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Asymmetric shocks in a currency union: The role of central bank collateral policy

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Abstract

Currency unions limit the ability of the central bank to use interest rate policy to accommodate asymmetric shocks. I show that collateral policy can serve to dampen asymmetric shocks in a currency area when these shocks also affect the collateral held by banks and when collateral portfolios of banks differ systematically across countries. In my model banks from 2 countries use collateral to borrow from the money market or a central bank that targets a level of interest rate (or investment) in each economy. The distressed bank may enter a "collateral crunch" regime where it is constrained in its access to funding due to a moral hazard problem. The central bank faces an heterogeneous transmission of its interest rate: a unit change in rate has a smaller effect on the economy rate of the distressed country. The central bank therefore sets a high interest rate which is well transmitted in the booming economy and relaxes the haircut on the collateral owned by the distressed bank.

Keywords: Central banking, currency union, collateral policy, repo, monetary policy.

JEL Codes: E58, G01, G20.

Résumé

Une union monétaire limite la capacité de la banque centrale à utiliser la politique de taux d'intérêt pour répondre à des chocs asymétriques. Nous démontrons que la politique de collatéral peut servir à amortir les chocs asymétriques lorsque ces chocs affectent également les actifs détenus par les banques et lorsque les portefeuilles d'actifs diffèrent entre les banques des différentes régions de l'union monétaire. Dans le modèle, des banques de 2 pays utilisent des actifs en garantie (appelés "collatéral") pour emprunter sur le marché interbancaire ou auprès de la banque centrale. Cette dernière vise un niveau de taux d'intérêt (ou d'investissement) dans chaque économie. La banque en difficulté peut être contrainte dans son accès au financement en raison d'un problème d'aléa moral et d'un manque de collatéral. La banque centrale fait alors face à une transmission hétérogène de son taux d'intérêt: un changement de taux a un effet moindre sur le pays en difficulté économique. La banque centrale utilise donc un taux d'intérêt élevé qui est entièrement transmis à l'économie en plein essor et relâche la politique de collatéral sur les actifs détenus principalement par la banque en difficulté afin de restaurer la transmission de sa politique monétaire.

Mots clés: Banque centrale, union monétaire, politique de collatéral, politique monétaire.

Codes JEL: E58, G01, G20.

Non-technical summary

The fragmentation of financial markets impaired the transmission of monetary policy in the euro area during the European sovereign debt crisis. From 2010 to 2013, countries that were negatively affected by the debt crisis experienced high interest rates, while countries that were relatively unaffected by the shock experienced low interest rates. Data from the European Central Bank (ECB) suggest for instance that the funding cost of firms in Greece, Ireland, Italy, Portugal and Spain was 3% above the risk free rate versus 1.7% for firms in other Euro area countries from 2010 to 2013 (Horny et al. (2014) show that these differences remain even after accounting for the riskiness of borrowers). The differences in the funding cost of firms contrast with local economic conditions. For example, the Taylor rates, which provides an indicator of the desired level of policy rates given economic conditions, suggest that the central bank rate should have been 2.7% in low yield countries versus roughly 0% in high yield countries over the same period.

In this paper, we build a microeconomic model of the bank lending channel of monetary policy in a two-country currency union. Our model includes an explicit role for collateral requirements, both in private markets and at the central bank. When lending to banks, central banks and most private market participants require collateral in order to be protected from counterparty risk. By explicitely including collateral in the model, we can study how changes in the collateral policy of the central bank can affect the transmission mechanism and influence the monetary policy stance across the currency union.

We first show that if an asymmetric shock is accompanied by a large fall in the value of the assets held by the representative bank in that country, this bank may enter a "collateral crunch" regime where it is constrained in its access to the interbank market. The interest rate in the distressed economy then increases above the policy rate of the central bank. Since the bank from the stable country keeps its access to the interbank market, the central bank faces a situation of high interest rates in the distressed country and low interest rates in the stable country.

We then derive the optimal collateral policy of the central bank. In our model, the central bank wishes to achieve a unique target interest rate in each country, consistent with its monetary policy stance. The central bank is also risk averse and puts a negative weight on being exposed to its counterparties (for example in case of default). We show that the fragmentation of financial markets in the collateral crunch regime affects the transmission of monetary policy: while there is a perfect pass through of the interest rate in the stable country, the effect of changes of the policy rate on the economy rate in the distressed country are dampened. The central bank in our model therefore chooses interest rates that are close to the target level for the stable country but relaxes collateral requirements to alleviate the lack of collateral in the distressed country.

We close the paper by a discussion of the model. We first show that our model and its assumptions are in line with several stylized facts of the European sovereign debt crisis. We then argue that the presence of dynamic moral hazard where banks may reduce their collateral in response to the policy of the central bank might justify the combined use of collateral policy with macro-prudential instruments such as the liquidity coverage ratio.

1 Introduction

The fragmentation of financial markets during the European sovereign debt crisis complicated the conduct of monetary policy in the euro area. From 2010 to 2013, countries that were negatively affected by the debt crisis experienced high interest rates, while countries that were relatively unaffected by (or benefited from) the shock experienced low interest rates. Data from the European Central Bank (ECB) suggest for instance that the funding cost of firms in Greece, Ireland, Italy, Portugal and Spain was 3% above the risk free rate versus 1.7% for firms in other Euro area countries from 2010 to 2013.[1] These differences contrast with the Taylor rate, which provides an indicator of the desired level of policy rates given economic conditions. According to this indicator, the central bank rate should have been 2.7% in low yield countries versus roughly 0% in high yield countries over the same period.

According to the literature on optimum currency area, there is little that the central bank can do to respond to such asymmetric economic conditions. The reason is that its policy instrument, the interest rate, has to be the same for the whole currency area. If a country is hit by a negative demand shock while another is hit by a positive shock, the central bank must therefore choose between high interest rates to avoid overheating in the booming economy, or low interest rates to accommodate the shock in the distressed economy. The central bank however cannot pursue both policies since its policy rate must be the same for both countries (Mundell, 1961; McKinnon, 1963; Kenen, 1969).

In this paper, I build a microeconomic model of the bank lending channel of monetary policy to study an alternative instrument available to the central bank: collateral requirements. When lending to banks, central banks have collateral requirements that vary with the type of asset used. Can collateral policy help the central bank to respond to asymmetric shocks? The first contribution is to show how asymmetric shocks can lead to high rates in the distressed country and low rates in the booming one. I build a model where banks have asymmetric collateral portfolios due to a domestic bias. When economic shocks also affect the value of the collateral, there exists a "collateral crunch" regime where banks are constrained in their access to the money market. When the shock is asymmetric, the bank in the distressed country reduces the allocation of funds to the economy which increases the economy rate above the policy rate, whereas the economy rate in the stable country remains equal to the policy rate.

The second contribution is to derive the interest rate and collateral policy of the central bank consistent with the objective of steering the real economy rate to a target level. If the distressed bank experiences a

[1] Gilchrist and Mojon (2014), Horny et al. (2014) and Pianeselli and Zaghini (2014) show that the differences in funding cost of firms remain when controlling for firm and risk characteristics.

collateral crunch, I show that there is an imperfect pass-through of the interest rate of the central bank in the distressed country. By lowering its policy rate by one unit, the central bank reduces the interest rate in the booming country by one unit (away from the objective of the central bank) while the economy rate of the distressed country falls by less than a unit (so the positive effect is dampened). This implies that the interest rate policy of the central bank should remain closer to the target level, while collateral policy may be used to relax the funding constraint of the distressed bank. The mechanism is that lower collateral requirements increase the profit of the banks and relaxes their incentive constraint. This allows the bank to increase its lending to the real economy. The policy of lending against low collateral requirements comes at a cost, however, since the central bank increases its exposure to the borrower in case of default.

The theoretical model extends the work of Koulischer and Struyven (2014) to two banks and two collateral types and introduces a central bank deposit facility. This allows to compare the interest rate instrument with the collateral policy instrument in the context of a currency area hit by asymmetric shocks. Macroeconomic shocks impact the investment opportunities available to banks and the value of their collateral. The central bank seeks to steer economy rates towards a target level. The game is in two stages: the central bank first set its rate and collateral policy, then the banks choose where to borrow.

As a benchmark, I first consider the case without moral hazard. I show that in this first best world the central bank can perfectly control the interest rate in the economy using only its deposit facility, where money market participants can place their funds at the policy rate. The central bank then simply sets its policy rate at the target level.

I then introduce moral hazard so that collateral becomes necessary to incentivize banks. As in Holmström and Tirole (2011), collateral requirements provide a penalty to banks in case of mismanagement since it is seized in case of default. I show that each bank is in one of two regimes depending on its stock of collateral. If a bank has sufficient collateral available, it can borrow the first-best amount and the regime is similar to the first best. If the available collateral falls below a threshold, the bank is constrained in its access to funding and enters the "collateral crunch" regime. The amounts lent fall and the economy rate increases relative to the policy rate of the central bank.

If the bank in the distressed economy is in a collateral crunch, the central bank faces a situation of high rates in the distressed country and low rates in the stable country. In this case the policy of the central bank consists in setting its policy rate close to the optimum of the booming country while reducing haircuts on the collateral owned mostly by the distressed bank. The rationale for a relatively high rate is that a reduction in the policy rate reduces the economy rate of country A by one unit while the reduction of the economy rate in the distressed country is less than one, so the positive effect of low rates in the distressed country is dampened relative to the negative effect of low rates in the booming country.

The degree of asymmetry in the collateral portfolios of banks plays a key role in the policy of the central bank since both banks benefit from lower haircuts. If the portfolio are concentrated on specific assets (for example due to a home bias), the central bank can ensure that the increased exposure is done at the benefit of the distressed bank, so that the cost of lowering rate in the distressed country is reduced.

My model suggests that collateral policy can be useful in a situation of high rates in the distressed country and low rates in the stable one. However, it does not suggest that collateral policy can engineer a situation of high rates in the booming country and low rates in the depressed country. The reason is that, in using collateral policy, the central bank is competing with market participants. If it were to set collateral requirements above those of the market, banks would borrow from the market so collateral policy would have no effect on the final equilibrium. The best that collateral policy can do is to achieve a same economy rate across the currency area. This is useful in times of crisis such as the European sovereign debt crisis, however it may be less useful in episodes where spreads across countries are too low, as was the case from 2002 to 2007.

1.1 Related literature

The key novelty of this paper is to study the collateral policy of the central bank when banks are heterogeneous, which allows to study collateral policy in a currency union. Ashcraft et al. (2011), Bindseil (2013) and Koulischer and Struyven (2014) for example use a representative agent to analyze the effect of collateral policy. The mechanism proposed in this paper for the real effects of collateral policy builds on the work of Holmström and Tirole (2011) and Koulischer and Struyven (2014) and differs somewhat from that of Ashcraft et al. (2011). In their paper, securities are subject to exogenous margin requirements which may be directly changed by the central bank. In our model, haircut requirements are endogenous and the central bank may incurr losses when lending to banks, so we include both the costs and benefits of collateral policy.

Our model includes both the interest rate and the collateral policy instrument. Focusing on the interest rate, Freixas et al. (2011) expand the model of Bhattacharya and Gale (1987) to study the optimal central bank policy when a crisis causes a disparity in the liquidity held among banks. They suggest, as we do, that the interest rate policy of the central bank should take into account developments in the financial sector (on this point see also Kashyap and Stein (2000); Freixas and Jorge (2008); Freixas et al. (2011);

Jimenez et al. (2012)). One contribution of this paper is to show that heterogeneity in the health of the financial sector can affect the optimal policy of the central bank in a currency union, a point also made by Kashyap and Stein (1997).

Collateral policy is also closely related to the issue of the lender of last resort (LOLR). Bagehot (1873) for instance argued that central banks should lend to illiquid banks against high quality collateral (see also Goodhart (1999)). Freixas et al. (2000, 2004) revisit the LOLR role of the central bank and show that the desirability of a LOLR depends on the type of moral hazard faced by banks or the structure of the payments system. In our model, the central bank may wish to lend against low collateral requirements if it relaxes the collateral constraint of the bank in the distressed country and reduces its economy rate.

The literature on optimum currency area (OCA) has focused on the use of fiscal policy to respond to asymmetric shocks (Mundell, 1961; McKinnon, 1963; Kenen, 1969; Farhi and Werning, 2012, 2013). Our paper shows that collateral policy may alleviate the "one instrument, two objectives" dilemma of the central bank. Our paper sheds light on the mechanism used by collateral policy to influence the real economy which took a central role in the European sovereign debt crisis (Acharya and Steffen, 2013; Dreschler et al., 2013).

Section 2 describes the different elements of the model. Section 3 then considers the "first-best" case without moral hazard and discusses how symmetric and asymmetric shocks affect the equilibrium. I then consider symmetric countries with moral hazard in section 4. Section 5 then considers the full model with asymmetric shocks and moral hazard. Section 6 makes the link between the model and the Sovereign debt crisis and discusses policy implications.

2 Setup

Banks. Two cashless banks are located in two different countries (A and B) that share a single currency. Each bank $i \in \{A, B\}$ has undertaken illiquid investments in the economy that need refinancing. For an investment q, the bank earns $R(q)$ with a probability p_i and zero with a probability $1 - p_i$. The return function $R(\cdot)$ takes the Cobb-Douglas form $R(q) = q^\alpha$ with $\alpha < 1$.[2]

Banks can borrow from the money market or from the central bank (as illustrated in Figure 1) but there is moral hazard in the process: instead of channeling the funds to the real economy, banks can shirk and obtain a private benefit Hq that varies with the investment size q. If a bank shirks, it defaults with probability 1.[3] To prevent shirking, lenders can require collateral that is transferred in case of default. Since the bank defaults for sure when shirking, high collateral requirements reduce the payoff in case of default and therefore incentivize the bank not to shirk. There are two types of collateral in the economy, indexed 1 and 2. Bank i owns each collateral for a total worth of $\theta_{i,1}$ and $\theta_{i,2}$ respectively. The 2 collaterals are perfect substitutes and we denote the aggregate stock of collateral as $\theta_i = \theta_{i,1} + \theta_{i,2}$. The stocks of assets are exogenous and may differ across banks (for instance if banks have a domestic bias).

For a total investment q_i, the expected output is $p_i R(q_i)$. The interest rate in the economy r_i is defined as the marginal return on investment

$$r_i = p_i R'(q_i).$$

This is the minimum return required for investment projects to be funded.

Money market. The money market is composed of corporations with excess cash, money market funds or other financial institutions with excess liquidity wishing to lend their cash short term. There are no impediments to arbitrage and market participants may either fund the bank or place their cash at the central bank and earn the policy rate r^{cb} for sure.[4] A contract in the money market is characterized by a (gross) interest rate r_i^p so that the bank pays $r_i^p q_i^p$ in case of success when borrowing q_i^p.[5] The contract also specifies a collateral requirement or "haircut" $h_{i,j}^p$ by bank i so that the bank must pledge $h_{i,j}^p$ units of collateral j to borrow 1 unit. Since market participants and the borrowing banks are indifferent between either type of collateral, the no-arbitrage condition will cause haircuts for both assets to be the same, i.e.

[2] As in Gertler and Kiyotaki (2013), this can be interpreted as banks having an equity stake in firms in the real economy.

[3] An alternative interpretation is that the bank makes an unobservable choice of the probability of success p_B so that if it chooses $p_B = 0$ it obtains some private benefit which could result from lower monitoring costs or excessive perks (Holmström and Tirole, 2011).

[4] In practice only banks have access to the deposit facility of the central bank. This is equivalent to our model if banks can place funds at the central bank on behalf of non-banks.

[5] The gross interest rate is equal to 1 plus the net interest rate, $r_i^p = 1 + \tilde{r}_i^p$ where $\tilde{r}_i^p$ is the net interest rate. Similarly, I use the gross haircut $h_i^p = 1 + \tilde{h}_i^p$ so a net haircut of $\tilde{h}_i^p = 20\%$ is equivalent to a gross haircut of $h_i^p = 1.2$.

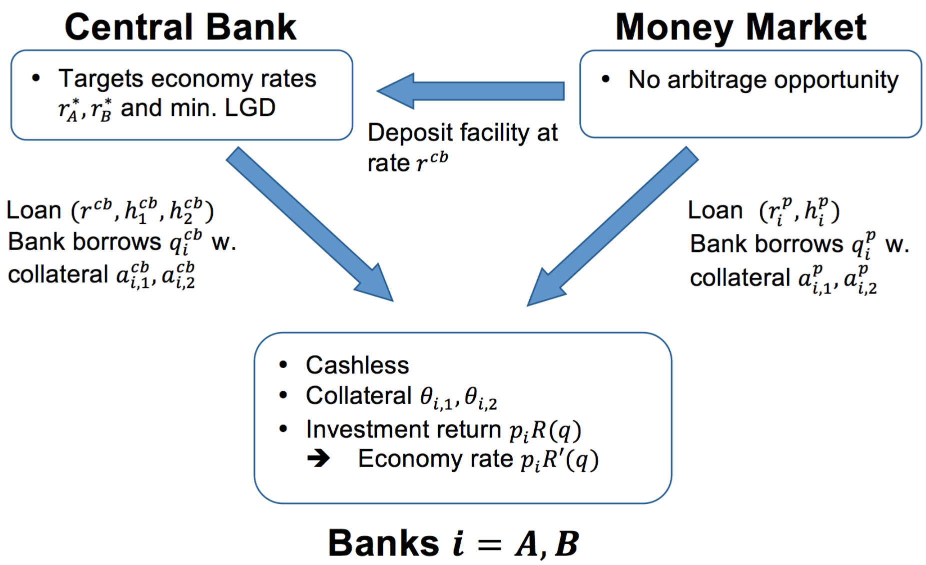

Figure 1: Overview of model

$h_{i,1}^p = h_{i,2}^p = h_i^p$.[6] Let $a_{i,1}^p$ and $a_{i,2}^p$ be the amount of collateral 1 and 2 pledged by the bank to the private money market and q_i^p the amount borrowed. A contract (r_i^p, h_i^p) requires that:

$$q_i^p = \frac{a_{i,1}^p + a_{i,2}^p}{h_i^p}. \tag{1}$$

Central bank. The central bank has two facilities (illustrated in Figure 1). First, the deposit facility allows money market participants to place their cash and earn the policy rate r^{cb} for each unit deposited. Second, the central bank offers loans to banks through open market operations. Loans are at the policy rate r^{cb} and are collateralized: for each unit borrowed the bank must pledge an amount h_1^{cb} of collateral 1 or h_2^{cb} of collateral 2, where h_1^{cb} and h_2^{cb} are the (gross) haircuts of the central bank. Let $a_{i,1}^{cb}$ and $a_{i,2}^{cb}$ be the amounts of collateral pledged by the bank to the central bank. To borrow an amount q_i^{cb}, the collateral pledged by bank i must satisfy

$$q_i^{cb} = \frac{a_{i,1}^{cb}}{h_1^{cb}} + \frac{a_{i,2}^{cb}}{h_2^{cb}}. \tag{2}$$

At this point it is useful to compare and contrast the types of contracts offered by the central bank and money market participants. First, the money market can offer bank-specific contracts while the central bank must offer the same contract to all banks. This captures a key operational constraint faced by central

[6]The reason for this is that for a given interest rate r_i^p the return of a loan with collateral 1 must be the same as for collateral 2, i.e.

$$p_i r_i^p + (1-p)\, h_{1,i}^p = p_i r_i^p + (1-p)\, h_{2,i}^p$$

which implies that $h_{1,i}^p = h_{2,i}^p = h_i^p$. Note that this does not necessarily imply that the two collateral have the same risk. Suppose that collateral 1 and 2 pay one with probability θ_1 and θ_2 respectively and such that $\theta_1 < \theta_2$. In this case to borrow one unit with full collateralization $h_i^p = 1$, the bank must pledge $1/\theta_1$ units of collateral 1 and $1/\theta_2$ units of collateral 2.

banks who must often lend to a very large number of counterparties amid tight time constraints.[7] Second, the central bank can set asset-specific haircuts while competition between money market participants causes haircuts to be the same for both assets.

The central bank follows a policy rule that attempts to steer the real interest rate in each country to a target level r^*. The distinction between the real economy rate r_i and a "target" or "neutral" interest rate r^* is meant to capture in a reduced form the influence of monetary policy on real activity (Svensson, 2011).[8] The central bank also wishes to minimize its operating losses which we measure as the loss given default LGD^{cb}, i.e. the difference between the amount owed $r^{cb}q_i^{cb}$ and the collateral seized in case of default:

$$LGD^{cb} = \left[\sum_{i=A,B}\left(r^{cb}q_i^{cb} - \sum_{j=1}^{2} a_{i,j}^{cb}\right)\right] 1_{\sum_{i=A,B}\left(r^{cb}q_i^{cb} - \sum_{j=1}^{2} a_{i,j}^{cb}\right)\geq 0}.$$

The indicator function $1_{\sum_{i=A,B}\left(r^{cb}q_i^{cb} - \sum_{j=1}^{2} a_{i,j}^{cb}\right)\geq 0}$ ensures that a potential profit (negative LGD) has no effect on the loss function. The central bank minimizes the following loss function:

$$L = \lambda \sum_{i=A,B} |r_i - r^*| + LGD^{cb}. \tag{3}$$

The parameter λ determines the relative importance to the central bank of minimizing rate deviations relative to minimizing losses. I assume that the central bank puts equal weight on each country.[9]

The objective function followed by the central bank in (3) can be given several interpretations. One is that such a policy rule closely resembles practical monetary policy, such as the rule used by Taylor (1993) to describe the policy of the Federal Reserve. This work spurred a large literature on the use of monetary policy targeting the nominal rate (Benhabib et al., 2001). In such a context, the target rate in (3) would thus represent the real equivalent of the nominal target.

Another interpretation of the policy rule (3) is that it is welfare maximizing. As noted by Woodford (2003),

[7] One could also argue that if the central bank used bank specific contracts this may increase the risk of runs on the banks that are assessed as risky by the central bank.

[8] Svensson (2011), page 5: "What monetary policy in the real world can do by setting a short nominal policy rate is only to temporarily make the short real interest rate deviate from the neutral real interest rate, which in turn is beyond the control of monetary policy. The effects that are attributed to monetary policy should only be the effects the deviation between the short real rate and the neutral rate, not the effects of the whole level of the short real rate, the sum of the deviation and the level of the neutral real rate. The neutral real rate is affected by many things and can be low for many years for several reasons, including global imbalances, fiscal policy, and shocks to aggregate demand and supply." The target rate in our model could be interpreted as the neutral rate.

[9] Benigno (2004) studies the optimal choice of the Pareto weights for the conduct of monetary policy in a two-region model of currency area in the presence of region-specific price rigidities.

> [Central] Banks around the world have committed themselves more explicitly to relatively straightforward objectives with regard to the control of inflation, and have found when they do so that not only is it easier to control inflation than previous experience might have suggested, but that price stability creates a sound basis for real economic performance as well.

Woodford (2003) shows that when prices are sticky, deviations of output from its "natural rate" are proportional to the unexpected component of inflation. Under such a setting, the rate r^* in (3) can be interpreted as the rate that achieves a "potential" investment level q_i^* so that output is at its natural rate $p_i R'(q_i^*) = r^*$. An economy rate r_i above r^* could then be interpreted as expansionary whereas $r_i < r^*$ would suggest that output is close to or above potential, with an increased risk of inflation.

Yet another interpretation is that of Williamson (2012) who argues that in practice the actions of the central bank are constrained by those of the fiscal authority. He shows that if the fiscal authority fixes the real deficit forever, the optimal monetary policy may deviate from the Friedman rule. The policy rule (3) would then correspond to the rule that balances the costs and benefits of inflation.

Finally, another interpretation is related to the bank-lending channel of monetary policy. Bernanke and Gertler (1995) and Kashyap and Stein (2000) have shown that bank lending conditions are instrumental in transmitting the monetary policy stance to the real economy. The central bank objective could thus be interpreted as the central bank targeting a specified amount of lending in the economy or a level of lending rates to firms and households.

This paper takes the objective of minimizing deviations $|r_i - r^*|$ as given and does not attempt to distinguish which interpretation is the most appropriate.

The second component of the policy rule (3) is the loss given default. This term is used to reflect the possibility that losses may endanger the independence of the central bank and its ultimate goal of price stability (Del Negro and Sims, 2014). The use of the realized losses instead of the expected losses ($(1 - p_i) LGD^{cb}$) captures in a reduced form the risk aversion of the central bank. Finally, our loss function implicitly assumes that the central bank is indifferent about potential redistributive effects of its interest rate policy, for example if one country is a net creditor while the other is a net debtor.

2.1 Funding choice

The borrowing process has two stages (illustrated in figure 2). In the first stage, the central bank chooses its lending and borrowing policy and proposes a contract $\left(r^{cb}, h_1^{cb}, h_2^{cb}\right)$. In the second stage, the money market opens and the bank chooses how much to borrow from each venue (q_i^p and q_i^{cb}) as well as how

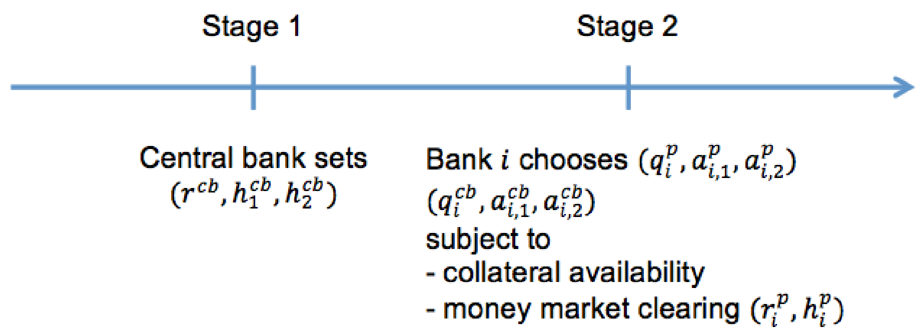

Figure 2: Structure of the borrowing process

much collateral to pledge to each venue $a_{i,1}^p, a_{i,2}^p, a_{i,1}^{cb}, a_{i,2}^{cb}$ to maximize its profit:

$$\max_{q_i^p, q_i^{cb}, a_{i,1}^p, a_{i,2}^p, a_{i,1}^{cb}, a_{i,2}^{cb}} p_i R\left(q_i^p + q_i^{cb}\right) - p_i\left(r_i^p q_i^p + r^{cb} q_i^{cb}\right) - (1-p_i) \sum_{s=\{p,cb\}} \sum_{j=1}^{2} a_{i,j}^s. \tag{4}$$

With probability p_i, the bank earns the return from its investments $R\left(q_i^p + q_i^{cb}\right)$ and pays back $\left(r_i^p q_i^p + r^{cb} q_i^{cb}\right)$. With probability $(1-p_i)$, the bank defaults and loses the collateral pledged $\sum_{s=\{p,cb\}} \sum_{j=1}^{2} a_{i,j}^s$. The bank's chosen outcome $\left(q_i^p, q_i^{cb}, a_{i,1}^p, a_{i,2}^p, a_{i,1}^{cb}, a_{i,2}^{cb}\right)$ must satisfy 6 conditions. The first two are that the collateral pledged must be lower than or equal to the collateral available to the bank:

$$a_{i,j}^p + a_{i,j}^{cb} \leq \theta_{i,j} \ (j = 1;\ 2). \tag{5}$$

The third and fourth are that it must satisfy the collateral requirements of the central bank and the money market contracts (equations (1) and (2)). Fifth, the contract in the money market (r_i^p, h_i^p) must be offered by money market participants who require a return equal to the central bank's policy rate r^{cb}:

$$p_i r_i^p + (1-p_i)\, h_i^p = r^{cb}. \tag{Mkt clearing}$$

Finally, money market participants and the central bank only lend to bank i if it does not shirk, i.e. the chosen outcome across lending venues satisfies the bank's incentive compatibility constraint:

$$p_i R(q_i) - p_i \sum_s r_i^s q_i^s - (1-p_i) \sum_{j,s} h_{i,j}^s q_i^s \geq H q_i - \sum_{j,s} h_{i,j}^s q_i^s. \tag{IC}$$

where the left hand side term is the bank's expected profit when the bank properly channels the funds to the real economy and the right hand side term is the payoff in case of shirking (where the bank loses the collateral pledged $\sum_{j,s} h^s_{i,j} q^s_i$ and earns a private benefit $\sum_s H q^s_i$).

Finally, the bank cannot borrow or pledge negative amounts,

$$q_i^{cb}, q_i^p, a^p_{i,1}, a^p_{i,2}, a^{cb}_{i,1}, a^{cb}_{i,2} \geq 0.$$

3 First best: no moral hazard

We begin with the model without moral hazard (thus ignoring the (IC)) and study the response of the central bank to economic shocks in this setting. To derive the equilibrium of the two stage game, we proceed by backward induction: in subsection 3.1 we first derive the optimal borrowing scheme of the bank and the contract offered by the money market (r^p_i, h^p_i) for a given central bank policy $\left(r^{cb}, h^{cb}_1, h^{cb}_2\right)$ and economic environment $(p_i, \theta_{i,1}, \theta_{i,2}, r^*)$. We then derive the optimal policy of the central bank in the first stage in case of symmetric and asymmetric shocks in subsection 3.2.

3.1 Optimal bank borrowing (Stage 2)

Bank $i \in \{A, B\}$ chooses the amounts to borrow q^p_i and q^{cb}_i and the collateral to pledge in each venue $\left(a^p_{i,1}, a^p_{i,2}, a^{cb}_{i,1}, a^{cb}_{i,2}\right)$ in order to maximize its profit

$$\max_{q^p_i, q^{cb}_i, a^p_{i,1}, a^p_{i,2}, a^{cb}_{i,1}, a^{cb}_{i,2}} p_i R\left(q^p_i + q^{cb}_i\right) - p_i\left(r^p_i q^p_i + r^{cb} q^{cb}_i\right) - (1-p_i) \sum_{s=\{p,cb\}} \sum_{j=1}^{2} a^s_{i,j}$$

subject to the collateral constraints (for $j = 1, 2$)

$$a^p_{i,j} + a^{cb}_{i,j} \leq \theta_{j,i}$$

the contract requirements of the central bank:

$$q^{cb}_i = \frac{a^{cb}_{i,1}}{h^{cb}_1} + \frac{a^{cb}_{i,2}}{h^{cb}_2},$$

the contract requirements of the private market:

$$p_i r^p_i + (1-p_i)\, h^p_i = r^{cb} \tag{6}$$

$$q^p_i = \frac{a^p_{i,1} + a^p_{i,2}}{h^p_i} \tag{7}$$

and the non-negativity constraints:

$$q_i^{cb}, q_i^p, a_{i,1}^p, a_{i,2}^p, a_{i,1}^{cb}, a_{i,2}^{cb} \geq 0.$$

Since there is no moral hazard we may abstract from equation (IC).

In order to simplify the problem, we can break down central bank borrowing in two terms $q_i^{cb} = q_{i,1}^{cb} + q_{i,2}^{cb}$ where $q_{i,j}^{cb} = a_{i,j}/h_j^{cb}$ is the amount borrowed using collateral j. We also define the cost of borrowing from the central bank using collateral j as

$$r_{i,j}^c = p_i r^{cb} + (1 - p_i) h_j^{cb}.$$

By plugging (6) and (7) in the maximization problem we may rewrite the problem as

$$\max_{q_i^p, q_{i,1}^{cb}, q_{i,2}^{cb}, a_{i,1}^p, a_{i,2}^p} p_i R \left(q_i^p + q_i^{cb} \right) - r^{cb} q_i^p - r_{i,1}^c q_{i,1}^{cb} - r_{i,2}^c q_{i,2}^{cb} \tag{8}$$

subject to the collateral constraints (for $j = 1, 2$)

$$a_{i,j}^p + q_{i,j}^{cb} h_j^{cb} \leq \theta_{j,i}$$

and the non-negativity constraints:

$$q_i^p, q_{i,1}^{cb}, q_{i,2}^{cb}, a_{i,1}^p, a_{i,2}^p \geq 0$$

This formulation shows that each funding source has a constant marginal cost, equal to r^{cb} for the private market and $r_{i,j}^c$ to borrow from the central bank using collateral j. Investors are indifferent between a collateral transfer or an interest payment as long as they earn the policy rate in expectation (equation (6)) so collateral is not a constraint to private borrowing. This is not the case however for the central bank which requires collateral so that the bank can borrow up to a maximum of $\overline{q}_{i,j}^{cb} = \theta_{i,j}/h_j^{cb}$ using collateral j.

Assume without loss of generality that $h_1^{cb} \leq h_2^{cb}$ so that $r_{i,1}^c \leq r_{i,2}^c$. The problem can be further simplified using the following marginal cost function:

$$
c_i(q) = \begin{cases} \min\left\{r^{cb}, r^c_{i,1}\right\} & \text{if } q < \overline{q}^{cb}_{i,1} \\ \min\left\{r^{cb}, r^c_{i,2}\right\} & \text{if } q \leq \overline{q}^{cb}_{i,1} + \overline{q}^{cb}_{i,2} \\ r^{cb} & \text{else} \end{cases} \tag{9}
$$

The optimal aggregate investment level equalizes the marginal cost $c_i(q_i)$ and the marginal return $p_i R'(q_i)$.

Proposition 1 (Bank strategy in stage 2 without moral hazard)**.** *The optimal aggregate investment level* q_i *solves* $\lim_{\epsilon \to 0} \; c_i(q_i - \epsilon) \leq p_i R'(q_i) \leq c_i(q_i + \epsilon)$*. The borrowing source is given by:*

$$
\left(q^p_i, q^{cb}_{i,1}, q^{cb}_{i,2}\right) = \begin{cases} (q_i, 0, 0) & \text{if } r^{cb} < h^{cb}_1 \\ \left(\max\left\{q_i - \overline{q}^{cb}_{i,1}, 0\right\}, \min\left\{q_i, \overline{q}^{cb}_{i,1}\right\}, 0\right) & \text{if } h^{cb}_1 < r^{cb} < h^{cb}_2 \\ \left(\max\left\{q_i - \overline{q}^{cb}_{i,1} - \overline{q}^{cb}_{i,2}, 0\right\}, \min\left\{q_i, \overline{q}^{cb}_{i,1}\right\}, \min\left\{q_i - \overline{q}^{cb}_{i,1}, \overline{q}^{cb}_{i,2} - \overline{q}^{cb}_{i,1}, 0\right\}\right) & \text{if } h^{cb}_2 < r^{cb} \end{cases}.
$$

The collateral pledged to the central bank is $a^{cb}_{i,j} = q^{cb}_{i,j} h^{cb}_j$ *and any* $a^p_{i,j} \in \left[0, \theta_{i,j} - a^{cb}_{i,j}\right]$ *can be an optimal amount of collateral pledged to the private market.*

Proof. We first show that $c_i(q_i)$ is the funding cost function with lowest funding cost for an investment level q_i. The general funding cost function is

$$
r^{cb} q^p_i + r^c_{i,1} q^{cb}_{i,1} + r^c_{i,2} q^{cb}_{i,2}.
$$

The funding scheme uses the cheapest funding source, subject to the collateral constraints. If $r^{cb} < h^{cb}_1$, the best option is to borrow only from the private market. Collateral availability cannot constrain the investment level since investors are indifferent between any contract that satisfies (6) (they are willing to fund the bank without collateral if the interest rate is sufficiently high). If $r^c_{i,1} < r^{cb} < r^c_{i,2}$, the bank first borrows from the central bank at cost r^c_1 using collateral 1 until it has exhausted all its collateral ($q < \overline{q}^{cb}_{i,1}$). It then borrows from the private market at cost r^{cb}. If $r^c_{i,1} < r^c_{i,2} < r^{cb}$, the bank borrows using collateral 1 until $\overline{q}^{cb}_{i,1}$, then collateral 2 until $\overline{q}^{cb}_{i,1} + \overline{q}^{cb}_{i,2}$ and then borrows from the private market. This shows that $c_i(q_i)$ is the minimum funding cost function.

Consider now the optimal aggregate investment level. The function $c(q_i)$ provides the marginal funding cost for a given investment level q_i. The condition $\lim_{\epsilon \to 0} \; c_i(q_i - \epsilon) \leq p_i R'(q_i) \leq c_i(q_i + \epsilon)$ is a first-order condition suited to the non-linearity of the cost function $c_i(q_i)$. □

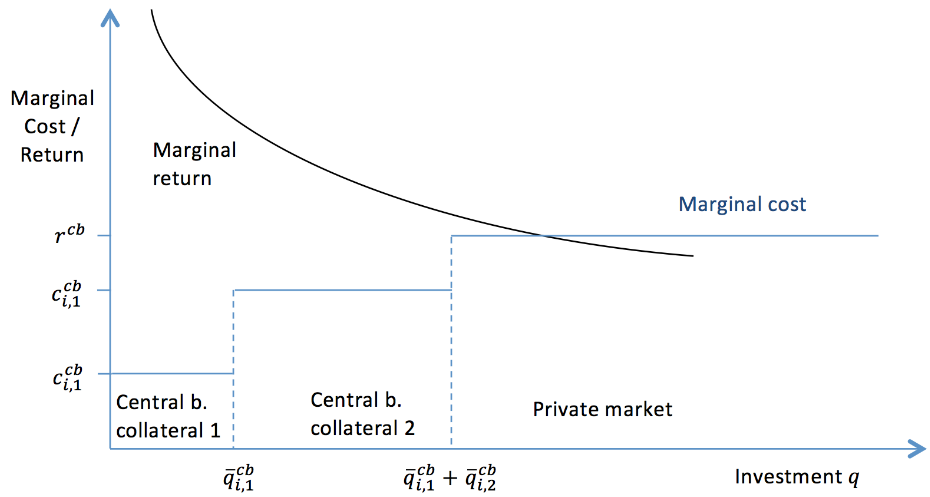

Figure 3: Optimal funding schedule with $h_1^{cb} < h_2^{cb} < r^{cb}$.

Figure 3 illustrates proposition 1 for the case where $h_1^{cb} < h_2^{cb} < r^{cb}$. The optimum marginal cost function faced by the bank is a step function, with discrete jumps at the thresholds $\overline{q}_{i,1}^{cb}$ and $\overline{q}_{i,1}^{cb} + \overline{q}_{i,2}^{cb}$. The marginal revenue $p_i R'(q_i)$ is a standard downward sloping and convex curve and the optimum investment level is at the intersection of the marginal revenue and the marginal cost. The value of the marginal revenue at this point is the interest rate in the economy. The quantities borrowed are to the left of the intercept - in the figure the bank pledges all its collateral 1 to the central bank and part of its collateral 2. It does not borrow from the private market.

The use collateral by banks can be interpreted as a following Gresham's law: banks use primarily the collateral that is overvalued by the central bank and keep the undervalued collateral. There is a continuum of contracts in the private market since private market participants are indifferent between collateral or interest payments.

Corollary 1 (Money Market Contract)**.** *Any contract* (r_i^p, h_i^p) *that satisfies* $r^{cb} = p_i r_i^p + (1 - p_i) h_i^p$ *and for which the bank has sufficient collateral* ($\left(\theta_{i,1} + \theta_{i,2} - q_{i,1}^{cb} h_1^{cb} - q_{i,2}^{cb} h_2^{cb}\right) / q_i^p \geq h_i^p$) *can be an equilibrium, where* q_i^p, $q_{i,1}^{cb}$ *and* $q_{i,2}^{cb}$ *are defined in proposition 1.*

Corollary 1 implies that contracts with less collateral command higher interest rates, which is what we observe in practice where interest rate on secured loans are lower than the unsecured rates. The premium between covered and uncovered loans is increasing in the probability of default $(1 - p_i)$, as observed in practice.

3.2 Optimal central bank policy (Stage 1)

We now derive the optimal policy of the central bank in stage 1 given the behaviour of the bank in stage 2 that we derived above. Economic shocks affect the probabilities of success of projects in each country, the value of the collateral held by banks and the target rate r^*. In particular we consider the case where countries are hit by an asymmetric shock that lowers the probability of success in country B relative to A, $p_A > p_B$. It also increases the value of collateral 1 and reduces that of collateral 2.

If the central bank chooses not to lend to banks and sets $h_{i,j}^{cb} \geq r^{cb}$, each bank i borrows a quantity q_i in the private market so that the economy rate is equal to the policy rate of the central bank.

$$r^{cb} = p_i R' (q_i) = r_i. \tag{10}$$

Since there is no spread between the policy rate and the target rate, there is a perfect pass-through of the rate policy of the central bank and the central bank can reach its target $r_i = r^*$ by setting its policy rate equal to the target rate. Notice in particular that the economy rate is independent of the probability of success of real economy projects p_i: while riskier entrepreneurs will pay a higher interest rate without collateral, the risk-adjusted cost of funding will be the same for firms in the booming or distressed country.

Since banks only borrow from the central bank when it has a risk exposure ($h_j^{cb} < r^{cb}$), it is never in the interest of the central bank to lend against collateral to banks when the two countries are hit by a symmetric shock. The central bank adjusts its rate policy to the change in the target rate and is indifferent to changes in the probability of success p_i or collateral values $\theta_{i,j}$.

Proposition 2 (Central Bank Policy Without Moral Hazard)**.** *Without moral hazard and when countries are identical the central bank can perfectly gear the interest rate in the economy and sets its policy rate* $r^{cb} = r^*$*. All lending goes through the money market (*$h_j^{cb} > r^{cb}\ \forall j$*).*

Proof. If the central bank sets $h_j^{cb} > r^{cb}\ \forall j$, the bank only borrows from the private market by proposition 1 and the economy rate is equal to the policy rate. In case of symmetric shock, the target rates are equal so that by setting the policy rate at $r^{cb} = r^*$ the central bank's loss function is worth zero. This is optimal since any change of policy with $h_j^{cb} < r^{cb}$ would involve a positive LGD^{cb} and cannot improve on the rate targeting objective. □

In the absence of moral hazard there is a perfect transmission of monetary policy so the interest rate in the economy is equal to the policy rate of the central bank. Since the economy rate (i.e. the return required by banks to its borrowers) does not depend on local economic conditions such the probability of success p_i, the central bank therefore sets its policy rate at its target in proposition 2. This rate is uniformly transmitted accross the currency area. The central bank does not lend to banks since they only borrow from the central bank if the required return is lower than that of the market, which increases the exposure of the central bank and increases its loss function.

4 Bank borrowing with moral hazard (Stage 2)

We now introduce moral hazard so collateral becomes necessary to incentivize the bank. We start in this section with the second stage and derive the optimal borrowing of each bank i for a given central bank policy $\left(r^{cb}, h_1^{cb}, h_2^{cb}\right)$ and we derive in the next section the policy of the central bank in stage 1.

The main difference with moral hazard is that money market participants and the central bank are no longer indifferent between contracts (r_i^p, h_i^p) that solve

$$p_i r_i^p + (1 - p_i)\, h_i^p = r^{cb}. \tag{11}$$

The reason is that not all contracts induce compliance by the bank. To ensure that the bank effectively invests the funds in the project, the incentive compatibility constraint (IC) must be satisfied:

$$\underbrace{p_i R\,(q_i) - p_i \sum_{s=p;cb} r_i^s q_i^s}_{\text{Net return if success}} - \underbrace{(1 - p_i) \sum_{j,s} a_{i,j}^s}_{\text{Loss if failure}} \geq \underbrace{H q_i - \sum_{j,s} a_{i,j}^s}_{\text{Net benefit from shirking}} . \tag{12}$$

Notice in particular that unless the net returns in case of success are especially high (so that the incentive compatibility constraint is automatically satisfied), collateral will help meet the IC constraint. This means that the available quantity of collateral will possibly act as a constraint to lending.

The problem faced by the bank is thus identical to the previous case but now includes the IC. The bank maximizes its profit

$$\max_{q_i^p, q_i^{cb}, a_{i,1}^p, a_{i,2}^p, a_{i,1}^{cb}, a_{i,2}^{cb}} p_i R\left(q_i^p + q_i^{cb}\right) - p_i \left(r_i^p q_i^p + r^{cb} q_i^{cb}\right) - (1 - p_i) \sum_{s=\{p,cb\}} \sum_{j=1}^{2} a_{i,j}^s$$

subject to the collateral constraints (for $j = 1, 2$)

$$a_{i,j}^{p} + a_{i,j}^{cb} \leq \theta_{i,j}, \tag{13}$$

the contract requirements of the central bank:

$$q_i^{cb} = \frac{a_{i,1}^{cb}}{h_1^{cb}} + \frac{a_{i,2}^{cb}}{h_2^{cb}},$$

and the non-negativity constraints:

$$q_i^{cb}, q_i^{p}, a_{i,1}^{p}, a_{i,2}^{p}, a_{i,1}^{cb}, a_{i,2}^{cb} \geq 0.$$

The bank must also provide an expected return of r^{cb} to money market participants and satisfy the collateral requirements:

$$p_i r_i^p + (1 - p_i)\, h_i^p = r^{cb},$$

$$q_i^p = \frac{a_{i,1}^{p} + a_{i,2}^{p}}{h_i^p}.$$

Finally, the chosen outcome must satisfy the IC constraint:

$$p_i R(q_i) - p_i \sum_s r_i^s q_i^s - (1 - p_i) \sum_{j,s} a_{i,j}^s \geq H q_i - \sum_{j,s} a_{i,j}^s. \tag{14}$$

The incentive compatibility constraint reduces the set of contracts available to the bank: for a given q, the bank must pledge a minimum amount of collateral $\sum_{j,s} a_{i,j}^s$ in order to satisfy the IC. In particular the larger the investment level q, the more collateral will have to be pledged by the bank.

The solution to the unconstrained problem (without the IC) always yields a higher surplus than the solution to the constrained problem with moral hazard. Consider an outcome $\left(q_i^p, q_i^{cb}, a_{i,1}^p, a_{i,2}^p, a_{i,1}^{cb}, a_{i,2}^{cb}\right)$ that maximizes the unconstrained problem, which we will call the "first best outcome". If this outcome satisfies the IC, it remains the equilibrium in the case with the IC constraint.

If the IC is violated at the first-best investment level, then the collateral constraints will have to bind at the optimum. To see this, suppose that there exists an optimum where the bank borrows below the first best amount so that the IC is binding but not the collateral capacity constraint. If the bank increases the collateral pledged, this relaxes the IC constraint so that the bank can increase its investment level closer to the first-best, which increases its profit. This implies that the initial outcome cannot be optimal.

In this case the collateral constraints and the IC bind so that the optimal strategy of the bank solves

$$\max_{q_i^p, q_i^{cb}, a_{i,1}^p, a_{i,2}^p, a_{i,1}^{cb}, a_{i,2}^{cb}} p_i R\left(q_i^p + q_i^{cb}\right) - p_i\left(r_i^p q_i^p + r^{cb} q_i^{cb}\right) - (1-p_i) \sum_{s=\{p,cb\}} \sum_{j=1}^{2} a_{i,j}^s$$

such that

$$\begin{aligned}
a_{i,1}^p + a_{i,1}^{cb} &= \theta_{1,i} && (15)\\
a_{i,2}^p + a_{i,2}^{cb} &= \theta_{2,i} && (16)\\
q_i^{cb} &= \frac{a_{i,1}^{cb}}{h_1^{cb}} + \frac{a_{i,2}^{cb}}{h_2^{cb}} && (17)\\
q_i^p &= \frac{a_{i,1}^p + a_{i,2}^p}{h_i^p} && (18)\\
p_i r_i^p + (1-p_i) h_i^p &= r^{cb} && (19)\\
p_i R(q_i) - p_i \sum_s r_i^s q_i^s - (1-p_i) \sum_{j,s} a_{i,j}^s &= H q_i - \sum_{j,s} a_{i,j}^s && (20)
\end{aligned}$$

Since the constraints form a system of 6 equations with 6 unknowns, the solution is entirely determined by the constraints (15) to (20). Plug the equations (15) to (19) into the IC:

$$p_i R(q_i) - \left(H + r^{cb}\right) q_i + \theta_i + (1-p_i) \sum_{j=1}^{2} \left(r^{cb} - h_j^{cb}\right) q_{i,j}^{cb} = 0. \tag{21}$$

The last term is the risk exposure taken by the central bank, the difference between the amount owed and the value of collateral seized in case of bank default. Depending on the parametric specification, two broad cases should be considered: (1) If the bank borrows both from the private market and the central bank (as in the example of Figure 3), the amount borrowed from the central bank is pinned down by the collateral constraints so that $q_{i,j}^{cb} = \theta_{i,j}/h_j^{cb}$. Plugging this in (21) allows to recover the total investment of the bank. (2) If the bank only borrows from the central bank, the investment level can also be recovered from (21) but the solution must take into account non-linearities in the funding cost function. We focus here on the first case where the bank borrows both from the private market and the central bank and cover all the parametric cases in appendix A.

Since banks borrow both from the central bank and the private market, the amount borrowed if $h_j^{cb} < r^{cb}$ is determined by their stock of collateral. Defining LGD_i^{cb} as the loss incurred by the central bank in case of a default of bank i,

$$LGD_i^{cb} = \sum_{j=1}^{2} \left[r^{cb} - h_j^{cb} \right] \frac{\theta_{i,j}}{h_j^{cb}} 1_{r^{cb} < h_j^{cb}}, \tag{22}$$

we may rewrite equation (21) as:

$$p_i R(q_i) - \left(H + r^{cb} \right) q_i + (1 - p_i) LGD_i^{cb} + \theta_i = 0. \tag{23}$$

The solution for q_i of this equation provides the total investment level of bank i. When the bank lacks collateral, this can constrain its investment capacity. The exposure taken by the central bank relaxes the constraint as would an increase in the available collateral.

We have now derived the investment level in the 2 regimes under moral hazard: first-best and collateral crunch. The final step is to derive the parametric regions of each regime. Let $\tilde{q}_i$ be the aggregate investment level of the first-best case of proposition 1. The collateral crunch occurs if the IC is violated when the bank pledges all its collateral and borrows $\tilde{q}_i$. We call this parametric region the "condition 1" (which we refer to in short as C1).

Condition 1. C1: The parametric region where the first-best investment level satisfies the IC is such that

$$p_i R(\tilde{q}_i) - \left(H + r^{cb} \right) \tilde{q}_i + (1 - p_i) LGD_i^{cb} + \theta_i \geq 0 \tag{24}$$

where $\tilde{q}_i$ is the first-best investment of proposition 1. We label the case where $r^{cb} = r^*$ as $C1^*$.

If C1 is violated, the IC is violated at the first-best investment level $\tilde{q}_i$. The bank then enters the collateral crunch regime where it is constrained by the availability of collateral. The investment level falls below the unconstrained, first-best level.

Proposition 3 (Stage 2: Bank borrowing with moral hazard)**.** *The optimal strategy* $\left(q_i^p, q_i^{cb}, a_{i,1}^p, a_{i,2}^p, a_{i,1}^{cb}, a_{i,2}^{cb} \right)$ *of the bank in stage 2 is*

[First-best] If C1 holds, the strategy is the same as in the first best case of proposition 1.

[Collateral crunch] Else, the equilibrium aggregate investment level solves

$$p_i R(q_i) - \left(H + r^{cb} \right) q_i + (1 - p_i) LGD_i^{cb} + \theta_i = 0$$

where LGD_i^{cb} *is defined in* (22)*. The central bank borrowing is* $q_j^{cb} = \overline{q}_j^{cb} 1_{h_j^{cb} < r^{cb}}$ *and the private market* *borrowing is* $q_i - q_1^{cb} - q_2^{cb}$. *The collateral pledged is* $a_{i,j}^{cb} = \theta_{i,j} 1_{h_j^{cb} < r^{cb}}$ *and* $a_{i,j}^{p} = \theta_{i,j} - a_{i,j}^{cb}$.

Proof. [First-best] Suppose that C1 holds. In this case by definition of C1 the bank has enough collateral to borrow the first best amount of proposition 1 without violating the IC. Since the bank is indifferent between collateral transfers or interest payment to private investors, the first-best investment level maximizes the profit of the bank so q_i^p, $q_{i,1}^{cb}$ and $q_{i,2}^{cb}$ are the same as in proposition 1 (we derive in corollary 2 below the private market contract terms).

[Collateral crunch] Suppose that C1 does not hold. Two functions will be useful in this case. (1) Let $\theta^{\min}(q)$ be the minimum collateral required to borrow a total quantity q using the optimal funding scheme of proposition 1. From (20):

$$\theta^{\min}(q) = \min\left\{p_i R(q_i) - \left(H + r^{cb}\right) q_i + (1 - p_i)\, LGD_i^{cb}, 0\right\}.$$

This is an increasing function of q_i: for very low values of q_i, the profit of the bank is very high relative to the private benefit so the bank could borrow the amount even without collateral. However as q_i increases, the marginal profit decreases whereas the marginal benefit from shirking H stays constant, so the minimum required collateral increases with q. (2) The second function is the profit of the bank along the optimal funding path which minimizes costs for a given investment q. This is

$$\pi(q_i) = p_i R(q_i) - c_i(q_i)$$

where $c_i(q_i)$ is the minimum cost function (9). The profit $\pi(q_i)$ is a concave function with a maximum in the first-best investment level.

Suppose then that C1 does not hold. This implies that $\theta^{\min}\left(q^{FB}\right) > \theta$ where q^{FB} is the first-best investment level. Let $q^{\max}$ be the maximum amount that can be borrowed by the bank given its stock of collateral, i.e. $\theta^{\min}(q^{\max}) = \theta$. Since the profit function is strictly increasing for $q < q^{\max}$, the optimal strategy for the bank is to pledge all its collateral and borrow the maximum amount. This implies that the collateral capactity constraints (13) and the IC (14) will bind in the bank's problem, which therefore consists of 6 equations with 6 unknowns as in equations (15) to (20), for which we showed that the solution for aggregate investment q_i solves (23).

Finally, in the collateral crunch the bank borrows from the central bank if the cost of doing so is lower than in the private market ($h_j^{cb} < r^{cb}$). In that case it borrows the maximum amount given its available collateral since it also borrows from the private market, so $q_j^{cb} = \overline{q}_j^{cb} 1_{h_j^{cb} < r^{cb}}$ (see appendix A for the case where the bank only borrows from the central bank). The collateral pledged is such that the bank pledges all its available collateral. Collateral j is fully pledged to the central bank if the cost of doing so is lower than the funding cost on the private market, i.e. $h_j^{cb} < r^{cb}$. □

The contract in the private market is similar to the first best case with the addition of the incentive constraint.

Corollary 2 (Money market contract). *If C1 holds and $q_i^p > 0$ in proposition 3, any contract (r_i^p, h_i^p) that satisfies $r^{cb} = p_i r_i^p + (1 - p_i) h_i^p$, for which the bank has sufficient collateral ($\left(\theta_{i,1} 1_{h_1^{cb} > r^{cb}} + \theta_{i,2} 1_{h_2^{cb} > r^{cb}}\right) \leq q_i^p h_i^p$) and that is incentive compatible, i.e. satisfies*

$$p_i R(q_i) - \left(H + r^{cb}\right) q_i + (1 - p_i) LGD_i^{cb} + h_i^p q_i^p + \theta_{i,1} 1_{h_1^{cb} > r^{cb}} + \theta_{i,2} 1_{h_2^{cb} > r^{cb}} \geq 0$$

is part of an equilibrium.

Comparative statics. We now study how shocks to the economic environment affect the outcome for a given central bank policy $\left(r^{cb}, h_1^{cb}, h_2^{cb}\right)$. Economic shocks affect the parameters p_i, θ_i and r^* so that high values of ϵ_s are associated with high p_i, θ_i and r^*. High values of ϵ_s also relax the IC constraint, i.e.

$$\left| \frac{\partial W_i}{\partial \epsilon_s} \right| \leq \left| \frac{\partial \theta_i}{\partial \epsilon_s} \right| \tag{25}$$

where $W_i = p_i R(\tilde{q}_i) - \left(H + r^{cb}\right) \tilde{q}_i$ and $\tilde{q}_i$ is the first-best investment level of proposition 1. Equation (25) ensures that incentive problems worsen in a downturn, when investment opportunities are less attractive and collateral values are low. We let E_i be the value of the shock ϵ_s for which (24) holds so that C1 holds for $\epsilon_s > E_i$ and C1 is violated if $\epsilon_s < E_i$.

The economic shock can be interpreted as a common factor that influences the level of the economic parameters. We can show that, when the central bank does not lend to banks, the collateral crunch regime creates a spread between the economy rate and the policy rate.

Lemma 1 (Interest rates and economic shocks). *For a given central bank policy $\left(r^{cb}, h_1^{cb}, h_2^{cb}\right)$ such that $h_j^{cb} > r^{cb}$,*

If $\epsilon_s > E_i$ (C1 holds), the interest rate in the economy i is equal to the policy rate of the central bank.

If $\epsilon_s < E_i$ (C1 does not hold), the interest rate in the economy i is above the policy rate of the central bank.

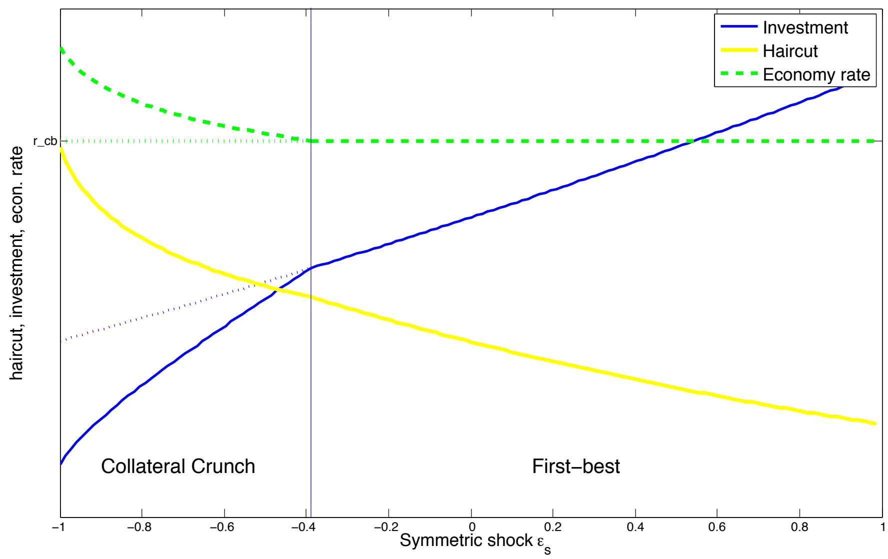

Figure 4: Stage 2: Different regimes with moral hazard when the bank borrows exclusively from the money market. Specification used: $R(q) = \sqrt{q}$, $\theta_i = 0.015 + 0.01 \times \epsilon_s$, $p = 0.4 + 0.1 \times \epsilon_s$, $H = 1.5$, $r^{cb} = 1.08$.

Proof. Since $h_j^{cb} > r^{cb}$, the bank does not borrow from the central bank. In the first-best, the investment level solves $r^{cb} = p_i R'(q_i) = r_i$ so the economy rate is equal to the policy rate. In the collateral crunch regime (C1 does not hold), the investment level is below the first best level (see proof of Lemma (3)) so the economy rate is above the policy rate since $R'(q_i)$ is decreasing in q_i. □

Figure 4 illustrates this result for the case where $R(q) = \sqrt{q}$, $\theta_i = 0.015 + 0.01 * \epsilon_s$, $p = 0.4 + \epsilon * 0.1$, $H = 1.5$, $r^{cb} = 1.08$ and ϵ_s goes from -1 to 1. In the first best, the investment level q_i falls and the minimum haircut increases as the shock ϵ_s falls. More importantly, the economy rate is not influenced by the shock and remains equal to the central bank's policy rate. If the symmetric shock is too negative and below a threshold E_1 (so that condition 1 is not satisfied any more), the banks enter the "collateral crunch" regime. In this case, the lack of available collateral leads money participants to restrict the amount lent to banks below the first best level in order to ensure that they remain sufficiently incentivized. This causes the interest rate in the economy $r_i = p_i R'(q_i^p)$ to increase relative to the central bank's policy rate.

5 Central bank policy (Stage 1)

We now consider how the central bank responds to an asymmetric shock. Bank A is located in the booming country and bank B is in the distressed country. The probability of success of projects in economy A is higher than in economy B, $p_A > p_B$. The shock also has an asymmetric effect on the value of the collateral owned by banks. Collateral 1, owned mostly by bank A, gains in value whereas the value of collateral 2 falls. This implies that $\theta_{1,A} > \theta_{1,B}$, $\theta_{2,A} < \theta_{2,B}$ and $\theta_{1,A} + \theta_{2,A} > \theta_{1,B} + \theta_{2,B}$. We will focus here on the case where the collateral owned by bank A is such that bank A is always in the first-best regime (which is equivalent to say that $\theta_A \to \infty$).

We solved in section 4 the bank's borrowing scheme in the second stage. In this section, we derive the policy of the central bank in stage 1. For a given central bank policy, banks are in a collateral crunch or the first-best depending on whether C1 is satisfied or not in their case. so that bank A is in the first-best regime for common values of the policy rate. In stage 1, the central bank solves

$$\min_{r^{cb}, h_1^{cb}, h_2^{cb}} L = \lambda \left(|r_A - r^*| + |r_B - r^*| \right) + LGD^{cb}.$$

The central bank faces 2 possible situations. If both banks are in the first best regime, the problem is similar to the case without moral hazard of section 3. There is a perfect transmission of the policy rate of the central bank in both countries, and the central bank sets its rate at the target level so $r^{cb} = r^*$.

If bank B is in a collateral crunch while bank A is in the first-best regime, the economy rate in country B is higher than the policy rate whereas the economy rate in country A is equal to the central bank rate. When r^{cb} is between r_A and r_B, the loss function can be rewritten as

$$\min_{r^{cb}, h_1^{cb}, h_2^{cb}} L = \lambda \left(r_B - r_A \right) + LGD^{cb} \tag{26}$$

so the spread between the economy rates strictly increases the loss function.

To derive the policy of the central bank in this case, we rely on an intermediate result that explains how the spread changes with the policy rate of the central bank. We will show that the spread is a decreasing function of the policy rate because of an imperfect pass-through of the policy rate in the distressed country. When the central bank reduces its policy rate by one unit, the economy rate in the collateral crunch country falls by less than a unit, so the spread increases.

Lemma 2 (Imperfect pass-through)**.** *In the collateral crunch regime (i) there is an imperfect pass-through of the policy rate so that a unit change in the policy rate of the central bank changes the economy rate by less than a unit,*

$$\frac{\partial r_i}{\partial r^{cb}} = \frac{\partial \left(p_i R'(q) \right)}{\partial r^{cb}} < 1,$$

where q solves $p_i R(q) - r^{cb} q - Hq + \theta_i = 0$. (ii) The spread between the policy rate and the economy rate *is decreasing in the policy rate of the central bank;*

Proof. Let us begin with point (i). In the first-best case, investment is given by

$$p_i R'(q) = r^{cb} \tag{27}$$

In the collateral crunch, investment solves

$$p_i R(q) - r^{cb} q - Hq + \theta_i = 0,$$

which may be rewritten as

$$\frac{p_i R(q) + \theta_i}{q} = r^{cb} + H. \tag{28}$$

Let $F(q) = p_i R'(q)$ and

$$G(q) = \frac{p_i R(q) + \theta_i}{q}$$

denote the left hand side of equations (27) and (28) respectively. Since $R'(q) > 0$ and $R''(q) < 0$, the average output is always higher than the marginal output so $F(q) < G(q)$. Moreover since $R(q) = q^{\alpha}$ we have $F'(q) > G'(q)$, i.e. the (negative) slope of $G(q)$ is larger than that of $F(q)$ for a given q.

To see this point, note that

$$\frac{\partial [G(q) - F(q)]}{\partial q} = \frac{p_i R'(q) q - (p_i R(q) + \theta_i)}{q^2} - p_i R''(q)$$

Since $R(q) = q^{\alpha}$:

$$\frac{p_i \alpha q^{\alpha-1} q - (p_i q^{\alpha} + \theta_i)}{q^2} - p_i \alpha (\alpha - 1) q^{\alpha-2}$$

This may be rewritten as:

$$\frac{-p_i q^{\alpha} (\alpha - 1)^2 - \theta_i}{q^2} < 0$$

which is always negative. Hence $F'(q) > G'(q)$.

Consider now a decrease of the policy rate from r_0^{cb} to r_1^{cb} in the collateral crunch regime. Let q_0 and q_1 be the investment levels before and after the rate change. We want to show that the economy rate decreases less than the change in policy rate, i.e. $F(q_0) - F(q_1) < r_0^{cb} - r_1^{cb}$ (since $F(q)$ is the economy rate for a given q). The investment level in the collateral crunch solves (28), which implies that $G(q_0) - G(q_1) = r_0^{cb} - r_1^{cb}$. Since $F(q)$ decreases more slowly than $G(q)$, $F'(q) > G'(q)$, we have that

$G(q_0) - G(q_1) > F(q_0) - F(q_1)$ which proves that the change in economy rate in the collateral crunch is lower than the change in policy rate, which implies a pass-through of interest rate lower than one.

Finally, the imperfect pass-through implies that the spread is decreasing in r^{cb}. To see this, suppose that the policy rate is at the limit case where $r^{cb} = \overline{r_i^{cb}}$. If the central bank lowers its policy rate, the bank enters the collateral crunch and the economy rate falls by less than the policy rate. The larger the fall, the larger the spread between the economy rate and the policy rate. □

The intuition for Lemma 2 is that when the central bank increases its policy rate, this has 2 opposing effects. First, it reduces the investment level required for the bank to be in the first-best regime. Second, it increases the funding cost of the bank which reduces the amount that the bank can invest. Lemma 2 shows that when $R(q) = q^{\alpha}$ the first effect is larger than the second one so an increase in the policy rate moves bank B closer to the first-best.

Consider now the policy of the central bank and assume that its policy rate is at the target level r^* and bank B is in the collateral crunch. In country A, the economy rate is equal to the target rate. In country B, the economy rate is above target. A first observation is that the central never increases its policy rate above r^* because this increases the economy rate both in A and B away from the target level.

If the currency union only consisted of bank B, the central bank would only have to lower its policy rate below the target until the economy rate in country B is on target. A lower rate indeed relaxes the incentive compatibility constraint since it reduces the cost of funding of banks. The central bank would therefore take the spread into account when setting its policy rate, however that would not affect its ability to reach the target.

In a heterogeneous currency union, the central bank faces a dilemma: to reach the target in country A, it would need to set the policy rate on target r^* whereas for country B only lower rates would allow to reach the target. Lemma 2 shows however that the central bank does not reduce its policy rate below r^* because this increases the spread. A lower policy rate therefore strictly increases the Loss function in (26), so the central bank keeps its policy rate at r^*.

Collateral policy can however be used to alleviate the funding constraints on bank B and increase the investment level in that country. By setting $h_j^{cb} < r^{cb}$, the central bank increases its exposure to banks. This increases the expect profit of the distressed bank which relaxes its incentive constraint and allows it to borrow more from the money market. An important difference with the symmetric case however is that while below market haircuts can have positive welfare consequences by lowering rates in B, it also creates a transfer to bank A, which is already in the first best so these transfers only increase the loss function. This implies that the central bank will reduce its haircut on the asset that is owned mostly by the distressed bank (asset 2 in our case), in contrast to the symmetric case where it lowered haircut for both assets. The portfolio asymmetry will thus play a key role in the response of the central bank. If it is large, the central bank can target its exposure to the distressed bank which lowers the total amount of exposure required to lower rates in country B.

Proposition 4. *If $C1^*$ holds for bank B, the central bank sets $r^{cb} = r^*$ and does not lend to banks ($h_j^{cb} > r^{cb}\ \forall j = 1, 2$).*

Else, the central bank sets $r^{cb} = r^*$, $h_1^{cb} > r^{cb}$ and relaxes collateral policy on collateral 2 by setting

$$h_2^{cb} = \sqrt{r_A^* \frac{\theta_{A,2} + \theta_{B,2}}{\lambda \xi_{r,h,B}}}$$

where $\xi_{r,h,B} = \partial r_B / \partial h_2^{cb}$; $r_B = p_B R'(q_B)$ and q_B solves $p_B R(q_B) - r^{cb} q_B - H q_B + \theta_B - \left(r^{cb} - h_2^{cb}\right) \frac{\theta_{B,2}}{h_2^{cb}} = 0$.

Proof. The central bank solves:

$$\min_{h_2^{cb}} L = \lambda \left(r_A - r_A^* + r_B - r_B^*\right) + LGD^{cb}.$$

The proof is in two steps. In step 1, I show that the policy of setting $r^{cb} = r_A^*$ is optimal. In step 2, I derive the optimal haircut policy.

Step 1: Interest rate policy

Suppose that the central bank sets its policy rate at r_A^*.

If it increases the policy rate, the central bank increases its loss function since this increases the interest rate in both countries above the target rates.

If it decreases its policy rate, the economy rate in country A decreases with r^{cb} whereas the economy rate in B decreases less than r^{cb}. This implies that the value of the loss function increases since the negative effect of lower rates in A outweighs the gain from lower rates in B.

Step 2: Collateral policy

We have shown that it is always optimal for the central bank to set its policy rate at $r^{cb} = r_A^*$ if $\overline{r_B^{cb}} > r_A^*$. We now derive the optimal collateral policy when bank B in that case. First, it is important to see that the central bank will always relax its policy on collateral 2 because this asset is the one owned mostly by bank B. This allows the central bank, for a unit of risk taken, to maximize the effect on the economy rate in country B. The central bank therefore solves:

$$\min_{h_2^{cb}} L = \lambda \left(r_A - r_A^* + r_B - r_B^*\right) + LGD^{cb}.$$

Taking out the constant terms and since $r^{cb} = r_A^* = r_A$, we may rewrite this as

$$\min_{h_2^{cb}} L = \lambda r_B + LGD^{cb}.$$

where $r_B = p_B R'(q_B)$, q_B solves

$$p_B R(q_B) - r^{cb} q_B - H q_B + \theta_B + LGD_B^{cb} = 0, \tag{29}$$

$$LGD_i^{cb} = \left(r^{cb} - h_2^{cb}\right) \frac{\theta_{i,2}}{h_2^{cb}},$$

and $LGD^{cb} = \sum_{i=A,B} LGD_i^{cb}$. The objective function can be written as

$$\min_{h_2^{cb}} L = \lambda r_B + \left(r^{cb} - h_2^{cb}\right) \frac{\theta_{A,2} + \theta_{B,2}}{h_2^{cb}}$$

where $r_B = p_B R'(q_B)$ and q_B solves (29). The FOC yields:

$$\lambda \xi_{r,h_2^{cb},B} - r^{cb} \frac{\theta_{A,2} + \theta_{B,2}}{\left(h_2^{cb}\right)^2} = 0$$

where $\xi_{r,h,B} = \partial r_B / \partial h_2^{cb}$. This can be written as:

$$h_2^{cb} = \sqrt{r_A^* \frac{\theta_{A,2} + \theta_{B,2}}{\lambda \xi_{r,h,B}}}$$

This is indeed a minimum since L is a convex function of h_2^{cb}. The second-order derivative of L is:

$$\lambda \xi'_{r,h,B} + 2r^{cb} \frac{\theta_{A,2} + \theta_{B,2}}{\left(h_2^{cb}\right)^3}$$

The second term is positive. Regarding the first term, we have

$$\xi_{r,h,B} = \frac{\partial p R'(q_B)}{\partial h_2^{cb}}$$

i.e.

$$\xi_{r,h,B} = pR''(q_B)\, q'_B$$

where $q'_B = \partial q_B / \partial h_2^{cb}$. The second order derivative is:

$$\partial \xi_{r,h,B} / \partial h_2^{cb} = pR'''(q_B)\left(q'_B\right)^2 + pR''(q)\, q''_B$$

The first term is positive since $R''' > 0$. The second term is also positive since q''_B is negative. To see this, take the implicit differentiation of the investment level function

$$p_B R(q_B) - r^{cb} q_B - H q_B + \theta_B - \left(r^{cb} - h_2^{cb}\right) \frac{\theta_{B,2}}{h_2^{cb}} = 0$$

with respect to h_2^{cb}. This yields:

$$p_B R'(q_B)\, q'_B - r^{cb} q'_B - H q'_B + r^{cb} \frac{\theta_{B,2}}{\left(h_2^{cb}\right)^2} = 0.$$

Derive again:

$$p_B R''(q_B)\left(q'_B\right)^2 + p_B R'(q_B)\, q''_B - r^{cb} q''_B - H q''_B - 2r^{cb} \frac{\theta_{B,2}}{\left(h_2^{cb}\right)^3} = 0.$$

Rearrange:

$$q''_B = \frac{p_B R''(q_B)\left(q'_B\right)^2 - 2r^{cb} \frac{\theta_{B,2}}{\left(h_2^{cb}\right)^3}}{-p_B R'(q_B) + r^{cb} + H}.$$

This is always negative since $R''(\cdot) < 0$ and $p_B R'(q_B) < +r^{cb} + H$ in the collateral crunch (else the bank could increase its borrowing and generate a return higher than the cost of funding r^{cb} and the private benefit H). □

The interest rate policy of the central bank is determined by the imperfect pass through result of Lemma 2. The haircut policy reflects the trade-off faced by the central bank when using this instrument: it avoids an excessive increase in the economy rate of B on the one hand but it increases the exposure of the central bank to the banking system. In particular, while increased exposure to the distressed bank allows to improve lending conditions in that country, the exposure towards bank A only increases the loss function. This implies that, all else equal, an increase in the collateral 2 owned by bank A increases the haircut of the central bank. The reason is that it increases the cost of using collateral policy since, for a unit of exposure to bank B, the central bank also increases its exposure to bank A. The portfolio asymmetry will therefore play a key role in determining to what extent the central bank can use collateral policy. Ex-post, once the shock has struck, the central bank prefers a high degree of asymmetry since this reduces the cost of using collateral policy. However ex-ante the central bank prefers a low degree of asymmetry since this helps reduce the negative impact of the shock on bank B (if collateral values are correlated to the local economic conditions).

6 Discussion

Collateral availability during the Sovereign debt crisis. The model suggests that the availability of collateral is one potential mechanism driving the "high-rate / low rate" equilibrium during the Sovereign debt crisis. In practice it is challenging to observe the collateral available to banks for refinancing. In an empirical exploration of the collateral policy of the ECB from 2009 to 2011, Cassola and Koulischer (2014) use a specificity of the ECB's operating framework to construct a proxy for collateral availability. The proxy is based on the fact that the Eurosystem uses a "pooling system" for collateral management, so banks pledge any amount of collateral they desire and obtain a credit line that may or may not use. In practice, banks tend to pledge more collateral than required for their borrowing need. However, one can expect that when banks are short on collateral they will save on the use of the extra collateral and pledge only the amount required for their borrowing need.

Figure 5 (from Cassola and Koulischer (2014)), plots the ratio of the total amount of collateral pledged (after haircut) to the amount borrowed against the interest rate spread of non-financial corporations (NFC). Each observation is a country in a given quarter. As predicted by the model, the figure suggests a non-linear correlation pattern between the economy rate and the availability of collateral. When banks have plenty of collateral, the economy rate does not seem to correlate with collateral availability. However if collateral is scarce, we observe an almost vertical relationship between the economy rate and the availability of collateral.

The figure also illustrates the trade-off faced by the central bank when using collateral policy. It is when banks are short on collateral that a relaxation of requirements (i.e. moving the vertical red bar to the left) is likely to have the largest effect on economic conditions in the distressed country. But this comes at the cost of increasing the exposure of the central bank to the riskiest banks.

Collateral valuation and central bank exposure. In the model, the key mechanism of collateral policy is that there is an "overvaluation" of the collateral by the central bank relative to the market. This creates

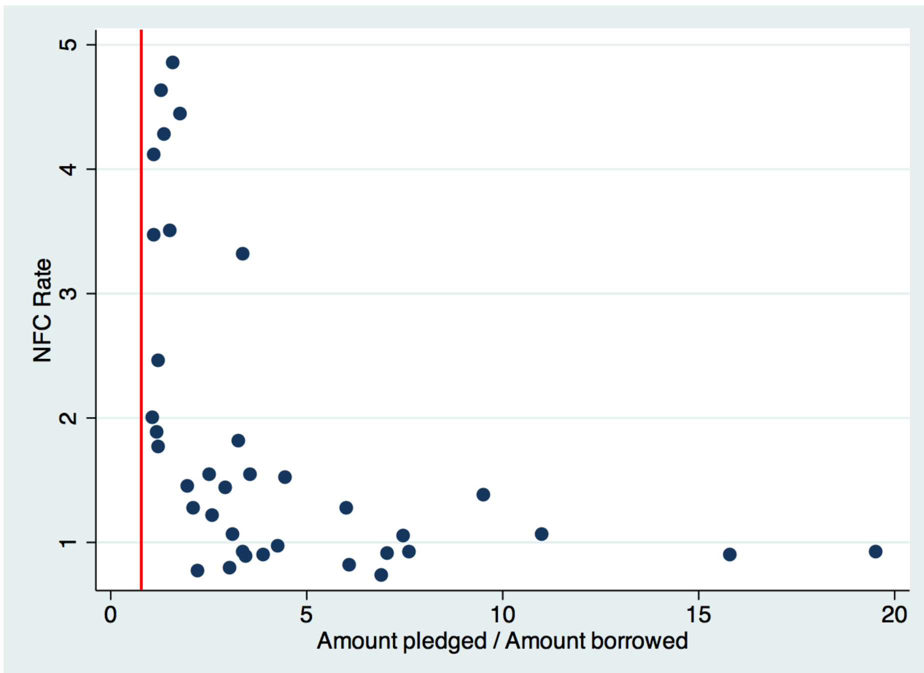

Figure 5: Interest rate on loans to non-financial corporations (NFC) and overcollateralization rate by country and quarter. The NFC rate is a proxy for the real economy rate in the theoretical model. The overcollateralization rate is the ratio of the amount of assets pledged to the amount borrowed and can be interpreted as a proxy for the availability of collateral. We exclude cases where the overcollateralization rate is higher than 5 and countries with less than € 4 billion of assets pledged. Source: Cassola et al. (2013).

an exposure of the central bank to the banking system. In practice, the type of exposure will depend on how one interprets the value premium assigned by the central bank. If the true value is the market value, then the central bank may be exposed to credit risk and a loss in case of default. However the value of the collateral could also be owner-specific (as in e.g. Parlatore Siritto (2012)). If market prices include a large liquidity discount, which can be plausible in a severe turmoil, the central bank could have an advantage with respect to market participants since it is a long-term market participant. Our model is thus compatible with the Bagehot (1873) principle that the central bank may take on liquidity risk but not credit risk, however it also highlights that distinguishing the two may be difficult in case of turmoil (Goodhart, 1999).

The exposure of the central bank also makes clear that collateral policy is not a "free" policy instrument like the policy rate. This contrasts with the work of Ashcraft et al. (2011) or Acharya et al. (2012) where the central bank is not explicitely included in the model and simply influences the economic fundamentals. In our model, the central bank directly competes with the private market and must make a more attractive offer (i.e. take on risk) if it wants to influence the market outcome. In particular if the value of the collateral was endogenous to the collateral policy of the central bank, the conclusions would be reinforced: lower haircuts on an asset increases its attractiveness and incentivizes banks to hold this asset. This increases the value of the collateral so if it is owned mostly by the distressed bank, this further reduces rates in the distressed economy.

Collateral and macro-prudential policy. The fact that the central bank competes with the money market also suggests that collateral policy is not ideally suited to "lean against the wind". The reason is that if the central bank sets its haircuts above market levels, banks will simply choose to borrow from the market so the haircut does not influence the final outcome. Of course the model abstracts from potential effects on asset prices which is the main mechanism behind the proposed used of collateral policy as a macro-prudential tool (Buiter and Sibert, 2005; Brunnermeier, 2010). Still, it seems reasonable to think that it is in case of severe turmoil that collateral policy takes the center stage, as was the case during the Sovereign debt crisis. This "one-sidedness" of the collateral policy instrument to alleviate downturns also raises questions of moral hazard and of the link to other macro-prudential tools. These are discussed in detail by Koulischer and Struyven (2014).

The fact that collateral policy is a costly instrument also raises the question of whether there exists other, better instruments that could be used to respond to the "high rate / low rate" policy conundrum. One could for example cite the need for a banking union or a fiscal union or a European-wide unemployment insurance. While these elements are arguably necessary to help the euro area to respond to future asymmetric shocks, these tools were mostly unavailable during the sovereign debt crisis. Collateral policy on the other hand was an established component of the Eurosystem's operational framework and the policy changed during the crisis. Our model therefore helps us understand the policy responses during the crisis.

Collateral policy versus asset purchases. An alternative policy could be for the central bank to purchase the collateral directly. This would provide cash for the bank to invest, however, in general the bank will still need to borrow from the market. The reason is that banks generally borrow both covered and uncovered so that in effect one unit of collateral is used to borrow more than one unit (and $h_i^p < 1$ in our model). In that case the final equilibrium remains unchanged: the bank uses the cash received as collateral and remains constrained by the lack of skin in the game. The central bank could however choose to purchase the assets above their market value. This could be justified for example if the assets are illiquid and the central bank attaches a lower liquidity premium than the market. In that case the value differential will relax the incentive constraint and increase lending, just as in the model. A recapitalization or a direct transfer would similarly help to relax the incentive problem.

In the model the real effect of this policy would depend on how it influences the budget constraint. If the bank purchases the collateral at its market values, this exchanges the collateral against currency. This however does not change the problem faced by the bank, since it uses collateral to solve the moral hazard problem.

Imperfect pass-through. My paper shows that a collateral crunch can distort the transmission of monetary policy in a currency union. This may in turn affect the optimum interest rate policy: if the policy rate is a weighted average of the target rates in various countries (as in Benigno (2004)), our paper suggests that the weights used should take into account financial market conditions in each country. A similar point was made by Kashyap and Stein (1997) who argued that differences in the structure of national banking systems has important implications for the conduct of monetary policy in the euro area.

Home bias and financial integration. A key assumption in our model is that the banking systems of the two countries of the currency union can be described as two representative banks. This assumption seems suited to the case of the European sovereign debt crisis which strongly affected banking systems at the country level. One reason for this is that banks are more exposed towards their home country - Acharya and Steffen (2013) in fact show that the home bias increased during the crisis. However this assumption may be less suited to study the case of an integrated currency union, where banks from the stable country could lend directly to entrepreneurs in the country where banks are constrained.

OCA. The literature on optimum currency area emphasizes that the central bank faces a "one tool / two objectives" dilemma when responding to an asymmetric shock in a currency union. I show in this paper that collateral policy can be used to overcome this dilemma in a specific case: when the distressed country experiences a collateral crunch. In this situation, where rates are high in the distressed country (where they should be low) and low where they should be high, collateral policy can help by reducing the interest rate in the distressed economy. However, the best that the central bank can do with collateral policy is to bring back interest rates to a same level in both countries. It cannot engineer high rates in the booming country and low rates in the distressed country. In that sense our model is most suited to the Sovereign debt crisis period, but not to the preceding years (2002-2007) when interest rate spreads were similar for all countries whereas they should have been high in booming countries.

Incentive problems and zombie lending. Finally, the model underscores the difficulties of using collateral policy in the presence of moral hazard. In the model, the central bank is able to ensure that its lending contract remains incentive compatible by increasing the reward in case of success. Insufficient haircuts levels could however provide wrong incentives for banks to invest in socially inefficient projects. In practice, when the central bank deals with a large number of counterparties on a daily basis it may be difficult to ensure that the collateral requirements are incentive compatible for all banks. The danger of supporting weak banks has been emphasized by Acharya and Tuckman (2013).

Dynamic moral hazard and links to fiscal policy. As discussed above, the model captures several features of collateral policy that may be relevant in practice. However there are potentially two aspects that are not captured by the model. The first one is the fact that using collateral policy in case of turmoil could lower banks' incentives to hold collateral. This could lead to increased intervention by the central bank in the long run. As explained above, one option to minimize the negative incentives created by collateral policy could be to combine its use with macro-prudential tools such as the Liquidity Coverage Ratio. Another aspect that the model does not capture is the link between collateral policy and fiscal policy. The eligibility criteria may be an important factor for the decision of banks to invest in an asset. Collateral requirements could then be viewed as a tool to ensure fiscal discipline by governments (Buiter and Sibert, 2005). In practice it is likely that both the mechanism described in this paper and the interaction with fiscal discipline are active. This paper focuses on the former and leaves the latter for future research.

7 Conclusion

I build a model of the transmission of monetary policy where banks can borrow from the central bank or the money market using collateral. I show that in the presence of moral hazard a lack of collateral can constrain banks' access to the interbank market which increases the funding cost of firms and households in the real economy.

In the case of two banks and two assets, I show how asymmetric shocks that have a different effect on collateral values can lead to a situation with high rates in the distressed country and low rates in the booming country. The collateral crunch in the distressed country also distorts the transmission of monetary policy so that a unit reduction in the policy rate translates into less than a unit reduction in the economy rate. This blunts the effectiveness of the policy rate instrument and suggests that higher policy rates that are closer to the optimum of the stable country are more desirable.

Collateral policy can nevertheless be used to improve lending conditions in the distressed country. By reducing the haircut on assets owned mostly by the distressed bank, the central bank can relax the incentive problem and lower the interest rate in that economy. The asymmetry of banks' collateral portfolios plays a key role in the effectiveness of the haircut instrument. If banks have significant differences in their collateral portfolios, the central bank can better target its policy to economic conditions across regions. From an ex-ante point of view, however, portfolio asymmetries may worsen the extent of the shock by increasing the correlation between banks' collateral stocks and economic conditions.

The paper suggests several fruitful avenues for future research. One is to make asset prices endogenous to the haircut policy of the central bank which would allow to distinguish the role played by asset price changes versus increased risk of the central bank that is studied in the paper. A second one is to benchmark the collateral policy instrument to other instruments that could be used to respond to asymmetric shocks. Finally, the issue of collateral policy in practice would deserve an empirical study - this is the focus of ongoing work by Cassola and Koulischer (2014).

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A Bank borrowing with moral hazard - parametric cases

In the main text we focus on the case where the bank borrows both from the central bank and the private market. In this section, we develop all parametric cases. Without loss of generality we assume that $h_1^{cb} < h_2^{cb}$.

Case 1: Borrow both from private market and central bank

Suppose that $\frac{\theta_{i,1}}{h_1^{cb}} \times 1_{\left(h_1^{cb} < r^{cb}\right)} + \frac{\theta_{i,2}}{h_2^{cb}} \times 1_{\left(h_2^{cb} < r^{cb}\right)} \leq q^{CCrunch}$, where $q^{CCrunch}$ solves

$$p_i R\left(q_i\right) - \left(H + r^{cb}\right) q_i + \theta_i + \underbrace{(1 - p_i) \sum_{j=1}^{2} \left(r^{cb} - h_j^{cb}\right) \left(\frac{\theta_{i,1}}{h_1^{cb}} \times 1_{\left(h_1^{cb} < r^{cb}\right)} + \frac{\theta_{i,2}}{h_2^{cb}} \times 1_{\left(h_2^{cb} < r^{cb}\right)}\right)}_{LGD_i^{cb}} = 0. \quad (30)$$

(where the last equation is the IC if the bank uses all its collateral at the central bank when $h_j^{cb} < r^{cb}$). In that case the IC is satisfied when the bank borrows the maximum amount at the central bank given its collateral capacity constraint. The bank therefore also borrows from the private market, and the total borrowing is given by (30).

Case 2: Borrow only from the central bank using collateral 1

Suppose that $\frac{\theta_{i,1}}{h_1^{cb}} \times 1_{\left(h_1^{cb} < r^{cb}\right)} + \frac{\theta_{i,2}}{h_2^{cb}} \times 1_{\left(h_2^{cb} < r^{cb}\right)} \geq q^{CCrunch}$ and $\frac{\theta_{i,1}}{h_1^{cb}} \times 1_{\left(h_1^{cb} < r^{cb}\right)} \geq q^{CCrunch}$. In that case the IC is violated if the bank were to borrow using all its collateral at the central bank - the total investment level in (30) would be smaller than the maximum amount that could be borrowed from the central bank. The solution is for the bank to pledge all its remaining collateral in the private market against a negligible sum $\epsilon \to 0$ and for a low interest rate (so with a high haircut but a low interest rate so that the expected payoff remains r^{cb}). Since the bank only borrows with collateral 1, the investment amount solves:

$$p_i R\left(q_{i,1}^{cb}\right) - \left(H + r^{cb}\right) q_{i,1}^{cb} + \theta_i + \underbrace{(1 - p_i) \sum_{j=1}^{2} \left(r^{cb} - h_j^{cb}\right) q_{i,1}^{cb}}_{LGD_i^{cb}} = 0.$$

Case 3: Borrow only from the central bank using collateral 1 and 2

Suppose that $\frac{\theta_{i,1}}{h_1^{cb}} \times 1_{\left(h_1^{cb} < r^{cb}\right)} + \frac{\theta_{i,2}}{h_2^{cb}} \times 1_{\left(h_2^{cb} < r^{cb}\right)} \geq q^{CCrunch}$ and $\frac{\theta_{i,1}}{h_1^{cb}} \times 1_{\left(h_1^{cb} < r^{cb}\right)} \leq q^{CCrunch}$. In that case the bank pledges all its collateral 1 to the central bank and part of its collateral 2 to the central bank.

The total investment amount solves:

$$p_i R(q_i) - \left(H + r^{cb}\right) q_i + \theta_i + (1-p_i) \underbrace{\sum_{j=1}^{2} \left(r^{cb} - h_j^{cb}\right) q_i}_{LGD_i^{cb}} = 0.$$

General formulation

Note that in all cases 1, 2 and 3, the total investment amount solves

$$p_i R(q_i) - \left(H + r^{cb}\right) q_i + \theta_i + (1-p_i) LGD_i^{cb} = 0.$$

where LGD_i^{cb} takes the form of the last term in the equation of each case above.

B Lower Bound Constraint on the Policy Rate in the Symmetric Case

In this section we compute the optimal policy of the central bank when the bank is in the collateral crunch regime and the policy rate of the central bank is constrained by a lower bound, so $r^{cb} \geq 1$ (we normalize the lower bound to one for simplicity). If the spread between the economy rate and the policy rate is too large, the central bank may be unable to lower its policy rate sufficiently.

Suppose that the bank is in a collateral crunch and that the target rate in the economy is 1.01. Because of the collateral crunch, there is a spread between the economy rate and the policy rate so that the policy rate required to reach the target is 0.9. In this case the central bank is constrained by the lower bound and must set $r^{cb} = 1$. There is a positive loss L because the economy rate is different from the target rate, $r_i - r^* > 0$.

To increase investment, the central bank can use collateral policy to relax the incentive compatibility constraint. The only way to achieve this is to take on risk, i.e. to set $h_j^{cb} < r^{cb}$. The central bank's loss in case of default (or "risk") is

$$LGD^{cb} = \sum_{i=A,B} \left(r^{cb} q_i^{cb} - \sum_{j=1}^{2} h_j^{cb} q_{i,j}^{cb} \right).$$

Since the required return in the money market is r^{cb} and the bank pledges all its available collateral in the collateral crunch, the bank's IC

$$p_i R(q_i) - p_i \sum_s r_i^s q_i^s - (1-p_i) \sum_{j,s} h_{j,i}^s q_i^s \geq H q_i - \sum_{j,s} h_{j,i}^s q_i^s$$

can be rewritten as:

$$p_i R(q_i) - r^{cb} q_i + (1 - p_i) LGD_i^{cb} \geq H q_i - \theta_i \tag{31}$$

where LGD_i^{cb} is the exposure of the central bank to bank i, $LGD_i^{cb} = r^{cb} q_i^{cb} - \sum_{j=1}^{2} h_j^{cb} q_{i,j}^{cb}$.

The risk taken by the central bank, measured by LGD_i^{cb} in equation (31), has a positive impact on the bank's incentive problem because it increases its payoff if it does not shirk. Lower central bank haircuts therefore alleviate the lack of collateral faced by the bank. This increases investment and lowers the interest rate in the economy.

Proposition 5. *If C1 is violated and the economy rate is higher than the target rate when $r^{cb} = 1$, the central bank relaxes its collateral policy such that its loss given default LGD^{cb} solves*

$$\xi_{r,L,i} = -\frac{1}{\lambda}$$

where $\xi_{r,L,i}$ is the sensitivity of the economy rate in i to an increase in the LGD,

$$\xi_{r,L,i} = \frac{\partial r_i \left(LGD_i^{cb}\right)}{\partial LGD_i^{cb}}.$$

If the central bank chooses to relax the haircut on collateral j only, the optimal haircut level solves:

$$h^{cb} = \sqrt{\frac{\sum_{i=A}^{B} \theta_i}{\lambda \sum_{i=A}^{B} \xi_{r,j,i}}}$$

where

$$\xi_{r,j,i} = \frac{\partial r_i \left(h_j^{cb}\right)}{\partial h_j^{cb}}.$$

Proof. The policy rate of the central bank is at the lower bound constraint ($r^{cb} = 1$) and the interest rate in the economy is above target, $r_i > r^*$. The loss function is positive, $L > 0$. The central bank would like to lower its policy rate further but is constrained by the lower bound. It solves

$$\min_{h_1^{cb}, h_2^{cb}} L = \lambda \sum_{i=A}^{B} \left(r_i(h_1, h_2) - r^*\right) + LGD^{cb} \tag{32}$$

such that

$$r_i = p_i R'(q_i)$$

and q_i solves

$$p_i R(q_i) - r^{cb} q_i + (1 - p_i) LGD_i^{cb} = H q_i - \theta_i. \tag{33}$$

Note that the central bank is indifferent whether to reduce the haircut on collateral 1, 2 or both at the same time: the key variable is the loss given default implied by the collateral policy, LGD_i^{cb}. The first-order condition with respect to LGD_i^{cb} yields

$$\lambda \xi_{L,r,i} + 1 = 0,$$

i.e. the LGD must be such that the marginal benefit of an increase in the LGD must be equal to its cost, -1. This is a minimum since r_i is a decreasing and convex function of LGD^{cb}. Indeed, the economy rate is given by

$$r_i = p_i R'(q_i)$$

where q_i solves

$$p_i R(q_i) - r^{cb} q_i - H q_i + \theta + (1 - p_i) LGD_i^{cb} = 0.$$

We want to show that the economy rate is a convex function of LGD_i^{cb}:

$$\frac{\partial^2 (p_i R'(q_i))}{\left(\partial LGD_i^{cb}\right)^2} > 0$$

Let $q_{i,L} = \partial q_i / \partial LGD_i^{cb}$ and $q_{i,LL} = \partial q_{i,L} / \partial LGD_i^{cb}$. We have

$$\frac{\partial (p_i R'(q_i))}{\partial LGD_i^{cb}} = p_i R''(q_i) q_{i,L} > 0$$

The second order derivative is:

$$\frac{\partial^2 (p_i R'(q))}{\left(\partial LGD_i^{cb}\right)^2} = p_i \underbrace{R'''(q_i)}_{>0} \underbrace{q_{i,L}^2}_{>0} + p_i \underbrace{R''(q_i)}_{<0} \underbrace{q_{i,LL}}_{<0} > 0. \tag{34}$$

To see that $q_{i,LL} < 0$, take the implicit derivative of (33):

$$p_i R'(q_i) q_{i,L} - r^{cb} q_{i,L} + (1 - p_i) = H q_{i,L},$$

Using the same notation the second-order implicit differentiation yields:

$$p_i R''(q_i)(q_{i,L})^2 + p_i R'(q_i) q_{i,L,L} - r^{cb} q_{i,L,L} = H q_{i,L,L},$$

so that

$$q_{i,L,L} = \frac{p_i R''(q_i)(q_{i,L})^2}{H + r^{cb} - p_i R'(q_i)}$$

The denominator is always negative since $R''(q) < 0$. The numerator is always positive since the economy rate $p_i R'(q_i)$ can never be below $H + r^{cb}$ (else by increasing investment the bank's profit increases by more than the cost of funding and the moral hazard benefit H). This shows that the objective function of the central bank is indeed convex in LGD_i^{cb} so the FOC is a minimum of the Loss function.

In the special case where the central bank chooses to reduce the haircut on both collateral similarly ($h_1^{cb} = h_2^{cb} = h^{cb}$), the LGD is given by

$$\begin{aligned} LGD^{cb} &= \sum_{i=A}^{B} \left(\theta_i - q_i^{cb}\right) \\ &= \sum_{i=A}^{B} \theta_i \left(1 - \frac{1}{h^{cb}}\right) \end{aligned}$$

The FOC of (32) yields:

$$h_j^{cb} = \sqrt{\frac{\sum_{i=A}^{B} \theta_i}{\lambda \sum_{i=A}^{B} \xi_{r,i}}}. \tag{35}$$

As above and in proposition 4, one can verify that the economy rate function is a convex function of h^{cb} so that this is a minimum. □

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