint (25) is positive at $\gamma = \gamma^1$ if $u'(q) - 1 > 0$. From (43), this is the case if $\gamma/\beta - 1 - b\pi_1(\eta_1 - \varepsilon) - b\pi_2(\eta_2 - \varepsilon) > 0$ at $\gamma = \gamma^1 = \gamma^{1\prime}$. Since $\gamma \geq 1$ and $\varepsilon \geq 0$, this inequality always holds if $1/\beta - 1 - b\pi_1\eta_1 - b\pi_2\eta_2 > 0$. Since $\eta_1 - c - \varepsilon > 0$ and $\eta_2 > \eta_1$, given (15) this condition implies that the multiplier of the borrowing constraint is also positive for the agent who consumes the foreign good. It follows that if β is sufficiently low agents are credit-constrained for all realizations of the preference shock for an interval of values of $\gamma \geq \gamma^1$.

Subcase $\eta_1 - c - \varepsilon \leq 0$: For an agent with preference shock η_1, given (15) the multiplier of the borrowing constraint (25) is positive at $\gamma = \gamma^1$ if $u'(q) + \eta_1 - 1 - c - \varepsilon > 0$. From (43), this is the case if $(\gamma/\beta - 1)/b - \pi_1(\eta_1 - \varepsilon) - \pi_2(\eta_2 - \varepsilon) + \eta_1 - c - \varepsilon > 0$ at $\gamma = \gamma^1 = \gamma^{1\prime}$. Since $\gamma \geq 1$, $\varepsilon \geq 0$ and $\eta_1 > \varepsilon + (1-b)c$, this inequality always holds if $(1/\beta - 1)/b - \pi_1\eta_1 - \pi_2\eta_2 - b\eta_1/(1-b) > 0$. Since $\eta_2 > \eta_1$ and $\eta_1 - c - \varepsilon \leq 0$, this condition implies that the multiplier of the borrowing constraint is also positive for the agent whose preference shock is η_2 and for the agent who consumes the home good given (11). It follows that if β is sufficiently low agents are credit-constrained for all realizations of the preference shock for an interval of values of $\gamma \geq \gamma^1$.

Therefore there is an interval $\left[\gamma^1, \gamma^2\right]$ such that if $\gamma \in \left[\gamma^1, \gamma^2\right]$ then a fully constrained equilibrium in which $i \geq 0$ exists.

Case $\varepsilon < \eta_1 \leq \varepsilon + (1-b)\, c < \eta_2$.

Using the solutions for η^* and $\hat{\eta}^*$ stated in (22) and (26), $\eta^* > \eta_1$ and $\hat{\eta}^* < \eta_1$. (20) and (23) can be rewritten as

$$\gamma/\beta = bu'(q) + b\pi_2 (\eta_2 - \varepsilon) + (1-b)(1+i) \tag{50}$$

and

$$\gamma/\beta - 1 = b\left[u'(\hat{q}) + \pi_1 (\eta_1 - \varepsilon) + \pi_2 (\eta_2 - \varepsilon) - 1\right]. \tag{51}$$

For a constrained equilibrium to exist, it must be that $i \geq 0$. Denote as $\gamma^{1\prime}$ the value of γ such that $\hat{q} = q$ and as γ^1 the value of γ such that $i = 0$ in a fully constrained equilibrium. From (50) and (51) it follows that $(1-b)\, i = b\pi_1 (\eta_1 - \varepsilon)$ at $\gamma = \gamma^{1\prime}$ so $i > 0$. Rewrite the borrowing constraint (25) by conjecturing a fully constrained equilibrium for the case $\eta_1 \leq \eta^* < \eta_2$ and $\hat{\eta}^* < \eta_1$. Using (42) equation (25) becomes

$$\begin{aligned} & -i(1-b)\, q - q + \frac{\beta b}{1-\beta}\left\{u(q) - q + \pi_2 \left[\eta_2 - \varepsilon - (1-b)\, c\right] q\right\} \\ & = \frac{\beta b}{1-\beta}\left\{u(\hat{q}) + \left[-1 + \pi_1 (\eta_1 - \varepsilon) + \pi_2 (\eta_2 - \varepsilon)\right] \hat{q}\right\} - \frac{(\gamma - \beta)\, \hat{q}}{1-\beta}. \end{aligned} \tag{52}$$

From (52) it follows that

$$\gamma^{1\prime} = 1 + \beta b \pi_2 (1-b)\, c + b\pi_1 (\eta_1 - \varepsilon). \tag{53}$$

Next we must ensure that $\partial i / \partial \gamma \geq 0$ for $\gamma \geq \gamma^{1\prime}$. Differentiate (52) with respect to γ to get

$$- \frac{\partial i}{\partial \gamma} (1-b) q - [i(1-b)+1] \frac{\partial q}{\partial \gamma} + \frac{\beta b}{1-\beta} \left\{ u'(q) - 1 + \pi_2 [\eta_2 - \varepsilon - (1-b)c] \right\} \frac{\partial q}{\partial \gamma} \tag{54}$$

$$= \frac{\beta b}{1-\beta} \left\{ u'(\hat{q}) - 1 + \pi_1 (\eta_1 - \varepsilon) + \pi_2 (\eta_2 - \varepsilon) \right\} \frac{\partial \hat{q}}{\partial \gamma} - \frac{\gamma - \beta}{1-\beta} \frac{\partial \hat{q}}{\partial \gamma} - \frac{\hat{q}}{1-\beta}.$$

From (50),

$$(1-b) \frac{\partial i}{\partial \gamma} = 1/\beta - bu''(q) \frac{\partial q}{\partial \gamma}. \tag{55}$$

Use (50), (51), (54) and (55) to get

$$\frac{\partial q}{\partial \gamma} = \frac{(1-\beta) q/\beta - \hat{q}}{(1-\beta) bu''(q) q + \gamma - 1 - (1-b) i - \beta b \pi_2 (1-b) c}$$

and

$$(1-b) \beta \frac{\partial i}{\partial \gamma} = \frac{\beta bu''(q) \hat{q} + \gamma - 1 - (1-b) i - \beta b \pi_2 (1-b) c}{(1-\beta) bu''(q) q + \gamma - 1 - (1-b) i - \beta b \pi_2 (1-b) c}. \tag{56}$$

From (52), we get

$$\gamma - i(1-b) - 1 - \beta b \pi_2 (1-b) c \tag{57}$$

$$= \gamma - \beta b \pi_2 (1-b) c + \frac{\beta b}{1-\beta} \frac{u(\hat{q}) + [-1 + \pi_1 (\eta_1 - \varepsilon) + \pi_2 (\eta_2 - \varepsilon)] \hat{q}}{q} - \frac{(\gamma - \beta) \hat{q}/q}{1-\beta}$$

$$- \frac{\beta b}{1-\beta} \frac{u(q) - q + \pi_2 [\eta_2 - \varepsilon - (1-b) c] q}{q}.$$

By the mean value theorem, $u(q) - u(\hat{q}) > u'(q)(q - \hat{q})$ for $q > \hat{q}$. Therefore, for $q > \hat{q}$ (or $(1-b) i > b\pi_1 (\eta_1 - \varepsilon)$) we verify from (57) that

$$\gamma - i(1-b) - 1 - \beta b \pi_2 (1-b) c < \beta [b\pi_1 (\eta_1 - \varepsilon) - (1-b) i] \hat{q}/q$$

so $\gamma - i(1-b) - 1 - \beta b \pi_2 (1-b) c$ is unambiguously negative for $(1-b) i > b\pi_1 (\eta_1 - \varepsilon)$ and given $\gamma^{1\prime}$ it is equal to zero at $(1-b) i = b\pi_1 (\eta_1 - \varepsilon)$. Therefore from (56) it follows that $\partial i / \partial \gamma > 0$ for $i \geq b\pi_1 (\eta_1 - \varepsilon) / (1-b) > 0$ provided that the borrowing constraint binds. Since $\partial i / \partial \gamma > 0$ at $\gamma = \gamma^{1\prime}$, $(1-b) i$ is slightly higher than $b\pi_1 (\eta_1 - \varepsilon)$ for γ slightly higher than $\gamma^{1\prime}$. In turn, this implies that $\partial i / \partial \gamma > 0$ for γ slightly higher than $\gamma^{1\prime}$. Therefore $i > b\pi_1 (\eta_1 - \varepsilon) / (1-b) > 0$ for a higher value of γ. Since $i > 0$ at $\gamma = \gamma^{1\prime}$, $\gamma^1 < \gamma^{1\prime}$ and there is an interval of values of $\gamma \geq \gamma^1$ for which $i \geq 0$.

To conclude, we must ensure that the representative agent is credit-constrained for all values of η as we conjectured at the beginning of the proof. We show that he is credit-constrained for a range of values of γ. Since $\varepsilon < \eta_1 \leq \varepsilon + (1-b) c < \eta_2$, two subcases may exist: $\eta_2 - c - \varepsilon > 0$ and $\eta_2 - c - \varepsilon \leq 0$.

Subcase $\eta_2 - c - \varepsilon > 0$: For the agent who consumes the home good, given (11) the multiplier of the borrowing constraint (25) is positive if $u'(q) - 1 - i > 0$. From (50), at $\gamma = \gamma^{1\prime}$ this is the case if $\gamma/\beta - 1 - b\pi_2 (\eta_2 - \varepsilon) - b\pi_1 (\eta_1 - \varepsilon) / (1-b) > 0$. Since $\gamma \geq 1$ and $\varepsilon \geq 0$, this always holds if $1/\beta - 1 - b\pi_2\eta_2 - b\pi_1\eta_1 / (1-b) > 0$. It is straightforward that this condition also implies that $u'(q) - 1 - i > 0$ for $0 \leq (1-b) i \leq b\pi_1 (\eta_1 - \varepsilon)$ and $\gamma \in \left[\gamma^1, \gamma^{1\prime}\right]$. Since $\eta_2 - c - \varepsilon > 0$, given (15) this condition also implies that the multiplier of the borrowing constraint is also positive for the agent who consumes the foreign good. It follows that if β is sufficiently low agents are credit constrained for all realizations of the preference shock for an interval of values of $\gamma \geq \gamma^1$.

Subcase $\eta_2 - c - \varepsilon \leq 0$: For an agent with preference shock η_2, given (15) the multiplier of the borrowing constraint (25) is positive if $u'(q) - 1 - i + \eta_2 - c - \varepsilon > 0$. From (50), at $\gamma = \gamma^{1\prime}$ this is the case if $(\gamma/\beta - 1)/b - \pi_2 (\eta_2 - \varepsilon) - \pi_1 (\eta_1 - \varepsilon) / (1-b) + \eta_2 - c - \varepsilon > 0$. Since $\gamma \geq 1$, $\varepsilon \geq 0$ and $\eta_2 > (1-b) c + \varepsilon$, this always holds if $(1/\beta - 1)/b - \pi_2\eta_2 - \pi_1\eta_1 / (1-b) + b\eta_2 / (1-b) > 0$. It is straightforward that this condition also implies that $u'(q) - 1 - i + \eta_2 - c - \varepsilon > 0$

for $0 \leq (1-b)\, i \leq b\pi_1 (\eta_1 - \varepsilon)$ and $\gamma \in \left[\gamma^1, \gamma^{1\prime}\right]$. Since $\eta_2 - c - \varepsilon < 0$, given (11) this condition also implies that the multiplier of the borrowing constraint is also positive for the agent who consumes the foreign good. It follows that if β is sufficiently low agents are credit constrained for all realizations of the preference shock for an interval of values of $\gamma \geq \gamma^1$.

Finally note that an agent with $\eta = \eta_1$ could prefer to consume the foreign good by using only his money holdings instead of borrowing and consuming the home good, but this possibility can be dismissed. That is, the following condition is satisfied

$$u(q) - \phi\ell\,(1+i) \geq u(m_{-1}) + (\eta_1 - \varepsilon)\, m_{-1}.$$

From (42), this expression can be written as

$$u(q) - (1-b)\, q\, (1+i) \geq u(bq) + (\eta_1 - \varepsilon)\, bq.$$

Since $u(q) - u(bq) > u'(q)(1-b)\, q$ and in a fully constrained equilibrium $i \leq u'(q) - 1$, it follows that it is always possible to define a value $\bar{\eta}_1$ such that if $\eta_1 \leq \bar{\eta}_1$ the above inequality holds.

Therefore there is an interval $\left[\gamma^1, \gamma^2\right]$ such that if $\gamma \in \left[\gamma^1, \gamma^2\right]$ then a fully constrained equilibrium in which $i \geq 0$ exists.

Case $\eta_2 < \varepsilon + (1-b)\, c.$

Using the solutions for η^* and $\hat{\eta}^*$ stated in (22) and (26), $\eta^* > \eta_2$ and $\hat{\eta}^* < \eta_1$. (20) and (23) can be rewritten as

$$\gamma/\beta - 1 = b\left[u'(q) - 1\right] + (1-b)\, i \tag{58}$$

and

$$\gamma/\beta - 1 = b\left[u'(\hat{q}) + \pi_1(\eta_1 - \varepsilon) + \pi_2(\eta_2 - \varepsilon) - 1\right]. \tag{59}$$

For a constrained equilibrium to exist, it must be that $i \geq 0$. Denote as $\gamma^{1\prime}$ the value of γ such that $\hat{q} = q$ and as γ^{1} the value of γ such that $i = 0$ in a fully constrained equilibrium. From (58) and (59), $(1-b)\, i = b\left[\pi_1 (\eta_1 - \varepsilon) + \pi_2 (\eta_2 - \varepsilon)\right]$ at $\gamma = \gamma^{1\prime}$. Rewrite the borrowing constraint (25) by conjecturing a fully constrained equilibrium for the case $\eta^* > \eta_2$ and $\hat{\eta}^* < \eta_1$. Using (42) it becomes

$$\begin{aligned}
&- i\,(1-b)\, q - q + \frac{\beta b}{1-\beta}\left[u(q) - q\right] \\
&= \frac{\beta b}{1-\beta}\left[u(\hat{q}) - \hat{q} + \pi_1 (\eta_1 - \varepsilon)\,\hat{q} + \pi_2 (\eta_2 - \varepsilon)\,\hat{q}\right] - \frac{(\gamma - \beta)\,\hat{q}}{1-\beta}. \qquad (60)
\end{aligned}$$

From (60) it follows that

$$\gamma^{1\prime} = 1 + b\left[\pi_1 (\eta_1 - \varepsilon) + \pi_2 (\eta_2 - \varepsilon)\right].$$

Next we must ensure that $\partial i/\partial\gamma \geq 0$ for $\gamma \geq \gamma^{1\prime}$. Differentiate (60) with respect to γ to get

$$\begin{aligned}
&- \frac{\partial i}{\partial \gamma}(1-b)\, q - \left[i\,(1-b) + 1\right]\frac{\partial q}{\partial \gamma} + \frac{\beta b}{1-\beta}\left[u'(q) - 1\right]\frac{\partial q}{\partial \gamma} \\
&= \frac{\beta b}{1-\beta}\left\{u'(\hat{q}) - 1 + \pi_1 (\eta_1 - \varepsilon) + \pi_2 (\eta_2 - \varepsilon)\right\}\frac{\partial \hat{q}}{\partial \gamma} - \frac{\gamma - \beta}{1-\beta}\frac{\partial \hat{q}}{\partial \gamma} - \frac{\hat{q}}{1-\beta}. \qquad (61)
\end{aligned}$$

From (58),

$$(1-b)\frac{\partial i}{\partial \gamma} = 1/\beta - b u''(q)\frac{\partial q}{\partial \gamma}. \qquad (62)$$

Use (58), (59), (61) and (62) to get

$$\frac{\partial q}{\partial \gamma} = \frac{(1-\beta)\, q/\beta - \hat{q}}{\gamma - 1 - i\,(1-b) + (1-\beta)\, b u''(q)\, q}$$

and

$$(1-b)\,\beta\frac{\partial i}{\partial\gamma}=\frac{\gamma-1-i\,(1-b)+\beta bu''\,(q)\,\hat{q}}{\gamma-1-i\,(1-b)+(1-\beta)\,bu''\,(q)\,q}. \tag{63}$$

From (60), we get

$$\begin{aligned}&\gamma-i\,(1-b)-1+\frac{\beta b}{1-\beta}\frac{u\,(q)-q}{q}\\&=\gamma+\frac{\beta b}{1-\beta}\frac{u\,(\hat{q})-\hat{q}+\pi_1\,(\eta_1-\varepsilon)\,\hat{q}+\pi_2\,(\eta_2-\varepsilon)\,\hat{q}}{q}-\frac{(\gamma-\beta)\,\hat{q}/q}{1-\beta}.\end{aligned} \tag{64}$$

By the mean value theorem, $u\,(q)-u\,(\hat{q})>u'\,(q)\,(q-\hat{q})$ for $q>\hat{q}$. Therefore, for $q>\hat{q}$ (or $i>b\,[\pi_1\,(\eta_1-\varepsilon)+\pi_2\,(\eta_2-\varepsilon)]\,/\,(1-b)$) we verify from (64) that

$$\gamma-i\,(1-b)-1<\beta\,\{b\,[\pi_1\,(\eta_1-\varepsilon)+\pi_2\,(\eta_2-\varepsilon)]-(1-b)\,i\}\,\hat{q}/q$$

so $\gamma-i\,(1-b)-1$ is unambiguously negative for $(1-b)\,i>b\,[\pi_1\,(\eta_1-\varepsilon)+\pi_2\,(\eta_2-\varepsilon)]$ and given $\gamma^{1\prime}$ it is equal to zero at $(1-b)\,i=b\,[\pi_1\,(\eta_1-\varepsilon)+\pi_2\,(\eta_2-\varepsilon)]$. Therefore from (63) it follows that $\partial i/\partial\gamma>0$ for $i\geq b\,[\pi_1\,(\eta_1-\varepsilon)+\pi_2\,(\eta_2-\varepsilon)]>0$ provided that the borrowing constraint binds. Since $\partial i/\partial\gamma>0$ at $\gamma=\gamma^{1\prime}$, $(1-b)\,i$ is slightly higher than $b\,[\pi_1\,(\eta_1-\varepsilon)+\pi_2\,(\eta_2-\varepsilon)]$ for γ slightly higher than $\gamma^{1\prime}$. In turn, this implies that $\partial i/\partial\gamma>0$ for γ slightly higher than $\gamma^{1\prime}$. Therefore $i>b\,[\pi_1\,(\eta_1-\varepsilon)+\pi_2\,(\eta_2-\varepsilon)]\,/\,(1-b)>0$ for a higher value of γ. Since $i>0$ at $\gamma=\gamma^{1\prime}$, $\gamma^1<\gamma^{1\prime}$ and there is an interval of values of $\gamma\geq 0$ for which $i\geq 0$.

To conclude, we must ensure that the representative agent is credit-constrained for all values of η as we conjectured at the beginning of the proof. When $\eta_2<\varepsilon+(1-b)\,c$, the agent consumes the home good for all realizations of the preference shock. Given (11), the multiplier of the borrowing constraint (25) is positive if $u'\,(q)-1-i>0$. From (58), at $\gamma=\gamma^{1\prime}$ this is the case if $\gamma/\beta-1-b\,[\pi_1\,(\eta_1-\varepsilon)+\pi_2\,(\eta_2-\varepsilon)]\,/\,(1-b)>0$. Since $\gamma\geq 1$ and $\varepsilon\geq 0$, this always holds if $1/\beta-1-b\,(\pi_1\eta_1+\pi_2\eta_2)\,/\,(1-b)>0$. It is straightforward that this condition also implies that $u'(q)-1-i>0$ for $0\leq(1-b)\,i\leq b\left[\pi_1(\eta_1-\varepsilon)+\pi_2(\eta_2-\varepsilon)\right]$ and $\gamma\in\left[\gamma^1,\gamma^{1\prime}\right]$. It follows that if β is sufficiently low agents are credit constrained for all realizations of the preference shock for an interval of values of $\gamma\geq\gamma^1$.

Finally note that an agent with $\eta=\eta_1,\eta_2$ could prefer to consume the foreign good by using only his money holdings instead of borrowing and consuming the home good, but this possibility can be dismissed. That is, the following condition is satisfied

$$u(q)-\phi\ell(1+i)\geq u(m_{-1})+(\eta_2-\varepsilon)\,m_{-1}.$$

From (42), this expression can be written as

$$u(q)-(1-b)\,q\,(1+i)\geq u(bq)+(\eta_2-\varepsilon)\,bq.$$

Since $u(q)-u(bq)>u'(q)(1-b)\,q$ and in a fully constrained equilibrium $i\leq u'(q)-1$, it follows that it is always possible to define a value $\bar{\eta}_2$ such that if $\eta_2\leq\bar{\eta}_2$ the above inequality holds. Further, the above inequality implies that $u(q)-(1-b)\,q\,(1+i)\geq u(bq)+(\eta_1-\varepsilon)\,bq$ since $\eta_2>\eta_1$.

Therefore there is an interval $\left[\gamma^1,\gamma^2\right]$ such that if $\gamma\in\left[\gamma^1,\gamma^2\right]$ then a fully constrained equilibrium in which $i\geq 0$ exists.

Proof of Proposition 3. Given (35), (20) can be written as

$$\gamma/\beta=u'\left(q_f^\eta\right)+\eta-\varepsilon-\left(1-b\sum_{\eta>\eta^*}\pi_\eta\right)c.$$

Hence

$$\frac{\partial q_f^\eta}{\partial\varepsilon}=\frac{1}{u''\left(q_f^\eta\right)} \tag{65}$$

so that $\partial q_f^\eta/\partial\varepsilon < 0$. From (35) and (65), $\partial q_h/\partial\varepsilon = 0$. From (37) and (38), we get

$$\frac{\partial(\phi\ell_h)}{\partial\varepsilon} = -b\sum_{\eta>\eta^*}\pi_\eta\frac{\partial q_f^\eta}{\partial\varepsilon}$$

and

$$\frac{\partial\left(\phi\ell_f^\eta\right)}{\partial\varepsilon} = \left(1 - b\sum_{\eta>\eta^*}\pi_\eta\right)\frac{\partial q_f^\eta}{\partial\varepsilon}.$$

Given (65), we get $\partial(\phi\ell_h)/\partial\varepsilon > 0$ and $\partial\left(\phi\ell_f^\eta\right)/\partial\varepsilon < 0$.

Differentiating (27) with respect to ε yields

$$\frac{\partial\mathcal{W}}{\partial\varepsilon} = \frac{b}{1-\beta}\sum_{\eta>\eta^*}\pi_\eta\left\{\left[u'\left(q_f^\eta\right) - 1 + \eta - \varepsilon - c + b\sum_{\eta>\eta^*}\pi_\eta c\right]\frac{\partial q_f^\eta}{\partial\varepsilon} - q_f^\eta\right\}.$$

Since $u'\left(q_f^\eta\right) - 1 + \eta - \varepsilon - c > 0$ for all $\eta > \eta^*$ from (35) and $\partial q_f^\eta/\partial\varepsilon < 0$ from (65), it follows that $\partial\mathcal{W}/\partial\varepsilon < 0$.

Proof of Proposition 4. Let $c < (\eta_1 - \varepsilon)/(1-b)$ and consider a fully constrained equilibrium in which $\lambda_h^\eta, \lambda_f^\eta > 0$ and the borrowing constraint (25) holds with equality. As in the proof of Proposition 2, we can set $\phi\ell_h = \phi\ell_f^\eta = \phi\ell$ and $q_h = q_f^\eta = q$ for all η. Given (26) and (42), welfare defined in (27) becomes

$$\mathcal{W}\left(\frac{b}{1-\beta}\right)^{-1} = u(q) + (-1 + \pi_1\eta_1 + \pi_2\eta_2)\,q - (\pi_1 + \pi_2)\left[\varepsilon + (1-b)\,c\right]q. \quad (66)$$

Differentiate the borrowing constraint for the case $c < (\eta_1 - \varepsilon)/(1-b)$ stated in (45) with respect to ε to get

$$
\begin{aligned}
&-\left[1+(1-b)\,i\right]\frac{\partial q}{\partial \varepsilon}-(1-b)\frac{\partial i}{\partial \varepsilon}q-\frac{\beta b\,(\pi_1+\pi_2)}{1-\beta}q \qquad (67)\\
&+\frac{\beta b}{1-\beta}\left\{u'(q)-1+\pi_1\eta_1+\pi_2\eta_2-(\pi_1+\pi_2)\left[\varepsilon+(1-b)\,c\right]\right\}\frac{\partial q}{\partial \varepsilon}\\
&=\frac{\beta b}{1-\beta}\left[u'(\hat{q})-1+\pi_1(\eta_1-\varepsilon)+\pi_2(\eta_2-\varepsilon)\right]\frac{\partial \hat{q}}{\partial \varepsilon}-\frac{\beta b\,(\pi_1+\pi_2)}{1-\beta}\hat{q}-\frac{\gamma-\beta}{1-\beta}\frac{\partial \hat{q}}{\partial \varepsilon}.
\end{aligned}
$$

Differentiating (43) with respect to ε yields

$$
(1-b)\frac{\partial i}{\partial \varepsilon}=-bu''(q)\frac{\partial q}{\partial \varepsilon}+b\,(\pi_1+\pi_2)\,. \qquad (68)
$$

Rewrite (67) using (43), (44) and (68) to get

$$
\frac{\partial q}{\partial \varepsilon}=\frac{b\,(\pi_1+\pi_2)\left[(1-\beta)\,q+\beta\,(q-\hat{q})\right]}{\gamma-1-(1-b)\,i-\beta b\,(\pi_1+\pi_2)\,(1-b)\,c+(1-\beta)\,bqu''(q)}. \qquad (69)
$$

From (43) and (44), $q=\hat{q}$ when $i=0$ and $q>\hat{q}$ when $i>0$. From the proof of Proposition 2, when $c<(\eta_1-\varepsilon)/(1-b)$ the value of γ such that $i=0$ and $q=\hat{q}$ is $\gamma^1=1+\beta b\,(\pi_1+\pi_2)\,(1-b)\,c$. Further, $q\geq\hat{q}$ for $\gamma\in\left[\gamma^1,\gamma^2\right]$. Therefore, the numerator at the right-hand side in (69) is positive for $\gamma\in\left[\gamma^1,\gamma^2\right]$. From the proof of Proposition 2, it can be deduced that the denominator at the right-hand side in (69) is negative for $\gamma\in\left[\gamma^1,\gamma^2\right]$. It follows that in a fully constrained equilibrium $\partial q/\partial\varepsilon<0$ for $\gamma\in\left[\gamma^1,\gamma^2\right]$. Since $\phi\ell_h=\phi\ell_f^\eta=\phi\ell=(1-b)\,q$ for all η from (42), $\partial\,(\phi\ell)\,/\partial\varepsilon<0$.

Differentiating (66) with respect to ε yields

$$
\frac{\partial \mathcal{W}}{\partial \varepsilon}\left(\frac{b}{1-\beta}\right)^{-1}=\left\{u'(q)-1+\pi_1\eta_1+\pi_2\eta_2-(\pi_1+\pi_2)\left[\varepsilon+(1-b)\,c\right]\right\}\frac{\partial q}{\partial \varepsilon}-(\pi_1+\pi_2)\,q.
$$

Since $\eta_1,\eta_2>\varepsilon+(1-b)\,c$ and $\partial q/\partial\varepsilon<0$ from (69), it follows that $\partial\mathcal{W}/\partial\varepsilon<0$.

Proof of Proposition 5. For this proof we distinguish two cases, $\varepsilon+(1-b)\,c>\eta_2$

and $\varepsilon < \eta_1 \leq \varepsilon + (1-b)\, c < \eta_2$. In the first case showing that $\partial \mathcal{W}/\partial \varepsilon > 0$ is straightforward since the non-defaulter consumes only the home good and hence does not incur conversion costs. For the second case it is shown that $\partial \mathcal{W}/\partial \varepsilon > 0$ holds for π_2 sufficiently low.

Consider a fully constrained equilibrium in which $\lambda_h^\eta, \lambda_f^\eta > 0$ and the borrowing constraint (25) holds with equality. As in the proof of Proposition 2, we can set $\phi\ell_h = \phi\ell_f^\eta = \phi\ell$ and $q_h = q_f^\eta = q$ for all η.

Case $\varepsilon + (1-b)\, c > \eta_2$.

Given (26), welfare defined in (27) becomes

$$\mathcal{W}\left(\frac{b}{1-\beta}\right)^{-1} = u\,(q) - q. \tag{70}$$

Differentiate the borrowing constraint for the case $\varepsilon + (1-b)\, c > \eta_2$ stated in (60) with respect to ε to get

$$\begin{aligned} & -(1-b)\,\frac{\partial i}{\partial \varepsilon} q + \frac{\beta b\,(\pi_1+\pi_2)}{1-\beta}\hat{q} - [1+(1-b)\, i]\,\frac{\partial q}{\partial \varepsilon} \\ & + \frac{\beta b}{1-\beta}\left[u'\,(q) - 1\right]\frac{\partial q}{\partial \varepsilon} \\ & = \frac{\beta b}{1-\beta}\left\{u'\,(\hat{q}) - 1 + [\pi_1\,(\eta_1-\varepsilon) + \pi_2\,(\eta_2-\varepsilon)]\right\}\frac{\partial \hat{q}}{\partial \varepsilon} - \frac{\gamma-\beta}{1-\beta}\frac{\partial \hat{q}}{\partial \varepsilon}. \end{aligned} \tag{71}$$

Differentiating (58) with respect to ε yields

$$(1-b)\,\frac{\partial i}{\partial \varepsilon} = -bu''\,(q)\,\frac{\partial q}{\partial \varepsilon}. \tag{72}$$

Rewrite (71) using (58), (59) and (72)

$$\frac{\partial q}{\partial \varepsilon} = \frac{-\beta b\,(\pi_1+\pi_2)\,\hat{q}/\,(1-\beta)}{bu''\,(q)\, q + [\gamma - 1 - (1-b)\, i]\,/\,(1-\beta)}. \tag{73}$$

From the proof of Proposition 2, it can be deduced that the denominator at the right-hand side of (73) is negative. Since the numerator at the right-hand side of (73) is also negative, it follows that in a fully constrained equilibrium $\partial q/\partial \varepsilon > 0$. Since $\phi \ell_h = \phi \ell_f^{\eta} = \phi \ell = (1-b)\, q$ for all η from (42), it follows that $\partial\left(\phi \ell\right)/\partial \varepsilon > 0$.

Differentiating (70) with respect to ε yields

$$\frac{\partial \mathcal{W}}{\partial \varepsilon}\left(\frac{b}{1-\beta}\right)^{-1} = \left[u'\left(q\right)-1\right]\frac{\partial q}{\partial \varepsilon}. \tag{74}$$

Since $\partial q/\partial \varepsilon > 0$ from (73), (74) implies that $\partial \mathcal{W}/\partial \varepsilon > 0$.

Case $\varepsilon < \eta_1 \leq \varepsilon + (1-b)\, c < \eta_2$.

Given (26) and (42), welfare defined in (27) becomes

$$\mathcal{W}\left(\frac{b}{1-\beta}\right)^{-1} = u\left(q\right) - q + \pi_2\left[\eta_2 - \varepsilon - (1-b)\, c\right] q. \tag{75}$$

Differentiate the borrowing constraint for the case $\varepsilon < \eta_1 \leq \varepsilon + (1-b)\, c < \eta_2$ stated in (52) with respect to ε to get

$$\begin{aligned} &-(1-b)\frac{\partial i}{\partial \varepsilon}q - \frac{\beta b}{1-\beta}\left[\pi_2 q - (\pi_1+\pi_2)\,\hat{q}\right] \\ &-\left[1+(1-b)\, i\right]\frac{\partial q}{\partial \varepsilon} + \frac{\beta b}{1-\beta}\left\{u'\left(q\right) - 1 + \pi_2\left[\eta_2 - \varepsilon - (1-b)\, c\right]\right\}\frac{\partial q}{\partial \varepsilon} \\ &= \frac{\beta b}{1-\beta}\left\{u'\left(\hat{q}\right) - 1 + \left[\pi_1\left(\eta_1-\varepsilon\right) + \pi_2\left(\eta_2 - \varepsilon\right)\right]\right\}\frac{\partial \hat{q}}{\partial \varepsilon} - \frac{\gamma-\beta}{1-\beta}\frac{\partial \hat{q}}{\partial \varepsilon}. \end{aligned} \tag{76}$$

Differentiating (50) with respect to ε yields

$$(1-b)\frac{\partial i}{\partial \varepsilon} = -bu''\left(q\right)\frac{\partial q}{\partial \varepsilon} + b\pi_2. \tag{77}$$

Rewrite (76) using (50), (51) and (77)

$$\frac{\partial q}{\partial \varepsilon}=\frac{b\pi_2 q+\beta b\left[\pi_2 q-(\pi_1+\pi_2)\hat{q}\right]/(1-\beta)}{bu''(q)\,q+\left[\gamma-1-(1-b)\,i-\beta b\,(1-b)\,\pi_2 c\right]/(1-\beta)}. \tag{78}$$

From the proof of Proposition 2, it can be deduced that the denominator at the right-hand side of (78) is negative. At $\gamma=\gamma^{1\prime}$, $q=\hat{q}$ and hence the numerator at the right-hand side of (78) is negative if $\pi_2-\beta\pi_1/(1-\beta)<0$. Therefore, $\partial q/\partial\varepsilon>0$ at $\gamma=\gamma^{1\prime}$ if π_2 is sufficiently low. Since q is increasing in ε as long as the numerator at the right-hand side of (78) is negative and $\hat{q}$ is decreasing in ε given (51), the numerator at the right-hand side of (78) is increasing in ε. Define $\gamma^{2\prime}$ the value of γ such that the numerator at the right-hand side of (78) is zero given $\{q,\hat{q},i\}$ that solve (50), (51) and (52). In addition, let $\bar{\gamma}^2=\min\left(\gamma^2,\gamma^{2\prime}\right)$. Then in a fully constrained equilibrium $\partial q/\partial\varepsilon>0$ for $\gamma\in\left[\gamma^1,\bar{\gamma}^2\right]$. Since $\phi\ell_h=\phi\ell_f^{\eta}=\phi\ell=(1-b)\,q$ for all η from (42), $\partial(\phi\ell)/\partial\varepsilon>0$ for $\gamma\in\left[\gamma^1,\bar{\gamma}^2\right]$.

Differentiating (75) with respect to ε yields

$$\frac{\partial\mathcal{W}}{\partial\varepsilon}\left(\frac{b}{1-\beta}\right)^{-1}=\left\{u'(q)-1+\pi_2\left[\eta_2-\varepsilon-(1-b)\,c\right]\right\}\frac{\partial q}{\partial\varepsilon}-\pi_2 q. \tag{79}$$

Using (78) for $\gamma\in\left[\gamma^1,\gamma^{1\prime}\right]$ with $\gamma^{1\prime}$ stated in (53) it follows that

$$\frac{\partial\mathcal{W}}{\partial\varepsilon}\left(\frac{b}{1-\beta}\right)^{-1}>\frac{u'(q)-1+\pi_2\left[\eta_2-\varepsilon-(1-b)\,c\right]}{-u''(q)}\left(\frac{\beta\pi_1}{1-\beta}-\pi_2\right)-\pi_2 q.$$

Since assumed preferences satisfy $-u''(q)\,q\leq u'(q)$ and $\eta_2-\varepsilon-(1-b)\,c>0$, a sufficient condition for $\partial\mathcal{W}/\partial\varepsilon>0$ for $\gamma\in\left[\gamma^1,\gamma^{1\prime}\right]$ is

$$\frac{u'(q)-1}{u'(q)}\left(\frac{\beta\pi_1}{1-\beta}-\pi_2\right)-\pi_2>0. \tag{80}$$

The left-hand side at (80) is positive at $\pi_2=0$ and given (50) is decreasing in π_2 for $\gamma\in\left[\gamma^1,\gamma^{1\prime}\right]$. Therefore there is a value $\bar{\pi}_2>0$ such that if $\pi_2\leq\bar{\pi}_2$ the left-hand side in (80) is positive for $\gamma \in \left[\gamma^1, \gamma^{1\prime}\right]$. Since condition (80) is sufficient (but not necessary) there is $\hat{\pi}_2 > \bar{\pi}_2 > 0$ and $\hat{\gamma}^2 > \gamma^{1\prime}$ such that if $\pi_2 \leq \hat{\pi}_2$ then $\partial \mathcal{W}/\partial \varepsilon > 0$ for $\gamma \in \left[\gamma^1, \hat{\gamma}^2\right]$.

Proof of Proposition 8. We proceed in three steps to show that the amount of credit is decreasing in c. Consider two cases: $\varepsilon = 0$ and $\varepsilon > 0$. First, we show that q is decreasing in c from some value c_0 up to some value $c < \eta_1 / (1-b)$ in the case $\varepsilon = 0$ and up to some value $c < (\eta_1 - \varepsilon) / (1-b)$ in the case $\varepsilon > 0$. To prove that an increase in c entails a decrease in credit in a fully constrained equilibrium in which $\varepsilon + (1-b)c \leq \eta_1$ with $\varepsilon \geq 0$, differentiate the borrowing constraint stated in (45) with respect to c:

$$\begin{aligned} &-[(1-b)i+1]\frac{\partial q}{\partial c} - (1-b)q\frac{\partial i}{\partial c} - \frac{\beta b}{1-\beta}(\pi_1+\pi_2)(1-b)q \\ &+\frac{\beta b}{1-\beta}\left\{u'(q) - 1 + \pi_1\eta_1 + \pi_2\eta_2 - (\pi_1+\pi_2)[\varepsilon+(1-b)c]\right\}\frac{\partial q}{\partial c} = 0. \end{aligned} \tag{81}$$

From (43) we get

$$(1-b)\frac{\partial i}{\partial c} = -bu''(q)\frac{\partial q}{\partial c}. \tag{82}$$

Use (43) and (82) to rewrite (81) as follows

$$\frac{\partial q}{\partial c} = \frac{\beta b(\pi_1+\pi_2)(1-b)q/(1-\beta)}{bu''(q)q + [\gamma - 1 - (1-b)i - \beta b(\pi_1+\pi_2)(1-b)c]/(1-\beta)}. \tag{83}$$

From the proof of Proposition 2, in the case $\varepsilon + (1-b)c < \eta_1$ of a fully constrained equilibrium the denominator at the right-hand side in (83) is negative. Since in the fully constrained equilibrium $\ell_h = \ell_f^\eta = \ell$ for all η and $\phi\ell = (1-b)q$ from (42), it follows that $\partial(\phi\ell)/\partial c < 0$ for $c < (\eta_1 - \varepsilon)/(1-b)$. Since $\partial q/\partial c < 0$ for all $c < (\eta_1 - \varepsilon)/(1-b)$ it follows that, in the case $\varepsilon = 0$, q and $(\phi\ell)$ are decreasing in c up to $c = \eta_1/(1-b)$ and, in the case $\varepsilon > 0$, q and $\phi\ell$ are decreasing in c up to $c = (\eta_1 - \varepsilon) / (1 - b)$.

Second, we show that the function $q = q(c)$ is not continuous. For this, we evaluate the function $q = q(c)$ at a particular value of γ and infer that its properties hold for a range of values of γ. Consider the case $\varepsilon = 0$. The function $q = q(c)$ jumps below at $c = \eta_1 / (1 - b)$; i.e., $q(c^-) > q(c^+)$ with $c^- = \eta_1 / (1 - b) - dc$, $c^+ = \eta_1 / (1 - b) + dc$ and $dc \to 0$. From (43) and (50), it follows that

$$b\left[u'\left(q\left(c^-\right)\right) + \pi_1\eta_1\right] + (1 - b)\, i\left(c^-\right) = bu'\left(q\left(c^+\right)\right) + (1 - b)\, i\left(c^+\right) \tag{84}$$

where $q(c^-)$ and $i(c^-)$ solve (45) and (43) (with $\hat{q}$ being determined by (44)), whereas $q(c^+)$ and $i(c^+)$ solve (52) and (50) (with $\hat{q}$ being determined by (51)). At $\gamma = \gamma^1(c^+, \varepsilon = 0) = 1 + \beta b\pi_2(1 - b)\, c^+ + b\pi_1\eta_1$, $i(c^+) = b\pi_1\eta_1 / (1 - b)$. Hence, at $\gamma = \gamma^1(c^+, \varepsilon = 0)$ (84) becomes

$$bu'\left(q\left(c^-\right)\right) + (1 - b)\, i\left(c^-\right) = bu'\left(q\left(c^+\right)\right). \tag{85}$$

Note that $\gamma^1(c^+, \varepsilon = 0) > \gamma^1(c^-, \varepsilon = 0) = 1 + \beta b(\pi_1 + \pi_2)(1 - b)\, c^-$ provided that $\beta < 1$. Thus, at $\gamma = \gamma^1(c^+, \varepsilon = 0)$, $i(c^-) > 0$ since $i(c^-) = 0$ at $\gamma^1(c^-, \varepsilon = 0)$ and $\partial i / \partial \gamma > 0$ in a fully constrained equilibrium with $c < \eta_1 / (1 - b)$ from Proposition 2. Hence, from (85) $q(c^+) < q(c^-)$. It follows that the function is discontinuous at $c = \eta_1 / (1 - b)$ with $q(c^+) < q(c^-)$. Since all functions in (84) ($u'(q(c^-))$ and $i(c^-)$ which solve (45) and (43), and $u'(q(c^+))$ and $i(c^+)$ which solve (52) and (50)) are continuous, we can infer that there is a range of values of γ for which the function $q = q(c)$ is not continuous at $c = \eta_1 / (1 - b)$ with $q(c^+) < q(c^-)$. From (42), it follows that at $c = \eta_1 / (1 - b)$ the function $\phi\ell$ also jumps below.

Similarly, in the case $\varepsilon > 0$, the function $q = q(c)$ jumps below at $c = (\eta_1 - \varepsilon) / (1 - b)$; i.e., $q(c^-) > q(c^+)$ with $c^- = (\eta_1 - \varepsilon) / (1 - b) - dc$, $c^+ =$

$(\eta_1 - \varepsilon) / (1 - b) + dc$ and $dc \to 0$. From (43) and (50), it follows that

$$b\left[u\left(q\left(c^{-}\right)\right) + \pi_1\left(\eta_1 - \varepsilon\right)\right] + (1 - b)\, i\left(c^{-}\right) = bu\left(q\left(c^{+}\right)\right) + (1 - b)\, i\left(c^{+}\right) \qquad (86)$$

where $q(c^{-})$ and $i(c^{-})$ solve (45) and (43) (with $\hat{q}$ being determined by (44)), whereas $q(c^{+})$ and $i(c^{+})$ solve (52) and (50) (with $\hat{q}$ being determined by (51)). At $\gamma = \gamma^1(c^{+}, \varepsilon > 0) = 1 + \beta b \pi_2 (1 - b)\, c^{+} + b\pi_1(\eta_1 - \varepsilon)$, $i(c^{+}) = b\pi_1(\eta_1 - \varepsilon) / (1 - b)$. Hence, at $\gamma = \gamma^1(c^{+}, \varepsilon > 0)$ (86) becomes

$$bu\left(q\left(c^{-}\right)\right) + (1 - b)\, i\left(c^{-}\right) = bu\left(q\left(c^{+}\right)\right). \qquad (87)$$

At $\gamma = \gamma^1(c^{+}, \varepsilon > 0)$, $i(c^{-}) > 0$ since $i(c^{-}) = 0$ at $\gamma^1(c^{-}, \varepsilon > 0)$, $\partial i / \partial \gamma > 0$ in a fully constrained equilibrium with $c < (\eta_1 - \varepsilon) / (1 - b)$ from Proposition 2, and $\gamma^1(c^{+}, \varepsilon > 0) > \gamma^1(c^{-}, \varepsilon > 0)$. Thus, from (87) $q(c^{+}) < q(c^{-})$. It follows that the function is discontinuous at $c = (\eta_1 - \varepsilon) / (1 - b)$ with $q(c^{+}) < q(c^{-})$. Since all functions in (86) are continuous, we can infer that there is a range of values of γ for which the function $q = q(c)$ is not continuous at $c = (\eta_1 - \varepsilon) / (1 - b)$ with $q(c^{+}) < q(c^{-})$. From (42), it follows that the function $\phi\ell$ also jumps below at $c = (\eta_1 - \varepsilon) / (1 - b)$.

Third, we show that q is decreasing in c for $c > \eta_1 / (1 - b)$ in the case $\varepsilon = 0$ and for $c > (\eta_1 - \varepsilon) / (1 - b)$ in the case $\varepsilon > 0$. To prove that this increase in c entails a decrease in credit in a fully constrained equilibrium in which $\varepsilon + (1 - b)\, c > \eta_1$ with $\varepsilon \geq 0$, differentiate the borrowing constraint stated in (52) with respect to c:

$$\begin{aligned} &- \left[(1 - b)\, i + 1\right] \frac{\partial q}{\partial c} - (1 - b)\, q \frac{\partial i}{\partial c} - \frac{\beta b}{1 - \beta} \pi_2 (1 - b)\, q \\ &+ \frac{\beta b}{1 - \beta} \left\{u'(q) - 1 + \pi_2\left[\eta_2 - \varepsilon - (1 - b)\, c\right]\right\} \frac{\partial q}{\partial c} = 0. \end{aligned} \qquad (88)$$

From (50) we get

$$(1-b)\frac{\partial i}{\partial c}=-bu''(q)\frac{\partial q}{\partial c}. \tag{89}$$

Use (50) and (89) to rewrite (88) as follows

$$\frac{\partial q}{\partial c}=\frac{\beta b\pi_2(1-b)q/(1-\beta)}{bu''(q)q+[\gamma-1-(1-b)i-\beta b\pi_2(1-b)c]/(1-\beta)}. \tag{90}$$

As shown in the proof of Proposition 2, in a fully constrained equilibrium in the case $\varepsilon<\eta_1\leq(1-b)c+\varepsilon<\eta_2$ the denominator at the right-hand side in (90) is negative, so $\partial q/\partial c<0$. Since in the fully constrained equilibrium $\ell_h=\ell_f^\eta=\ell$ for all η and $\phi\ell=(1-b)q$ from (42), it follows that $\partial(\phi\ell)/\partial c<0$.

Finally, from Proposition 4 for $\varepsilon+(1-b)c<\eta_1$, in a fully constrained equilibrium q and $\phi\ell$ are decreasing in ε. In addition, from Proposition 5 for $(1-b)c>\eta_1$, there is a range of values of γ for which q and $\phi\ell$ are increasing in ε. Then it is straightforward to verify that if c increases from c_0 to c_1 and a fully constrained equilibrium exists for c_0 and c_1 for this range of values of γ, the decrease in q and $\phi\ell$ is stronger in the case $\varepsilon=0$ than in the case $\varepsilon>0$.

Differentiating (66) with respect to c yields

$$\begin{aligned}\frac{\partial\mathcal{W}}{\partial c}\left(\frac{b}{1-\beta}\right)^{-1}&=\left\{u'(q)-1+\pi_1\eta_1+\pi_2\eta_2-(\pi_1+\pi_2)[\varepsilon+(1-b)c]\right\}\frac{\partial q}{\partial c}\\&\quad-(\pi_1+\pi_2)(1-b)q.\end{aligned}$$

Thus $\partial\mathcal{W}/\partial c>0$ for all $(1-b)c<\eta_1-\varepsilon$ since $\partial q/\partial c<0$ for $(1-b)c<\eta_1-\varepsilon$. Similarly, after differentiating (75) it is straightforward to verify that $\partial\mathcal{W}/\partial c<0$ for all $\varepsilon<\eta_1\leq(1-b)c+\varepsilon<\eta_2$ since $\partial q/\partial c<0$ in this case as well. Further, since $q(c^+)<q(c^-)$ for $c^-=(\eta_1-\varepsilon)/(1-b)-dc$, $c^+=(\eta_1-\varepsilon)/(1-b)+dc$ and $dc\to 0$, comparison of (66) and (75) demonstrates that welfare at c^+ is lower than welfare at c^-.

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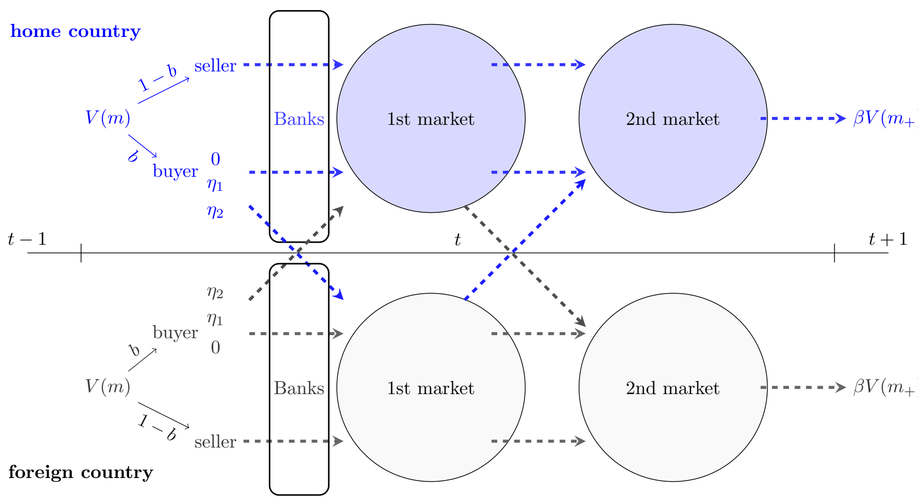

Figure 1: Sequence within a period

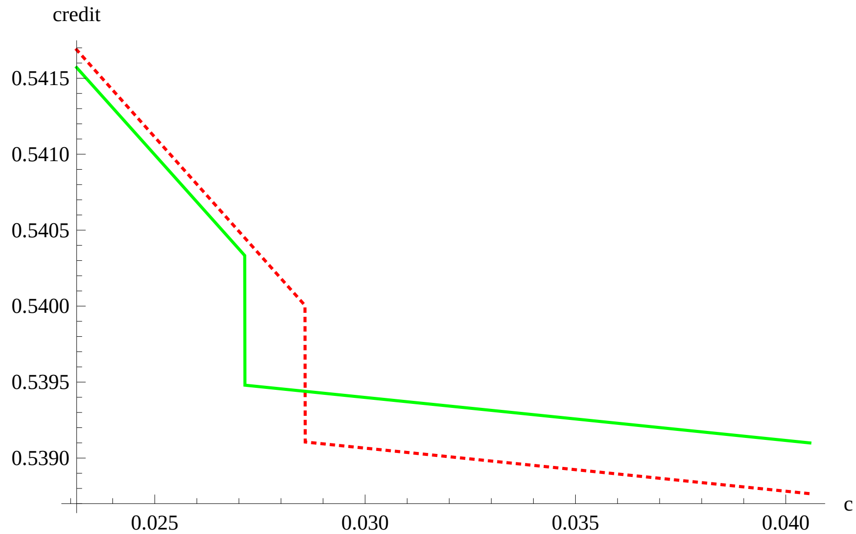

Figure 2: Quantity of credit as a function of the cross-border credit premium c in a currency union (dashed line) and in a regime of separate currencies (solid line)

7.1 Conversion cost and optimal inflation

Proposition 5 shows that under appropriate conditions a strictly positive conversion cost—separate currencies—may relax the borrowing constraint and improve welfare compared to the benchmark case of a currency union. This result is obtained taking the inflation rate (γ) as given. However, previous studies of economies with credit and limited commitment show that inflation can be used to curb default incentives.[28] In this section, we present a parametrization in which using positive conversion costs in combination with the inflation rate is necessary to maximize welfare.[29] Given parameter values, in a regime of currency union a fully constrained equilibrium exists up to the threshold value of γ equal to $\gamma^2 = 1.021$. The unconstrained equilibrium exists for values of γ higher than $\widetilde{\gamma} = 1.026$. For intermediate values the equilibrium is partially constrained since agents with preference shocks η_0 and η_1 are credit constrained whereas agents with preference shock η_2 are not.

[28]In this type of environment default is a cash-intensive activity. A positive inflation rate thus acts as a tax that discourages default. In the setup we consider, default is a conversion-intensive activity.

[29]Figure 3 is drawn assuming that $u(q) = (q^{\alpha})/\alpha$ and parameter values $\alpha = 0.2$, $\beta = 0.9$, $b = 0.3$, $c = 0.1$, $\eta_1 = 0.02$, $\eta_2 = 0.05$, $\pi_1 = 0.7$, $\pi_2 = 0.02$ and $\varepsilon = 0.015$ for the regime of separate currencies. Notice that in our example η_1 is lower than $(1-b)\,c$ and that the condition on β stated in Lemma 1 is verified. The maximum level of welfare is 1.19688 with no conversion costs and 19.691 with positive conversion costs. The software program Mathematica was used to check that the conditions for the existence of the different equilibria are satisfied.

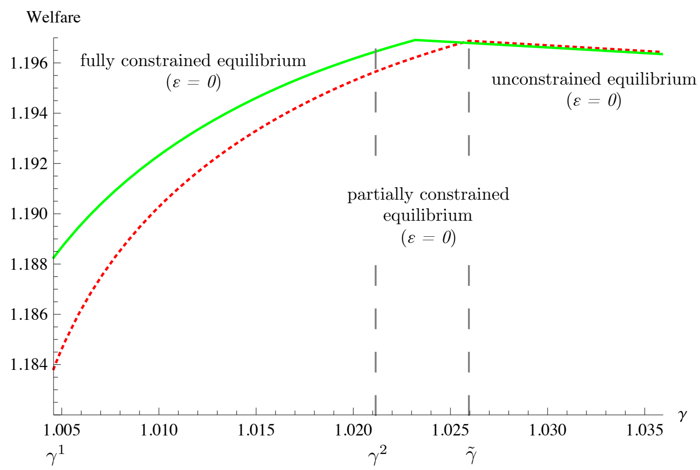

Figure 3: Welfare as a function of inflation in a currency union (dashed line) and in a regime of separate currencies (solid line)

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