NOTES D'ÉTUDES

ET DE RECHERCHE

DOES CORRELATION BETWEEN STOCK RETURNS REALLY INCREASE DURING TURBULENT PERIOD?

François Chesnay and Eric Jondeau

April 2000

NER # 73

BANQUE DE FRANCE

DIRECTION GÉNÉRALE DES ÉTUDES ET DES RELATIONS INTERNATIONALES

DIRECTION GÉNÉRALE DES ÉTUDES ET DES RELATIONS INTERNATIONALES
DIRECTION DES ÉTUDES ÉCONOMIQUES ET DE LA RECHERCHE

DOES CORRELATION BETWEEN STOCK RETURNS REALLY INCREASE DURING TURBULENT PERIOD?

François Chesnay and Eric Jondeau

April 2000

NER # 73

Does Correlation Between Stock Returns Really Increase During Turbulent Period?

François Chesnay and Eric Jondeau*

Banque de France, 41-1391 Centre de recherche

31 rue Croix des Petits Champs, 75049 Paris, France.

April 2000

Abstract

Correlations between international equity markets are often claimed to increase during periods of high volatility, therefore the benefits of international diversification are reduced when they are most needed, i.e. during crises. In this paper, we investigate the relationship between international correlation and stock-market turbulence. We estimate a multivariate Markov-switching model, in which the correlation matrix is allowed to vary across regimes. Subsequently, we test the null hypothesis that correlations are regime independent. Using weekly stock returns for the S&P, the DAX and the FTSE over the period 1988-1999, we find that international correlations significantly increased during turbulent periods.

Résumé

Certaines études empiriques ont mis en évidence que les corrélations entre marchés boursiers internationaux croissent en période de forte volatilité. Les gains apportés par la diversification internationale des portefeuilles sont alors réduits lorsqu'ils sont le plus nécessaires, c'est-à-dire en période de crise. Dans ce papier, nous examinons la relation entre les corrélations internationales et les turbulences sur les marchés boursiers. Nous estimons un modèle à changement de régimes multivarié, dans lequel la matrice de corrélations dépend du régime. Nous testons alors l'hypothèse nulle selon laquelle les corrélations sont indépendantes du régime. A partir de rendements boursiers hebdomadaires pour le S&P, le DAX et le FTSE au cours de la période 1988-99, nous obtenons que les corrélations internationales augmentent significativement au cours des périodes de turbulence.

Keywords: Stock returns, International correlation, Markov-switching model.

JEL Classification: C53, G15.

*Corresponding author. Tel: +33/(0)1 42 92 49 89, e-mail: ejondeau@banque-france.fr.

1 Introduction

Correlations between international equity markets are often claimed to increase during periods of high volatility. This issue is truly important for both portfolio managers and regulators, since international diversification benefits seem to decrease when they are most needed, i.e. during periods of market turbulence. Modern portfolio theory, since the seminal work of Markowitz (1952), underlined that not only returns and volatilities are important in the portfolio selection process, but also that correlations between assets are really a key to a good asset allocation. Therefore to perform an optimal allocation, one needs to determine precisely correlations between assets. If correlations are time-varying, and more precisely if correlations increase during periods of high volatility, then the allocation process is biased. Under this hypothesis, the portfolio is not diversified enough during periods of high volatilities as correlations increase.

Thus, as pointed out by Ang and Bekaert (1999), this increase in correlation may partly explain the home bias puzzle, one of the most challenging puzzles in international finance. This home bias puzzle represents the fact that investors tend to diversify far less internationally than what theory would predict. French and Poterba (1991) report that, at the end of the 1980s, domestic ownership shares in the stock market were 94% for the US, 98% for Japan and 82% for the UK.

The aim of our paper is to investigate the relationship between international correlation and stock-market turbulence. We want to assess empirically whether the claim that correlations increase during turbulent period is true.

The existing literature actually found rather mixed empirical evidence on the link between international correlation and stock-market turbulence. A first approach examined the stability of the correlation between returns over different periods of time. Kaplanis (1988) for instance did not reject the null hypothesis of constant correlation of monthly returns of 10 markets over the 1967-82 period. Ratner (1992) obtained a similar result over the 1973-89 period. Koch and Koch (1991) obtained a growing market interdependence in 1980 and 1987 as compared to 1972. Some papers focused more precisely on the effect of the 1987 crash: King and Wadhwani (1990), Bertero and Mayer (1990), Lee and Kim (1993) claimed that correlations increased significantly after the US stock-market crash. Similarly, King, Sentana, and Wadhwani (1994) found that the increase in correlation is only a transitory effect caused by the 1987 crash. See also Roll (1989) for a survey. Most of papers cited above consider changes in correlation by comparing unconditional correlation across different subperiods. However the breakpoint is generally exogenously selected. This approach implies that two subperiods corresponding to low and high volatilities have to be identified a priori. However, recent evidence by Boyer, Gibson, and Loretan (1997) as well as Forbes and Rigobon (1999) showed that testing unconditional correlation coefficient may be misleading. Indeed, this coefficient is biased when volatility shifts over time.

Another abundant and recent literature is based on the autoregressive conditional heteroskedasticity (ARCH) framework: Hamao, Masulis, and Ng (1990) estimated a two-step multivariate GARCH model allowing to measure interdependence of returns and volatilities across the New York, Tokyo and London stock markets. When they include the October 1987 crash period in the data set, they obtained significant spillovers in almost all directions, in terms of both return and volatility. Using a similar GARCH approach to study the interrelation between the New York and London stock markets, Susmel and Engle (1994) focused on hourly data. Even for the period including the 1987 crash, they did not find strong evidence of international volatility spillovers. Longin and Solnik (1995) specifically tested the hypothesis of a constant international conditional correlation between a large number of monthly stock returns over the large period 1960-90. Using bivariate GARCH models, they explored several potential sources of deviation from the constant conditional correlation model. In particular, they tested the hypothesis of higher international correlation during turbulent periods. They found that correlation generally rises in periods of high volatility. Using daily stock returns, Bera and Kim (1996) strongly rejected the conditional correlation between US market and Japan, German, UK, France and Italy markets over the period 1990-95.

Although the GARCH approach clearly improves our comprehension of the link between international correlation and stock markets turbulence, it also raises two serious difficulties. First, in most of the empirical studies, stock volatility is found to be too persistent, implying an explosive conditional variance. For instance, some estimations performed by Hamao, Masulis, and Ng (1990), Susmel and Engle (1994), or Hamilton and Susmel (1994), display excessive volatility persistence. Such a persistence gives rise to another related problem: Lamoureux and Lastrapes (1990) showed that GARCH models are strongly affected in case of structural breaks. As highlighted by Hamilton and Susmel (1994), after a large shock on the stock market such as the 1987 crash, the forecast volatility decreased much more slowly than the true volatility (as measured, for instance, by implied volatility extracted from stock-option prices).

Second, in previous papers, when modelling the relationship between international correlation and turbulent periods, it is implicitly assumed that the estimated relation is stable over the sample period. Thus parameters are held constant whatever the regime, calm or turbulent. Bera and Kim (1996) proposed a formal test of constancy of correlation within such a framework. Assuming a constant correlation multivariate GARCH model as the DGP under the null hypothesis, they derive a score test for correlation constancy in a bivariate normal model. Interestingly, this test only requires the estimation of the model under the null hypothesis. It is not adapted for our problem, however, since only one regime is allowed under the null hypothesis. An alternative way would be to treat turbulent periods as essentially different from calm periods. This can easily be done using a Markov-switching model as introduced by Hamilton (1989) (MS model).

An interesting empirical feature of MS models is that thus-estimated volatility appears to be significantly less persistent than standard GARCH-model estimated volatility. Early papers on MS model assumed only few parameters to be regime dependent, in order to deal with computational burden. More generally this model has been generalized in a GARCH context by Cai (1994) and Hamilton and Susmel (1994) (MS-GARCH model). For instance, Hamilton and Susmel (1994) proposed within-regime volatility specifications that differ by a multiplicative scaling parameter. Gray

(1996) developed a generalized regime-switching model, in which all of the GARCH parameters are regime dependent.

However, these previous papers only considered univariate MS or MS-GARCH models. Therefore, these models were not designed to measure regime-dependent international correlations. Ramchand and Susmel (1998) develop a multivariate MS model to test the hypothesis of a constant international conditional correlation between stock markets. They assume for each regime a constant-correlation bivariate ARCH model, in which volatility regime shifts are captured by a scale parameter. In a bivariate setting, they test the hypothesis of a constant conditional correlation between the US market and Japan, UK, Germany and Canada markets, using weekly returns from January 1980 to January 1990. In this framework, correlation is assumed to depend only on the state of the domestic (US) return. The null hypothesis is rejected in two over the four cases: between the US and the UK and between the US and Canada.

The approach develop in this paper differ from Ramchand and Susmel (1998) in different respects. First, since Ramchand and Susmel were unable to obtain significant within-regime ARCH effects, we adopt a simple MS model with constant within-regime volatilities. But we also estimate a generalized ARCH, allowing a more complex dynamics for volatility. Second, we assume that volatilities shift in all markets at the same date. This is a more constraining assumption, but it allows to distinguish unambiguously between calm and turbulent regimes. This approach is justified in Section 4 on empirical grounds. Moreover, we take into account some empirical features of stock returns. In particular, we address the non-normality feature of innovations, by allowing for a Student-t distribution, along the lines of Bollerslev (1987) and Baillie and DeGennaro (1990).[1]

In this paper, we focus on US, German, and UK weekly stock returns, over the 1988-99 period. In section 2, we describe the data used and provide some preliminary evidence on unconditional correlation between stock markets. Section 3 is devoted to the econometric methodology. We briefly present multivariate GARCH model with constant correlation and two-regime MS models. Next we indicate how to generalize the MS model to a multivariate context and how to test the null hypothesis of constant conditional correlation. Empirical results and economic implications are presented in section 4. Our conclusions are summarized in Section 5.

2 Data and preliminary evidence

2.1 Data

In this paper, we study the effect of turbulent periods on the international correlation between stock markets. We use weekly (from Friday to Friday) stock returns for New

[1] Stock returns are also known to affect asymmetrically subsequent volatility; this so-called leverage effect has been highlighted by Black (1976). Following Engle and Ng (1991), we estimated the specification proposed by Glosten, Jagannathan, and Runkle (1989), but we failed to obtain significant leverage effect.

York, Frankfurt and London stock markets.[2] For New York we use observations from the Standard and Poor's 500 Composite Index (S&P). The index represents approximately 75% of the investment-grade stocks held by most institutional investors. For Germany we use the DAX Share Index, which includes 30 of the most heavily traded stocks listed on the Frankfurt Stock Exchange, representing over 75% of the total turnover in German equities. For London we use the Financial Times 100 Share Index (FTSE), which also represents about 75% of the total equity turnover in the UK. The three indices are capitalization-weighted. The data cover the period from January 1988 to December 1999, and consist of 620 observations. Unlike most previous studies, our sample period excludes the October 1987 crash. Indeed the 1987 crash is shown to have dramatically affected stock markets and increased, at least transitory, international correlations (King and Wadhwani, 1990, Hamilton and Susmel, 1994).

Let r_{it}, $t = 1, ..., T$, denotes the weekly stock (log) return of market i. As a preliminary look at the data, Table 1 reports summary statistics on stock returns, including the mean, standard deviation, skewness and kurtosis.

The average weekly return is positive, ranging from 0.21% to 0.27% for the three stock returns. Standard deviations are ranging from 1.92% for the S&P to 2.62% for the DAX. Skewness (Sk) and its standardized version (Sk^*) are measure of the distribution's asymmetry of returns. US and German stock returns are negatively skewed, indicating that crashes are more likely to occur than booms. For UK stock returns conversely, the skewness is positive, although non-significantly different from 0 at a five percent significance level. Excess kurtosis measures the heaviness of distribution's tails compared to the normal one. The kurtosis of the normal distribution is 3. The kurtosis significantly exceeds 3 for all markets, therefore the distribution has fatter tails than the normal one. The Jarque-Bera test statistic strongly rejects the normality hypothesis of stock returns. Those preliminary statistics confirm some widespread results in the financial literature on stock returns: positive return, negative skewness and fat tails.

We next consider heteroskedasticity by regressing squared returns on past squared returns (up to 4 and 12 lags). The TR^2 Engle statistic, where R^2 is the coefficient of determination, is distributed as a χ^2_K under the null hypothesis of homoskedasticity ($K = 4$ and 12 respectively). The Engle statistic takes very large values for each market, indicating strong non-linear (second-moment) dependencies. We therefore conclude that there is a fair amount of heteroskedasticity in the data.

We now wish to test for the presence of return serial correlation. Given the high level of heteroskedasticity, we consider the usual Ljung-Box statistic as well as a version of the Ljung-Box statistic which corrects for heteroskedasticity (see White, 1980). For 4 (resp. 12) lags, the Ljung-Box statistic (LB) and the corrected Ljung-Box statistic (LB_c) are distributed as a χ^2_4 (resp. χ^2_{12}). LB and LB_c statistics for returns do not indicate significant linear dependencies of returns, for all markets investigated.

[2] We prefer weekly returns to daily returns, because weekly data is less noisy than daily data. Moreover we did not consider monthly data, because the number of observations would have been too small.

2.2 Preliminary evidence on international correlations

Table 2 reports unconditional correlation coefficients between stock returns estimated over the whole sample 1988-99. Correlation between stock returns is quite high: the lowest correlation is 0.45 (between S&P and DAX), whereas the highest correlation is 0.58 (between DAX and FTSE).

To get some additional insight on international correlation, Fig. 1 displays unconditional variances and unconditional correlations across markets (S&P-DAX, S&P-FTSE and DAX-FTSE). Variances and correlations are computed over a sliding window of one year.[3] The first subperiod (1988-91) has been affected by the German reunification at the mid-1990 and the Gulf war at the beginning of 1991. S&P and DAX variances appear to be very low over the 1992-95 subperiod. The major financial event occurring during this subperiod is the EMS crisis, at the mid-1992, which appears to have strongly affected the FTSE. The last subperiod is associated with a strong S&P volatility increase. The increase took place in Germany and the UK at the mid-1997. Two major events have impacted on stock markets: the South-East Asian crisis at the mid-1997 and the Russian crisis at the mid-1998. Therefore at first glance, the second subperiod can be seen as a calm period, whereas the first and last periods can be seen as turbulent periods.

Correlations present a somewhat different pattern. First S&P-DAX and S&P-FTSE correlations attain a minimum in 1994, during the so-called calm period. Moreover correlations are rather high during the last subperiod, especially the S&P-DAX correlation. However, we note that an increase in correlation cannot be systematically related to an increase in variance in our data sample. Two events are particularly worth noting from this point of view: first, the S&P-DAX correlation strongly decreased between 1993 and 1994 (from about 0.4 to 0.1). Second, the S&P-FTSE correlation peaked markedly in 1995 (from 0 to 0.6). Both events cannot be related to particular shocks on the variance of stock markets.

Table 2 also reports unconditional correlation matrices and variances computed over the three identified subperiods (1988-91, 1992-95 and 1996-99). The first and last subperiods can be seen as high-volatility episodes, whereas the second subperiod is characterized by a low volatility. Therefore testing for a constant unconditional correlation over these subperiods can be interpreted as a test of the link between correlation increase and stock-market turbulence. A formal test for a constant unconditional correlation can be performed using the Jennrich (1970) test of equality of two correlation matrices computed over independent subsamples. This test has been performed for instance by Kaplanis (1988), Ratner (1992) or Longin and Solnik (1995). Table 3 reports results of the Jennrich test. For an (n, n)-dimensional correlation matrix, the test statistic is distributed as a chi-square with $n\,(n-1)\,/2$ degrees of freedom. Each subsample contains 206 observations. First, the null hypothesis cannot be rejected over the 1988-91 and 1992-95 subperiods. Even if we consider pairwise correlations, none is found to have significantly changed. Second, the correlation matrices estimated over the 1992-95 and 1996-99 subperiods are found to

[3] We notice that such a computation allows to identify large swings in variance as well as in correlation, but not structural breaks in the series, since the series are smoothed.

be significantly different at any usual level. Over the last subperiod (1996-99), international linkages increased dramatically: correlations are higher than 0.6 for the three stock markets under study. Correlation between DAX and FTSE even reached 0.7. These results confirm the empirical evidence by Kaplanis (1988) and Longin and Solnik (1995), who found lower p-values for the Jennrich test over the more recent period. This increase in correlations may be indicative of a growing integration between stock markets.[4]

Finally, our results confirm only partly the presumed relationship between international correlation and stock-market turbulence. On one hand, the agitated period beginning in 1997 has rightly lead to a significant increase in correlation. But, on the other hand, the decrease in volatility in 1992 has not been accompanied by a significant decrease in correlation.

Recently, however, Boyer, Gibson, and Loretan (1997) and Forbes and Rigobon (1999) argued that the test of unconditional correlation constancy across various subperiods may be misleading. This is because the unconditional correlation estimate is biased in case of variance shift. Therefore, even when the breaking date is assumed to be known (corresponding to a well-established crash, for instance), unconditional correlation estimates have to be corrected before any testing procedure. Moreover, as pointed out by Boyer, Gibson, and Loretan (1997), when the breaking date cannot be considered as a clear structural break, "changes in correlations over time or across 'regimes' cannot be detected reliably by splitting a sample according to the realized values of the data." This result is a consequence of the selection bias that occurs when subsamples are chosen a priori, according to the data. In order to test for a change in correlation, it is therefore necessary (1) to use a data generating process allowing for the possibility of structural changes, (2) to estimate the model's parameters and (3) to test changing correlations (and possibly other structural breaks).

In the following section, we test the null hypothesis of a constant conditional correlation in a model where the variance regime is determined endogenously. More precisely, we test whether a change in volatility regime (from a calm regime to a turbulent regime) can affect significantly the conditional correlation between stock returns.[5] As a base model, we adopt a constant-correlation multivariate GARCH model, as in Longin and Solnik (1996) or Ramchand and Susmel (1998). In this framework, time-varying volatility is modelled by univariate GARCH specifications and correlations are assumed to be constant over time or across regimes. We then consider a MS model, in which volatilities and correlations vary across regimes but are constant within regime.

[4]However, it is worth noting that the period studied by Longin and Solnik (1995) ended with the 1987 crash, whereas our sample ended with the 1997-98 South-East Asian and Russian crises. These events may be largely responsible for the increasing international correlation obtained in both papers.

[5]Another way to compute correlations conditional to the regime has been advocated by Longin and Solnik (1998), in the context of the multivariate extreme value theory.

3 Econometric methodology

In this section, we recall some methodological aspects of two well-known econometric models: the constant-correlation multivariate GARCH model and the multivariate MS model.

3.1 The constant-correlation multivariate GARCH model

A standard approach to modelling time-varying volatility is the ARCH and GARCH models, formulated by Engle (1982) and Bollerslev (1986) respectively. Several multivariate extensions have been proposed in the literature. Two parsimonious specifications are the BEKK representation (Engle and Kroner, 1995) and the constant-correlation ARCH (Bollerslev, 1990). In the BEKK model, the conditional covariances are modelled in a similar way as conditional variances. Testing for a constant correlation in such a framework would be rather difficult, since this hypothesis cannot be expressed in terms of estimated parameters only. Conversely, in the constant-correlation model, time-varying conditional covariances are parametrized to be proportional to the product of corresponding conditional standard deviations. Therefore, the conditional correlation is a parameter to be estimated and it can be easily tested.

Let $r_t = \{r_{1t}, ..., r_{nt}\}$ denotes the $(n, 1)$ vector of returns. Then the multivariate process for returns can be written as:

$$\begin{aligned} r_t &= \mu + \varepsilon_t \\ \mu &= E\left[r_t | I_{t-1}\right] \\ \varepsilon_t | I_{t-1} &\sim N\left(0, H_t\right) \end{aligned} \tag{1}$$

where I_{t-1} is the information set available at time $t-1$ and H_t is the time-varying conditional covariance matrix. Let h_{ijt} denote the ij^{th} element of H_t and h_{it} the ii^{th} element of H_t. ε_t is the innovation process with mean zero and covariance matrix H_t, and it is assumed to be normally distributed. The standard representation of the constant-correlation GARCH(1,1) model is the following (Bollerslev, 1990):

$$h_{it} = \omega_i + \alpha_i \varepsilon_{it-1}^2 + \beta_i h_{it-1} \qquad i = 1, ..., n \tag{2}$$

$$h_{ijt} = \rho_{ij} \sqrt{h_{it} h_{jt}} \qquad i, j = 1, ..., n, j \neq i \tag{3}$$

where the conditional correlation, ρ_{ij}, is assumed to be constant over time.

Assuming conditional normality, the log-likelihood function for model (1) to (3) is

$$L\left(\theta_1\right) = -\frac{Tn}{2} \ln\left(2\pi\right) - \frac{1}{2} \sum_{t=1}^{T} \left(\ln\left|H_t\right| + \varepsilon_t' H_t^{-1} \varepsilon_t\right) \tag{4}$$

where $\theta_1 = \left\{\mu_i, \omega_i, \alpha_i, \beta_i, \rho_{ij}; i, j = 1, ..., n, j > i\right\}$ denotes the vector of parameters to be estimated. The log-likelihood function is maximized by the BHHH algorithm (Berndt, Hall, Hall, and Haussman, 1974) using numerical derivatives.

In order to account for non-normality of the residual distribution, we also estimate a GARCH model, in which standardized innovations, $\varepsilon_{it}/\sqrt{h_{it}}$, are assumed to be drawn from a Student-t distribution with ν degrees of freedom (Bollerslev, 1987). Such an assumption is designed to account for excess kurtosis in the residuals. The log-likelihood function for such a model is therefore

$$L(\theta_2) = \sum_{t=1}^{T} \ln \left(\Gamma\left(\frac{\nu+n}{2}\right) \left[\sqrt{\pi(\nu-2)}\Gamma\left(\frac{\nu}{2}\right)\right]^{-n} \left(1 + \frac{\varepsilon_t' H_t^{-1} \varepsilon_t}{(\nu-2)}\right)^{-\frac{\nu+n}{2}} |H_t|^{-\frac{1}{2}} \right) \quad (5)$$

where $\theta_2 = \left\{\mu_i, \omega_i, \alpha_i, \beta_i, \rho_{ij}, \nu; i, j = 1, ..., n, j > i\right\}$. Normality is obtained when $\nu \to +\infty$.

3.2 The multivariate Markov-switching model

Several authors argued that the MS model may be an alternative challenging way of modelling persistence in volatility. The MS model has been developed by Hamilton (1988, 1989). In this model, time series are assumed to have different values of the mean and variance in a small number of regimes. In the following basic model, the r_{it} process is assumed to depend on two underlying regimes, with constant mean and variance in both regimes:

$$r_{it} = \mu_i^0 S_t + \mu_i^1 (1 - S_t) + \sqrt{h_i^0 S_t + h_i^1 (1 - S_t)}\varepsilon_{it}$$

where $\varepsilon_{it} \sim iid\ N(0,1)$. μ_i^k and h_i^k are respectively the mean and variance of r_{it} in regime k. S_t denotes the unobserved regime of the system. S_t is assumed to follow a two-state Markov process:

$$\begin{aligned}
\Pr[S_t = 0 | S_{t-1} = 0] &= p \\
\Pr[S_t = 1 | S_{t-1} = 0] &= 1 - p \\
\Pr[S_t = 1 | S_{t-1} = 1] &= q \\
\Pr[S_t = 0 | S_{t-1} = 1] &= 1 - q.
\end{aligned}$$

Assuming conditional normality for each regime, the conditional distribution of r_{it} is expressed as a mixture of distributions:

$$r_{it} | I_{t-1} \sim \begin{cases} N(\mu_i^0, h_i^0) & \text{with probability } \pi_t \\ N(\mu_i^1, h_i^1) & \text{with probability } 1 - \pi_t \end{cases}$$

where $\pi_t = \Pr[S_t = 0 | I_{t-1}]$ is the conditional probability of being in regime 0.

As pointed out by Sola and Timmermann (1994), this model, although very simple, is able to generate persistence in the aggregated over regimes conditional variance process, defined as $h_{it} = E[r_{it}^2 | I_{t-1}] - E[r_{it} | I_{t-1}]^2$:

$$h_{it} = \pi_t \left[\left(\mu_i^0\right)^2 + h_{it}^0\right] + (1 - \pi_t)\left[\left(\mu_i^1\right)^2 + h_{it}^1\right] - \left[\pi_t \mu_i^0 + (1 - \pi_t)\mu_i^1\right]^2. \quad (6)$$

To see this, assume that r_{it} depends on two regimes, one regime characterized by a low variance and the other regime by a high variance. Then, according to eq. (6), if regimes are persistent, this model is sufficient to obtain persistence in volatility. On the contrary, a one-regime GARCH model is not capable of capturing the persistence of regimes. It will therefore imply a strong volatility persistence, even if volatility is constant within regime. Several empirical studies (in particular, Ramchand and Susmel, 1998) found that assuming further time-varying within-regime volatility is useless. In other words, the constant within-regime volatility MS model is sufficient to take into account most time-variability of volatility.

Generalizing this model to the multivariate case is quite easy. Assuming that innovations are correlated, with a constant conditional correlation within each regime, the covariance matrix within regime k is defined as $H^k = \left(h_{ij}^k\right)$, where $h_{ii}^k = h_i^k$ and $h_{ij}^k = \rho_{ij}^k \sqrt{h_i^k h_j^k}$, with ρ_{ij}^k the correlation coefficient in regime k.

To keep the model parsimonious, we adopt the following specification. First stock markets are assumed to switch from one regime to the other at the same time. Therefore regimes in various markets are perfectly correlated and transition probabilities are identical for all stock returns. This assumption is intended to distinguish unambiguously between calm and turbulent regimes.[6] This can be related to the widely reported empirical evidence on the existence of volatility spillover between stock markets (Hamao, Masulis, and Ng, 1990). Second we do not consider time-varying transition probabilities, in order to deal with computational burden. Under normality, the vector of parameters to be estimated is then $\theta_3 = \left\{\mu_i, h_i^0, h_i^1, \rho_{ij}^0, \rho_{ij}^1, p, q; i, j = 1, ..., n, j > i\right\}$. When innovations are assumed to be Student-t distributed, the degree of freedom, ν, is added to the parameter vector.

Estimation of a MS model is usually performed using (quasi) maximum likelihood (QML) estimation. The sample log-likelihood function of the multivariate MS model is

$$\begin{aligned}
\ln L(\theta_3) &= \sum_{t=1}^{T} \ln\left(f\left(r_t | I_{t-1}\right)\right) = \sum_{t=1}^{T} \ln\left(\sum_{k=0}^{1} f\left(r_t | S_t = k, I_{t-1}\right) \Pr\left[S_t = k | I_{t-1}\right]\right) \\
&= \sum_{t=1}^{T} \ln\left(\sum_{k=0}^{1} g_t^k \pi_t\right)
\end{aligned}$$

where $\pi_t = \Pr\left[S_t = 0 | I_{t-1}\right]$ is computed as

$$\pi_t = (1-q) \frac{g_{t-1}^1 (1-\pi_{t-1})}{g_{t-1}^0 \pi_{t-1} + g_{t-1}^1 (1-\pi_{t-1})} + p \frac{g_{t-1}^0 \pi_{t-1}}{g_{t-1}^0 \pi_{t-1} + g_{t-1}^1 (1-\pi_{t-1})}$$

and $g_t^k = f\left(r_t | S_t = k, I_{t-1}\right)$ is computed as

$$g_t^k = \frac{1}{\sqrt{2\pi}} \left|H_t^k\right|^{-1/2} \exp\left(-\frac{1}{2}\left(r_t - \mu^k\right)' \left(H_t^k\right)^{-1} \left(r_t - \mu^k\right)\right) \qquad k = 0, 1.$$

The log-likelihood function can be computed recursively. Reported standard errors are heteroskedasticity consistent. See Gray (1996) for additional details on the estimation method for the MS models.

[6] Alternatively, in a bivariate setting, Ramchand and Susmel (1998) consider a model with four regimes. But to keep the system tractable, they then assume that correlations only depend on the state of the US return.

3.3 A multivariate GARCH model with changes in regime

Conditional on the regime, the MS model assumes that mean and variance are constant over time. In particular, this rules out within-regime conditional heteroskedasticity. Models allowing for time-varying conditional moments within each regime have been proposed by Cai (1994), Hamilton and Susmel (1994) and Gray (1996). In MS-GARCH models, some or all of the parameters are regime dependent. Assuming that the conditional mean remains constant within each regime but that the conditional variance has a GARCH dynamics, we obtain:

$$r_{it} = \mu_i^0 S_t + \mu_i^1 (1 - S_t) + \sqrt{h_{it}^0 S_t + h_{it}^1 (1 - S_t)} \varepsilon_t \tag{7}$$

where h_{it}^0 and h_{it}^1 have the same functional form as eq. (2). But, as suggested by Gray (1996), h_{it}^0 and h_{it}^1 are constructed using lagged aggregated over regimes conditional variance, h_t, which is not path dependent (see eq. (6)), so that

$$\begin{aligned} h_{it}^0 &= \omega_i^0 + \alpha_i^0 \varepsilon_{it-1}^2 + \beta_i^0 h_{it-1} \qquad i = 1, ..., n \\ h_{it}^1 &= \omega_i^1 + \alpha_i^1 \varepsilon_{it-1}^2 + \beta_i^1 h_{it-1}. \end{aligned}$$

Measuring the link between an increase in conditional correlation and an increase in volatility requires a further generalization of the MS-GARCH model to the multivariate case. We then adopt a multivariate MS-GARCH model with a constant conditional correlation within each regime. This model can be summarized as follows:

Mean equation:

$$\varepsilon_t = r_t - \mu_t$$

with

$$\mu_t = E\left[r_t | I_{t-1}\right] = \pi_t \mu^0 + (1 - \pi_t) \mu^1.$$

Variance and covariance equations:

$$h_{it}^0 = \omega_i^0 + \alpha_i^0 \varepsilon_{it-1}^2 + \beta_i^0 h_{it-1} \qquad i = 1, ..., n \tag{8}$$

$$h_{it}^1 = \omega_i^1 + \alpha_i^1 \varepsilon_{it-1}^2 + \beta_i^1 h_{it-1} \tag{9}$$

and

$$h_{ijt}^0 = \rho_{ij}^0 \left(h_{it}^0 h_{jt}^0\right)^{1/2} \qquad i, j = 1, ..., n, j \neq i \tag{10}$$

$$h_{ijt}^1 = \rho_{ij}^1 \left(h_{it}^1 h_{jt}^1\right)^{1/2}. \tag{11}$$

The covariance matrices within each regime are: $H_t^k = \left(h_{ijt}^k\right)$, $k = 0, 1$. The aggregated over regimes covariance matrix at time t is then defined as

$$\begin{aligned} H_t &= E\left[r_t r_t' | I_{t-1}\right] - E\left[r_t | I_{t-1}\right] E\left[r_t | I_{t-1}\right]' \\ &= \pi_t \left(\mu^0 \mu^{0\prime} + H_t^0\right) + (1 - \pi_t) \left(\mu^1 \mu^{1\prime} + H_t^1\right) - \mu_t \mu_t' \end{aligned} \tag{12}$$

with $H_t = (h_{ijt})$.

Matrix of transition probabilities:

$$\begin{aligned}
\Pr\left[S_t = 0|S_{t-1} = 0\right] &= p \\
\Pr\left[S_t = 1|S_{t-1} = 0\right] &= 1 - p \\
\Pr\left[S_t = 1|S_{t-1} = 1\right] &= q \\
\Pr\left[S_t = 0|S_{t-1} = 1\right] &= 1 - q.
\end{aligned}$$

Under normality, the vector of parameters to be estimated is $\theta_4 = \{\mu_i, \omega_i^0, \omega_i^1, \alpha_i^0, \alpha_i^1, \beta_i^0, \beta_i^1, \rho_{ij}^0, \rho_{ij}^1, p, q; i, j = 1, ..., n, j > i\}$. When innovations are assumed to be Student-t distributed, the degree of freedom, ν, is added to the parameter vector.

The MS and MS-GARCH models described above are designed to test the null hypothesis of a conditional correlation constant across regimes. Indeed, regimes are only characterized by their covariance matrix, since returns are assumed to be regime-independent. Then it is generally possible to identify a low-volatility regime and a high-volatility regime.[7] In this case, one only has to compare conditional correlations obtained for both regimes. The test of the null hypothesis of a conditional correlation matrix constant across regimes is based on the likelihood-ratio (LR) test statistic $\xi = 2\left(L(\theta) - L\left(\theta^0\right)\right)$, where θ^0 corresponds to the vector of parameters for $\rho_{ij}^0 = \rho_{ij}^1$, $i, j = 1, ..., n, j > i$. Under the null, the test statistic ξ is distributed as a $\chi^2_{n(n-1)/2}$.

4 Empirical results

4.1 The multivariate GARCH models

We begin with estimating multivariate GARCH models. First, we wish to show that stock-return conditional volatility is time varying and, therefore, that a GARCH or MS representation is relevant. Second, we want to illustrate some drawbacks of the GARCH approach. In particular, GARCH models generally induce excessive persistence in volatility dynamics.

Table 4 reports several summary statistics for various constant-correlation multivariate GARCH models. The statistics include the log-likelihood as well as the model selection statistics proposed by Akaike (1976) and Schwartz (1978), the Ljung-Box statistic corrected for heteroskedasticity and the TR^2 Engle statistic for conditional heteroskedasticity. First, the standard Gaussian GARCH model clearly dominates the model with a constant covariance matrix. Under the null hypothesis of a constant covariance matrix, the LR test statistic is distributed as a χ^2_6. Since the LR test statistic is equal to 126.2, the null hypothesis is rejected at any usual significance level. For all countries, the volatility persistence (measured by $\lambda_i = \alpha_i + \beta_i$) is ranging from 0.964 to 0.993. Therefore, for all indices, the dynamics of conditional volatility is very close to non-stationarity.

Many authors (following Black, 1976) have argued that a stock-return decrease tends to increase subsequent volatility by more than would a stock-return increase

[7] Note however that, in a multivariate context, it is not always possible to identify low- and high-volatility regimes, since all stock returns have not necessarily their low volatility in the same regime.

of the same magnitude. Estimating a model incorporating this so-called leverage effect reduces only slightly the volatility persistence. Indeed, we estimated a Glosten, Jagannathan, and Runkle (1993) model (GJR), in which the volatility dynamics in eq. (2) is replaced by

$$h_{it} = \omega_i + \alpha_i \varepsilon_{it-1}^2 + \beta_i h_{it-1} + \gamma_i \varepsilon_{it-1}^2 1_{\{\varepsilon_{it-1}<0\}} \qquad i = 1, ..., n$$

where $1_{\{\text{condition}\}}$ is the indicator variable taking the value 1 if the condition is true and 0 otherwise. In this model, volatility is non-stationary when $\lambda_i = \alpha_i + \beta_i + \gamma_i/2 \geq 1$. When the GJR model is estimated, leverage effects are particularly significant for the UK stock return, but not for other markets.[8] However, introducing asymmetric effects does not allow the conditional volatility persistence to diminish significantly, since, as shown in Table 4, the volatility persistence is ranging between 0.945 and 0.989.

When turning to the GARCH model with Student-t distributed innovations, we obtain that the parameter estimate for the degrees of freedom, ν, is rather large, at about 13. The null hypothesis of normality is strongly rejected (the t-stat for $1/\nu$ is equal to 4.2). LR tests also indicate that the Student-t distribution improves significantly the model likelihood at the 1 percent level. Once again, however, assuming Student-t residuals does not reduce the persistence in conditional volatility.

Last incorporating both the Student-t distribution and the asymmetric effect in the model (Student-t GJR) does not improve the previous results significantly. The null hypothesis of non-significant asymmetry parameters ($\gamma_i = 0$, $i = 1, 2, 3$) is not rejected at the 5 percent level.

To conclude on one-regime GARCH models, the Student-t distribution appears to be more appropriate than the Gaussian distribution for modelling innovations. Besides, unlike some previous univariate results (such as Hamilton and Susmel, 1994) the GARCH model is not rejected in favor of the GJR model. Over these various one-regime models, the Schwartz criterion is maximized for the Student-t GARCH model. It is worth noting that this model is not without drawbacks, since volatility persistence is systematically very close to one, implying near non-stationary volatilities for each stock return.

4.2 The multivariate MS models

In this subsection, we examine results obtained with various multivariate MS models. Summary statistics are reported in Table 6. Each model is estimated assuming Gaussian or Student-t distributed innovations. Panel A is devoted to model with regime-dependent correlations, whereas results for the model with correlations constant across regimes are displayed in Panel B. In Table 7, we present LR tests for various null hypotheses.

We first consider different specification tests in order to rank our models. The LR test overwhelmingly rejects the Gaussian distribution in favor of the Student-t

[8] Unlike previous studies, we do not find GJR effects on the US market. This contrasting result may be explained by the day-of-the-week issue. In fact, we also carried out estimates using other days of the week, which have yielded a significant GJR effect.

distribution. The degree of freedom, ν, is large (above 17) but in each case, $1/\nu$ is found to be significantly different from 0. Moreover, the LR test statistics as well as the AIC and Schwartz criteria reject the Gaussian formulation in favor of the Student-t one.

Another interesting feature would be to test the statistical significance of the second regime. However this cannot be performed using a LR test, because parameters associated with the second regime are not identified under the null hypothesis. Therefore regularity conditions justifying the χ^2 approximation to the LR test do not hold. Hansen (1992, 1996) has proposed a LR test procedure that gets around this problem. But, even for simple models, the computational burden can be important. We thus report the usual LR test statistics as a descriptive summary of the fit of the alternative models.[9] The p-values we obtained are so tiny that there is little doubt about the existence of a second regime. In particular, when comparing the MS model with the constant-variance model, the quasi-LR test statistics are as high as 155.6 and 106.1 for Gaussian and Student-t distributed innovations respectively. Moreover as discussed later, estimated variances and covariances substantially differ in each regime. We can therefore consider the addition of the second regime to be economically significant.

Maximum-likelihood parameter estimates of the MS model with Student-t innovations are reported in table 8. For aim of comparison, the first column reports the parameter estimates of the one-regime constant-variance model. The conditional mean and variance terms are very close to the unconditional means and variances shown in Table 1. The conditional correlations are also very close to the unconditional correlation coefficients reported in Table 2. The degree of freedom for the Student-t distribution is equal to 8.5. All parameters are strongly significant. However, diagnostic tests indicate that there is serial correlation in the squared standardized residuals.

The second column of Table 8 shows parameter estimates for the MS model. Since conditional means are assumed to be regime-independent, they are close to those displayed in the first column. The two regimes are strongly persistent since the transition probabilities p and q are very large, at 0.991 and 0.990 respectively. Both regimes would be expected to last on average for $(1-p)^{-1} = 100$ weeks. The first regime is characterized by lower variances and lower correlations. Indeed, the regime-1 variances are between 2 and 4 times the regime-0 variances. Moreover correlation coefficients increase by about 0.2 from regime 0 to regime 1. For instance, the conditional correlation between the DAX and FTSE returns is 0.46 during calm periods and 0.67 during turbulent periods.

Last, column 3 reports parameter estimates for the MS model with correlations constant across regimes. Estimates for conditional means and variances are almost unchanged as compared to the MS model with correlations varying across regimes. Conditional correlations are estimated to be close to the unconditional correlations in Table 2. The LR test statistic for regime-independent correlations is equal to

[9] Using Monte Carlo experiments, Garcia (1998) has shown that the LR asymptotic distribution approximates the empirical distribution very well, for some simple MS models. Ang and Bekaert (1999) also use Monte Carlo simulations to find the small sample distribution of LR test statistics.

17.9. Since it is distributed as a chi-square with 3 degrees of freedom, the null hypothesis is strongly rejected at any usual significance level. In the MS model with regime-independent correlations, estimated correlations are $(0.42; 0.51; 0.55)$. The corresponding correlations in the MS model with correlations varying across regimes are $(0.34; 0.42; 0.46)$ in regime 0 and $(0.53; 0.63; 0.67)$ in regime 1.

Summary statistics indicate that, at least for the DAX, residuals display heteroskedasticity. Therefore volatility persistence is shown to have two sources: (1) persistence of regimes, which is modelled with MS model, and (2) within-regime volatility clustering, a feature which is not incorporated in this model. Then MS-GARCH models are well designed to deal with this problem.

4.3 The multivariate MS-GARCH model

We turn now to MS-GARCH model's results. Summary statistics are displayed in Table 6 and LR tests are reported in Table 7. First, with a LR test statistic equal to 17.8, the null hypothesis of a Gaussian distribution is strongly rejected at any usual level.

Second, although this is not a formal test, the quasi-LR test statistics for the null hypothesis of only one regime are so large (53.8 and 43.3 for Gaussian and Student-t distributed innovations respectively) that we can confidently accept the existence of a second regime.

Testing the significance of GARCH parameters gives mixed results. In effect, according to LR tests, the MS model is rejected in favor of the MS-GARCH model, at the 5 percent level. But the p-value is only 1.8% for the Gaussian model and 0.3% for the model with Student-t distribution innovations. Moreover the Schwartz criterion clearly rejects the MS-GARCH model in favor of the MS model.

In Table 9, we present maximum likelihood parameter estimates of GARCH models with Student-t innovations. The first column reports estimates of the one-regime GARCH model. All parameters of interest are significant. Results highlight a strong volatility persistence, a common feature of GARCH models. Engle test statistics indicate that standardized residuals are broadly homoskedastic.

The second column reports parameter estimates of the MS-GARCH model with regime-dependent correlations. For the US, the high-volatility regime (regime 1) displays more sensitivity to recent shocks ($\alpha_1^0 < \alpha_1^1$) but less persistence ($\beta_1^0 > \beta_1^1$) than the low-volatility regime. This result has been already pointed out by Gray (1996) for characterizing US interest-rate volatility. However, other stock markets under study only partly display a similar pattern. Indeed, in Germany as well as in the UK, volatility is strongly persistent within both regimes.

Further scrutiny of Table 9 shows that several volatility-equation parameters are not significantly different from 0 in the MS-GARCH model. This is a rather disappointing result, because it is then difficult to clearly characterize within-regime volatility dynamics. More precisely, even if the effect of recent shocks (α_i^k) is generally quite reasonable and close to previous findings (mainly in the US and in Germany), it is systematically non significant. This result may be due to the large number of parameters and/or to some multicolinearity effects.

The last column of Table 9 reports parameter estimates of the MS-GARCH model with correlations constant across regimes. Some autoregressive parameters in volatility equations (β_i^k) markedly decrease as compared with the MS-GARCH model with regime-dependent correlations. In particular, this is the case for the calm regime in the US and Germany. Conditional correlations are very close to those obtained with the MS model.

The LR test statistic for regime-independent correlations in the MS-GARCH model is 13.7, with a p-value of 0.3%. In the MS-GARCH model with regime-independent correlations, correlations are estimated to be $(0.43; 0.52; 0.56)$. Assuming regime-dependent correlations, the corresponding correlations are $(0.26; 0.40; 0.51)$ in regime 0 and $(0.57; 0.62; 0.62)$ in regime 1. We note that estimated within-regime correlations are very similar for MS models and MS-GARCH models.

4.4 The economic importance of switching models

4.4.1 Characterizing time-varying regime probabilities and correlations

At this stage, we get a further insight into the economic importance of switching models, in particular by studying international correlations. Fig. 2 plots weekly stock return series r_{it} for each stock market. As in Fig. 1, beginning of 1992 and 1996 are marked with vertical lines. The S&P and the DAX appear more volatile over the first and the last subperiod. When turning to the FTSE return, the three subperiods display a similar pattern. In particular the second period does not appear to be less volatile than the other ones, since the two largest returns occurred in April and September 1992.

Fig. 3 contains plots of the ex-ante probabilities $\Pr[S_t = 0|I_{t-1}]$ and the smoothed probabilities $\Pr[S_t = 0|I_T]$. These probabilities are computed as derived in Gray (1995), whose smoothing algorithm directly relates ex-ante probabilities and corresponding smoothed probabilities. Top panel of Fig. 3 plots ex-ante and smoothed probabilities for the MS model. The high-volatility regime (regime 1) can be associated with three periods, ignoring the very beginning of the sample: (1) from end-1989 to mid-1991, (2) from beginning of 1997 to beginning of 1998 and (3) since the end of 1998. The first of these periods (from October 1989 to May 1991) begins with the mini-crash of October 13, 1989 in the US and also corresponds to the Kuwait crisis from Iraq's invasion on August 2, 1990 through the conclusion of the Gulf war on March 3, 1991. The second period (April 1997-March 1998) is clearly driven by the South-East Asian crisis, which actually started in June 1997. The last period (from August 1998 to now) has been clearly initiated by the Russian crisis, which started with the collapse of the bond market at the beginning of August. We also note a short-lasting spike in September 1992 corresponding the EMS crisis, which implied a strong increase in the FTSE volatility.

Bottom panel of Fig. 3 plots ex-ante probabilities and smoothed probabilities for the MS-GARCH model. Although probabilities obtained using the MS and the MS-GARCH models display a broadly similar pattern, some differences are worth noting. First, considering ex-ante probabilities, the second half of 1988 is now considered as a turbulent period. Moreover, the MS-GARCH model implies a much longer lasting high-volatility episode at the beginning of the 90s than the MS model. Indeed the probability of being in regime 0 remains lower than 50% until April 1992, just after the FTSE boom, when the UK entered the EMS. Last the low-volatility regime ended at the end of 1996, almost one year before what was found with the MS model. Moreover the Russian crisis in 1998 is no longer considered as a turbulent period.

Smoothed probabilities are even more clear-cut, since the whole period is characterized with only two regime shifts. A first, turbulent, subperiod ends at the end of 1990, after the invasion of Kuwait by Iraq. The second shift occurred at the beginning of 1996 more than one year before the South-East Asian crisis started. This is due to the fact that the smoothed probability at date t is computed using information on the whole sample. Therefore, as in the usual Kalman smoother, smoothed probabilities seem to precede ex-ante probabilities.

Conditional correlations implied by the MS model are plotted in the left panel of Fig. 4. For each stock market, we display correlations estimated using the MS model with regime-dependent as well as regime-independent correlations. Since correlations are assumed to be constant within regime, there are only two possible levels of correlation and therefore the conditional correlation mimics the ex-ante regime probabilities. However, unlike most previous tests, our test procedure is not based on data-mining. Indeed our low-volatility and high-volatility regimes are determined endogenously during the estimation. They have no need to be chosen beforehand.

Last an interesting feature offered by the MS-GARCH model is that the transition from the low correlation to the high correlation is much more smoothed than with the MS model (right panel of Fig. 4). For instance, the US-German correlation decreased very gradually from 0.53 to 0.3 from the end of 1990 to the beginning of 1992. Conversely, with the MS model, the same change in correlation was attained in less than two months.

4.4.2 Volatility-forecasting ability tests

It is interesting to compare the volatility-forecasting ability of the different models studied. In particular this comparison can be seen as a test of over-parameterization of switching models. Moreover we can measure the usefulness of switching models to forecast the covariance matrix.

We adopt the methodology used, for instance, by Hamilton and Susmel (1994). For a particular model, we compute a time series of conditional variances and covariances over the sample. We then compare the conditional variance to the actual corresponding squared innovations and the conditional covariance to the cross-product of the actual corresponding innovations. Subsequently we measure the following loss functions:

$$RMSE_{ij} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(\hat{\varepsilon}_{it}\hat{\varepsilon}_{jt} - \widehat{h}_{ijt}\right)^2} \qquad i,j = 1,...,n$$

and

$$MAE_{ij} = \frac{1}{T}\sum_{t=1}^{T}\left|\hat{\varepsilon}_{it}\hat{\varepsilon}_{jt} - \widehat{h}_{ijt}\right|$$

where $\hat{\varepsilon}_{it} = r_{it} - \bar{r}_i$, with $\bar{r}_i = \frac{1}{T}\sum_{t=1}^{T} r_{it}$ and $\widehat{h}_{ijt}$ is the ij^{th} element of the estimated conditional covariance matrix, $\widehat{H_t}$. $\widehat{H_t}$ is defined by eq. (2) and (3) for the GARCH model, and by eq. (12) for the MS and MS-GARCH models.

Table 10 reports results of in-sample one-period-ahead forecasts using the different models studied. In Panel A, parameters are estimated over the whole sample, as reported in Table 8 and Table 9, and forecasts are performed over the same sample for each model. According to RMSE, the MS-GARCH with regime-dependent correlations performs very well in forecasting S&P and DAX variances and the S&P-DAX correlation. Conversely, the MS model appears to out-perform other models in forecasting the FTSE variance and the S&P-FTSE and DAX-FTSE correlations. Turning to the MAE, results are less clear-cut, since the MS model with correlations constant across regimes performs better in forecasting the S&P-FTSE and DAX-FTSE correlations.

A second test examines the forecasting ability of various models when models are estimated over the period from January 1988 to March 1998. Therefore parameters are identified using only one episode of each regime. As shown in Panel B, in-sample one-period-ahead forecasts give basically the same ranking as in Panel A.

Last but not least, Panel C reports out-of-sample one-period-ahead forecasts computed over the period from April 1998 to December 1999. Parameter estimates are estimated over the period from January 1988 to March 1998. According to RMSE, the MS model with regime-dependent correlation clearly out-performs all other models in forecasting correlations. As before, MAE statistics do not allow to conclude in favor of a specific model.

In summary, it is worth noting that these results are not indicative of any overfitting of MS and MS-GARCH models. Indeed, although a large number of parameters have to be estimated for these models, out-of-sample forecasts are fairly good as compared to one-regime models.

5 Conclusion

In this paper, we consider the relationship between international correlation and stock-market turbulence. We assume that stock markets are driven by two regimes, characterized by a low and a high volatility. We then estimate multivariate regime-switching models and test the null hypothesis that correlations are constant across regimes. The reference model is the standard multivariate GARCH model with constant correlation. Two alternative regime-switching models are studied: a simple MS model, in which variances are constant within regime, and a MS-GARCH model, in which variances are modeled as GARCH processes.

Using weekly stock return series for the S&P, the DAX and the FTSE over the period from January 1988 to December 1999, we find that MS and MS-GARCH specifications offer a better statistical fit to the data than standard multivariate GARCH models. Turning to the hypothesized relationship between international correlation and stock-market turbulence, we actually obtain that correlations are much higher during the high-volatility regime than during the low-volatility regime. We perform a LR test which confirms that an increase in volatility is usually associated with an increase in correlation. Broadly speaking, our sample can be split into three subperiods corresponding to different levels of volatility: before 1992, stock markets faced a high-volatility regime, associated in particular with the Gulf crisis; the second agitated period started in 1997 and was characterized by the South-East Asian crisis and the Russian crisis. The 1992-96 period is found to be a low-volatility regime.

Our test procedure improves previous tests based on a data-driven selection of high- and low-volatility subperiods. Indeed these tests are shown to be biased because of the regime selection (Boyer, Gibson, and Loretan, 1997). Conversely, in the approach developed in this paper, regimes are determined endogenously and are consistent with the data generating process. Therefore, our test procedure does not suffer from any selection bias.

Our work may be extended in two ways. First, the econometric model may be improved to incorporate further statistical features of stock returns. As already indicated, we did not succeed in introducing leverage effects in our models. Another improvement on our Markov-switching models would be to allow transition probabilities to be different across markets and/or to vary over time. Hamilton and Lin (1996) incorporated the first extension and Gray (1996) incorporated the second one. However, in a multivariate framework, such extensions would dramatically increase the computational burden.

Second, our test for a regime-independent conditional correlation may be performed for other groups of markets. In particular it would be interesting to assess whether correlation between emerging-market returns really increased during the well-documented Mexican (1994), South-East Asian (1997) and Russian (1998) crises. Many papers focused on these episodes (for instance, Baig and Goldfajn, 1998, and Forbes and Rigobon, 1999), but most of them considered unconditional correlations computed over subperiods selected ex post and therefore incorporating all information about past crises. In order to avoid a data-mining approach, we choose to develop an unbiased test for a regime-independent conditional correlation, which is consistent with the data generating process and thus free from selection bias.

Appendix 1: Test of constant unconditional correlation matrix

A convenient way to test the null hypothesis of a constant unconditional correlation matrix is to test for the equality of the correlation matrices computed over two subsamples. Different test statistics have been proposed in the literature to perform such a test. One of the most popular is the test developed by Jennrich (1970), based on the normalized difference between the two correlation matrices.[10] The test for the equality of the correlation matrices, denoted R_1 and R_2, over two independent subsamples of equal size $n_1 = n_2 = n$ is based on the statistics:

$$\chi^2 = \frac{1}{2} tr\left(Z^2\right) - \text{diag}\left(Z\right)' S^{-1} \text{diag}\left(Z\right)$$

where $Z = \sqrt{\frac{n}{2}} R^{-1} \left(R_1 - R_2\right)$; $R = \frac{1}{2}\left(R_1 + R_2\right)$ is the average correlation matrix over the two subsamples; $S = \left(\delta_{ij} + r_{ij} r^{ij}\right)$ with $R = \left(r_{ij}\right)$, $R^{-1} = \left(r^{ij}\right)$ and

$$\delta_{ij} = \begin{cases} 1 \text{ if } i = j \\ 0 \text{ otherwise} \end{cases}$$

and diag(X) denotes the diagonal of the square matrix X in a column form. The Jennrich test statistic has an asymptotic chi-square distribution with $p\left(p-1\right)/2$ degrees of freedom, if the correlation matrix is computed for p variables.

We note that the test statistic for constant correlation between two variables $(p = 2)$ is simply:

$$\chi^2 = \frac{n}{2} \frac{\left(r_1 - r_2\right)^2}{\left(1 - r^2\right)^2}$$

where r_1 and r_2 are the estimated correlation over the two subsamples and $r = \frac{1}{2}\left(r_1 + r_2\right)$.

[10] Box (1949) also proposed a statistic for testing the equality of two covariance matrices. His test however can not be adapted for testing the equality of two correlation matrices.

References

[1] Akaike, H. (1974), A New Look at the Statistical Model Identification, *I.E.E.E. Transaction on Automatic Control*, 19(6), 716-723.

[2] Ang, A., and G. Bekaert (1999), International Asset Allocation with Time-Varying Correlations, NBER Working Paper 7056.

[3] Baillie, R.T., and R.P. DeGennaro (1990), Stock Returns and Volatility, *Journal of Financial and Quantitative Analysis*, 25(2), 203-214.

[4] Bera, A.K., and S. Kim (1996), Testing Constancy of Correlation with an Application to International Equity Returns, CIBER Working Paper 96-107.

[5] Berndt, E.K., H.B. Hall, R.E. Hall, and J.A. Haussman (1974), Estimation and Inference in Nonlinear Structural Models, *Annals of Economic and Social Measurement*, 4, 653-666.

[6] Bertero, E., and C. Mayer (1990), Structure and Performance: Global Interdependence of Stock markets around the Crash of October 1987, *European Economic Review*, 34(6), 1155-1180.

[7] Black, F. (1976), Studies in Stock Price Volatility Changes, *Proceedings of the 1976 Business Meeting of the Business and Economic Statistics Section*, American Statistical Association.

[8] Bollerslev, T. (1986), Generalized Autoregressive Conditional Heteroskedasticity, *Journal of Econometrics*, 31(3), 307-327.

[9] Bollerslev, T. (1987), A Conditional Heteroskedastic Time Series Model for Speculative Prices and Rates of Return, *Review of Economics and Statistics*, 69(3), 542-547.

[10] Bollerslev, T. (1990), Modelling the Coherence in Short-Run Nominal Exchange Rates: A Multivariate Generalized ARCH Model, *Review of Economics and Statistics*, 72(3), 498-505.

[11] Boyer, B.H., M.S. Gibson, and M. Loretan (1997), Pitfalls in Tests for Changes in Correlations, Board of Governors of the Federal Reserve System, International Finance Discussion Paper 597.

[12] Box, G.E.P. (1949), A General Distribution Theory for a Class of Likelihood Criteria, *Biometrika*, 36, 317-346.

[13] Cai, J. (1994), A Markov Model of Switching-Regime ARCH, *Journal of Business and Economic Statistics*, 12(3), 309-316.

[14] Engle, R.F. (1982), Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of UK Inflation, *Econometrica*, 50(4), 987-1007.

[15] Engle, R.F., and K. Kroner (1995), Multivariate Simultaneous Generalized ARCH, *Econometric Theory*, 11(1), 122-150.

[16] Engle, R.F., and V.K. Ng (1993), Measuring and Testing the Impact of News on Volatility, *Journal of Finance*, 48(5), 1749-1778.

[17] Eun, C., and S. Shim (1989), International Transmission of Stock Market Movements, *Journal of Financial and Quantitative Analysis*, 24(2), 241-256.

[18] Forbes, K., and R. Rigobon (1999), No Contagion, Only Interdependence: Measuring Stock Market Co-Movements, NBER Working Paper 7267.

[19] Friedman, B.M., and D.I. Laibson (1989), Economic Implications of Extraordinary Movements in Stock Prices, *Brookings Papers on Economic Activity*, 2, 137-189.

[20] Garcia, R. (1998), Asymptotic Null Distribution of the Likelihood Ratio Test in Markov Switching Models, *International Economic Review*, 39(3), 763-788.

[21] Glosten, R.T., R. Jagannathan, and D. Runkle (1993), On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks, *Journal of Finance*, 48(5), 1779-1801.

[22] Gray, S.F. (1995), An Analysis of Conditional Regime-Switching Models, Working Paper, Fuqua School of Business, Duke University.

[23] Gray, S.F. (1996), Modeling the Conditional Distribution of Interest Rates as a Regime-Switching Process, *Journal of Financial Economics*, 42(1), 27-62.

[24] Hamao, Y., R.W. Masulis, and V. Ng (1990), Correlations in Price Changes and Volatility Across International Stock Markets, *Review of Financial Studies*, 3(2), 281-307.

[25] Hamilton, J.D. (1988), Rational Expectations Econometric Analysis of Changes in Regime: An Investigation of the Term Structure of Interest Rates, *Journal of Economic and Dynamics and Control*, 12(2/3), 385-423.

[26] Hamilton, J.D. (1989), A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle, *Econometrica*, 57(2), 357-384.

[27] Hamilton, J.D., and G. Lin (1996), Stock Market Volatility and the Business Cycle, *Journal of Applied Econometrics*, 11, 573-593.

[28] Hamilton, J.D., and R. Susmel (1994), Autoregressive Conditional Heteroscedasticity and Changes in Regime, *Journal of Econometrics*, 64(1-2), 307-333.

[29] Hansen, B.E. (1992), The Likelihood Ratio Test under Non-standard Conditions: Testing the Markov Switching Model of GNP, *Journal of Applied Econometrics*, 7, S61-S82.

[30] Hansen, B.E. (1996), Inference When a Nuisance Parameter Is Not Identified under the Null Hypothesis, *Econometrica*, 64(2), 413-430.

[31] Jennrich, J.I. (1970), An Asymptotic Chi-square Test for the Equality of Two Correlation Matrices, *Journal of the American Statistical Association*, 65(330), 904-912.

[32] Kaplanis, E.C. (1988), Stability and Forecasting of the Co-movement Measures of International Stock Market Return, *Journal of International Money and Finance*, 8(1), 63-95.

[33] King, M., and S. Wadhwani (1990), Transmission of Volatility between Stock Markets, *Review of Financial Studies*, 3(1), 5-33.

[34] King, M., E. Sentana, and S. Wadhwani (1994), Volatility and Links between National Stock Markets, *Econometrica*, 62(4), 901-933.

[35] Koch, P.D., and T.W. Koch (1991), Evoluation in Dynamic Linkages across National Stock Indexes, *Journal of International Money and Finance*, 10(2), 231-251.

[36] Lee, S.B., and K.J. Kim (1993), Does the October 1987 Crash Strengthen the Co-Movements among National Stock Markets? *Review of Financial Economics*, 3(1), 89-102.

[37] Longin, F., and B. Solnik (1995), Is the Correlation in International Equity Returns Constant: 1960-1990?, *Journal of International Money and Finance*, 14(1), 3-26.

[38] Longin, F., and B. Solnik (1998), Correlation Structure of International Equity Markets during Extremely Volatile Periods, HEC Working Paper CR 646.

[39] Loretan, M., and W.B. English (1999), Evaluate Correlation Breakdowns during Periods of Market Volatility, Board of Governors of the Federal Reserve System, mimeo.

[40] Markowitz, H.M. (1952), Portfolio Selection, *Journal of Finance*, 7(1), 77-91.

[41] Ramchand, L., and R. Susmel (1998), Volatility and Cross Correlation across Major Stock Markets, *Journal of Empirical Finance*, 5, 397-416.

[42] Ratner, M. (1992), Portfolio Diversification and the Inter-temporal Stability of International Indices, *Global Finance Journal*, 3, 67-78.

[43] Roll, R. (1989), Price Volatility, International Market Links and their Implications for Regulatory Policies, *Journal of Financial Services Research*, 3, 211-236.

[44] Schaller, H., and S. van Norden (1997), Regime Switching in Stock Market Returns, *Applied Financial Economics*, 7(2), 177-191.

[45] Schwartz, G. (1978), Estimating the Dimension of a Model, *Annals of Statistics*, 6(2), 461-464.

[46] Sola, M., and A. Timmermann (1994), Fitting the Moments: A Comparison of ARCH and Regime Switching Models for Daily Stock Returns, Discussion Paper in Financial Economics, FE 1/94, Birkbeck College.

[47] Susmel, R., and R.F. Engle (1994), Hourly Volatility Spillovers between International Equity Markets, *Journal of International Money and Finance*, 13(1), 3-25.

[48] Von Furstenberg, G.M., and B.N. Jeon (1989), International Stock Prices Movements: Links and Messages, *Brookings Papers on Economic Activity*, 1, 125-179.

[49] White, H. (1980), A Heteroscedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroscedasticity, *Econometrica*, 48(4), 817-838.

Table 1: Summary statistics on weekly stock returns

	S&P	DAX	FTSE
Mean	0.274	0.281	0.210
Std dev.	1.920	2.627	2.034
Sk	-0.528	-0.589	0.157
*Sk**	-5.371	-5.986	1.598
XKu	1.579	1.289	0.916
*XKu**	8.027	6.549	4.658
J-B	93.276	78.730	24.252
Engle(4)	26.317**	52.298**	23.511**
Engle(12)	50.924**	83.626**	34.919**
$LB(4)$	6.219	7.222	4.496
$LB(12)$	27.155**	19.594	13.381
$LB_c(4)$	5.639	4.603	4.117
$LB_c(12)$	22.779*	14.986	13.280

Note: The sample period is January 1988 to December 1999, a total of 620 observations. Std dev. is the standard deviation of returns. *Sk* (*Sk**) and *Xku* (*Xku**) represent the skewness (its standardized version) and its analogues for excess kurtosis. The Jarque-Bera (J-B) statistic is defined as $(Sk^*)^2+(XKu^*)^2$. It is distributed as a χ_2^2 under the null hypothesis of normality. Engle(K) is the TR^2 test statistic for conditional heteroskedasticity obtained by regressing squared returns on 4 (12) lags. $LB(K)$ is the Ljung-Box test statistic for serial correlation. $LB_c(K)$ is the Ljung-Box test statistic corrected for heteroskedasticity. These test statistics are all distributed under the corresponding null hypothesis as a χ_K^2. * and ** indicate that the statistic is significant at the 5% and 1% level respectively.

Table 2: Unconditional correlation matrices and variances over various subperiods

	Correlation matrix			Variance
	S&P	DAX	FTSE	
1988-1999				
S&P	1.000	0.565	0.585	3.672
DAX	0.565	1.000	0.595	6.902
FTSE	0.585	0.595	1.000	4.115
1988-1991				
S&P	1.000	0.296	0.509	4.034
DAX	0.296	1.000	0.468	7.112
FTSE	0.509	0.468	1.000	4.003
1992-1995				
S&P	1.000	0.310	0.381	1.415
DAX	0.310	1.000	0.512	4.454
FTSE	0.381	0.512	1.000	3.151
1996-1999				
S&P	1.000	0.626	0.627	5.263
DAX	0.626	1.000	0.702	9.566
FTSE	0.627	0.702	1.000	4.866

Note: Correlation matrices and variances of weekly stock returns for the S&P. the DAX and the FTSE. The sample period is January 1988 to December 1999. a total of 620 observations.

Table 3: Jennrich test of equality of correlation matrices over various subperiods

Model	Degree of freedom	1988-1991 compared to 1992-1995		1992-1995 compared to 1995-1999	
		Statistics	*p*-value	Statistics	*p*-value
S&P-DAX-FTSE	3	4.006	0.2608	20.802	0.0001
S&P DAX	1	0.024	0.8758	16.726	0.0000
S&P-FTSE	1	2.659	0.1030	11.202	0.0008
DAX-FTSE	1	0.358	0.5497	9.253	0.0024

Note: Correlation matrices of weekly stock returns for the S&P. the DAX and the FTSE are computed over various subperiods. The Jennrich test statistic is asymptotically distributed as a chi-square with a degree of freedom equal to the number of independent correlation coefficients.

Table 4: Summary statistics for various one-regime GARCH models

	No. of parameters (k)	Log-likelihood (L^*)	AIC	Schwartz	Degrees of freedom (ν)	Persistence (λ_i)
Constant variance						
Gaussian innovations	9	-3813.44	-3822.44	-3842.37	-	0.000 0.000 0.000
Student-*t* innovations	10	-3782.59	-3792.59	-3814.73	8.54 (5.44)	0.000 0.000 0.000
GARCH(1.1)						
Gaussian innovations	15	-3750.33	-3765.33	-3798.55	-	0.993 0.964 0.965
Student-*t* innovations	16	-3736.17	-3752.17	**-3787.61**	12.76 (4.18)	0.993 0.960 0.966
GJR(1.1)						
Gaussian innovations	18	-3743.44	-3761.44	-3801.31	-	0.989 0.945 0.964
Student-*t* innovations	19	-3732.31	**-3751.31**	-3793.39	14.05 (3.77)	0.989 0.947 0.965

Note: The sample period is January 1988 to December 1999. a total of 620 observations. AIC and Schwartz model selection criteria are computed as L^*-k and L^*-0.5$k$ln(T) respectively. where k is the number of parameters and T the number of observations. The degree of freedom parameter is the estimate of ν in eq. (5). for the Student-*t* distribution. The persistence parameter (λ_i) is equal to 0 for the constant-variance model; it is defined by $\alpha_i+\beta_i$ for the GARCH model and by $\alpha_i+\beta_i+\gamma_i/2$ for the GJR model.

Table 5: LR test for various null hypotheses concerning one-regime GARCH models

	Test statistics	Degree of freedom (k)	p-value
Testing for distribution of innovations			
Constant variance – Gaussian vs Student-*t*	61.70	1	4e-15
GARCH – Gaussian vs Student-*t*	28.32	1	1e-07
GJR – Gaussian vs Student-*t*	22.26	1	2e-06
Testing for variance specification			
Gaussian - Constant variance vs GARCH	126.22	6	0.0000
Gaussian - GARCH vs GJR	13.78	3	0.0032
Student-*t* - Constant variance vs GARCH	92.83	6	0.0000
Student-*t* - GARCH vs GJR	7.72	3	0.0521

Note: The LR test statistics are defined as $2(L^*-L_0^*)$. where L^* and L_0^* are the log-likelihoods under the alternative and null hypotheses respectively. Log-likelihoods are found in Table 4. The degree of freedom. k. corresponds to the number of parameters constrained under the null. The LR test statistics are distributed as a χ_p^2.

Table 6: Summary statistics for various two-regime MS models

	No. of parameters (k)	Log-likelihood (L^*)	AIC	Schwartz	Degrees of freedom (ν)	Persistence λ_i^0	λ_i^1
Regime-dependent correlation							
Constant variance							
Gaussian innovations	17	-3735.65	-3752.65	-3790.30	-	0.00	0.00
						0.00	0.00
						0.00	0.00
Student-*t* innovations	18	-3729.53	-3747.53	-3787.40	17.64	0.00	0.00
					(2.83)	0.00	0.00
						0.00	0.00
GARCH							
Gaussian innovations	29	-3723.43	-3752.43	-3816.66	-	0.58	0.09
						0.56	0.99
						0.99	0.95
Student-*t* innovations	30	-3715.16	-3745.16	-3811.60	16.37	0.87	0.16
					(3.17)	0.92	0.95
						0.99	0.94
Constant correlation							
Constant variance							
Gaussian innovations	14	-3743.49	-3757.49	-3788.50	-	0.00	0.00
						0.00	0.00
						0.00	0.00
Student-*t* innovations	15	-3738.48	-3753.48	**-3786.70**	18.05	0.00	0.00
					(2.70)	0.00	0.00
						0.00	0.00
GARCH							
Gaussian innovations	26	-3728.65	-3754.65	-3812.24	-	0.46	0.10
						0.47	1.00
						0.99	0.99
Student-*t* innovations	27	-3722.00	-3749.00	-3808.80	16.76	0.12	0.17
					(3.13)	0.64	0.96
						0.99	0.98

Note: The sample period is January 1988 to December 1999. a total of 620 observations. AIC and Schwartz model selection criteria are computed as L^*-k and L^*-0.5$k$ln(T) respectively. where k is the number of parameters and T the number of observations. The degree of freedom parameter is the estimate of ν in eq. (5). for the Student-*t* distribution. The persistence parameter (λ_i^k) for regime k. k=0.1. is equal to 0 for the constant-variance model; it is defined by $\alpha_i^k+\beta_i^k$ for the GARCH model.

Table 7: LR test for various null hypotheses concerning two-regime MS models

	Test statistics	Degrees of freedom	*p*-value
Testing for distribution of innovations			
Regime-dependent correlation			
Constant variance – Gaussian vs Student-*t*	12.24	1	0.0005
GARCH - Gaussian vs Student-*t*	16.54	1	0.0000
Constant correlation			
Constant variance – Gaussian vs Student-*t*	10.02	1	0.0015
GARCH – Gaussian vs Student-*t*	13.30	1	0.0003
Testing for the two-regime model			
Gaussian constant variance	155.58	1	-
Student-*t* constant variance	106.12	1	-
Gaussian GARCH	53.80	1	-
Student-*t* GARCH	42.02	1	-
Testing for variance specification			
Regime-dependent correlation			
Gaussian – Constant variance vs GARCH	24.44	12	0.0179
Student-*t* – Constant variance vs GARCH	28.74	12	0.0043
Constant correlation			
Gaussian – Constant variance vs GARCH	29.68	12	0.0031
Student-*t* – Constant variance vs GARCH	32.96	12	0.0010
Testing for constant correlation			
Gaussian constant variance model	15.67	3	0.0013
Student-*t* constant variance model	17.90	3	0.0005
Gaussian GARCH model	10.44	3	0.0152
Student-*t* GARCH model	13.68	3	0.0034

Note: The LR test statistics are defined as $2(L^*-L_0^*)$. where L^* and L_0^* are the log-likelihoods under the alternative and null hypotheses respectively. Log-likelihoods are found in Tables 4 and 6. The degree of freedom. *k*. corresponds to the number of parameters constrained under the null. The LR test statistics are distributed as a χ_p^2.

Table 8: Parameter estimates for one-regime and two-regime constant-variance models

Parameter	One-regime Estimates	One-regime Student	Two-regime with regime-dependent correlations Estimates	Two-regime with regime-dependent correlations Student	Two-regime with regime-independent correlations Estimates	Two-regime with regime-independent correlations Student
μ_1	0.301	4.032	0.300	4.518	0.298	4.365
h_1^0	3.607	14.526	2.102	13.424	2.262	12.536
h_1^1	-		6.396	10.712	5.732	11.143
μ_2	0.373	3.660	0.354	3.922	0.354	3.775
h_2^0	6.849	13.764	3.798	12.947	4.064	11.577
h_2^1	-		12.932	9.120	11.488	9.945
μ_3	0.225	2.840	0.227	3.170	0.222	2.972
h_3^0	3.976	14.813	2.803	11.996	3.100	10.868
h_3^1	-		6.007	10.009	5.145	10.638
ν	8.540	6.141	17.635	2.831	18.049	2.697
ρ_{12}^0	0.447	12.462	0.342	7.761	0.415	10.468
ρ_{12}^1	-		0.528	8.133	-	
ρ_{13}^0	0.525	17.500	0.421	9.246	0.506	13.724
ρ_{13}^1	-		0.628	10.743	-	
ρ_{23}^0	0.573	18.926	0.462	10.297	0.546	13.992
ρ_{23}^1	-		0.673	10.892	-	
p	-		0.991	7.650	0.991	6.736
q	-		0.990	4.788	0.988	4.799
Log Likelihood	-3782.586		-3729.534		-3738.475	
	Statistics	p-value	Statistics	p-value	Statistics	p-value
Engle(4) for r_{1t}	41.969	0.00	4.329	0.36	8.585	0.07
Engle(4) for r_{2t}	74.185	0.00	12.599	0.01	18.516	0.00
Engle(4) for r_{3t}	12.836	0.01	3.929	0.42	4.827	0.31
$\text{LB}_\text{c}(4)$ for r_{1t}	12.830	0.01	13.823	0.01	13.705	0.01
$\text{LB}_\text{c}(4)$ for r_{2t}	0.435	0.98	0.989	0.91	0.906	0.92
$\text{LB}_\text{c}(4)$ for r_{3t}	2.245	0.69	2.878	0.58	2.826	0.59

Note: The sample period is January 1988 to December 1999. a total of 620 observations. Engle(4) is the TR^2 test statistic for conditional heteroskedasticity obtained by regressing squared returns on 4 lags. $LB_c(4)$ is the Ljung-Box test statistic corrected for heteroskedasticity. These test statistics are distributed under the null hypothesis as a χ_4^2.

Table 9: Parameter estimates for one-regime and two-regime GARCH models

	One-regime		Two-regime with regime-dependent correlations		Two-regime with regime-independent correlations	
Parameter	Estimates	Student	Estimates	Student	Estimates	Student
μ_1	0.287	4.298	0.268	4.261	0.286	4.349
ω_1^0	0.035	1.559	0.200	1.429	1.336	1.747
α_1^0	0.038	2.420	0.107	0.345	0.081	0.121
β_1^0	0.955	55.941	0.764	3.193	0.037	0.006
ω_1^1	-		4.340	2.193	3.622	2.982
α_1^1	-		0.155	0.505	0.169	0.653
β_1^1	-		0.005	0.000	0.004	0.000
μ_2	0.371	4.000	0.337	3.715	0.354	3.870
ω_2^0	0.293	2.398	0.322	1.363	1.711	0.414
α_2^0	0.084	3.489	0.041	0.112	0.015	0.008
β_2^0	0.876	34.771	0.875	3.223	0.620	0.356
ω_2^1	-		0.540	1.537	0.297	1.798
α_2^1	-		0.066	0.237	0.091	0.478
β_2^1	-		0.881	3.487	0.864	5.216
μ_3	0.235	3.183	0.216	2.902	0.231	3.051
ω_3^0	0.136	2.019	0.015	0.964	0.000	0.002
α_3^0	0.034	2.369	0.001	0.000	0.002	0.000
β_3^0	0.932	37.390	0.988	2.241	0.992	1.480
ω_3^1	-		0.317	1.128	0.089	1.733
α_3^1	-		0.002	0.001	0.003	0.002
β_3^1	-		0.936	2.169	0.978	3.643
ν	12.760	4.204	16.369	3.172	16.758	3.135
ρ_{12}^0	0.427	12.124	0.259	2.128	0.426	4.837
ρ_{12}^1	-		0.570	5.492	-	
ρ_{13}^0	0.516	16.807	0.397	3.388	0.522	5.659
ρ_{13}^1	-		0.621	4.342	-	
ρ_{23}^0	0.563	18.742	0.505	4.322	0.557	6.376
ρ_{23}^1	-		0.617	4.492	-	
p	-		0.998	5.480	0.995	6.273
q	-		0.999	0.412	0.999	2.844
Log Likelihood	-3736.170		-3715.157		-3722.003	
	Statistics	p-value	Statistics	p-value	Statistics	p-value
Engle(4) for r_{1t}	8.424	0.08	3.983	0.41	7.740	0.10
Engle(4) for r_{2t}	5.879	0.21	3.932	0.42	4.274	0.37
Engle(4) for r_{3t}	4.516	0.34	7.835	0.10	8.764	0.07
$LB_c(4)$ for r_{1t}	15.091	0.00	17.444	0.00	17.220	0.00
$LB_c(4)$ for r_{2t}	1.143	0.89	1.187	0.88	1.078	0.90
$LB_c(4)$ for r_{3t}	2.956	0.57	2.814	0.59	2.812	0.59

Note: The sample period is January 1988 to December 1999. a total of 620 observations. Engle(4) is the TR^2 test statistic for conditional heteroskedasticity obtained by regressing squared returns on 4 lags. $LB_c(4)$ is the Ljung-Box test statistic corrected for heteroskedasticity. These test statistics are distributed under the null hypothesis as a χ_4^2.

Table 10: One-period-ahead forecasts of various models

		one-regime const.-var. model	one-regime GARCH model	two-regime constant-variance model (MS)		two-regime GARCH model (MS-GARCH)	
				regime-dep. correlations	regime-indep. correlations	regime-dep. correlations	regime-indep. correlations
Panel A: In sample - 1988-99							
RMSE	h_{1t}	6.034	5.822	5.789	5.801	5.775	5.815
	h_{2t}	6.952	6.750	6.757	6.801	6.746	6.805
	h_{3t}	14.012	13.358	13.409	13.450	13.315	13.326
	h_{12t}	4.646	4.531	4.497	4.542	4.532	4.573
	h_{13t}	7.671	7.456	7.425	7.496	7.442	7.477
	h_{23t}	6.794	6.697	6.662	6.684	6.681	6.703
MAE	h_{1t}	3.721	3.526	3.529	3.481	3.490	3.455
	h_{2t}	7.330	6.916	6.994	6.864	6.749	6.751
	h_{3t}	4.113	4.032	4.016	3.969	3.996	3.978
	h_{12t}	3.983	3.752	3.824	3.755	3.725	3.731
	h_{13t}	2.881	2.756	2.789	2.739	2.783	2.766
	h_{23t}	4.367	4.198	4.211	4.118	4.137	4.124
Panel B: In sample - 1988-March 1998							
RMSE	h_{1t}	5.039	4.925	4.902	4.904	4.880	4.881
	h_{2t}	5.033	4.977	4.966	4.968	4.977	4.983
	h_{3t}	9.441	9.044	9.011	9.009	8.989	8.989
	h_{12t}	3.627	3.553	3.523	3.549	3.557	3.574
	h_{13t}	5.684	5.563	5.541	5.561	5.561	5.565
	h_{23t}	6.419	6.337	6.304	6.315	6.321	6.336
MAE	h_{1t}	3.142	3.057	3.035	3.000	2.973	2.950
	h_{2t}	6.109	5.917	5.902	5.826	5.736	5.732
	h_{3t}	3.784	3.722	3.688	3.655	3.655	3.658
	h_{12t}	3.105	3.035	3.039	3.004	3.003	2.988
	h_{13t}	2.411	2.337	2.342	2.317	2.345	2.328
	h_{23t}	3.693	3.613	3.600	3.536	3.543	3.535
Panel C: Out-of-sample – April 1998-1999							
RMSE	h_{1t}	10.158	9.555	9.528	9.636	9.663	9.741
	h_{2t}	13.663	13.070	13.181	13.370	13.140	13.292
	h_{3t}	28.891	27.319	27.693	27.873	27.307	27.344
	h_{12t}	8.499	8.220	8.157	8.339	8.248	8.343
	h_{13t}	14.692	14.150	14.147	14.391	14.209	14.203
	h_{23t}	8.597	8.371	8.359	8.408	8.367	8.371
MAE	h_{1t}	5.671	6.082	5.720	5.660	5.679	5.621
	h_{2t}	11.694	12.351	11.828	11.691	12.259	12.348
	h_{3t}	5.134	5.565	5.613	5.401	5.500	5.529
	h_{12t}	7.615	7.369	7.450	7.495	7.431	7.467
	h_{13t}	4.705	4.761	4.839	4.735	4.761	4.718
	h_{23t}	6.945	6.967	6.913	6.848	6.915	6.927

Note: The table reports root mean squared prediction errors (RMSE) and mean absolute errors (MAE) for various models. In panel A. parameters are estimated over the 1988-99 period and RMSE and MAE are computed over the same period. In panel B. parameters are estimated over the period from January 1988 to March 1998 and RMSE and MAE are computed over the same period. In panel C. parameters are estimated over the period from January 1988 to March 1998 and RMSE and MAE are computed from April 1998 to December 1999.

Legends for Figures.

Fig. 1: This figure illustrates the evolution of unconditional variance and correlation across markets. They are computed over sliding window of one year. Beginning of 1992 and 1995 are marked with vertical lines.

Fig. 2: This figure illustrates the evolution of weekly stock returns series for each stock market. Beginning of 1992 and 1995 are marked with vertical lines.

Fig. 3: The top panel represents a time series of the ex-ante and smoothed probabilities that stock returns are in low-volatility regime (regime 0) at time *t* according to the within-regime constant-variance MS model. The bottom panel represents a time series of the ex-ante and smoothed probabilities that stock returns are in low-volatility regime (regime 0) at time *t* according to the within-regime MS-GARCH model.

Fig. 4: This figure contains a time series plot of conditional correlation across markets. In Fig. 4a. parameter estimates are based on the MS model. In Fig. 4b. parameter estimates are based on the MS-GARCH model.

Fig. 1a: Unconditional variances over time

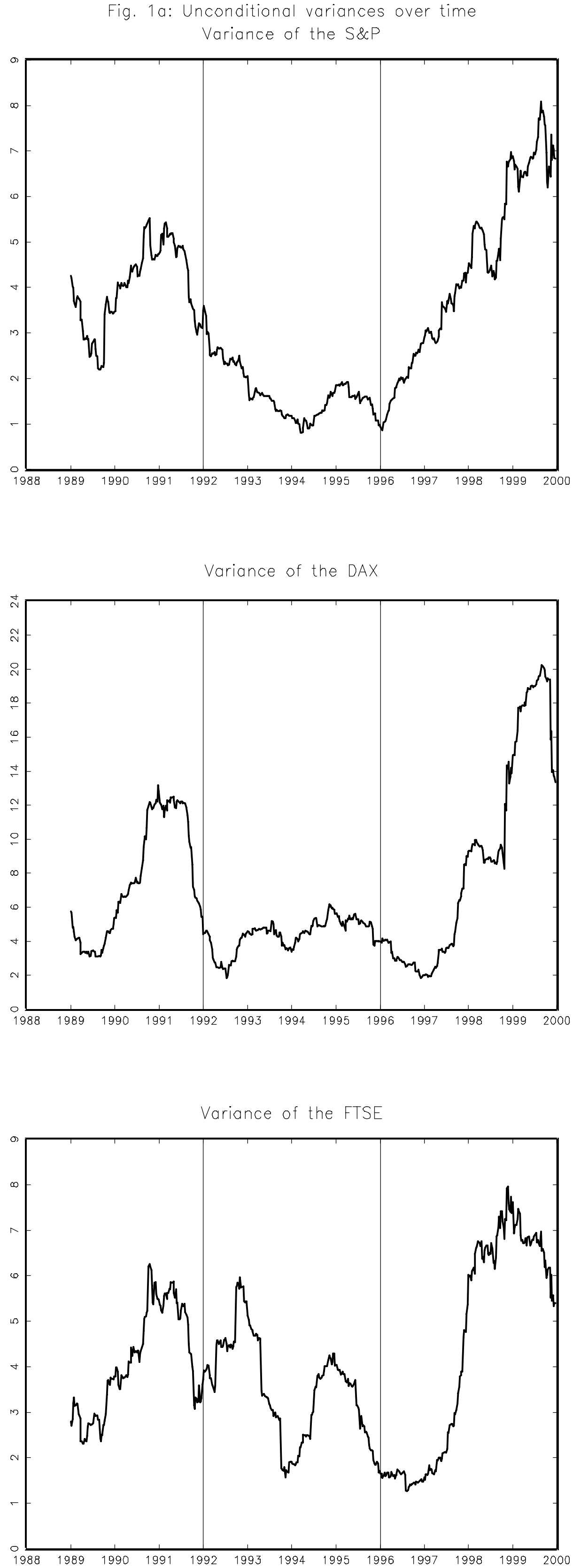


Fig. 1b: Unconditional covariances over time

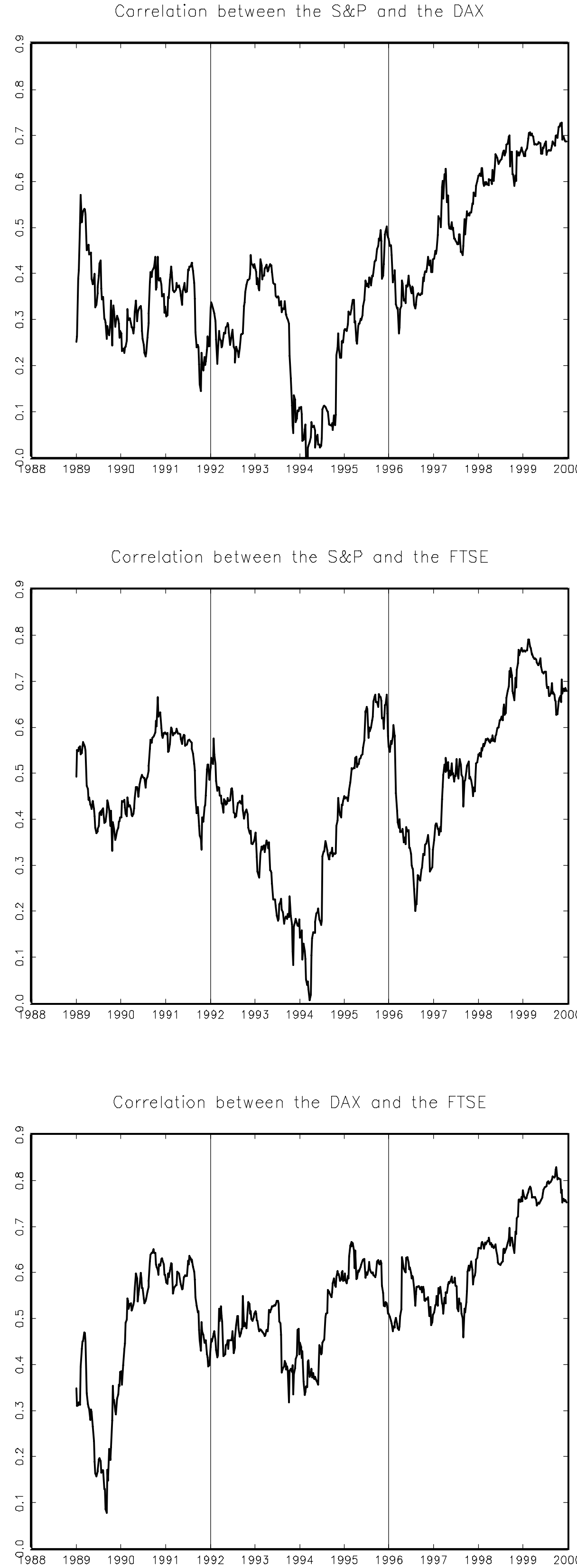


Fig. 2: Weekly stock returns

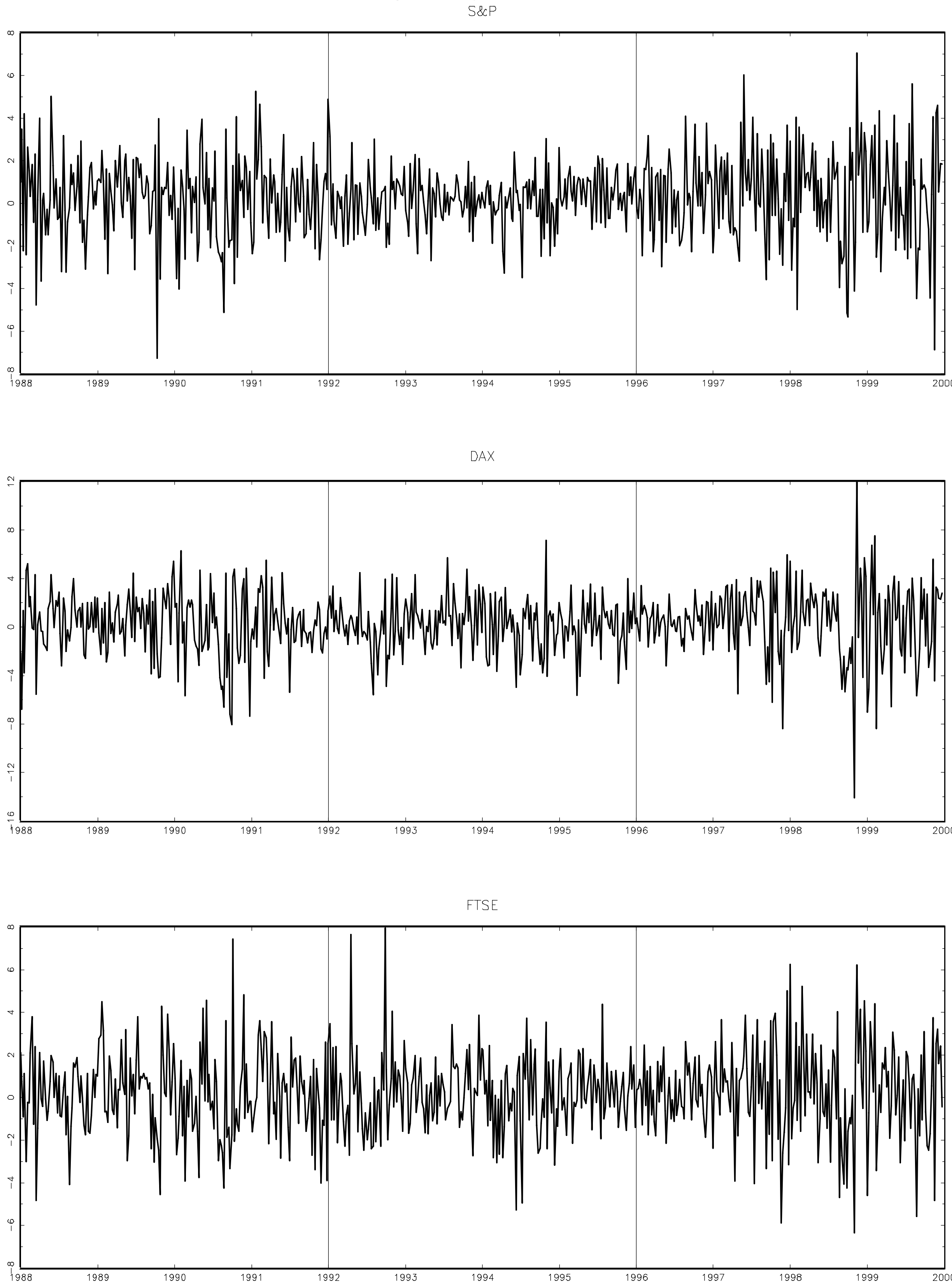
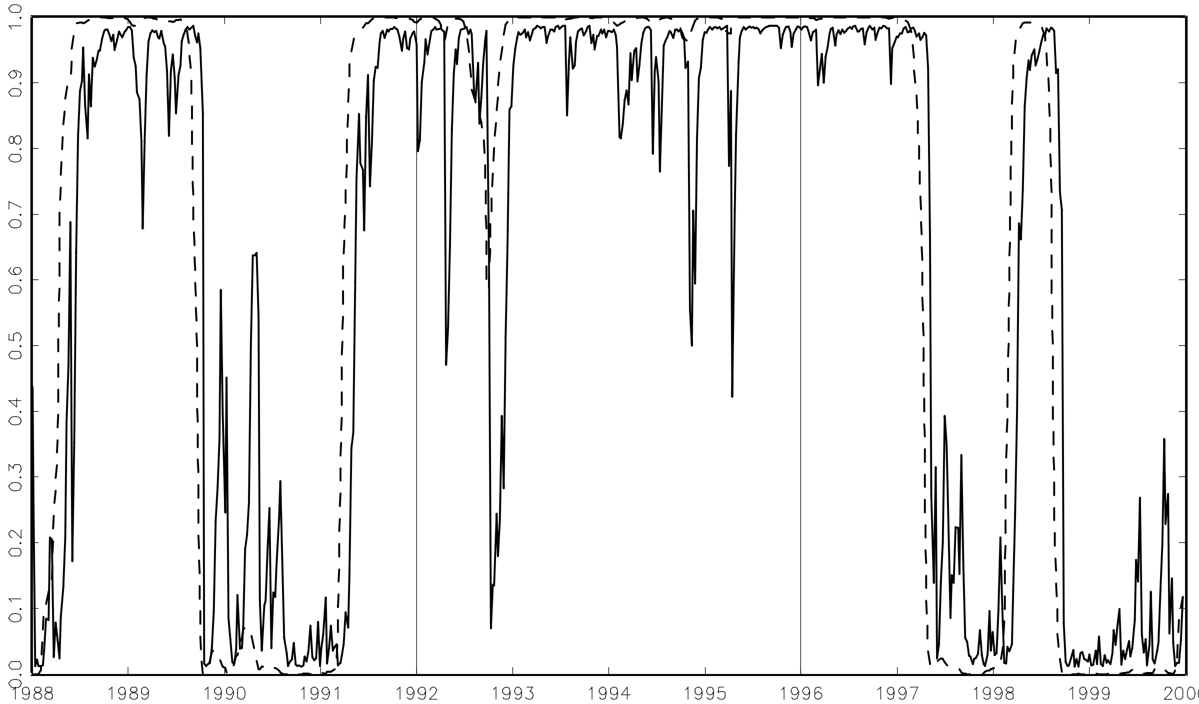


Fig. 3: Ex–ante and smoothed probabilities

MS model



MS–GARCH model

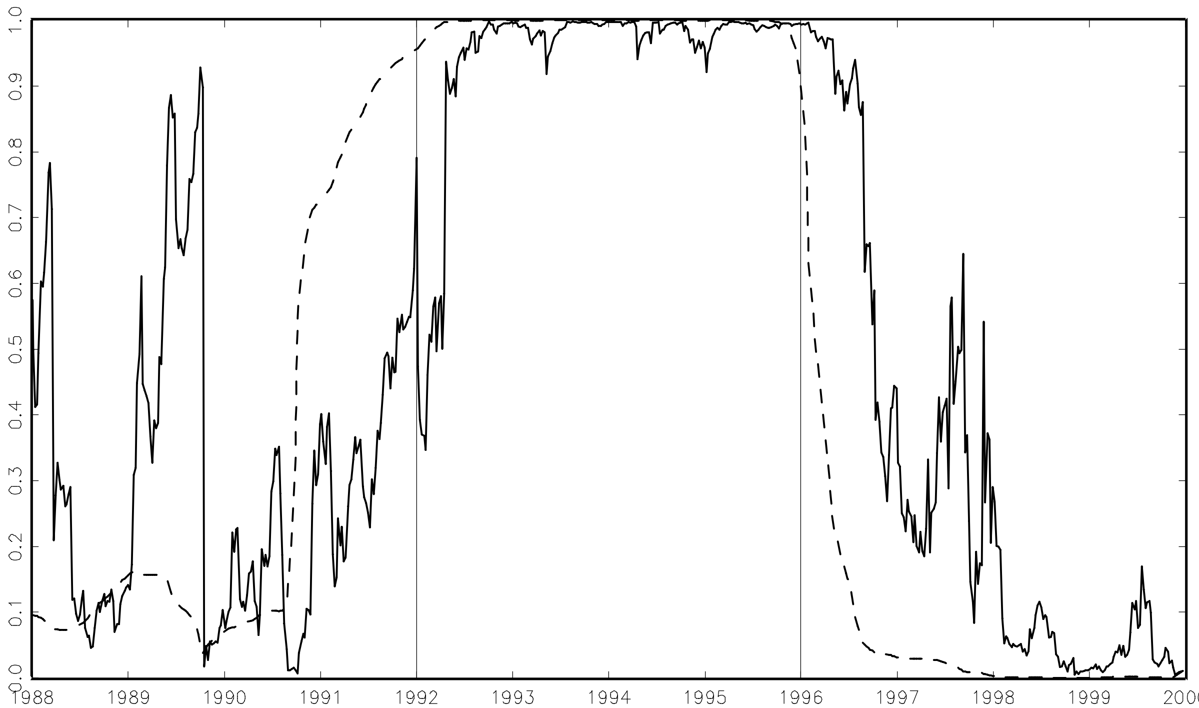


Fig. 4a: Conditional correlation estimates: MS model

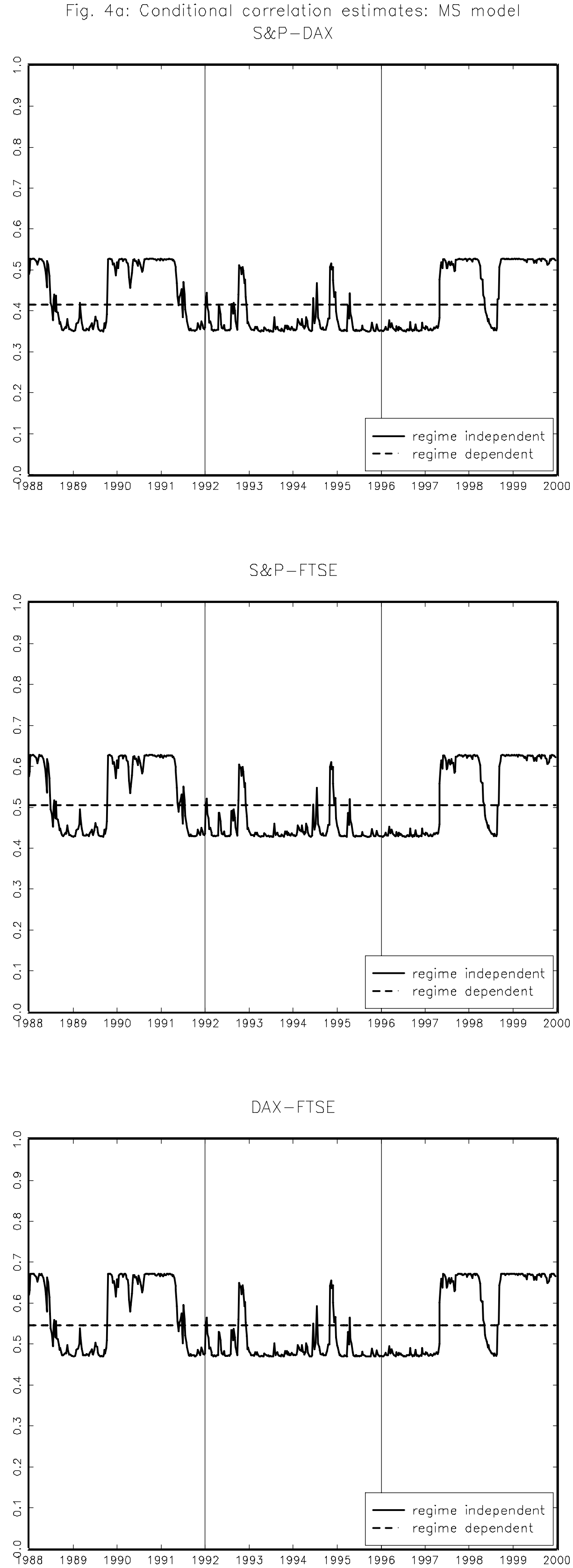
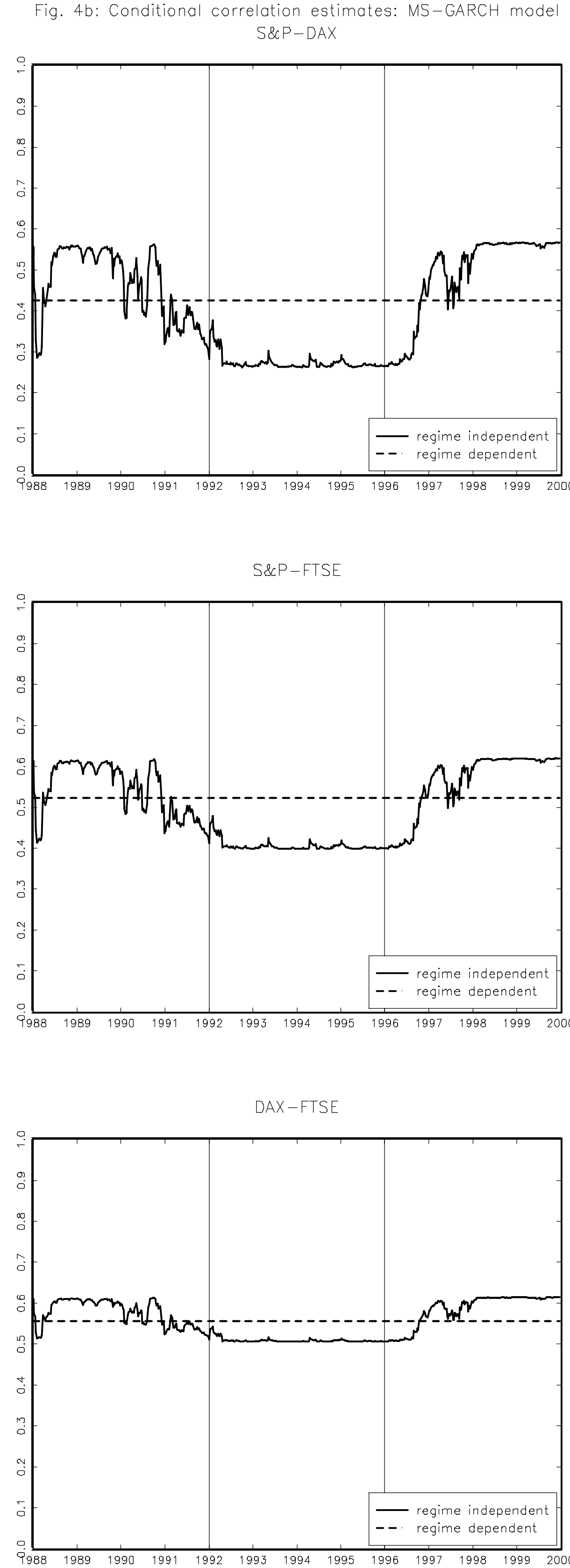


Fig. 4b: Conditional correlation estimates: MS-GARCH model



Notes d'Études et de Recherche

1. C. Huang and H. Pagès, “Optimal Consumption and Portfolio Policies with an Infinite Horizon: Existence and Convergence,” May 1990.

2. C. Bordes, « Variabilité de la vitesse et volatilité de la croissance monétaire : le cas français », février 1989.

3. C. Bordes, M. Driscoll and A. Sauviat, “Interpreting the Money-Output Correlation: Money-Real or Real-Real?,” May 1989.

4. C. Bordes, D. Goyeau et A. Sauviat, « Taux d'intérêt, marge et rentabilité bancaires : le cas des pays de l'OCDE », mai 1989.

5. B. Bensaid, S. Federbusch et R. Gary-Bobo, « Sur quelques propriétés stratégiques de l’intéressement des salariés dans l'industrie », juin 1989.

6. O. De Bandt, « L'identification des chocs monétaires et financiers en France : une étude empirique », juin 1990.

7. M. Boutillier et S. Dérangère, « Le taux de crédit accordé aux entreprises françaises : coûts opératoires des banques et prime de risque de défaut », juin 1990.

8. M. Boutillier and B. Cabrillac, “Foreign Exchange Markets: Efficiency and Hierarchy,” October 1990.

9. O. De Bandt et P. Jacquinot, « Les choix de financement des entreprises en France : une modélisation économétrique », octobre 1990 (English version also available on request).

10. B. Bensaid and R. Gary-Bobo, “On Renegotiation of Profit-Sharing Contracts in Industry,” July 1989 (English version of NER n° 5).

11. P. G. Garella and Y. Richelle, “Cartel Formation and the Selection of Firms,” December 1990.

12. H. Pagès and H. He, “Consumption and Portfolio Decisions with Labor Income and Borrowing Constraints,” August 1990.

13. P. Sicsic, « Le franc Poincaré a-t-il été délibérément sous-évalué ? », octobre 1991.

14. B. Bensaid and R. Gary-Bobo, “On the Commitment Value of Contracts under Renegotiation Constraints,” January 1990 revised November 1990.

15. B. Bensaid, J.-P. Lesne, H. Pagès and J. Scheinkman, “Derivative Asset Pricing with Transaction Costs,” May 1991 revised November 1991.

16. C. Monticelli and M.-O. Strauss-Kahn, “European Integration and the Demand for Broad Money,” December 1991.

17. J. Henry and M. Phelipot, “The High and Low-Risk Asset Demand of French Households: A Multivariate Analysis,” November 1991 revised June 1992.

18. B. Bensaid and P. Garella, “Financing Takeovers under Asymetric Information,” September 1992.

19. A. de Palma and M. Uctum, “Financial Intermediation under Financial Integration and Deregulation,” September 1992.

20. A. de Palma, L. Leruth and P. Régibeau, “Partial Compatibility with Network Externalities and Double Purchase,” August 1992.

21. A. Frachot, D. Janci and V. Lacoste, “Factor Analysis of the Term Structure: a Probabilistic Approach,” November 1992.

22. P. Sicsic et B. Villeneuve, « L'Afflux d'or en France de 1928 à 1934 », janvier 1993.

23. M. Jeanblanc-Picqué and R. Avesani, “Impulse Control Method and Exchange Rate,” September 1993.

24. A. Frachot and J.-P. Lesne, “Expectations Hypothesis and Stochastic Volatilities,” July 1993 revised September 1993.

25. B. Bensaid and A. de Palma, “Spatial Multiproduct Oligopoly,” February 1993 revised October 1994.

26. A. de Palma and R. Gary-Bobo, “Credit Contraction in a Model of the Banking Industry,” October 1994.

27. P. Jacquinot et F. Mihoubi, « Dynamique et hétérogénéité de l'emploi en déséquilibre », septembre 1995.

28. G. Salmat, « Le retournement conjoncturel de 1992 et 1993 en France : une modélisation V.A.R. », octobre 1994.

29. J. Henry and J. Weidmann, “Asymmetry in the EMS Revisited: Evidence from the Causality Analysis of Daily Eurorates,” February 1994 revised October 1994.

30 O. De Bandt, “Competition Among Financial Intermediaries and the Risk of Contagious Failures,” September 1994 revised January 1995.

31. B. Bensaid et A. de Palma, « Politique monétaire et concurrence bancaire », janvier 1994 révisé en septembre 1995.

32. F. Rosenwald, « Coût du crédit et montant des prêts : une interprétation en terme de canal large du crédit », septembre 1995.

33. G. Cette et S. Mahfouz, « Le partage primaire du revenu : constat descriptif sur longue période », décembre 1995.

34. H. Pagès, “Is there a Premium for Currencies Correlated with Volatility ? Some Evidence from Risk Reversals,” January 1996.

35. E. Jondeau and R. Ricart, “The Expectations Theory : Tests on French, German and American Euro-rates,” June 1996.

36. B. Bensaid et O. De Bandt, « Les stratégies “stop-loss” : théorie et application au Contrat Notionnel du Matif », juin 1996.

37. C. Martin et F. Rosenwald, « Le marché des certificats de dépôts. Écarts de taux à l'émission : l'influence de la relation émetteurs-souscripteurs initiaux », avril 1996.

38. Banque de France - CEPREMAP - Direction de la Prévision - Erasme - INSEE - OFCE, « Structures et propriétés de cinq modèles macroéconomiques français », juin 1996.

39. F. Rosenwald, « L'influence des montants émis sur le taux des certificats de dépôts », octobre 1996.

40. L. Baumel, « Les crédits mis en place par les banques AFB de 1978 à 1992 : une évaluation des montants et des durées initiales », novembre 1996.

41. G. Cette et E. Kremp, « Le passage à une assiette valeur ajoutée pour les cotisations sociales : Une caractérisation des entreprises non financières “gagnantes” et “perdantes” », novembre 1996.

42. S. Avouyi-Dovi, E. Jondeau et C. Lai Tong, « Effets “volume”, volatilité et transmissions internationales sur les marchés boursiers dans le G5 », avril 1997.

43. E. Jondeau et R. Ricart, « Le contenu en information de la pente des taux : Application au cas des titres publics français », juin 1997.

44. B. Bensaid et M. Boutillier, « Le contrat notionnel : Efficience et efficacité », juillet 1997.

45. E. Jondeau et R. Ricart, « La théorie des anticipations de la structure par terme : test à partir des titres publics français », septembre 1997.

46. E. Jondeau, « Représentation VAR et test de la théorie des anticipations de la structure par terme », septembre 1997.

47. E. Jondeau et M. Rockinger, « Estimation et interprétation des densités neutres au risque : Une comparaison de méthodes », octobre 1997.

48. L. Baumel et P. Sevestre, « La relation entre le taux de crédits et le coût des ressources bancaires. Modélisation et estimation sur données individuelles de banques », octobre 1997.

49. P. Sevestre, “On the Use of Banks Balance Sheet Data in Loan Market Studies : A Note,” October 1997.

50. P.-C. Hautcoeur and P. Sicsic, “Threat of a Capital Levy, Expected Devaluation and Interest Rates in France during the Interwar Period,” January 1998.

51. P. Jacquinot, « L’inflation sous-jacente à partir d’une approche structurelle des VAR : une application à la France, à l’Allemagne et au Royaume-Uni », janvier 1998.

52. C. Bruneau et O. De Bandt, « La modélisation VAR structurel : application à la politique monétaire en France », janvier 1998.

53. C. Bruneau and E. Jondeau, “Long-Run Causality, with an Application to International Links between Long-Term Interest Rates,” June 1998.

54. S. Coutant, E. Jondeau and M. Rockinger, “Reading Interest Rate and Bond Futures Options’ Smiles: How PIBOR and Notional Operators Appreciated the 1997 French Snap Election,” June 1998.

55. E. Jondeau et F. Sédillot, « La prévision des taux longs français et allemands à partir d'un modèle à anticipations rationnelles », juin 1998.

56. E. Jondeau and M. Rockinger, "Estimating Gram-Charlier Expansions with Positivity Constraints," January 1999.

57. S. Avouyi-Dovi and E. Jondeau, "Interest Rate Transmission and Volatility Transmission along the Yield Curve," January 1999.

58. S. Avouyi-Dovi et E. Jondeau, « La modélisation de la volitilité des bourses asiatiques », janvier 1999.

59. E. Jondeau, « La mesure du ratio rendement-risque à partir du marché des euro-devises », janvier 1999.

60. C. Bruneau and O. De Bandt, "Fiscal policy in the transition to monetary union: a structural VAR model," January 1999.

61. E. Jondeau and R. Ricart, "The Information Content of the French and German Government Bond Yield Curves: Why Such Differences?" February 1999.

62. J.-B. Chatelain et P. Sevestre, « Coûts et bénéfices du passage d'une faible inflation à la stabilité des prix », février 1999.

63. D. Irac et P. Jacquinot, « L'investissement en France depuis le début des années 1980 », Avril 1999.

64. F. Mihoubi, « Le partage de la valeur ajoutée en France et en Allemagne », mars 1999.

65. S. Avouyi-Dovi and E. Jondeau, "Modelling the French Swap Spread," April 1999.

66. E. Jondeau and M. Rockinger, "The Tail Behavior of Stock Returns: Emerging Versus Mature Markets," June 1999.

67. F. Sédillot, « La pente des taux contient-elle de l'information sur l'activité économique future ? », juin 1999.

68. E. Jondeau, H. Le Bihan et F. Sédillot, « Modélisation et prévision des indices de prix sectoriels », septembre 1999.

69. H. Le Bihan and F. Sédillot, "Implementing and Interpreting Indicators of Core Inflation : The French Case," September 1999.

70 R. Lacroix, "Testing for Zeros in the Spectrum of an Univariate Stationary Process: Part I," December 1999.

71 R. Lacroix, "Testing for Zeros in the Spectrum of an Univariate Stationary Process: Part II," December 1999.

72 R. Lacroix, "Testing the Null Hypothesis of Stationarity in Fractionally Integrated Models," December 1999.

73 F. Chesnay and E. Jondeau, "Does correlation between stock returns really increase during turbulent period?," April 2000.

Pour tous commentaires ou demandes sur les Notes d'Études et de Recherche, contacter la bibliothèque du Centre de recherche à l'adresse suivante :

For any comment or enquiries on the Notes d'Études et de Recherche, contact the library of the Centre de recherche at the following address :

BANQUE DE FRANCE
41.1391 - Centre de recherche
75 049 Paris CEDEX
tél : 01 42 92 49 55
fax :01 42 92 62 92
email : thierry.demoulin@banque-france.fr