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CONDITIONAL VOLATILITY, SKEWNESS, AND KURTOSIS: EXISTENCE AND PERSISTENCE

Eric Jondeau and Michael Rockinger

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Abstract

Recent portfolio choice, asset pricing, and option valuation models highlight the importance of skewness and kurtosis. Since skewness and kurtosis are related to extreme variations, they are also important for Value-at-Risk measurements. Our framework builds on a GARCH model with a conditional generalized-t distribution for residuals. We compute the skewness and kurtosis for this model and compare the range of these moments with the maximal theoretical moments. Our model, thus, allows for time-varying conditional skewness and kurtosis. We implement the model as a constrained optimization with possibly several thousand restrictions on the dynamics. A sequential quadratic programming algorithm successfully estimates all the models, on a PC, within at most 50 seconds. Estimators, obtained with logistically-constrained dynamics, have different properties. We apply this model to daily and weekly foreign exchange returns, stock returns, and interest-rate changes. We show that skewness exists for many dates and for almost all series except short-term interest-rate changes. This finding is consistent with findings from extreme value theory. Kurtosis exists on fewer dates and for fewer series. There is little evidence, at the weekly frequency, of time-variability of conditional higher moments. Transition matrices document that agitated states come as a surprise and that there is a certain persistence in moments beyond volatility. For exchange-rate and stock-market data, cross-sectionally and at daily frequency, we also document co-variability of moments beyond volatility.

Keywords: GARCH, Stock indices, Exchange rates, Interest rates, SNOPT, VaR.

JEL classification: C22, C51, G12.

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Abstract: Recent portfolio choice, asset pricing, and option valuation models highlight the importance of skewness and kurtosis. Since skewness and kurtosis are related to extreme variations, they are also important for Value-at-Risk measurements. Our framework builds on a GARCH model with a conditional generalized-t distribution for residuals. We compute the skewness and kurtosis for this model and compare the range of these moments with the maximal theoretical moments. Our model, thus, allows for time-varying conditional skewness and kurtosis. We implement the model as a constrained optimization with possibly several thousand restrictions on the dynamics. A sequential quadratic programming algorithm successfully estimates all the models, on a PC, within at most 50 seconds. Estimators, obtained with logistically-constrained dynamics, have different properties. We apply this model to daily and weekly foreign exchange returns, stock returns, and interest-rate changes. We show that skewness exists for many dates and for almost all series except short-term interest-rate changes. This finding is consistent with findings from extreme value theory. Kurtosis exists on fewer dates and for fewer series. There is little evidence, at the weekly frequency, of time-variability of conditional higher moments. Transition matrices document that agitated states come as a surprise and that there is a certain persistence in moments beyond volatility. For exchange-rate and stock-market data, cross-sectionally and at daily frequency, we also document co-variability of moments beyond volatility.

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1 Introduction

The following paper questions the existence and persistence of conditional skewness and kurtosis of various financial series taken at daily and weekly frequency. It also addresses the issue whether modeling higher moments affects the dynamics of volatility. To address this question, we build on Hansen (1994) who proposes a GARCH model where conditional residuals are modeled as a generalized Student-t distribution. The generalized-t distribution is asymmetric and allows for fat-tailedness. We first express skewness and kurtosis of Hansen's GARCH model as a function of the underlying parameters. For a given dynamic structure of the underlying parameters, these computations characterize the conditional evolution of skewness and kurtosis.

A further theoretical contribution is the characterization, conditional on kurtosis being finite, of the largest possible domain of skewness and kurtosis for which a density exists with a zero mean and a unit variance. We achieve this characterization using results from the so-called Hamburger (1920) problem. These results go back to Stieltjes (1894) and are also related with the little Hausdorff (1921a,b) problem. Conditional on the assumption that kurtosis and hence skewness exist, we show that the moments of the generalized Student-t distribution are way within the maximal set of skewness-kurtosis. An advantage of using the generalized-t distribution is that moments may become infinite. Hence, it is possible to determine those periods when higher moments do not exist.

The model is inspired by Engle (1982) and Bollerslev (1986) in the way volatility is determined. For the generalized Student-t distribution, both the asymmetry and the fat-tailness parameters are modeled as a function of lagged innovations. Given that the generalized Student-t distribution will only be defined for certain parameters, it is necessary to impose constraints on the dynamic specification. Given the dynamic nature of the problem, this results in a number of constraints proportional to the number of observations. For instance, our daily stock-index dataset involves $T = 7158$ observations, yielding an estimation with $2 + 3(T - 2) = 21470$ restrictions. This difficult estimation problem is solved using a recent sophisticated sequential quadratic optimization algorithm implemented in SNOPT, see Gill, Murray, and Saunders (1997, 1999).[1]

We estimate the model for six foreign exchange returns, five stock returns, four short-term interest-rate changes, and five long-term interest-rate changes. The foreign exchange series start on July 26, 1991, the stock indices on August 23, 1971, the 3-month rates on January 3, 1975, and the 10-year rates on Mai 20, 1986. All series end on September 3, 1999. We investigate the existence of conditional skewness and kurtosis both at daily and weekly frequencies. For exchange-rate and stock-market data, we also investigate the persistence of moments in a time-series context and cross-sectionally.

Our findings are of importance for various strands of literature that recently emphasized the importance for asset returns of moments beyond the second one. Within a Capital Asset Pricing Model, interest in moments beyond volatility goes back to the work of Kraus and Litzenberger (1976). Further work

[1] In the econometrics literature, Gallant and Tauchen (1989) have used the package NPSOL, developed by Gill *et al.* for dense matrices.

in that area is by Friend and Westerfield (1980), Barone-Adesi (1985), Sears and Wei (1985, 1988), and more recently by Fang and Lai (1997), Tan (1991), Kan and Zhou (1999), Harvey and Siddique (1999, 2000). For emerging countries, Hwang and Satchell (1999) model risk premia using higher moments. With our model and estimation technique, it is possible to extend these models to conditional versions.

Another strand of literature initiated by Mandelbrot (1963) and Fama (1963) recognizes the possibility that asset returns have such fat tails as to prevent existence of moments beyond the first one. Even though the non-existence of a second moment appears questionable, there remains the question of the existence of conditional moments. This type of question may be easily addressed with our model. Our model involves a set of parameters that are related to skewness and kurtosis. We estimate our model under the constraints that a density exists. The data then decides if, furthermore, a skewness and also possibly a kurtosis exist. We show that, for 3-month interest-rate changes, the third moment does not appear to exist. For the other series, skewness appears to exist most of the time but not kurtosis. This type of result is also related to extreme value theory. In that field, it has been shown by Loretan and Phillips (1998) that moments beyond the third one do not appear to exist.

Our model has also implications from a purely econometric point of view. In econometric applications the presence of heteroskedasticity is dealt with by an adjustment of standard errors, i.e., White (1980), Bollerslev and Wooldridge (1982), Gourieroux, Monfort and Trogon (1984), and Newey and West (1987). For a general discussion of this issue see Wooldridge (1994). So far it has not been possible to adjust for heteroskedasticity of moments beyond variance. The technology proposed in this work may be adapted to situations where higher moments need to be explicitly modeled, and, thus, a gain in efficiency may be attained.

The structure of this paper is as follows. In section 2, we present our model and the moments generated by it. In that section, we also indicate how such a model may be estimated. We relate our problem to the little Hausdorff problem and discuss the set of skewness-kurtosis pairs that are generated by our model. In section 3, we present the data. In section 4, we discuss the parameter estimates. In section 5, we address the issue of existence and persistence of conditional skewness and kurtosis. Then, we consider co-variability between markets of moments beyond volatility. In section 6, we conclude with directions for further research. Analytical results are given in the appendix.

2 A model for conditional skewness and kurtosis

2.1 The generalized Student-t distribution

Our model builds on the GARCH model of Engle (1982) and of Bollerslev (1986).[2] Within this class of models, it is well known that residuals are non-normal. This result has led to the introduction of fat-tailed distributions. Nelson (1994) considers the generalized error distribution. Bollerslev and Wooldridge

[2] The literature concerning GARCH models is huge. Several reviews of the literature are available, i.e.,Bollerslev, Chou, and Kroner (1992), Bera and Higgins (1993), and Bollerslev, Engle, and Nelson (1994).

(1992) consider the case of a Student-t distribution.[3] Engle and Gonzalez-Rivera (1991) model residuals non-parametrically. Even though these contributions recognize the fact that errors have fat tails, they do not render the tails time varying, i.e., the parameters of the error distribution are assumed to be constant.

Hansen (1994) is the first to propose a model that allows for conditional higher moments. He achieves this by introducing a generalization of the Student-t distribution where asymmetries may occur, while maintaining the assumption of a zero mean and unit variance. By assuming that parameters are dependent on past realizations, he shows that parameters may be made time varying, and thus that higher-moments may be made time varying.[4] In the finance literature, Harvey and Siddique (1999) introduce a non-central Student-t distribution.

Hansen's Student-t distribution is defined by

$$g(z|\eta,\lambda) = \begin{cases} bc\left(1+\frac{1}{\eta-2}\left(\frac{bz+a}{1-\lambda}\right)^2\right)^{-\frac{\eta+1}{2}} & \text{if } z < -a/b, \\ bc\left(1+\frac{1}{\eta-2}\left(\frac{bz+a}{1+\lambda}\right)^2\right)^{-\frac{\eta+1}{2}} & \text{if } z \geq -a/b \end{cases} \tag{1}$$

where

$$a \equiv 4\lambda c\frac{\eta-2}{\eta-1}, \qquad b^2 \equiv 1+3\lambda^2-a^2, \qquad c \equiv \frac{\Gamma\left(\frac{\eta+1}{2}\right)}{\sqrt{\pi(\eta-2)}\Gamma\left(\frac{\eta}{2}\right)}.$$

If a random variable Z has the density $g(z|\eta,\lambda)$, we will write $Z \sim HT(z|\eta,\lambda)$. Inspection of the various formulas reveals that this density is defined for $2 < \eta < \infty$ and $-1 < \lambda < 1$. Furthermore, this density encompasses a large set of conventional densities. For instance, if $\lambda = 0$, Hansen's distribution reduces to the traditional Student-t distribution. We recall that the traditional Student-t distribution is not skewed. If in addition $\eta = \infty$, the Student-t distribution collapses to a normal density. Figure 1 displays various densities obtained for different values of λ and η. We notice that λ controls skewness: if λ is positive, the probability mass concentrates in the right tail.

It is well known that a traditional Student-t with η degrees of freedom allows for the existence of all moments up to the ηth. Therefore, given the restriction $\eta > 2$, Hansen's skewed t distribution is well defined and its second moment exists. The higher moments are not given directly by the parameter η, but formulas exist for these moments. We establish now the formulas of the higher moments of Hansen's generalized-t distribution.

We show in appendix A that, if $Z \sim HT(z|\eta,\lambda)$, then Z has zero mean and unit variance. Furthermore, defining a random variable X with mean a and standard deviation b, obtained with $X = bZ + a$, and $m_j \equiv E[X^j]$, we find that

$$\mathrm{E}[Z^3] = [m_3 - 3a\,m_2 + 2a^3]/b^3, \tag{2}$$

$$\mathrm{E}[Z^4] = [m_4 - 4a\,m_3 + 6a^2 m_2 - 3a^4]/b^4, \tag{3}$$

[3] For a definition of the traditional Student-t distribution, see for instance Mood, Graybill, and Boes (1982).

[4] Hansen does not discuss the link between parameters and higher moments.

where

$$
\begin{aligned}
m_2 &= 1 + 3\lambda^2 \\
m_3 &= 16c\,\lambda(1+\lambda^2)\frac{(\eta-2)^2}{(\eta-1)(\eta-3)} && \text{if } \eta > 3, \\
m_4 &= 3\frac{\eta-2}{\eta-4}(1+10\lambda^2+5\lambda^4) && \text{if } \eta > 4.
\end{aligned}
$$

Since Z has zero mean and unit variance, we obtain that skewness (Sk) and kurtosis (Ku) are directly related to the third and fourth moments:

$$\mathrm{Sk}[Z] = \mathrm{E}[Z^3], \qquad \mathrm{Ku}[Z] = \mathrm{E}[Z^4].$$

Excess kurtosis is defined as $\mathrm{XKu} = \mathrm{Ku} - 3$.

We notice at this stage that the density and the various moments do not exist for all parameters. Given the way asymmetry is introduced, we must have $-1 < \lambda < 1$. The density g is meaningful if $\eta > 2$. Careful scrutiny of the algebra yielding equation (2) shows that skewness exists if $\eta > 3$. Last, kurtosis in equation (3) is well defined if $\eta > 4$.[5]

Given these restrictions on the underlying parameters, it is clear that skewness and kurtosis will also be restricted to certain domains. Figure 2 (3) traces the skewness (respectively kurtosis) surface for given values of λ and η. Focusing on figure 2, we notice that for small values of η the range of possible skewness is large. On the other hand, when one slightly increases η beyond 4, the surface strongly levels out. When we consider the case of kurtosis in figure 3, we verify a degeneracy as η reaches its boundary value of 4. To get a better feel for the possible range of skewness that one can obtain as λ varies between -1 and 1, we trace in figure 4 various curves corresponding to selected values of η. For the case where η takes the value 4.5, hence when kurtosis exists, we notice a strong restriction for skewness ranging between -3 and 3. Clearly, for even higher values of η, the possible range of skewness decreases even further.

This last picture illustrates the fact that for a given level of kurtosis only a finite set of skewness may exist. This raises the more general question of existence of a density for given moments. We address this issue in the next section.

2.2 The moment problem

The question of the existence of a non-decreasing function α for a sequence of scalars μ_j such that

$$\mu_j = \int_a^b x^j d\alpha(x) \tag{4}$$

has already been addressed in the functional analysis literature. There are essentially two approaches. The first approach is discussed in Widder (1946). The case $a = 0$, $b = \infty$ has been investigated by Stieltjes (1894) and was motivated by a problem issued from physics.[6] The case $a = 0$ and $b = 1$ has

[5] In empirical applications, we will only impose that $\eta > 2$ and let the data decide for itself if for a given time period a given moment exists.

[6] The first moment appears as a center of gravity and the second moment is interpreted as the inertia.

been studied by Hausdorff (1921a, 1921b), and is called the little Hausdorff problem. The situation of interest for us, $a = -\infty$ and $b = +\infty$, has been studied by Hamburger (1920). A second approach to the moment problem is discussed in Baker and Graves-Morris (1996) and involves Padé approximants.

A first result is that a solution to equation (4) is not unique. Widder (1946) provides a counter-example. Next, there is the question of conditions that must be satisfied by μ_j to ensure existence of a solution to equation (4). The answer to this question is that the sequence μ_j must be positive definite (Widder, 1946, p. 134, Theorem 12.a). This means that the following sequence of numbers and determinants must satisfy

$$\mu_0 \geq 0 \qquad \begin{vmatrix} \mu_0 & \mu_1 \\ \mu_1 & \mu_2 \end{vmatrix} \geq 0, \qquad \begin{vmatrix} \mu_0 & \mu_1 & \mu_2 \\ \mu_1 & \mu_2 & \mu_3 \\ \mu_2 & \mu_3 & \mu_4 \end{vmatrix} \geq 0 \cdots .$$

In particular, for the four-moment problem, with $\mu_0 = 1$, $\mu_1 = 0$, and $\mu_2 = 1$, this implies the following relation between skewness μ_3 and kurtosis μ_4:

$$\mu_3^2 < 1 + \mu_4, \text{ with } \mu_4 > 0. \tag{5}$$

This relation shows that, for a given level of kurtosis, only a finite set of skewness may be reached. Given that, for a normal density, μ_4 takes the value 3, this shows that a density will exist for densities that may have significantly thinner tails than the normal density.

Figure 5 illustrates the skewness-kurtosis boundary ensuring the existence of a density. The curve ABC corresponds to the domain (5) for the general case. The curve DEFG corresponds to the domain of attainable skewness and kurtosis, assuming $\eta > 0$. We notice that the kurtosis cannot be below 3, indicating that the generalized Student-t distribution does not allow for tails thinner than the normal distribution. The maximum value for skewness is attained when $\eta \to 0$ and $\lambda \to 1$ (or -1). In this case, upper bound for skewness is

$$\bar{\mu}_3 = \frac{4\bar{c}\left(9 + 32\bar{c}^2\right)}{\left(9 - 16\bar{c}^2\right)^{3/2}}$$

with $\bar{c} = \Gamma(5/2) / \left(\sqrt{2\pi}\Gamma(2)\right)$. Approximately, one finds $\bar{\mu}_3 = 3.9978$. Note that kurtosis has to take very large values for $\bar{\mu}_3$ to be attainable.

Given these conditions on the existence of moments, we may now consider our general model.

2.3 A model for time-varying skewness and kurtosis

Let r_t, for $t = 1, \cdots, T$, be realizations of a variable of interest. For exchange-rate and stock-market data, this variable will be a log-return. For interest-rate data, we will consider changes that are defined as $100\,(R_t - R_{t-1})$ where R_t is the interest rate prevailing at t. We assume that

$$r_t = \mu_t + y_t, \tag{6}$$

$$y_t = \sigma_t \epsilon_t, \tag{7}$$

$$\sigma_t^2 = a_0 + b_0^+ (y_{t-1}^+)^2 + b_0^- (y_{t-1}^-)^2 + c_0 \sigma_{t-1}^2, \tag{8}$$

$$\epsilon_t \sim HT(\epsilon_t | \eta_t, \lambda_t). \tag{9}$$

Equation (6) decomposes the return of time t into a conditional mean, μ_t, and an innovation, y_t. Equation (7) defines this innovation as the product between conditional volatility, σ_t, and a residual, ϵ_t. The next equation (8) determines the dynamics of volatility. We use the notation $y^+ = \max(y, 0)$ and $y^- = \max(-y, 0)$. Such a specification has been suggested by Glosten, Jagannathan, and Runkel (1993), and by Zakoïan (1994). In equation (9), we specify that residuals follow a generalized Student-t with time-varying parameters (η_t, λ_t).

Our stated aim is to test for persistence and existence of conditional moments. This means that we must allow for the parameters of the generalized Student-t distribution to have a dynamic specification. It is tempting to use for η_t and λ_t a specification similar to an ARMA(1,1), thus resembling equation (8). Such a specification is, however, hazardous. Indeed, for financial data, there exist outliers (such as the October 1987 crash). This in turn may lead to spuriously significant parameters. To see how such spurious parameters may arise let us proceed with a thought experiment. We assume an ARMA(1,1) type specification for the parameters such as

$$\lambda_t = a + b r_{t-1} + c \lambda_{t-1}.$$

Furthermore, we consider that the data results from i.i.d. normal data. Now, the estimates of b and c will be small and statistically non-significant. Because of random variation, b and c will not be equal to zero, assume that they take positive values. For our thought experiment we consider now the replacement of r_{t-1}, the return at time $t-1$, by a large positive perturbation. Had such an event existed in reality, it would have created heavy tailedness. Because at time t, λ_t needs to be bounded above by 1, the program will converge at a solution where the impact at time t is undone. This is achieved with the choice of a large negative c that may appear statistically significant even if robust estimates, i.e., White (1980), of the standard error get used. For this reason, we will use a specification without lagged parameter:

$$\eta_t = a_1 + b_{11} r_{t-1} + b_{12} r_{t-2}, \tag{10}$$

$$\lambda_t = a_2 + b_{21} r_{t-1} + b_{22} r_{t-2}. \tag{11}$$

We also have the following constraints on the parameters

$$b_0^+ + c_0 < 1, \tag{12}$$

$$b_0^- + c_0 < 1, \tag{13}$$

$$2 < \eta_t, \tag{14}$$

$$1 < \lambda_t < 1. \tag{15}$$

The first two equations, (12) and (13), guarantee stationarity for the volatility process given by equation (8). The following $T-2$ constraints are necessary to guarantee that the density is well defined (the first two values η_1 and η_2 are not needed). Last, equation (15) involves $2(T-2)$ inequality constraints that guarantee that skewness will be well defined.

The estimation of model (6) to (9) and (10) to (11) under the constraints (12) to (15) represents a formidable task. Furthermore, given that the likelihood is defined unless the constraints are binding, it is necessary to use an optimization algorithm, where the constraints are always satisfied. This implies using an interior optimization algorithm. Furthermore, the sample is of a rather large size. For this reason, speed becomes a very important factor. Given the structure of our problem, we use a program developed for sparse matrices: SNOPT, developed by Gill, Murray, and Saunders (1997, 1999).

For a given set of initial values, this program first verifies that all initial values satisfy the restrictions. In case of non satisfaction, it searches initial values satisfying the restrictions and that are closest to the proposed initial values with respect to the Euclidean norm. Next, it uses a sequential quadratic programming algorithm whereby it is guaranteed that the linear constraints are always satisfied.

As mentioned, optimization under tens of thousand of restrictions is a formidable task. For this reason, it is tempting to impose artificial constraints forcing parameters into the authorized domain via a non-linear function. This non-linear function may, however, introduce distortions in the problem. To investigate this issue, we consider logistic transforms mapping unconstrained dynamics into constrained ones. This yields:

$$\tilde{\eta}_t = a_1 + b_{11} r_{t-1} + b_{12} r_{t-2}, \tag{16}$$

$$\tilde{\lambda}_t = a_2 + b_{21} r_{t-1} + b_{22} r_{t-2}, \tag{17}$$

$$\eta_t = g_{[2,+\infty]}(\tilde{\eta}_t),\ \lambda_t = g_{[-1,1]}(\tilde{\lambda}_t),$$

where $g_{[a,b]}(x) \equiv a + (b-a)(1+e^{-x})^{-1}$ is defined as the logistic map.[7]

3 Empirical results

3.1 The dataset

3.1.1 Description

In this study, we investigate the time series behavior of six foreign exchange-rate series, of five stock indices, of four 3-month Euro-rate changes, and of five long-term interest-rate changes. We use the following symbols: SFR-DM for the Swiss Franc-Deutsche Mark and CAN-US for the Canadian dollar to US dollar. Then we use DM-US, YEN-US, UK-US, FF-US for the amount of Deutsche Mark, Yen, British Pound, and French Francs necessary to purchase one US dollar. S&P, NIK, DAX, CAC, and FTSE correspond to the S&P 500, the Nikkei, the Deutsche Aktien Index, the CAC40, and the FTSE 100 stock-market indices. All those indices have also been used in other studies, i.e., Jorion (1995), Ait-Sahalia (1999), Gray (1996). Data for the week of the October 1987 crash have been suppressed

[7] For the case $+\infty$, we set b to a large positive value such as 30.

from the data set. The exchange rates have been provided by a large bank. They cover the period from July 26, 1991 to September 3, 1999, representing 1969 observations. The stock-market indices have been obtained from Datastream. They cover the period from August 23, 1971 to September 3, 1999 for a total of 7159 observations. For interest-rate data we use the symbols US3M, UK3M, FR3M, and GE3M for the 3-month Euro-rate changes for the US, the UK, France, and Germany. Similarly, we use USLT, UKLT, FRLT, and GELT for bonds with ten years to maturity for the US, UK, France, and Germany. Our short-term interest-rate data runs from January 3, 1975 to September 3, 1999, for a total of 6535 observations. Our long-term interest rates cover the period from May 20, 1986 to September 3, 1999.[8]

3.1.2 Descriptive Statistics

Exchange rates and stock-market indices

Table 1 displays several sample statistics, first for exchange rates and then for stock-market indices. We notice that the standard deviation of exchange rates tends to be smaller than for indices. Exchange rates display a wide range of possible skewness. This ranges from -1.3889 for the YEN-US to 0.3868 for the UK-US. This translates the fact that, over the sample considered, on certain occasions, the Yen appreciated sharply whereas the pound depreciated. Skewness is significant for all series except for the DM-US and the FF-US. Given the effort to align both currencies, we can expect the two series to have a similar behavior. For the stock indices, we find a negative skewness for the S&P, DAX, and CAC, indicating the presence of sharp drops in stock prices. However, when standard errors are computed using the Generalized Method of Moments, as suggested by Richardson and Smith (1993), we find that no skewness is significant at the 5 percent level. Turning to excess kurtosis, we find that all countries have a strongly significant statistic. This translates the fact that the tails of exchange rates and of stock returns are fatter than tails of the normal distribution. Considering the Jarque-Bera statistic, which is distributed as a χ^2 with two degrees of freedom, we reject normality for all series.

The Engle test statistics with lags 1 and 5, obtained by regressing squared returns on one lagged, respectively five lagged, squared returns is distributed as a χ^2 with the degree of freedom equal to the number of lags. The strong significance of the statistics reveals the presence of heteroskedasticity in the data.

The Box-Ljung statistic, corrected for heteroskedasticity, tests for the existence of serial correlation among the first 5 or 10 observations. Even though we are able to detect serial correlation, the coefficient of correlation is always small.[9]

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[8] We did not do any data-snooping in this study. That is we did not try to select a particular sample length to obtain *nicer* empirical results. Also, we did not drop any data-series for which our model may not have worked. The inclusion of little investigated datasets such as the Swiss Franc-Deutsche Mark or the Can-US exchange rate was motivated by the question whether less liquid markets are subject to different dynamics.

[9] We filtered the data with an AR(5) auto-regression and estimated various specifications with and without the filtering. Since the estimations, involving filtered or non-filtered data, yielded similar results, we decided to report the results obtained for non-filtered data.

Three-month and ten-year interest-rate changes

Table 2 displays descriptive statistics for short-term and long-term interest-rate changes. We find that the standard deviation of short rates is larger than for long rates. Skewness is significant at the 5 percent level for all countries except for the German 3-month rate. Economic theory provides few hints what the sign of skewness should be for interest-rate data, and indeed, the sign pattern of skewness is not clear-cut. The positive skewness for the UK short rate and the German long rate suggests that these countries had on average more sharp upward movements than downward movements. At usual significance levels, no skewness is statistically significant.

Excess kurtosis is always strongly significant. This suggests that the distribution of interest-rate changes has a thicker tail than the normal distribution. We notice at the short end a magnitude of kurtosis at least three times larger than at the long end. In particular, for the French short rate, we find an excess kurtosis of 331.53 and an associated standardized excess kurtosis of 5470.65. The magnitude of this statistics questions the existence of a fourth moment for this series. The Jarque-Bera statistics takes very large values, suggesting that the data is non-normally distributed.

Turning to the Engle test for heteroskedasticity, we notice again large coefficients, indicating that the data is highly heteroskedastic. The Box-Ljung statistics for serial correlation reveals that serial correlation may exist for US and UK data at the short end. At the long end, only US rates display a certain level of serial correlation.

These descriptive statistics suggest that moments beyond variance may have an important role to play. Our model will help in understanding the dynamics of skewness and kurtosis.

4 Estimation of the general model

4.1 ...using daily foreign exchange-rate and stock-index data...

Table 3 reports parameter estimates of the general model at daily frequency. The first row of the table, labeled info, displays consistently a 0. This indicates that the sequential quadratic programming algorithm converged and found a solution. [10] We performed all estimations using the same set of initial values for the GARCH equation (8) ($a_0 = 0.05$, $b_0^+ = 0.05$, $b_0^- = 0.05$, $c_0 = 0.85$). This choice was guided by our prior knowledge of estimates reported in the literature. Furthermore, we imposed $a_1 = 5$, $b_{11} = b_{12} = a_2 = b_{21} = b_{22} = 0$. This vector of initial values is an interior point, i.e., a vector for which all constraints are satisfied. When we perturbed these initial values and selected non-interior points, the program performed a phase-I run of the simplex algorithm to find an interior point, closest to the proposed initial value, with respect to the Euclidean distance. The second row of table 3 displays the

[10] A solution may be local rather than global; for this reason, we also estimated the model with different initial values. For all series, we found convergence to the same values with the exception of the SFR-DM. For that series we initialized the model with the parameters obtained from the model without the second lag, i.e., $b_{12} = b_{22} = 0$. For weekly data, where our samples are much smaller, the problem of multiplicity of a solution appears more pronounced than for daily data.

time required before convergence was reached. For a precision of 10^{-6}, we obtained convergence under 18 seconds for all exchange-rate data and under 57 seconds for the roughly three times larger stock-market database. This suggests that our algorithm can be used for real-time applications.

We now turn to interpreting the volatility equation (8). Since the volatility equation allows asymmetries, we first test their relative importance. The statistics LRT_1 corresponds to the likelihood-ratio test of the null hypothesis $H_0 : b_0^+ = b_0^-$. Considering exchange rates, we notice that, except for the SFR-DM, no asymmetric impact of news exists. For the SFR-DM, a rather thinly traded currency, we find that a decrease of the exchange rate (an appreciation of the SFR with respect to the DM) is followed by an increase of volatility.[11] For all series, inspection of the magnitude of b_0^+ and b_0^- indicates that a large return will lead to a subsequent increase in volatility.

Considering stock-market data, we find strong differences between "bad" negative residuals and "good" positive residuals. Negative returns will have a larger impact on the volatility of future returns than positive returns. This observation has been well documented, e.g., Nelson (1990), Campbell and Hentschel (1992), Glosten, Jagannathan, and Runkle (1993), Zakoïan (1994), Sentana (1995), and may be explained by Black's (1976) leverage hypothesis.

We now turn to the dynamics of those parameters that drive higher moments. That is, we consider estimates of the degree of freedom parameter η_t (equation (10)) and of the asymmetry parameter λ_t (equation (11)). To do so, we first turn to the issue of how long time it takes before a large event is incorporated in the data. A hint is given by the likelihood-ratio test statistic LRT_2 of the null hypothesis $H_0 : b_{12} = b_{22} = 0$, i.e., if the second order lag for both η_t and λ_t matters. For exchange rates, we uniformly find that the second lag is not required. We present, in table 3, the parameters b_{12} and b_{22} even though they are non significant, because, a reestimation of our model imposing $b_{12} = b_{22} = 0$ showed that the lag-one parameter estimates were not significantly altered. For stock returns, the smallest likelihood ratio statistics is 8.30 for the Nikkei, whereas the critical value for a 5 percent level is 5.99. This means that a specification with two lags is required for stock returns.

Inspection of parameter b_{11} indicates, for exchange rates, that an excessively large positive return (depreciation of a currency with respect to its reference currency) will increase the tails of the distribution on the subsequent day. We also find that all estimates of the parameter b_{12} are negative. This suggests that a correction occurs during the second day. Hence, in the short run (say one day), extremes will be followed by extremes, then markets calm down after the second day.

Turning to stock returns, we find that all parameters b_{11} and b_{12} are negative (except b_{11} for the CAC return) and that both b_{11} and b_{12} are significant for three out of five series. Again, an interpretation may be given. Consider an extreme event consisting of a crash. Occurrence of this crash, combined with the negative coefficients b_{11} and b_{12} suggests that on two consecutive days η_t will be abnormally high. This implies the presence of conditional fat-tailedness. This finding also indicates that the "removal"

[11] An explanation of this finding without further considering cross-country interest-rate differentials would be hazardous and will not be attempted here.

of volatility through a GARCH model leaves conditional information, of an higher order, that may be important to model situations where the tail behavior of distributions is crucial.

Stock market data is skewed, and casual evidence suggests that extreme events will consist of crashes. Let us therefore consider the occurrence at time $t-1$ of a large negative return. Given the form of equation (10), a negative coefficient b_{11} implies that the day after, at time t, the market will be associated with a large event. Since η_t is a measure of the tail of the distribution, it is not possible at this stage to make a statement whether the large event will be positive or negative. The second lag, b_{12}, indicates that even two days after a large negative event, there will be a higher probability of a tail event.

Now, we turn to the interpretation of the dynamics of λ_t. First, we observe that the sign of a_2 tends to be of the sign of skewness, as reported in table 1. This confirms that λ_t captures directional movements driven by large events. Next, we observe that for exchange rates as well as for stock returns, parameters b_{21} and b_{22} are positive. Exceptions are b_{22} for the YEN-US with a non-significant estimate of -0.0161 and b_{21} for the CAC with an estimate of -0.0236.

This positive sign for the two lagged parameters indicates that there is a tendency for extremes of a given sign to be followed by large events of the same sign for several days. For exchange rates, for which only lag-one parameters are significant, this implies that after a depreciation of the currency, the day after there is an increased probability of a further depreciation. For stock returns, we find that for four out of five series, in case of a large drop of the market, skewness will continue to be negative, and hence the probability of a large negative event is increased.

These results display intuitive and reasonable patterns for conditional fat-tailedness of returns and of skewness. Now, we turn to the interpretation of interest-rate changes.

4.2 ... using daily short-term and long-term interest-rate changes

As we will show in interpreting table 4, the dynamic of interest-rate changes differs in important ways from the dynamic behavior of foreign exchange and stock returns. In particular, we will show that higher moments for short rates do not exist, and when they do they cannot be predicted. First, we consider the volatility equation of the short rate. Inspection of LRT_1, which is the test statistics of the hypothesis $b_0^+ = b_0^-$, reveals that there is an asymmetric impact of positive and negative interest-rate changes on subsequent volatility. Inspection of the relative size of the parameters b_0^+ and b_0^- indicates that a positive shock on interest rates induces a stronger increase in volatility than a negative shock. This finding is in accordance with the casual observation that increases in interest rate represent “bad news” for the economy given their negative impact on investment and given that they are triggered by other “bad news” often related to increases in inflation. We also find that there is persistence in volatility as testified by the large estimates of c_0.

We turn now to the dynamics of η_t and λ_t. First, the likelihood-ratio test statistic LRT_2 of the null hypothesis $b_{11} = b_{12} = b_{21} = b_{22} = 0$ indicates that, with the exception of the German long rates, there is little evidence for a dynamic involving two lags. Inspection of the heteroskedasticity-consistent standard errors, associated with b_{12} and b_{22}, sheds further insights on the lag-two dynamic. For the 3-month interest-rate changes, none of the coefficients, with the exception of b_{12} for FR3M and b_{22} for GE3M, are significant. On the other hand, we notice for the long rate, even though $b_{12} = 0$ cannot be rejected excepted for Germany, that the hypothesis $b_{22} = 0$ may be soundly rejected for all long rates except for the US where $b_{22} = 0.0041$ with a standard error of 0.0028. This finding suggests that a certain persistence exists in higher moments, driven by b_{22}.

Given that none of the coefficients involved in the dynamics of η_t and λ_t for the short rate are significant, we focus now on the interpretation of the dynamics for the long rates. We notice that coefficients b_{11}, b_{12}, b_{21}, and b_{22} tend in general to be positive. This means that after a "bad event" occurred for interest rates, i.e., an increase, subsequently the distribution of returns will be fat tailed. Furthermore, it is possible to state that after a positive shock, skewness will be positive. This shows that after an event of a given sign occurred, the market will tend to be followed by a similar event.

We may conclude this section by stating that very little can be said about the dynamics of higher moments of short-term interest-rate changes. There does not seem to exist a particular relation. On the contrary, long-term interest-rate changes follow dynamics similar to exchange rates or stock indices. After a bad news, i.e., a large increase of the rate, the probability of a further large augments. This feature is measured through the fat-tailedness of returns as well as through the increased skewness.

So far we interpreted only briefly the behavior of the parameters λ_t and η_t. The reason for doing so is that their actual, simultaneous impact on the generalized-t distribution is sometimes difficult to interpret. For instance, an increase of λ_t will lead to higher skewness, yet a contemporaneous decrease of kurtosis via η_t may offset the possible increase in fat-tailedness. The link of λ_t and η_t with skewness and kurtosis is highly non-linear. In a later section we will return to the direct analysis of skewness and kurtosis.

As aggregation of the data occurs, the dynamic at a different frequency is known to be rather different, i.e. Drost and Nijman (1993). For this reason we consider now the estimates obtained at a weekly frequency.

4.3 ... and using weekly data

The interpretation of weekly results follows a similar logic as the interpretation of daily data. The set of series for which predictability of higher moments exists is strongly reduced. For this reason, we discuss the results for all series simultaneously. In tables 5 and 6, we display the results for our estimations at a weekly frequency. Concerning the volatility dynamics, we notice for exchange rates that decreases of an exchange rate with respect to the dollar are likely to trigger increased volatility. Our reference currency is the US dollar, hence it is not surprising that all exchange-rate returns will display similar patterns.

For the stock-market indices, we find volatility impact patterns that are compatible with Black's (1976) leverage hypothesis: Bad news, i.e., crashes, will trigger larger volatility than good news.

Turning to interest rates, both at the short and the long end, we find that interest-rate increases tend to trigger larger subsequent volatility than interest-rate drops.

Turning now to the dynamics of η_t and λ_t, we first observe that the likelihood-ratio test statistic LRT_3 testing for any dynamics at all is not significant, except for a few series. Furthermore, rather few coefficients reach statistical significance, and when they do, the signs are ambiguous. Those observations suggest that there is little evidence of persistence of skewness and kurtosis at a weekly frequency. For these reasons, our further investigations will only focus on daily data.

4.4 Remaining specification issues

We now address the question whether the allowance of a dynamic of higher moments destroys asymmetries in the volatility equation of stock-market data. To do so, we compared the estimates reported in table 3 with those obtained in a standard GARCH(1,1) model, i.e., where $b_{11} = b_{12} = b_{21} = b_{22} = 0$. Even though we do not report the values obtained in this estimation, our finding is that the explicit modeling of the asymmetry and fat-tailedness of residuals does not significantly alter the value taken by the estimates that appear in the volatility equation. For instance, for the SFR-DM, the largest deviation of a parameter is by an amount of 0.01 for c_0. For the DM-US, estimates of a_0, b_0^+, b_0^-, and c_0, were 0.0138, 0.0471, 0.0786, and 0.9099 respectively, indicating that the difference with the estimates reported in table 3 is truly a small one. For all other series, the deviations of the parameters with respect to the GARCH(1,1) estimates are of a similar magnitude.

The estimation of a model involving several thousand inequality constraints is a unusual task in finance. Often, the choice of a logistic transform is made, such as in equations (16) and (17). To measure the importance of this type of non-linear map, we also estimated the model after imposing the logistic transform. Again, we will not report the estimates. Clearly, because of the transformation, parameter estimates will no longer be the same. More importantly, we found that many times a parameter that turned out to be significant with one model, was no longer significant with the other model. Also, the dynamics obtained for the resulting series of constrained, time-varying parameters, η_t and λ_t turned out to be different. For this reason, we decided to report the results obtained without the logistic transform.

5 Analysis of the dynamics of skewness and kurtosis

Several issues are outstanding. In the previous section, we estimated the dynamics of parameters η_t and λ_t. Even though these parameters are related to skewness and kurtosis, the relation is a highly non-linear one. For this reason, in order to proceed one step further, we now consider the evolution of skewness and kurtosis through time, obtained from equations (2) and (3). Next, we analyze cross-sectional movements between various markets in terms of skewness as well as kurtosis. Our model, therefore, extends in a certain sense the one by Kroner and Ng (1998).

5.1 Existence of moments

Inspection of formulas (2) and (3) suggests that third and fourth moments will only exist for $\eta_t > 3$ and $\eta_t > 4$ respectively. We first address the issue of the existence of skewness and kurtosis.[12]

Table 7 reports at daily frequency, for each series, the number of times when constraints $\eta_t > 3$ and $\eta_t > 4$ were binding. This counts the number of days when skewness or kurtosis does not exist over the sample under study.

We first consider foreign exchange returns and stock-market returns. The number of dates during which the skewness does not exist is very low. The largest proportion of infinite skewness is obtained for the YEN-US and the Nikkei, with 0.3% of the sample in both cases.

The number of cases where the kurtosis does not exist is, by construction, larger than for the skewness, since the constraint $\eta_t > 4$ is more often binding than $\eta_t > 3$. It is as high as 5.3% for the Nikkei and 4.4% for the YEN-US exchange rate. Therefore, for these markets, assuming the existence of the fourth moment may be misleading.

Turning now to interest-rate changes, we find that skewness and kurtosis do not exist for many dates. In particular, for three-month interest-rate changes, kurtosis never exists whatever the country. Moreover, for France and Germany, even skewness is found to be infinite in most cases. Concerning long-term interest-rate changes, we obtain that German and UK kurtosis do not exist for a large number of dates (between 12% and 60% of the sample). Yet, skewness exists for almost all dates for each country.

5.2 Persistence of skewness and kurtosis

We turn now to the issue of the persistence of skewness and kurtosis. As an illustration, we display the evolution of η_t and λ_t, and the first four moments for the DM-US exchange rate in figures 6 and 7 and for the S&P in figures 8 and 9. Since equations (10) and (11) describe both η_t and λ_t as depending on lagged returns, we obtain that both η_t and λ_t display a pattern similar to returns (figures 6 and 8). But this is not the case for skewness and kurtosis, since relations between (η_t, λ_t) and (m_{3t}, m_{4t}) are highly non-linear ones. In figure 7, we first note that conditional volatility is highly persistent, but stationary. Major events during the sample period (as the September 1992 EMS crisis or the September 1998 Russian crisis) are associated with a long-lasting increase in volatility. The mean-reverting time period can be as long as one month. A particularly interesting feature of skewness and kurtosis is that they are far less persistent than volatility. Stars in figures for skewness and kurtosis indicate dates when kurtosis was found to be infinite $(\eta_t < 4)$. We notice that a strong increase in volatility is generally accompanied by infinite skewness and kurtosis. Moreover, these figures do not reveal strong serial correlation of skewness and kurtosis. Closer inspection of kurtosis for the S&P reveals some kurtosis clustering.

For the S&P, we also display in figure 10 a scatterplot of past and current skewness in the top graph as well as of past and current kurtosis in the bottom graph. At first glance, we notice a positive relationship

[12] In fact, there are only very few cases where skewness exists but kurtosis does not exist, $(3 < \eta_t < 4)$.

between past and current skewness. The serial correlation coefficient is 0.2 and the sum of diagonal element in the transition matrix is 30.8%.

To gain further insight on persistence in third and fourth moments, tables 8 and 9 report transition probability matrices for skewness and kurtosis respectively. We rank the value of a higher moment into one of five possible categories. First, entries with infinite values were isolated (in an interval denoted by I_5). For the remaining entries, we constitute four intervals, corresponding to the quartiles (I_j, $j = 1, ...4$). Elements (a, b) of transition probability matrices measure the percentage of times that the series moves from a skewness (or kurtosis) in quartile b at time $t-1$ to quartile a at time t. Considering transition probability allows to circumvent some drawbacks of serial correlation coefficients in presence of outliers.[13] In absence of serial correlation, each element of the transition matrix would have the same value ($1/16 = 6.25\%$). In case of a positive correlation, elements along the principal diagonal are larger than off-diagonal elements.

We first consider transition matrices for skewness, for exchange rates and stock returns (table 8). In most cases, we obtain a positive relation between m_{3t} and m_{3t-1}. Let us for instance consider the UK-US exchange rate. At any date $t-1$, the skewness falls in quartile 1 with a probability of 25 percent. Then, conditionally on the fact that m_{3t-1} belonged to quartile 1, the skewness at date t turns out to be in quartile 1 in 11.7 out of the 25 cases, whereas it is in quartile 4 in only 3 out of 25 cases. If we sum probabilities on the diagonal, we obtain 41.6%. This is much higher than 25%, as it would prevail in case of absence of serial correlation. We can conclude that skewness of the UK-US exchange rate has a strong positive correlation. Interestingly, we notice that skewness in low and high quartiles (quartiles 1 and 4) at date $t-1$ displays a larger probability to stay in the same state at time t. This persistence-result of states is rather general. For instance, the sum of elements on the first diagonal is high for the DM-US and FF-US exchange rates (35.8%, and 39.35% respectively) and for the DAX and FTSE returns (39.3% and 37.5% respectively). Moreover, elements $(1, 1)$ and $(4, 4)$ of the transition matrix have probability larger than 10%.

Finally, for two series, the YEN-US exchange rate and the CAC return, we obtain a negative relationship between m_{3t} and m_{3t-1}.

Now, we turn to the persistence of kurtosis. For some markets, we obtain a strong positive relation between past and current kurtosis. The sum of diagonal elements is 41.1% for the S&P, 40.1% for the Nikkei and 35.5% for the FTSE. Inspection of figure 10, bottom panel, confirms that the S&P displays a rather persistent kurtosis. On the contrary, we obtain a negative relationship for most exchange rates (DM-US, UK-US, and FF-US) and for the CAC return.

5.3 Coskewness and cokurtosis

An abundant literature has documented volatility co-movements (see, e.g., Hamao, Masulis, and Ng, 1990, or Susmel and Engle, 1994, for stock markets). More recently, some authors stated that correlation

[13] This is a well-known problem. The correlation coefficient can be biased, if two extreme values occurs at two consecutive dates. With transition matrices, outliers cannot bias correlation measures, since they only appear as elements of an interval.

between markets may increase during periods of high volatility (Longin and Solnik, 1995, Ramchand and Susmel, 1998). Now, we wish to address the issue of co-movements between markets under investigation in terms of skewness and kurtosis. A positive coskewness between two markets indicates an increase in the probability of occurrence of a large event in the same direction on both markets. A positive cokurtosis reveals an increase in the probability of occurrence of a large event on both markets, whatever the direction of the shock.

To illustrate the strong relationship between higher moments of DM-US and FR-US, figure 11 displays a scatterplot of skewness and kurtosis for the exchange rates. The relation is clearly positive. In absence of outliers, the estimation of the correlation coefficient is not biased. Therefore, we regress the DM-US moment on the corresponding FR-US moment. We find a parameter estimate of 1.118 (with standard error of 0.010) for skewness and 0.921 (with standard error of 0.011) for kurtosis. Corrected R^2 are as high as 0.87 and 0.77 respectively.

Tables 10 and 11 report joint probability matrices for coskewness and cokurtosis for some pairs of markets. We consider relationships between the DM-US exchange rates vis-à-vis other currencies, and relationships between the S&P and other stock-market indices.

First, considering exchange rates, we find large positive co-movements of skewness between European currencies. The sum of diagonal elements is as high as 73% for DM-US and FR-US and 50% for DM-US and UK-US. This indicates, for instance, that the occurrence of a DM-US skewness in a given quartile is associated, in 73 out of 100 cases, with a FR-US skewness in the same quartile. Moreover, the strong link between both skewness mainly comes from large events, since elements $(1,1)$ and $(4,4)$ of the transition matrix exceed 20% for the DM-US and FR-US and 15% for the DM-US and UK-US.

Inspection of table 11 reveals a similar pattern for cokurtosis. The sum of diagonal elements is 78% for DM-US and FR-US and 38% for DM-US and UK-US.

Turning to stock markets, we obtain positive relations between skewness and between kurtosis. But links are weaker than those obtained with exchange rates. The largest co-movements are found for the S&P and FTSE. The sum of diagonal elements is 35% for skewness and 33% for kurtosis. For other markets, we notice some co-movements, but only for the first and fourth quartiles, thus for more extreme events. Therefore, once again, our results indicate that co-movements of moments beyond volatility are more intensive during agitated periods.

6 Conclusion

This work has shown how to implement directly Hansen's (1994) model without ad-hoc parameter restrictions. This implementation involves a sequential quadratic programming algorithm where even several thousand constraints may arise.

The model is run over a large number of series at daily and weekly frequency. For daily data we find evidence of persistence for foreign exchange data and stock indices, less so for interest rate changes.

Especially for short term interest rates little evidence of persistence of higher moments is found. Turning to a weekly frequency, even foreign exchange and stock market higher-moments loose their predictability.

We extend the finding of Harvey and Siddique (1999) that the modeling of asymmetries of volatility has no impact on the dynamics of the skewness parameter to the case of kurtosis.

Turning to cross-sectional skewness and kurtosis, we document that higher moments of foreign exchange data and stock returns are strongly correlated.

The methodology developed in this paper is important for dynamic portfolio allocation and the testing of conditional financial models. Given that we model the tails of a distribution, our model is also of relevance for value at risk models. The model considered gives a conditional description of the tail behavior of returns. That financial returns are fat-tailed is not new. Mandelbrot (1973) mentions that returns should be modeled with stable laws. Stable distributions do not admit second moments. Without being so drastic, we are able to quantify those dates where kurtosis and possibly even skewness ceases to exist. Our model provides, thus, an alternative to stable laws.

The models is presently univariate. Multivariate extensions may be achieved with copula functions, i.e., Nelsen (1999). So far we emphasized the importance of this model for economic applications, yet the modeling of a certain type of parametric heteroskedasticity may be of relevance to gain in efficiency.

Appendix A

In the following we derive a certain number of theoretical results concerning Hansen's (1994) generalized-t distribution. We will extensively use the following result of Gradshteyn and Ryzhik (1994, p. 341, 3.241.4):

$$\int_0^\infty x^\mu (p+qx^\nu)^{-(n+1)} dx = \frac{1}{\nu p^{n+1}} \left(\frac{p}{q}\right)^{\frac{\mu}{\nu}} \frac{\Gamma\left(\frac{\mu}{\nu}\right)\Gamma\left(1+n-\frac{\mu}{\nu}\right)}{\Gamma\left(1+n\right)}$$

defined for $0 < \mu/\nu < n+1$, $p \neq 0$, and $q \neq 0$ where Γ is the gamma function for which $\Gamma(x) = (x-1)\Gamma(x-1)$ and $\Gamma(1/2) = \sqrt{\pi}$. We first use this lemma to verify that the expression (1), given in the text truly defines a density.

The starting point is the conventional Student's t distribution with η degrees of freedom defined by

$$g(x|\eta) = c\left(1+\frac{x^2}{\eta-2}\right)^{-\frac{\eta+1}{2}} \qquad x \in \mathcal{R}.$$

The constant c has to be set in such a manner that the probability mass integrates to 1. This requires that

$$\int_{x\in\mathcal{R}} g(x|\eta)\, dx = c\int_{-\infty}^0 \left(1+\frac{x^2}{\eta-2}\right)^{-\frac{\eta+1}{2}} dx + c\int_0^\infty \left(1+\frac{x^2}{\eta-2}\right)^{-\frac{\eta+1}{2}} dx = 1.$$

A change of variable in the left integral, from x into $-x$, shows that the two integrals in the center are equal. A straightforward application of the lemma implies that

$$c = \frac{\Gamma\left(\frac{\eta+1}{2}\right)}{\sqrt{\pi(\eta-2)}\Gamma\left(\frac{\eta}{2}\right)}.$$

In order to introduce an asymmetry, Hansen considers the new random variable

$$Y = \begin{cases} (1-\lambda)X & \text{if } X \leq 0, \\ (1+\lambda)X & \text{if } X > 0. \end{cases}$$

The mean and variance are defined as $a \equiv \mathrm{E}[Y]$ and $b^2 \equiv \mathrm{V}[Y]$.

$$g(y|\eta,\lambda) = \begin{cases} c\left(1+\frac{1}{\eta-2}\left(\frac{y}{1-\lambda}\right)^2\right)^{-\frac{\eta+1}{2}} & \text{if } y \leq 0, \\ c\left(1+\frac{1}{\eta-2}\left(\frac{y}{1+\lambda}\right)^2\right)^{-\frac{\eta+1}{2}} & \text{if } y > 0. \end{cases}$$

The first moment of Y, $m_1 \equiv \mathrm{E}[Y]$ follows from

$$\begin{aligned} m_1 \equiv \mathrm{E}[Y] &= c\int_{-\infty}^0 y\left(1+\frac{1}{\eta-2}\left(\frac{y}{1-\lambda}\right)^2\right)^{-\frac{\eta+1}{2}} dy + c\int_0^\infty y\left(1+\frac{1}{\eta-2}\left(\frac{y}{1+\lambda}\right)^2\right)^{-\frac{\eta+1}{2}} dy \\ &= I_a + I_b. \end{aligned}$$

We perform the change of variables $y = (1-\lambda)x$ and $y = (1+\lambda)x$ in the two integrals. It follows from the lemma that

$$\begin{aligned} I_a &= c(1-\lambda)^2 \int_{-\infty}^{0} x \left(1 + \frac{x^2}{\eta-2}\right)^{-\frac{\eta+1}{2}} dx \\ &= -(1-\lambda)^2(\eta-2)\frac{c}{2}\frac{\Gamma\left(\frac{\eta-1}{2}\right)}{\Gamma\left(\frac{\eta+1}{2}\right)}. \end{aligned}$$

And similarly $I_b = (1+\lambda)^2(\eta-2)\,(c/2)\,\Gamma\left(\frac{\eta-1}{2}\right)/\Gamma\left(\frac{\eta+1}{2}\right)$. Putting everything together yields

$$a \equiv \mathrm{E}[Y] = 4\lambda c \frac{\eta-2}{\eta-1}.$$

The second moment of Y follows using similar computations: $m_2 \equiv \mathrm{E}[Y^2] = I_a + I_b$. The same change of variables as previously yields

$$I_a = \frac{c}{2}(1-\lambda)^3(\eta-2)^{\frac{3}{2}}\frac{\Gamma\left(\frac{3}{2}\right)\Gamma\left(\frac{\eta-2}{2}\right)}{\Gamma\left(\frac{\eta+1}{2}\right)}.$$

Also $I_b = (1+\lambda)^3/(1-\lambda)^3 I_a$ and, therefore, after several simplifications we get that $\mathrm{E}[Y^2] = 1 + 3\lambda^2$. Since $\mathrm{V}[Y] = \mathrm{E}[Y^2] - (\mathrm{E}[Y])^2$, we obtain that $b^2 \equiv \mathrm{V}[Y] = 1 + 3\lambda^2 - a^2$. Since conditional residuals are assumed to have zero mean and unit variance, we introduce the random variable $Z = (Y-a)/b$ which will be centered, i.e., with mean 0, and reduced, i.e., with variance 1. The passage from Y to Z will not change the constant c, it is only necessary to multiply the density by the Jacobian of the transformation b^{-1}. Clearly, the random variable $Z \equiv (Y-a)/b$ has zero mean and unit variance. The density of Z follows from the change of variable $Y = bZ + a$ and is displayed in formula (1) of the text.

Those computations verify Hansen's. Our model involves, however, higher order moments that we compute now. The third moment of Y is given by $m_3 \equiv \mathrm{E}[Y^3] = I_a + I_b$. The same change of variables as previously yields

$$\begin{aligned} I_a &= -c(1-\lambda)^4 \int_{0}^{\infty} x^3 \left(1 + \frac{x^2}{\eta-2}\right)^{-\frac{\eta+1}{2}} dx \\ &= -\frac{c}{2}(1-\lambda)^4(\eta-2)\frac{\Gamma(2)\Gamma\left(\frac{\eta-3}{2}\right)}{\Gamma\left(\frac{\eta+1}{2}\right)} \\ &= -2c\frac{(1-\lambda)^4(\eta-2)^2}{(\eta-1)(\eta-3)} \end{aligned}$$

where the first equality follows from a straightforward application of the lemma. The second equality follows from simple algebra. Also $I_b = (1+\lambda)^4/(1-\lambda)^4 I_a$. Eventually we obtain

$$m_3 = 16c\lambda(1+\lambda^2)\frac{(\eta-2)^2}{(\eta-1)(\eta-3)}$$

defined if $\eta > 3$. The third moment of Z may be obtained as a function of the various moments of Y. We obtain

$$E[Z^3] = \left[\mathrm{E}[Y^3] - 3a\mathrm{E}[Y^2] + 2a^3\right]/b^3.$$

We now turn to the computation of the last moment of interest for this paper. A generalization for even higher moments can be easily obtained. The fourth moment may again be written as the sum of integrals: $m_4 = I_a + I_b$.

We have

$$\begin{aligned} I_a &= c(1-\lambda)^5 \int_0^\infty x^4 \left(1 + \frac{x^2}{\eta - 2}\right)^{-\frac{\eta+1}{2}} dx \\ &= \frac{c}{2}(1-\lambda)^5(\eta-2)^{\frac{5}{2}} \frac{\Gamma\left(\frac{5}{2}\right)\Gamma\left(\frac{\eta-4}{2}\right)}{\Gamma\left(\frac{\eta+1}{2}\right)}. \end{aligned}$$

The various steps involved in the computation use the same techniques as previously. Also, it can be shown that $I_b = (1+\lambda)^5/(1-\lambda)^5 I_a$. The second equality follows from the lemma and the third one from simple algebra. Regrouping terms we obtain

$$m_4 = 3\frac{\eta - 2}{\eta - 4}(1 + 10\lambda^2 + 5\lambda^4)$$

defined if $\eta > 4$. We also obtain the associated moment of Z as:

$$\mathrm{E}[Z^4] = \left[\mathrm{E}[Y^4] - 4a\mathrm{E}[Y^3] + 6a^2\mathrm{E}[Y^2] - 3a^4\right]/b^4.$$

We verified those formulas and their numerical implementation by computing the various moments via numerical integration of a generalized-t.

Appendix B

In the following appendix, we present the computations of the gradient of the log-likelihood. To simplify notations, we focus on the gradient of a single observation. Summation of these gradients yields the sample gradients. We define $d = (br/\sigma + a)/(1 - \lambda s)$ where s is a sign dummy taking the value of 1 if $br/\sigma + a < 0$ and $s = -1$ otherwise. We also define $v_1 = 1 + d^2/(\eta - 2)$. We recall that the likelihood of an observation is

$$l = \ln(b) + \ln\left(\Gamma(\frac{\eta+1}{2})\right) - \frac{1}{2}\ln(\pi) - \frac{1}{2}\ln(\eta-2) - \ln\left(\frac{\eta}{2}\right) - \ln(\sigma) - \frac{\eta+1}{2}\ln(v_1).$$

To obtain the gradients with respect to the various parameters a_0, b_0^+, b_0^-, c_0, a_1, b_{11}, b_{12}, a_1, b_{21}, b_{22} we decompose the problem and make frequent use of the chain rule of differentiation. The necessary ingredients to obtain the gradients are:

$$\frac{\partial l_t}{\partial \sigma} = -\frac{1}{\sigma} + \frac{\eta+1}{2}\frac{1}{v_1}\frac{2d}{\eta-2}\frac{br}{(1-\lambda s)\sigma^2}.$$

Next we have $\partial a/\partial\lambda = 4c(\eta-2)(\eta-1)^{-1}$, $\partial b/\partial\lambda = (3\lambda - a\,\partial a/\partial\lambda)/b$,

$$\begin{aligned}
\frac{\partial d}{\partial \lambda} &= \left(\frac{\partial b}{\partial \lambda}\frac{r}{\sigma} + \frac{\partial a}{\partial \lambda}\right)(1-\lambda s)^{-1} + sz(1-\lambda s)^{-2}, \\
\frac{\partial v_1}{\partial \lambda} &= \frac{2}{\eta-2}d\frac{\partial d}{\partial \lambda}, \text{ so that } \frac{\partial l_t}{\partial \lambda} = \frac{1}{b}\frac{\partial b}{\partial \lambda} - \frac{\eta+1}{2}\frac{1}{v_1}\frac{\partial v_1}{\partial \lambda}
\end{aligned}$$

To obtain $\partial l_t/\partial\eta$ we proceed similarly. First, we notice that $\partial c/\partial\eta = c\,\partial\ln(c)/\partial\eta$ and

$$\begin{aligned}
\frac{\partial \ln(c)}{\partial \eta} &= \frac{1}{2}\Psi(\frac{\eta+1}{2}) - \frac{1}{2}\frac{1}{\eta-2} - \frac{1}{2}\Psi(\frac{\eta}{2}), \\
\frac{\partial a}{\partial \eta} &= 4\lambda(\eta-2)(\eta-1)^{-1}\frac{\partial c}{\partial \eta} + 4\lambda c[(\eta-1)^{-1} - (\eta-2)(\eta-1)^{-2}], \\
\frac{\partial b}{\partial \eta} &= -\frac{a}{b}\frac{\partial a}{\partial \eta}, \qquad \frac{\partial d}{\partial \eta} = \left(\frac{\partial b}{\partial \eta}\frac{r}{\sigma} + \frac{\partial a}{\partial \eta}\right)/(1-\lambda s), \\
\frac{\partial v_1}{\partial \eta} &= -(\eta-2)^{-2}d^2 + 2(\eta-2)^{-1}d\frac{\partial d}{\partial \eta}, \\
\frac{\partial l_t}{\partial \eta} &= \frac{1}{b}\frac{\partial b}{\partial \lambda} + \frac{\partial \ln(c)}{\partial \eta} - \frac{1}{2}\ln(v_1)6\frac{\eta+1}{2}\frac{1}{v_1}\frac{\partial v_1}{\partial \eta},
\end{aligned}$$

where $\Psi(\cdot)$ is the derivative of the log of the gamma function. This derivative is known as the di-gamma function, which may be implemented with desired accuracy. The Fortran library IMSL also implements this function.

Now, we can compute the partials with respect to the actual parameters by using:

$$\begin{aligned}
\frac{\partial l_t}{\partial a_0} &= \frac{\partial l_t}{\partial \sigma}(1 + c_0\frac{\partial h_{t-1}}{\partial a_0}), \quad \frac{\partial l_t}{\partial b_0} = \frac{\partial l_t}{\partial \sigma}\frac{1}{2\sigma}(r_{t-1}^2 + c_0\frac{\partial h_{t-1}}{\partial b_0}), \quad \frac{\partial l_t}{\partial c_0} = \frac{\partial l_t}{\partial \sigma}\frac{1}{2\sigma}(h_{t-1} + c_0\frac{\partial h_{t-1}}{\partial c_0}), \\
\frac{\partial h_1}{\partial a_0} &= 1 + c_0\frac{\partial h_0}{\partial a_0} = 1, \quad \frac{\partial h_2}{\partial a_0} = 1 + c_0,
\end{aligned}$$

$$\frac{\partial l_t}{\partial a_1} = \frac{\partial l_t}{\partial \eta}, \quad \frac{\partial l_t}{\partial b_{11}} = \frac{\partial l_t}{\partial \eta} r_{t-1}, \quad \frac{\partial l_t}{\partial b_{12}} = \frac{\partial l_t}{\partial \eta} r_{t-2},$$
$$\frac{\partial l_t}{\partial a_2} = \frac{\partial l_t}{\partial \lambda}, \quad \frac{\partial l_t}{\partial b_{21}} = \frac{\partial l_t}{\partial \lambda} r_{t-1}, \quad \frac{\partial l_t}{\partial b_{22}} = \frac{\partial l_t}{\partial \lambda} r_{t-2}.$$

For the problem with logistic transform, the gradients are obtained in a similar manner.

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	SFR-DM	CAN-US	DM-US	YEN-US	UK-US	FF-US	S&P	NIK	DAX	CAC	FTSE
Day 1	26/07/91	26/07/91	26/07/91	26/07/91	26/07/91	26/07/91	23/08/71	23/08/71	23/08/71	23/08/71	23/08/71
N. Obs	1969	1969	1969	1969	1969	1969	7159	7159	7159	7159	7159
mean	-0.0032	0.0119	0.0025	-0.0060	0.0021	0.0018	0.0388	0.0262	0.0350	0.0391	0.0427
	0.0062	0.0077	0.0149	0.0175	0.0139	0.0144	0.0111	0.0125	0.0124	0.0136	0.0135
std. dev.	0.2947	0.4398	0.6903	0.8163	0.6078	0.6555	0.8923	1.0568	1.0542	1.0526	1.0176
	0.0126	0.0198	0.0198	0.0534	0.0212	0.0192	0.0161	0.0265	0.0271	0.0251	0.0238
sk	-0.3234	0.2389	-0.0015	-1.3893	0.3869	0.0139	-0.1815	0.2276	-0.5495	-0.4298	0.1247
	0.4462	0.2794	0.1683	0.8108	0.2041	0.1597	0.1331	0.2438	0.3039	0.3419	0.2022
ku	8.6814	6.5852	2.8583	15.9406	3.5221	2.7526	4.1538	8.5862	9.4163	8.9365	5.0643
	2.6728	1.6963	0.5754	9.0290	0.7601	0.5819	0.8587	2.2301	3.6092	3.7793	1.0719
J-B	6209.44 [a]	3571.26 [a]	668.89 [a]	21455.01 [a]	1064.94 [a]	620.36 [a]	5188.49 [a]	22059.01 [a]	26812.92 [a]	24046.18 [a]	7671.41 [a]
min	-2.6011	-2.5231	-3.9346	-9.954	-2.8076	-3.8343	-7.1127	-6.8267	-13.7099	-13.9100	-7.5609
max	2.2939	3.5879	3.8580	4.6928	4.0173	3.4857	4.9887	12.4303	7.2875	8.2254	8.9434
Engle(1)	47.25 [a]	120.68 [a]	12.58 [a]	358.41 [a]	78.36 [a]	18.46 [a]	201.57 [a]	214.15 [a]	346.98 [a]	50.87 [a]	941.82 [a]
Engle(5)	52.10 [a]	224.21 [a]	67.32 [a]	370.18 [a]	187.36 [a]	90.35 [a]	363.34 [a]	621.18 [a]	485.05 [a]	367.62 [a]	1355.17 [a]
ac(1)	-0.0309	-0.2138	-0.0596	-0.0199	-0.0187	-0.0372	0.0785	0.0272	0.0507	0.1336	0.1566
ac(2)	-0.0301	-0.0657	0.0228	-0.0173	0.0621	0.0255	-0.0027	-0.0441	-0.0656	-0.0169	-0.0091
ac(3)	-0.0367	0.0011	-0.0056	-0.0537	-0.0188	-0.0059	-0.0186	0.0038	-0.0165	-0.0121	0.0108
QW(5)	4.66	39.45 [a]	12.54 [a]	3.48	9.75 [b]	9.45 [b]	26.56 [a]	7.88	20.21 [a]	70.20 [a]	58.07 [a]
QW(10)	6.26	45.47 [a]	18.92 [a]	6.71	17.10 [b]	16.41 [b]	37.95 [a]	17.74 [b]	27.76 [a]	77.21 [a]	70.46 [a]

Table 1: Descriptive statistics at daily frequency. Data are foreign exchange returns and stock returns.
Day 1 indicates when a given series starts. All series end with September 1999. N. Obs is the number of observations in a given series. Mean, std. dev., min, and max stand for the mean, the standard deviation, the minimum, and the maximum. Sk and ku stand for skewness and kurtosis. These statistics are asymptotically normally distributed. The number below a statistics represents its standard error. J-B is the Jarque-Bera statistics. Engle(1) and Engle(5) are the Lagrange multiplier statistics to test for heteroskedasticity in the data. The statistics ac(1), ac(2), and ac(3) are the first three autocorrelations. QW(5) and QW(10) are the Box-Ljung statistics for autocorrelation, robustified following White. These statistics follow a χ^2 with 5, respectively 10 degrees of freedom.
[a] corresponds to 5% significance. [b] corresponds to 10% significance.
The critical values at a significant level of 5% of a χ^2 with 1, 2, 5, and 10 degrees of freedom are: 3.84, 5.99, 11.1, and 18.3.

	US3M	UK3M	FR3M	GE3M	USLT	UKLT	FRLT	GELT
Day 1	03/01/75	03/01/75	03/01/75	03/01/75	20/05/86	20/05/86	20/05/86	20/05/86
N. Obs	6535	6535	6535	6535	3568	3568	3568	3568
mean	-0.0834	-0.2144	-0.2469	-0.0846	-0.0592	-0.1251	-0.1056	-0.0492
	0.2040	0.2363	0.6666	0.1155	0.1131	0.1256	0.1094	0.0825
std. dev.	17.9306	20.1330	51.1081	9.8146	6.3770	7.0558	6.4107	4.6625
	0.7532	0.7646	6.3568	0.4943	0.1923	0.1956	0.3310	0.1323
sk	-0.1791	0.2529	-6.2772	-0.0474	-0.1802	-0.1713	-0.8440	0.3986
	0.6127	0.3929	5.8161	0.6036	0.4217	0.2194	0.6818	0.2393
ku	23.0913	17.2369	322.6777	32.2050	7.1129	5.1772	15.3650	5.7374
	5.6483	2.0346	143.4799	13.6382	3.7972	1.1038	8.0095	1.1783
J-B	$152\ 10^3$ [a]	$84\ 10^3$ [a]	$29972\ 10^3$ [a]	$287\ 10^3$ [a]	$7\ 10^3$ [a]	$3\ 10^3$ [a]	$33\ 10^3$ [a]	$4\ 10^3$ [a]
min	-237.50	-218.75	-1850.00	-160.94	-71.00	-55.60	-87.00	-37.80
max	250.00	175.00	850.00	148.44	39.60	35.00	46.00	32.60
Engle(1)	344.49 [a]	463.59 [a]	27.52 [a]	1351.08 [a]	12.29 [a]	42.90 [a]	243.02 [a]	17.22 [a]
Engle(5)	636.27 [a]	825.87 [a]	346.61 [a]	1538.66 [a]	134.18 [a]	160.66 [a]	627.63 [a]	192.25 [a]
ac(1)	-0.1326	-0.0778	0.0070	-0.0970	0.0404	0.0238	0.0296	0.0330
ac(2)	0.0615	-0.0244	0.0310	0.0237	0.0355	0.0172	-0.0378	0.0051
ac(3)	-0.0086	0.0171	0.0105	-0.0052	-0.0334	0.0051	-0.0091	-0.0217
QW(5)	63.41 [a]	19.61 [a]	1.63	7.05	13.25 [a]	3.67	1.70	11.84 [a]
QW(10)	85.10 [a]	20.39 [a]	12.90	8.55	23.77 [a]	13.39	6.77	16.95 [b]

Table 2: Descriptive statistics at daily frequency. Data are short-term (3-month) and long-term (10-year) interest-rate changes. Interest rate changes are defined as $100(r_t - r_{t-1})$. The meaning of the various statistics is the same as in table 1.

	SFR-DM	CAN-US	DM-US	YEN-US	UK-US	FF-US	S&P	NIK	DAX	CAC	FTSE
Info	0	0	0	0	0	0	0	0	0	0	0
Time	8.30	16.36	17.08	12.36	15.76	14.78	52.90	36.74	54.33	37.68	56.68
a_0	0.0071	0.0078	0.0149	0.0127	0.0056	0.0122	0.0062	0.0120	0.0163	0.0240	0.0171
	0.0027	0.0053	0.0062	0.0050	0.0031	0.0056	0.0020	0.0028	0.0040	0.0056	0.0038
b_0^+	0.0164	0.2142	0.0536	0.0774	0.0928	0.0674	0.0277	0.0587	0.0653	0.0732	0.0635
	0.0181	0.0887	0.0174	0.0173	0.0302	0.0190	0.0048	0.0106	0.0109	0.0114	0.0087
b_0^-	0.1277	0.1470	0.0905	0.1022	0.0818	0.0965	0.0651	0.1773	0.1059	0.1251	0.0893
	0.0369	0.0661	0.0194	0.0146	0.0214	0.0214	0.0102	0.0229	0.0162	0.0195	0.0106
c_0	0.8380	0.7989	0.9005	0.8976	0.9069	0.8944	0.9468	0.8820	0.9005	0.8867	0.9047
	0.0436	0.0868	0.0232	0.0172	0.0261	0.0255	0.0079	0.0146	0.0136	0.0152	0.0116
a_1	6.1194	5.2767	6.2127	4.7115	4.5047	5.9992	6.5661	4.5827	8.9715	5.1477	15.2433
	0.8858	0.5940	0.7992	0.5284	0.4638	0.7315	0.5678	0.2825	1.4266	0.3819	2.7006
b_{11}	1.1267	0.9162	1.1180	-0.1372	0.1828	1.0875	-0.5709	-0.2546	-0.2352	0.2190	-0.5314
	0.6378	0.1873	0.2210	0.2334	0.5184	0.2186	0.3231	0.0296	1.5604	0.0301	0.5735
b_{12}	-1.4556	-0.5153	-0.4214	-0.4994	-0.2579	-0.2232	-0.4772	-0.1815	-0.9204	-0.1828	-0.8166
	0.7673	0.1124	0.2192	0.1203	0.3596	0.2396	0.2218	0.0261	0.1851	0.2113	0.6170
a_2	-0.0265	0.0079	-0.0616	-0.0648	0.0279	-0.0521	-0.0061	-0.0486	-0.0275	-0.0065	-0.0552
	0.0307	0.0343	0.0323	0.0278	0.0253	0.0309	0.0130	0.0124	0.0168	0.0119	0.0173
b_{21}	0.0476	0.1888	0.1188	0.0995	0.0839	0.0812	0.0072	0.0353	0.0269	-0.0236	0.0217
	0.0999	0.0636	0.0465	0.0277	0.0385	0.0405	0.0142	0.0150	0.0135	0.0127	0.0176
b_{22}	0.0020	0.0112	0.0746	-0.0161	0.0558	0.0834	0.0476	0.0046	0.0462	0.0426	0.0575
	0.1066	0.0620	0.0563	0.0378	0.0574	0.0548	0.0160	0.0129	0.0169	0.0122	0.0175
N.Obs.	1969	1969	1969	1969	1969	1969	7159	7159	7159	7159	7159
Likelihood	-192.49	-816.07	-1915.96	-2035.13	-1547.66	-1802.42	-8576.48	-8544.33	-9440.69	-9504.99	-9318.18
LRT_1	13.09 [a]	3.40 [b]	0.48	1.52	0.24	2.37	24.93 [a]	85.43 [a]	15.40 [a]	10.49 [a]	9.09 [a]
LRT_2	1.25	3.31	2.88	1.63	1.61	3.05	9.58 [a]	8.30 [a]	17.15 [a]	11.22 [a]	12.61 [a]
LRT_3	2.52	14.60 [a]	14.01 [a]	20.49 [a]	5.68	10.81 [a]	14.11 [a]	23.56 [a]	22.03 [a]	14.77 [a]	18.79 [a]

Table 3: Estimates of the general model for daily frequency. Data are foreign exchange returns and stock returns.
This table presents the results of a GARCH with asymmetries and conditional residuals that are distributed as a Generalized-t. The volatility equation is $\sigma_t^2 = a_0 + b_0^+(y_{t-1}^+)^2 + b_0^-(y_{t-1}^-)^2 + c_0\sigma_{t-1}^2$. Numbers under a given statistics always represent a standard error. The dynamics of the parameters are $\eta_t = a_1 + b_{11}r_{t-1} + b_{12}r_{t-2}$ and $\lambda_t = a_2 + b_{21}r_{t-1} + b_{22}r_{t-2}$.
LRT_1 represents the likelihood-ratio statistic for the null hypothesis that $b_0^+ = b_0^-$. This statistics is distributed as a χ^2 with one degree of freedom. LRT_2 corresponds to the likelihood-ratio statistics to test if a second lag is required, i.e., if $b_{22} = b_{12}$. This statistics is distributed as a χ^2 with two degrees of freedom. LRT_3 is the likelihood-ratio test if there is any dynamics at all, i.e., if $b_{11} = b_{12} = b_{21} = b_{22} = 0$. This statistic follows a χ^2 with 4 degrees of freedom.
[a] corresponds to 5% significance. [b] corresponds to 10% significance.
The critical values at a significant level of 5% of a χ^2 with 1, 2, 4 degrees of freedom are: 3.84, 5.99, and 9.49.

	US3M	UK3M	FR3M	GE3M	USLT	UKLT	FRLT	GELT
Info	0	0	0	0	0	0	0	0
Time	32.35	39.71	17.79	29.17	15.05	15.6	19.17	12.14
a_0	0.0001	0.0001	1.8240	1.5130	0.9830	0.9493	1.0859	0.6079
	4.4125	0.1728	0.7005	2.0646	0.2746	0.3729	0.3130	0.1441
b_0^+	0.0773	0.0818	0.2120	0.1645	0.0509	0.0810	0.1476	0.0819
	0.3610	0.0241	0.0178	0.0457	0.0112	0.0220	0.0226	0.0134
b_0^-	0.0457	0.0579	0.0412	0.1332	0.0261	0.0294	0.0651	0.0653
	0.2934	0.0205	0.0205	0.0579	0.0075	0.0119	0.0159	0.0132
c_0	0.9385	0.9302	0.8734	0.8512	0.9358	0.9284	0.8695	0.9025
	0.4987	0.0280	0.0224	0.0964	0.0118	0.0200	0.0209	0.0125
a_1	3.2120	3.2011	2.6858	2.7095	4.7826	4.2391	5.3566	4.2547
	5.3924	0.2625	0.0745	0.2264	0.3995	0.3468	0.5033	0.3298
b_{11}	-0.0008	0.0010	0.0003	0.0004	0.0430	-0.0290	0.0367	0.0556
	0.0043	0.0017	0.0001	0.0015	0.0085	0.0665	0.0113	0.0141
b_{12}	0.0035	0.0012	0.0003	0.0042	-0.0302	0.0163	0.0008	0.0430
	0.0186	0.0021	0.0001	0.0024	0.0204	0.0162	0.0173	0.0077
a_2	-0.0120	-0.0127	-0.0012	-0.0085	0.0310	0.0440	0.0087	0.0387
	0.0271	0.0077	0.0044	0.0033	0.0183	0.0180	0.0172	0.0180
b_{21}	-0.0004	0.0004	0.0000	0.0002	0.0066	0.0036	0.0020	0.0048
	0.0013	0.0004	0.0001	0.0006	0.0032	0.0025	0.0028	0.0046
b_{22}	0.0004	0.0006	0.0001	0.0009	0.0041	0.0055	0.0069	0.0119
	0.0020	0.0004	0.0002	0.0005	0.0028	0.0026	0.0028	0.0037
N.Obs.	6356	6356	6356	6356	3391	3391	3391	3391
Likelihood	-23024.05	-23790.40	-26065.49	-21060.52	-10750.43	-10987.84	-10487.71	-9574.34
LRT_1	15.66 [a]	8.48 [a]	93.64 [a]	2.48	5.48 [a]	14.35 [a]	18.51 [a]	1.03
LRT_2	3.65	1.45	1.98	4.54	2.48	4.76 [b]	4.89 [b]	8.69 [a]
LRT_3	4.60	2.35	3.64	4.68	10.77 [a]	7.44	7.86 [b]	13.20 [a]

Table 4: Estimates of the general model for daily frequency. Data are short-term and long-term interest-rate changes. The model estimated and the meaning of the parameters is the same as the one presented in table 3.

	SFR-DM	CAN-US	DM-US	YEN-US	UK-US	FF-US	S&P	NIK	DAX	CAC	FTSE
Info	0	0	0	0	0	0	0	0	0	0	0
Time	2.69	3.08	2.42	2.91	3.13	2.64	8.24	10.82	8.85	7.41	5.88
a_0	0.0503	0.0429	0.3047	0.1457	0.0972	0.2339	0.1748	0.1400	0.1743	0.3939	0.1913
	0.2030	0.0263	0.1214	0.0708	0.0783	0.1045	0.0657	0.0570	0.0788	0.1139	0.0736
b_0^+	0.0128	0.2142	0.0001	0.1048	0.0643	0.0204	0.0348	0.0741	0.0743	0.0551	0.0551
	0.1471	0.1178	0.0322	0.0684	0.0610	0.0285	0.0166	0.0213	0.0177	0.0179	0.0185
b_0^-	0.0903	0.0471	0.1442	0.0711	0.1574	0.1546	0.1369	0.2283	0.1298	0.1259	0.1305
	0.3442	0.0613	0.0552	0.0271	0.0723	0.0538	0.0296	0.0533	0.0293	0.0275	0.0322
c_0	0.8171	0.8126	0.7990	0.8738	0.8545	0.8121	0.8695	0.8356	0.8666	0.8452	0.8760
	0.6318	0.0927	0.0585	0.0437	0.0634	0.0525	0.0284	0.0350	0.0286	0.0282	0.0293
a_1	18.2326	6.2583	9.8262	4.4287	4.3454	10.6342	12.6021	6.5474	14.9127	9.7021	17.9827
	22.5959	1.7473	4.6432	0.8507	1.1443	5.0512	3.5086	1.1227	5.1127	2.2520	5.7319
b_{11}	-0.5766	-0.2517	0.4386	-0.0236	0.4027	0.1533	-0.7139	0.0666	-0.7593	-0.1280	-0.7210
	44.3557	0.3441	0.2670	0.1224	0.2777	0.1566	0.2522	0.1208	0.5837	0.1489	0.3462
b_{12}	6.9412	-1.3329	1.6998	-0.3649	-0.1708	2.1551	0.2836	-0.3212	0.4480	0.4485	0.9469
	19.2166	0.5718	1.0216	0.1333	0.1085	1.2929	0.3213	0.0992	0.2737	0.1453	0.3777
a_2	0.0122	0.0569	0.1217	-0.2859	0.1813	0.1164	-0.1553	-0.0743	-0.0489	-0.0892	-0.0935
	0.1507	0.0893	0.0556	0.0574	0.0545	0.0575	0.0419	0.0319	0.0644	0.0395	0.0415
b_{21}	0.0825	0.1012	-0.0281	0.0102	0.0464	-0.0461	-0.0416	0.0066	-0.0059	-0.0176	0.0555
	0.3040	0.0652	0.0441	0.0508	0.1289	0.0542	0.0151	0.0226	0.0983	0.0145	0.0131
b_{22}	-0.1162	0.0766	-0.0797	0.0208	-0.0165	-0.0822	0.0065	-0.0094	0.0196	0.0118	-0.0058
	0.2651	0.1179	0.0355	0.0520	0.0665	0.0436	0.0232	0.0173	0.0838	0.0186	0.0178
N.Obs.	393	393	393	393	393	393	1431	1431	1431	1431	1431
Likelihood	-353.82	-422.53	-690.39	-713.25	-610.44	-682.59	-2945.30	-2975.76	-3085.96	-3247.62	-3207.91
LRT_1	0.76	4.13 [a]	5.85 [a]	0.38	0.86	4.32 [a]	13.12 [a]	22.48 [a]	4.74 [a]	6.63 [a]	9.80 [a]
LRT_2	1.99	2.25	7.04 [a]	3.75	2.12	5.28 [b]	0.28	3.49	1.99	1.15	1.50
LRT_3	2.55	3.08	7.07	3.75	3.63	5.54	7.36	3.64	3.48	5.86	17.84 [a]

Table 5: Estimates the general model for weekly frequency. Data are foreign exchange returns and stock returns. The model and labels are as in table 3.

	US3M	UK3M	FR3M	GE3M	USLT	UKLT	FRLT	GELT
Info	0	0	0	0	0	0	0	0
Time	4.99	6.97	6.48	9.11	3.41	4.01	3.79	4.4
a_0	1.6596	4.0852	12.3724	19.9983	20.2322	12.7798	5.6767	8.6839
	1.2969	2.0662	11.0293	9.0187	26.6638	5.1011	1.9274	7.5794
b_0^+	0.2098	0.1451	0.2771	0.4239	0.0778	0.0945	0.1248	0.1026
	0.0337	0.0299	0.1120	0.0909	0.0762	0.0231	0.0313	0.0527
b_0^-	0.1092	0.0827	0.0001	0.2345	0.0053	0.0036	0.0001	0.0150
	0.0270	0.0205	0.0554	0.0588	0.0328	0.0268	0.0214	0.0493
c_0	0.8405	0.8861	0.8614	0.6708	0.8472	0.8987	0.9019	0.8532
	0.0279	0.0244	0.0790	0.0733	0.1707	0.0356	0.0248	0.1120
a_1	5.1599	3.1975	2.8732	3.5688	9.2926	8.4602	12.9223	7.1682
	0.5382	0.2187	0.2513	0.3107	1.3155	2.4986	4.0568	1.8130
b_{11}	-0.0104	0.0021	0.0003	-0.0045	-0.1607	-0.0291	-0.0416	0.0787
	0.0018	0.0009	0.0001	0.0010	0.0291	0.0103	0.0241	0.0450
b_{12}	0.0100	0.0011	0.0003	0.0058	0.0086	0.0972	-0.1810	0.0153
	0.0018	0.0007	0.0001	0.0017	0.0046	0.0382	0.0727	0.0457
a_2	0.0264	0.0030	0.0810	-0.0324	0.0786	0.1122	0.1383	0.1922
	0.0290	0.0289	0.0261	0.0226	0.0515	0.0540	0.0482	0.0616
b_{21}	0.0027	0.0002	0.0000	0.0010	-0.0024	0.0056	0.0020	-0.0072
	0.0022	0.0004	0.0001	0.0008	0.0028	0.0037	0.0033	0.0069
b_{22}	-0.0001	0.0001	0.0000	0.0016	-0.0021	0.0052	0.0111	-0.0048
	0.0006	0.0005	0.0005	0.0004	0.0024	0.0030	0.0037	0.0068
N.Obs.	1271	1271	1271	1271	678	678	678	678
Likelihood	-5512.02	-5807.17	-6347.03	-5184.63	-2701.72	-2786.35	-2650.27	-2474.20
LRT_1	11.43 [a]	5.42 [a]	48.03 [a]	7.30 [a]	4.64 [a]	6.71 [a]	20.47 [a]	3.70 [b]
LRT_2	6.25 [a]	2.16	1.39	4.28	0.50	3.02	7.80 [a]	0.96
LRT_3	9.71 [b]	4.54	4.85	7.04	8.73 [b]	6.98	8.82 [b]	3.45

Table 6: Estimates of the general model for weekly frequency. Data are short-term and long-term interest-rate changes. The model estimated and the meaning of the parameters is the same as the one presented in table 3.

	Panel A: Daily frequency										
	SFR-DM	CAN-US	DM-US	YEN-US	UK-US	FF-US	S&P	NIK	DAX	CAC	FTSE
N. Obs.	1969	1969	1969	1969	1969	1969	7158	7158	7158	7158	7158
Nb no skewness	1	7	1	1	0	2	2	16	5	1	0
Nb no kurtosis	6	39	8	59	27	20	11	258	9	16	0
Nb eta active	0	2	0	0	0	1	0	1	1	1	0
Nb lda active	0	0	0	1	0	0	0	0	0	0	0
	US3M	UK3M	FR3M	GE3M	USLT	UKLT	FRLT	GELT			
N. Obs.	6357	6357	6357	6357	3392	3392	3392	3392			
Nb no skewness	82	6	6356	6352	3	0	1	6			
Nb no kurtosis	6356	6357	6357	6357	45	409	3	655			
Nb eta active	0	0	2	1	0	0	1	1			
Nb lda active	0	0	0	0	0	0	0	0			
	Panel B: Weekly frequency										
	SFR-DM	CAN-US	DM-US	YEN-US	UK-US	FF-US	S&P	NIK	DAX	CAC	FTSE
N. Obs.	394	394	394	394	394	394	1432	1432	1432	1432	1432
Nb no skewness	1	2	3	6	2	3	1	1	0	1	1
Nb no kurtosis	2	10	5	72	104	8	1	6	0	1	1
Nb eta active	1	0	2	1	0	3	0	0	0	0	0
Nb lda active	0	0	0	0	0	0	0	0	0	0	1
	US3M	UK3M	FR3M	GE3M	USLT	UKLT	FRLT	GELT			
N. Obs.	1271	1271	1271	1271	678	678	678	678			
Nb no skewness	6	37	1257	8	5	2	2	0			
Nb no kurtosis	28	1271	1271	1254	11	4	3	0			
Nb eta active	1	1	1	1	5	2	1	0			
Nb lda active	0	0	0	0	0	0	0	0			

Table 7: Binding constraints and existence of higher moments.
This table summarizes for a given series and frequency the number of observations N. Obs. Then it indicates the number of times skewness and kurtosis does not exist. The last two rows of each group indicate how often the constraints $2 < \eta_t$ and $-1 < \lambda_t < 1$ are binding.

	DM-US					YEN-US					UK-US				
	$I_1(-1)$	$I_2(-1)$	$I_3(-1)$	$I_4(-1)$	$I_5(-1)$	$I_1(-1)$	$I_2(-1)$	$I_3(-1)$	$I_4(-1)$	$I_5(-1)$	$I_1(-1)$	$I_2(-1)$	$I_3(-1)$	$I_4(-1)$	$I_5(-1)$
I_1	10.000	6.497	5.025	3.096	0.254	3.401	3.756	6.802	10.964	0.000	11.726	6.193	4.010	2.995	0.000
I_2	6.650	7.462	6.244	4.569	0.000	5.076	6.142	7.360	6.396	0.000	6.294	8.782	6.599	3.299	0.000
I_3	5.584	6.244	7.157	5.939	0.000	6.497	8.934	6.244	3.299	0.000	3.858	7.157	8.173	5.787	0.000
I_4	2.538	4.721	6.497	11.218	0.000	9.949	6.142	4.569	4.315	0.051	3.046	2.843	6.193	12.995	0.000
I_5	0.102	0.000	0.000	0.152	0.000	0.000	0.000	0.000	0.051	0.000	0.000	0.000	0.000	0.000	0.000

	FF-US					S&P					NIK				
	$I_1(-1)$	$I_2(-1)$	$I_3(-1)$	$I_4(-1)$	$I_5(-1)$	$I_1(-1)$	$I_2(-1)$	$I_3(-1)$	$I_4(-1)$	$I_5(-1)$	$I_1(-1)$	$I_2(-1)$	$I_3(-1)$	$I_4(-1)$	$I_5(-1)$
I_1	11.726	6.802	3.706	2.538	0.102	8.897	7.388	4.958	3.729	0.000	7.821	6.089	5.014	5.880	0.098
I_2	7.107	7.665	6.041	4.112	0.000	5.978	6.983	6.187	5.838	0.000	5.335	7.668	7.053	4.791	0.070
I_3	3.756	6.345	8.122	6.701	0.000	5.293	6.201	6.774	6.718	0.000	5.140	6.369	6.927	6.466	0.014
I_4	2.183	4.112	7.056	11.777	0.000	4.791	4.413	7.067	8.715	0.028	6.564	4.777	5.880	7.723	0.028
I_5	0.102	0.000	0.000	0.000	0.000	0.014	0.000	0.000	0.014	0.000	0.042	0.014	0.042	0.112	0.070

	DAX					CAC					FTSE				
	$I_1(-1)$	$I_2(-1)$	$I_3(-1)$	$I_4(-1)$	$I_5(-1)$	$I_1(-1)$	$I_2(-1)$	$I_3(-1)$	$I_4(-1)$	$I_5(-1)$	$I_1(-1)$	$I_2(-1)$	$I_3(-1)$	$I_4(-1)$	$I_5(-1)$
I_1	12.053	6.704	3.925	2.263	0.014	4.581	4.050	6.187	10.140	0.014	11.439	6.788	4.092	2.654	0.000
I_2	6.285	7.737	6.606	4.344	0.000	4.539	5.726	7.570	5.196	0.000	6.494	7.277	6.662	4.553	0.000
I_3	3.673	6.816	7.807	6.676	0.000	5.964	7.416	8.101	5.461	0.000	4.162	6.718	7.500	6.606	0.000
I_4	2.919	3.715	6.634	11.690	0.056	9.888	5.838	5.070	4.232	0.000	2.877	4.204	6.732	11.229	0.000
I_5	0.028	0.000	0.000	0.042	0.000	0.000	0.000	0.014	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 8: Transition frequencies for skewness.
$I_j(t)$ designs the interval to which skewness belongs at time t. The state $j = 5$ corresponds to the situation where skewness is not defined, ($\eta_t < 3$). The intervals I_j for $j = 1, \cdots, 4$ correspond to the four quartiles starting with the smallest. An element in row a and column b of each matrix measures the percentage of times that one moves from a skewness in the quartile b to a quartile a.

	DM-US					YEN-US					UK-US				
	$I_1(-1)$	$I_2(-1)$	$I_3(-1)$	$I_4(-1)$	$I_5(-1)$	$I_1(-1)$	$I_2(-1)$	$I_3(-1)$	$I_4(-1)$	$I_5(-1)$	$I_1(-1)$	$I_2(-1)$	$I_3(-1)$	$I_4(-1)$	$I_5(-1)$
I_1	2.538	4.213	7.056	10.000	0.761	6.751	5.838	5.584	4.772	0.863	2.589	2.893	5.127	11.777	1.878
I_2	5.178	5.888	7.056	6.345	0.152	5.178	5.838	5.838	6.244	0.761	3.452	6.345	8.071	6.142	0.305
I_3	6.853	7.970	5.533	4.112	0.152	5.381	6.497	5.736	5.279	0.964	5.482	8.426	6.701	3.604	0.102
I_4	9.289	6.193	4.873	4.010	0.355	5.533	5.279	5.990	5.838	1.371	11.066	6.447	4.061	2.690	0.254
I_5	0.711	0.355	0.102	0.254	0.000	0.964	0.406	0.711	1.878	0.457	1.675	0.203	0.355	0.305	0.000

	FF-US					S&P					NIK				
	$I_1(-1)$	$I_2(-1)$	$I_3(-1)$	$I_4(-1)$	$I_5(-1)$	$I_1(-1)$	$I_2(-1)$	$I_3(-1)$	$I_4(-1)$	$I_5(-1)$	$I_1(-1)$	$I_2(-1)$	$I_3(-1)$	$I_4(-1)$	$I_5(-1)$
I_1	4.213	4.365	7.157	8.528	0.305	13.003	6.704	3.561	1.648	0.000	12.221	5.880	3.073	2.095	0.391
I_2	5.482	6.802	5.736	6.142	0.457	6.453	7.905	6.732	3.841	0.000	5.279	8.492	6.034	3.464	0.405
I_3	6.345	7.107	6.091	5.076	0.000	3.254	6.397	8.254	7.025	0.000	2.765	5.545	8.799	5.978	0.587
I_4	7.919	6.041	5.482	4.822	0.558	2.179	3.925	6.383	12.277	0.209	2.444	3.184	5.237	10.573	2.249
I_5	0.609	0.305	0.152	0.254	0.000	0.028	0.000	0.000	0.182	0.028	0.950	0.573	0.531	1.578	1.662

	DAX					CAC					FTSE				
	$I_1(-1)$	$I_2(-1)$	$I_3(-1)$	$I_4(-1)$	$I_5(-1)$	$I_1(-1)$	$I_2(-1)$	$I_3(-1)$	$I_4(-1)$	$I_5(-1)$	$I_1(-1)$	$I_2(-1)$	$I_3(-1)$	$I_4(-1)$	$I_5(-1)$
I_1	9.148	7.235	4.972	3.534	0.056	4.078	4.581	5.698	10.349	0.196	9.064	7.249	5.363	3.296	0.000
I_2	6.187	7.277	6.453	5.042	0.000	4.022	6.453	7.556	5.992	0.028	9.707	6.983	5.000	3.296	0.000
I_3	4.902	6.494	6.983	6.578	0.000	5.642	7.165	8.031	4.930	0.014	4.902	7.207	6.844	6.034	0.000
I_4	4.637	3.953	6.536	9.791	0.070	10.936	5.838	4.455	3.701	0.042	1.299	3.547	7.779	12.416	0.000
I_5	0.070	0.000	0.014	0.042	0.014	0.223	0.014	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 9: Transition probabilities for kurtosis.
$I_j(t)$ designs the interval to which kurtosis belongs at time t.The state $j = 5$ corresponds to the situation where kurtosis is not defined, $(\eta_t < 4)$. The intervals I_j for $j = 1, \cdots, 4$ correspond to the four quartiles starting with the smallest. An element in row a and column b of each matrix measures the percentage of times that one moves from a kurtosis in the quartile b to a quartile a.

	DM-US and Yen-US					DM-US and UK-US					DM-US and FF-US				
	I_1	I_2	I_3	I_4	I_5	I_1	I_2	I_3	I_4	I_5	I_1	I_2	I_3	I_4	I_5
I_1	10.914	6.701	4.924	2.335	0.000	15.025	6.244	1.878	1.726	0.000	20.863	3.503	0.457	0.051	0.000
I_2	5.685	7.513	6.447	5.279	0.000	5.787	9.442	7.208	2.487	0.000	3.909	15.685	4.975	0.355	0.000
I_3	4.670	5.736	7.005	7.513	0.000	3.147	6.904	10.051	4.822	0.000	0.102	5.482	15.838	3.503	0.000
I_4	3.401	5.025	6.599	9.949	0.051	0.863	2.335	5.838	15.990	0.000	0.051	0.305	3.706	20.964	0.000
I_5	0.254	0.000	0.000	0.000	0.000	0.102	0.051	0.000	0.102	0.000	0.000	0.000	0.000	0.152	0.102

	S&P and NIK					S&P and DAX					S&P and CAC				
	I_1	I_2	I_3	I_4	I_5	I_1	I_2	I_3	I_4	I_5	I_1	I_2	I_3	I_4	I_5
I_1	8.729	6.606	5.517	4.078	0.042	9.818	6.606	4.679	3.855	0.014	6.299	5.517	6.913	6.243	0.000
I_2	6.034	6.536	6.201	6.159	0.056	6.187	6.718	6.802	5.279	0.000	5.824	5.601	7.374	6.187	0.000
I_3	5.615	6.550	6.550	6.187	0.084	4.972	6.746	6.913	6.355	0.000	6.243	5.908	6.550	6.285	0.000
I_4	4.539	5.223	6.662	8.506	0.098	3.966	4.902	6.564	9.539	0.056	6.592	6.006	6.103	6.313	0.014
I_5	0.000	0.014	0.000	0.014	0.000	0.014	0.000	0.014	0.000	0.000	0.014	0.000	0.000	0.014	0.000

	S&P and FTSE				
	I_1	I_2	I_3	I_4	I_5
I_1	10.391	6.229	4.539	3.813	0.000
I_2	6.034	7.472	6.480	5.000	0.000
I_3	4.902	6.522	7.374	6.187	0.000
I_4	3.645	4.777	6.606	10.000	0.000
I_5	0.014	0.000	0.000	0.014	0.000

Table 10: Co-skewness classification

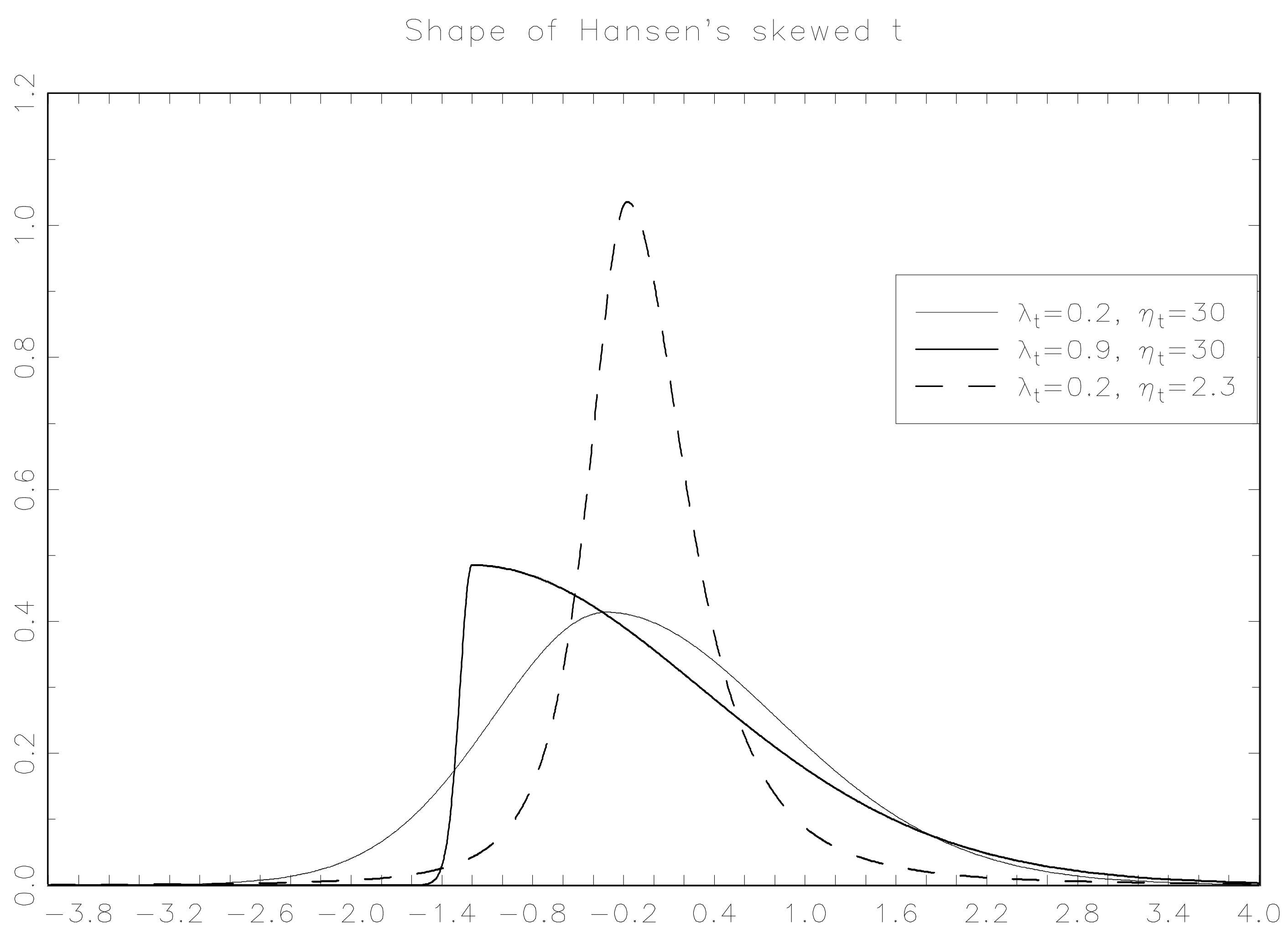
I_j designs the interval to which skewness of a given country belongs to. The state $j = 5$ corresponds to the situation where skewness is not defined, $(\eta_t < 3)$. The intervals I_j for $j = 1, \cdots, 4$ correspond to the four quartiles starting with the smallest. An element in row a and column b of each matrix measures the percentage of times one observes a skewness in quartile a for the first mentioned country and a skewness in quartile b for the second mentioned country.

	DM-US and Yen-US					DM-US and UK-US					DM-US and FF-US				
	I_1	I_2	I_3	I_4	I_5	I_1	I_2	I_3	I_4	I_5	I_1	I_2	I_3	I_4	I_5
I_1	7.208	6.193	5.431	5.178	0.558	12.335	7.310	3.198	1.574	0.152	21.117	3.249	0.102	0.102	0.000
I_2	5.685	6.701	5.381	5.939	0.914	6.650	6.853	7.107	3.858	0.152	3.350	17.310	3.604	0.355	0.000
I_3	5.381	5.736	6.244	5.939	1.320	3.096	6.751	7.817	6.853	0.102	0.102	4.010	18.071	2.437	0.000
I_4	5.178	4.924	6.396	6.853	1.421	2.234	3.452	6.244	11.320	1.523	0.051	0.102	2.893	21.574	0.152
I_5	0.355	0.305	0.406	0.152	0.203	0.000	0.000	0.000	0.812	0.609	0.000	0.000	0.000	0.254	1.168

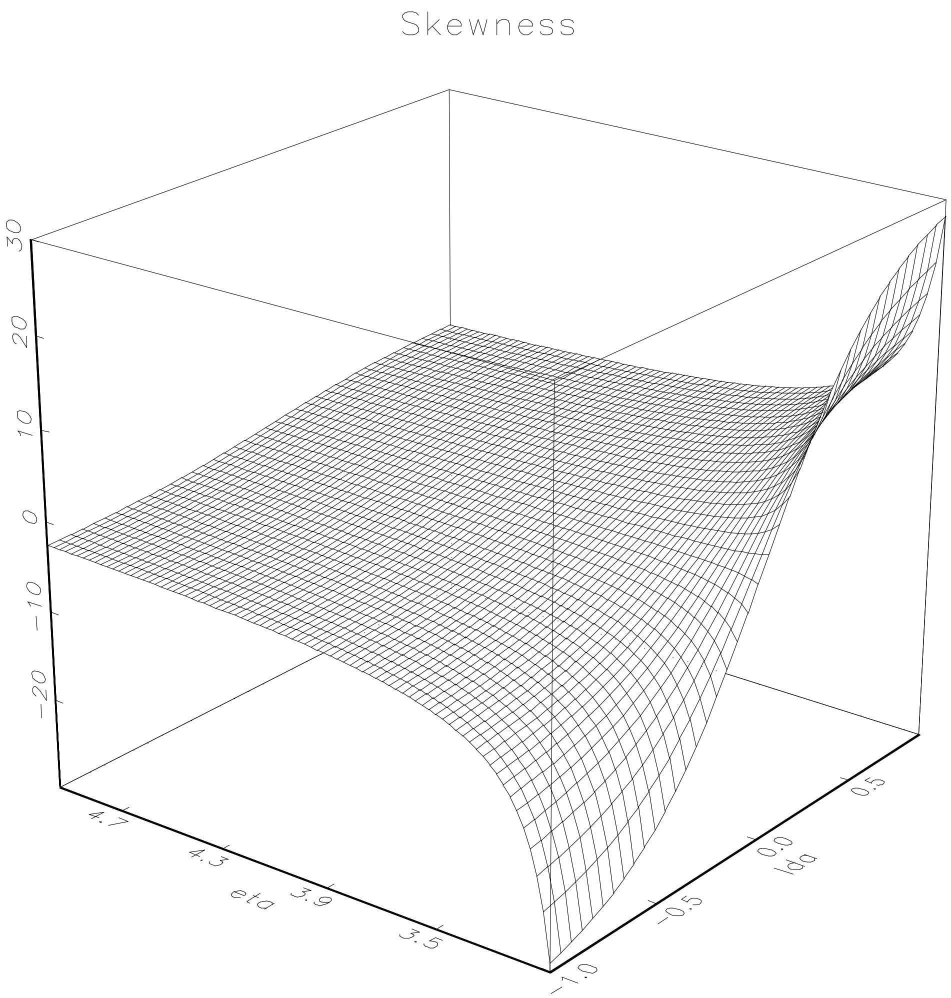
	S&P and NIK					S&P and DAX					S&P and CAC				
	I_1	I_2	I_3	I_4	I_5	I_1	I_2	I_3	I_4	I_5	I_1	I_2	I_3	I_4	I_5
I_1	8.031	6.508	5.293	4.441	0.642	8.017	6.550	5.335	5.000	0.014	4.693	5.377	6.760	7.989	0.098
I_2	5.922	6.075	5.950	5.726	1.257	5.950	6.858	6.830	5.279	0.014	5.503	6.159	7.249	5.978	0.042
I_3	4.916	6.061	6.201	6.480	1.271	5.824	6.257	6.341	6.508	0.000	6.383	6.089	6.397	6.006	0.056
I_4	4.721	5.014	6.229	6.955	2.067	5.112	5.265	6.425	8.087	0.098	8.240	6.355	5.391	4.930	0.070
I_5	0.070	0.014	0.000	0.098	0.056	0.042	0.028	0.028	0.126	0.014	0.098	0.056	0.028	0.042	0.014

	S&P and FTSE				
	I_1	I_2	I_3	I_4	I_5
I_1	9.413	5.936	5.279	4.288	0.000
I_2	6.844	7.318	5.880	4.888	0.000
I_3	5.363	6.606	7.025	5.936	0.000
I_4	3.366	5.126	6.788	9.707	0.000
I_5	0.000	0.014	0.028	0.196	0.000

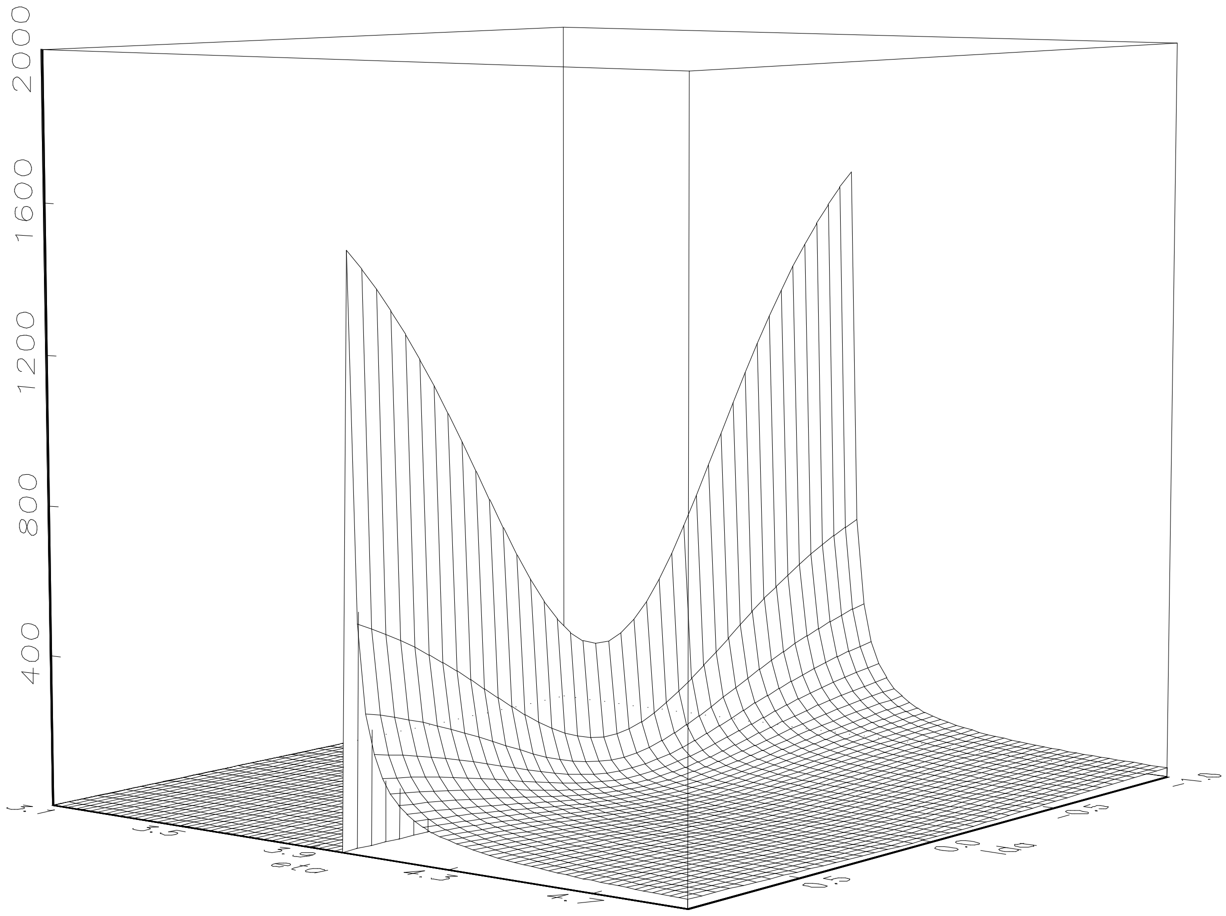
Table 11: Co-kurtosis classification
I_j designs the interval to which kurtosis of a given country belongs to. The state $j = 5$ corresponds to the situation where kurtosis is not defined, $(\eta_t < 4)$. The intervals I_j for $j = 1, \cdots, 4$ correspond to the four quartiles starting with the smallest. An element in row a and column b of each matrix measures the percentage of times one observes a kurtosis in quartile a for the first mentioned country and a kurtosis in quartile b for the second mentioned country.

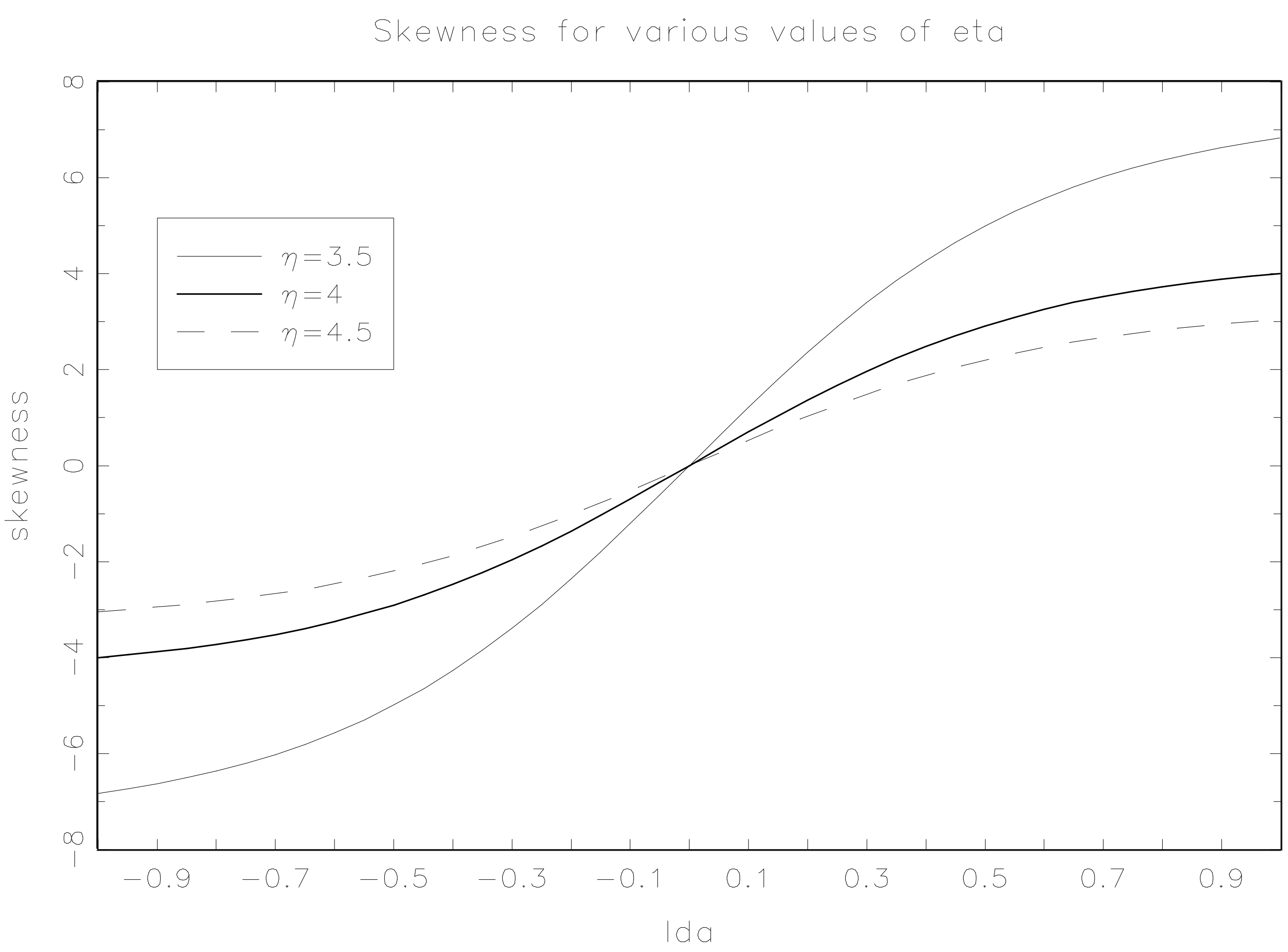


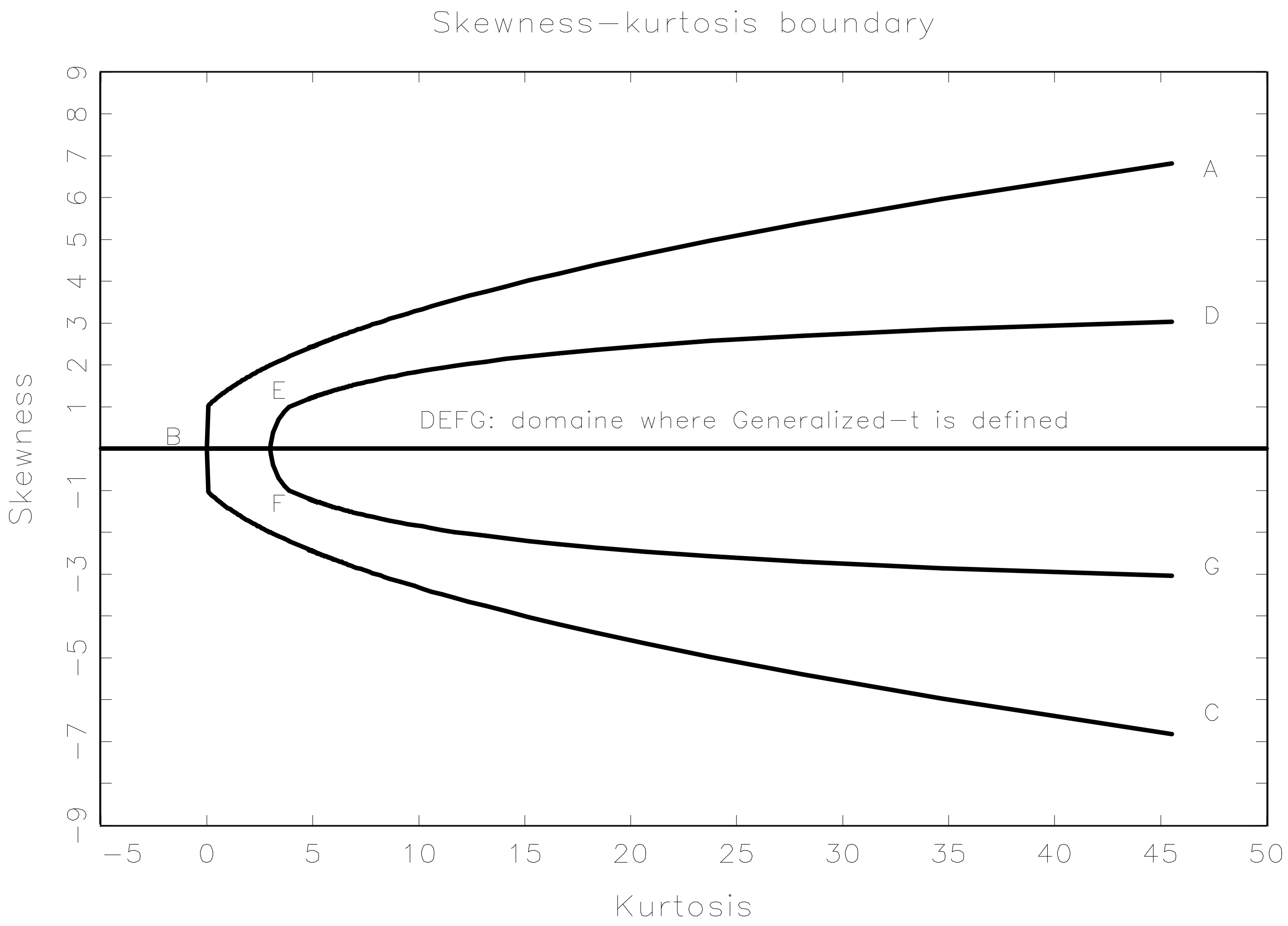
Shape of Hansen's skewed t
λt=0.2, ηt=30
λt=0.9, ηt=30
λt=0.2, ηt=2.3
0.0
0.2
0.4
0.6
0.8
1.0
1.2
−3.8
−3.2
−2.6
−2.0
−1.4
−0.8
−0.2
0.4
1.0
1.6
2.2
2.8
3.4
4.0


Skewness
30
20
10
0
−10
−20
4.7
4.3
eta
3.9
3.5
−1.0
−0.5
0.0
lda
0.5

Kurtosis

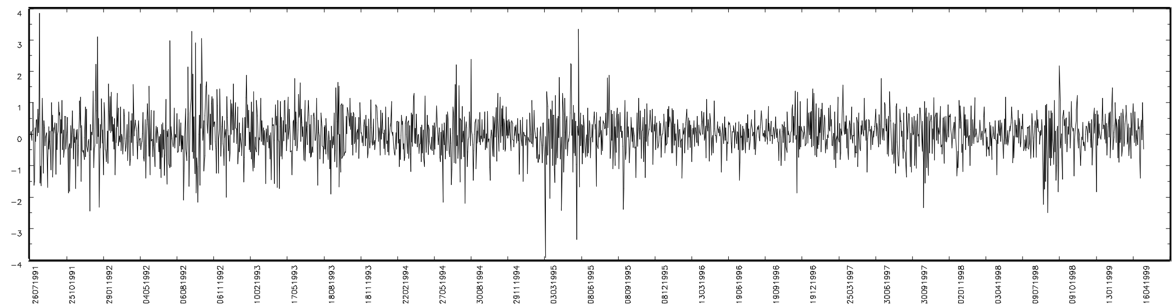



Skewness for various values of eta
skewness
lda
η=3.5
η=4
η=4.5
8
6
4
2
0
−2
−4
−6
−8
−0.9
−0.7
−0.5
−0.3
−0.1
0.1
0.3
0.5
0.7
0.9

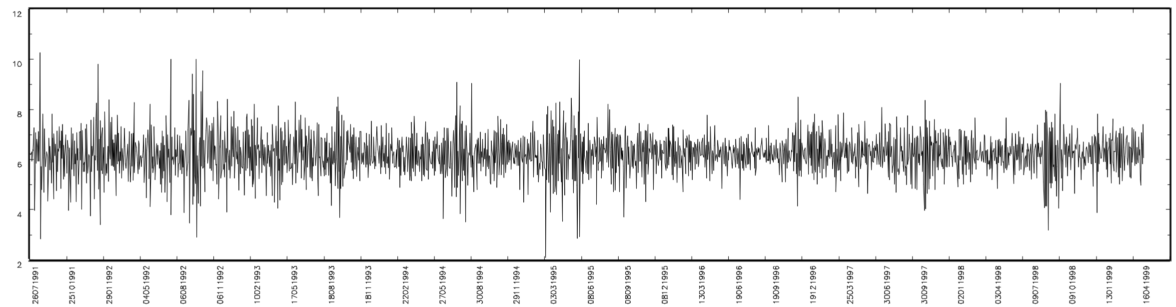


Skewness−kurtosis boundary
A
D
E
B
DEFG: domaine where Generalized−t is defined
F
G
C
Skewness
9
8
7
6
5
4
3
2
1
−1
−3
−5
−7
−9
−5
0
5
10
15
20
25
30
35
40
45
50
Kurtosis

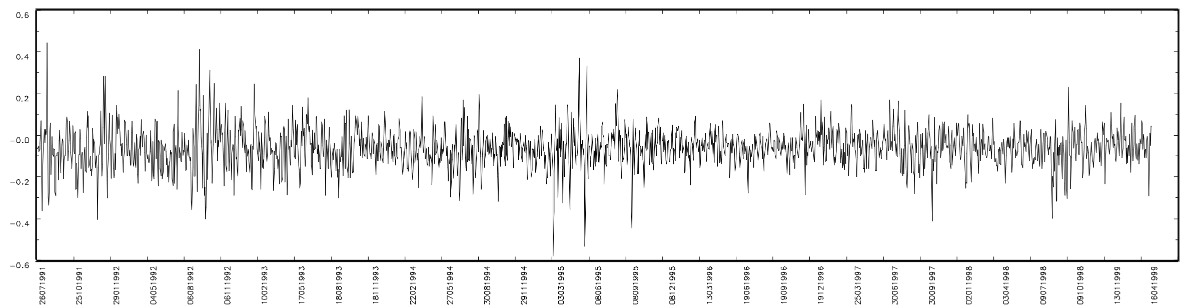
R_t for DM: Daily frequency

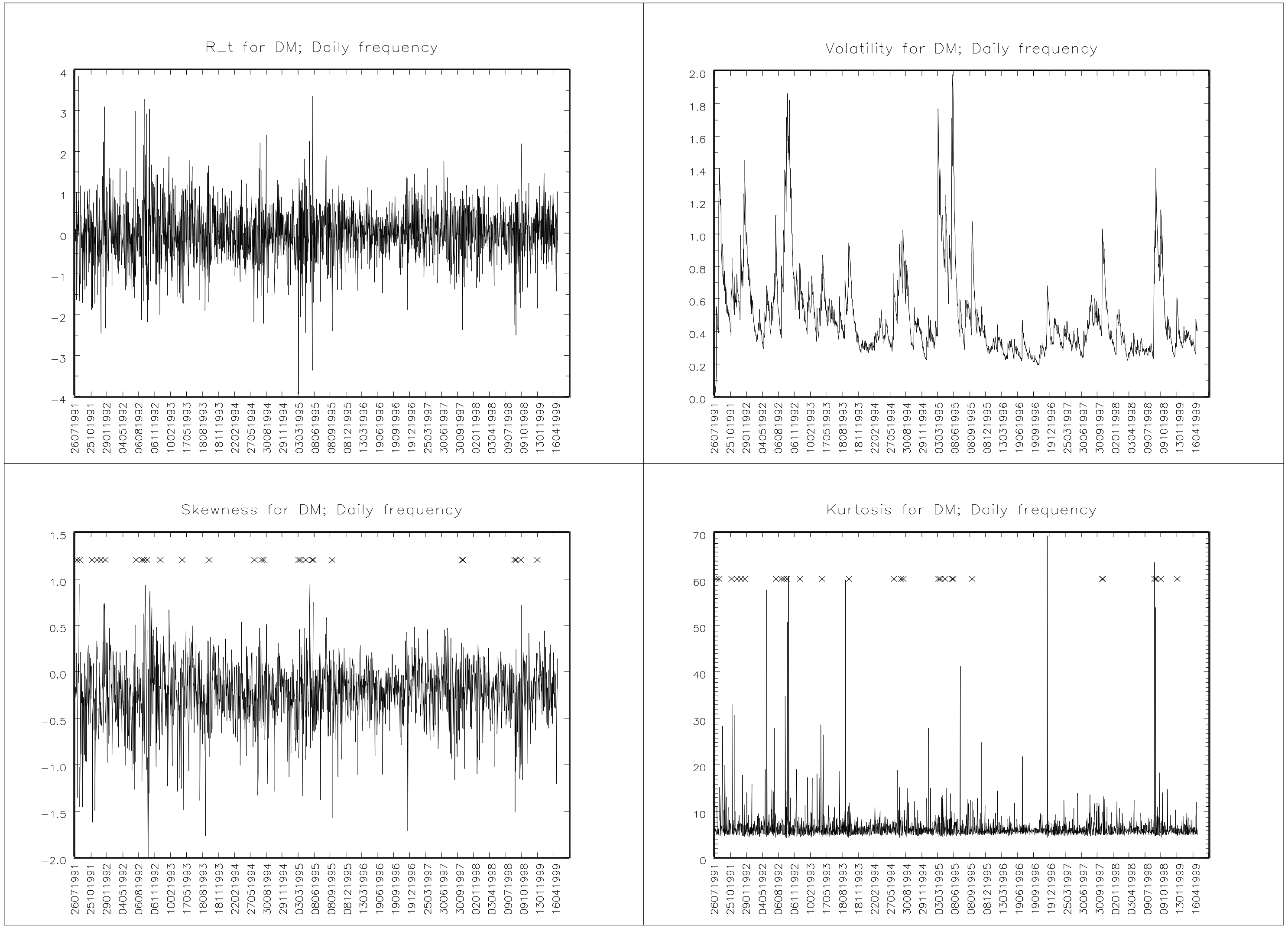


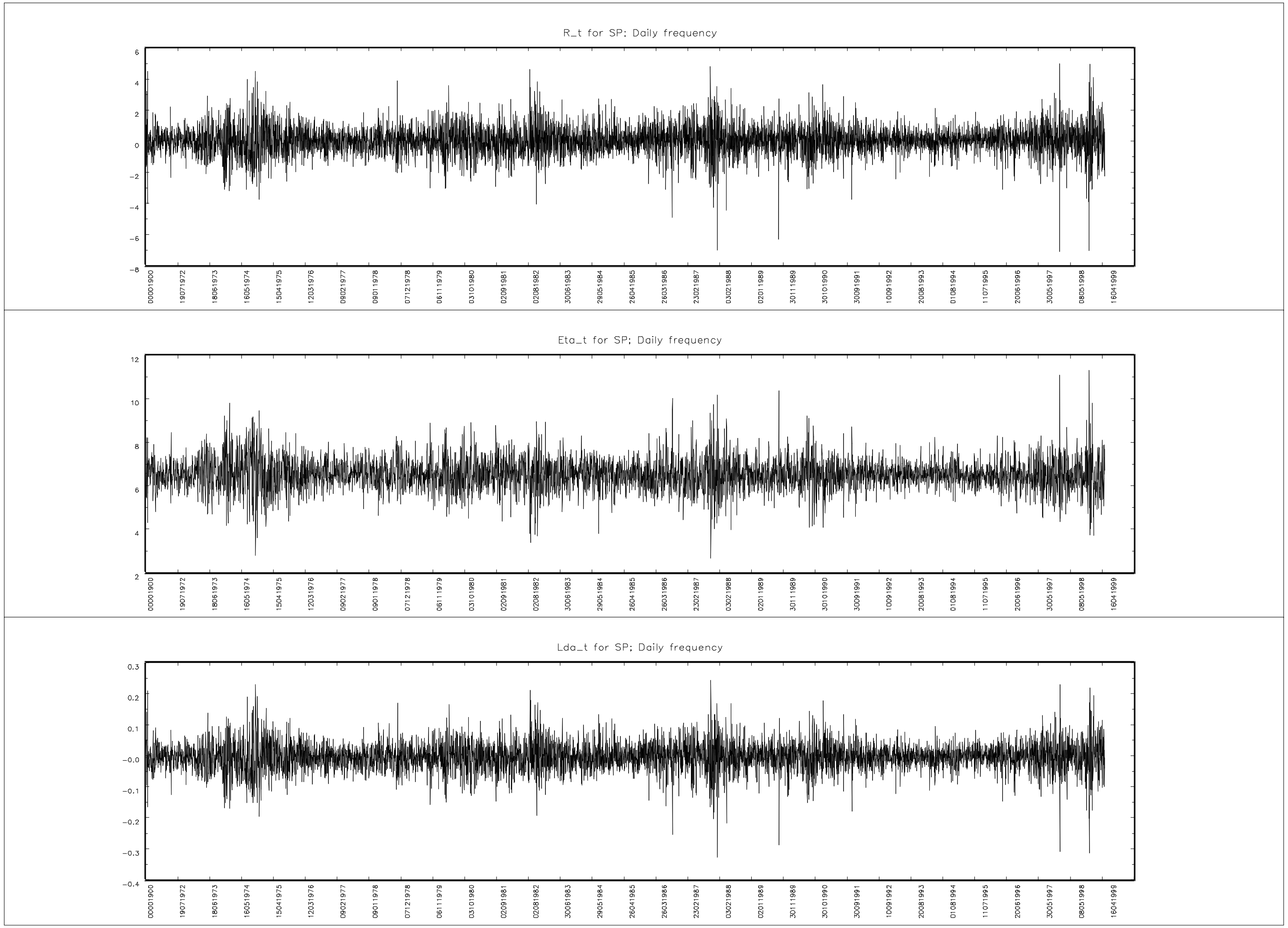
Eta_t for DM: Daily frequency

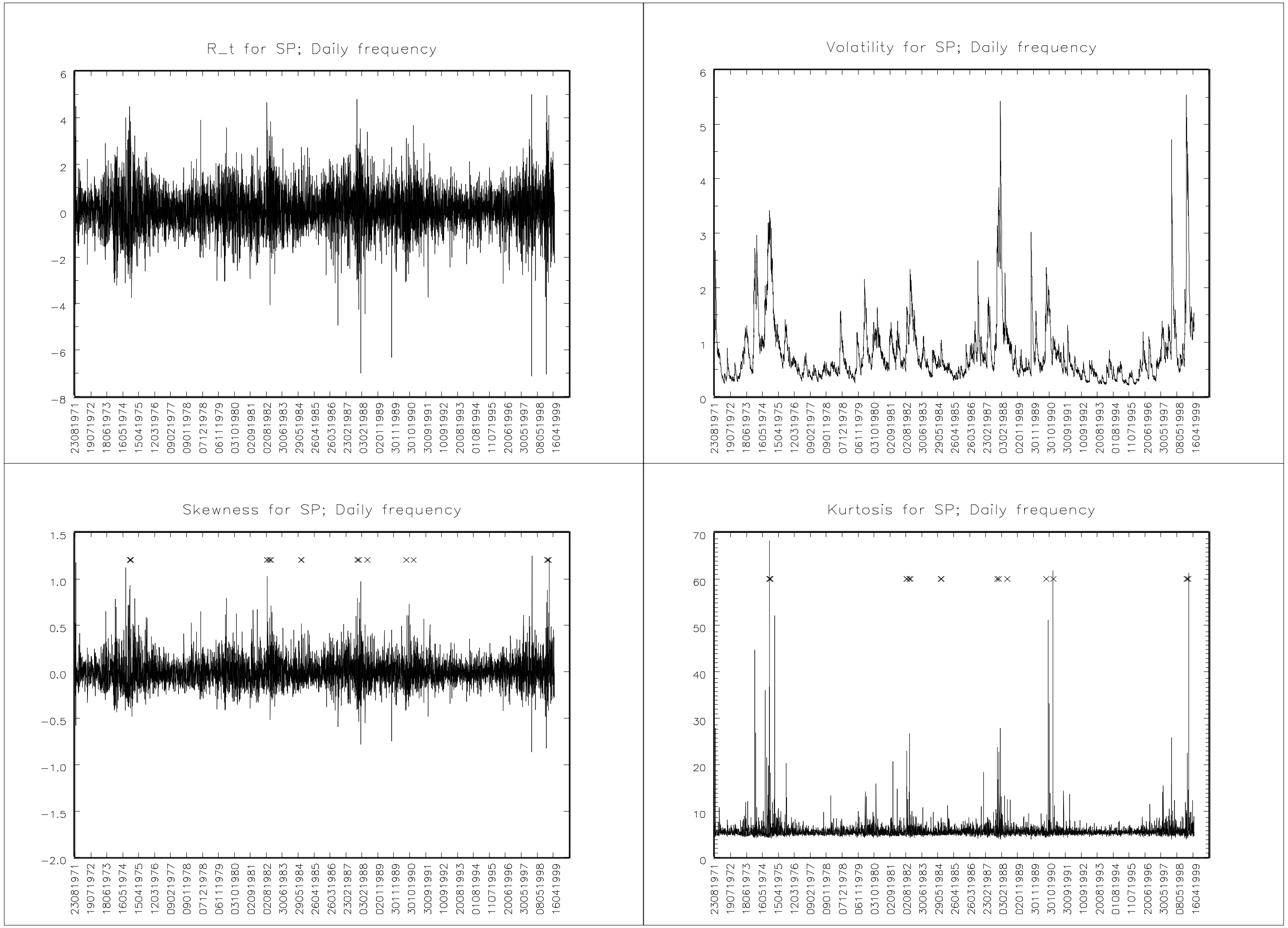


Lda_t for DM: Daily frequency

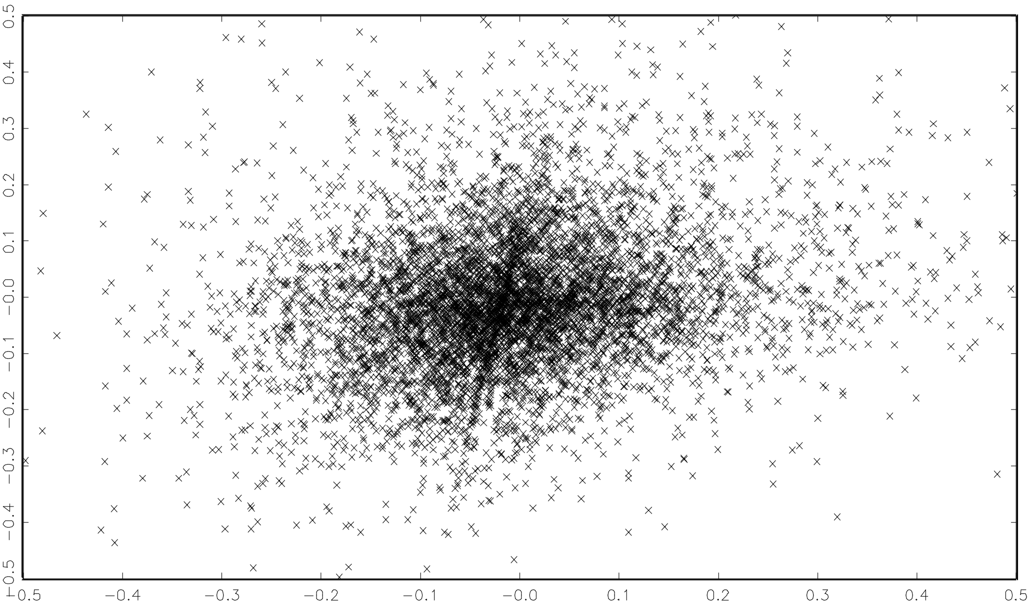



R_t for DM; Daily frequency
Volatility for DM; Daily frequency
Skewness for DM; Daily frequency
Kurtosis for DM; Daily frequency

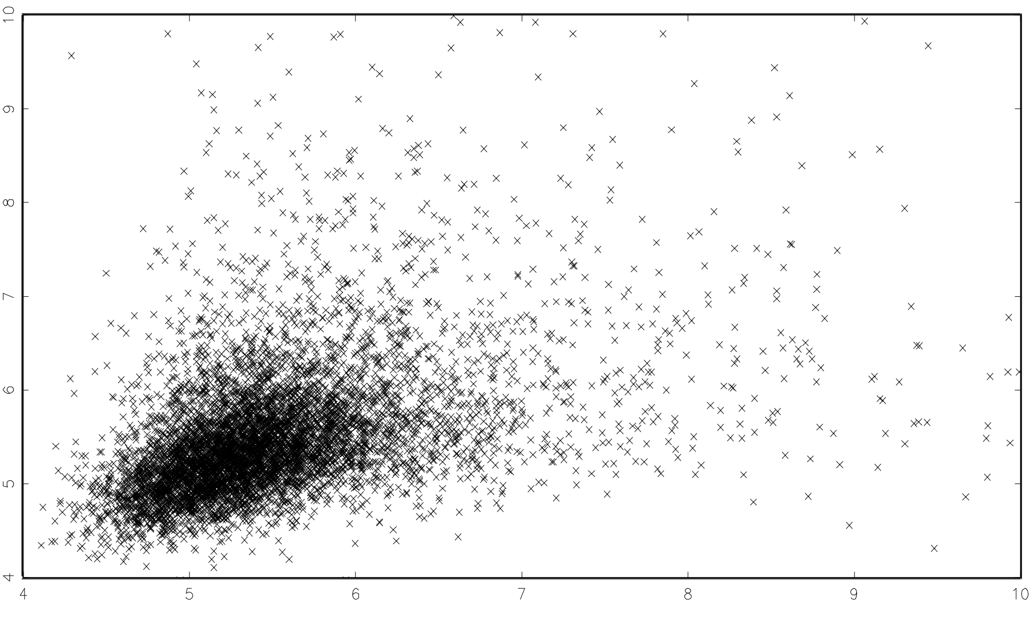

R_t for SP: Daily frequency
Eta_t for SP: Daily frequency
Lda_t for SP: Daily frequency


R_t for SP; Daily frequency
Volatility for SP; Daily frequency
Skewness for SP; Daily frequency
Kurtosis for SP; Daily frequency

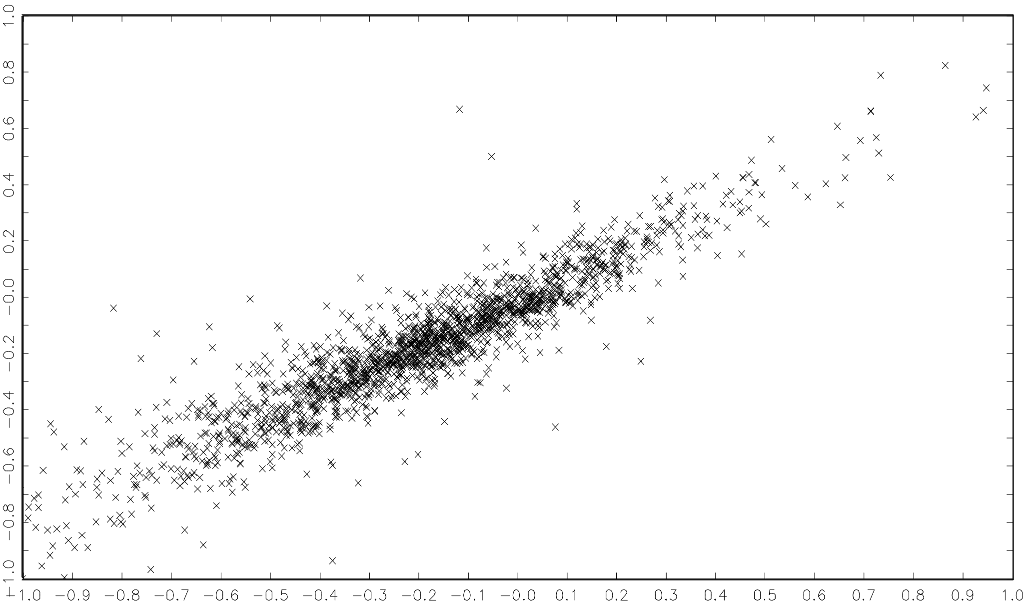
Skewness for SP; Daily frequency



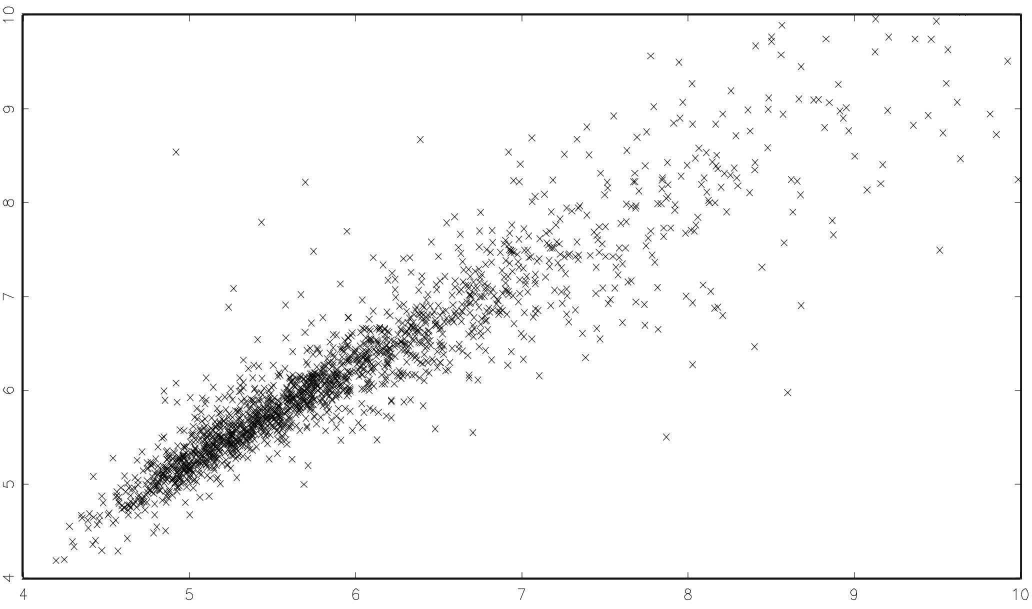
Kurtosis for SP; Daily frequency



Skewness for DM-FF; Daily frequency



Kurtosis for DM-FF; Daily frequency



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