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READING INTEREST RATE AND BOND FUTURES OPTIONS' SMILES: HOW PIBOR AND NOTIONAL OPERATORS APPRECIATED THE 1997 FRENCH SNAP ELECTION

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Reading Interest Rate and Bond Futures Options' Smiles: How PIBOR and Notional Operators Appreciated the 1997 French Snap Election

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Abstract

The aim of this paper is to compare various methods which extract a Risk Neutral Density (RND) out of PIBOR as well as of Notional interest rate futures options and to investigate how traders react to a political event. We first focus on 5 dates surrounding the 1997 snap election and several methods: Black (1976), a mixture of lognormals (as in Melik and Thomas (1997)), an Hermite expansion (as in Abken, Madan, and Ramamurtie (1996)), and a method based on Maximum Entropy (following Buchen and Kelly (1996)). The various methods give similar RNDs, yet, the Hermite expansion approach, by allowing for somewhat dirty options prices, by providing a good fit to options prices, and by being fast is the retained method for the data at hand. This approach also allows construction of options with a fixed time till maturity. A daily panel of options running from February 1997 to July 1997 reveals that operators in both markets anticipated the snap election a few days before the official announcement and that a substantial amount of political uncertainty subsisted even a month after the elections. Uncertainty evolved with polls' forecasts of the future government.

Keywords: Risk neutral density, Futures option pricing, PIBOR, Notional, Political risk.
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1 Introduction

Much of the literature following the seminal work on option pricing by Black-Scholes (1973) and Merton (1973) assumed that the price process of an option's underlying asset can be described by a lognormal diffusion. Empirical studies of options' implied volatilities, such as Rubinstein's (1994), demonstrate that options for a same maturity but with different strike prices have different volatilities, a feature called the *option's smile*. This feature may arise if the underlying asset has jumps or changes its volatility and is in contradiction with Black-Scholes' and Merton's assumptions. In this case an option may not be perfectly hedged which in turn implies that market operators' expectations about the future play an important role.

From the works of Harrison and Kreps (1979) as well as Harrison and Pliska (1981) it is known that, under rather mild assumptions, there exists a Risk Neutral Density (RND) which allows pricing of an option as a conditional expectation. This RND is related to market participants' expectations of the future price process in a risk neutral environment. As shown by Campa, Chang, and Reider (1997a), or Söderlind and Svensson (1997), once such a density is obtained it is possible to compute moments as well as confidence intervals, the evolution of which is indicative of market participants' perceptions of the future. Clearly, because of a possibly time-varying risk premium RNDs differ from real life probability distributions. Nonetheless, the RND plays an important role as a tool to evaluate the credibility of the Central Bank. RNDs are equally important for an investor who needs a measure how markets are thought to evolve through time. RNDs can also provide an objective measure of expected extreme variations in the underlying asset's price which is useful for risk management.[1]

In this study we investigate the information content of PIBOR (Paris Interbank Offered Rate) interest rate futures options and Notional bond futures options. Whereas PIBOR instruments capture short-term interest rate movements, the Notional bond, by being a virtual instrument with up to 10 years till maturity, captures long-run movements. Both futures and options are traded at MATIF (Marché à Terme International de France). Thus, a first contribution is the investigation of a dataset which has attracted less attention than US or UK data.[2]

A further contribution of this research is the comparison of four methods which extract a RND by applying them to actual options data. First, the benchmark model follows Black (1976) and assumes that the RND is a lognormal. Second, we consider the mixture of lognormals model

[1] Unlike some of the literature which has addressed the question how to price options under non constant volatility (e.g. Derman and Kani (1994), Dumas, Fleming, and Whaley (1996), Dupire (1994), Shimko (1993), as well as Stein and Stein (1991)) we address the question of what the information content in options of various maturities is.

[2] Possibly because this market is less liquid it is necessary to filter the data. These filters can be applied to data of similar quality in other markets.

developed by Melick and Thomas (1997) who also indicate how to price American options within this setup. This model is further used by Bahra (1996), Campa, Chang, and Reider (1997a). Third, we follow Abken, Madan, and Ramamurtie (1996) who implement the idea exposed in Madan and Milne (1994) which consists in approximating the underlying RND with Hermite polynomials. Fourth, we approximate the RND with a functional having the highest Entropy conditional on being consistent with the data. This last approach has been developed by Buchen and Kelly (1996). To our knowledge, this paper is the first to assess their method on interest-rate and bond futures-options. Within the Entropy literature, Stutzer (1996) suggests a multistep Bayesian procedure, where the initial step involves a density estimation from historical prices of the underlying asset.

Further literature builds upon structural models by assuming jump-diffusions such as Malz (1996, 1997) or stochastic volatility and/or jumps as in Bates (1991, 1996a, 1996b, 1997). Those methods have been successfully applied to foreign exchange and stock price data. See also Campa, Chang, and Reider (1997b), as well as Jondeau and Rockinger (1997) for a comparison of various existing methods on exchange data.

Several other approaches to obtain a RND are possible. Rubinstein (1994) as well as Jackwerth and Rubinstein (1996) suggest a method based on binomial trees. Aït-Sahalia and Lo (1995) provide a non-parametric method based on time series analysis and kernel estimates. Breeden and Litzenberger (1978) observed that applying Leibnitz's rule to options prices also yields the RND. This observation has lead Shimko (1993) to fit cubic splines to the original data whose derivatives approximate the RND. Similarly, Neuhaus (1995) works with cumulative distribution functions. We do not implement those methods, which are certainly valuable, because the main thrust of our paper is to focus on the information content of options whereas the comparison of methods assures us that the retained method is similar to others.

Since there exist PIBOR options for fixed maturities ranging between 3 and 12 months we discuss various approaches to construct *standardized* options, i.e. options with a constant time to maturity. This allows comparison of market participants' expectations through time while avoiding the difficulty that options with different maturities may also exhibit a premium.

By using data running from February 1997 to July 1997 we are able to investigate the impact of the 1997 snap election on the French financial market. This investigation allows us to show that markets reacted strongly to the announcement of the election, even though information appeared to have trickled into the market before the announcement. Also before the first and second electoral round, markets appear influenced by polls. Like the study by Bates (1997) on stock index options after the 1987 crash, or by Malz (1996) on foreign exchange data, we are able to exhibit a peso-problem: Market participants put a significant probability on an event which has not occurred in recent history such as a very strong increase in interest rates.

The structure of this research is as follows. In the next Section we describe the methods used to obtain the RND. In Section 3 we start by describing the data. Then we compare the various methods for five selected dates. After finding a satisfactory model (the Hermite polynomial approximation) we set to applying it to the entire database and analyze the message contained in the PIBOR and Notional options. Conclusions are presented in Section 4.

2 How to obtain a Risk Neutral Density

Let C and P be the generic notation for the price of a call and a put option. Let r, t, τ, T, K be the risk-free continuously compounded interest rate, the current date, the expiration date, the time till expiration, $(\tau = t + T)$, and a strike price, respectively. Let F_t be the current futures price or rate. For further use we want to introduce the notation C_i, $i = 1, \cdots, m$ as the price for the European call option with ith strike price written as K_i, and $c_i(x) \equiv (x - K_i)^+$ the associated payoff.[3] We will also denote PIBOR and Notional options with the superscripts Pi and No.

The results of Harrison and Kreps (1979), of Harrison and Pliska (1981), or at textbook level of Duffie (1988) p. 115, guarantee that there exists a Risk Neutral Density $q(\cdot)$ such that the call and put option's price can be computed as a conditional expectation:

$$C = e^{-rT} \int_{F_¿=K}^{+\infty} (F_\tau - K)^+ q(F_\tau, \theta) dF_\tau \tag{1}$$

$$P = e^{-rT} \int_{0}^{F_¿=K} (K - F_\tau)^+ q(F_\tau, \theta) dF_\tau \tag{2}$$

where θ is a vector of parameters characterizing the distribution.

There are different approaches to obtaining a RND. It is possible to adopt a structural model as in Black (1976) where it is assumed that the Forward price follows a geometric Brownian motion. The arbitrage arguments of Black-Scholes (1973) then yield a RND. The observed non-constancy of volatility across smiles suggests that the structural assumption concerning the underlying asset is erroneous and suggests that the direct modeling of the RND in a non-parametric fashion may shed additional insights.

2.1 Paradigm used to value interest and bond futures options

In this work we are dealing with PIBOR and Notional options which are American options on futures contracts where the underlying asset is either an interest rate, or a bond. As summarized in Musiela and Rutkowski (1997), chap. 14, structural pricing models have been developed for

[3] $(X)^+$ takes the value X if X > 0 and 0 otherwise.

derivative instruments on interest rate and bond options. For the options at hand anecdotal evidence suggests, however, that operators use Black's 1976 model (possibly with *ad-hoc* improvements) and seldomly use their right of early exercise. Since we are interested, in this study, in constructing a simple tool to indicate market participants expectations we decided to neglect the American type character of options and to focus on models in the same spirit as Black.[4]

2.2 Black's benchmark model

Black's benchmark option pricing model assumes that the dynamics of the underlying asset follows a geometric Brownian motion $dF_t = \sigma F_t dW_t$ where σ represents the instantaneous volatility and W_t is a Brownian motion in a risk neutral world.

For such a dynamic the risk neutral density can be written as

$$q(x) = \frac{1}{\sqrt{2\pi} x\sigma\sqrt{T}} \exp[-\frac{1}{2}\left(\frac{\ln(x) - \ln(F_t) + 0.5\sigma^2 T}{\sigma\sqrt{T}}\right)^2], \tag{3}$$

hence as a lognormal with mean $\ln(F_t) - 0.5\sigma^2 T$ and variance $\sigma^2 T$. As discussed above, it is possible to price Notional options following Black (1976). His formulas for European futures options are given by:

$$\begin{aligned} C^{No}(F_t, K, T, r, \sigma) &= e^{-rT}\left\{F_t \Phi(d_1^{No}) - K\Phi(d_2^{No})\right\} \\ P^{No}(F_t, K, T, r, \sigma) &= e^{-rT}\left\{-F_t \Phi(-d_1^{No}) + K\Phi(-d_2^{No})\right\} \\ d_1^{No} &= \frac{\ln(F_t/K) + 0.5\sigma^2 T}{\sigma\sqrt{T}} \\ d_2^{No} &= d_1^{No} - \sigma\sqrt{T} \end{aligned}$$

where Φ is the cumulative distribution function of a mean zero, unit variance normal variate.

Under Black's assumptions, these formulas apply for the Notional option. For the PIBOR option it is necessary to take into account the fact that we are dealing with an entity which is quoted in terms of deviations from 100. Using

$$\begin{aligned} C^{Pi}(F_t, K, T, r, \sigma) &\equiv P^{No}(100 - F_t, 100 - K, T, r, \sigma) \\ P^{Pi}(F_t, K, T, r, \sigma) &\equiv C^{No}(100 - F_t, 100 - K, T, r, \sigma) \end{aligned}$$

it is possible to price PIBOR options or to extract out of options an implied volatility σ.

As discussed in section 2.1 we neglect the early exercise possibility which is know to have

[4]We will check the empirical relevance of neglecting the American option feature.

value (see Stoll and Whaley (1993) p.184–190). This value is created by fact that the early exercise of an option gives its owner the futures contract and a possible margin adjustment. It is possible to (partially) correct Black's formulas for American type options following the approximation suggested by Barone-Adesi and Whaley (1987). To check the importance of the American option feature we implemented this correction. Then we computed call and put prices, using the implied volatility supplied by MATIF, with and without the correction. The percentage difference is found to be more important for lower strikes than for higher ones, and more important for higher rather than shorter maturities. For the PIBOR (Notional) options the difference in option prices being always smaller than 1.13% (0.13%) we decided to pursue by neglecting the American option feature.

2.3 Non-structural methods to extract a RND

2.3.1 Approximation of the RND with a mixture of lognormal densities

A first natural way to approximate the risk neutral density is to follow Bahra (1996), Melick and Thomas (1997), or Söderlind and Svensson (1997) and to describe it as a mixture of L weighted lognormal distributions

$$q(F_\tau, \theta) = \sum_{j=1}^{L} \alpha_j a_j(F_\tau) \tag{4}$$

where the α_j are constants.[5] The parameter θ contains the α_j as well as the parameters characterizing $a_j(\cdot)$ namely the mean μ_j and the annualized variance σ_j^2.

It then follows that the price of an European call option can be written as:

$$C = e^{-rT} \sum_{j=1}^{L} \alpha_j \int_{F_¿=K}^{+\infty} (F_\tau - K)^+ a_j(F_\tau) dF_\tau.$$

The expression for the density of a_j is given in formula (3). In this case the call option price

[5]If desired, other distributions than then lognormal could be used.

(1) can be written as:[6]

$$C = e^{-rT} \sum_{j=1}^{L} \alpha_j \left(\exp(\mu_j + \frac{1}{2}\sigma_j^2 T) \left[1 - \Phi\left(\frac{\ln(K) - \mu_j - \sigma_j^2 T}{\sigma_j \sqrt{T}} \right) \right] - K \left[1 - \Phi\left(\frac{\ln(K) - \mu_j}{\sigma_j \sqrt{T}} \right) \right] \right).$$

In addition, it is convenient to impose the constraint that under risk-neutrality the Forward price should be equal to the expected Forward price at maturity.

$$F_t = \sum_{j=1}^{L} \alpha_j \exp(\mu_j + \frac{1}{2}\sigma_j^2 T).$$

This constraint can be imposed with a Lagrangean penalty function. Once estimates of the parameters are obtained it is possible to construct the RND with (4).

2.3.2 A semi-parametric approach involving Hermite polynomials

The theoretical foundations of this method (HER) are elaborated in Madan and Milne (1994) and applied in Abken, Madan, and Ramamurtie (1996) to Eurodollar futures options. Their model operates as follows. First, they assume that the underlying asset follows a lognormal diffusion

$$dF_t = \mu F_t dt + \sigma F_t dW_t, \tag{5}$$

where W_t is a Brownian motion with respect to some abstract reference density $\phi(\cdot)$ assumed to be Normal with mean zero and variance 1. This implies, when we move to a discretization that

$$F_\tau = F_t \exp((\mu - \frac{1}{2}\sigma^2)T + \sigma\sqrt{T}z) \tag{6}$$

where $z \sim \mathcal{N}(0,1)$.

The density $p(\cdot)$ of the actual statistical (observed) price process is obtained after a first

[6] Johnson, Kotz, and Balakrishnan (1994), p.241 indicate that if $F \sim N(\mu; \sigma^2)$ then

$$E[F|F > K] = \exp(\mu + \frac{1}{2}\sigma^2)\frac{1 - \Phi(U_0 - \sigma)}{1 - \Phi(U_0)}; \text{where } U_0 = \frac{\ln(K) - \mu}{\sigma}.$$

The observation that

$$\int_{F_\tau = K}^{+\infty} (F_\tau - K)^+ a(F_\tau) dF_\tau = (E[F_\tau|F_\tau > K] - K)\Pr[F_\tau|F_\tau > K]$$

then yields a formula, equivalent, from the point of view numerical complexity, to Black's formula.

change of probability: $p(z) = \nu(z)\phi(z)$. Given actual data, $p(z)$ could be estimated and thus $\nu(z)$ inferred. The risk neutral density is given by another change of probability so that $q(z) = \lambda(z)\phi(z)$. The main thrust of their work aims at estimating $\lambda(z)$.

The key observation of their approach is that the reference measure being a normal one, the various components involved in the option pricing can be expressed as linear combinations of Hermite polynomials. Let $\{h_k\}_{k=1}^{\infty}$ be those polynomials. Such polynomials are known to form an orthogonal basis with respect to the scalar product $< f, g >= \int f(z)g(z)\phi(z)dz$.[7]

Since under the reference measure, $\phi(z)$, the dynamics of the underlying asset are perfectly defined, Madan and Milne show how it is possible to write any payoff, such as for instance the payoff of a call option as:

$$(z - K)^+ = \sum_{k=0}^{\infty} a_k h_k(z).$$

The a_k are well defined and their expression depends on μ, σ, T, τ.

On the other hand, it is also possible to write $\lambda(z)$ with respect to the basis as $\lambda(z) = \sum_{j=0}^{\infty} b_j h_j(z)$. Following (1) and given the orthogonality property of Hermite polynomials, the price of a call option can then be written as

$$C = \sum_{k=0}^{\infty} a_k \pi_k,$$

where the $\pi_k = e^{-rT} b_k$ are interpreted as the implicit price of polynomial risk h_k. Since the Hermite polynomial of order k will depend on a k-th moment we will also refer to π_3 and π_4 as the price of skewness and kurtosis.

For practical purposes the infinite sum can be truncated up to the fourth order. Madan and Milne (1994) then show that the risk neutral density can be written as

$$q(x) = \phi(x)\left[b_0 - \frac{b_2}{\sqrt{2}} + \frac{3b_4}{\sqrt{24}} + (b_1 - \frac{3b_3}{\sqrt{6}})x + (\frac{b_2}{\sqrt{2}} - \frac{6b_4}{\sqrt{24}})x^2 + \frac{b_3}{\sqrt{6}}x^3 + \frac{b_4}{\sqrt{24}}x^4\right], \qquad (7)$$

where the b_i are parameters to be estimated. One can either estimate $\pi_k, k = 1, \cdots, 4$ or follow Abken, Madan, and Ramamurtie (1996) and impose $\pi_0 = e^{rT}$, $\pi_1 = \pi_2 = 0$ and estimate μ and σ. In the empirical part of this work we will further pin down μ by imposing the martingale restriction $\mathrm{E}_q[F_\tau|F_t] = F_t$ and estimate only σ and the future value of the third and fourth price

[7] The Hermite polynomial of order k is defined as the solution to the differential equation $H_k(x) = (-1)^k \frac{\partial^k \phi}{\partial x^k} \frac{1}{\phi(x)}$ where ϕ is the mean zero and unit variance normal density. After standardization of the polynomials H_k to unit norm, one obtains that the first four standardized Hermite polynomials are $h_0(x) = 1; h_1(x) = x$, $h_2(x) = (x^2 - 1)/\sqrt{2}$, $h_3(x) = (x^3 - 3x)/\sqrt{6}$, $h_4(x) = (x^4 - 6x^2 + 3)/\sqrt{24}$.

of risk. The actual risk neutral density of F_τ can then be inferred using a change of variable.

They also show that for two times till maturity T and T' the following restriction must hold

$$\frac{\pi_k(T)e^{rT}}{T^{k/2}} = \frac{\pi_k(T')e^{rT'}}{T'^{k/2}}. \tag{8}$$

Even though they reject this restriction, one can use it as an approximation to extrapolate from given options information for options of a fixed horizon.

2.3.3 Estimation of the RND with the Maximum Entropy Principle

A method based on the Principle of Maximum Entropy (ME) is introduced to Finance by Buchen, Kelly (1996), and Stutzer (1996). Its origin resides in the work of Shannon (1948) who related the notion of Entropy of thermodynamics to information. Jaynes (1957, 1982) later on extended this concept to statistical inference. As Buchen and Kelly recall: 'Jaynes argued along the lines of Gibbs, that the distribution that maximizes the Entropy, subject to the constraints, is the one that is least committal with respect to unknown or missing information and is, hence, also the least prejudiced'.

The general definition of Entropy is

$$E = -\int_0^{+\infty} q(x)\ln(q(x))dx \tag{9}$$

where as before $q(\cdot)$ will represent a RND. The Entropy can be viewed as a metric, the maximization of which will give a RND with the *maximum* possible information content.

With the notations introduced in Section 2 the constraints can be written as

$$\int_0^{+\infty} q(x)dx = 1, \tag{10}$$

$$\int_0^{+\infty} c_i(x)q(x)dx = e^{rT}C_i \tag{11}$$

where (10) insures that q is a density and (11) relates the RND with the ith call option.

The martingale restriction imposes the further condition

$$\int_0^{+\infty} xq(x)dx = F_t \tag{12}$$

where F_t is the forward price. Condition (12) can be put in the same form as (11) with the notation $c_0(x) = x$ and $C_0 = e^{-rT}F_t$.

Following Shannon, maximization of the Entropy under the constraints (10) to (12) is a natural way to obtain the RND. To implement this approach one considers maximization of the

Hamiltonian

$$H(q) = -\int_0^{+\infty} q(x)\ln(q(x))dx + (1+\gamma)(\int_0^{+\infty} q(x)dx - 1) + \sum_{i=0}^{m}\lambda_i(\int_0^{+\infty} c_i(x)q(x)dx - e^{rT}C_i).$$

Defining

$$\mu \equiv \frac{1}{\exp(\gamma)} = \int_0^{+\infty} \exp(\sum_{i=0}^{m}\lambda_i c_i(x))dx \tag{13}$$

the RND is then found to be equal to

$$q(x) = \frac{1}{\mu}\exp(\sum_{i=0}^{m}\lambda_i c_i(x)), \tag{14}$$

in which case the Entropy becomes

$$\begin{aligned} E(\lambda_0, \cdots, \lambda_m) &= \int_0^{+\infty} q(x)[-\ln(\mu) + \sum_{i=0}^{m}\lambda_i c_i(x)]dx \\ &= -\ln(\mu) + \sum_{i=0}^{m}\lambda_i e^{rT}C_i. \end{aligned} \tag{15}$$

At this stage we have to indicate how this Entropy can be estimated. Buchen and Kelly (1996) show that partial differentiation of E yields

$$\frac{\partial E}{\partial \lambda_i} = -\int_0^{+\infty} c_i(x)q(x)dx + e^{rT}C_i, \qquad \forall i = 0, \cdots, m. \tag{16}$$

This means that when we have an extremum for equation (15) we get conditions (10) and (11).

Inversely, the RND can be obtained by solving a system of $m+2$ nonlinear equations obtained by plugging (14) into (10)-(11). Further simplifications of the resulting expressions are obtained by explicit integration of the various terms which is done in an Appendix.

From (14) we notice that the ME method will give by construction RNDs composed by segments of exponential curves. Also, the ME method is an exact one in the sense that given m options and the martingale restriction, there will be exactly $m+1$ segments composing the RND. A consequence of this fact is that if one of the options is somewhat misspriced then this misspricing will be completely reflected in the corresponding RND segment.

2.3.4 RND for standardized options

As we will show in the empirical section, options are subject to a term structure of volatility as well as to complex changes of higher moments as time to maturity evolves. This means that the comparison of RNDs is difficult across days and deserves further thoughts to which we turn now.

Butler and Davies (1998), who construct RNDs with the MIX approach, suggest linear interpolation as the preferred way to obtain information for options with a constant time to maturity. This means that they first construct moments for fixed maturities with a linear interpolation of existing moments. Secondly, to obtain RNDs with constant time to maturity they use a convex combination of existing RNDs where the weights involve the times to maturity. Since the MIX approach is non-structural there is no obviously better approach. Also, using this technique is delicate to obtain information for options with maturities outside existing ones since this would involve extrapolation yielding a possibly negative density.

Among the other methods of this study, ME is also a completely non-structural model. This suggests that for ME information for fixed horizons could be obtained using a linear interpolation of moments or RNDs.

As for the Hermite polynomial approximation, because that approach is more structural, once parameters are estimated for one single maturity, by using (8) it is possible to infer information for all other maturities.[8] In theory it is even possible to make inference for options with maturity above the longest existing maturity and similarly for the shortest maturity. For our data, without performing a formal test, for a given day, the left and right side of (8) took different values as maturities changed. This suggests that it is possible to improve the extrapolation by using (8) to get forecasts for the π_k for all existing maturities and by averaging.

3 Empirical Results

Below we introduce the data used. Then, by focusing on five dates surrounding the 1997 snap election we compare the various methods presented earlier. Eventually, we will discuss what kind of information is contained in PIBOR and Notional Options in a time-series context.

3.1 The data

Data on PIBOR and Notional futures-options as well as on the underlying futures contract has been kindly provided by MATIF and covers the period between February 3rd 1997 and July

[8] It is also possible to use data on many maturities and options and to formally test (8). Abken, Madan, and Ramamurtie (1996) do so and actually reject this restriction.

30th 1997.[9]

3.1.1 Facts concerning PIBOR options...

There are five market makers for the 3-Month PIBOR Futures contract.[10] Along with a permanent presence on the floor, they have the duty to quote all the open strike prices at the request of either a market participant or an exchange supervisor. They must quote a maximum of 100 contracts with a bid/ask spread of 5 basis points for all the strike prices on the first two expirations. The nominal is 5 million francs.

The market makers for the 3-month PIBOR Futures-options are the same five as for the underlying contract. All options are American type. Price quotation is in 100 minus the percent of nominal value with 3 decimals. Strike prices are integer multiples of 10bp. There are at least quotes for the 15 closest ones to the money. At each point of time there are 4 maturities with expiration month March, June, September, and December. In-the-money options are automatically exercised on the last trading day. The last trading day is the 2nd business day preceding the 3rd Wednesday of delivery month at 11:00am (Paris time) for the expirations used in this study. The expiration date of the contracts is the same as of the underlying.

3.1.2 ...and Notional options

For the dates considered in this study, the MATIF Notional bond futures has a 10% coupon and is based on a fictitious 7- to 10- year French Government bond with 500'000F face value.[11] Notional contracts are the most liquid ones traded at MATIF.

The Notional option is an American-type option. Its price quotation is in percent of the nominal with 2 decimals. The strike price is in integer multiples of 100bp, for at least 9 options closest to the money. Expiration dates are the same as for PIBOR.

There are also five market makers.[12] For Notional Futures options the market makers have the duty to display continuously the strike prices surrounding the at-the-money price for the front-month option, and upon the request from either a market participant or an exchange supervisor, they must quote the other strike prices. To insure fair trading, for a maximum of 100 contracts, the market makers must quote a maximum bid/ask spread of 10bp for the 9 strikes surrounding the at-the-money price and for the front month, and 20bp for the other strike prices.

[9]Information on how the MATIF operates can be found under the web-page: `http://www.matif.fr/`.

[10]BNP, Caisse Nationale du Crédit Agricole, CPR, Société Générale, and Transoptions Finance.

[11]This fictitious bond can be obtained as a portfolio of OAT bonds where OAT stands for "Obligations Assimilables du Trésor". Since December 1997 the Notional bond is assumed to pay 5.5% coupons.

[12]Which are the same as for PIBOR with the exception of Indosuez replacing the Caisse Nationale du Crédit Agricole.

The fact that market-makers for the PIBOR and Notional options are compelled to give quotes with a reasonable spread, assures that even if the trading volume of those options is small that there is information about market participants' anticipation.

MATIF follows the CME and uses traditional compensation whereby the option premium must be paid at the time an option is bought. This differs from the LIFFE where options are futures-style margined.[13]

3.1.3 The role of MATIF S.A.

MATIF besides its role as clearing house obtains each day implicit volatilities for certain reference strike prices. Then, in order to cover a wider range of strike prices they perform a linear interpolation between the range of reference strike prices. Outside the reference strikes they assume a constant volatility by choosing the closest reference strike. Then, using those volatilities and Black's model they infer an option price. Even though this approach may be of use for compensation purposes, for empirical purposes the data is likely to contain lots of noise. Figures 1a to 1d display volatilities against strike prices for two selected dates.[14]

Symbols on the various curves correspond to data provided by MATIF. Whenever symbols are lying on a straight line they can be assumed to be obtained with a linear interpolation. In other words, only symbols where the line has a link are likely to correspond to actual information. We also notice that MATIF does not systematically provide interpolated volatilities. For instance, on April 14th '97 and the PIBOR option with highest maturity, a volatility was quoted for the 96.0 and the 96.2 strike but not for the 96.1 strike. Those observations suggest that the methods used to extract information should allow for possible misspricing of the options, and also that one should consider filters before using this raw data.

Those figures also have an economic meaning. When we consider PIBOR options of a same maturity in Fig 1a and 1b, we notice that options with different strikes have different implied volatilities. This feature is precisely the *option smile*. We also notice the vertical shift of smiles as maturities change. This corresponds to a *term structure of volatilities*. For April 14th '97 a date one week before the official announcement of the snap election, higher maturities are associated with higher volatility.[15] We will refer to April 14th as the *normal* date. When we take May 26th, the day after the first election round, we notice the reversal of the term structure of volatility, translating the idea that operators had lots of uncertainty concerning the short run.

[13]A nice feature of futures-style margining is that the early exercise feature drops and an American option can be priced as an European one (see Chen and Scott (1993)).

[14]To simplify the figures, the extrapolations at constant volatility level to the left and to the right have already been discarded.

[15]As shown later, on this date market makers did not anticipate, yet, the election.

Examples of Notional options' implied volatilities are displayed in Fig 1c and 1d. We notice that there exist less options on Notional options. The smaller level of volatility translates the fact that Notional prices have a smaller volatility than PIBOR rates. Next, we observe that for both dates the relative slope of the smile is not very steep. This suggests that the Notional might not react to political events as strongly as the PIBOR options.

3.2 Comparison of methods to extract RNDs

In this section we wish to compare along several lines the various methods. For practical purposes the method should be fast, should take into account the fact that the data is noisy, and should give results which are stable, i.e. there should be no difficulties to finding a global optimum during the numerical estimations.

Rather than using the entire database we consider at this stage five selected dates. Monday April 14th 1997 which is one week before the official announcement of Dissolution of Parliament. Monday April 21th, the day after the official announcement. May 26th, the first Monday when trading occurred after the first round of legislative elections. Anecdotal evidence suggests that the then opposition party might win the elections. Next, we consider June 2nd, the first Monday after the second round of elections. At this stage it was known that the Government of Alain Juppé was defeated and that the Socialist-Communist parties had made it back to government. Last we choose June 9th which is a Monday one week after the elections.

3.2.1 Implementation of the various methods

We implement the benchmark model (LN) and HER in a NLLS framework by minimizing separately for each day and maturity

$$\sum_{i=1}^{m}(C_i - \hat{C}_i)^2, \tag{17}$$

where C_i is the actual price of a call option and $\hat{C}_i$ is the price given by a certain model. This means that we first transform put option prices into call option prices using the put-call parity.[16] Also for options with a same strike we use the in-the-money option.

[16]The fact that option prices deviate from the Black-Scholes option price suggests that there are market imperfections implying that options might not be completely hedged and that the put-call parity might not hold. To check for robustness of our results we used separately put and call options and experimented with several methods. We obtained results similar to the ones reported. A possible explanation is that the put-call parity holds rather well since its derivation only involves the payoffs at maturity whereas the hedging argument involves locally riskless portfolios which may not exist if the price process of the underlying jumps or changes its volatility.

To implement the model involving a mixture of lognormal distributions (MIX) we add to (17) a Lagrangean penalty involving the distance between the actual futures prices and the expected one.

Implementation of the maximum Entropy method (ME) on the raw data turned out to be problematic. Since this method is an exact one, and since some of the prices are only linear interpolations, we were often unable to obtain convergence of the method. When the method converged we found that the obtained RND contained huge humps. To improve this situation we decided to filter the data by removing all possible linear interpolations in the smiles and to keep only those options where the smile has a kink. This greatly improved the performance of the ME method.

To remain coherent, we also applied the other methods to the data filtered in this way. Now, those methods turned out to show poor convergence properties because of the small number of remaining strikes.

At this stage we decided to implement the ME method with filtered data and the other methods with the raw MATIF data (making sure that we have for each maturity at least as much data as parameters).

To further check for robustness of our parameter estimates, we estimate for each date and maturity each method with a set of 20 different starting values obtained as random variates with enough standard deviation to insure coverage of a large spectrum of possible initial values.[17] At this stage we wish to report the difficulty to find a global minimum for the MIX model.

3.2.2 The fit of the models

In Tables 1 and 2 we compare the mean squared error (defined as MSE=$\frac{10^4}{m-n}\sum_{i=1}^{m}(C_i - \hat{C}_i)^2$ where n is the number of parameters involved in the method) and the average relative error (defined as ARE=$\frac{1}{m-n}\sum_{i=1}^{m}((C_i - \hat{C}_i)/C_i)^2)$. The MSE is expected to be larger if the option prices are larger and allows comparison across methods for a given type of option. The ARE further allows comparison across data sets by relating the error to the size of the option prices.

Tables 1 and 2 do not contain the errors for the ME method since, by construction, this method is an exact one and there is no error term. We notice in both tables that the benchmark model (LN) provides only a very poor fit. This fit improves when we shift to the mixtures of lognormal model. When we turn to HER we find an even better fit both in the MSE and the ARE.

[17]Computations were performed on a Pentium II computer running at 300MHz.

3.2.3 Who is the Champion?

At this stage our preferred methods were HER and ME. To shed further light on the relative contributions of the methods, we consider in Table 3 the patterns of volatility, skewness, and kurtosis for the PIBOR options. We notice a very similar magnitude of volatility across methods. Skewness and kurtosis have different magnitudes across methods but for a given date, the pattern of how skewness and kurtosis vary across maturities is similar for MIX and HER but not for ME. For instance for the latter, there seems to be a systematical decrease in kurtosis for options with higher maturity, whereas with HER there is first an increase then a decrease (for all dates after the announcement of the election, but June 9th). If we associate with volatility *general* economic uncertainty and with kurtosis *occurrence of an extreme variation of the underlying asset* the ME suggests the occurrence of a large variation in the short run, whereas HER postpones such a variation subsequently to the elections which, on economic grounds, appears reasonable.

Further inspection of Table 3 shows that on the *normal* day, April 14th, implied volatility increases with maturity but that skewness as well as kurtosis decrease. This suggests that a variation of interest rates, which might be considered extreme in the short run, actually becomes normal in the longer run.

Last, we notice, for all days and maturities, that skewness is always negative (and more negative than for the LN-benchmark). This suggests that operators anticipated an increase in interest rates rather than a decrease. This also holds for April 14th suggesting that there is also a Peso-problem in interest rates.

We further compare the various methods by displaying in Figures 2a-2d the various RNDs for April 14th and May 26th. Careful inspection of Figs 2a and 2b for the PIBOR option shows that whatever the date, the one or the other method gives a shape of the RND which is unlike the others. (Whatever the method considered, we notice a strong deviation from lognormality). It appears that the various methods have more similar RNDs for the earlier maturities than for the latter ones for possible liquidity reasons.

Focusing on the ME we notice that this method produces RNDs with sharp kinks. As discussed earlier, this is a direct consequence of the way the method operates. Table 3 shows how the tail-behavior of the RNDs differs.

When we turn to Fig 2c-2d corresponding to the Notional options, we notice that now the various other than lognormal distributions behave more similarly. Again, since the ME does not allow any possible misspricing we notice weird kinks in the distribution.

To conclude this section we recognize the difficulty to select a champion among the various methods. The rather popular method involving a mixture of lognormals, which appears to do a good job for exchange rate data (see Jondeau and Rockinger (1997)), has difficulties to converge to a global minimum: Its residual error is larger than for HER and ME. On the other hand the RNDs for HER sometimes take negative probabilities and the ME gives raise to strangely shaped densities and is moreover slower than HER to find a solution. In our quest of a quick and numerically robust method which also allows simple construction of options with fixed maturities as discussed in section 2.3.4 we decided to pursue our investigations with HER only.

It should be noticed that in Jondeau and Rockinger (1997) HER was found to give RNDs which were very similar to RNDs obtained with Edgeworth expansions as suggested by Jarrow and Rudd (1982) and implemented by Corrado and Su (1996). In this work we did not attempt to use Edgeworth expansions because they are obtained in a less-structural framework than HER and it would have been difficult to obtain RNDs for constant maturities.

3.3 Market operators' expectations through time

In this section we consider the Hermite polynomial approximation and the entire database to investigate how the expectations of traders in PIBOR and Notional options evolved through time.

At this stage it is necessary to insist on the fact that we are interpreting moments derived from the RNDs as if they were obtained in the actual world. This is based on the remark by Rubinstein (1994) that: *..., despite warnings to the contrary, we can justifiably suppose a rough similarity between the risk-neutral probabilites implied in option prices and subjective beliefs*. Rubinstein justifies this with a numerical example. This means in particular for the graphs presented below that any number should be considered as an approximation to the actual subjective one. Thus, changes in the numbers are more informative than the actual levels.

3.3.1 PIBOR options

Fig 3a displays the evolution through time of the ARE. We notice that the model fits better for the option with greater maturity.[18]

Fig 3b displays the various standard errors of the estimated parameters. We notice that for all dates volatility is rather well estimated. Skewness has higher errors and kurtosis is even worse. This confirms the findings of Corrado and Su (1996) who notice that kurtosis is highly correlated with volatility which renders its precise estimation difficult. We further notice that many dates when the ARE was found to be large in Fig 3a are also associated with worse estimates of skewness and kurtosis but not with volatility.

[18] We also report that when we initially estimated the model in a time-series context we obtained much larger ARE for certain dates. This resulted from the fact that some prices appear to be badly quoted. We decided to implement a loop in the program where we check for possible misspricing of certain strikes. Such misspriced strikes then get eliminated. This method gives us a certain control over the ARE. The ME being an exact method, would have yielded a very strangely shaped RND for such a price without providing to the econometrician an indication on how to objectively filter the data.

Fig 3c shows us in a nice way how global uncertainty of market operators evolved through time. We notice the relative calm in the market before April 14th. Then volatility builds up in the week before the official announcement occurred. After the official announcement on April 21th, uncertainty remained constant. Then came the surprise on May 26th that the government might change. This uncertainty rose even further as polls revealed the possibility of a socialist victory. On June 2nd it became clear that the socialists had won. As they held reassuring talks about their stance on the European Monetary Union and their general economic policy markets appeared to calm down.

In Fig 3d we turn to (the negative of) the price of skewness which further indicates that operators anticipated directional moves of interest rates. Clearly this measure has greater variability than volatility.

In Fig 3e we investigate the evolution through time of the price of kurtosis. Comparison with volatility, displayed in Fig 3c, shows that there appears to exist a trade off between the two. Dates before or after the period of elections had lower volatility but higher kurtosis and vice versa during the election period. Also, the trend towards higher skewness after the elections reveals that under the previous government, traders were somewhat worried about abnormal variations towards higher interest rates. During the election, global uncertainty increased and decreased after. However, traders were then more worried about an abnormal variation than before the election. This suggests that by the end of July 1997 the new government was unable to dissolve all fears.

It is possible to analyze this information in an alternative manner by considering confidence intervals obtained from the RND. Let b_i and b_s be the 5- and 95-percentiles of the RND. We refer to $b_i, b_s, b_s - b_i$ as *absolute deviations*. We also obtain percentage deviations with

$$b_{inf} = 100 \cdot (\frac{F}{b_i} - 1), \quad b_{sup} = 100 \cdot (\frac{b_s}{F} - 1), \quad rge = 100 \cdot (\frac{b_s - b_i}{F}).$$

Percentage deviations are relative to the forward price and are indicative towards which side the RND tends.

Also, as discussed in section 2.3.4, the availability of options with several maturities allows construction of standardized options by interpolation of the parameters. We focus now on options standardized arbitrarily for 3 months (90 days) and 9 months (270 days). In Table 4 we display confidence intervals as a percentage and in absolute terms. Figures 3f and 3g display for each day such intervals. To make results as easy to interpret as possible, we transform the $100 - F$ quote into a forward rate F.

For the normal date we notice that operators tend to believe that rates are likely to deviate with 90% down to 8.25% and up to 15.64% around the current forward rate within 90 days. This range increases substantially for the 9 month standardized option.

As news hit the market by April 21st fears of large movements of rates appeared. Those fears where greatest after the first round of elections. One week after the second round uncertainty had decreased but was still high.

Fig 3f gives a more intuitive picture of this evolution. We notice, going directly to rates, that operators put a lower bound on rates with ranges between 2.8 and 3.0%. Even though the forward rate does not move very much, we see huge variations in the upper bound. Even though, after the elections, the upper bound decreases, it remains high. This figure also confirms that the market knew about the election before the official announcement occurred.

Fig 3g displays the confidence intervals for a 9 month horizon. We notice a slight widening from below and a large widening from above of the confidence interval. The interval has widened over the sample and shifted upwards.

This suggests that investigating RNDs can shed additional light on how financial markets operate and that they contain information which is not contained in traditional instruments such as term structures of forward rates.

3.3.2 Notional options

We now turn to the analysis of the Notional option contracts. A first difficulty stems from the fact that for certain dates we only have information for options for one maturity and on other dates for two maturities. Further, since bond options obey a particular pattern for volatility, a jump occurs in the prices when one switches from one maturity to the other. For this reason we focus on options with the higher maturity since those instruments appear to be more liquid.

In Figure 4a we display the evolution through time of the ARE for the most liquid option. The mean of the ARE for this situation is 0.0012. For PIBOR options this variable takes the mean value of 0.0235 for the shorter maturity and 0.0016 for the longest maturity. This indicates that in general the fit for Notional options tends to be better than for the PIBOR option.

In Figure 4b we display the evolution of the standard errors for the various parameters. We find a similar pattern as for PIBOR options, namely that volatility is much better estimated than higher moment parameters. The standard errors for the price of skewness and of kurtosis are not as good.

In Figure 4c we consider the plot of volatility. As for the PIBOR the implied volatility increased in the week before the official announcement of the snap election. Unlike the PIBOR, volatility then decreased sharply. As the first round of elections approached this volatility builds up again but decreased even before the second round took place. After the elections volatility decreased too. Clearly, the pattern of volatility is more irregular and we find higher volatility in the Notional between mid till end of March.

Similar to the PIBOR case it is possible, by using risk-neutral confidence intervals to gauge how market participants' expectations evolved through time. This is done in Table 5 where we present confidence intervals for various maturities. We notice that the forward price is higher for the shorter maturity than for the longer one when both maturities are available. The width of confidence intervals increased and traders expected notional prices to become lower rather than higher as uncertainty increases.

Clearly, some of the findings are possibly due to the fact that the time till maturity changes. For this reason, we focus now on standardized 90 days options. For days with enough information for two maturities we choose the more liquid one (second maturity) and use formula (8) to extrapolate fixed horizon parameters. While doing so, we clearly neglect the possibility of a term structure of volatility. Figure 4d displays the evolution through time of an interpolated 90 days forward price and the 5- and 95- percent confidence bounds. We notice that by and large the lower bound in Figure 4d behaves similarly to the upper bound of Figure 3f and 3g. Since if Notional bond prices decrease this implies a rise in long-run rates, we conclude that the information contained in PIBOR and Notional options is similar. Already by mid-March, well before the official announcement of the elections, the lower bound became smaller. After some normalization after the announcement, as the first round approached the lower bound dropped again. Exactly as for the PIBOR contracts markets calmed down before the second electoral round.

We can summarize our experiments with Notional options as follows: For many days there is only enough information for one maturity and when there is information for two maturities then estimates are usually not as good for one maturity as for the other one. This makes the estimation of standardized options more difficult. Globally, we get the impression that Notional options convey a similar information as PIBOR options, but the message comes through with more noise.

4 Conclusion

In this paper we investigate the information context in PIBOR and National futures options during a period when financial markets were subject to great uncertainty, namely the 1997 snap election.

We first consider several methods to extract risk neutral densities. Whatever the method, we obtain a better fit to option's prices than with the traditional benchmark lognormal distribution. Eventually, we settle for an approximation based on Hermite polynomials developed originally by Madan and Milne (1994). This method yields accurate estimates and is able to deal with somewhat dirty data. It is fast and does not suffer from difficulties to yield a global optimum. Also, this method can be used to obtain standardized options, i.e. with a fixed time till maturity.

Then we apply this method to data ranging between February 3rd and July 30th 1997. For PIBOR options we notice that the market anticipated that elections were about to be called even before the official announcement. As polls suggested a possible change of government before the first electoral round, uncertainty increased. After the second round of the elections, even though uncertainty about the future decreased, the market still appeared to have fears about a future increase of interest rates. This can be explained by the possibility that markets accepted the fact of a socialist victory but were still uncertain about the future political program. By the end of July '97 the new government was still unable to reassure completely the financial market about its intentions.

In contrast to the information extraction out of PIBOR options, information extraction out of Notional options is more delicate. For a given date and maturity there are only fewer strikes which get quoted. Also, since the market for this type of instrument is less liquid, at most, options with two given maturities on a given date are available. As a consequence, the construction of standardized options is more difficult and likely to be prone to errors. Given those caveats, the information contained in Notional options seems correlated to the one contained in PIBOR options.

Appendix

To actually implement the method of maximum Entropy we define $a_i \equiv \lambda_0 + \lambda_1 + \cdots + \lambda_i$, $b_i \equiv -(\lambda_1 K_1 + \lambda_2 K_2 + \cdots + \lambda_i K_i)$, and set $b_0 = 0$, $K_0 = 0$, $K_{m+1} = +\infty$. It then follows that

$$\begin{aligned}\mu &= \frac{1}{a_0}\left(\exp(a_0 K_1 + b_0) - \exp(a_0 K_0 + b_0)\right) + \frac{1}{a_1}\left(\exp(a_1 K_2 + b_1) - \exp(a_1 K_1 + b_1)\right)\\ &\quad \frac{1}{a_2}\left(\exp(a_2 K_3 + b_2) - \exp(a_2 K_2 + b_2)\right) + \cdots\\ &\quad + \frac{1}{a_m}\left(\exp(a_m K_{m+1} + b_m) - \exp(a_m K_m + b_m)\right).\end{aligned}$$

The following lemma simplifies computations

Lemma:

$$\int_{K_a}^{K_b} x \exp(ax+b)dx = \left(\frac{K_b}{a} - \frac{1}{a^2}\right)\exp(aK_b + b) - \left(\frac{K_a}{a} - \frac{1}{a^2}\right)\exp(aK_a + b)$$

Straightforward but tedious computations then give:

$$\begin{aligned}\mu d_i &= \int_0^{+\infty} c_i(x)\exp(\sum_{j=0}^{m} \lambda_j c_j(x))dx\\ &= \sum_{k=i}^{m} \left(\frac{K_{k+1} - K_i}{a_k} - \frac{1}{a_k^2}\right)\exp(a_k K_{k+1} + b_k) - \left(\frac{K_k - K_i}{a_k} - \frac{1}{a_k^2}\right)\exp(a_k K_k + b_k)\end{aligned}$$

where we have set $K_{m+1} = +\infty$ as before. Clearly, one must select parameters such that $a_m < 0$.

To obtain good convergency properties the choice of initial values is crucial. We found the following procedure to give good starting values:

First, we estimate using NLLS the parameters of a log-normal density $\hat{q}(x)$, which is an approximate risk neutral density. Second, we evaluate this density over a finite support by taking a grid. Since we would like to obtain λ_i so that the estimated Entropy density comes close to $\hat{q}(x)$ we consider the system:

$$\begin{aligned} \hat{q}(x) &= \frac{1}{\mu} \exp(\sum_{i=0}^{m} \lambda_i c_i(x)) \\ \Rightarrow \ln(\hat{q}(x)) &= -\ln(\mu) + \sum_{i=0}^{m} \lambda_i c_i(x). \end{aligned}$$

Since the $c_i(\cdot)$ are perfectly known (at least if we have taken x over a finite grid) it is possible to estimate as a last step λ_i using OLS.

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Table 1. Let C_i, $\hat{C}_i$ be the actual and the theoretical option price for a given maturity and strike price K_i, $i = 1, \cdots, m$ then $MSE = \frac{10^4}{m-n} \sum_{i=1}^{m} (C_i - \hat{C}_i)^2$ where n is the number of parameters involved in a given estimation, and $ARE = \frac{1}{m-n} \sum_{i=1}^{m} ((C_i - \hat{C}_i)/C_i)^2)$. n takes the values 1,4,3 for LN, MIX, and HER respectively.

Table 2. The error measures MSE and ARE are the same as in Table 1.

Table 3. The moments for PIBOR options presented here for the various methods are directly obtained from the Risk Neutral Densities.

Table 4. Confidence intervals for the PIBOR forward rate. If F is the forward rate, b_i and b_s the 5- and 95-percentiles of the RND, then

$$b_{inf} = 100 \cdot (\frac{F}{b_i} - 1), \quad b_{sup} = 100 \cdot (\frac{b_s}{F} - 1), \quad rge = 100 \cdot (\frac{b_s - b_i}{F}).$$

Confidence intervals are constructed for virtual options with 90 and 270 days to maturity.

Table 5. Confidence intervals for the Notional forward price. Construction of variables as in Table 4. Maturity is as given by options.

Figure 1a. Smiles for PIBOR options of several maturities on April 14th 1997. The symbols $\circ$, 2 , $\triangle$, $+$ correspond to options with 63, 154, 245, and 336 days to maturity.

Figure 1b. Smiles for PIBOR options of several maturities on May 26th 1997. The symbols $\circ$, 2 , $\triangle$, $+$ correspond to options with 56, 147, 238, and 329 days to maturity.

Figure 1c. Smiles for Notional options of several maturities on April 14th 1997. The symbols $\circ$, and 2 correspond to options with 45 and 136 days to maturity.

Figure 1d. Smile for Notional options on May 26th 1997. The symbol 2 corresponds to options with 94 days to maturity. An option with shorter maturity existed but was not retained because of insufficient quoted strikes.

			MSE			ARE		
date	T	m	LN	MIX	HER	LN	MIX	HER
970414	63	5	1.109	0.322	0.147	0.065	0.002	0.006
	154	13	0.789	0.376	0.045	0.009	0.118	0.006
	245	15	1.219	0.342	0.032	0.008	0.004	0.001
	336	12	1.662	0.394	0.253	0.009	0.003	0.004
970421	56	8	4.782	0.579	0.045	0.018	0.001	0.000
	147	12	10.039	0.016	0.005	0.049	0.002	0.000
	238	15	3.786	3.649	0.217	0.005	0.002	0.003
	329	12	2.439	1.789	0.162	0.001	0.061	0.000
970526	21	11	6.329	0.903	0.076	0.047	0.003	0.001
	112	15	5.959	1.178	0.071	0.132	0.004	0.000
	203	13	3.875	0.300	0.034	0.030	0.014	0.000
	294	16	3.299	0.190	0.019	0.040	0.017	0.000
970602	14	11	1.614	0.066	0.016	0.256	0.011	0.004
	105	15	9.504	0.871	0.236	0.281	0.004	0.001
	196	15	4.937	11.649	0.022	0.041	0.006	0.000
	287	15	3.540	8.516	0.015	0.022	0.009	0.000
970609	98	13	10.116	3.291	0.447	0.375	0.033	0.003
	189	14	4.276	1.101	0.146	0.049	0.000	0.002
	280	16	4.389	0.614	0.063	0.039	0.006	0.000

Table 1: Comparison of MSE and ARE for various methods and PIBOR options

			MSE			ARE		
date	T	m	LN	MIX	HER	LN	MIX	HER
970414	45	7	19.653	0.688	0.793	0.050	0.001	0.000
970414	136	7	40.982	0.090	0.030	0.012	0.018	0.000
970421	38	7	30.588	0.212	0.715	0.050	0.014	0.002
970421	129	7	73.165	4.151	3.435	0.016	0.005	0.000
970526	94	7	64.172	5.733	4.116	0.024	0.020	0.001
970602	87	7	74.584	0.049	0.286	0.033	0.004	0.000
970609	80	7	64.046	0.150	0.436	0.059	0.000	0.001

Table 2: Comparison of MSE and ARE for various methods and Notional options

		LN			MIX			HER			ME		
date	T	σ	SK	KU	σ	SK	KU	σ	Sk	KU	σ	SK	KU
970414	63	0.390	-0.146	0.038	0.434	-1.367	4.873	0.423	-1.485	3.960	0.436	-2.588	12.147
	154	0.467	-0.267	0.127	0.506	-1.351	6.147	0.497	-1.001	3.334	0.496	-1.639	6.973
	245	0.631	-0.442	0.348	0.665	-1.247	3.739	0.660	-1.175	2.922	0.654	-1.203	3.166
	336	0.744	-0.564	0.472	0.743	-0.905	1.308	0.732	-1.113	1.375	0.740	-0.847	1.149
970421	56	1.500	-0.477	0.392	1.523	-1.539	2.791	1.465	-1.727	1.905	1.569	-1.468	3.978
	147	1.047	-0.537	0.461	1.064	-1.794	4.143	1.075	-1.883	3.405	1.048	-1.703	4.237
	238	0.921	-0.580	0.471	0.914	-1.193	2.727	0.939	-1.277	3.409	0.934	-1.090	2.625
	329	0.918	-0.588	0.303	0.844	-0.364	1.518	0.852	-0.455	1.763	0.885	-0.684	1.605
970526	21	2.371	-0.450	0.350	2.430	-1.629	3.301	2.373	-1.727	2.547	2.416	-1.578	3.628
	112	1.241	-0.532	0.426	1.260	-1.367	2.153	1.247	-1.506	2.041	1.240	-1.391	2.468
	203	0.931	-0.536	0.427	0.949	-1.353	2.621	0.963	-1.407	2.520	0.926	-1.304	2.764
	294	0.899	-0.566	0.345	0.881	-1.078	1.523	0.883	-1.141	1.564	0.876	-1.031	1.492
970602	14	1.442	-0.235	0.098	1.524	-1.632	4.146	1.506	-1.460	2.838	1.580	-1.885	7.533
	105	1.145	-0.507	0.437	1.201	-1.732	3.830	1.206	-1.877	3.605	1.189	-1.686	3.703
	196	0.898	-0.538	0.470	0.924	-0.997	1.699	0.954	-1.596	3.698	0.921	-1.471	3.557
	287	0.820	-0.567	0.460	0.828	-0.874	1.395	0.824	-1.285	2.378	0.815	-1.157	2.185
970609	98	0.932	-0.406	0.294	1.037	-2.199	7.210	1.014	-2.077	4.749	1.023	-2.036	5.932
	189	0.833	-0.502	0.434	0.880	-1.522	3.927	0.883	-1.581	3.487	0.867	-1.515	3.661
	280	0.786	-0.557	0.482	0.794	-1.379	2.768	0.801	-1.425	2.673	0.789	-1.307	2.731

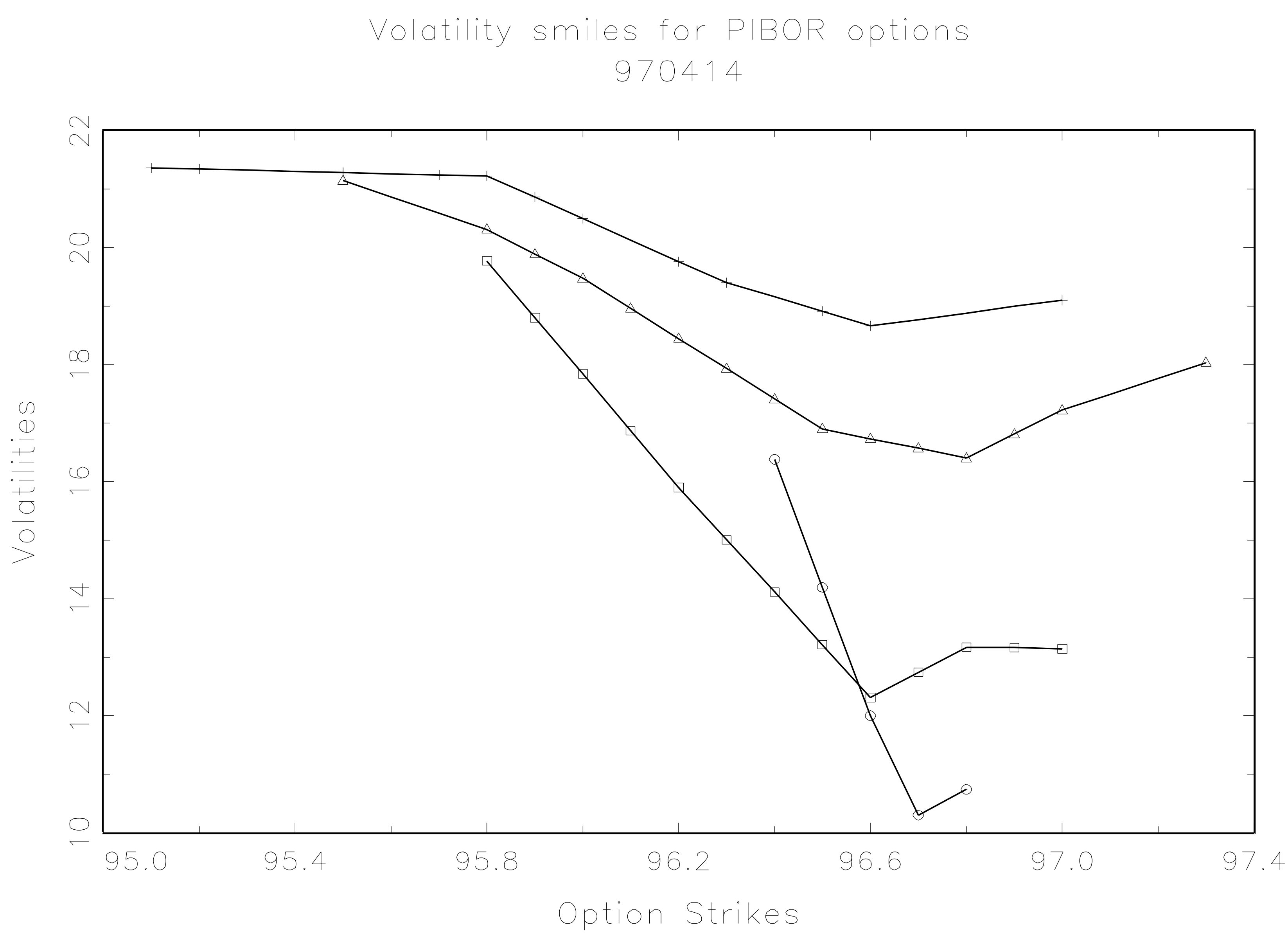
Table 3: Volatility, skewness, and kurtosis for various methods applied to PIBOR options.

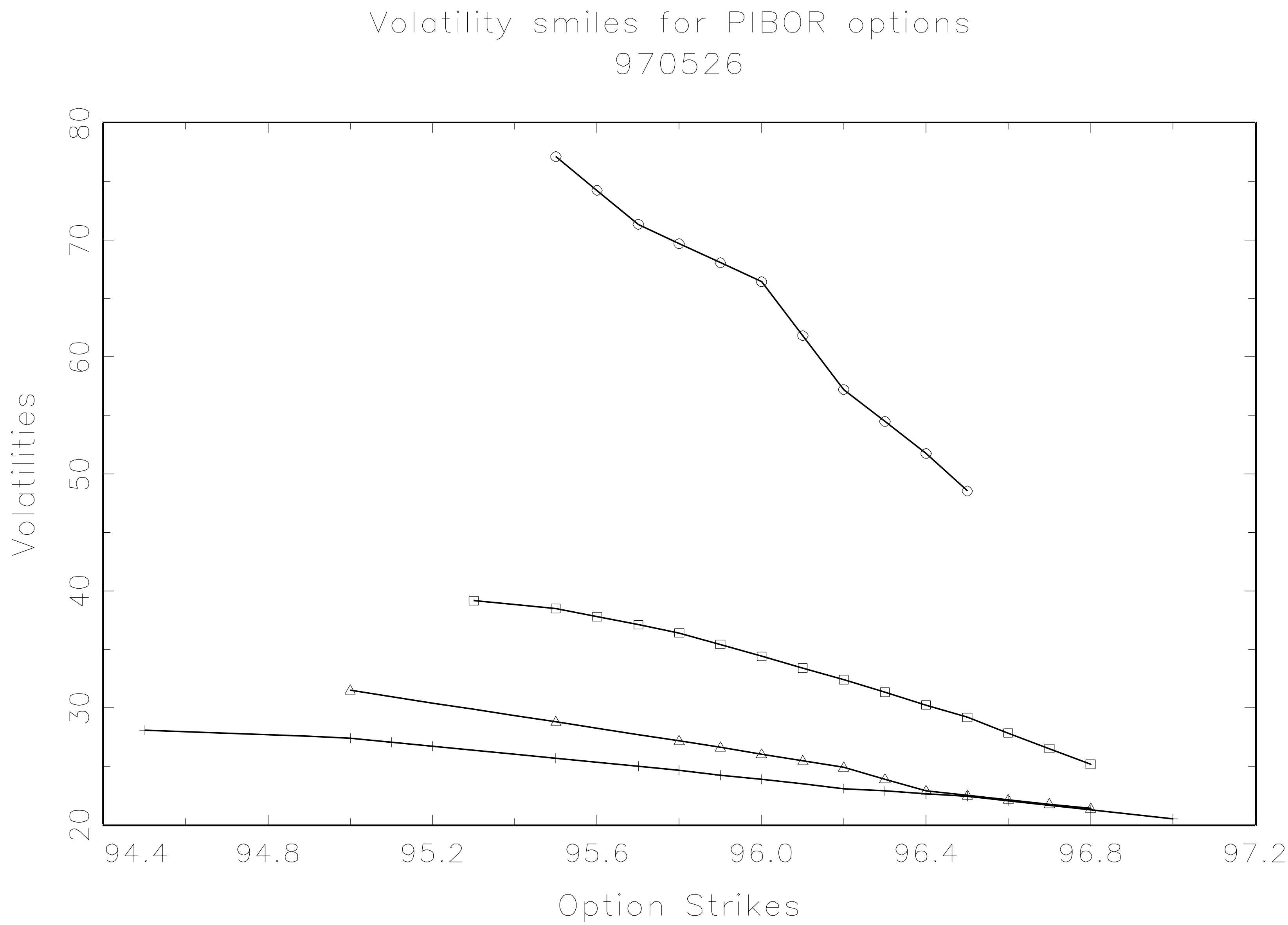
			Percentage deviation			Absolute deviation		
Date	T	F	b_{inf}	b_{sup}	rge	b_i	b_s	$b_s - b_i$
970414	90	3.35	8.25	15.64	23.27	3.10	3.88	0.78
	270	3.57	26.16	31.20	51.93	2.83	4.68	1.85
970421	90	3.68	24.61	40.64	60.39	2.95	5.17	2.22
	270	3.69	41.39	36.38	65.66	2.61	5.03	2.42
970526	90	3.78	35.13	48.77	74.77	2.80	5.62	2.83
	270	3.77	30.31	37.08	60.34	2.90	5.17	2.28
970602	90	3.62	25.25	41.68	61.85	2.89	5.13	2.24
	270	3.67	30.48	40.08	63.44	2.81	5.13	2.33
970609	90	3.58	16.89	35.47	49.92	3.06	4.85	1.79
	270	3.62	30.73	39.62	63.13	2.77	5.06	2.29

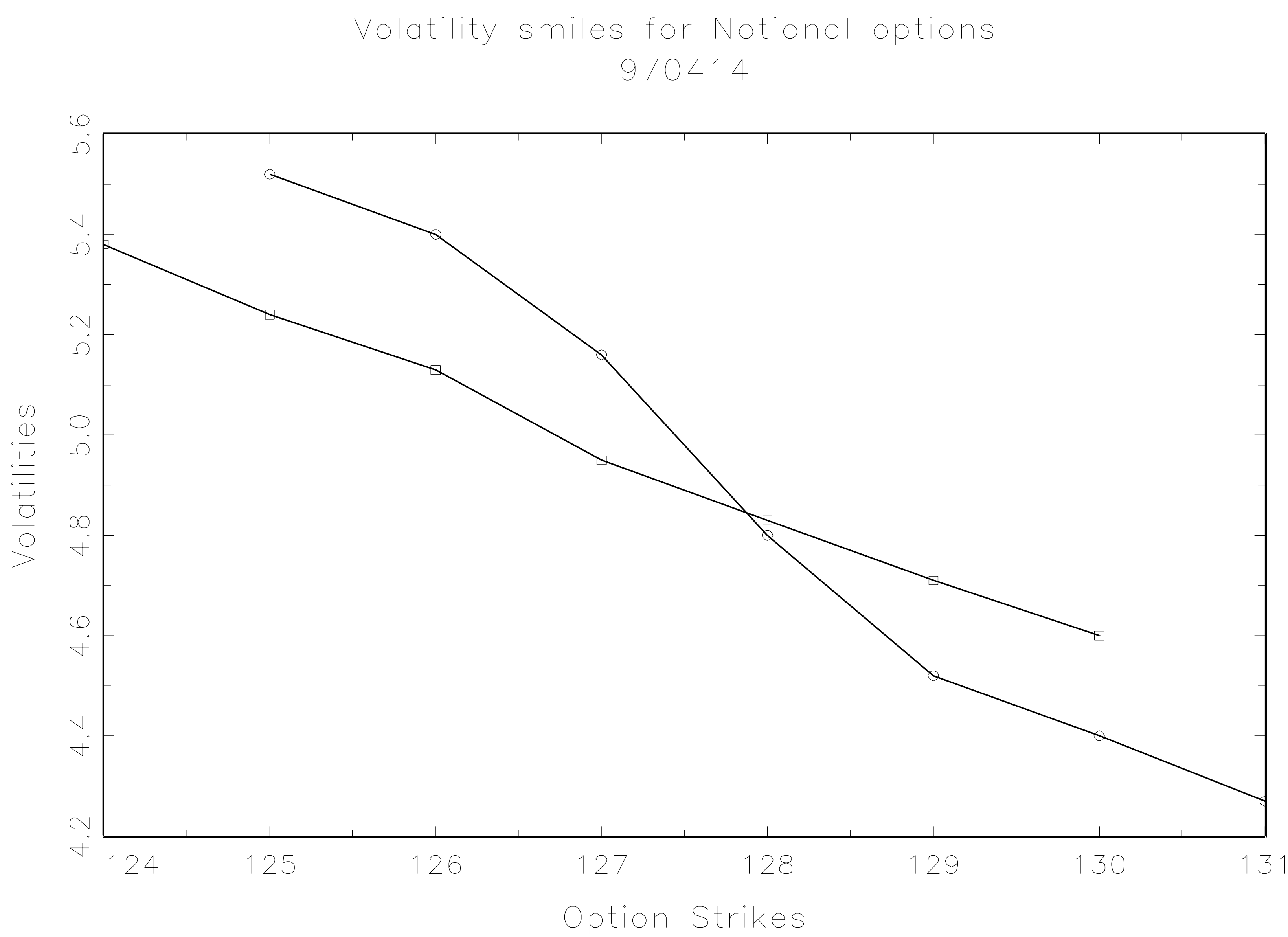
Table 4: Risk neutral confidence intervals in percent and absolute for standardized PIBOR options with Hermite polynomial adjustment.

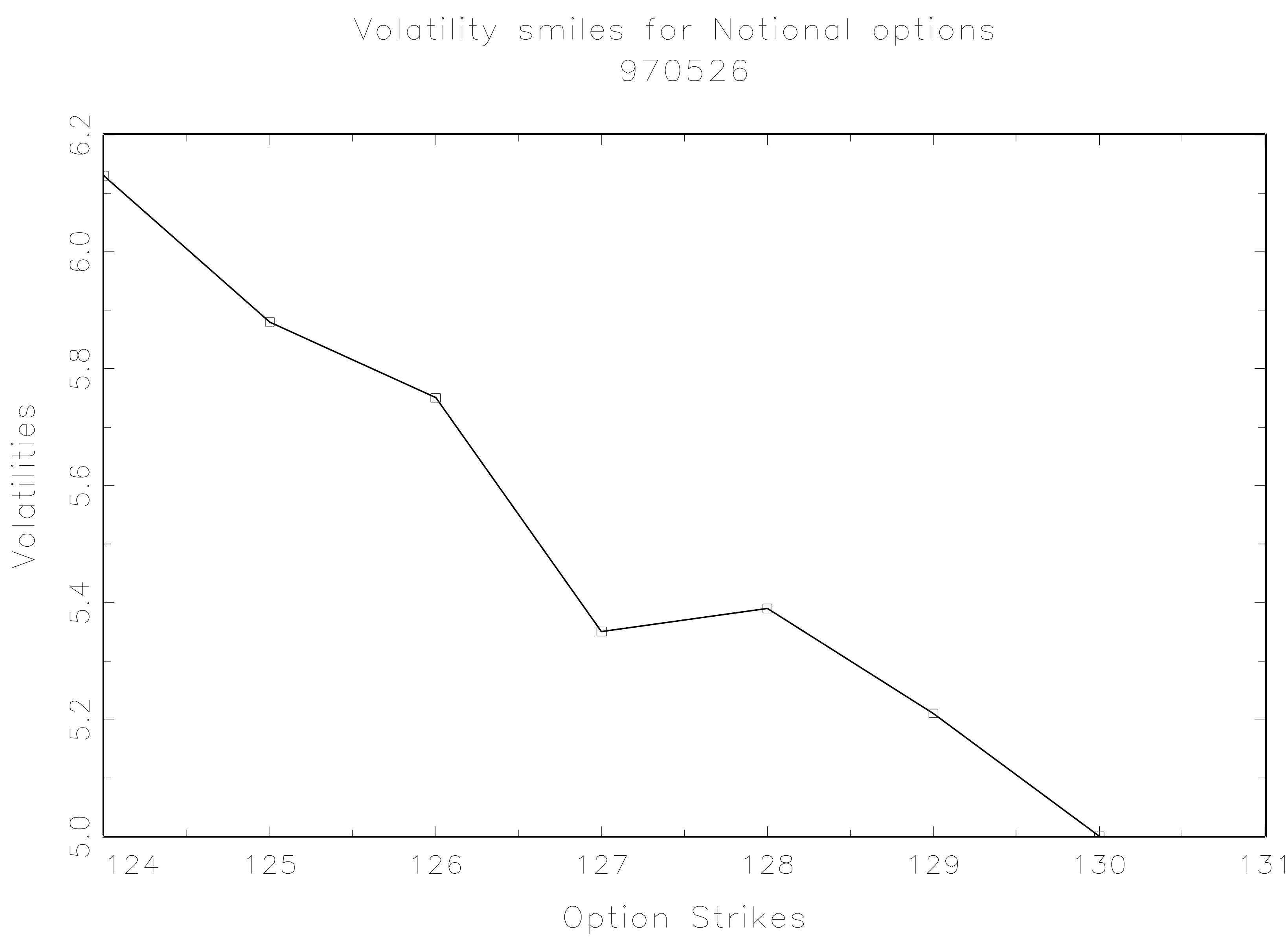
			Percentage deviation			Absolute deviation		
Date	T	F	b_{inf}	b_{sup}	rge	b_i	b_s	$b_s - b_i$
970414	45	128.24	3.24	2.53	5.66	124.22	131.48	7.26
	136	126.58	5.92	4.71	10.30	119.50	132.54	13.04
970421	38	128.28	3.52	2.63	6.03	123.92	131.66	7.74
	129	126.74	6.43	4.73	10.78	119.08	132.74	13.66
970526	94	126.78	5.79	4.23	9.70	119.84	132.14	12.30
970602	87	126.96	5.41	3.89	9.03	120.44	131.90	11.46
970609	80	127.92	4.73	3.39	7.91	122.14	132.26	10.12

Table 5: Risk neutral confidence intervals in percent and absolute for Notional options with Hermite polynomial adjustment.


Volatility smiles for PIBOR options
970414
Volatilities
22
20
18
16
14
12
10
95.0
95.4
95.8
96.2
96.6
97.0
97.4
Option Strikes

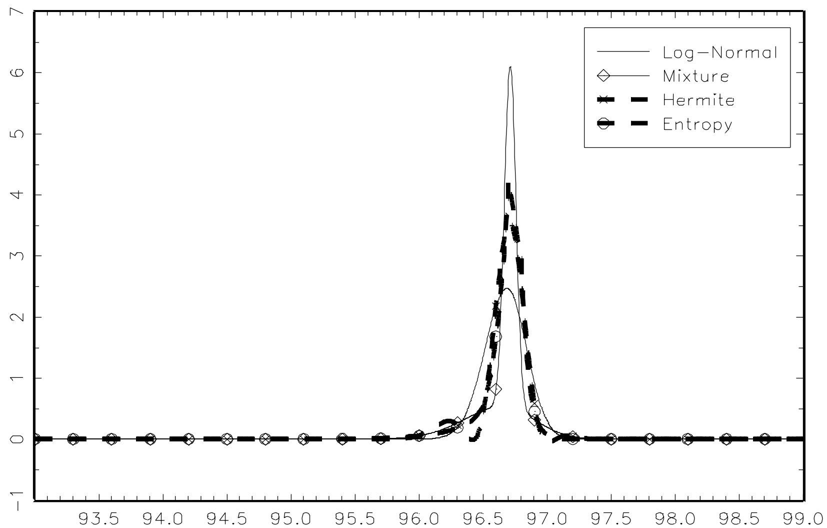

Volatility smiles for PIBOR options
970526
Volatilities
80
70
60
50
40
30
20
94.4
94.8
95.2
95.6
96.0
96.4
96.8
97.2
Option Strikes


Volatility smiles for Notional options
970414
Volatilities
5.6
5.4
5.2
5.0
4.8
4.6
4.4
4.2
124
125
126
127
128
129
130
131
Option Strikes



Volatility smiles for Notional options
970526
Volatilities
6.2
6.0
5.8
5.6
5.4
5.2
5.0
124
125
126
127
128
129
130
131
Option Strikes

RND comparison for Pibor options
Date: 970414 Maturity: 1



RND comparison for Pibor options
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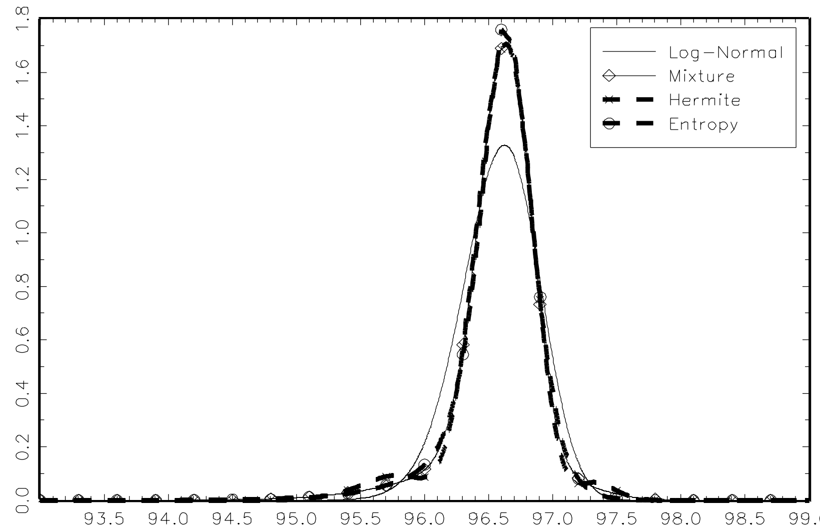

RND comparison for Pibor options
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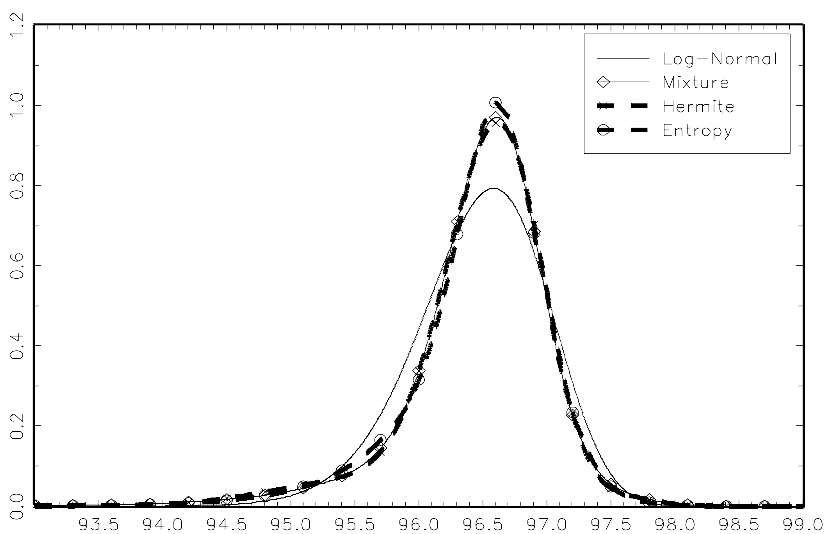

RND comparison for Pibor options
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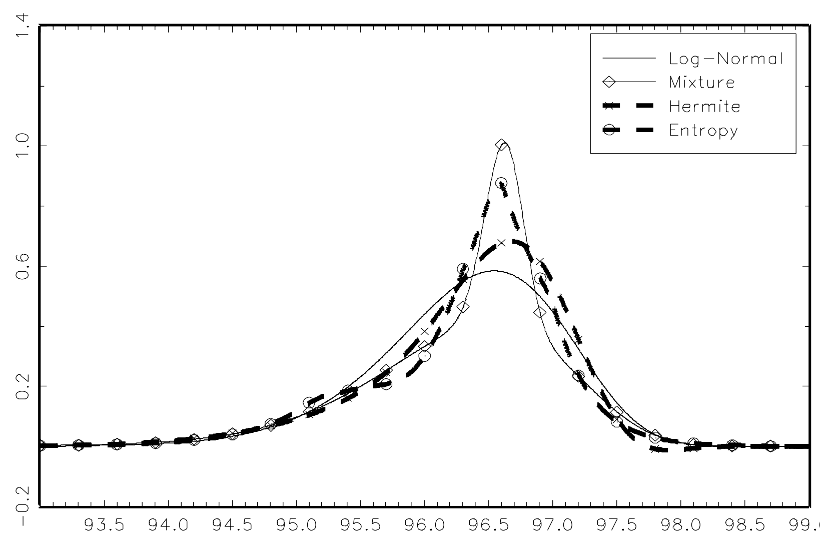

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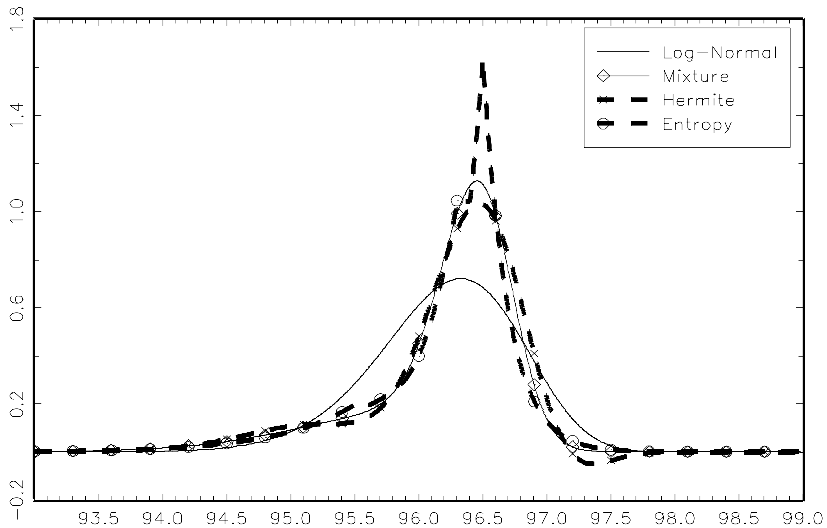

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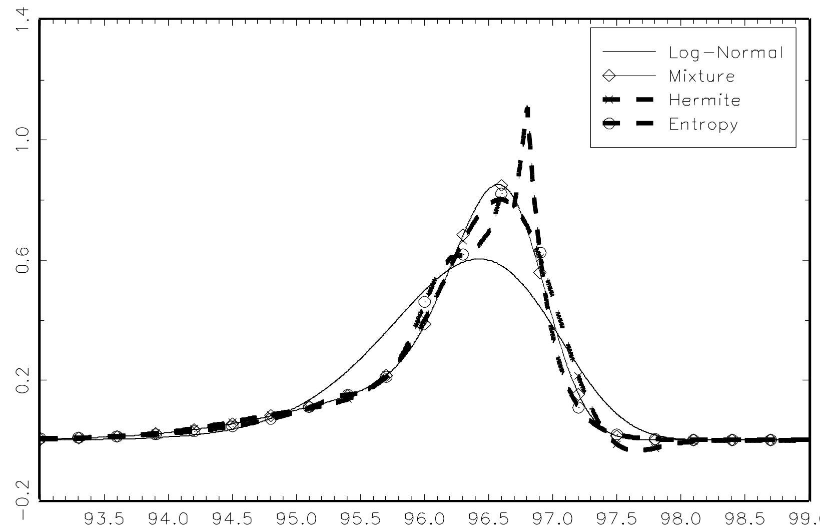

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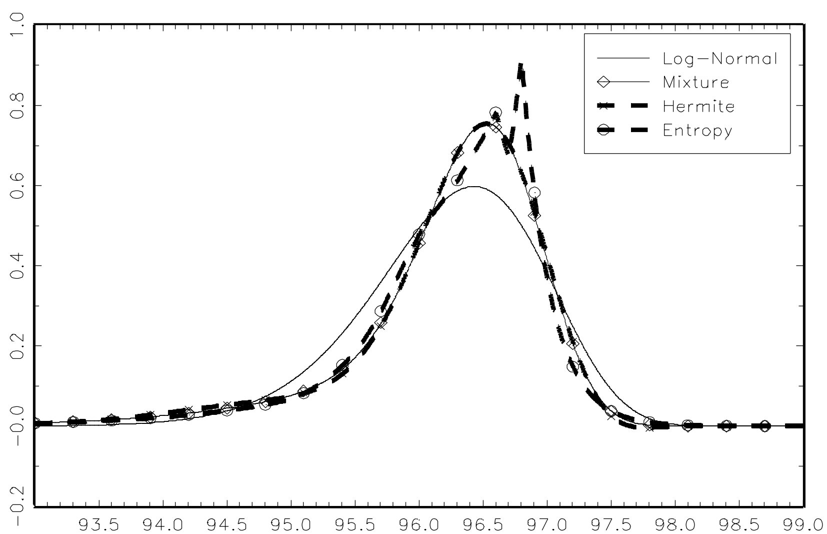

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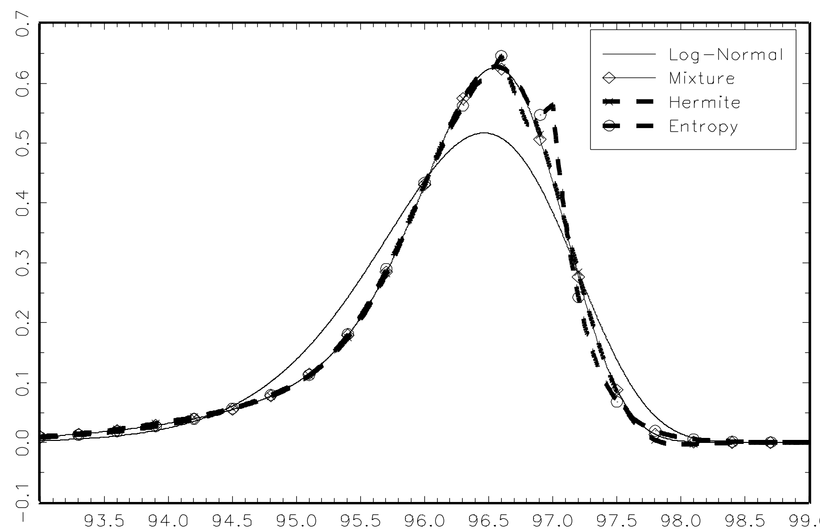

RND comparison for Notional options
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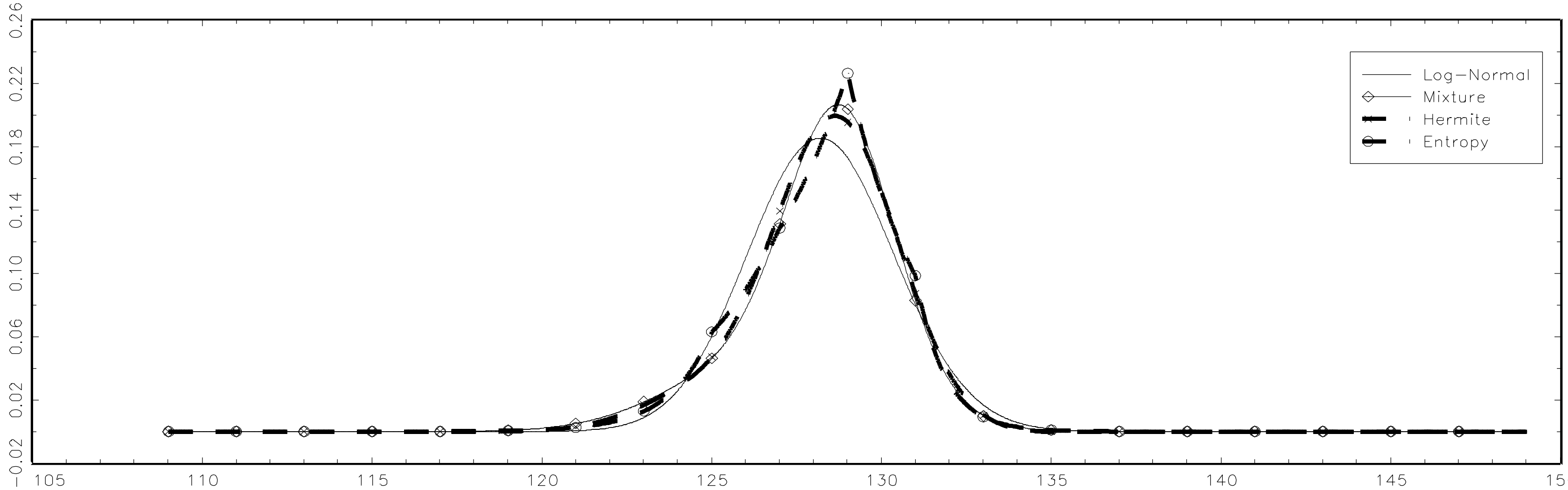
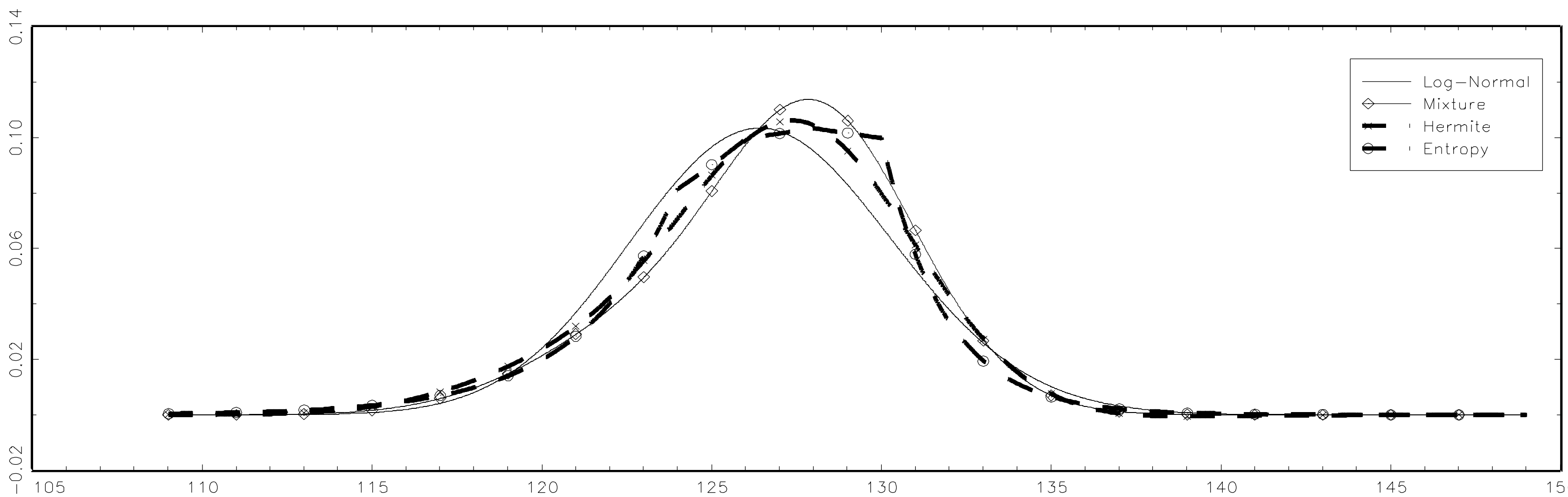
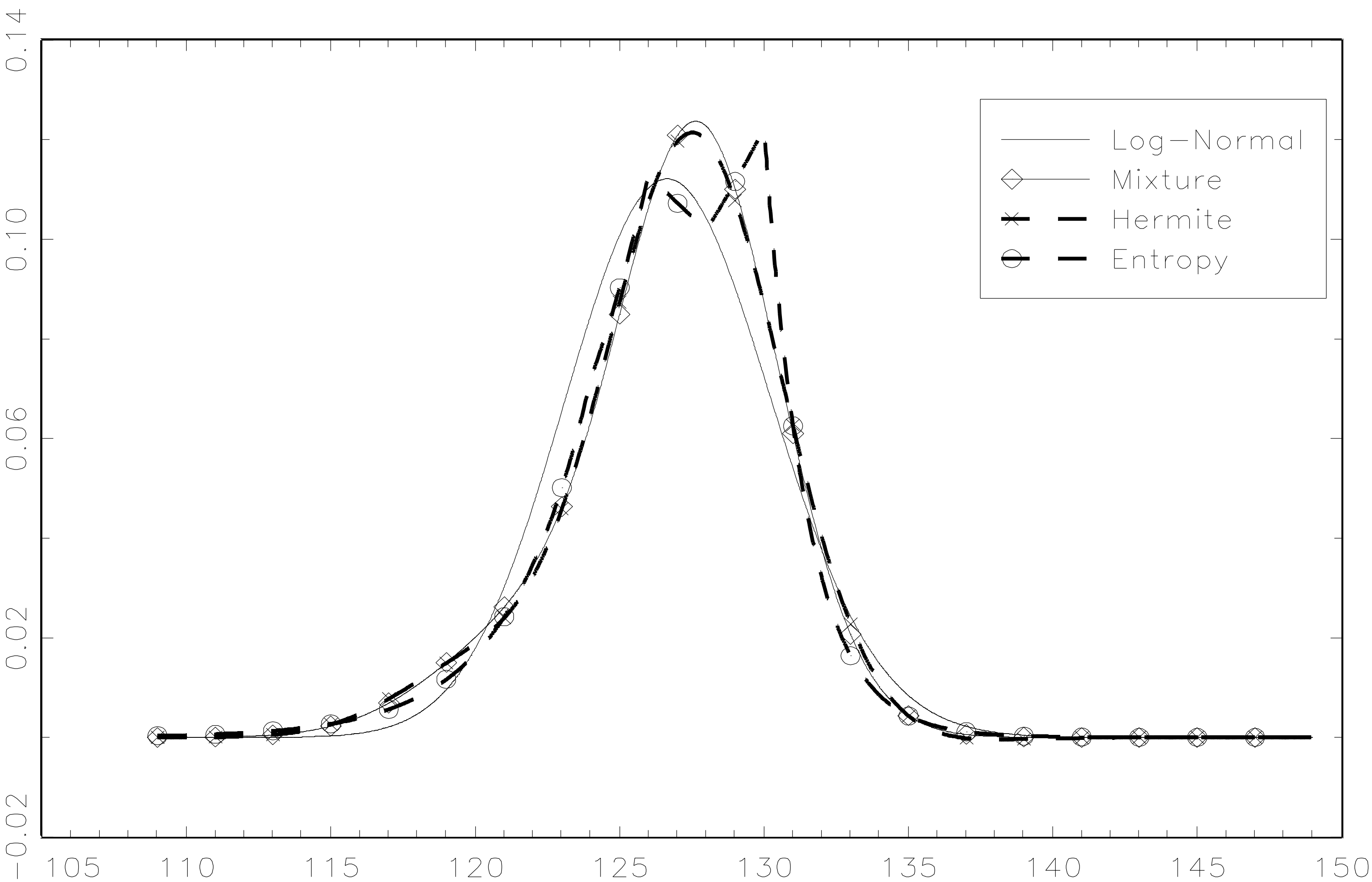


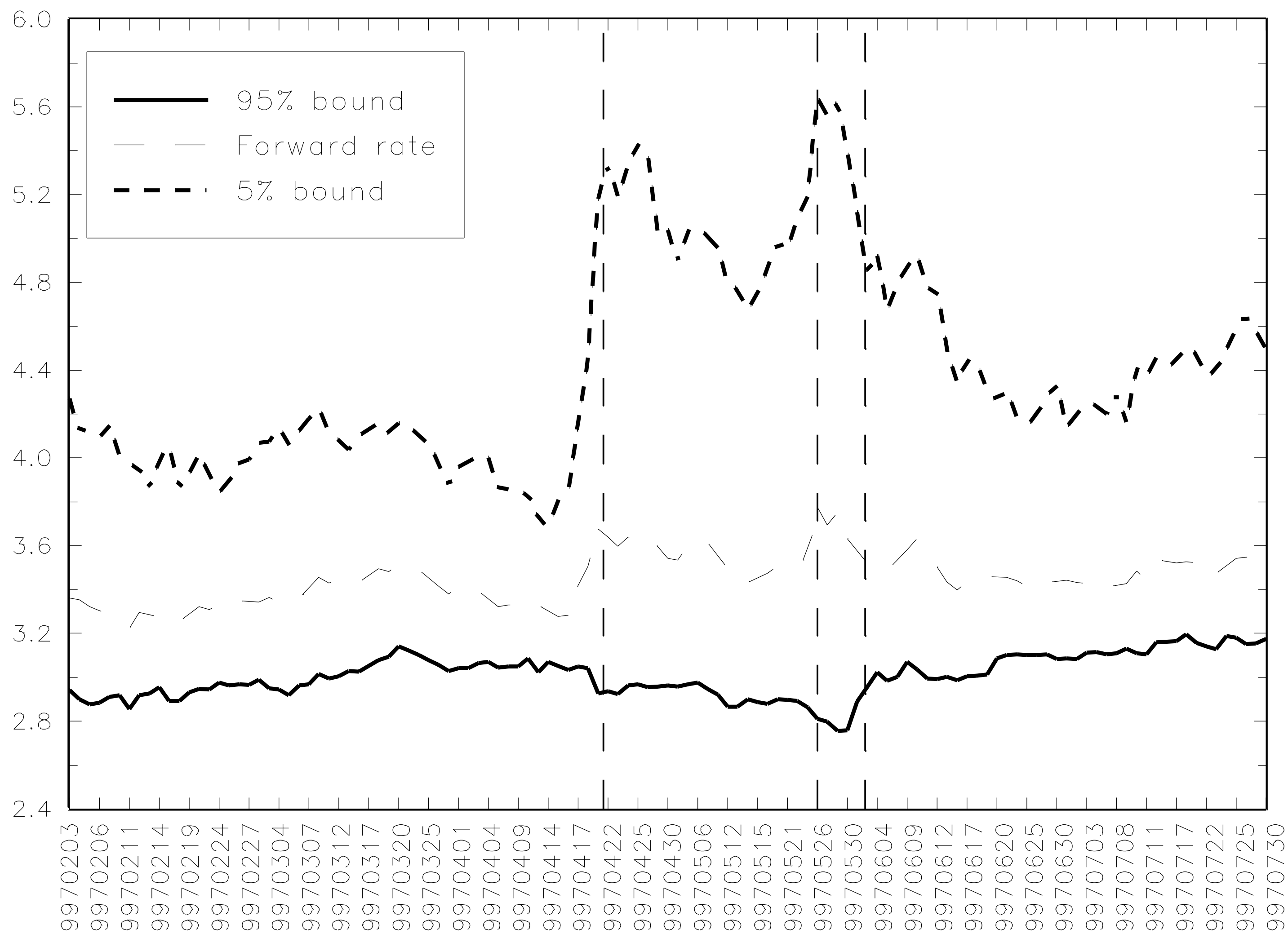
RND comparison for Notional options
Date: 970414 Maturity: 2



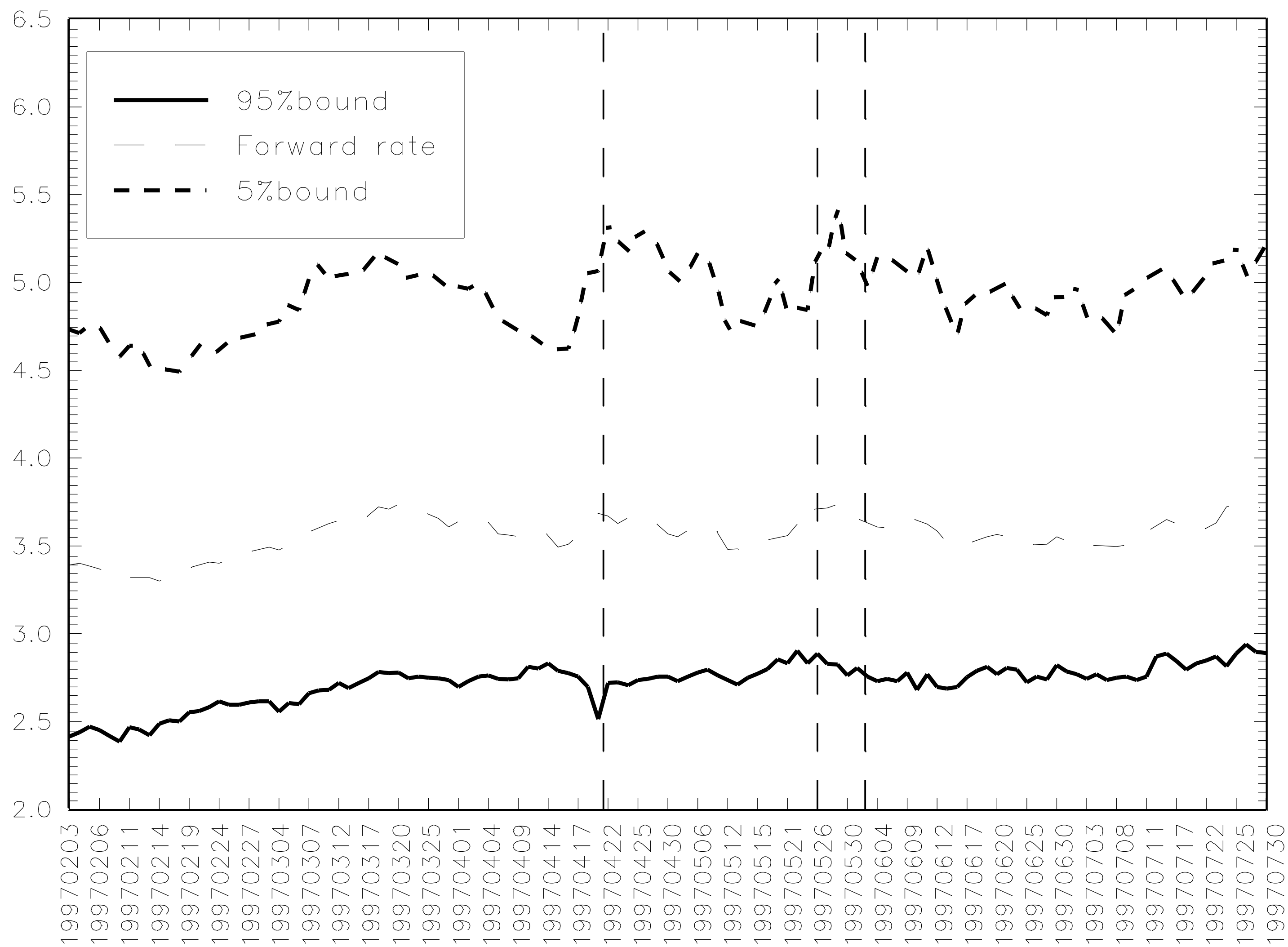
RND comparison for Notional options
Date: 970526 Maturity: 1

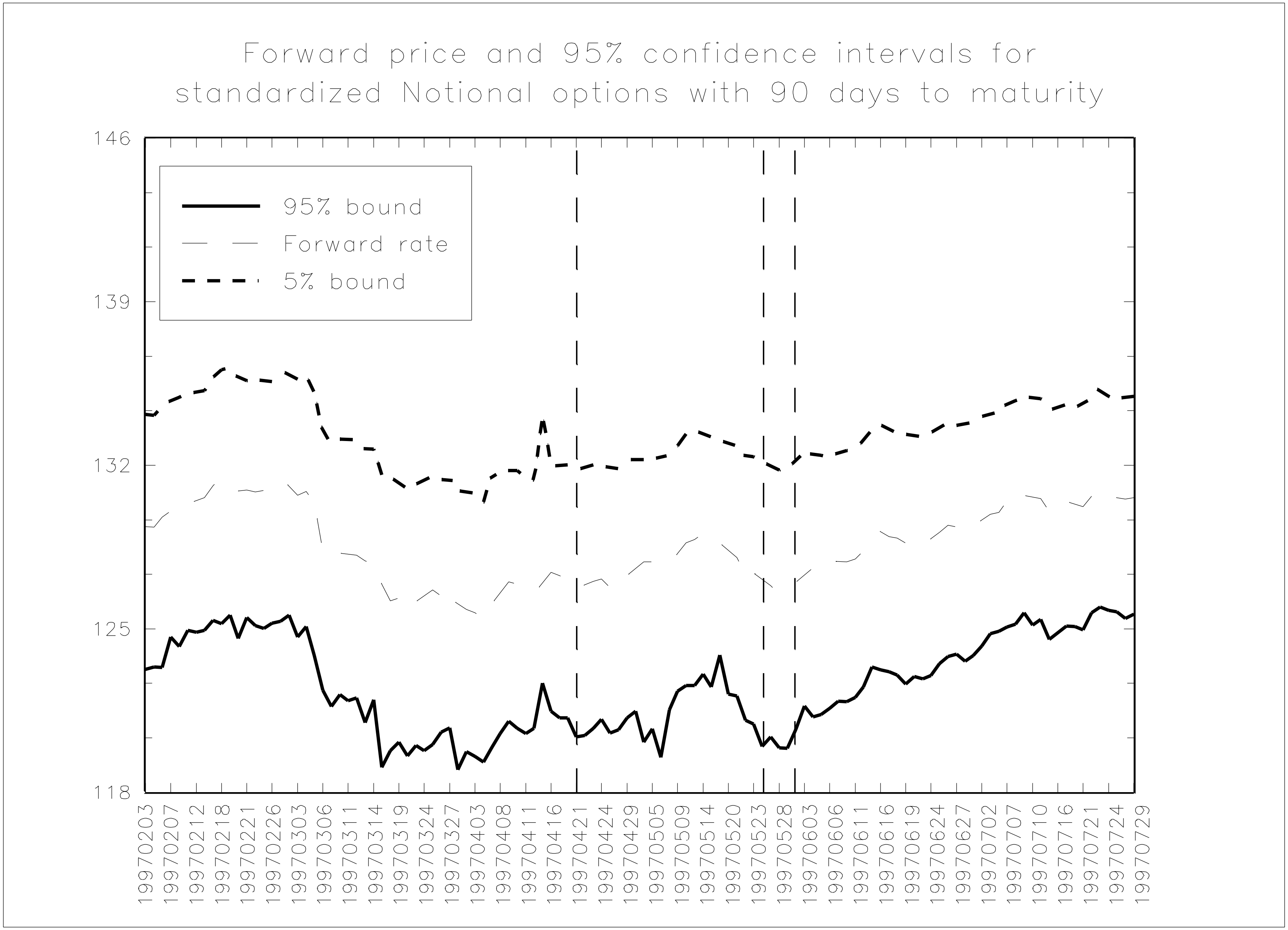


Forward rate and 95% confidence intervals for standardized PIBOR options with 90 days to maturity



Forward rate and 95% confidence intervals for standardized PIBOR options with 270 days to maturity




Forward price and 95% confidence intervals for
standardized Notional options with 90 days to maturity
95% bound
Forward rate
5% bound
146
139
132
125
118
19970203
19970207
19970212
19970218
19970221
19970226
19970303
19970306
19970311
19970314
19970319
19970324
19970327
19970403
19970408
19970411
19970416
19970421
19970424
19970429
19970505
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19970514
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