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**NOTES D'ÉTUDES**

**ET DE RECHERCHE**

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FEDERAL RESERVE REACTION  
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# Assessing GMM Estimates of the Federal Reserve Reaction Function

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## Abstract

Estimating a forward-looking monetary policy rule by the Generalized Method of Moments (GMM) has become a popular approach since the influential paper by Clarida, Gali, and Gertler (1998). However, an abundant econometric literature underlines to the unappealing small-samples properties of GMM estimators. Focusing on the Federal Reserve reaction function, we assess GMM estimates in the context of monetary policy rules. First, we show that three usual alternative GMM estimators yield substantially different results. Then, we compare the GMM estimates with two Maximum-Likelihood (ML) estimates, obtained using a small model of the economy. We use Monte-Carlo simulations to investigate the empirical results. We find that the GMM are biased in small sample, inducing an overestimate of the inflation parameter. The two-step GMM estimates are found to be rather close to the ML estimates. By contrast, iterative and continuous-updating GMM procedures produce more biased and more dispersed estimators.

Keywords: Forward-looking model, monetary policy reaction function, GMM estimator, FIML estimator, small-sample properties of an estimator.

JEL classification: E52, E58, F41.

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# 1 Introduction

The benchmark Taylor rule states that the central banks should set the short-term interest rate in proportion of the inflation rate and the output gap. However, since Taylor's (1993) prominent contribution, an abundant empirical as well as theoretical literature has claimed that central banks may have a forward-looking behavior (Clarida and Gertler, 1997, Clarida, Gali, and Gertler, 1998, 2000, Haldane and Batini, 1998). This assumption implies the use of an adequate estimation method to overcome the presence of expected inflation in the policy rule. Estimating forward-looking monetary policy rules by Generalized Method of Moments (GMM) has become a popular approach since the influential work by Clarida, Gali, and Gertler (1998).<sup>1</sup>

In this paper, we assess the robustness of GMM estimates in the context of a forward-looking reaction function. This investigation is motivated by the growing literature on the small-sample properties of the GMM procedure. Using Monte-Carlo experiments, Tauchen (1986), Fuhrer, Moore, and Schuh (1995) and Andersen and Sørensen (1996) provided evidence that GMM estimators can be strongly biased and widely dispersed in small samples. These studies have investigated small-sample properties of the GMM, in the context of asset-pricing, inventories, or consumption models respectively. Forward-looking monetary policy rules offer an original field for investigating GMM properties. Moreover, it suggests to focus specifically on the inflation parameter in the reaction function.

We contrast the GMM estimators with an alternative estimation procedure for forward-looking models, namely the Maximum Likelihood (ML) method. The ML approach involves the estimation of a structural model of the economy. For this purpose, we use a version of the Rudebusch and Svensson (1998) model and solve the whole rational-expectations model using the Anderson and Moore (1985) procedure. The properties of ML and GMM estimates of the reaction function are then compared using Monte Carlo simulations.

While we focus on the widely-studied Federal Reserve reaction function over the period 1979 to 1998, we present some original empirical results. In the literature on the reaction function, estimation has been typically performed using the two-step GMM estimator. Here, we estimate the reaction function using three alternative GMM procedures, as well as two ML procedures. ML and "two-step" GMM are found to produce similar results, from which the estimates produced by "iterative" and "continuous-updating" GMM differ markedly. We conduct Monte-Carlo simulations to investigate these results. The GMM exhibit a small-sample bias, towards overestimating the response of interest rate to expected inflation. The bias is limited, however, so that our overall assessment of GMM in the case of a monetary policy rule is less critical than that found by Fuhrer, Moore, and Schuh (1995) in the case of inventories. Nevertheless, GMM parameter estimates are found to be more imprecise than FIML estimates, especially in the case of continuous-updating GMM.

The paper is organized as follows. The benchmark specification of a forward-

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<sup>1</sup>Examples include Clarida, Gali, and Gertler (2000), Mehra (1999), Orphanides (2000) for US data, or Peersman and Smets (1998), Gerlach and Schnabel (2000), Angeloni and Dedola (1999), Nelson (2000) for European data.

looking reaction function is presented in section 2. Section 3 summarizes GMM estimation procedures and presents empirical evidence obtained with this approach. In section 4, ML estimation is introduced and implemented. Section 5 provides some interpretation of the empirical evidence, using Monte-Carlo simulations. Section 6 concludes.

## 2 The monetary-policy reaction function

In the baseline Taylor rule, the central bank is assumed to set the level of the nominal short-term interest rate as a function of the rate of inflation and output gap:

$$i_t = i^* + \beta (\bar{\pi}_t - \pi^*) + \gamma y_t \quad (1)$$

where  $i_t$  denotes the short-term nominal interest rate,  $\bar{\pi}_t$  the inflation rate,  $y_t$  the output gap. The inflation rate is the four-quarter moving average of inflation in the implicit GDP deflator. The output gap is defined as the percent deviation of real GDP from its deterministic trend. The constant  $i^*$  is the long-run equilibrium nominal interest rate and  $\pi^*$  is the inflation target. The output-gap target is assumed to be zero. The coefficient on inflation ( $\beta$ ) is a crucial parameter, since in most macroeconomic models based on the Phillips curve and the I-S curve,  $\beta > 1$  is a relevant condition for stability (Taylor, 1999b, and in the context of forward-looking models, Kerr and King, 1996, Clarida, Gali, and Gertler, 2000).

The Taylor rule has received a widespread attention in the empirical literature. In particular, it has been shown to provide a rough description of US monetary policy during the Greenspan and Volcker tenures (Taylor, 1993, 1999a, Judd and Rudebusch, 1998). However, most estimates consider “modified” Taylor rules. First, central banks often appear to smooth changes in interest rates. Several motivations for such an interest-rate smoothing have been proposed (Sack and Wieland, 1999, Woodford, 1999). For instance, in case of uncertainty concerning the model’s parameters, it is optimal for the central bank to adjust interest rates only gradually. Therefore, the short-term rate is allowed to adjust gradually to its target, which is defined by equation (1).

A second issue is whether the central bank sets the level of the interest rate as a function of observed or expected inflation. Though the terminology is not uncontroversial, we will refer to the policy rule as being “forward-looking” if the central bank reacts to some expectation of inflation. Several authors have claimed that the forward-looking reaction function is consistent with the observed behavior of central banks (Mehra, 1999, Clarida and Gertler, 1997, Clarida, Gali, and Gertler, 1998, 2000, Orphanides, 1998).<sup>2</sup> Most central banks explicitly claim that they do not only consider past or current economic conditions, but they also include economic forecasts in their macroeconomic condition statement. In addition, from the normative viewpoint, that policy rules should be forward-looking has been advocated by Haldane

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<sup>2</sup>Note, however, that Fair (2001) strongly rejects the forward-looking specification for the Fed reaction function. He argues that this outcome occurs when estimating the policy rule over the period from 1982:Q4 through 1999:Q3, which is assumed to be more stable than the sample usually used (beginning in 1979).

and Batini (1998) and Svensson (1997). Therefore, a large number of recent studies estimate the following more general specification, which incorporates the expected inflation rate:

$$i_t = \rho i_{t-1} + (1 - \rho) (i^* + \beta (E_{t-1} \bar{\pi}_{t+4} - \pi^*) + \gamma y_{t-1}) + \varepsilon_t \quad (2)$$

where  $\varepsilon_t$  denotes a random policy shock, and  $E_{t-1}$  denotes the mathematical expectation conditional to the information set containing all variables dated  $t - 1$ . The parameter  $\rho$  represents the degree of interest-rate smoothing. We do not assume that the central bank reacts to expected output gap. It is worth emphasizing that current inflation and output gap are not supposed to be observed in real time by the central bank. Rather, we assume that, at date  $t$ , only inflation and output gap at date  $t - 1$  are in the information set of the central bank.<sup>3</sup>

Estimating equation (2) provides estimates of the weights on inflation and output gap in the monetary policy rule and the speed of adjustment to the rule. The long-run inflation target  $\pi^*$  is not identified however, since the constant term is equal to  $r^* + \beta\pi^*$ . Note that, if we assume a value for the long-run equilibrium real interest rate (for instance, the sample average real rate), we easily obtain an estimate of the long-run inflation target. Last, note that in line with most of the reaction function literature, we assume stationarity of the variables.

### 3 GMM estimation

Since expectations are unobserved, the standard approach is to substitute  $E_t \bar{\pi}_{t+4}$  with the actual value  $\bar{\pi}_{t+4}$  in equation (2).<sup>4</sup> This approach rises two problems. First, since the expected inflation is measured with error by observed inflation, we face an error-in-variable problem. This is because the error term is likely to be correlated with one of the explanatory variables, namely the future inflation rate. Second, since the current interest rate shock is likely to affect future inflation, there is an endogeneity bias. Thus, the OLS procedure provides biased estimators, whereas GMM provides a consistent estimation procedure. GMM estimators have been shown to be strongly consistent, asymptotically normal (Hansen, 1982) and have been applied to rational-expectation models along the lines of Cumby, Huizinga, and Obstfeld (1983) and Hansen and Singleton (1982). In the rational-expectation context, the GMM approach is very appealing, because it does not require strong assumptions concerning the underlying model. In fact, it only requires identifying relevant instrument variables, strongly correlated with RHS variables, but uncorrelated with innovations. The central bank's information set at time  $t$  is here assumed to include four lagged

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<sup>3</sup>To some extent, even this assumption is questionable, since the output gap is measured precisely after many quarters only. See Orphanides (2000) who claims that, if monetary policy during Chairman Burns tenure appears non-optimal a posteriori, this is because the output-gap measure has been dramatically corrected since the end of the 70s.

<sup>4</sup>An alternative approach for models with expectations, not pursued here, is to use actual inflation forecasts. McNees (1985, 1992) and Orphanides (1998) used the Board of Governors' staff forecasts presented at each FOMC meeting.

values of interest rate, inflation and output gap. (We reduce intentionally the information set to the variables used in the structural model discussed in section 4.) These instruments are plausibly correlated with future inflation.<sup>5</sup>

### 3.1 GMM estimators

Let equation (2) be expressed in standard regression notation as

$$y = X\theta + \varepsilon$$

with  $y$  a  $(T \times 1)$  vector and  $X$  a  $(T \times n)$  matrix.  $X_t = (x_{1t}, \dots, x_{nt})$  is a vector of observation and  $\theta$  is the  $(n \times 1)$  vector of unknown parameters. Let  $Z$  be the  $(T \times q)$  matrix of instrumental variables, with  $q > n$ . All the  $q$  instruments are assumed to be predetermined, in the sense that they are orthogonal to the current error term:  $E(Z_{it}\varepsilon_t) = 0, \forall t$  and  $i = 1, \dots, q$ . This can be written as

$$Eg_t(\theta) = 0$$

where  $g_t(\theta) = Z_t \cdot (y_t - X_t'\theta) = Z_t \cdot \varepsilon_t$ .

The GMM estimator, denoted  $\hat{\theta}_{GMM}$ , is the value of  $\theta$  that minimizes the scalar

$$Q_T(\theta) = \bar{g}_T(\theta)' W_T \bar{g}_T(\theta)$$

where the  $(q \times 1)$  vector  $\bar{g}_T(\theta) = \frac{1}{T} \sum_{t=1}^T g_t(\theta)$  denotes the sample mean of  $g_t(\theta)$ .  $W_T$  denotes the  $(q \times q)$  GMM weighting matrix. An asymptotically efficient estimator is obtained by choosing as a weighting matrix a consistent estimator of  $W = V^{-1}$ , where  $V = E(g_t(\theta)g_t(\theta)') = E(\varepsilon_t^2 Z_t Z_t')$  is the covariance matrix of  $g_t(\theta)$ . The GMM estimator is then defined by:

$$\hat{\theta}_T = (X'ZW_TZ'X)^{-1} X'ZW_TZ'y. \quad (3)$$

When innovations are likely to be heteroskedastic and serially correlated, an optimal GMM estimator is obtained when the weighting matrix  $W_T$  is estimated by the inverse of the long-run covariance matrix

$$W_T = (\hat{S}_T)^{-1}. \quad (4)$$

The long-run covariance matrix can be consistently estimated by (Newey and West, 1987)

$$\hat{S}_T = S_0 + \sum_{l=1}^L w(l)(S_l + S_l') \quad \text{with} \quad S_l = \frac{1}{T} \sum_{t=l+1}^T \hat{\varepsilon}_t \hat{\varepsilon}_{t-l} (Z_t Z_{t-l}') \quad (5)$$

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<sup>5</sup>We also estimated equation (2) using the same instrument set as Clarida, Gali, and Gertler (2000), which, in addition to lags of inflation, output gap and interest rate, includes lags of commodity price inflation, M2 growth and the spread between the long rate and the short rate. Our results are essentially unaltered. For instance, the two-step GMM estimate of parameter  $\beta$  is 1.85 (versus 1.92 for our baseline instrument set).

where  $\hat{\varepsilon}_t = y_t - X_t' \hat{\theta}_T$  and  $w(l) = 1 - \frac{l}{L+1}$  denotes the Bartlett kernel. Hence, the asymptotic covariance matrix of  $\hat{\theta}_T$  is given by  $\hat{V} = (X' Z W_T Z' X)^{-1}$ .

Note that we need an estimate of  $W_T$  before estimating  $\hat{\theta}_T$  and we need an estimate of  $\hat{\theta}_T$  before estimating  $W_T$ . In the following, we implement three alternative versions of the GMM estimator often considered in the theoretical literature.

In the first approach, the parameter vector is estimated with the two-step two-stage least squares, or “two-step GMM”, initially proposed by Cumby, Huizinga, and Obstfeld (1983) and Hansen and Singleton (1982).<sup>6</sup> Assume an initial guess for the weighting matrix. Usually, one sets  $\hat{W}_T^{(0)} = \frac{1}{T} \sum_{t=1}^T Z_t Z_t'$ . An initial estimate of the parameters,  $\hat{\theta}_T^{(0)}$ , is obtained using two-stage least squares. Then, one constructs an estimate of the optimal weighting matrix  $\hat{W}_T^{(1)}$  using equation (4) with  $\hat{\varepsilon}_t = y_t - X_t' \hat{\theta}_T^{(0)}$ . Last, the two-step GMM estimator, denoted  $\hat{\theta}_T^{(1)}$ , is obtained from equation (3) with  $\hat{W}_T^{(1)}$  as weighting matrix.

The second approach, suggested by Hansen (1982), Ferson and Foester (1994) or Hansen, Heaton, and Yaron (1996), relies on estimating parameters and the weighting matrix recursively. Thus, equations (3) and (4) are estimated repeatedly, until convergence of parameter  $\hat{\theta}_T$  is reached, such that  $(\hat{\theta}_T^{(i)} - \hat{\theta}_T^{(i-1)})$  is less than a convergence criterion (chosen here to be equal to  $10^{-5}$ ).  $\hat{\theta}_T^{(i)}$  denotes the GMM estimator at step  $i$ . This procedure is called “iterative GMM” in the following.

In the last approach, called “continuous-updating GMM”, developed by Hansen, Heaton, and Yaron (1996) and studied in Stock and Wright (2000), the weighting matrix is continuously altered as  $\theta$  is changed in the minimization. Therefore, the continuous-updating GMM estimator is the minimizer of

$$\bar{g}_T(\theta)' W_T(\theta) \bar{g}_T(\theta).$$

GMM estimates are justified on asymptotic grounds. Small-samples properties of the GMM procedure have been studied in a number of papers. The general result is that the asymptotic theory provides a poor approximation in finite samples.<sup>7</sup> Using Monte-Carlo experiments, Tauchen (1986), Kocherlakota (1990), and Andersen and Sørensen (1996) provided evidence that GMM estimates can be strongly biased in small samples. More specifically, there is a strong variance/bias trade-off as the number of instruments is increased. Including more moment restrictions (more information) improves the estimation performance, but, in small samples, it also deteriorates the precision of the estimated weighting matrix. Hansen, Heaton, and Yaron (1996) compared the small-sample properties of alternative GMM estimators. The two-step estimator and the iterative estimator are shown to be more widely median biased than

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<sup>6</sup>This approach has been used, for instance, by Clarida, Gali, and Gertler (1998, 2000) and Peersman and Smets (1998) in the context of the reaction function.

<sup>7</sup>One reason for this result is that estimating an efficient GMM estimator uses a Newey and West (1987) weighting matrix, which is a function of estimated fourth moments of innovations. Sample fourth moments are known to converge toward the true value only very slowly. Therefore, the Newey and West weighting matrix is an optimal estimator of the true matrix, although it is likely to be a poor estimator in small sample.



the continuous-updating estimator. But, the distribution of the continuous-updating estimator has much fatter tails.

The alternative GMM estimators have the same asymptotic distribution. Nevertheless, the continuous-updating GMM offers the advantage over the two-step and iterative GMM approaches that estimates are invariant with respect to the initial weighting matrix for  $W_T$ . Moreover, in an asset-pricing context, the iterative GMM approach is found to have superior small-sample properties as compared with the two-step GMM (Ferson and Foerster, 1994).

As stressed by Nelson and Startz (1990) and Maddala and Jeong (1992) in the context of the Instrumental Variable estimator or by Stock and Wright (2000) in the context of the GMM estimator, the poor performance of these estimators can be related to the weak correlation between instruments and the relevant first-order conditions.

Another important feature of GMM estimation is that the information set contains more instruments than parameters to be estimated (provided  $q > n$ ). Therefore, when the model is true, all the elements of the sample moments  $\bar{g}_T(\hat{\theta}_T)$  are close to zero, but they cannot be set to zero exactly. It turns out that if the weighting matrix  $W_T$  is chosen optimally, so that  $W_T = (\hat{S}_T)^{-1}$ , then the minimized distance

$$J_T = T\bar{g}_T(\hat{\theta}_T)' W_T \bar{g}_T(\hat{\theta}_T)$$

is asymptotically distributed as a  $\chi^2$  with  $q - n$  degrees of freedom. This provides us with the Hansen's test of the over-identifying restrictions (Hansen, 1982). A rejection of these restrictions would indicate that some variables in the information set fail to satisfy the orthogonality conditions.

### 3.2 Empirical results

We now present estimation of the monetary policy reaction function based on equation (2). We consider monetary policy for the Federal Reserve over the period from 1979:Q3 to 1998:Q4.<sup>8</sup> Our sample period covers P. Volcker (1979:Q3-1987:Q2) and A. Greenspan (1987:Q3 up to now) tenures. The assumption that a unique monetary policy regime has prevailed over this period is controversial. However, statistical evidence in favor of reaction function instability within this sample is at most mixed (Judd and Rudebusch, 1998, Estrella and Fuhrer, 1999). We use quarterly data, drawn from the OECD databases BSDB and MEI. The Federal funds rate is used as the central bank's instrument. The output gap is defined by the deviation of (log) real GDP from (log) potential GDP. Potential GDP is computed using a linear trend with a break in trend growth rate in 1974.

Table 1 reports parameter estimates of the forward-looking reaction function. Estimates obtained using the two-step GMM, the iterative GMM, and the continuous-updating GMM are reported in Panel A, B, and C respectively. The lag length in the weighting matrix is chosen equal to the conventional value  $L = 4$  (consistently with

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<sup>8</sup>While our data set ends in 1999:Q4, our estimates end in 1998:Q4, because our GMM estimates use data until the end of 1999.

the quarterly frequency of our data) or determined using the optimal bandwidth as suggested by Andrews (1991), in which case we have  $L = L^*$ .

First, we consider the two-step GMM approach with  $L = 4$ . The point estimate of the inflation parameter ( $\beta = 1.92$ ) is markedly larger than the Taylor 1.5 benchmark coefficient, although the Taylor coefficient falls in the confidence interval. Furthermore, the output-gap parameter is not significant, in contrast with the standard Taylor rule. When reestimating the model with lagged inflation in place of expected inflation, we find the output-gap parameter ( $\gamma = 0.38$ ) to be significant and the inflation parameter to ( $\beta = 1.56$ ) be remarkably close to the Taylor coefficient. These results indicate that current output gap appears in the usual reaction function as a predictor of future inflation only. The J-statistic supports the over-identifying restrictions implied by the model.

The iterative GMM and the continuous updating GMM yield even larger estimates for the inflation parameter:  $\beta$  is as high as 3.1 for the iterative GMM and 3.5 for the continuous-updating GMM. Note that examples of large inflation parameters can be found in the forward-looking reaction function literature. For instance, using ex-post revised data and the Instrumental-Variable estimator, Orphanides (1998) finds that the inflation parameter is equal to 3.7, with a standard error of 3. As with the two-step GMM, the output-gap parameter is not significant in the reaction functions reported in Panels B and C. Standard errors of parameter are larger than with the two-step GMM. The continuous-updating GMM estimator yields estimates which are close to the iterative GMM estimates, but standard errors are much larger.

For the three GMM approaches, the optimal bandwidth in the weighting matrix is found to be  $L^* = 1$ . Computing the weighting matrix with  $L = 4$  or  $L = L^*$  does not materially alter the estimated parameters for the iterative and continuous-updating GMM approaches. By contrast, in the case of two-step GMM, the inflation parameter obtained with  $L = L^*$  is much larger than the parameter obtained with  $L = 4$ . Moreover, it is closer to the estimates obtained with the iterative and continuous-updating approaches. Note that computing the weighting matrix with  $L = L^*$  provides a much larger Hansen's test statistic for the two-step GMM. Although over-identifying restrictions are not rejected even in this case, this may indicate that residuals are slightly serially correlated. This may explain the dramatic change in the inflation parameter.

It is noteworthy that the three GMM procedures display large differences in the standard errors of parameter. Considering the inflation parameter with  $L = 4$ , we find that the standard error obtained with iterative GMM is much larger than the standard error obtained with the two-step GMM, but smaller than the standard error obtained with the continuous-updating GMM.

To sum up, several empirical results are worth emphasizing. First, according to the various GMM estimates, the weight of inflation in the reaction function is quite large while the weight of output gap is insignificant. Second, empirical results provided by iterative GMM and continuous-updating GMM contrast markedly with those provided by the usual two-step GMM approach. The former approaches produce higher parameter standard errors and higher standard errors of estimates.

## 4 ML estimation

### 4.1 The ML approach

In this section, we focus on an alternative estimation procedure, namely the Maximum Likelihood (ML) approach. The ML approach requires that an auxiliary model is estimated for the forcing variables (here, the output gap and the inflation rate). The auxiliary model, together with the equation of interest is then solved for forward-looking variables, yielding cross-equation restrictions. An appealing advantage of ML over GMM, in forward-looking models, is that expectations obtained with ML estimation are fully model-consistent. Thus, the expected inflation which appears in the reaction function (2) is consistent with the inflation equation of the model. The ML approach is of course demanding since a structural model has to be estimated for variables other than interest rate. However, in the present case, the widely-used I-S curve / Phillips curve framework provides us with a reliable benchmark model of the output-inflation joint dynamics.

For the problem at hand, the estimation can be performed in two ways: First, the FIML procedure involves the joint estimation of the Phillips curve, the I-S curve and the reaction function. Second, the two-step ML procedure is based on a preliminary estimation of the Phillips curve and the I-S curve. The latter implies a loss of efficiency, but reduces the computational burden dramatically. We will provide evidence on both procedures in the following.

Both ML estimations of the forward-looking reaction function are implemented using the procedure developed by Anderson and Moore (1985). This procedure computes the reduced form of a forward-looking model. The forward-looking model can be written in the format

$$\sum_{j=-\tau}^0 H_j x_{t+j} + \sum_{j=1}^{\theta} H_j E_t(x_{t+j}) = \varepsilon_t \quad (6)$$

where  $x_t = (\pi_t, y_t, i_t)'$  and  $H_i$  are conformable square matrices containing the model's parameters. The innovations  $\varepsilon_t$  are i.i.d.  $N(0, \Sigma)$ .  $\tau$  and  $\theta$  denote leads and lags respectively ( $\tau = \theta = 4$ , in our empirical application). In the present context, inflation leads are the only forward-looking terms.

Using the generalized saddlepath procedure of Anderson and Moore (1985), the expectation of future terms in equation (6) is expressed as a function of expectations of lagged terms:

$$E_t(x_{t+k}) = \sum_{j=-\tau}^{-1} B_j E_t(x_{t+k+j}) \quad k > 0. \quad (7)$$

Then, equation (7) is used to derive the expectation of future terms as a function of the present and past terms. Substituting expectations into equation (6) gives the so-called observable structure

$$\sum_{j=-\tau}^0 S_j x_{t+j} = \varepsilon_t. \quad (8)$$

This procedure is very efficient and can be applied to a wide range of applications. It has been widely used in the empirical literature (see, e.g., Fuhrer, Moore, and Schuh, 1995, Fuhrer and Moore, 1995a and b).

Finally, the concentrated log-likelihood function is computed using the observable structure (8):

$$\ln L = -\frac{1}{2}nT \ln(2\pi) - \frac{1}{2} \sum_{t=1}^T \left( \ln |\hat{\Sigma}| + \hat{\varepsilon}_t \hat{\Sigma}^{-1} \hat{\varepsilon}_t' \right)$$

where  $\hat{\Sigma} = \frac{1}{T} \sum_{t=1}^T \hat{\varepsilon}_t \hat{\varepsilon}_t'$  is the estimated covariance matrix of residuals. The log-likelihood function is maximized using the BFGS algorithm. The parameter covariance matrix is computed as the inverse of the Hessian of the log-likelihood function.

## 4.2 Estimation results

As a structural model, we estimate a version of the model proposed by Rudebusch and Svensson (1998). It includes a Phillips curve (PC) and an I-S curve. This model embodies the main features of the standard macroeconomic paradigm and has proved to be a robust representation of the U.S. economy. The backward-looking nature of this model can be pointed as one potential source of mis-specification. However, no compelling empirical forward-looking counterpart of the RS model has so far emerged (see Estrella and Fuhrer, 1998). We estimate the following PC/I-S model:

$$\pi_t = \alpha_{\pi 1} \pi_{t-1} + \alpha_{\pi 2} \pi_{t-2} + \alpha_{\pi 3} \pi_{t-3} + \alpha_{\pi 4} \pi_{t-4} + \alpha_y y_{t-1} + u_t \quad (9)$$

$$y_t = \beta_{y1} y_{t-1} + \beta_{y2} y_{t-2} + \beta_r (i_{t-1} - \pi_{t-1} - \beta_0) + v_t. \quad (10)$$

The Phillips curve relates quarterly inflation  $\pi_t$  to lags of inflation and to lagged output gap. Since we do not reject the assumption that the four autoregressive parameters freely estimated sum to one, we impose this restriction in equation (9), so that  $\alpha_{\pi 4} = 1 - \alpha_{\pi 1} - \alpha_{\pi 2} - \alpha_{\pi 3}$ . This restriction is consistent with an accelerationist form of the Phillips curve. Moreover, we do not include a constant term in the equation, so that the output gap is assumed to be null in the long run. The I-S curve relates the output gap to its own lags and to the difference between the lagged short nominal rate and the lagged inflation rate. This last term is a proxy of the short real interest rate. Note that, unlike Rudebusch and Svensson, we include the lagged short real rate rather than a four-quarter moving average of the short real rate.

Parameter estimates are reported in Table 2. Standard errors are shown in parentheses. To be consistent with our estimate of the reaction function, we use the sample period 1979:Q3 to 1998:Q4. The empirical model is very close to the model estimated by Rudebusch and Svensson (1998) over the period 1961:Q1 to 1996:Q2. Considering first the ML structural model (Panel A), we obtain that the sensitivity of inflation to output gap and the responsiveness of output gap to the short real interest rate are slightly lower than those obtained by Rudebusch and Svensson (1998), but they have right signs and are significant.

If we turn to the FIML estimation (Panel B), the Phillips curve, the I-S curve, and the reaction function are estimated simultaneously, with a free covariance matrix

of innovations. As compared with the individually estimated Phillips curve reported above, we notice that parameter estimates remain unaltered, when the model is estimated using the FIML procedure. Concerning the I-S curve, the lagged output-gap parameters  $\beta_{y1}$  and  $\beta_{y2}$  change slightly, although the persistence is unchanged. More importantly, the sensitivity of output gap to the short real interest rate decreases: the point estimate is  $-0.058$  with a standard error equal to  $0.039$ .

We turn now to the reaction function parameters. Two-step ML is obtained by using the estimated parameters of equations (9) and (10) to solve for inflation expectations and estimate the monetary policy rule. The inflation parameter  $\beta$  is equal to  $1.668$ . Therefore, the two-step ML estimate is lower than the estimates obtained by GMM. Standard error of  $\beta$  is equal to  $0.18$ . Therefore, it is much smaller than those obtained with GMM procedures. Remind that it is  $0.29$  for the two-step GMM with lag length  $L = 4$ .

The output-gap parameter  $\gamma$  is essentially zero and non significant. The autoregressive parameter  $\rho$  is equal to  $0.71$  only. This is much lower than estimates obtained with GMM procedures. The standard error of residuals is  $1.071$ . The empirical fit appears to be better than with GMM procedures.

In Panel B of Table 2, we report FIML estimates of the reaction function. We obtain the following results. As compared to the two-step ML estimation, the inflation parameter in the reaction function is slightly larger. We obtain  $\beta = 1.71$  to be compared with the two-step ML estimate  $\beta = 1.67$ . The output-gap parameter  $\gamma$  and the autoregressive parameter  $\rho$  are essentially the same as the parameters obtained with two-step ML.

## 5 Monte-Carlo evidence

Previous sections have provided contrasting estimates of the forward-looking reaction function. The GMM parameter estimates differ substantially from one approach to the other and from ML estimates. Estimates seem to be less precise with the GMM approaches than with the ML approaches. In this section, we investigate these results, with a special focus on the expected-inflation parameter. We conduct Monte-Carlo experiments to illustrate the small-sample properties of the various estimation procedures.

### 5.1 Monte-Carlo design

In this section, we conduct Monte-Carlo experiments to assess the small- and large-sample properties of estimators obtained using the GMM and ML estimators. The design of the experiment is the following.

The data generating process (DGP) is given by the complete macroeconomic model estimated by FIML. The DGP is thus constituted of equations (9), (10) together with the reaction function estimated by FIML (Table 3). The innovation covariance matrix,  $\hat{\Sigma}$ , used in the Monte-Carlo experiments is the residual covariance matrix of  $(\hat{u}_t, \hat{v}_t, \hat{\varepsilon}_t)$ .

Each Monte-Carlo experiment is based on  $N = 500$  replications.<sup>9</sup> For a given sample size  $T$ , a sequence of  $T + 50$  random innovations are drawn from the normal distribution  $N(0, \hat{\Sigma})$ . In the simulations, two sample sizes are used:  $T = 78$ , and 1500 observations. The sample size  $T = 78$  corresponds to our estimation sample. The sample size  $T = 1500$  illustrates whether biases are likely to disappear in large samples. The random innovations are used to simulate the macroeconomic model (9), (10) and (2). Initial conditions are set equal to the average values over the sample. The first 50 entries are discarded to reduce the effects of initial conditions on the solution path. For each simulated dataset, the reaction function is then estimated using the three GMM estimators and the two ML estimators. We also report results for the OLS estimators, expected to be inconsistent in our set-up, in order to evaluate the magnitude of the OLS bias. For each sample size and each estimator, we have 500 parameter estimates, and the distribution of parameter estimates can then be analyzed.<sup>10</sup>

In some experiments, for  $T = 78$  observations, the continuous-updating GMM estimator failed to converge. The number of crashes is 2 percent of our samples. Hansen, Heaton, and Yaron (1996) also reported some difficulties to obtain reasonable parameter estimates with the continuous-updating GMM. First, the numerical search for the minimizer sometimes fails. In addition, even when convergence is reached, the distribution of estimates can be severely distorted, because of a few unrealistic samples. For this reason, in Table 6, for  $T = 78$ , two rows are devoted to the continuous-updating GMM estimator distribution. In the first row, we report distribution statistic after we discarded only estimations which reached the maximum number of iterations (here, 150). In the second row, we selected estimations which satisfied the additional criterion that the smoothing parameter  $\rho$  lies inside the reasonable interval  $[-1; 1]$ . In our Monte-Carlo experiments, 5.4 percent of estimations fell outside of this parameter space.

The two-step ML and FIML estimators are implemented as follows in the Monte-Carlo experiments. For the two-step ML estimator, for each replication, we estimate the Phillips curve and the I-S curve with simulated data, and then we estimate the reaction function using the procedure described in section 4.1. For the FIML, all macroeconomic parameters are estimated jointly with the reaction function. Therefore, FIML is expected to perform better in this context.

## 5.2 Results

The distribution of the alternative reaction-function parameter estimators is summarized in Table 3 for the small-sample case ( $T = 78$ ), and in Table 4 for the large-sample case ( $T = 1500$ ). Fig. 1 and 2 also display the parameter  $\beta$  distribution for the various estimation approaches. In the following, the assumed true value of the parameter

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<sup>9</sup>An upper bound on the Monte-Carlo standard error of parameter, denoted  $\sigma_\theta$ , is  $N^{-1/2}\sigma_\theta$ . Therefore, for  $N = 500$ , the upper bound on the Monte-Carlo standard error is 0.045 times the computed standard error.

<sup>10</sup>Simulations were performed using GAUSS version 3.2 on a Pentium III platform. We used the BFGS algorithm of the OPTMUM procedure for optimization. We found no discrepancies when we used different algorithms. All experiments were performed using numerical derivatives.

is  $\beta = 1.712$  (see Panel B of Table 2).

Table 3 reveals that, in small samples, the two-step ML procedure is unbiased. The median  $\beta$  parameter is equal to 1.704. This estimator is precisely estimated, since the standard deviation is equal to 0.213 and the 90 percent confidence interval is [1.425; 1.930]. As regards the FIML estimators, even for small samples, the estimator of  $\beta$  is also unbiased, with a median value equal to 1.694.

Turning to GMM procedures, we find that estimators are somewhat biased and very imprecise in small samples. The GMM bias on  $\beta$  is positive. The smallest bias is obtained for iterative GMM, with an estimator mean equal to 1.817, and a median equal to 1.767. The bias is more pronounced in the case of the two-step GMM estimator (with a 1.842 mean and a 1.767 median). The continuous-updating estimator provides very imprecise estimates, even though the median is equal to 1.875. When “unreasonable” outcomes are excluded, the mean is equal to 1.949 and median to 1.795. While the magnitude of the bias is not very large, one notices that it is in fact as large as the bias obtained with the inconsistent OLS estimator.

The dispersion of GMM estimator is at least twice as large as that of ML estimators. The iterative GMM estimator has a standard deviation as high as 0.586, so that the 90 percent confidence interval is [1.325; 2.334]. The distribution of GMM estimators is markedly asymmetric, since the upward boundary is much more distant from the median than the downward boundary. This provides a rationale for the very large  $\beta$  estimates obtained with iterative and continuous-updating GMM on the actual data. This finding is consistent with some previous results, which indicate that the GMM estimator may have very unusual properties (see, e.g., Tauchen, 1986, Fuhrer, Moore, and Schuh, 1995, and Nelson and Startz, 1990, in the case of the Instrumental Variable estimator). Noticeably, the continuous-updating GMM estimator has very fat tails and yields a non-negligible proportion of implausible estimate.

For large samples (Table 4), the bias obtained with GMM procedures disappears, as expected.<sup>11</sup> However, the standard error of GMM estimators remains about 25 percent higher than the standard error of ML estimators. This result is illustrated in Fig. 2. The OLS estimator is asymptotically biased. Interestingly, in our set-up, the bias on the inflation parameter is rather small.

The above results point in favor of ML estimate. However, an important caveat is in order. On one hand, since the true specification is assumed to be given by our FIML parameter estimates, the ML approach is undoubtedly favored in the experiments. On the other hand, in the Monte-Carlo experiments, we only use relevant instruments, and lags of relevant instruments. In actual GMM empirical estimate of reaction function, it is a common practice to include a large number of instruments (such as the exchange rate or raw material prices), some of which may be irrelevant, thus accentuating the small-sample undesirable properties of GMM.

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<sup>11</sup>Note that the difference between the alternative GMM estimators vanishes, even for intermediate sample sizes ( $T = 300$ , not reported here).

## 6 Conclusion

This paper has investigated the robustness of GMM estimates of the Federal Reserve reaction function, relying on a dynamic forward-looking Taylor rule. Our main findings are the following. First, focusing on the inflation parameter of the dynamic Taylor rule, the various GMM procedures are found to provide very different estimates. Iterative and continuous-updating GMM, which have not been often considered in the reaction-function literature, produce particularly high inflation parameters in our sample.

Second, ML is a feasible alternative to GMM for estimating a forward-looking reaction function. A traditional drawback with ML is that it requires estimating a structural model for forcing variables. But in the present context, a I-S curve / Phillips curve model, such as the Rudebusch and Svensson's (1998) model, provides a fairly reliable simple model of the economy. In our sample, the inflation parameters estimated using ML are lower than those estimated using GMM, and they are more in line with the Taylor rule. Moreover, ML estimates are more precise than GMM estimates. It is worth emphasizing, however, that results obtained using the two-step GMM estimator are rather close to those obtained using the ML approach. Monte-Carlo experiments support these outcomes. We find evidence that, in small sample, GMM estimates tend to overstate the degree to which interest rates respond to future inflation. The size of this bias is limited however.

Finally, our assessment of GMM in the case of a reaction function is therefore less critical than that of Fuhrer, Moore, and Schuh (1995) in the case of inventories. The simple approach usually adopted in empirical studies of the reaction function, i.e. the two-step GMM, does not provide strongly biased parameter estimates. Other GMM estimates are strongly biased. Last, GMM estimates exhibit an excessive dispersion.



## References

- [1] Andersen, T. G., and B. E. Sørensen (1996), GMM Estimation of a Stochastic Volatility Model: A Monte Carlo Study, *Journal of Business and Economic Statistics*, 14(3), 328–352.
- [2] Anderson, G. A., and G. R. Moore (1985), A Linear Algebraic Procedure for Solving Perfect Foresight Models, *Economics letters*, 17, 247–252.
- [3] Angeloni, I., and L. Dedola (1999), From the ERM to the Euro: New Evidence on Economic and Policy Convergence among EU countries, ECB Working Paper 4.
- [4] Andrews, D. W. K. (1991), Heteroskedasticity and Autocorrelation Consistent Matrix Estimation, *Econometrica*, 59(3), 817–858.
- [5] Clarida, R., J. Gali, and M. Gertler (1998), Monetary Policy Rules in Practice: Some International Evidence, *European Economic Review*, 42(6), 1033–1067.
- [6] Clarida, R., J. Gali, and M. Gertler (2000), Monetary Policy Rules and Macroeconomic Stability: Evidence and some Theory, *Quarterly Journal of Economics*, 115(1), 147–180.
- [7] Clarida, R., and M. Gertler (1997), How the Bundesbank Conducts Monetary Policy, in Romer, C. D., and D. H. Romer (eds), *Reducing Inflation: Motivation and Strategy*, NBER Studies in Business Cycles, vol. 30, University of Chicago Press, 363–406.
- [8] Cumby, R. E., J. Huizinga, and M. Obstfeld (1983), Two-Step Two-Stage Least Square Estimation in Models with Rational Expectations, *Journal of Econometrics*, 21(3), 333–355.
- [9] Estrella, A., and J. C. Fuhrer (1999), Are Deep Parameters Stable? The Lucas Critique as an Empirical Hypothesis, Working Paper 99-04, Federal Reserve Bank of Boston, May.
- [10] Ferson, W. E., and S. R. Foerster (1994), Finite Sample Properties of the Generalized Method of Moments in Tests of Conditional Asset Pricing Models, *Journal of Financial Economics*, 36(1), 29–55.
- [11] Fuhrer, J. C. (1996), Monetary Policy Shifts and Long-Term Interest Rates, *Quarterly Journal of Economics*, 61(4), 1183–1209.
- [12] Fuhrer, J. C., and G. R. Moore (1995a), Monetary Policy Trade-Off and the Correlation Between Nominal Interest Rates and Real Output, *American Economic Review*, 85(1), 219–239.
- [13] Fuhrer, J. C., and G. R. Moore (1995b), Inflation Persistence, *Quarterly Journal of Economics*, 110(1), 127–160.

- [14] Fuhrer, J. C., G. R. Moore, and S. D. Schuh (1995), Estimating the Linear Quadratic Inventory Model. Maximum Likelihood versus Generalized Method of Moments, *Journal of Monetary Economics*, 35(1), 115–157.
- [15] Gerlach, S., and G. Schnabel (1999), The Taylor Rule and Interest Rates in the EMU Area: a Note, *Economics Letters*, 67(2), 165–171.
- [16] Haldane, A., and N. Batini (1998), Forward-Looking Rule for Monetary Policies, NBER Working Paper 6543.
- [17] Hansen, L. P. (1982), Large Sample Properties of Generalized Method of Moments Estimator, *Econometrica*, 50(4), 1029–1054.
- [18] Hansen, L. P., J. Heaton, and A. Yaron (1996), Finite-Sample Properties of Some Alternative GMM Estimators, *Journal of Business and Economic Statistics*, 14(3), 262–280.
- [19] Hansen, L. P., and K. J. Singleton (1982), Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models, *Econometrica*, 50(5), 1269–1286.
- [20] Judd, J. P., and G. D. Rudebusch (1998), Taylor’s Rule and the Fed: 1970-1997, *FRBSF Economic Review*, 3, 3–16.
- [21] Kerr, W., and R. G. King (1995), Limits on Interest Rate Rules in the IS Model, *Economic Quarterly*, 82(2), 47–75.
- [22] Kocherlakota, N. R. (1990), On Tests of Representative Consumer Asset Pricing Models, *Journal of Monetary Economics*, 26(2), 285–304.
- [23] McNeese, S. K. (1985), Modeling the Fed: A Forward-Looking Monetary Policy Reaction Function, Federal Reserve Bank of Boston *New England Economic Review*, November/December, 3–8.
- [24] McNeese, S. K. (1992), A Forward-Looking Monetary Policy Reaction Function: Continuity and Change, Federal Reserve Bank of Boston *New England Economic Review*, November/December, 3–13.
- [25] Maddala, G. S., and J. Jeong (1992), On the Exact Small Sample Distribution of the Instrumental Variable Estimator, *Econometrica*, 60(1), 181–183.
- [26] Mehra, Y. P. (1999), A Forward-Looking Monetary Policy Reaction Function, Federal Reserve Bank of Richmond, *Economic Quarterly*, 85(2), 33–53.
- [27] Nelson, E. (2000), UK Monetary Policy 1972-97: A Guide using Taylor Rules, Bank of England Working Paper 120.
- [28] Nelson, C. R., and R. Startz (1990), Some Further Results on the Exact Small Sample Properties of the Instrumental Variable Estimator, *Econometrica*, 58(4), 967–976.

- [29] Newey, C. R., and K. D. West (1987), A Simple, Positive Definite, Heteroscedasticity and Autocorrelation Consistent Covariance Matrix, *Econometrica*, 55(3), 703–708.
- [30] Orphanides, A. (1998), Monetary Policy Rules Based on Real Time Data, Board of Governors of the Federal Reserve System, Finance and Economics Discussion Series 1998-03.
- [31] Orphanides, A. (2000), The Quest for Prosperity without Inflation, E.C.B. Working Paper 15.
- [32] Peersman, G., and F. Smets (1998), Uncertainty and the Taylor Rule in a Simple Model of the Euro Area Economy, mimeo presented at the conference "The political economy of fiscal and monetary policy in EMU", Barcelona, December 1998.
- [33] Rudebusch, G. D., and L. E. O. Svensson (1998), Policy Rules for Inflation Targeting, NBER Working Paper 6512.
- [34] Sack, B., and V. Wieland (1999), Interest-Rate Smoothing and Optimal Monetary Policy: A Review of Recent Empirical Evidence, Board of Governors of the Federal Reserve System. Finance and Economics Discussion Series 1999-39.
- [35] Stock, J. H., and J. Wright (2000), GMM with Weak Identification, *Econometrica*, 68(5), 1055–1096.
- [36] Svensson, L. E. O. (1997), Inflation Forecast Targeting: Implementing and Monitoring Inflation Targets, *European Economic Review*, 41(6), 1111–1146.
- [37] Tauchen, G. (1986), Statistical Properties of Generalized Method-of-Moments Estimators of Structural Parameters Obtained from Financial Market Data, *Journal of Business and Economic Statistics*, 4(4), 397–425.
- [38] Taylor, J. B. (1993), Discretion Versus Policy Rules in Practice, *Carnegie-Rochester Conference Series on Public Policy*, 39, 195–214.
- [39] Taylor, J. B. (1999a), An Historical Analysis of Monetary Policy Rules, in J. B. Taylor (ed), *Monetary Policy Rules*, University of Chicago Press, 319–341.
- [40] Taylor, J. B. (1999b), The Robustness and Efficiency of Monetary Policy Rules as Guidelines for Interest Rate Setting by the European Central Bank, *Journal of Monetary Economics*, 43(3), 655–679.
- [41] Woodford, M. (1999), Optimal Monetary Policy Inertia, *Manchester School*, 67, supplement 1999, 1–35.

**Table 1: Parameter estimates using GMM procedures**

Parameter	Panel A: Two-step GMM		Panel B: Iterative GMM		Panel C: Continuous-updated GMM	
	Estimate	Std Err.	Estimate	Std Err.	Estimate	Std Err.
<b><math>L=L^* (L^*=1)</math></b>						
$\beta$	2.510	0.536	3.143	0.526	3.113	0.417
$\gamma$	0.006	0.257	-0.118	0.236	-0.127	0.214
$\alpha$	-0.464	1.595	-2.007	1.640	-1.909	1.270
$\rho$	0.832	0.049	0.844	0.055	0.829	0.065
see	1.303		1.445		1.481	
$J_T$ (stat/p-value)	11.154	0.265	7.410	0.595	7.405	0.595
<b><math>L=4</math></b>						
$\beta$	1.918	0.289	3.077	0.456	3.468	0.603
$\gamma$	0.143	0.210	-0.023	0.232	0.094	0.150
$\alpha$	1.108	1.078	-1.642	1.615	-2.644	2.019
$\rho$	0.808	0.048	0.833	0.052	0.744	0.089
see	1.241		1.468		2.354	
$J_T$ (stat/p-value)	6.864	0.651	7.020	0.635	6.524	0.687

**Table 2: Parameter estimates using ML procedures**

Parameter	Panel A: Two-step ML		Panel B: FIML	
	Estimate	Std Err.	Estimate	Std Err.
<b>Phillips curve</b>				
$\alpha_{\pi 1}$	0.499	0.117	0.482	0.111
$\alpha_{\pi 2}$	0.212	0.127	0.187	0.119
$\alpha_{\pi 3}$	0.268	0.126	0.261	0.119
$\alpha_{\pi 4}$	0.021	-	0.070	-
$\alpha_y$	0.100	0.048	0.116	0.047
see	0.868		0.910	
<b>I-S curve</b>				
$\beta_{y1}$	1.309	0.104	1.196	0.102
$\beta_{y2}$	-0.393	0.103	-0.271	0.099
$\beta_r$	-0.071	0.037	-0.058	0.038
$\beta_0$	3.731	1.155	4.078	1.159
see	0.704		0.753	
<b>Reaction function</b>				
$\beta$	1.668	0.180	1.712	0.190
$\gamma$	-0.059	0.162	-0.090	0.210
$\alpha$	1.607	0.758	1.480	0.793
$\rho$	0.705	0.056	0.708	0.061
see	1.071		1.142	
lnL	-281.908		-280.888	

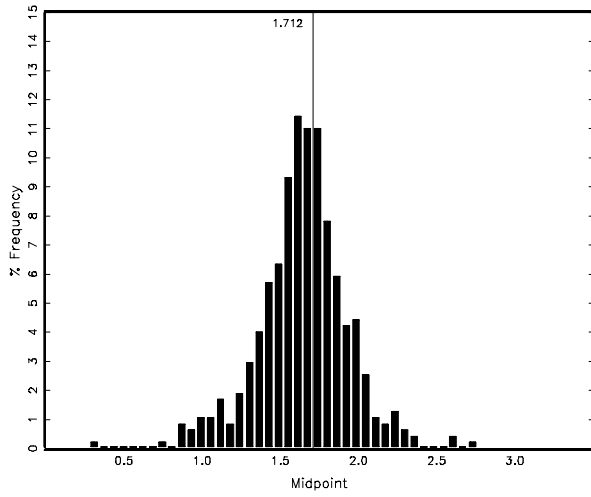
**Table 3: Parameter distribution statistics**  
**( $T=78$  observations,  $N=500$  draws)**

Parameter	Method	Mean	Median	Std dev.	5% CI	10% CI	90% CI	95% CI
$\beta=1.712$	Two-step GMM	1.842	1.790	0.445	1.308	1.457	2.284	2.525
	Iterative GMM	1.817	1.767	0.586	1.133	1.325	2.334	2.623
	Continuous-updating GMM	9.400	1.785	166.441	0.790	1.217	2.704	3.320
	Continuous-updating GMM (truncated)	1.949	1.795	1.521	1.040	1.244	2.685	3.124
	Two-step ML	1.699	1.704	0.213	1.336	1.425	1.930	2.007
	FIML	1.708	1.694	0.217	1.377	1.452	1.967	2.049
$\gamma=-0.090$	Two-step GMM	-0.179	-0.166	0.314	-0.685	-0.549	0.165	0.270
	Iterative GMM	-0.196	-0.178	0.519	-0.893	-0.642	0.238	0.390
	Continuous-updating GMM	8.329	-0.190	195.404	-1.302	-0.904	0.395	0.759
	Continuous-updating GMM (truncated)	-0.292	-0.197	0.989	-1.276	-0.904	0.290	0.516
	Two-step ML	-0.130	-0.137	0.238	-0.510	-0.420	0.178	0.253
	FIML	-0.124	-0.125	0.226	-0.479	-0.410	0.159	0.246
$\rho=0.708$	Two-step GMM	0.659	0.672	0.104	0.463	0.522	0.769	0.791
	Iterative GMM	0.628	0.647	0.136	0.355	0.445	0.783	0.816
	Continuous-updating GMM	-76850.3	0.578	1721461.4	-0.492	-0.077	0.783	0.840
	Continuous-updating GMM (truncated)	0.478	0.579	0.333	-0.257	0.037	0.777	0.810
	Two-step ML	0.642	0.648	0.083	0.490	0.530	0.744	0.771
	FIML	0.639	0.648	0.087	0.476	0.521	0.749	0.768
$\alpha=1.480$	Two-step GMM	2.491	2.036	3.783	-2.111	-0.918	6.507	8.980
	Iterative GMM	2.213	1.804	5.080	-3.989	-1.750	6.955	9.503
	Continuous-updating GMM	68.757	1.993	1449	-6.955	-2.618	10.434	16.104
	Continuous-updating GMM (truncated)	3.526	2.021	15.175	-4.344	-1.974	9.576	13.597
	Two-step ML	1.499	1.503	1.089	-0.199	0.313	2.705	3.209
	FIML	1.449	1.483	1.085	-0.381	0.180	2.568	3.069

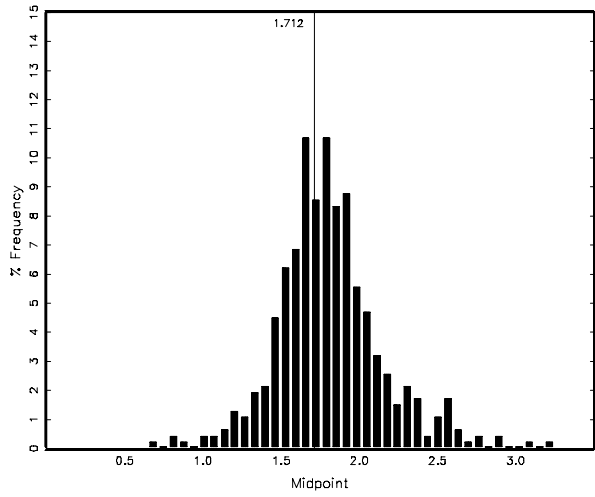
**Table 4: Parameter distribution statistics  
( $T=1500$  observations,  $N=500$  draws)**

Parameter	Method	Mean	Median	Std dev.	5% CI	10% CI	90% CI	95% CI
$\beta=1.712$	Two-step GMM	1.718	1.716	0.034	1.665	1.675	1.761	1.779
	Iterative GMM	1.718	1.716	0.034	1.665	1.675	1.761	1.779
	Continuous-updating GMM	1.718	1.716	0.034	1.665	1.675	1.762	1.779
	Two-step ML	1.713	1.711	0.027	1.668	1.678	1.747	1.758
	FIML	1.711	1.711	0.027	1.667	1.676	1.746	1.758
$\gamma=-0.090$	Two-step GMM	-0.096	-0.096	0.051	-0.176	-0.162	-0.030	-0.014
	Iterative GMM	-0.096	-0.096	0.051	-0.176	-0.163	-0.030	-0.013
	Continuous-updating GMM	-0.098	-0.097	0.051	-0.177	-0.164	-0.031	-0.017
	Two-step ML	-0.089	-0.090	0.045	-0.165	-0.148	-0.034	-0.016
	FIML	-0.092	-0.093	0.046	-0.163	-0.148	-0.033	-0.015
$\rho=0.708$	Two-step GMM	0.706	0.707	0.020	0.672	0.681	0.732	0.736
	Iterative GMM	0.706	0.707	0.020	0.672	0.680	0.732	0.736
	Continuous-updating GMM	0.705	0.706	0.021	0.670	0.678	0.731	0.735
	Two-step ML	0.706	0.706	0.015	0.681	0.686	0.726	0.730
	FIML	0.705	0.705	0.015	0.679	0.685	0.724	0.728
$\alpha=1.480$	Two-step GMM	1.531	1.511	0.294	1.097	1.183	1.914	2.039
	Iterative GMM	1.531	1.510	0.294	1.098	1.181	1.914	2.044
	Continuous-updating GMM	1.535	1.510	0.296	1.094	1.180	1.919	2.053
	Two-step ML	1.474	1.482	0.139	1.241	1.302	1.647	1.731
	FIML	1.478	1.480	0.243	1.097	1.170	1.787	1.909

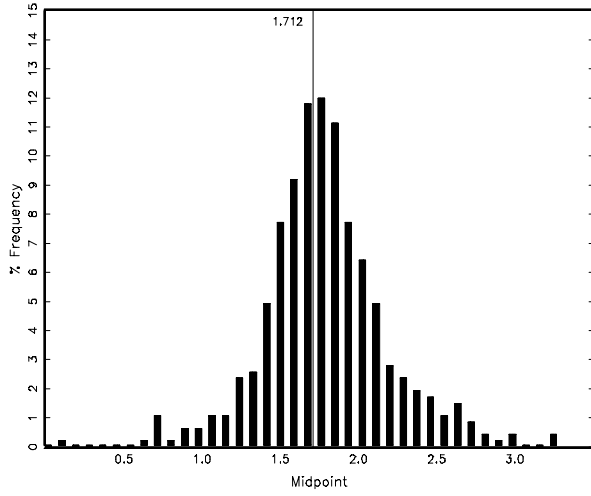
OLS estimator distribution



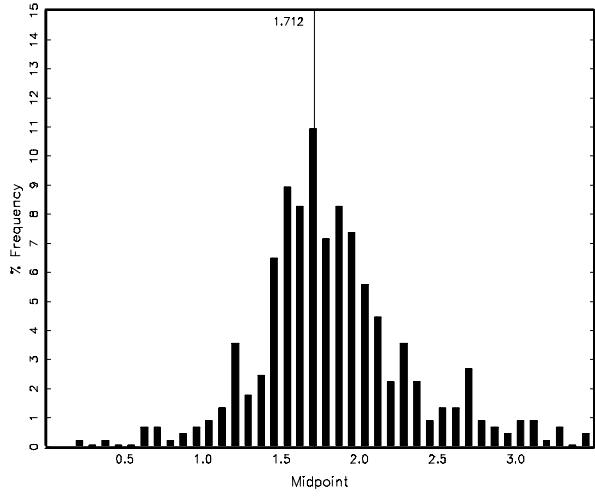
Two-step GMM estimator distribution



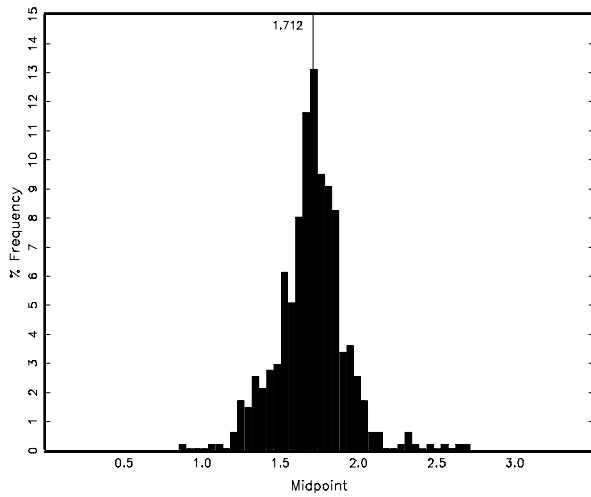
Iterative GMM estimator distribution



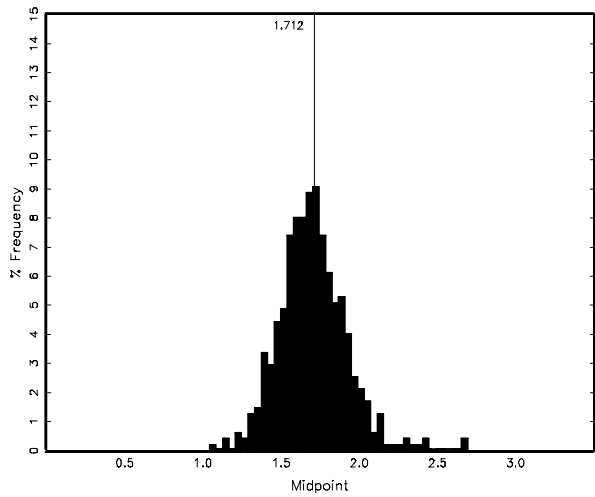
Continuous-updated GMM estimator distribution



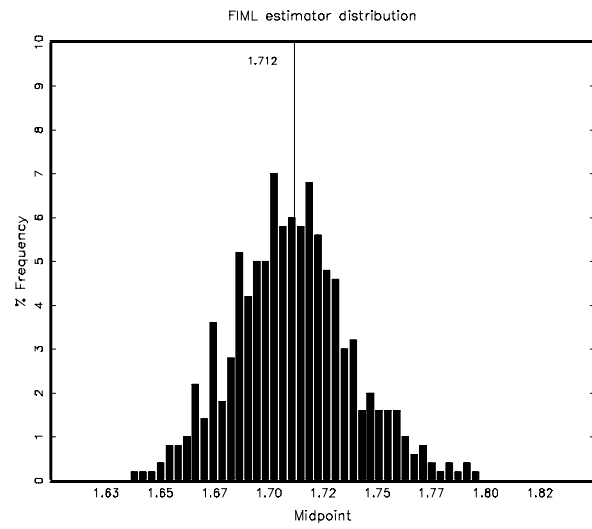
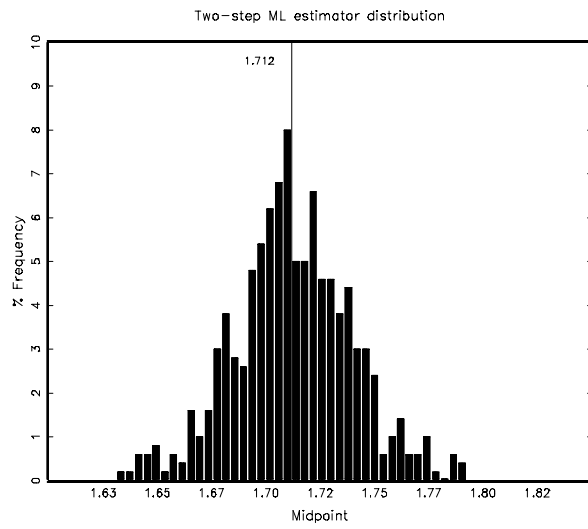
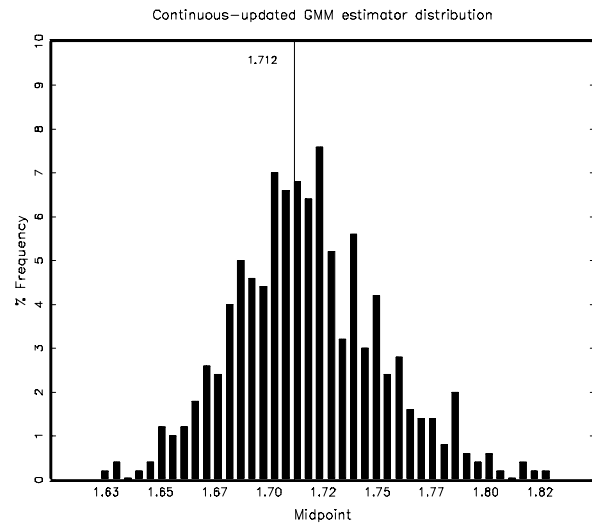
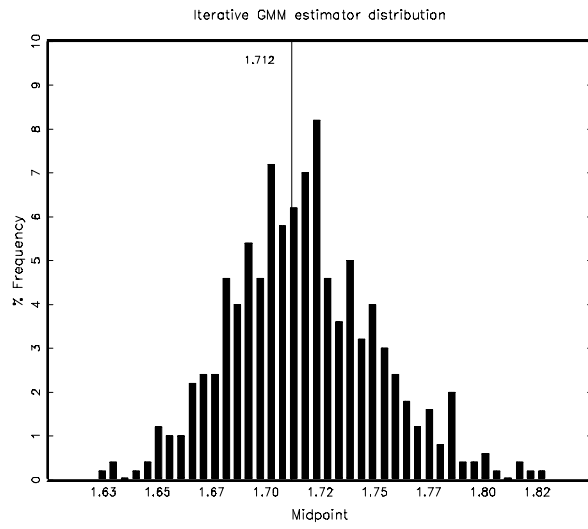
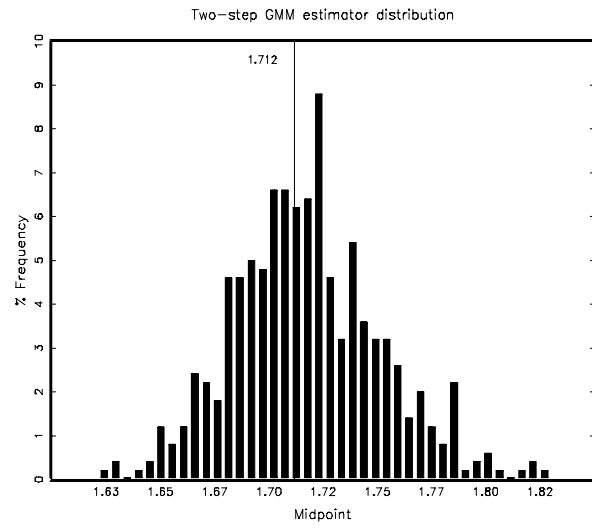
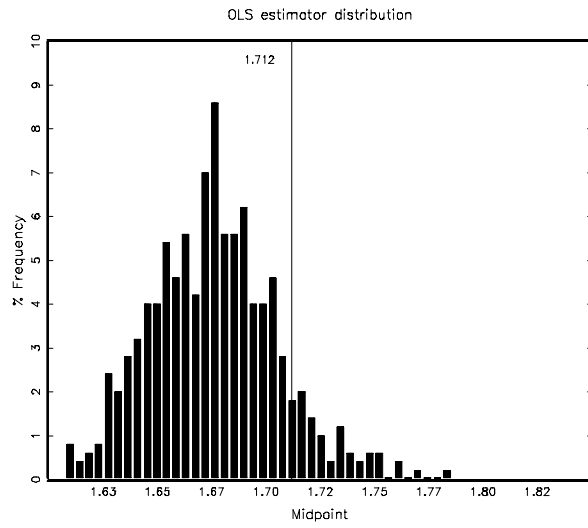
Two-step ML estimator distribution



FIML estimator distribution







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