NOTES D'ÉTUDES

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# TESTING FOR ZEROS IN THE SPECTRUM OF AN UNIVARIATE STATIONARY PROCESS: PART II

Renaud Lacroix

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**NER # 71**

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# Testing for zeros in the spectrum of an univariate stationary process : Part II*

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*The author acknowledges helpful comments from Stephane Gregoir.

**Corresponding address : Banque de France, 41-1391, DGEI-DEER-Centre de Recherche, 31 Rue Croix des Petits Champs, 75049 Paris, tel 01-42-92-48-32, email renaud.lacroix@banque-france.fr.

Résumé

On présente dans cet article une analyse théorique et empirique de la puissance des tests non-paramétriques de nullité de la densité spectrale en une fréquence $\theta$ introduits récemment par Lacroix (1999). Ces tests reposent sur le comportement du périodogramme pour une suite de fréquences $\theta_n$ qui converge vers $\theta$. Nous explicitons d'abord le comportement des statistiques pour une suite d'alternatives locales.

Ensuite, une analyse par méthode de Monte Carlo pour deux modèles ARMA particuliers permet, pour un échantillon de taille moyenne, d.étudier empiriquement le comportement des statistiques sous l'hypothèse nulle, ainsi que le niveau et la puissance des tests.

Abstract

It is well known that traditional inference do not apply when the spectral density of a stationary process vanishes for some frequency. This paper examines some properties of several new non parametric tests of this hypothesis which have been recently proposed by Lacroix (1999). These tests exploit the asymptotic behavior of the periodogram for some well-chosen sequence of frequencies. In particular, we investigate the power properties of the tests from both theoretical and empirical approach. We first derive the limiting properties of the tests under a sequence of local alternatives. Then, we use Monte Carlo experiments to study the size and power of the tests for two particular ARMA models. The distribution of the statistics in finite sample is also investigated.

Mots-clés : stationnarité, densité spectrale, racine moyenne « moyenne mobile », tests non-paramétriques, alternatives locales.

Keywords: stationarity; spectral density; moving average unit root; non parametric tests; local alternatives.

JEL Classication: C12, C14, C22

# 1 Introduction

Let $(X_t)_{t\in\mathbb{Z}}$ be a stationary process with autocovariance function $\gamma(h)$ and spectral density $f(\omega) = \frac{1}{2\pi}\sum_{k=-\infty}^{+\infty}\gamma(k)e^{-ik\omega}$. We suppose a sample of size $n$ is available to the econometrician. We consider in this paper the following hypothesis:

$$\mathbf{H}_0 : f(\theta) = 0, \mathbf{H}_a : f(\theta) > 0$$

where $\theta$ is a fixed frequency. The motivation for this problem is not purely theoretical. Indeed, it is well-known now that seasonal adjustment procedures, including X11-Arima and SEATS can induce spurious moving-averages unit root in seasonally adjusted series (see Maravall (1995)). But it is important to note that any macroeconomic aggregate is obtained by summation of seasonal adjusted series, and then is <u>not</u> the direct result of some seasonal adjustment procedure. It means that, although disaggregated series may have some moving average unit roots, it is not clear whether they are still present after aggregation.. So, in the case of monthly data, testing lack of power at the seasonal frequencies $\pi\backslash 6$ , $\pi\backslash 3$ , $\pi\backslash 2$ , $2\pi\backslash 3$ , $5\pi\backslash 6$ and $\pi$ is an important issue prior to econometric modelling.

We study in this paper the performance of non parametric tests of $\mathbf{H}_0$ which have been proposed by Lacroix (1999) under some classical regularity assumptions. The paper is organized as follows. Section 2 recalls the test statistics used in this exercise and their associated asymptotic distribution. Section 3 examines the power of the tests against sequences of local alternatives. Section 4 is devoted to small sample distribution of the tests under $\mathbf{H}_0$ through a Monte Carlo experiment. In section 5 the empirical power of the tests is investigated through a limited Monte Carlo experiment. A brief application to French GDP is presented in section 6. An appendix collects technical details.

We turn now to the statistical framework used throughout the paper. Though our theoretical results do not suppose that $f$ is indefinitely differentiable, we will only consider the special case where $X_t$ follows an ARMA model. Then, under the null hypothesis, we recall that $X_t$ can be expressed as:

$$\begin{aligned} X_t &= \left(1 - 2\cos\theta B + B^2\right) u_t \text{ if } \theta \neq 0\,[\pi] \\ &= \left(1 - \cos\theta B\right) u_t \text{ if } \theta = 0\,[\pi] \end{aligned}$$

and $u_t$ follows also an ARMA model. We write the Wold representation of $X_t$ as $X_t = C_X(B)\,\varepsilon_t$ where the rational complex function $C_X(z)$ can be factorized as:

$$C_X(z) = -\left(1 - ze^{i\theta}\right) C_X(\theta, z) = \left(1 - ze^{i\theta}\right)\left(1 - ze^{-i\theta}\right) D(\theta, z) \tag{1}$$

$D(\theta, z)$ is a polynomial. It should be noted that the hypothesis to be tested is in fact more restrictive:

$$\mathbf{H}_0 : f(\theta) = 0, f_u(\theta) \neq 0 \tag{2}$$

In other words, we test for a single moving-average unit root for $X_t$. We denote by $\theta_n$ a sequence converging to $\theta$, written as:

$$\theta_n = \theta + \frac{1}{e(n)} \text{ with } e(n) \to +\infty$$

We precise now some notations. For any discrete-time variable $(Z_t)_{t\in\mathbb{Z}}$, we denote by $J_Z(\omega)$ its finite Fourier transform with $n$ observations:

$$J_Z(\omega) = \frac{1}{\sqrt{n}} \sum_{k=1}^{n} Z_k e^{-ik\omega}, \ \omega \in [-\pi, \pi]$$

and $I_Z(\omega)$ its periodogram:

$$I_Z(\omega) = |J_Z(\omega)|^2$$

If necessary, we add a supercript to indicate precisely the range of values used. For instance:

$$J_Z^m(\omega) = \frac{1}{\sqrt{m}} \sum_{k=1}^{m} Z_k e^{-ik\omega}$$

$\mathbb{W}_c(t)$ will denote a complex Brownian motion defined on $[0,1]$. The sign $\Rightarrow$ always means "convergence in law when $n$ goes to infinity". Let $\Delta_n(u) = \sum_{k=0}^{n-1} e^{iku}$ be the Dirichlet kernel,; it satisfies $\int_{-\pi}^{\pi} |\Delta_n(x)|^2 dx = n$ and:

$$|x\Delta_n(x)| \leq 2 \text{ for } 0 \leq |x| < \pi \tag{3}$$

# 2 The test statistics

In this part, the reader is referred to Lacroix (1999) for details and properties of the statistics listed below. We keep the same notations as in this paper. First:

$$\xi_n^p = \frac{1}{K(\theta)} \times \frac{e^2(n)\mathbb{I}_X(\theta_n)}{\widehat{f_u(\theta)}}. \tag{4}$$

where

$$\widehat{f_u(\theta)} = \frac{1}{2\pi} \times \frac{1}{2m+1} \sum_{j=-m}^{m} \mathbb{I}_u^*\left(\theta + \frac{2\pi j}{n}\right), m = o(n) \tag{5}$$

$\mathbb{I}_u^*(\omega)$ is an estimate of the periodogram of $(u_t)$ ,and:

$$K(\theta) = 4\pi \sin^2(\theta) \ \text{ if } \theta \in ]0,\pi[\,, \ K(\pi) = K(0) = \pi$$

With $e(n) = \sqrt{n}$ we get the statistic:

$$\xi_n^{p*} = \frac{1}{K(\theta)} \times \frac{e^2(n)\left|\mathbb{J}_X^*(\theta_n)\right|^2}{\widehat{f_u(\theta)}}$$

where

$$\mathbb{J}_X^*(\theta_n) = \mathbb{J}_X(\theta_n) - e^{in(\theta-\theta_n)}\mathbb{J}_X(\theta) - \frac{1}{\sqrt{n}}\left[e^{in(\theta-\theta_n)} - 1\right] \times \widehat{u_{0,n}}$$

With $e(n) = n$ we get the statistic:

$$\xi_n^{s**} = \frac{1}{K^*(\theta)} \times \frac{n^2\left|\mathbb{J}_X\left(\theta + \frac{2\pi}{n}\right) - \mathbb{J}_X\left(\theta - \frac{2\pi}{n}\right)\right|^2}{\left[\widehat{f_X(\theta)} + \widehat{f_u(\theta)}^{-1}\right]^{-1}}$$

where $K^*(\theta) = 4\pi^2 K(\theta)$

If $\widetilde{\theta}_n = \theta + \frac{1}{\widetilde{e}(n)}$ is another sequence converging to $\theta$ then let $\xi_n^r$ be the ratio:

$$\xi_n^r = \left|\frac{e(n)}{\widetilde{e}(n)}\right| \sqrt{\frac{\mathbb{I}_X(\theta_n)}{\mathbb{I}_X\left(\widetilde{\theta}_n\right)}} \tag{6}$$

Lastly

$$\xi_n^{r*} = \frac{\sqrt{n}}{2\pi} \frac{\left|\mathbb{J}_X\left(\theta + \frac{2\pi}{n}\right) - \mathbb{J}_X\left(\theta - \frac{2\pi}{n}\right)\right|}{\left|\mathbb{J}_X^*(\theta)\right|}$$

with

$$\mathbb{J}_X^*(\theta) = \mathbb{J}_X\left(\theta + \frac{\pi}{\sqrt{n}}\right) - e^{-i\pi\sqrt{n}}\mathbb{J}_X(\theta) - \frac{1}{\sqrt{n}}\left[e^{-i\pi\sqrt{n}} - 1\right] \times \widehat{u_{0,n}}$$

$\widehat{u_{0,n}}$ is a least square estimation of the random variable $u_0$ (see Lacroix (1999)). The limiting properties of these statistics are recalled in the next theorem, which is stated here without proof. We denote by $\mathbf{L}_p, \mathbf{L}_s, \ldots$ some limit laws which depend upon nuisance parameters, and that we do not need to specify in this paper.

**Theorem 1** *i) If* $e(n) = o(\sqrt{n})$, *then* $\xi_n^p \Rightarrow \chi_2(2)$ *under* $\mathbf{H}_0$, *and* $\left(\frac{n}{e(n)}\right)^2 \times \xi_n^p \Rightarrow \mathbf{L}_p$ *under* $\mathbf{H}_a$.
*ii)* $\xi_n^{p*} \Rightarrow \chi_2(2)$ *under* $\mathbf{H}_0$, *and* $\xi_n^{p*} = O_p\left(\frac{1}{n}\right)$ *under* $\mathbf{H}_a$.
*iii)* $\xi_n^{s**} \Rightarrow \chi_2(2)$ *under* $\mathbf{H}_0$, *and* $\frac{1}{n^2} \times \xi_n^{s**} \Rightarrow \mathbf{L}_{s*}$ *under* $\mathbf{H}_a$.
*iv) If* $e(n) = o(\sqrt{n})$, $\widetilde{e}(n) = o(\sqrt{n})$ *and* $\widetilde{e}(n) = o(e(n))$ *then* $\xi_n^r \Rightarrow \sqrt{F_{2,2}}$ *under* $\mathbf{H}_0$, *and* $\left|\frac{\widetilde{e}(n)}{e(n)}\right| \times \xi_n^r \Rightarrow \mathbf{L}_r$ *under* $\mathbf{H}_a$.
*v)* $\xi_n^{r*} \Rightarrow \sqrt{F_{2,2}}$ *under* $\mathbf{H}_0$, *and* $\frac{1}{\sqrt{n}} \times \xi_n^{r*} \Rightarrow \mathbf{L}_r$ *under* $\mathbf{H}_a$.

# 3 Power of the test

Under the null hypothesis, we have seen that $X_t$ can be expressed as:

$$\begin{array}{l} X_t = \left(1 - 2\cos\theta B + B^2\right) u_t \text{ if } \theta \neq \pi \\ \text{or } X_t = (1 + B)\, u_t \text{ if } \theta = \pi \end{array} \quad \text{with } f_u(\theta) > 0 \tag{7}$$

We consider the sequence of local alternatives $\mathbf{H}_a^{loc}$ defined by the double indexed process:

$$\begin{array}{l} X_{t,n} = \left[1 - 2\left(1 + \frac{a}{n^\beta}\right)\cos\theta B + \left(1 + \frac{2a}{n^\beta}\right)B^2\right] u_t \text{ if } \theta \neq \pi \\ X_{t,n} = \left[1 + \left(1 + \frac{a}{n^\beta}\right)B\right] u_t \text{ if } \theta = \pi \end{array} \tag{8}$$

for $t = 1, 2, \ldots n$, $\beta > 0$ and $a$ fixed. The parameter "$a$" summarizes the closeness between $X_{t,n}$ and $X_t$. We prioritize in this part the statistics $\xi_n^p$. So, we suppose that $e(n) = \frac{n^\zeta}{C}$ for some $\zeta < \frac{1}{2}$ and $C$ constant. The following technical lemma gives the behavior of the periodogram under the local alternatives under consideration. Let $\mathbb{J}_{\overline{X},n}(\omega) = \frac{1}{\sqrt{n}} \sum_{k=1}^n e^{-ik\omega} X_{k,n}$.

**Lemma 2** *i)* $\underline{\textit{If } \theta \neq \pi:}$
*If* $\beta > \zeta$, $e^2(n)\mathbb{I}_{\overline{X}}(\theta_n) \Rightarrow 4\pi f_u(\theta)\sin^2\theta\chi_2(2)$
*If* $\beta = \zeta$, $e^2(n)\mathbb{I}_{\overline{X}}(\theta_n) \Rightarrow 4\pi f_u(\theta)\sin^2(\theta/2)\left[\left(\sin\theta + \frac{2a}{C}\right)^2 + 4\cos^4(\theta/2)\right]\chi_2(2)$
*If* $\beta < \zeta$, $n^{2(\beta-\zeta)}e^2(n)\mathbb{I}_{\overline{X}}(\theta_n) \Rightarrow 16\pi a^2 f_u(\theta)\frac{\sin^2(\theta/2)}{C^2}\chi_2(2)$
*ii)* $\underline{\textit{If } \theta = \pi:}$
*If* $\beta > \zeta$, $e^2(n)\mathbb{I}_{\overline{X}}(\theta_n) \Rightarrow \pi f_u(\theta)\chi_2(2)$
*If* $\beta = \zeta$, $e^2(n)\mathbb{I}_{\overline{X}}(\theta_n) \Rightarrow \pi f_u(\theta)\left[\left(\frac{a}{C}\right)^2 + 1\right]\chi_2(2)$
*If* $\beta < \zeta$, $n^{2(\beta-\zeta)}e^2(n)\mathbb{I}_{\overline{X}}(\theta_n) \Rightarrow \pi f_u(\theta)\frac{a^2}{C^2}\chi_2(2)$

**Proof**: see the appendix.
■

We turn now to the denominator of $\xi_n^p$.
Let $Z_t$ be a non-stationary process such as [1]:

$$u_t = \left(1 - 2\cos\theta B + B^2\right) Z_t \text{ if } t > 0 \text{ and } Z_t = 0 \text{ if } t \leq 0$$

We calculate the estimator $\widehat{f_u(\theta)}$ from:

$$\overline{u_{t,n}}^* = S_t[X_{t,n};\theta] = u_t + u_0^* e^{it\theta} + u_{-1}^* e^{-it\theta} + \frac{2a}{n^\beta}(Z_{t-2} - \cos\theta Z_{t-1})$$

$u_0^*$ and $u_{-1}^*$ are new "initial conditions" which are derived from $u_0$ and $u_{-1}$. We define:

$$\overline{u_{t,n}} = \overline{u_{t,n}}^* - u_0^* e^{it\theta} - u_{-1}^* e^{-it\theta}$$

$$\mathbb{J}_{\overline{u}}(\omega) = \frac{1}{\sqrt{n}}\sum_{k=1}^{n} e^{-ik\omega}\overline{u}_{k,n}, \mathbb{J}_{\overline{u}^*}(\omega) = \frac{1}{\sqrt{n}}\sum_{k=1}^{n} e^{-ik\omega}\overline{u}_{k,n}^*$$

The filtering procedure for deterministic terms yields the modified Fourier transform:

$$\mathbb{J}_{\overline{u}^*}^*(\omega_j) = \mathbb{J}_{\overline{u}}(\omega_j) - \frac{1}{n}\left(1 - \frac{|\Delta_n(2\theta)|^2}{n^2}\right)^{-1} \times \Delta_n\left(-2\theta - \frac{2\pi j}{n}\right) \times \left\{\overline{\mathbb{J}_{\overline{u}}(\theta)} - \frac{\Delta_n(2\theta)}{n}\mathbb{J}_{\overline{u}}(\theta)\right\}$$

It is clear that:

$$\mathbb{J}_{\overline{u}}(\omega) = \mathbb{J}_u(\omega) + \frac{2a}{n^\beta}\left[\left(e^{-2i\omega} - \cos\theta e^{-i\omega}\right)\mathbb{J}_Z(\omega) - e^{-i\omega n}\cos\theta\frac{Z_{n-1}}{\sqrt{n}}\right] \tag{9}$$

Now, if $\omega = \theta$, we know that:

$$\frac{\sqrt{2}\sin\theta}{n}\mathbb{J}_Z(\theta) = \frac{ie^{i\theta}}{2n}\sum_{k=1}^{n}\mathbb{W}_n\left(-\theta, \frac{t}{n}\right) + o_p(1) \Rightarrow \frac{ie^{i\theta}}{2}\int_0^1 \mathbb{W}_c(r)dr$$

[1] $Z_t$ is obtained explicitely by the recurrence : $Z_1 = u_1$, $Z_2 = u_2 + 2\cos\theta Z_1$, $Z_3 = u_3 + 2\cos\theta Z_2 - Z_1$ and so on.

whereas $\mathbb{J}_u(\theta) = \mathbb{W}_n(-\theta, 1) \Rightarrow \mathbb{W}_c(1)$ and $\frac{Z_{n-1}}{\sqrt{n}} = O_p(1)$.
From (9), for $\beta \leq 1$, $\mathbb{J}_{\overline{u}}(\theta) = O_p\left(n^{1-\beta}\right)$, and for $\beta > 1$, $\mathbb{J}_{\overline{u}}(\theta) = O_p(1)$.
Suppose now $\beta \leq 1$. Then, if $j \neq 0$, then we get:

$$\mathbb{J}^*_{\overline{u}^*}(\omega_j) = \mathbb{J}_{\overline{u}}(\omega_j) + O_p\left(\frac{1}{n^\beta}\right) \text{ uniformly in } j \tag{10}$$

Remember that, for $j = 0$ :

$$\begin{aligned} \mathbb{J}^*_{\overline{u}^*}(\theta) &= \frac{\mathbb{J}_{\overline{u}^*}(\theta)}{n^\alpha} = \frac{1}{n^\alpha} \times \left[\mathbb{J}_{\overline{u}}(\theta) + \frac{u_0^*}{\sqrt{n}}\Delta_{n-1}(\theta) + \frac{u_{-1}^*}{\sqrt{n}}\left(\Delta_{n-1}(\theta) - 1 - e^{-i\theta}\right)\right] \\ &= \frac{\mathbb{J}_{\overline{u}}(\theta)}{n^\alpha} + g_n \text{ with } \mathbb{E}\left|g_n^2\right| = O\left(\tfrac{1}{n}\right) \end{aligned}$$

From these considerations, we have:

$$\widehat{f_{\overline{u}^*}(\theta)} = \widehat{f_{\overline{u}}(\theta)} + O_p\left(\frac{1}{n^\beta}\right) \tag{11}$$

Now, (9) yields:

$$\begin{aligned} \mathbb{J}_{\overline{u}}(\omega_j) &= \mathbb{J}_u(\omega_j) + \tfrac{2a}{n^\beta}\left[\left(e^{-2i\omega_j} - \cos\theta e^{-i\omega_j}\right)\mathbb{J}_Z(\omega_j) - e^{-i\omega_j n}\cos\theta\tfrac{Z_{n-1}}{\sqrt{n}}\right] \\ \tfrac{\mathbb{J}_{\overline{u}}(\omega_j)}{n^{1-\beta}} &= O_p\left(\tfrac{1}{n^{1-\beta}}\right) + 2a\left(e^{-2i\omega_j} - \cos\theta e^{-i\omega_j}\right)\tfrac{\mathbb{J}_Z(\omega_j)}{n} + O_p\left(\tfrac{1}{\sqrt{n}}\right) \\ &= O_p\left(\tfrac{1}{n^{1-\beta}} + \tfrac{1}{\sqrt{n}}\right) + 2a\left(e^{-2i\theta} - \cos\theta e^{-i\theta}\right)\tfrac{\mathbb{J}_Z(\omega_j)}{n} + O\left(\tfrac{j}{n}\right)\tfrac{\mathbb{J}_Z(\omega_j)}{n} \end{aligned}$$

But $O\left(\frac{j}{n}\right)\frac{\mathbb{J}_Z(\omega_j)}{n} = O\left(\frac{m}{n}\right) \times O_p(1) = O_p\left(\frac{m}{n}\right)$. Let $C = 2a\left(e^{-2i\theta} - \cos\theta e^{-i\theta}\right)$.
$\frac{\mathbb{J}_{\overline{u}}(\omega_j)}{n^{1-\beta}} = O_p\left(\frac{1}{n^{1-\beta}} + \frac{1}{\sqrt{n}} + \frac{m}{n}\right) + C\frac{\mathbb{J}_Z(\omega_j)}{n}$, and, for $\beta < 1$:
$\frac{\mathbb{I}_{\overline{u}}(\omega_j)}{n^{2-2\beta}} = o_p(1) + |C|^2\frac{\mathbb{I}_Z(\omega_j)}{n^2}$. Hence, from the definition of the spectral estimator:

$$\frac{\widehat{f_{\overline{u}}(\theta)}}{n^{2-2\beta}} = o_p(1) + |C|^2\frac{\widehat{f_Z(\theta)}}{n^2}$$

Under the assumptions of theorem 12 of Lacroix (1999), $\frac{\widehat{f_Z(\theta)}}{n^2}$ converges in law to $C^{ste} \times \int_0^1 |\mathbb{W}_c(t)|^2 dt$; $\widehat{f_{\overline{u}}(\theta)}$ diverge with rate $n^{2-2\beta}$. It follows from lemma 2, (11) and the continuous mapping theorem that:

$$\begin{aligned} &\textit{If } \zeta \leq \beta < 1,\ e^2(n)\frac{\mathbb{I}_{\overline{X}}(\theta_n)}{\widehat{f_{\overline{u}^*}(\theta)}} = O_p\left(\tfrac{1}{n^{2-2\beta}}\right) \\ &\textit{If } \beta < \zeta,\ e^2(n)\frac{\mathbb{I}_{\overline{X}}(\theta_n)}{\widehat{f_{\overline{u}^*}(\theta)}} = O_p\left(\tfrac{1}{n^{2-2\zeta}}\right) \end{aligned} \tag{12}$$

(12) shows that at least asymptotically, we don't make false decision for alternatives of the form $n^{-\beta}$ with $\beta < 1$.
We turn now to the case $\beta = 1$. Now $\mathbb{J}_{\overline{u}}(\theta)$ verify:

$$\sqrt{2}\mathbb{J}_{\overline{u}}(\theta) \Rightarrow \mathbb{W}_c(1) + \frac{\left(e^{-2i\theta} - \cos\theta e^{-i\theta}\right)}{\sqrt{2}\sin\theta}a\int_0^1 \mathbb{W}_c(r)dr \tag{13}$$

This result seems to indicate that $\widehat{f_{\overline{u}^*}(\theta)}$ converges, which is actually true.

**Lemma 3** *$\widehat{f_{\overline{u}^*}(\theta)}$ converges in probability to $f_u(\theta)$.*

**Proof**: see the appendix.

$\blacksquare$.

From lemmas (2) and (3) we obtain immediately:

**Theorem 4** *Under $\mathbf{H}_a^{loc}$, $\xi_n^p$ has the same limit law than under $\mathbf{H}_0$.*

The same method can now be applied to the statistic $\xi_n^{s**}$ for $\theta < \pi$, so we omit details. From lemma 3:

$$\left[\widehat{f_{\overline{X}^*}(\theta)} + \widehat{f_{\overline{u}^*}(\theta)}^{-1}\right]^{-1} \to f_u(\theta)$$

in probability under $\mathbf{H}_a^{loc}$, whereas (we note $\theta_{n,t} = \theta + \frac{t}{n}, t \in [-\pi, \pi]$):

$$n\mathbb{J}_{\overline{X},n}\left(\theta_{n,t}\right) = n\mathbb{J}_{X,n}\left(\theta_{n,t}\right) + \frac{2ae^{-i\theta_{n,t}}}{Cn^{\beta-1}}\left[\left(e^{-i\theta_{n,t}} - 1\right)\mathbb{J}_{u,n-2}\left(\theta_{n,t}\right) + O_p\left(\frac{1}{\sqrt{n}}\right)\right]$$

If $\beta > 1$, then $n\mathbb{J}_{\overline{X},n}\left(\theta_{n,t}\right) = n\mathbb{J}_{X,n}\left(\theta_{n,t}\right) + o_p\left(1\right)$, the terms $o_p$ is uniform in $t$.
If $\beta = 1$: from the proof of theorem 8, and theorem 17 of Lacroix (1999), we have the convergence in $C\left[-\pi, \pi\right]$:

$$\sqrt{2}\begin{pmatrix} n\mathbb{J}_X\left(\theta_{n,t}\right) \\ \mathbb{J}_{u,n-2}\left(\theta_{n,t}\right) \end{pmatrix} \Rightarrow \begin{pmatrix} i\sigma C\left(\theta, e^{-i\theta}\right)\mathbb{V}(t) \\ \sigma D\left(\theta, e^{-i\theta}\right)\mathbb{V}(t) \end{pmatrix}$$

It yields $\sqrt{2}n\mathbb{J}_{\overline{X},n}\left(\theta_{n,t}\right) \Rightarrow \sigma\left[iC\left(\theta, e^{-i\theta}\right) + \frac{2ae^{-i\theta}}{C}D\left(\theta, e^{-i\theta}\right)\left(e^{-i\theta} - 1\right)\right]\mathbb{V}(t)$
or $\sqrt{2}n\mathbb{J}_{\overline{X},n}\left(\theta_{n,t}\right) \Rightarrow \sigma\left[i\left(1 + e^{-i\theta}\right) - \frac{2ae^{-i\theta}}{C}\right]\left(1 - e^{-i\theta}\right)D\left(\theta, e^{-i\theta}\right)\mathbb{V}(t)$
Then we get, using the normality of $\mathbb{V}(t)$ :

$$\xi_n^{s**} \Rightarrow 16\pi^3 \underbrace{\sin^2\left(\theta/2\right)\left[\left(\sin\theta + \frac{2a}{C}\right)^2 + 4\cos^4\left(\theta/2\right)\right]}_{l(\theta)}\chi_2\left(2\right) \tag{14}$$

If $\beta < 1$, then $\xi_n^{s**}$ diverge with rate $n^{2-2\beta}$.

The power of $\xi_n^p$ is very low for alternatives which converge with rate $1/n$. Indeed, if $W_n\left(\alpha\right) = \left\{\xi_n^p < c_\alpha\right\}$, $\lim\limits_{n\to\infty} P_{H_a^{loc}}\left(W_n\left(\alpha\right)\right) = \alpha$ independently of $a$.

Now, for the test to be consistent against such local alternatives, the following condition should be satisfied:

$$\lim_{a\to\infty}\left(\lim_{n\to\infty} P_{H_a^{loc}}\left(W_n\left(\alpha\right)\right)\right) = 1 \tag{15}$$

(15) is not satisfied by $\xi_n^p$, but is fulfilled by $\xi_n^{s**}$ as it can be seen from (14). Note also that $l\left(\theta\right)$ is an increasing function on $[0, \pi]$ for $a > c$, which means that the power function converges faster to one when $\theta$ comes close to $\pi$ from above.

Let us consider now the statistic $\xi_n^r$ with $e(n) = \frac{n^\zeta}{C}$ ($\varsigma < \frac{1}{2}$) and $\widetilde{e(n)} = \frac{\log(n)}{\widetilde{C}}$, $C$ and $\widetilde{C}$ constant. It is easily seen that under the sequence of local alternatives (8), the test is consistent when $\beta \leq \zeta$. Indeed:

1. <u>If $\beta < \zeta$:</u> $\xi_n^r$ diverges with rate $n^{\zeta-\beta}$.

2. <u>If $\beta = \zeta$ and $\theta \neq \pi$</u>
$e^2(n)\mathbb{I}_{\overline{X}}(\theta_n) \Rightarrow 8\pi f_u(\theta)\sin^2(\theta/2)\left[\left(\sin\theta + \frac{2a}{C}\right)^2 + 4\cos^4(\theta/2)\right]|\mathbb{W}_c(1)|^2$
$\widetilde{e}^2(n)\mathbb{I}_{\overline{X}}\left(\widetilde{\theta}_n\right) \Rightarrow 8\pi f_u(\theta)\sin^2\theta\left|\widetilde{\mathbb{W}}_c(1)\right|^2$, and the convergence for the couple yields:
$$\xi_n^r \Rightarrow \frac{1}{2\cos(\theta/2)}\sqrt{\left(\sin\theta + \frac{2a}{C}\right)^2 + 4\cos^4(\theta/2)}\sqrt{F_{2,2}} \tag{16}$$
$\beta(\alpha, a) = \lim\limits_{n\to\infty} P_{H_a^{loc}}[W_n(\alpha)]$ satisfies the condition $\lim\limits_{a\to\infty}\beta(\alpha, a) = 1$.

3. <u>If $\beta = \zeta$ and $\theta = \pi$:</u> we obtain likewise:
$$\xi_n^r \Rightarrow \sqrt{1 + \left(\frac{a}{C}\right)^2}\sqrt{F_{2,2}} \tag{17}$$
and $\lim\limits_{a\to\infty}\beta(\alpha, a) = 1$.

# 4 Small sample properties of the statistics under $\mathbf{H}_0$

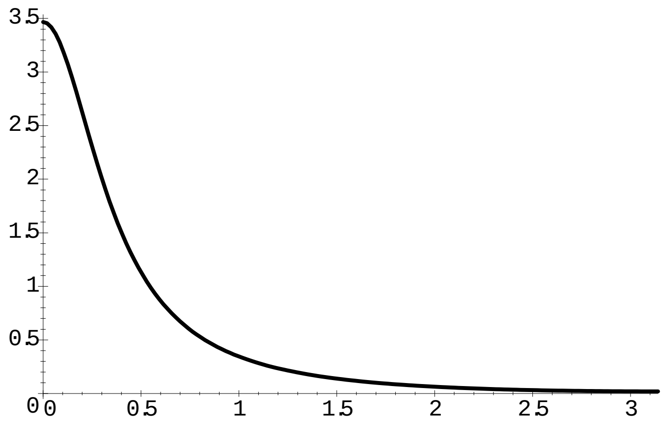
We consider two distinct processes for simulation purpose:

Model 1: $ARMA(1,1): X_{1t} = (1 - 0.7B)^{-1}(1 + 0.4B)\varepsilon_t$,
Model 2: $ARMA(2,1): X_{2t} = (1 - 0.7B + 0.49B^2)^{-1}(1 - 0.7B)\varepsilon_t$

The $\varepsilon_t$ are *iid* $T(5)$, Student law with 5 degrees of freedom[2]. The case of $ARCH$ or $GARCH$ models for $\varepsilon_t$ is left for future experiment (we recall that our tests are not asymptotically affected by conditional heteroskedasticity, see Lacroix (1999)). The

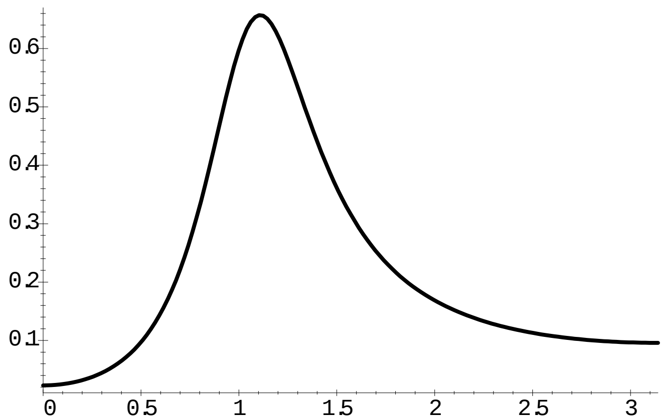
[2] We need at least five degrees of freedom to insure that moments of order 4 are finite. This hypothesis is sufficient for the convergence of our statistics.

spectrum of model 1 is concentrated at law frequencies...




$$f_1(\omega) = \frac{\sigma^2}{2\pi}\frac{1+2\times 0.4\times\cos(\omega)+0.4^2}{1-2\times 0.7\times\cos(\omega)+0.7^2}$$

...whereas the spectrum of model 2 has a peak around the frequency $\theta_0 = \pi \backslash 3$ :




$$f_2(\omega) = \frac{\sigma^2}{2\pi}\frac{\left|1-0.7e^{-i\omega}\right|^2}{\left|1-2\times 0.7\cos\left(\frac{\pi}{3}\right)e^{-i\omega}+0.7^2e^{-2i\omega}\right|^2}$$

We consider for a given value of $\theta$ the seasonal-differenced process $Y_{it}$:

$$Y_{it}(\theta) = \left(1 - 2\cos\theta B + B^2\right) X_{it}, i = 1, 2$$

The first question we have to deal with is: under $\mathbf{H}_0$, how accurate is the approximation of the finite sample distribution of our statistics by their asymptotic counterpart? To this end, we replicate 2000 samples of $n = 150$ points for both models. This is typically the sample size available for most macroeconometric models. Secondly, we want to investigate the robustness of the finite sample distribution to the presence of a seasonal intercept in the model, that is:

$$\widetilde{Y}_{it}(\theta,\nu) = a\cos(\nu t) + b\sin(\nu t) + Y_{1t}(\theta)$$

It is shown in Lacroix (1999) that, provided a preliminary regression of $\widetilde{Y}_{1t}$ on the variables $(\cos(\nu t), \sin(\nu t))'$ has been performed, the limit laws of the tests statistics are not asymptotically affected by the replacement of $Y_{1t}$ by the residuals of this regression. It is then interesting to examine whether this result is still valid in finite sample. For the simulation, we take $\nu = \theta$, and $(a,b) = (1,2)$.
In order to avoid edge-effects in the estimation of densities for a positive variable, we simulate under $\mathbf{H}_0$ the distribution of the following variables ($\Phi$ is the c.d.f of $\mathrm{N}(0,1)$):

$$\begin{aligned} \widetilde{\xi}_n^{p} &= \Phi^{-1}\left[1-\exp\left(-0.5\xi_n^{p}\right)\right] \\ \widetilde{\xi}_n^{s**} &= \Phi^{-1}\left[1-\exp\left(-0.5\xi_n^{s**}\right)\right] \\ \widetilde{\xi}_n^{r*} &= \Phi^{-1}\left[\frac{1}{1+(\xi_n^{r*})^2}\right] \end{aligned}$$

Note that the function which maps $\xi_n^p$ and $\xi_n^{s**}$ on to $\widetilde{\xi}_n^p$ and $\widetilde{\xi}_n^{s**}$ is strictly increasing, while it is strictly decreasing for $\xi_n^{r*}$. Asymptotically, $\widetilde{\xi}_n^{s**} \Rightarrow \mathrm{N}(0,1)$. The same task is achieved for $\widetilde{\xi}_n^p$. Moreover, for $\xi_n^p$ and $\xi_n^{s**}$ , we also draw the distribution of the nuisance parameter estimate $f_u(\theta)$, as well as the distribution of the statistic which uses the true value of $f_u(\theta)$ instead of $\widehat{f_u(\theta)}$: these "pseudo-estimators" are labelled "numerator" in the following figures. The distribution of $\widehat{f_u(\theta)} = \left[\widehat{f_X(\theta)} + \widehat{f_u(\theta)}^{-1}\right]^{-1}$ is also shown for $\xi_n^{s**}$.We begin with time series without seasonal intercepts.

## 4.1 Model 1, low frequency

In all graphics, the thin line represents the density of the standard normal distribution.

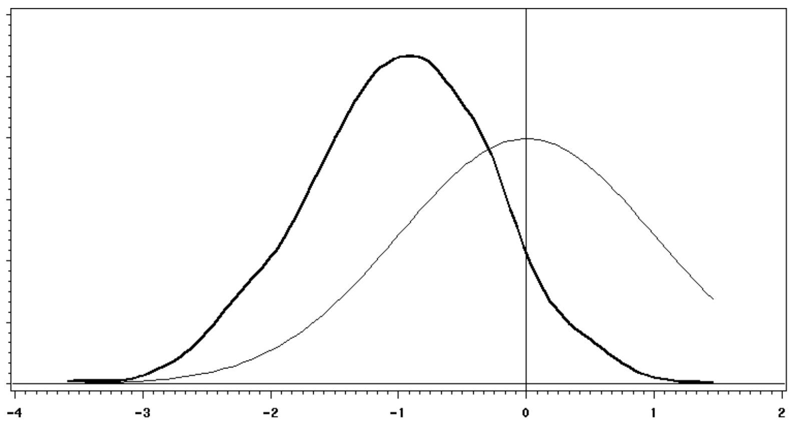



Fig. 1. $\theta = \frac{\pi}{6}, \widetilde{\xi}_n^p$

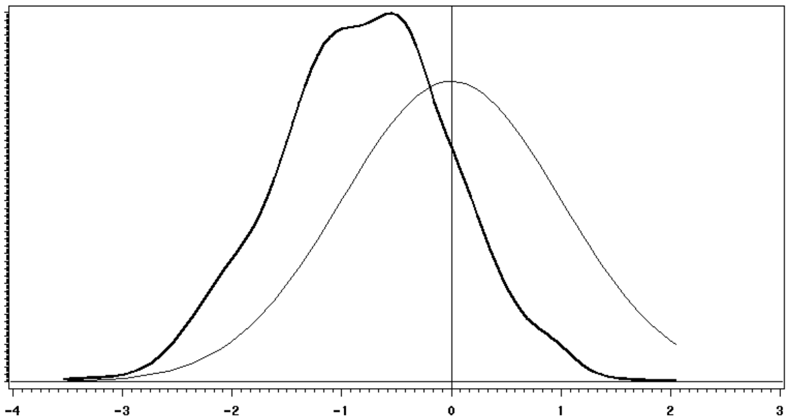



Fig. 2. $\theta = \frac{\pi}{6}$,Numerator $\widetilde{\xi}_n^p$

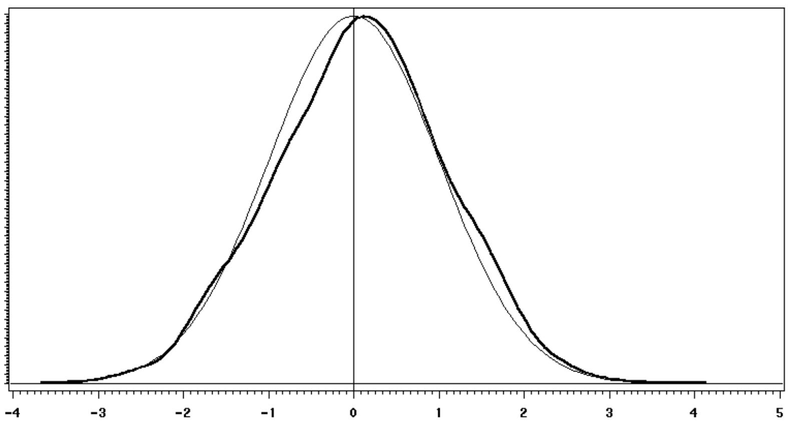



Fig. 3. $\theta = \frac{\pi}{6}$, $\widetilde{\xi}_n^{s**}$

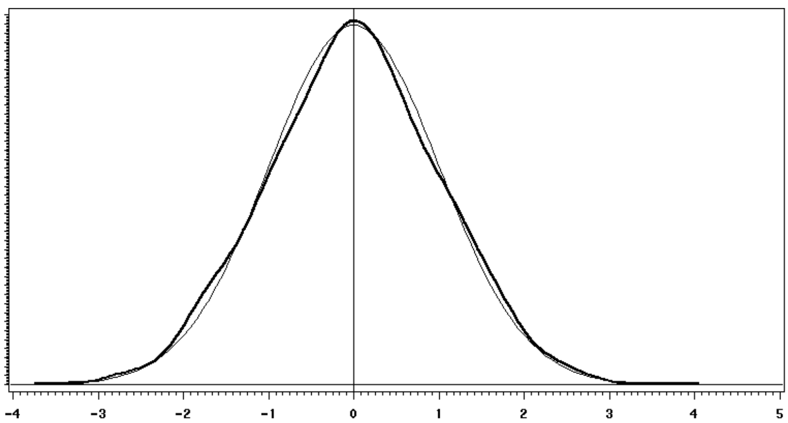



Fig. 4. $\theta = \frac{\pi}{6}$,Numerator $\widetilde{\xi}_n^{s**}$

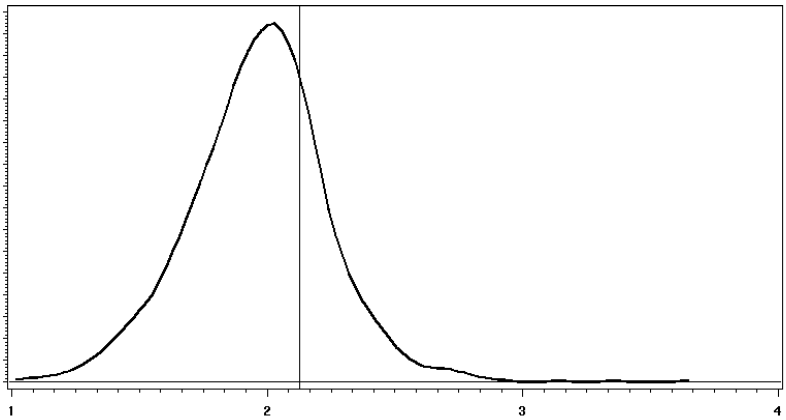



Fig. 5. Nuisance parameter (the vertical line refers to the true value of $f_u(\theta)$)

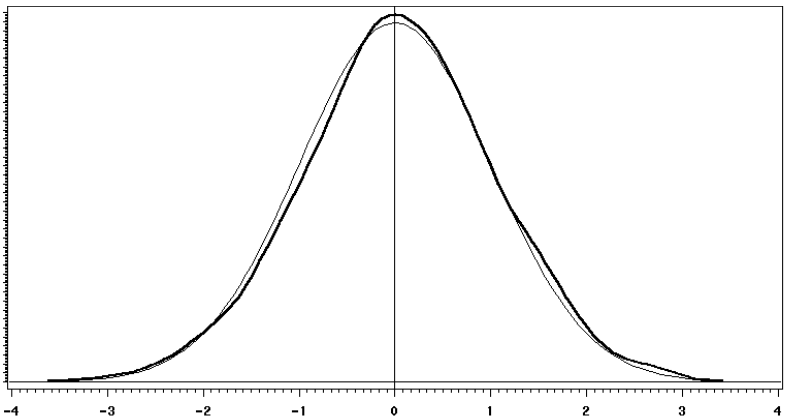



Fig. 6. $\theta = \frac{\pi}{6}$, $\widetilde{\xi}_n^{r*}$

It is clear that serious distortion are present for $\xi_n^p$, which can't be attributed to the poor quality of the nuisance parameter estimator (see figure 1 and 2). These distortions are quite reduced with $\xi_n^{s**}$ (recall that convergence to $\theta$ is assumed at the faster rate $n^{-1}$ for this test). $\xi_n^{r*}$, which is by construction free of nuisance parameters does not indicate size distortions due to finite sample properties.

## 4.2 Model 2, low frequency

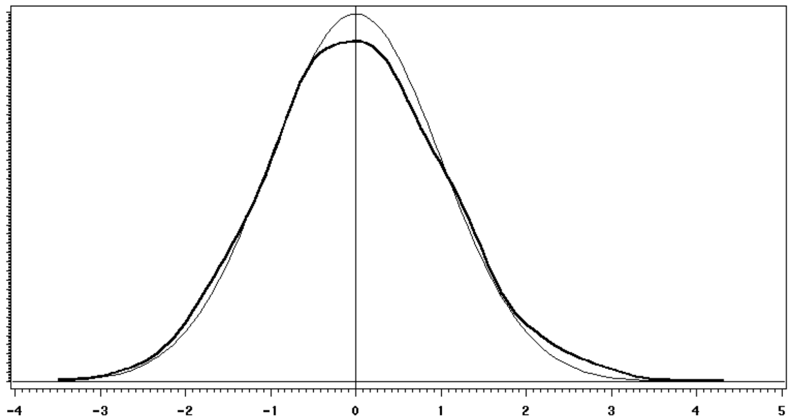



Fig 7. $\theta = \frac{\pi}{6}, \widetilde{\xi}_n^p$

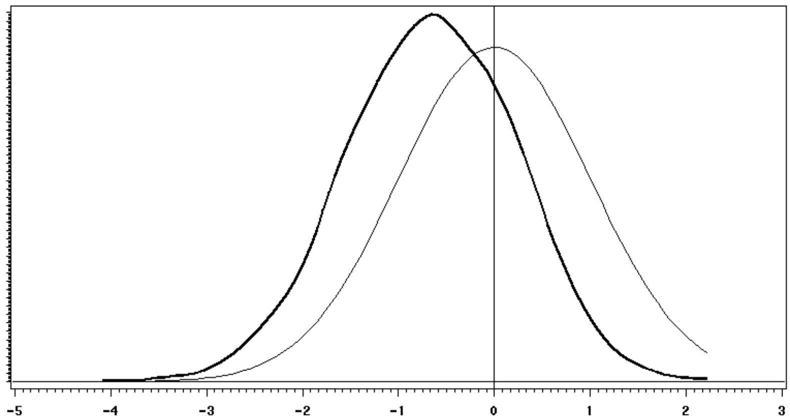



Fig 8. $\theta = \frac{\pi}{6}, \widetilde{\xi}_n^{s**}$

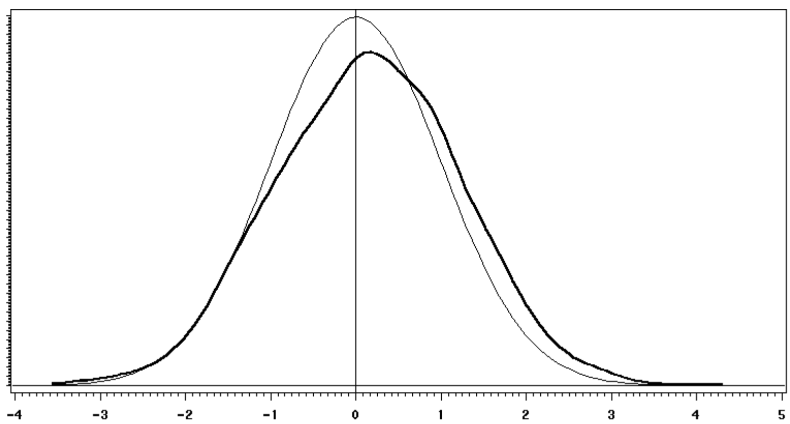



Fig. 9. $\theta = \frac{\pi}{6}$,Numerator $\widetilde{\xi}_n^{s**}$

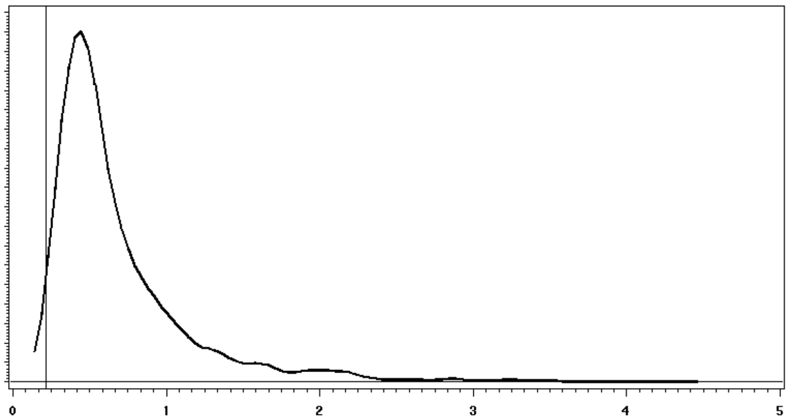



Fig. 10. Nuisance parameter (the vertical line refers to the true value of $f_u(\theta)$)

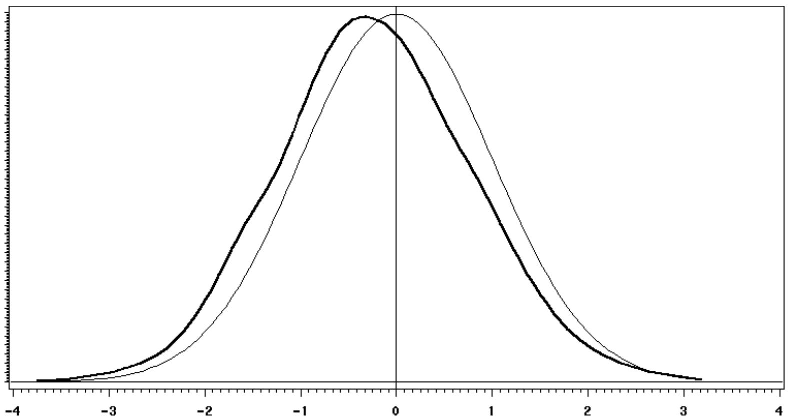



Fig. 11. $\theta = \frac{\pi}{6}$, $\widetilde{\xi}_n^{r*}$

In this case, the bias of the nuisance parameter makes $\xi_n^p$ close to the asymptotic distribution. On the contrary, the nuisance parameter used for $\xi_n^{s**}$ increases the distorsion.

### 4.3 Model 1, high frequency

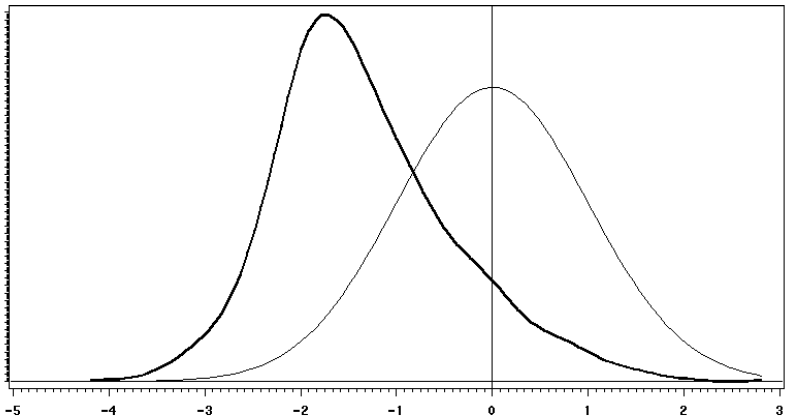



Fig. 12. $\theta = \frac{5\pi}{6}$, $\widetilde{\xi}_n^{s**}$

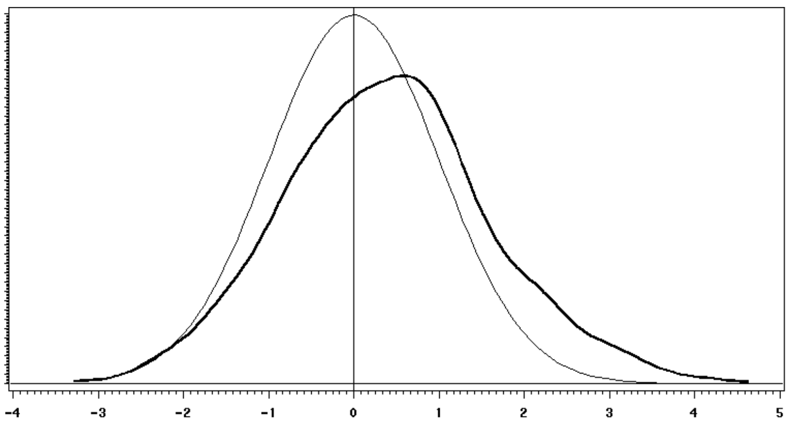



Fig. 13. $\theta = \frac{5\pi}{6}$,Numerator $\widetilde{\xi}_n^{s**}$

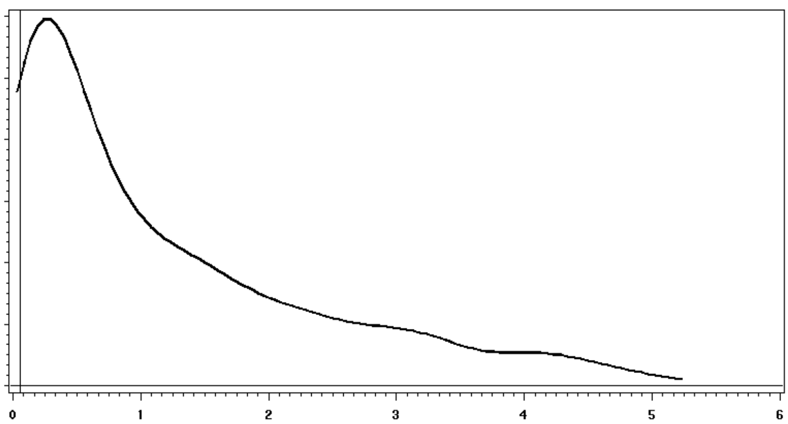



Fig. 14. Nuisance parameter (the vertical line refers to the true value of $f_u(\theta)$)

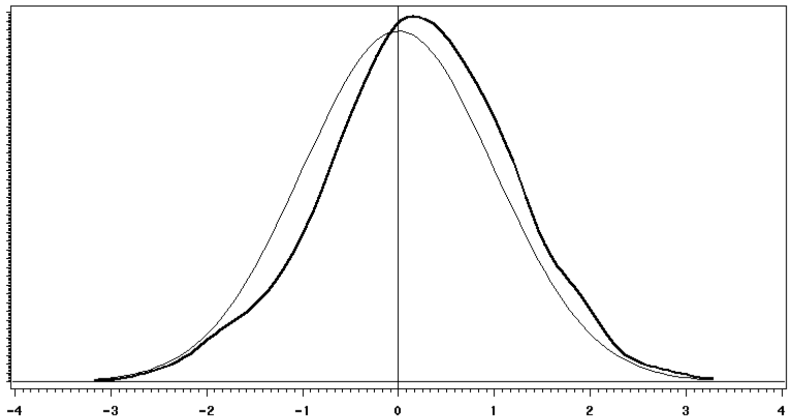



Fig. 15. $\theta = \frac{5\pi}{6}$, $\widetilde{\xi}_n^{r*}$

The behavior of the statistics are less satisfactory for frequencies associated with low values of the spectral density: of $u_t$. Indeed, one may expect the finite sample law to be close to the law associated with two roots in the spectrum. In this case, the asymptotic approximation may be quite poor. Note also that the error on the nuisance parameter, whose true value is small has an important effect on the distribution of $\xi_n^{s**}$

## 4.4 Model 2, frequency with high power

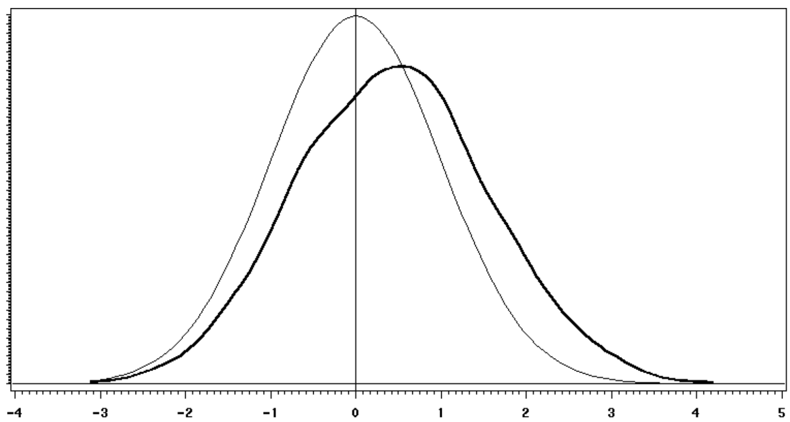



Fig. 16. $\theta = \frac{\pi}{3}$, $\widetilde{\xi}_n^{s**}$

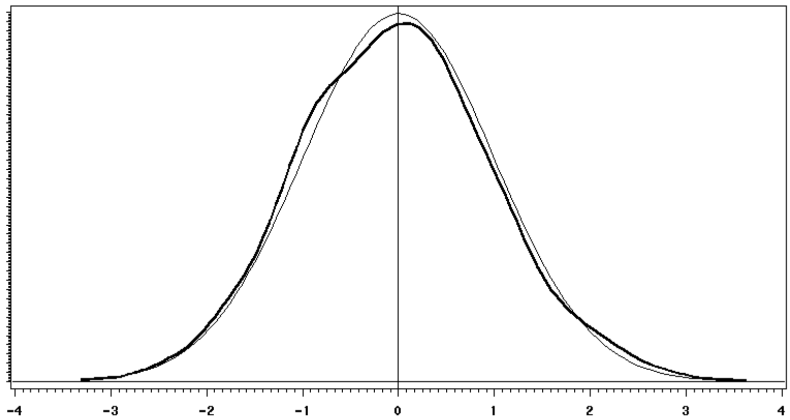


Fig. 17. $\theta = \frac{\pi}{3}$,Numerator $\widetilde{\xi}_n^{s**}$

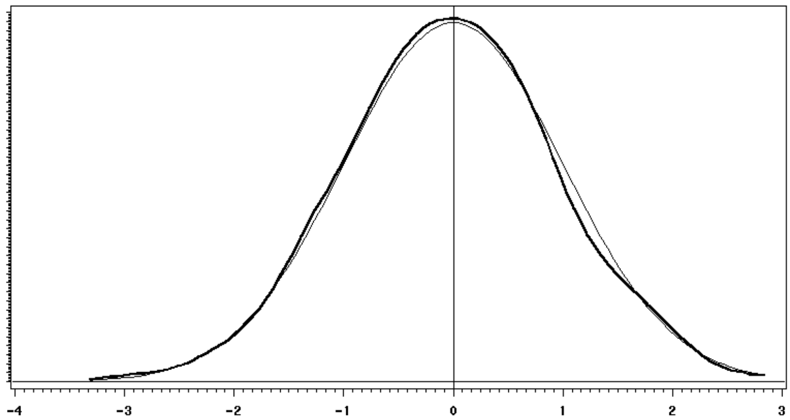


Fig. 18. $\theta = \frac{\pi}{3}$, $\widetilde{\xi}_n^{r*}$

The peak at $\theta$ affects the precision of $f_u(\theta)$, and then $\xi_n^{s**}$. On the contrary, $\xi_n^{r*}$, by definition, not sensitive to this problem, a property which appear here to be quite attractive. It is worth noting that non parametric bias correction, which would entail the estimation of the second derivative of $f$ isn't very useful here. Indeed, the order of magnitude of the sample size under consideration implies very poor properties of this estimator, whose best rate of convergence is about $n^{\frac{1}{7}}$(see Prewitt (1998)).

We turn now to the model with a seasonal intercept.

## 4.5 Model 1, low frequency

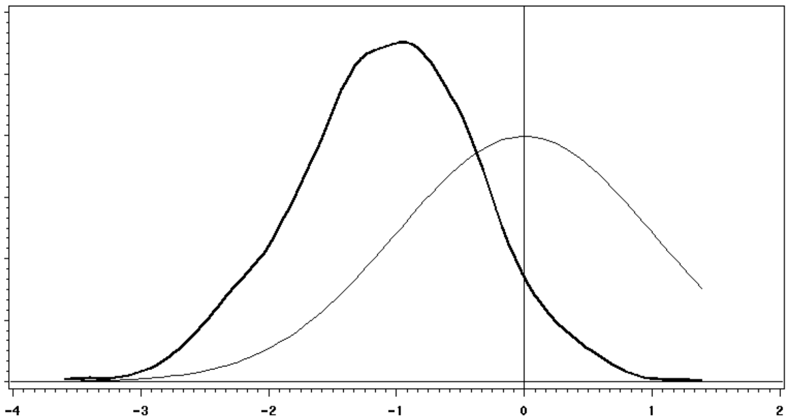



Fig. 19. $\theta = \frac{\pi}{6}$, $\widetilde{\xi}_n^p$

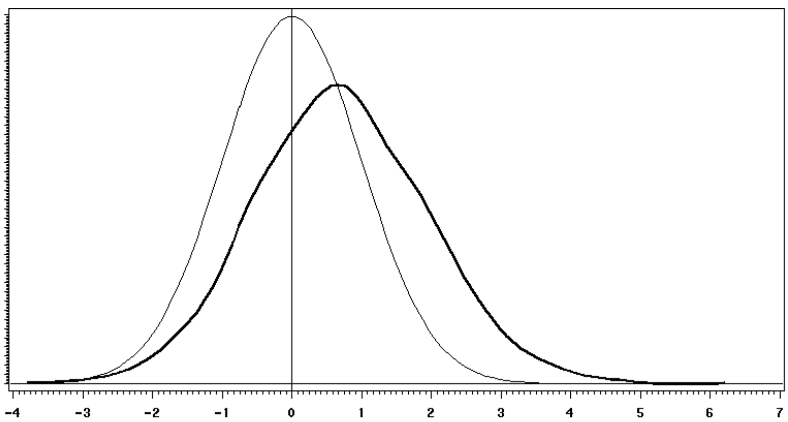



Fig. 20. $\theta = \frac{\pi}{6}$, $\widetilde{\xi}_n^{s**}$

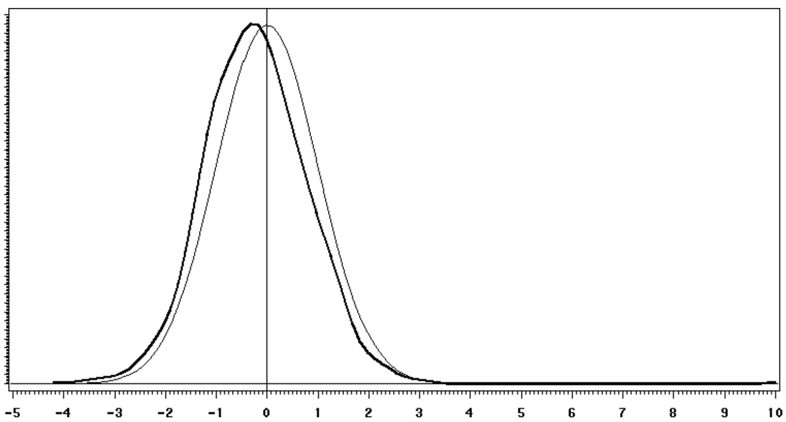



Fig. 21. $\theta = \frac{\pi}{6}$, $\widetilde{\xi}_n^{r*}$

## 4.6 Model 1, high frequency

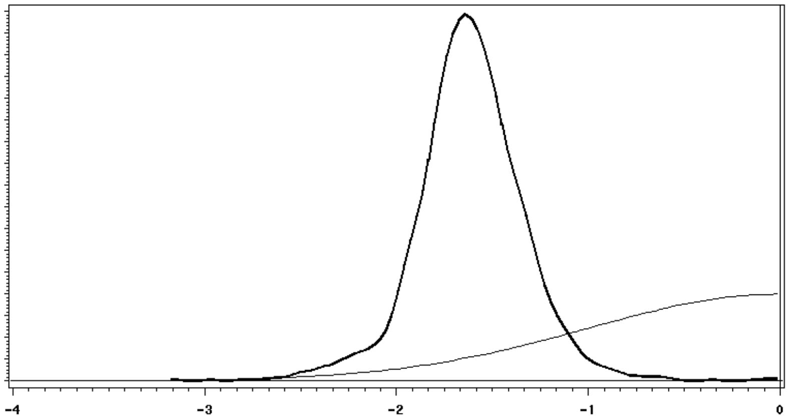



Fig. 22. $\theta = \frac{5\pi}{6}$, $\widetilde{\xi}_n^p$

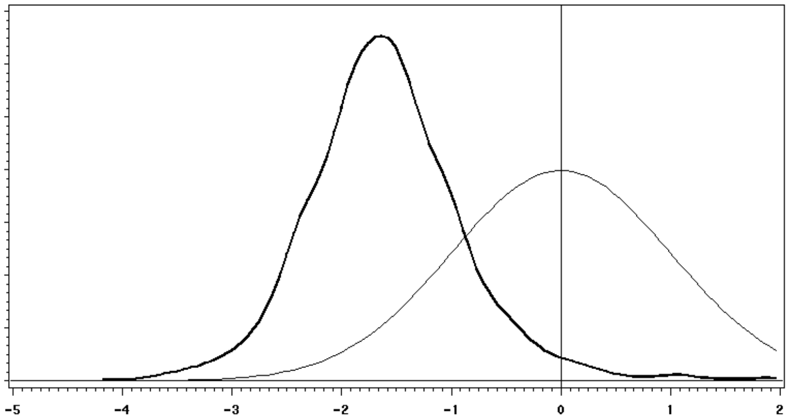



Fig. 23. $\theta = \frac{5\pi}{6}$, $\widetilde{\xi}_n^{s**}$

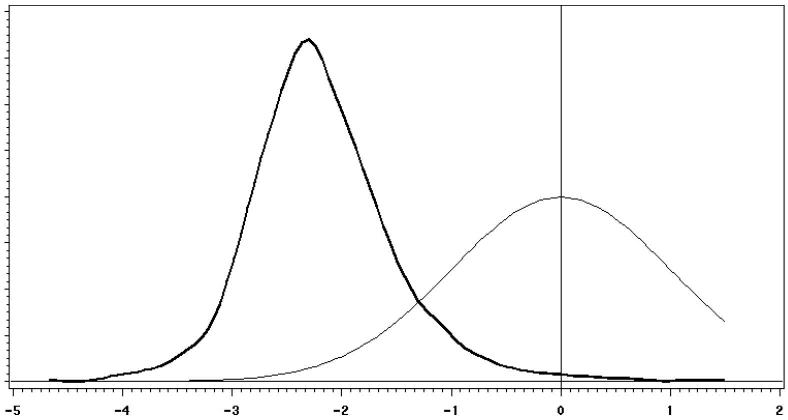



Fig. 24 $\theta = \frac{5\pi}{6}$, $\widetilde{\xi}_n^{r*}$

## 4.7 Model 2, high frequency

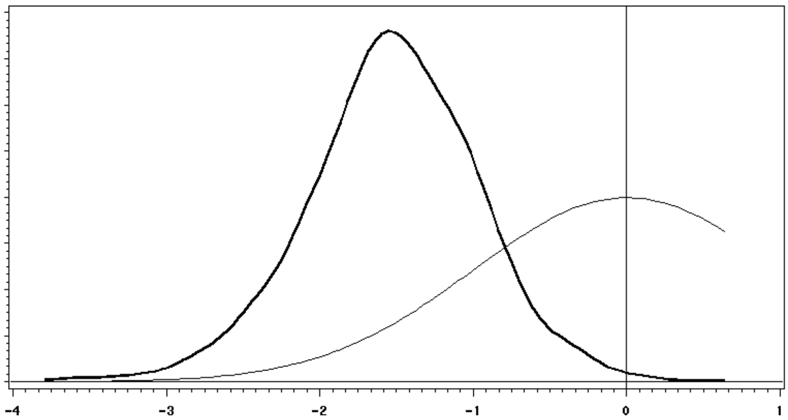



Fig. 25. $\theta = \frac{5\pi}{6}$, $\widetilde{\xi}_n^p$

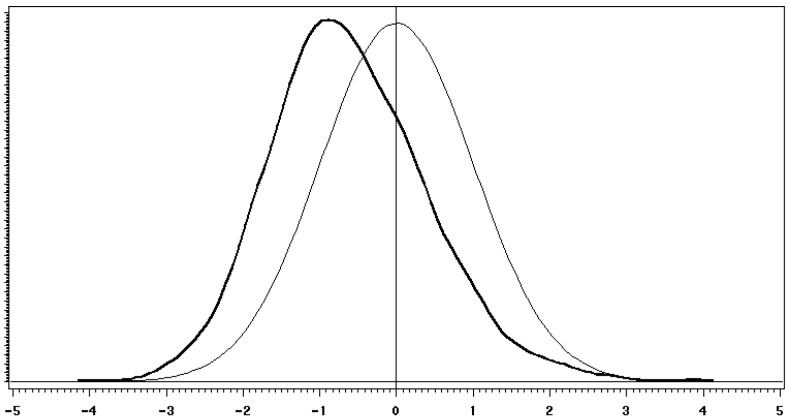



Fig. 26. $\theta = \frac{5\pi}{6}$, $\widetilde{\xi}_n^{s**}$

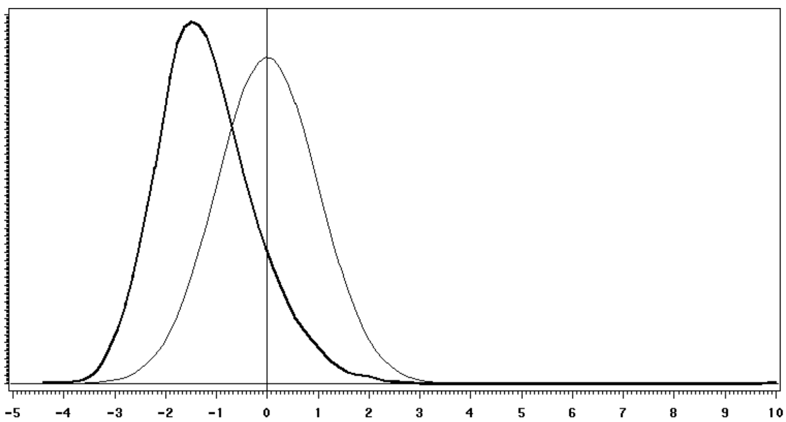



Fig. 27. $\theta = \frac{5\pi}{6}$, $\widetilde{\xi}_n^{r*}$

The statistics show dramatic distortions of the distribution especially for low value of the spectrum (see figures 22-24). In this case, the stochastic signal is clearly dominated by the deterministic component. We note in particular that the estimator of the nuisance parameter performs very poorly in the case of $\widetilde{\xi}_n^{s*}$. Note also that $\widetilde{\xi}_n^{p}$ is always the worst estimator.

## 4.8 Model 2, frequency with high power

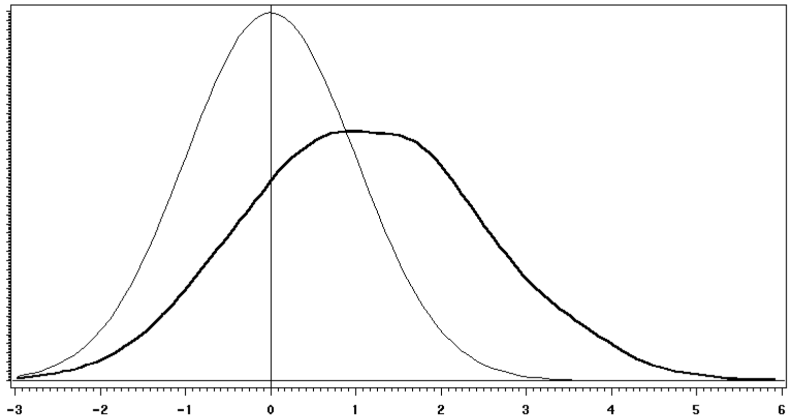



Fig. 28. $\theta = \frac{\pi}{3}$, $\widetilde{\xi}_n^{s**}$

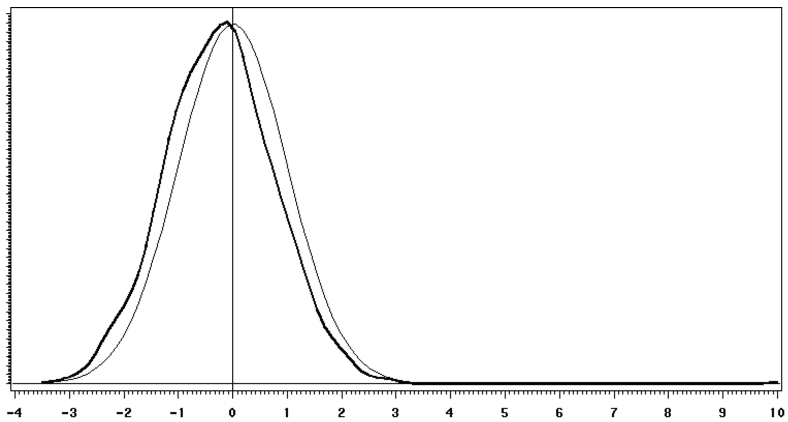



Fig. 29. $\theta = \frac{\pi}{3}$, $\widetilde{\xi}_n^{r*}$

In this case, the distortions appear to be less significant compared to the model without intercept. Indeed, the stochastic component has high spectral power at $\pi \backslash 3$, which makes the influence of the deterministic part for this frequency less influent.

# 5 Empirical power of the tests

We define, for $i \in \{1,2\}$ :

$$\begin{array}{ll} X_{it} & = \left(1 - 2\cos\theta B + B^2\right) X_{it} \\ X_{it}\left(\lambda\right) & = \left(1 - 2\lambda\cos\theta B + \lambda^2 B^2\right) X_{it} \end{array}$$

Tests $\xi_n^p, \xi_n^{s**}$ , and $\xi_n^{r*}$ are performed with $N_s = 2000$ replications of $(X_t, X_t(\lambda))$, each sample being made of size $N_p = 150$ points. We fix $e(N_p) = N_p^{0.40}$ for $\xi_n^p$ and $m = \left[\sqrt{N_p}\right]$ (bandwidth parameter) for $\xi_n^p$ and $\xi_n^{s**}$ . Next, we calculate for different values of $\lambda$ and $\theta$ :

$i)$ The empirical size of the asymptotic test with $\alpha = 5\%$ of $\mathbf{H}_0$ against $\mathbf{H}_a$

$ii)$ The empirical power of the test for the alternative $\mathbf{H}_a(\lambda)$, calculated from $X_{it}(\lambda)$.

Model 1

| Size, and power for $\mathbf{H}_a$ : theoretical size: 5%, 2000 replications | | | | | |
|---|---|---|---|---|---|
| $\xi_n^p$ | | | | | |
| | $\mathbf{H}_0(\lambda=1)$ | $\lambda=0.7$ | $\lambda=0.9$ | $\lambda=0.95$ | $\lambda=0.97$ |
| $\theta=\pi/6$ | .09 | .49 | .18 | .14 | .09 |
| $\theta=\pi/2$ | .11 | .50 | .21 | .13 | .12 |
| $\theta=5\pi/6$ | .06 | .18 | .05 | .06 | .06 |
| $\xi_n^{s**}$ | | | | | |
| | $\mathbf{H}_0(\lambda=1)$ | $\lambda=0.7$ | $\lambda=0.9$ | $\lambda=0.95$ | $\lambda=0.97$ |
| $\theta=\pi/6$ | .06 | .97 | .85 | .60 | .36 |
| $\theta=\pi/2$ | .01 | .90 | .72 | .39 | .20 |
| $\theta=5\pi/6$ | .01 | .70 | .26 | .10 | .04 |
| $\xi_n^{r*}$ | | | | | |
| | $\mathbf{H}_0(\lambda=1)$ | $\lambda=0.7$ | $\lambda=0.9$ | $\lambda=0.95$ | $\lambda=0.97$ |
| $\theta=\pi/6$ | .05 | .23 | .22 | .14 | .11 |
| $\theta=\pi/2$ | .04 | .23 | .22 | .15 | .12 |
| $\theta=5\pi/6$ | .07 | .25 | .20 | .16 | .12 |

Model 2

| Size, and power for $\mathbf{H}_a$ : theoretical size: 5%, 2000 replications | | | | | |
|---|---|---|---|---|---|
| | | | $\xi_n^p$ | | |
| | $\mathbf{H}_0(\lambda=1)$ | $\lambda=0.7$ | $\lambda=0.9$ | $\lambda=0.95$ | $\lambda=0.97$ |
| $\theta=\pi/6$ | .06 | .11 | .03 | .04 | .02 |
| $\theta=\pi/3$ | .05 | .40 | .11 | .06 | .05 |
| $\theta=\pi/2$ | .11 | .52 | .22 | .13 | .10 |
| $\theta=5\pi/6$ | .12 | .46 | .17 | .13 | .13 |
| $\theta=\pi$ | .07 | .34 | .13 | .08 | .07 |
| | | | $\xi_n^{s**}$ | | |
| | $\mathbf{H}_0(\lambda=1)$ | $\lambda=0.7$ | $\lambda=0.9$ | $\lambda=0.95$ | $\lambda=0.97$ |
| $\theta=\pi/6$ | .00 | .88 | .61 | .26 | .08 |
| $\theta=\pi/3$ | .14 | .97 | .87 | .70 | .49 |
| $\theta=\pi/2$ | .05 | .94 | .82 | .55 | .32 |
| $\theta=5\pi/6$ | .01 | .80 | .43 | .14 | .04 |
| $\theta=\pi$ | .04 | .75 | .56 | .34 | .19 |
| | | | $\xi_n^{r*}$ | | |
| | $\mathbf{H}_0(\lambda=1)$ | $\lambda=0.7$ | $\lambda=0.9$ | $\lambda=0.95$ | $\lambda=0.97$ |
| $\theta=\pi/6$ | .03 | .18 | .16 | .10 | .08 |
| $\theta=\pi/3$ | .05 | .22 | .18 | .15 | .09 |
| $\theta=\pi/2$ | .05 | .23 | .22 | .15 | .12 |
| $\theta=5\pi/6$ | .06 | .25 | .22 | .16 | .11 |
| $\theta=\pi$ | .04 | .23 | .18 | .13 | .09 |

As expected from the previous part, $\xi_n^{r*}$ shows the better properties in term of size, but its power is particularly low (this is a bit surprising, because the test diverges with rate $n$ under the alternative). The conclusions are reversed for $\xi_n^{s**}$ : the distortions of size may be quite significant, but its power is almost all the time the best among the three tests under consideration, and is comparable to corresponding results for unit roots tests, see e.g. DeJong *et al.*(1992). Results for $\xi_n^p$ are quite disappointing, for both size and power. This justifies, ex-post, the need for the introduction of statistics involving frequencies which converges fast enough to $\theta$. Note that the results depend on the value of $\theta$, except perhaps for $\xi_n^{r*}$. As in the previous section, the shape of the spectrum of $u_t$ explains certainly these variations. For instance, in model 1, the power of $\xi_n^{s**}$ is very low for $\theta = 5\pi/6$ ($f_u(\theta) \simeq 0$), but it is higher for $\theta = \pi/6$ ($f_u(\theta) \gg 0$).

# 6 An empirical application to French GDP

The series which we investigate now is French GDP released by the French Statistical Institute. It is well-known now that seasonal adjustment procedures, including X11-Arima and SEATS induce spurious moving-averages unit root in seasonally adjusted series (see Maravall (1995)). But it is important to note that any macroeconomic aggregate is obtained by summation of seasonal adjusted series, and then is not the direct result of some seasonal adjustment procedure. It means that, although disaggregated series may have some MA unit roots, it is not clear wether they are still present after aggregation.. So, we are left with the problem of testing lack of power at seasonal frequencies $\frac{\pi}{2}$ and $\pi$.

Granted to non stationarity at frequency zero, we work with the first difference of the logarithm of the series. The graph below shows an estimation of the spectrum . The estimator uses data-dependent bandwidth (see Robinson (1994)) and Parzen's kernel. Confidence intervals are omitted since they are very large, a situation commonly encountered in the field of applied spectral analysis.

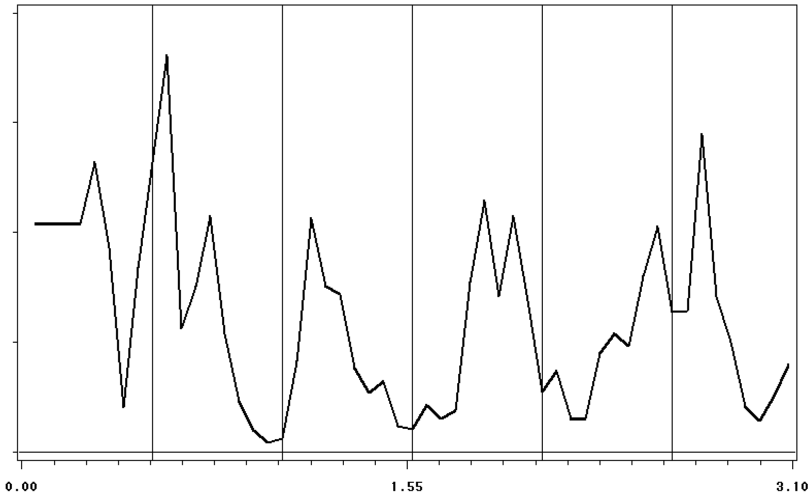



Fig. 30. Spectral density for Quarterly French GDP

The properties of the spectrum are not very easy to interpret. It is particularly difficult, at least visually, to make some decision about the dips at $\pi/2$ for $\Delta\log(PIB)$. Moreover, similar dip occur at $\theta \simeq \pi/3\,, 2\pi/3\,, 5\pi/6$. We perform now our tests $\xi_n^p$, $\xi_n^{r*}$ and $\xi_n^{s*}$:

Test for $\Delta$log(PIB)

| $\theta$ | $\xi_n^p$ | level | $\xi_n^{r*}$ | level | $\xi_n^{s*}$ | level |
|---|---|---|---|---|---|---|
| 0 | $3.5{\times}10^{-4}$ | 0 | 4.24 | 0.05 | 58.39 | 0 |
| $\pi/6$ | 0.02 | 0 | 2.10 | 0.18 | 27.73 | 0 |
| $\pi/3$ | 0.65 | 0.28 | 1.38 | 0.34 | 15.22 | 0 |
| $\pi/2$ | 1.00 | 0.39 | 1.53 | 0.29 | 28.66 | 0 |
| $2\pi/3$ | 0.13 | 0.06 | 1.22 | 0.39 | 9.48 | 0 |
| $5\pi/6$ | 0.13 | 0.06 | 1.33 | 0.35 | 14.28 | 0 |
| $\pi$ | 0.14 | 0.05 | 1.32 | 0.34 | $7.3{\times}10^{-3}$ | 0.99 |

Surprisingly, the test $\xi_n^{r*}$ fails to reject all the hypothesis. $\xi_n^p$ and $\xi_n^{s*}$ deliver contradictory results for $\theta = \pi/3\,, \pi/2$. Note that $\xi_n^{s*}$ rejects clearly all the hypothesis, apart $\theta = \pi$. From the previous section, we expect this test to have better power in finite samples. So, we may conclude to a moving average unit root frequency $\pi$ for GDP. This seasonal frequency is associated with semestrial cycles. The explication for such a surprising result probably lies in the statistical methods used in the the calculation of quarterly accounts: this subject is currently under investigation. Finally, we remark that the hypothesis of a unit root at frequency zero can't be rejected by the three tests at the conventional level $\alpha$ =5%.

# 7 Concluding remarks

This paper presents results related to some statistical tests of the moving average unit root hypothesis. Both theoretical study of sequences of local alternatives and (limited) finite sample experiment suggest that the simple test considered here, $\xi_n^p$, appears to be of little interest for econometric applications. The same conclusion is obtained for the statistic $\xi_n^{r*}$. Indeed, despite its good properties under the null hypothesis, its power is very low. On the contrary, $\xi_n^{s**}$ which uses sequences converging to $\theta$ with rate $n^{-1}$ may be useful. It is indeed convergent for local alternatives of the form $\theta + Cn^{-1}$, and its finite sample properties (size and power) are encouraging. However, under the null hypothesis, the properties of the test appear to be less satisfactory in three cases. Firstly, when the spectrum of the differenced series is very low, the series looks like is a model with two MA unit roots, and a different limit theory applies. Secondly, when this spectrum has a peak at $\theta$, the nuisance parameter estimator can be severely biased. Lastly, under the null hypothesis, deterministic seasonal intercept at frequency $\theta$ can seriously affect the law of the statistic. Naturally, the distorsion is particularly important when the differenced series has low spectral power at $\theta$ compared to the deterministic signal.

Now, it would be interesting to compare the performance of this test with other procedures which are known to be satisfactory. However, itspower properties appear to be less attractive in finite sample. Another simulation work is now needed in order to examine the power of the test in various specifications allowed by our theoretical results. For instance, seasonal intercept and conditional heteroskedasticity should be considered, as well as stationary linear models which are not ARMA. It seems also interesting to compare these results which other procedures which are known to be locally optimal under some simple ARMA specification (see Saikkonen and Luukkonen (1996)).

# 8 Appendix

## 8.1 Proof of lemma 2

Suppose $\theta \neq \pi$. Starting with $X_{t,n} = X_t + \frac{2a}{n^\beta}\left(u_{t-2} - \cos\theta u_{t-1}\right)$ we get immediately, with the notation

$$\mathbb{J}_{\overline{X}}(\omega) = \frac{1}{\sqrt{n}}\Sigma_{k=1}^{n} X_{k,n} e^{-ik\omega}$$

$$\begin{aligned} \mathbb{J}_{\overline{X}}(\omega) &= \mathbb{J}_X(\omega) + \tfrac{2a}{n^\beta}\left[\tfrac{1}{\sqrt{n}}\left(e^{-2i\omega}\textstyle\sum_{k=-1}^{n-2} - \cos\theta e^{-i\omega}\sum_{k=0}^{n-1}\right) u_k e^{-ik\omega}\right] \\ &= \mathbb{J}_X(\omega) + \tfrac{2ae^{-i\omega}}{n^\beta}\left[\sqrt{\tfrac{n-2}{n}}\left(e^{-i\omega} - 1\right)\mathbb{J}_u^{n-2}(\omega) + g_n(\omega)\right] \end{aligned}$$

with $g_n(\omega) = \frac{e^{i\omega}}{\sqrt{n}}\left[u_0 + u_{-1}e^{i\omega} - \cos\theta u_{n-1}e^{-i(n-1)\omega}\right]$ and $\mathbb{E}\left(\sup_\omega |g_n(\omega)|\right) = O\left(\frac{1}{\sqrt{n}}\right)$
But $\mathbb{J}_u(\theta_n) = O_p(1)$, so:

$$e(n)\,\mathbb{J}_{\overline{X}}(\theta_n) = e(n)\,\mathbb{J}_X(\theta_n) + \frac{2ae^{-i\theta_n}}{Cn^{\beta-\zeta}}\left[\left(e^{-i\theta_n} - 1\right)\mathbb{J}_u^{n-2}(\theta_n) + O_p\left(\frac{1}{\sqrt{n}}\right)\right]$$

If $\beta > \zeta$ then $e(n)\,\mathbb{J}_{\overline{X},n}(\theta_n) = e(n)\,\mathbb{J}_{X,n}(\theta_n) + o_p(1)$.
If $\beta = \zeta$ then from the proof of theorem 8 and lemma 5 of Lacroix (1999), we have:

$$\sqrt{2}\begin{pmatrix} e(n)\mathbb{J}_X(\theta_n) \\ \mathbb{J}_u^{n-2}(\theta_n) \end{pmatrix} \Rightarrow \begin{pmatrix} i\sigma C\left(\theta, e^{-i\theta}\right)\mathbb{W}_c(1) \\ \sigma D\left(\theta, e^{-i\theta}\right)\mathbb{W}_c(1) \end{pmatrix}$$

It yields $e(n)\sqrt{2}\mathbb{J}_{\overline{X}}(\theta_n) \Rightarrow \sigma\left[iC\left(\theta, e^{-i\theta}\right) + \frac{2ae^{-i\theta}}{C}D\left(\theta, e^{-i\theta}\right)\left(e^{-i\theta} - 1\right)\right]\mathbb{W}_c(1)$, or:

$$e(n)\sqrt{2}\mathbb{J}_{\overline{X}}(\theta_n) \Rightarrow \sigma\left[i\left(1 + e^{-i\theta}\right) - \frac{2ae^{-i\theta}}{C}\right]\left(1 - e^{-i\theta}\right)D\left(\theta, e^{-i\theta}\right)\mathbb{W}_c(1)$$

If $\beta < \zeta$, then $n^{\beta-\zeta}e(n)\,\mathbb{J}_{\overline{X}}(\theta_n) \Rightarrow \frac{2a\sigma e^{-i\theta}}{C\sqrt{2}}D\left(\theta, e^{-i\theta}\right)\left(e^{-i\theta} - 1\right)\mathbb{W}_c(1)$

If $\theta = \pi$, we have $X_{t,n} = X_t + \frac{a}{n^\beta}u_{t-1}$, and:
$\mathbb{J}_{\overline{X}}(\omega) = \mathbb{J}_{X,n}(\omega) + \frac{a}{n^\beta}\left[e^{-i\omega}\mathbb{J}_{u,n-1}(\omega) + O_p\left(\frac{1}{\sqrt{n}}\right)\right]$ hence

$$e(n)\,\mathbb{J}_{\overline{X}}(\theta_n) = e(n)\,\mathbb{J}_X(\theta_n) + \frac{a}{Cn^{\beta-\zeta}}\left[e^{-i\theta_n}\mathbb{J}_u^{n-2}(\theta_n) + O_p\left(\frac{1}{\sqrt{n}}\right)\right]$$

Thus, if $\beta = \zeta$ :

$$e(n)\sqrt{2}\mathbb{J}_{\overline{X}}(\theta_n) \Rightarrow \sigma\left[i - \frac{a}{C}\right]C(\pi, -1)\,\mathbb{W}_c(1)$$

and if $\beta < \zeta$ :

$$n^{\beta-\zeta}e(n)\sqrt{2}\mathbb{J}_{\overline{X}}(\theta_n) \Rightarrow -\frac{\sigma a}{C}C(\pi, -1)\,\mathbb{W}_c(1)$$

■

## 8.2 Proof of lemma 3

We write $\mathbb{J}_{\overline{u}}(\omega) = \mathbb{J}_u(\omega) + C_1(\omega)\frac{\mathbb{J}_Z(\omega)}{n} + C_2(\omega, n)\frac{Z_{n-1}}{n\sqrt{n}}$ with $C_1(\omega)$ and $C_2(\omega, n)$ uniformly bounded in $\omega$ and $n$.
From $u_t = (1 - 2\cos\theta B + B^2) Z_t$ we get:

$$\mathbb{J}_u(\omega) = \mathbb{J}_Z^{n-2}(\omega)\left(1 - 2\cos\theta e^{-i\omega} + e^{-2i\omega}\right) + \frac{1}{\sqrt{n}}\left[Z_{n-1}e^{-i(n-1)\omega} + e^{in\omega}(u_t - Z_{n-2})\right]$$

$$\mathbb{J}_u(\omega) = \mathbb{J}_Z^{n-2}(\omega)\left(1 - e^{-i(\omega-\theta)}\right)\left(1 - e^{-i(\omega+\theta)}\right) + \frac{e^{-in\omega}u_n}{\sqrt{n}} + \frac{e^{-in\omega}}{\sqrt{n}}\left[Z_{n-1}e^{i\omega} - Z_{n-2}\right]$$

$$\begin{aligned} \mathbb{J}_Z(\omega) &= \tfrac{1}{\left(1-e^{-i(\omega-\theta)}\right)\left(1-e^{-i(\omega+\theta)}\right)}\times \\ &\quad \left[\mathbb{J}_u^{n+2}(\omega) - \tfrac{e^{-i(n+2)\omega}u_{n+2}}{\sqrt{n+2}} + \tfrac{e^{-i(n+2)\omega}}{\sqrt{n+2}}\left[Z_{n+1}e^{i\omega} - Z_n\right]\right] \end{aligned}$$

$$\mathbb{J}_{\overline{u}}(\omega) = \mathbb{J}_u(\omega) + C_1(\omega)\frac{\mathbb{J}_Z(\omega)}{n} + C_2(\omega, n)\frac{Z_{n-1}}{\sqrt{n}}$$

with $\theta_{j,n} = \theta + \frac{2\pi j}{n}$ for $j = -m, \ldots, -1, 1 \ldots m$.
From $\left|\frac{\sin x}{x}\right| \geq \frac{2}{\pi}$ for all $x \in \left[0, \frac{\pi}{2}\right]$, and for $n$ large enough $\frac{m}{n} < \frac{1}{2}$ we get:

$$\left|\left(1 - e^{-i(\theta_{j,n}-\theta)}\right)\left(1 - e^{-i(\theta_{j,n}+\theta)}\right)\right| = 4\left|\sin\frac{j\pi}{n}\right|\left|\sin\left(\theta + \frac{j\pi}{n}\right)\right| \geq C\frac{j}{n}$$

$$\left|\frac{\mathbb{J}_Z(\theta_{j,n})}{n}\mathbb{J}_u(\theta_{j,n})\right| \leq \frac{\left|\mathbb{J}_u^{n+2}(\theta_{j,n})\mathbb{J}_u(\theta_{j,n})\right|}{Cj} + \frac{C'n}{jn^{\frac{3}{2}}}\left\{\left|u_{n+2}\mathbb{J}_u(\theta_{j,n})\right| + \left(|Z_{n+1}| + |Z_n|\right)\left|\mathbb{J}_u(\theta_{j,n})\right|\right\}$$

We treat separately each of the three terms of right-hand side of this inequality. First:

$$\mathbb{E}\left|J_u^{n+2}(\theta_{j,n})J_u(\theta_{j,n})\right| \leq \sqrt{\mathbb{E}\mathbb{I}_u^{n+2}(\theta_{j,n})}\sqrt{\mathbb{E}\mathbb{I}_u(\theta_{j,n})} = O(1)$$

uniformly in $j$ (see the proof of lemma 5 of Lacroix (1999)). Next:

$$\mathbb{E}\left|u_{n+2}\mathbb{J}_u(\theta_{j,n})\right| \leq \sqrt{\mathbb{E}u_{n+2}^2}\sqrt{\mathbb{E}\mathbb{I}_u(\theta_{j,n})} = O(1)$$

by stationarity of $u_t$.

$$\mathbb{E}\left(|Z_{n+1}| + |Z_n|\right)\left|\mathbb{J}_u^n(\theta_{j,n})\right| \leq C\sqrt{\mathbb{E}\left(|Z_{n+1}| + |Z_n|\right)^2} \leq C\sqrt{2\left(\mathbb{E}Z_{n+1}^2 + \mathbb{E}Z_n^2\right)}$$

But

$$\begin{aligned} \left(\sqrt{\tfrac{2}{n}}\sin\theta\right)Z_n &= \sin\left((n+1)\theta\right)C_n(\theta, 1) - \cos((n+1)\theta S_n(\theta, 1) \\ &= \tfrac{1}{\sqrt{n}}\textstyle\sum_{k=1}^{n}\sin(n+1-k)\,u_k \end{aligned}$$

Since $u_t$ is stationary we have $\mathbb{E}\left(Z_n^2\right) = O(n)$. It yields:

$$\mathbb{E}\left(|Z_{n+1}| + |Z_n|\right)\left|\mathbb{J}_u(\theta_{j,n})\right| = O\left(\sqrt{n}\right)$$

uniformly in $j$, and finally:

$$\mathbb{E}\left|\frac{\mathbb{J}_Z(\theta_{j,n})}{n}\mathbb{J}_u(\theta_{j,n})\right| \leq \frac{O(1)}{j} + \frac{1}{j}\left[O\left(\frac{1}{\sqrt{n}}\right) + O(1)\right] \leq \frac{C}{j}$$

and

$$\mathbb{E}\left|\frac{1}{2m+1}\sum_{|j|<m,\ j\neq 0}\left(\frac{\mathbb{J}_Z(\theta_{j,n})}{n}\mathbb{J}_u(\theta_{j,n})\right)\right| \leq \frac{2C}{2m+1}\sum_{j=1}^{m}\frac{1}{j} \leq C\frac{\log(m)}{m} = o(1)$$

So

$$\begin{aligned}\mathbb{E}\left|\tfrac{1}{2m+1}\sum_{|j|<m,\ j\neq 0}\left(\tfrac{Z_{n-1}}{n\sqrt{n}}\mathbb{J}_u(\theta_{j,n})\right)\right| &\leq \tfrac{1}{2m+1}\times\tfrac{1}{n}\sum_{|j|<m,\ j\neq 0}\sqrt{\mathbb{E}\mathbb{I}_u(\theta_{j,n})}\sqrt{\tfrac{\mathbb{E}Z_{n-1}^2}{n}} \\ &\leq \tfrac{C}{n} = o(1)\end{aligned}$$

The same argument yields $\mathbb{E}\left|\frac{1}{2m+1}\sum_{|j|<m,\ j\neq 0}\left(\frac{Z_{n-1}}{n\sqrt{n}}\frac{\mathbb{J}_Z(\theta_{j,n})}{n}\right)\right| = o(1)$. Now,:

$$\mathbb{I}_{\overline{u}}(\theta_{j,n}) = \mathbb{I}_u(\theta_{j,n}) + |C_1(\theta_{j,n})|^2\left|\frac{\mathbb{J}_Z(\theta_{j,n})}{n}\right|^2 + |C_2(\theta_{j,n},n)|^2\left|\frac{Z_{n-1}}{n\sqrt{n}}\right|^2 + R(j,n)$$

$$\begin{aligned}\text{with } R(j,n) &= \operatorname{Re}\left[C_1(\theta_{j,n})\tfrac{\mathbb{J}_Z(\theta_{j,n})}{n}\overline{\mathbb{J}_u^n(\theta_{j,n})}\right] + 2\operatorname{Re}\left[C_2(\theta_{j,n},n)\tfrac{Z_{n-1}}{n\sqrt{n}}\overline{\mathbb{J}_u(\theta_{j,n})}\right] \\ &+2\operatorname{Re}\left[C_1(\theta_{j,n})\tfrac{\mathbb{J}_Z(\theta_{j,n})}{n}\overline{C_2(\theta_{j,n},n)\tfrac{Z_{n-1}}{n\sqrt{n}}}\right]\end{aligned}$$

and

$$\widehat{f_{\overline{u}}(\theta)} = \frac{1}{2m+1}\sum_{|j|<m,j\neq 0}\mathbb{I}_{\overline{u}}(\theta_{j,n}) + \frac{1}{n}\times\frac{\mathbb{I}_{\overline{u}}(\theta)}{2m+1}$$

But $\widehat{f_{\overline{u}}(\theta)} = \widehat{f_u(\theta)} + A_n + B_n + \frac{1}{2m+1}\sum_{|j|<m,j\neq 0}R(j,n) + \frac{1}{n(2m+1)}\left[\mathbb{I}_{\overline{u}}(\theta) - \mathbb{I}_u(\theta)\right]$

Thus, we have proved that $\frac{1}{2m+1}\sum_{|j|<m,j\neq 0}R(j,n) \xrightarrow{L^1} 0$

Similarly, $A_n, B_n \xrightarrow{L^1} 0$. We give some details for $A_n$ :

$$\left|\frac{\mathbb{J}_Z(\theta_{j,n})}{n}\right|^2 \leq \frac{C}{j^2}\left[\mathbb{I}_u(\theta_{j,n}) + \frac{u_{n+2}^2}{n} + \frac{2\left(|Z_{n+1}|^2 + |Z_n|^2\right)}{n}\right]$$

$\mathbb{E}\left|\frac{\mathbb{J}_Z(\theta_{j,n})}{n}\right|^2 \leq \frac{C}{j^2}\left[O(1) + \frac{O(1)}{n} + O(1)\right] = O\left(\frac{1}{j^2}\right)$ uniformly in $j$.

$\mathbb{E}\left|\frac{1}{2m+1}\sum_{|j|<m,\ j\neq 0}\left|\frac{\mathbb{J}_Z(\theta_{j,n})}{n}\right|^2\right| \leq \frac{1}{2m+1}\times\sum_{k=1}^{m}\frac{1}{j^2} = O\left(\frac{1}{m}\right)$ and the result follows.

We know that $\mathbb{I}_{\overline{u}}(\theta)$ and $\mathbb{I}_u(\theta)$ converge in law, so

$$\frac{1}{n(2m+1)}\left[\mathbb{I}_{\overline{u}}(\theta) - \mathbb{I}_u(\theta)\right] = o_p(1)$$

Lastly, from lemma 11 of Lacroix (1999), $\widehat{f(\theta)}$ is a consistent estimator of $f_u(\theta)$, so $\widehat{f_{\overline{u}}(\theta)} \underset{n\to\infty}{\rightarrow} f_u(\theta)$. Then, the result for $\widehat{f^*_{\overline{u}^*}(\theta)}$ follows as in lemma 11.

We turn now to the case $\theta = \pi$. Details will be omitted

$X_{t,n} = X_t + \frac{a}{n^\beta} u_{t-1}$. Define $Z_t$ a process such as $u_t = (1+B)\, Z_t$ and $Z_t = 0$ if $t \leq 0$.

With $\overline{u_{t,n}} = S_t\left(\overline{X}, \theta\right) = u_t + \frac{a}{n^\beta} Z_{t-1}$, we get:

$$\mathbb{J}_{\overline{u}}\left(\omega\right) = \frac{1}{\sqrt{n}} \sum_{k=1}^{n} e^{-ik\omega} \overline{u}_{k,n} = \mathbb{J}_u^n\left(\omega\right) + \frac{ae^{-i\omega}}{n^\beta} \mathbb{J}_Z^{n-1}\left(\omega\right)$$

$u_t = (1+B)\, Z_t$ thus $\mathbb{J}_u\left(\omega\right) = \mathbb{J}_Z^{n-1}\left(\omega\right)\left(1+e^{-i\omega}\right) - \frac{1}{\sqrt{n}}\left[e^{in\omega} Z_n\right]$

For $\beta = 1$ we obtain for $\omega \neq \pi$:

$$\mathbb{J}_{\overline{u}}\left(\omega\right) = \mathbb{J}_u\left(\omega\right) - ae^{-i\omega}\left[\frac{\mathbb{J}_u\left(\omega\right) + e^{in\omega}\frac{Z_n}{\sqrt{n}}}{n\left(1+e^{-i\omega}\right)}\right]$$

Let $\theta_{j,n} = \pi + \frac{2\pi j}{n}$ for $j = -m, \ldots, -1, 1 \ldots m$.

$$\left|\frac{\mathbb{J}_u\left(\theta_{j,n}\right) + e^{in\theta_{j,n}}\frac{Z_n}{\sqrt{n}}}{n\left(1+e^{-i\theta_{j,n}}\right)}\right| \leq \frac{C}{j}\left[\left|\mathbb{J}_u\left(\theta_{j,n}\right)\right| + \left|\frac{Z_n}{\sqrt{n}}\right|\right]$$

As before, $\mathbb{E}\left(|Z_n|\right) = O\left(\sqrt{n}\right)$ and $\mathbb{E}\left(\left|\mathbb{J}_u^n\left(\theta_{j,n}\right)\right|\right) = O\left(1\right)$ uniformly in $j$

It yields $\left|\frac{\mathbb{J}_u(\theta_{j,n})+e^{in\theta_{j,n}}\frac{Z_n}{\sqrt{n}}}{n\left(1+e^{-i\theta_{j,n}}\right)}\right| \leq \frac{C'}{j}$ and then $\frac{1}{2m+1}\sum_{|j|<m,\ j\neq 0}\left(\frac{\mathbb{J}_u(\theta_{j,n})+e^{in\theta_{j,n}}\frac{Z_n}{\sqrt{n}}}{n\left(1+e^{-i\theta_{j,n}}\right)}\right) \overset{L^1}{\rightarrow} 0$

$\widehat{f(\theta)}$ is still a consistent estimator of $f_u(\theta)$, and the conclusion follows.

■

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