NOTES D'ÉTUDES

ET DE RECHERCHE

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What is the best approach to measure the interdependence between different markets?

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Abstract: In order to measure the interdependence between different markets, we investigate and compare different measures of dependence including cross-correlation, conditional correlation, concordance and correlation in tails. In the latter case, we use the notion of copula and we define two kinds of diagnoses which enable us to adjust the joint empirical tail distribution in the case of two or three markets for the best copulas. In particular, this approach makes it possible to understand the evolution of the interdependence of more than two markets in the tails, in particular, when extremal values (which correspond to a shock) induce some turmoil in the evolution of the markets.

Keywords: interdependence, conditional correlation, concordance, functions copulas.

Résumé : Quelle est la meilleure mesure du degré d'interdépendance entre les marchés ? Dans cet article, nous analysons et comparons diverses mesures de dépendance (corrélations croisées, corrélations conditionnelles, indices de concordance et corrélations dans les queues de distribution) permettant d'évaluer l'intensité de l'interdépendance entre différents marchés. En outre, à l'aide de la notion de copule, nous proposons deux types d'approche permettant d'ajuster la queue de distribution empirique dans le cas de deux ou trois marchés. On peut ainsi comprendre l'évolution de l'interdépendance dans les queues de distribution de plus de deux marchés lorsque les valeurs extrêmes (correspondant à des chocs) induisent des perturbations sur ces marchés.

Mots-clés: interdependance, corrélation conditionnelle, concordance, fonctions copules.

JEL Classification: C14, C22, G15

1 Introduction

Important issues debated in the literature include the existence of specific patterns characterizing macroeconomic phenomena, in particular the existence of cycles, their interactions in activity as well as existence of phenomena of co-movement in detrended series.

In order to investigate the existence of cycles inside specific series it is necessary to specify the notion of cycles. In two recent papers, Harding and Pagan (1999, 2002) present a methodological investigation into the notion of cycles. They give preference to business cycles, defined in terms of the turning points in the level of economic activity. They debate whether nonlinear models are required to make business cycles. To answer their question it is necessary to examine certain factors such as the duration of cycles and their phases, the amplitude of the cycles and their phases, the asymmetric behaviour of the phases and the cumulative movements within phases. The problem of the stationarity of series is also examined.

Here, rather than investigating the existence of cycles, we shall focus on the existence of the interaction between different markets. Then, we will use our results to analyze the co-movements of cycles between different markets. In order to determine interaction between markets it is necessary to first assess their correlation. But, in the literature, it has often been said that the correlation between national markets changes because the volatility of national markets evolves overtime and also because the interdependence across markets changes. In order to explain these phenomena, we consider different approaches based on the notion of correlation, conditional correlation, concordance and, finally, copula. Using these different measures, it is possible to assess whether or not contagion exists between markets. In all these cases we examine stock market co-movements.

Economists have developed a straightforward approach for measuring contagion across stock markets by comparing the correlation between the stock markets during stable periods to that during a period of turmoil. Then, contagion can be defined as a significant increase in the cross-market correlation during the period of turmoil. This means that if a shock affects one market (causing it to rise for example), it will have ripple effects on the other one (causing it to rise as well). This rise constitutes contagion. Based on this approach, contagion implies that cross-market linkages are fundamentally different after a shock to one market, while interdependence implies no significant change in the cross-market relationship. To examine this phenomenon of contagion between two or more markets we will consider different measures and their accuracy.

The first natural measure of dependence is linear correlation (or the Pearson correlation), but it is a measure of linear dependence. Linear correlation is widely used but is also an often misunderstood measure of dependence. Its popularity stems from the ease with which it can be calculated and it is a natural scalar measure of dependence in elliptical distributions. However, most financial instruments are not jointly elliptically distributed and using linear correlation as a measure of dependence in many situations might give misleading conclusions. On the one hand, if we use scenario using heavy-tailed distributions such as t_2- distributions in our modelisation, then the linear correlation coefficient is not even defined because of infinite second order moments. Finally, this linear measure cannot capture the nonlinear dependence relationships that exist between real data sets. On the other hand, this measure is static and does not take into account the time evolution of the series studied. For more details on the problems relating to the use of the linear correlation, please refer to the paper by Embrechts *et al.* (1999). For these reasons, we will not use the static measure in this paper.

The different measures that we can use to characterize the existence of co-movements between stock markets include:

- cross-correlation;
- conditional correlation;
- concordance;
- correlation in tails.

Since it is now well known that stock returns are serially correlated, see for instance Fama and French (1986) and Poterba and Summers (1987), we will investigate some links between several financial instruments over time. We will particularly focus on contagion from the American market to the French and Japanese markets by considering the returns on their MSCI indices in different periods of time including particular crises that we can interpret as specific shocks (the description of the common stock returns used in this paper can be found in Longin and Solnik (1995)).

First, in Section 3, we use the cross-correlation (time-varying) between two asset prices, assumed to be a stochastic process, to estimate the delay of this contagion and the coefficients that characterize it. We build accurate models of the returns on the MSCI indices of American, French and Japanese markets. These are called transfer function models. In particular, we will focus on periods with a high volatility and we will try to detect the influence of this behaviour using the cross-correlation coefficient. Here, we will discuss some ideas developed by King and Wadhwani (1990) even though we do not use the same approach as them. The details of the computations of the various models are given in the appendix in Section 8. In Section 4, we apply the approach of Boyer *et al.* (1999) on the use of the conditional correlation to detect the existence of switching behaviour inside data sets (and consequently of volatility in the sense that jumps imply volatility): we consider here mixing processes like autoregressive processes and non-mixing processes like long memory processes. Other measures like concordance and correlation in tails will be examined in other sections. This approach will enable us to give some insight into the concordance measure and the copula. In Section 5, we compute the degree of concordance and Kendall's tau, which are concordance measures, between the different markets. These two measures are very useful as they give general information and take into account the existence of non-linear features inside data sets. We specify the links between Kendall's tau and the notion of copula and we recall some important properties of this coefficient. In Section 6, we develop the notion of correlation in the tails. First of all, we show how to compare the empirical tail distribution with different copulas. We specify our diagnosis and illustrate our results using the classical QQ-plot method. Then, on the one hand, we compare the joint empirical tail distribution of the three markets with different copulas, using the notion of dependent copulas in the tails; on the other hand, we explain how we can obtain information on more than two markets, which is very important because the other measures of dependence do not enable us to obtain a similar result.

2 Data sets

The data sets used in this paper consist of the Morgan Stanley Capital International indices (MSCI), daily closing prices for the American market (MSCI-US), the French market (MSCI-FR) and the Japanese market (MSCI-JP), from January 1985 to 31 December 2001 (4435 observations). The data sets were collected from DataStream. The stock market crash of October 1987 generated a large number of reports and commentaries, as did the Asian crisis of 1997 and the Russian crisis of 1998. Since we want to take into account these different crises in our scenario, we investigate the different criteria introduced previously for these different sub-periods. In particular, for the crash of 1987, we consider the periods from the 22 July 1987 to the 13 October 1987 and from the 23 October 1987 to 14 January 1988. Finally, we include the crash by considering the period from the 22 July 1987 to the 14 January 1988. We also determine different sub-periods surrounding the Asian crisis in 1997. We consider the three periods from 25 July 1997 to 16 October 1997 (period before the crisis), from 28 October 1997 to 19 January 1998 (period after the crisis), and then we include the crisis and analyse the period from 25 July 1997 to January 1998. We also study the Russian crisis in 1998 and we consider the following three sub-periods: from 4 June 1998 to 26 August 1998 (period before the crisis), from 8 September to 30 November 1998 (period after the crisis), and from 4 June 1998 to 30 November 1998 (period including the crisis). Changes in logarithms of the MSCI indices i.e. the stocks' returns are used in order to achieve stationarity. More precisely, we assume that these indices follow stochastic processes and we consider the series of their log-returns that we denote $(X_t)_t$ for the American index, $(Y_t)_t$ for the French index and $(Z_t)_t$ for the Japanese index.

In Figure 1, we represent the curve and the empirical distribution of the log returns of the three indices over the full sample period from 1985 to 2001 with a total of 4434 points.

We give the first four empirical moments of these three indices over the full period in Table 1. We note that the empirical skewness is different from zero for each index. So, the data exhibit excess kurtosis compared with the normal distribution.

Series	mean	standard deviation	skewness	kurtosis
X_t	$4.34\ 10^{-4}$	$1.04\ 10^{-2}$	-2.67	59.93
Y_t	$5.27\ 10^{-4}$	$1.23\ 10^{-2}$	-0.37	7.03
Z_t	$1.94\ 10^{-4}$	$1.47\ 10^{-2}$	-0.10	12.69

Table 1: Statistics for the series X_t, Y_t and Z_t (full period 01/01/1985–31/12/2001).

The processes $(X_t)_t$, $(Y_t)_t$ and $(Z_t)_t$ are non Gaussian and follow a Log Laplace distribution, with parameters $a = 1.0005$ and $b = 7.062\ 10^{-3}$ for $(X_t)_t$, $a = 1.0006$ and $b = 9.455\ 10^{-3}$ for $(Y_t)_t$, and $a = 1.0003$ and $b = 1.048\ 10^{-2}$ for $(Z_t)_t$. Note that these distributions were adjusted using the Kolmogorov-Smirnov test with a 95% level.

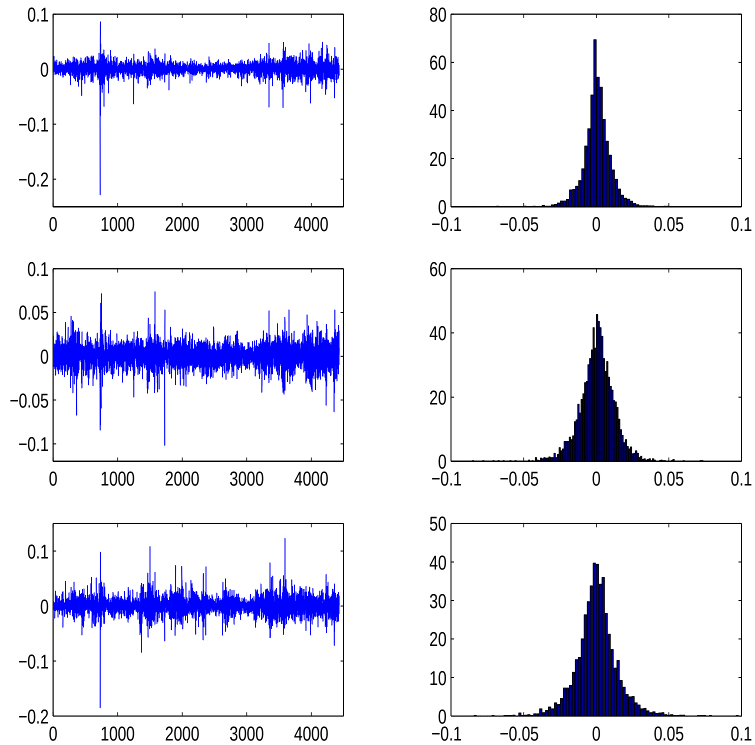

Figure 1: Trajectory and histogram for the log returns of the indices MSCI-US, MSCI-FR and MSCI-JAP during the full period 01/01/1985-31/12/2001

We also compute the empirical linear correlation coefficient ρ (see Table 2) between the three indices: overall, for the different periods considered, we obtain a positive value which confirms the dependence between the three markets.

	$\rho(X_t, Y_t)$	$\rho(X_t, Z_t)$	$\rho(Y_t, Z_t)$
Full period	0.29	0.08	0.26
Oct. 87			
Before	-0.10	-0.05	0.34
After	0.51	0.18	0.30
Including	0.52	0.08	0.30
Asian			
Before	0.26	-0.27	0.24
After	0.47	0.22	0.52
Including	0.32	0.06	0.44
Russian			
Before	0.49	0.22	0.45
After	0.57	0.09	0.34
Including	0.50	0.10	0.35

Table 2: Empirical correlation coefficients between the series X_t, Y_t and Z_t.

The values that we obtain are consistent with the idea that a form of contagion exists between these three processes. Analyzing in detail these correlations, we observe that in most cases, after each shock (crisis in 1987 or the two crises in 1997 and 1998), the values of the coefficient of correlation are higher than before the shock. We also observe a particular relationship between the American and the Asian markets: the correlation being very weak over the full period, then we can say that the two markets are uncorrelated; during the period of crash, in October 1987, this relationship is unchanged, both markets continue to behave in the same way.

To consider in greater depth the behaviour which governs these three markets, we will study the existence of an recursive cross-correlation over the full period and the sub-periods corresponding to the three crises.

3 The transfer function between the three markets: American, French and Japanese

First of all, we shall define the notion of cross-correlation used in this section. Then we will apply it to the three relevant data sets of interest over the full period and over different sub-periods. We analyse our results in terms of whether volatility is finite or not.

We will begin by specifing the general framework. Given two processes $(X_t)_t$ and $(Y_t)_t$, we can consider the bivariate process $(Z_t)_t$ defined for each t by $Z_t = (X_t, Y_t)^T$ whose components are respectively X_t and Y_t and compute the covariance matrix of this process defined by $\Gamma(h) = E[Z_{t+h}Z_t] = [\gamma_{ij}(h)]_{i,j=1,2}$, where the $\gamma_{12}(h)$'s and $\gamma_{21}(h)$'s represent the cross-covariance coefficients between the processes $(X_t)_t$ and $(Y_t)_t$. A^T denotes the transposition of the matrix A. When $i = j = 1$, we obtain the correlation function of the process $(X_t)_t$ and when $i = j = 2$, the correlation function of the process $(Y_t)_t$. We assume here that the two processes are centered. In fact, we will use the correlation matrix defined by $R(h) = [\rho_{ij}(h)]_{i,j=1,2}$, with

$$\rho_{ij}(h) = \frac{\gamma_{ij}(h)}{[\gamma_{ii}(0)\gamma_{jj}(0)]^{1/2}}.$$

$>$From a sample data set $Z_1, \cdots, Z_n$ of length n, a natural estimator of the covariance matrix $\Gamma(h)$ is given by:

$$\hat{\Gamma}(h) = n^{-1}\sum_{t=1}^{n-h}(Z_{t+h} - \overline{Z}_n)(Z_t - \overline{Z}_n)^T = [\hat{\gamma}_{ij}(h)]_{i,j=1,2}, \quad 0 \leq h \leq n-1.$$

The statistic $\overline{Z}_n = n^{-1}\sum_{t=1}^{n} Z_t$ denotes the vector of sample means. Then, we can estimate the coefficients of the correlation matrix by

$$\hat{\rho}_{ij}(h) = \frac{\hat{\gamma}_{ij}(h)}{[\hat{\gamma}_{ii}(0)\hat{\gamma}_{jj}(0)]^{1/2}}.$$

In general, deriving the large sample properties of $\hat{\gamma}_{ij}(h)$ and of $\hat{\rho}_{ij}(h)$ is quite complicated. Here, we are interested in testing the independence of the two component series. We will use the following result, see for instance Brockwell and Davis (1996).

Theorem 3.1 *Let $(Z_t)_t$ be the bivariate time series whose components are defined by:*

$$X_t = \sum_{k=-\infty}^{\infty} \alpha_k \varepsilon_{t-k},$$

where $(\varepsilon_t)_t$ is a white noise process $(0, \sigma_\varepsilon^2)$, and

$$Y_t = \sum_{k=-\infty}^{\infty} \beta_k \eta_{t-k},$$

where $(\eta_t)_t$ is a white noise process $(0, \sigma_\eta^2)$. The two sequences $(\varepsilon_t)_t$ and $(\eta_t)_t$ are independent, $\sum_{k=-\infty}^{\infty} |\alpha_k| < \infty$ and $\sum_{k=-\infty}^{\infty} |\beta_k| < \infty$.

Then if $h \geq 0$, $\hat{\rho}_{12}(h)$ is asymptotically normal with mean 0 and variance $n^{-1} \sum_{j=-\infty}^{\infty} \rho_{11}(j)\rho_{22}(j)$.

This theorem is useful in testing for correlation between two time series. If one of the two processes in Theorem 3.1 is a white noise, then it follows at once from the theorem that $\hat{\rho}_{12}(h)$ is asymptotically normally distributed with mean zero and variance n^{-1}, in which case it is straightforward to test the hypothesis that $\rho_{12}(h) = 0$. However, if neither process is white noise, then a value of $|\hat{\rho}_{12}(h)|$ which is large relative to $n^{-1/2}$ does not necessarily indicate that $\rho_{12}(h)$ is different from zero.

Since by Theorem 3.1 the large sample distribution of $\hat{\rho}_{12}(h)$ depends on both $\hat{\rho_{11}}(.)$ and $\hat{\rho_{22}}(.)$, any test for independence of the two components (X_t and Y_t) cannot be based solely on $\hat{\rho}_{12}(h), h \in \mathbb{Z}$, without taking into account the nature of the two components. This difficulty can be circumvented by "prewhitening" the two series before computing the cross correlations $\hat{\rho}_{12}(h)$, i.e. by transforming the two series to white noise by application of suitable filters. In the following, we will follow this procedure.

To make the interpretation as clear as possible, we will not provide all the models that we successively adjusted for the different processes: those are available from the authors on request. We will now describe the different steps we carry out for the two processes $(X_t)_t$ and $(Y_t)_t$.

- First step: we adjust (when it is possible) an AR(p) process for the two processes $(X_t)_t$ and $(Y_t)_t$. We denote respectively ε_t and η_t the noises which appear in the AR(p) representations.
- Second step: assuming that ε_t and η_t are white noises, we compute the cross-correlation between these two processes and we derive a model which makes possible to explain ε_t with respect to η_t. We call e_t^1 the noise which appears in this linear representation.
- Third step: If the noise e_t^1 appears to be non-white, we adjust an AR(p) for it and then we obtain a new noise called e_t^2. If e_t^2 is white, then the procedure stops, otherwise, return to the beginning of the third step.
- Fourth step: using all the previous models, we derive a model which explains the behaviour of process $(Y_t)_t$ with respect to process $(X_t)_t$

and noise $(e_t^2)_t$ (the last noise we use in our procedure). This allows us to build the transfer function between the two processes $(Y_t)_t$ and $(X_t)_t$.

We will now provide the results of this exercice.

3.1 The Full period: transfer function model between the three markets

- The transfer function model between $(X_t)_t$ and $(Y_t)_t$ is:

$$Y_t = 0.34X_t + 0.26X_{t-1} + 0.03X_{t-2} + 0.02X_{t-3} + 0.01X_{t-4} + e_t^2.$$

- The transfer function model between $(X_t)_t$ and $(Z_t)_t$ is:

$$Z_t = 0.36X_{t-1} + 0.02X_{t-3} + 0.01X_{t-4} - 0.02X_{t-7} + e_t^1 - 0.05e_{t-6}^1.$$

- The transfer function model between $(Y_t)_t$ and $(Z_t)_t$ is:

$$Z_t = 0.30Y_t + 0.16Y_{t-1} - 0.01Y_{t-2} - 0.02Y_{t-6} - 0.01Y_{t-7} + e_t^1 - 0.05e_{t-6}^1.$$

All the coefficients that appear are significant. The French index seems more strongly dependent vis-à-vis the American index than the Japanese one. Nevertheless, the first coefficient explaining the contagion during the current day is slightly identical in all models.

3.2 The period surrounding the October 1987 crisis

3.2.1 transfer function model between $(X_t)_t$ and $(Y_t)_t$

- Before the crash (22/07/1987 - 13/10/1987 (60 points)):

$$\begin{aligned} Y_t = {} & 0.28X_{t-2} + 0.12X_{t-3} - 0.05X_{t-4} + 0.03X_{t-5} + 0.01X_{t-6} \\ & +0.08X_{t-7} + 0.04X_{t-8} - 0.11X_{t-9} - 0.04X_{t-10} + 0.02X_{t-11} \\ & +e_t^1 + 0.44e_{t-1}^1 - 0.19e_{t-2}^1 + 0.09e_{t-3}^1 + 0.04e_{t-4}^1 + 0.01e_{t-5}^1. \end{aligned}$$

- After the crash (23/10/1987 - 14/01/1988 (60 points)):

$$Y_t = 0.55X_t + e_t^1. \quad (1)$$

- Including the crash (22/07/1987 - 14/01/1988 (127 points)):

$$Y_t = 0.37X_t + e_t^1. \quad (2)$$

We obtain a very simple model on the two last periods (including the crash or after it). In these two cases, the impact of the American market on the French one depends only on the same-day transactions. The French returns do not depend on the lagged values of $(X_t)_t$. The contagion is almost instantaneous and quite strong (after the crash, $a_0 = 0.55$).

3.2.2 transfer function model between $(X_t)_t$ and $(Z_t)_t$

• Before the crash (22/07/1987 - 13/10/1987 (60 points)): the two data sets seem to have an independent evolution since it is not possible to adjust a regression model between them.
• After the crash (23/10/1987 - 14/01/1988 (60 points)):

$$\begin{aligned} Z_t &= 0.20X_{t-1} - 0.06X_{t-3} + 0.02X_{t-5} \\ &\quad + e^1_t - 0.31e^1_{t-2} + 0.10e^1_{t-4} - 0.03e^1_{t-6}. \end{aligned} \tag{3}$$

• Including the crash (22/07/1987 - 14/01/1988 (127 points)):

$$Z_t = 0.58X_{t-1} + e^2_t - 0.21e^2_{t-1} + 0.04e^2_{t-2} - 0.01e^2_{t-3}.$$

The crash has created a new interaction between these two markets: this can be seen when we compare their behaviour before the crash and after it.

3.2.3 transfer function model between $(Y_t)_t$ and $(Z_t)_t$

• Before the crash (22/07/1987 - 13/10/1987 (60 points)):

$$Z_t = 0.46Y_t - 0.20Y_{t-1} + e^1_t.$$

• After the crash (23/10/1987 - 14/01/1988 (60 points)):

$$\begin{aligned} Z_t &= 0.20Y_t + 0.29Y_{t-1} - 0.06Y_{t-2} - 0.09Y_{t-3} + 0.02Y_{t-4} + 0.03Y_{t-5} \\ &\quad + e^1_t - 0.31e^1_{t-2} + 0.10e^1_{t-4} - 0.03e^1_{t-6}. \end{aligned}$$

• Including the crash (22/07/1987 - 14/01/1988 (127 points)):

$$Z_t = 0.38Y_t + 0.50Y_{t-1} + e^1_t. \tag{4}$$

The relationship between the Asian market and the French market is quite different from that observed between the French and the American markets. Indeed, before the crash, the two markets (Japanese and French) displayed the same pattern. Now, comparing (2) and (4) we note that after the crash the evolution between the three markets is slightly similar: the impact of the shock seems non-negligible.

It seems that the variation generated by this crisis resulted in greater disruption between the Asian and the American markets than between the others. Indeed, the Asian and the American markets appeared independent before the crash, and they appeared to be correlated after the crash.

3.3 The period surrounding the Asian crisis of 1997

3.3.1 transfer function model between $(X_t)_t$ and $(Y_t)_t$

- Before the crisis (25/07/1997 - 16/10/1997 (60 points)):

$$Y_t = 0.36X_{t-1} + e_t^1.$$

- After the crisis (28/10/1997 - 19/01/1998 (60 points)):

$$Y_t = 0.52X_t + 0.33X_{t-1} + e_t^1. \tag{5}$$

- Including the crisis (25/07/1997 - 19/01/1998 (127 points)):

$$Y_t = 0.35X_t + 0.50X_{t-1} - 0.10X_{t-10} - 0.14X_{t-11} + e_t^1.$$

After the crisis the French market and the American markets were correlated but this phenomenon disappears when we take into account the period including the crisis.

3.3.2 transfer function model between $(X_t)_t$ and $(Z_t)_t$

- Before the crisis (25/07/1997 - 16/10/1997 (60 points)):

$$Z_t = 0.58X_{t-1} + e_t^1.$$

- After the crisis (28/10/1997 - 19/01/1998 (60 points)):

$$\begin{aligned} Z_t &= 0.78X_{t-1} - 0.24X_{t-3} + 0.08X_{t-5} - 0.02X_{t-7} \\ &\quad + e_t^1 - 0.31e_{t-2}^1 + 0.10e_{t-4}^1 - 0.03e_{t-6}^1. \end{aligned} \tag{6}$$

- Including the crisis (25/07/1997 - 19/01/1998 (127 points)):

$$\begin{aligned} Z_t &= 0.56X_{t-1} - 0.15X_{t-3} + 0.04X_{t-5} - 0.01X_{t-7} + 0.16X_{t-11} - 0.04X_{t-13} \\ &\quad + e_t^1 - 0.26e_{t-2}^1 + 0.07e_{t-4}^1 - 0.02e_{t-6}^1. \end{aligned}$$

The two markets appear quite stable during this crisis. Here, there is some evidence of correlation between the two markets for the different periods under consideration.

3.3.3 transfer function model between $(Y_t)_t$ and $(Z_t)_t$

- Before the crisis (25/07/1997 - 16/10/1997 (60 points)):

$$Z_t = 0.35Y_{t-1} + e_t^1.$$

- After the crisis (28/10/1997 - 19/01/1998 (60 points)):

$$\begin{aligned} Z_t &= 1.01Y_t - 0.31Y_{t-2} + 0.10Y_{t-4} - 0.03Y_{t-6} + 0.01Y_{t-8} \\ &\quad + e_t^1 - 0.31e_{t-2}^1 + 0.10e_{t-4}^1 - 0.03e_{t-6}^1 + 0.01e_{t-8}^1. \end{aligned}$$

- Including the crisis (25/07/1997 - 19/01/1998 (127 points)):

$$\begin{aligned} Z_t &= 0.70Y_t - 0.18Y_{t-2} + 0.05Y_{t-4} - 0.01Y_{t-6} + \\ &\quad + e_t^1 - 0.26e_{t-2}^1 + 0.07e_{t-4}^1 - 0.02e_{t-6}^1. \end{aligned}$$

After the crisis, the Asian market is strongly influenced by the French one. We note this stronger influence for all the periods we examined for these three data sets. When we introduce the crisis, the influence of the French market persists.

3.4 The period surrounding the Russian crisis of 1998

3.4.1 transfer function model between $(X_t)_t$ and $(Y_t)_t$

- Before the crisis (04/06/1998 - 26/08/1998 (60 points)):

$$Y_t = 0.67X_t - 0.21X_{t-8} + e_t^1.$$

- After the crisis (08/09/1998 - 30/11/1998 (60 points)):

$$Y_t = 0.74X_t + e_t^1.$$

- Including the crisis (04/06/1998 - 30/11/1998 (127 points)):

$$Y_t = 0.55X_t + e_t^1.$$

It seems that this crisis did not change the link between the two markets.

3.4.2 transfer function model between $(X_t)_t$ and $(Z_t)_t$

For the three sub-periods: before the crisis (04/06/1998 - 26/08/1998 (60 points)), after the crisis (08/09/1998 - 30/11/1998 (60 points)) and with the sub-period including the crisis (04/06/1998 - 30/11/1998 (127 points)), it is not possible to adjust a regression model: the coefficients are not significantly different from zero. The two markets seem "independent". We do not observe a contagion phenomenon.

3.4.3 transfer function model between $(Y_t)_t$ and $(Z_t)_t$

- Before the crisis (04/06/1998 - 26/08/1998 (60 points)):

$$\begin{aligned} Z_t &= 0.61Y_t + 0.19Y_{t-1} - 0.06Y_{t-2} + 0.02Y_{t-3} \\ &\quad + e_t^1 + 0.31e_{t-1}^1 - 0.10e_{t-2}^1 + 0.03e_{t-3}^1. \end{aligned}$$

- After the crisis (08/09/1998 - 30/11/1998 (60 points)):

$$\begin{aligned} Z_t &= 0.38Y_t + 0.40Y_{t-1} - 0.11Y_{t-5} - 0.12Y_{t-6} + 0.03Y_{t-10} + 0.03Y_{t-11} \\ &\quad + e_t^2 - 0.31e_{t-4}^2 - 0.29e_{t-5}^2 + 0.10e_{t-8}^2 + 0.09e_{t-9}^2. \end{aligned}$$

- Including the crisis (04/06/1998 - 30/11/1998 (127 points)):

$$Z_t = 0.44Y_t + 0.38Y_{t-1} + e_t^1.$$

The crisis did not amplify the phenomenon of contagion between the French and the Asian markets. The relationship between the two markets remains similar (same models) during the three periods.

3.5 Some additional remarks

In order to try to explain why markets around the world fall simultaneously, we can consider for instance the different models given in (1), (3), (5) and (6). In all cases, there is an amplification of the relationship between the markets which are in competition. We observe that the cross-correlation structures between these returns change before and after the crises, and they also vary when we compare them over the long period. There is also some evidence of an increase in the correlation between the three markets after the different crises (see Table 2). Moreover, we note a significant change in the volatility for the French index after the crisis and during the crisis, but this kind of jump is not so significant when we use cross-correlations. In this case, the volatility seems self-sustained. The positive coefficients of these transfer function models explain the strong cross-correlations between the markets and demonstrate that the contagion is strong the same day and with a lag of one day. Besides, we also observe some asymmetry in the information process with the model that we adjust. The question to be considered is whether false information is produced as we know that this situation produces some volatility.

Series	full period	Before Oct. 87,	After Oct. 87
X_t	$1.04\ 10^{-2}$	$1.03\ 10^{-2}$	$2.29\ 10^{-2}$
Y_t	$1.23\ 10^{-2}$	$9.78\ 10^{-3}$	$2.50\ 10^{-2}$
Z_t	$1.47\ 10^{-2}$	$1.35\ 10^{-2}$	$1.74\ 10^{-2}$

Table 3: Standard deviation for the series X_t, Y_t, Z_t, over the full period and surrounding the crisis of 1987.

Table 3 shows that index volatility exists for each and increases under the impact of the crash, compared with its level for the full period. The following question now arises: does the existence of contagion between two markets increase the volatility or vice versa, i.e. does the change in the volatility of two markets imply a contagion phenomenon between them, inducing high positive coefficients in their transfer function? If we decide to measure the contagion by means of cross-correlations assessed between the different data sets for different periods, it then seems that when the contagion appears stronger, the volatility that characterizes the data sets increases also. However, it is important to recall that the empirical variance one of the measure of the volatility, appears in the computation of both the correlation and cross-correlation coefficients: thus it is very difficult to establish which factor influences the other. We can only observe that the correlation coefficient is positive and is consistent with the framework of contagion model.

The empirical cross-correlation between two markets can differ relative to the sub-periods considered (for instance, a smooth period or a period of turmoil): when this difference is detected it is called "correlation breakdown" in the literature. However, this is not the characteristic of switching in a model. Indeed, it is possible to show that the data drawn from a stationary process (which implies that the correlation coefficients are constant) can show the same relationship.

In order to illustrate this phenomenon, we now use another approach to try to measure the contagion between the markets when there is some volatility and if there is some evidence that price jumps have occurred during the periods. This approach focuses on the concept of conditional correlation which allows us to identify the presence of volatility.

4 Conditional correlation relative to the deciles of the distribution of returns

We have observed previously that during periods of high market volatility, correlations between asset prices can differ substantially from those seen in smoother markets. Such differences in correlations have been attributed either to structural breaks in the underlying distribution of returns or to "contagion" across markets that occurs only during periods of market turbulence (here we will try to ascertain whether these differences only reflect time-varying sampling volatility). We will now analyse the distribution of returns and compare them with a stationary distribution. Indeed, it is possible to observe changes in the correlations even when the distribution is stationary: thus, those changes cannot be attributed to the presence of high volatility! This means that if we observe an increase in the sampling correlations (like in the previous section), this does not necessarily mean that there is contagion between the two markets. This behaviour may be the result of high volatility within the data set. This also means that the change in the correlations does not imply the presence of a structural break in the data. The important question is to determine whether a change in the correlations provides any information or not. In this section we use the conditional correlation to try to understand this problem.

4.1 Presentation of the method

To better understand whether the increase in the volatility of returns varies together with an increase in sampling correlations even when the true correlations are constant, we will consider some data sets obtained from a stationary process (thus their correlation is constant over the period under consideration), and we will compute the conditional correlation relative to a specific information set. The choice of this information set can be used to characterize, on a market, the periods of calm and of turmoil. First, we explain the method theoretically, then we show how we can use it empirically.

Given two correlated Gaussian random variables X and Y, we denote ρ the non-null correlation between these two random variables. We assume that (X, Y) follows a bivariate Gaussian distribution with Gaussian margins. For a set A, we can compute the conditional correlation ρ_A between X and Y conditionally to an event $X \in A$ relative to ρ and we obtain:

$$\mathrm{corr}(X, Y|X \in A) = \rho_A = \rho \left(\rho^2 + (1-\rho^2) \frac{\mathrm{var}(X)}{\mathrm{var}(X|X \in A)} \right)^{-1/2}. \quad (7)$$

For different values of ρ, we can compute analytically this conditional correlation as soon as we specify the sets A. In the following, we will use the sets defined by the deciles of the Gaussian distribution of X. It is quite natural that the variances of the points which belong to the first decile set ($\text{var}(X|X \in D_1)$) and to the last decile set ($\text{var}(X|X \in D_{10})$) are higher than the others, because we are considering the tails of the distributions. The variances in the central decile sets ($\text{var}(X|X \in D_5)$ and $\text{var}(X|X \in D_6)$) are smaller. For fixed values of ρ, the relationship between the deciles and the conditional correlations $\text{corr}(X, Y|X \in D_i)$ are "U-shaped", which means that the higher the variance, the higher the conditional correlation and vice versa. Note that the relationship (7) is accurate if the random variables X and Y are Gaussian.

This approach can be generalized for a couple of random vectors $\mathbf{X} \in \mathbb{R}^n$ and $\mathbf{Y} \in \mathbb{R}^n$. If we denote $\Sigma_{\mathbf{XY}} = \text{cov}(\mathbf{X}, \mathbf{Y})$ the unconditional covariance and $\Sigma_{\mathbf{XY}|A}$ the conditional covariance relative to an event A, then the average correlation between $\mathbf{X}$ and $\mathbf{Y}$ may be defined by:

$$\rho = \frac{\text{tr}(\sum_{\mathbf{XY}})}{\sqrt{\text{tr}(\sum_{\mathbf{XX}})\text{tr}(\sum_{\mathbf{YY}})}}$$

where tr(.) is the trace operator and the corresponding conditional correlation relative to an event A is given by:

$$\rho_A = \frac{\text{tr}(\sum_{\mathbf{XY|A}})}{\sqrt{\text{tr}(\sum_{\mathbf{XX|A}})\text{tr}(\sum_{\mathbf{YY|A}})}}.$$

Then it is possible, in the vectorial setting up, to derive a similar formula between ρ and ρ_A, as in (7), using the following relationship: $\sum_{\mathbf{XY}|A} = \sum_{\mathbf{XY}} \sum_{\mathbf{XX}}^{-1} \sum_{\mathbf{XX}|A}$.

Now, to apply these results to different data sets $X_1, X_2, \cdots X_n$ and $Y_1, Y_2, \cdots Y_n$, we estimate the coefficients ρ and ρ_A by their empirical expression. For instance, for ρ_A, we use:

$$\hat{\rho}_A = \frac{\sum_{i \in A}(X_i - \overline{X}_n^A)(Y_i - \overline{Y}_n^A)}{\sqrt{\sum_{i \in A}(X_i - \overline{X}_n^A)^2 \sum_{i \in A}(Y_i - \overline{Y}_n^A)^2}} \tag{8}$$

where $\overline{X}_n^A$ represents the empirical mean in the set A. note that a similar expression is used in order to evaluate ρ.

4.2 Conditional correlation between $(X_t)_t$ and $(Y_t)_t$ over the full period 01/01/1985-31/12/2001

In this section we set out to compute (8) for our data sets, with a view to understanding the behaviour of their conditional correlation relative to different data sets representing the fluctuation in volatility. These results will be compared with the accurate computation (7) obtained from a bivariate Gaussian framework.

We use two of the three data sets previously investigated, i.e. the log returns of the MSCI-US index, denoted $(X_t)_t$, and of the MSCI-FR index, denoted $(Y_t)_t$. In Section 3, we showed that these two processes are dependent and in Section 2 we noted that they follow logLaplace distributions. In order to see if the independence between two series plays a fundamental role in the computation of the coefficient of the conditional correlation (the empirical formula (8) does not take account of the series' model), we compute this coefficient for the whitened series of our data sets initially, and subsequently, for the original series.

First as regards, the residuals, we use the following notation in this paragraph: $(\varepsilon_t)_t$ and $(\eta_t)_t$ denote the whitened series relative to $(X_t)_t$ and $(Y_t)_t$ defined in Section 3. In Table 4, we give the empirical values for the conditional correlations between $(\varepsilon_t)_t$ and $(\eta_t)_t$ relative to the deciles of the empirical law of $(\varepsilon_t)_t$. The correlation coefficient between these two processes is equal to $\hat{\rho} = 0.294$.

Decile	Interval	$\mathrm{Corr}(X, Y \| X \in D_i)$	90% Confidence Interval
1	$-22.187 < x \leq -1.017$	0.303	(0.056, 0.267)
2	$-1.017 < x \leq -0.567$	0.074	(-0.058, 0.135)
3	$-0.567 < x \leq -0.314$	0.097	(-0.070, 0.118)
4	$-0.314 < x \leq -0.127$	0.029	(-0.078, 0.104)
5	$-0.127 < x \leq 0.005$	-0.033	(-0.080, 0.098)
6	$0.005 < x \leq 0.164$	0.005	(-0.080, 0.098)
7	$0.164 < x \leq 0.363$	0.041	(-0.078, 0.104)
8	$0.363 < x \leq 0.617$	0.034	(-0.070, 0.118)
9	$0.617 < x \leq 1.047$	0.037	(-0.058, 0.135)
10	$1.047 < x \leq 6.920$	0.196	(0.056, 0.267)

Table 4: Conditional correlations between $X = (\varepsilon_t)_t$ and $Y = (\eta_t)_t$ ($\hat{\rho} = 0.294$).

In order to compare the results obtained here with the theory, we give, in Table 5, the theoretical conditional correlations computed from a stationary i.i.d. bivariate Gaussian distribution with the same correlation coefficient $\rho = 0.294$, using (7). In Figure 2, we give the representation of the conditional correlation calculated in Table 5. We obtain a U-curve form which shows that the higher the variance, the higher the conditional correlation. Although the correlation is constant because of the stationarity, we show that the conditional correlation is not constant. We recall that this conditioning is linked to the volatility of the process.

Decile	Interval	$\text{Var}(X \mid X \in D_i)$	$\text{Corr}(X, Y \mid X \in D_i)$
1	$-\infty < x \leq -1.282$	0.169	0.126
2	$-1.282 < x \leq -0.842$	0.016	0.039
3	$-0.842 < x \leq -0.524$	0.008	0.028
4	$-0.524 < x \leq -0.253$	0.006	0.024
5	$-0.253 < x \leq 0$	0.005	0.022
6	$0 < x \leq 0.253$	0.005	0.022
7	$0.253 < x \leq 0.524$	0.006	0.024
8	$0.524 < x \leq 0.842$	0.008	0.028
9	$0.842 < x \leq 1.282$	0.016	0.039
10	$1.282 < x \leq +\infty$	0.169	0.126

Table 5: Variances and conditional correlations for a bivariate Gaussian vector with $\rho = 0.294$.

Thus, a U-shaped pattern need not indicate a correlation breakdown, but may instead merely be a consequence of the "ex post" partitioning of the data, here, into deciles. The differences between the conditional correlations are caused by the choice of the sub-samples alone and not by any change in the parameters of the data generating process.

In order to compare the theoretical results (Table 5) with the empirical results (Table 4), we provide a 90% confidence interval for the theoretical conditional correlation, in Table 4. All these confidence intervals have been

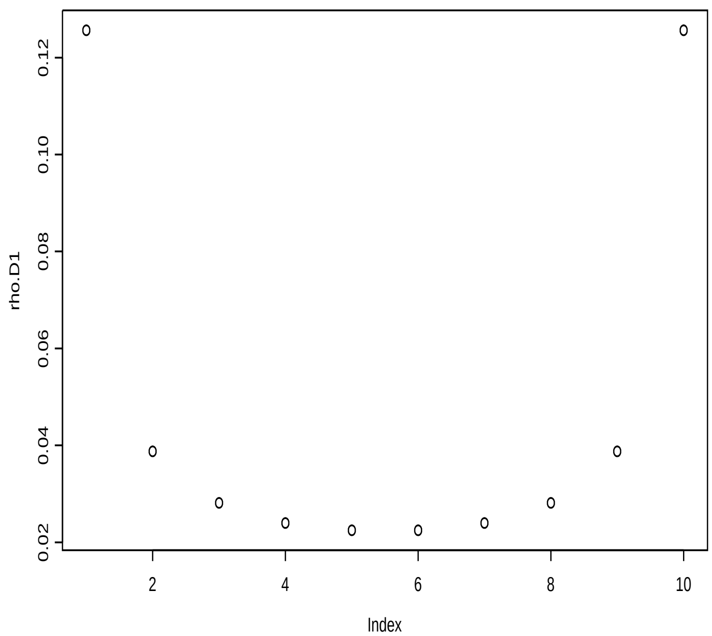

Figure 2: U-curve of the conditional correlation relative to the decile sets obtained from the bivariate Gaussian distribution with $\rho = 0.294$

obtained by means of Monte Carlo methods by simulating $s = 100$ realizations of length $n = 4434$ of a bivariate Gaussian process with theoretical correlation coefficient $\rho = 0.294$. From the results, we note that the empirical and theoretical conditional correlations follow virtually the same U-shaped pattern. The empirical conditional correlation is outside the 90% confidence interval for normally distributed data only once in decile 1. Thus, the previous remarks concerning the behaviour of the conditional correlations can be applied to these residual data sets $(\varepsilon_t)_t$ and $(\eta_t)_t$.

Now, we are interested in investigating this approach for the *observed* log returns of the MSCI-US index denoted $(X_t)_t$ and the *observed* log returns of the MSCI-FR index denoted $(Y_t)_t$. These two data sets are not i.i.d. and their empirical correlation coefficient is $\hat{\rho} = 0.295$. We give, in Table 6, the theoretical conditional correlations obtained using an i.i.d. bivariate Gaussian distribution with correlation coefficient $\rho = 0.295$. In Table 7, we present the empirical conditional correlations by empirical deciles between the two series

Decile	Interval	$\mathrm{Var}(X\|X \in D_i)$	$\mathrm{Corr}(X,Y\|X \in D_i)$
1	$-\infty < x \leq -1.282$	0.169	0.126
2	$-1.282 < x \leq -0.842$	0.016	0.039
3	$-0.842 < x \leq -0.524$	0.008	0.028
4	$-0.524 < x \leq -0.253$	0.006	0.024
5	$-0.253 < x \leq 0$	0.005	0.023
6	$0 < x \leq 0.253$	0.005	0.023
7	$0.253 < x \leq 0.524$	0.006	0.024
8	$0.524 < x \leq 0.842$	0.008	0.028
9	$0.842 < x \leq 1.282$	0.016	0.039
10	$1.282 < x \leq +\infty$	0.169	0.126

Table 6: Variances and conditional correlations for a bivariate Gaussian process ($\rho = 0.295$).

$(X_t)_t$ and $(Y_t)_t$, and we also provide 90% confidence intervals for the theoretical conditional correlations under the assumption of bivariate normality. Again, the results suggest that the empirical and theoretical conditional correlations are quite similar. The empirical conditional correlations are outside the 90% confidence interval only for the first decile as in the previous case.

Correlation breakdowns are still observed when taking into account the deciles even when the true data generating process has a constant correlation coefficient. Then, these features are not only true in the i.i.d. case but also for non-i.i.d. mixing processes as in the case of our series. This justifies our dealing with the returns and not the whitened series.

Decile	Interval	Corr($X, Y \| X \in D_i$)	90% Confidence Interval
1	$-21.919 < x \leq -1.016$	0.336	(0.060, 0.276)
2	$-1.016 < x \leq -0.567$	0.042	(-0.054, 0.138)
3	$-0.567 < x \leq -0.319$	0.052	(-0.065, 0.121)
4	$-0.319 < x \leq -0.128$	0.036	(-0.076, 0.105)
5	$-0.128 < x \leq -0.014$	-0.047	(-0.079, 0.098)
6	$-0.014 < x \leq 0.165$	0.036	(-0.079, 0.098)
7	$0.165 < x \leq 0.361$	0.005	(-0.076, 0.105)
8	$0.361 < x \leq 0.618$	0.006	(-0.065, 0.121)
9	$0.618 < x \leq 1.038$	0.049	(-0.054, 0.138)
10	$1.038 < x \leq 8.213$	0.202	(0.060, 0.276)

Table 7: Conditional correlations between $X = (X_t)_t$ and $Y = (Y_t)_t$ ($\hat{\rho} = 0.295$).

For both the series (ε_t/η_t and X_t/Y_t), the same pattern of correlations would arise if they were generated under bivariate Gaussian distributions with a constant correlation coefficient. Hence, the question of correlation breakdown cannot be decided on this basis, since the U-shaped pattern of conditional correlations presented in the data set cannot be used by itself to determine whether actual correlations differ across turbulent and calm subperiods.

We have seen in the previous section that these data sets can be modelized by autoregressive processes which are mixing processes. Thus, the correlation breakdowns that we observed in Tables 4 and 7 are irrespective of the actual stationarity properties of the data. In order to find a solution to the real problem of finding a good method, alternative investigations are required. Before looking at other methods, we suggest seeing if the same behaviour can be observed when the data sets follow long memory processes whose non-mixing properties are known, see Guégan and Ladoucette (2001). The importance of long memory behaviour is now well-known in a lot of financial data sets, see for instance the recent work of Avouyi-Dovi *et al.* (2002) and references therein, thus it is important to have some kind of information for this class of processes.

We investigate the previous approach for two kinds of processes: the FARIMA processes and the GARMA processes.

First, we simulated a bivariate process whose margins correspond to non-mixing Gaussian FARIMA processes. We use a theoretical value of the long memory parameter d equals 0.4 for the both processes. Because of this value of the parameter, the processes are stationary and highly persistent. We use the same correlation coefficient as we observed between $(X_t)_t$ and $(Y_t)_t$, i.e. $\rho = 0.295$, and we make Monte Carlo simulations to obtain the confidence intervals. The results are given in Table 8. We observe that the values of (8) are outside the confidence intervals for deciles 1, 2, 4 and 7. Thus, these results are quite different from those presented in Table 6.

Decile	Interval	$\mathrm{Corr}(X, Y \mid X \in D_i)$	90% Confidence Interval
1	$-3.392 < x \leq -1.291$	0.032	(0.060, 0.276)
2	$-1.291 < x \leq -0.900$	0.152	(-0.054, 0.138)
3	$-0.900 < x \leq -0.566$	0.081	(-0.065, 0.121)
4	$-0.566 < x \leq -0.268$	-0.160	(-0.076, 0.105)
5	$-0.268 < x \leq -0.007$	0.012	(-0.079, 0.098)
6	$-0.007 < x \leq 0.245$	0.025	(-0.079, 0.098)
7	$0.245 < x \leq 0.526$	-0.081	(-0.076, 0.105)
8	$0.526 < x \leq 0.857$	0.031	(-0.065, 0.121)
9	$0.857 < x \leq 1.358$	0.096	(-0.054, 0.138)
10	$1.358 < x \leq 4.021$	0.112	(0.060, 0.276)

Table 8: Conditional correlations between two Gaussian FARIMA processes (with long memory parameter $d = 0.4$) with $\rho = 0.295$.

Now, we carry out the same procedure using a Gegenbauer process which belongs to the class of GARMA processes and which is known to take into account both long memory behaviour and some kind of seasonality, see Gray *et al.* (1989). In Table 9, we report the results obtained from a bivariate stationary Gaussian Gegenbauer process whose correlation coefficient is also $\rho = 0.295$. These two processes have been simulated with the theoretical parameters $d = 0.4$ and $\nu = \cos(\pi/6)$.

Decile	Interval	$\mathrm{Corr}(X, Y \mid X \in D_i)$	90% Confidence Interval
1	$-3.703 < x \leq -1.284$	0.113	(0.060, 0.276)
2	$-1.284 < x \leq -0.822$	0.150	(-0.054, 0.138)
3	$-0.822 < x \leq -0.536$	0.211	(-0.065, 0.121)
4	$-0.536 < x \leq -0.269$	0.082	(-0.076, 0.105)
5	$-0.269 < x \leq -0.019$	-0.033	(-0.079, 0.098)
6	$-0.019 < x \leq 0.242$	-0.118	(-0.079, 0.098)
7	$0.242 < x \leq 0.508$	0.187	(-0.076, 0.105)
8	$0.508 < x \leq 0.850$	-0.030	(-0.065, 0.121)
9	$0.850 < x \leq 1.268$	0.073	(-0.054, 0.138)
10	$1.268 < x \leq 3.696$	0.020	(0.060, 0.276)

Table 9: Conditional correlations between two Gaussian Gegenbauer processes (with parameters $d = 0.4$ and $\nu = \cos(\pi/6)$) with $\rho = 0.295$.

We observe in that latter case that values are often outside the confidence intervals as with the FARIMA process (deciles 2,3,6 and 7). Thus, it seems that there is a difference in behaviour between mixing and non-mixing processes concerning the conditional correlations relative to the deciles of the distribution. For long memory processes we do not obtain the classical U-shaped pattern like in Figure 2.

Thus, working with non-mixing processes renders this approach irrelevant: taking into account the deciles of the distribution is not relevant because of the default of non-mixing, which does not make it possible to separate correctly the data into the different subsets under consideration.

To test whether the correlation between two series is constant or changing over time, we compared sampling correlations between two series calculated from sub-sets of the data. If these conditional correlations are found to be statistically different from each other, one might be tempted to conclude that the population correlation is not constant. We have shown analytically (following here Boyer *et al.*, 1999) and empirically (with a new approach) that this intuitively attractive approach to testing correlation breakdowns can be misleading unless the data are governed, possibly, by long memory processes.

Similar results have been obtained for the others couple of series (American/Japanese and French/Japanese) we therefore do not give them here for simplicity's sake. They are available from the authors on request.

5 Concordance measures

In the previous sections, we firstly investigated the non- conditional cross-correlations between three markets. This allowed us to define a transfer function between the markets as whole and to give an indication of the delay of response to some specific shocks within the markets. This does not make it possible to define a link between the presence of volatility and the different movements within the markets. Next, we studied the conditional correlation between two markets in order to understand the link between the change in correlation and the volatility. We show that the changes in correlation cannot be a good indicator of the variation of volatility within the markets because the same behaviour can be observed for strong stationary processes.

In this section, we use overall measures between the markets to detect their co-movements. These measures could be stronger than the previous ones in the sense that they make it possible to take into account the presence of non-linearity within the data sets.

One of the measures that have been developed in the literature is the conformity measure introduced by King and Plosser (1994). In their paper, they compare the evolution of different macro-economic data sets relative to a reference business cycle introduced by Burns and Mitchell (1943). Their measure is defined in the following way: to compute the conformity of a series during reference cycle expansions, a value of 1 is assigned to each expansion for which the average per month change in the cycle relative from trough to peak is positive. For those expansions where the average per month change is negative (that is the series falls during an expansion), a value of -1 is assigned. The average of this series of ones and minus ones (multiplied by 100) is the index of conformity. A conformity of $+100$ corresponds to a series that, on average rises, during the each reference cycle expansion and a conformity of -100 corresponds to a series, that on average, falls during the each reference cycle expansion. We do not investigate this index here as it requires a reference cycle to do so.

Thus, we will focus on the following concordance measures: the degree of concordance and Kendall's tau. We begin by defining the degree of concordance, then we study Kendall's tau and specify its properties.

Let $(X', Y')^T$ be an independent copy of the random vector $(X, Y)^T$. We say that $(X, Y)^T$ and $(X', Y')^T$ are concordant if $(X - X')(Y - Y') > 0$, and discordant if $(X - X')(Y - Y') < 0$. In particular, this notion will enable us to determine whether two time series co-move.

To determine whether a pattern exists in the evolution of the data, we use the degree of concordance introduced by Harding and Pagan (2002). S_X (respectively S_Y) denotes the series which takes the value unity when the series X (respectively Y) is in expansion and zero when it is in contraction, the degree of concordance is defined by:

$$C(X,Y) = n^{-1}(\sum_{i=1}^{n}(S_{i,X}.S_{i,Y}) + (1 - S_{i,X}).(1 - S_{i,Y})),$$

where n represents the sample size that we observe for the random variables X and Y. This degree summarizes the common phases of expansion and recession in X and Y but not the amplitude of the swings. Thus it may appear complementary to the method developed in Section 3, which provides the amplitude of the change with the transfer function. In this section, the log returns of the three MSCI indices are respectively denoted $X = (X_t)_t$ for the American market, $Y = (Y_t)_t$ for the French market and $Z = (Z_t)_t$ for the Japanese market.

In Table 10, we present the empirical degrees of concordance for the three indices X, Y and Z during the various sample periods we have already considered in Section 3.

	$C(X,Y)$	$C(X,Z)$	$C(Y,Z)$
Full period	0.54	0.48	0.57
Oct. 87			
Before	0.53	0.36	0.66
After	0.64	0.53	0.58
Including	0.60	0.44	0.60
Asian			
Before	0.59	0.31	0.54
After	0.68	0.58	0.63
Including	0.65	0.45	0.60
Russian			
Before	0.51	0.56	0.61
After	0.66	0.54	0.68
Including	0.59	0.55	0.61

Table 10: Empirical degrees of concordance for the three markets.

We observe that the degrees appear higher after a strong shock. This characterizes the existence of a co-movement within the three returns. Over the full period, the degrees are close to 0.5, which means that these markets seem to follow an independent evolution. After the different crises, the degree of concordance estimated for the American market and the French market increases. This is not the case between the Asian market and the American market. Specially during the Russian crisis, the degree of concordance is always close to 0.5. These results are close to those observed in Section 3. Thus, if we compare this measure with the transfer's method, it seems complementary: it indicates how often changes coincide inside the series.

We can also consider two other concordance measures close to this degree of concordance which are Kendall's tau and Spearman's rho. Like the previous degree, they provide alternatives to the linear correlation coefficient as a measure of dependence for non-elliptical distributions. We give their definitions and properties, using the same notations as before.

Kendall's tau for two random variables X and Y is defined as

$$\tau(X,Y) = \mathbb{P}[(X - X')(Y - Y') > 0] - \mathbb{P}[(X - X')(Y - Y') < 0],$$

where $(X', Y')^T$ is an independent copy of the vector $(X, Y)^T$. Hence, Kendall's tau is simply the probability of concordance minus the probability of discordance.

Spearman's rho for two random variables X and Y is defined as

$$\rho_S(X,Y) = 3(\mathbb{P}[(X - \tilde{X})(Y - Y') > 0] - \mathbb{P}[(X - \tilde{X})(Y - Y') < 0]),$$

where $(X', Y')^T$ and $(\tilde{X}, \tilde{Y})^T$ are also independent copies of the vector $(X, Y)^T$. For our purposes there is no difference between working with Kendall's tau or Spearman's rho. Here, we are going to work with Kendall's tau.

Recall that $-1 \leq \tau(X,Y) \leq 1$. Kendall's tau is invariant under strictly increasing transformations: that is, if f and g are strictly increasing functions then $\tau(f(X), g(Y)) = \tau(X,Y)$. This property does not hold for linear correlation. Note that if f and g are marginal distribution functions of X and Y, respectively, then $f(X)$ and $g(Y)$ are uniform. Now $\tau = 1$ $(= -1)$ if and only if $Y = f(X)$ for any monotone increase (or decrease) in the function. The coefficient τ is null if X and Y are independent.

If H denotes the joint distribution of the random vector $(X, Y)^T$, then:

$$\tau = \tau(X, Y) = 4E[H(X, Y)] - 1. \tag{9}$$

This relationship is derived from the following result: $\mathbb{P}[X > x, Y > y] = H(x, y) - F(x) - G(y) + 1$, where F and G are the marginal distribution functions of X and Y. *Thus, if the function H is known, τ is known and vice versa.* Now, we obtain a way to understand the strong link which exists between τ and H, and how to construct the well-known function H. For more details, see, for instance Lehmann (1966) and Schweizer and Sklar (1983).

We consider the following class of functions:

$$\Phi_\alpha = \Big\{\phi_\alpha : [0,1] \to [0,\infty],\ \phi_\alpha(1) = 0,\ \phi'_\alpha(t) < 0,\ \phi''_\alpha(t) > 0,\ \alpha \in [-1,1]\Big\}.$$

Classical functions $\phi_\alpha \in \Phi_\alpha$ are: $\phi_\alpha(t) = -\log t$, $\phi_\alpha(t) = (1-t)^\alpha$, $\phi_\alpha(t) = t^{-\alpha} - 1$ with $\alpha > 1$. Then, it is easy to show that for all convex functions $\phi_\alpha \in \Phi_\alpha$, there exists a function C_α such that:

$$C_\alpha(u, v) = \begin{cases} \phi_\alpha^{-1}(\phi_\alpha(u) + \phi_\alpha(v)), & \text{if } \ \phi_\alpha(u) + \phi_\alpha(v) \le \phi_\alpha(0) \\ \\ 0, & \text{otherwise.} \end{cases} \tag{10}$$

Thus, the function $C_\alpha(u, v)$ is a symmetric 2-dimensional distribution function whose margins are uniform on the interval $[0, 1]$. It is an *Archimedean copula generated by ϕ_α*. The notion of Archimedean copulas was introduced by Ling (1965). Amongst the Archimedean distributions, we have the Frank law, see Frank (1979), the Cook and Johnson law (and the Oakes law), see Cook and Johnson (1981) (and Oakes, 1982), the Gumbel law (and the Hougaard law), see Gumbel (1958) (and Hougaard, 1986), the Ali-Mikhail-Haq law, see Ali *et al.* (1978). Note that the Plackett, the Farlie and the Marda laws are not Archimedean, see respectively Plackett (1965), Farlie (1960) and Marda (1970): this derives from Abel's criteria (1826). The Archimedean property is fundamental in applications: indeed, this means that it is possible to construct this copula using a generator ϕ_α and that there exists a formula which makes it possible to compute Kendall's tau from this operator, i.e.:

$$\tau(C_\alpha) = 1 + 4\int_0^1 \frac{\phi_\alpha(t)}{\phi'_\alpha(t)} dt. \tag{11}$$

We will now provide for some of these laws, the relationship between the parameter α, the coefficient τ and the generator ϕ_α. This will be very useful to compute, in the next section, the copulas on the markets we investigate.

- The Gumbel law G_α is defined on the unit square by:

$$G_\alpha(u,v)^{\alpha+1} = \exp\Big(- \big(|\log u|^{\alpha+1} + |\log v|^{\alpha+1}\big)^{1/(\alpha+1)}\Big), \quad \alpha \geq 0.$$

  Then G_α is generated by $\phi_\alpha(t) = |\log t|^{\alpha+1}, 0 \leq t \leq 1$ and Kendall's tau is computed as follows:

$$\tau(G_\alpha) = \frac{\alpha}{\alpha+1}. \tag{12}$$

  The same generator is used to obtain Hougaard's copulas .

- The Cook and Johnson, also called Clayton, law J_α is defined on the unit square by:

$$J_\alpha(u,v) = \Big[\frac{1}{u^\alpha} + \frac{1}{v^\alpha} - 1\Big]^{-\frac{1}{\alpha}}, \quad \alpha > 0.$$

  We note that this class contains the following particular cases: the logistic function, see Satterthwaite and Hutchinson (1978), the Pareto function, see Marda (1962) and the Burr function, see Takahasi (1965). Then J_α is generated by $\phi_\alpha(t) = \frac{t^{-\alpha}-1}{\alpha}, 0 \leq t \leq 1$ and Kendall's tau is given by:

$$\tau(J_\alpha) = \frac{\alpha}{\alpha+2}. \tag{13}$$

  The same generator is used to obtain Oakes's copulas.

- The Ali-Mikhail-Haq law A_α is defined on the unit square by:

$$A_\alpha(u,v) = \frac{uv}{1-\alpha(1-u)(1-v)}, \quad -1 \leq \alpha \leq 1.$$

  Then, A_α is generated by $\phi_\alpha(t) = (1-\alpha)^{-1}\log\Big[\frac{1+\alpha(t-1)}{t}\Big], 0 \leq t \leq 1$. We note that when $\alpha = 1$ we obtain the Cook and Johnson law as a particular case of the Ali-Mikhail-Haq law. Also, we obtain the following Kendall's tau:

$$\tau(A_\alpha) = \frac{3\alpha^2 - 2\alpha - 2(1-\alpha)^2\log(1-\alpha)}{3\alpha^2}. \tag{14}$$

- The Frank law F_α is defined on the unit square by:

$$F_\alpha(u,v) = \log_\alpha\Big[1 + \frac{(\alpha^u-1)(\alpha^v-1)}{\alpha-1}\Big], \quad \alpha > 0.$$

Then F_α is generated by $\phi_\alpha(t) = -\log \frac{1-\alpha^t}{1-\alpha}, 0 \le t \le 1$. Also, we obtain the following expression for Kendall's tau:

$$\tau(F_\alpha) = 1 - \frac{4\big(1 - D_1(-\log \alpha)\big)}{-\log \alpha} \tag{15}$$

with $D_1(x) = \frac{1}{x}\int_0^x \frac{t}{e^t-1}dt$.

- The Dependent law D_α is defined on the unit square by:

$$D_\alpha(u,v) = \Big[1 + \big((u^{-1}-1)^\alpha + (v^{-1}-1)^\alpha\big)^{\frac{1}{\alpha}}\Big]^{-1}, \quad \alpha \ge 1.$$

  Then D_α is generated by $\phi_\alpha(t) = (t^{-1}-1)^\alpha, 0 \le t \le 1$ and Kendall's tau is given by:

$$\tau(D_\alpha) = 1 - \frac{2}{3\alpha}. \tag{16}$$

We note that the Cook-Johnson/Oakes copulas family and the Gumbel/Hougaard copulas family allow for only non-negative correlations. However, Frank's family allows for negative as well as positive dependence.

In Table 11, we present the empirical Kendall's tau computed for the indices X, Y and Z during the various sub-periods defined before.

	$\tau(X,Y)$	$\tau(X,Z)$	$\tau(Y,Z)$
Full period	0.09	-0.03	0.15
Oct. 87			
Before	0.07	-0.28	0.31
After	0.28	0.03	0.14
Including	0.20	-0.12	0.20
Asian			
Before	0.17	-0.38	0.10
After	0.38	0.17	0.24
Including	0.30	-0.09	0.20
Russian			
Before	0	0.14	0.24
After	0.31	0.07	0.34
Including	0.17	0.11	0.24

Table 11: Empirical Kendall's tau for the three markets.

These results are coherent with those presented in Table 10 (see column one) since the coefficients are higher after the shocks. This measure is interesting because, when we compare it with the values obtained for the linear correlation in Table 2, we observe contradictory results. For instance, before the Russian crisis, $\rho(X,Y) = 0.49$, which means that the two markets appear linearly correlated, whereas $\tau(X,Y) = 0$, which means that the markets are independent! As the distribution between the two markets is non-elliptical, we know that the linear correlation is not efficient.

We will use the relationship that exists between Kendall's tau and certain copulas to compute the copulas between the different markets under consideration. We will use this approach in the next section.

6 Tail correlation

The previous sections (Sections 3 and 4) show that we cannot determine the presence of volatility, jumps or switching within data sets from the analysis of non-conditional correlation and conditional correlation. To assess the conditional correlation necessitates knowledge of the conditional distribution that is unknown in general. Here, we try to bypass this problem looking at the conditional distribution in the tail. Some recent papers have investigated conditional tail behaviour, our approach is close to the work of Brummelhuis and Guégan (2000), Frey and McNeil (2000) and Longin and Solnik (2001), but our goal is different. The asymptotic conditional distribution could provide us another way to understand the behaviour of conditional correlation. To obtain this information, we need to calculate the bivariate distribution in the tails and we will use the notion of copula to do this. We will introduce another definition for the copulas, coherent with the previous one. This definition is more popular but it is restricted because it does not make it possible to construct the copula contrary to the former definition.

6.1 Bivariate case

Here, we are interested in measuring the dependence in the tail of the joint distribution between two markets. To measure this dependency, we proceed in several steps. First of all, using the POT method (Peak Over Threshold), we estimate the tail distribution of each market. In the second step, we compute the empirical Kendall's tau in the tails for each market pair. This will enable us to compute, for each copula, its parameter α. In the third step, we use the expression that links the joint distribution and the copula with the margins (Sklar Theorem). Finally, we carry out a diagnostic test between the empirical joint distribution and the estimated copula. For this final step, we need to give an important feature of the copulas in the tails: indeed, some copulas present what we call a tail dependence and others do not. Thus, the choice of the copulas to reconstruct the joint distribution in the tails is fundamental. So, there is a risk of misspecification if we are not careful about this choice.

1. In the first step, we estimate the distribution associated with each market when we fix a specific threshold, in order to determine their tail behaviour. To obtain these distributions, we use the POT method whose principle will be briefly recalled.

   If X follows a distribution function F, we define the associated distribution of excesses losses over a high threshold u as:

   $$F_u(y) = P[X - u \leq y | X > u] \tag{17}$$

   for $0 \leq y < x_0 - u$ where $x_0 \leq +\infty$ is the right endpoint of F. We note that F_u can be written in terms of the underlying distribution F as follows:

   $$F_u(y) = \frac{F(y+u) - F(u)}{1 - F(u)}. \tag{18}$$

   Now, the following theorem gives the asymptotic behaviour of the function F_u.

   Theorem 6.1 *For a large class of underlying distribution F, we can obtain a function $\beta(u)$ such that:*

   $$\lim_{u \to x_0} \sup_{0 \leq y < x_0 - u} |F_u(y) - G_{\xi,\beta(u)}(y)| = 0,$$

   where the function $G_{\xi,\beta(u)}(y)$ is called the Generalized Pareto Distribution (GPD). The GPD depends on two parameters, a shape parameter ξ and a scaling parameter β, and is expressed as follows:

   $$G_{\xi,\beta}(x) = \begin{cases} 1 - (1 + \xi x/\beta)^{-1/\xi}, \ \xi \neq 0 \\ \\ 1 - \exp(-x/\beta), \ \xi = 0 \end{cases} \tag{19}$$

   with $\beta > 0$, $x \geq 0$ when $\xi \geq 0$ and $0 \leq x \leq -\beta/\xi$ when $\xi < 0$.

If $\xi > 0$ it is a reparametrized version of the Pareto distribution, $\xi = 0$ corresponds to the exponential distribution and $\xi < 0$ is known as a Pareto type II distribution. When $\xi > 0$, the GPD is heavy-tailed and for $k \geq 1/\xi$, $\mathbb{E}[X^k]$ is infinite. For instance, we obtain an infinite variance when $\xi = 1/2$.

Thus, for a "large class" of distribution F, the excess function F_u converges to a generalized Pareto distribution as the threshold u is raised. All the common continuous distributions of statistics are included in this "large class". In fact, we can assume that the GPD models can approximate the unknown excess distribution F_u, i.e. for a certain threshold u and for some ξ and β (to be estimated), we obtain:

$$F_u(y) = G_{\xi,\beta}(y). \tag{20}$$

Now, we can define the link between a general one-dimensional distribution function F for a fixed threshold u and $G_{\xi,\beta}$. Combining the expressions (18), (20) and setting $x = u + y$ we obtain:

$$F(x) = (1 - F(u))G_{\xi,\alpha}(x - u) + F(u) \tag{21}$$

for $x > u$.

We estimate $F(u)$ using $1 - N_u/n$, where N_u is the number of data exceeding the fixed threshold u, and if we estimate the parameters ξ and β of the GPD, we obtain the tail estimator:

$$\hat{F}(x) = 1 - \frac{N_u}{n}\Big(1 + \hat{\xi}\frac{x - u}{\hat{\beta}}\Big)^{-1/\hat{\xi}},\ x > u. \tag{22}$$

which is only valid for $x > u$.

We will now consider the data sets $X_1, \cdots, X_n$ from the different markets. For these sets, we fit the GPD to the N_u excesses using the maximum likelihood estimation (MLE) of the parameters ξ and β and we compute the confidence intervals for the estimates of the parameters using a bootstrap procedure. To obtain these estimates, we need to choose the threshold u. On the one hand, it has to be chosen sufficiently high so that Theorem 6.1 can be applied, and, on the other hand, it has to be considered sufficiently low to have sufficient data for the estimation procedure. Here, we choose u relative to the quantiles.

The threshold u will represent the 90% and the 95% levels respectively. This means that to define the tails of the empirical distributions of the three indices we consider the upper 10% (respectively 5%) of the total number of observations (given the 4434 daily data, this implies $N_u = 443$ threshold exceedances (respectively $N_u = 222$ threshold exceedances)).

In the following, we consider the time series defined by the log returns of the MSCI indices, denoted $X = (X_t)_t$, $Y = (Y_t)_t$ and $Z = (Z_t)_t$ for the American market, the French market and the Japanese market respectively .

	90% ($N_u = 443$)	95% ($N_u = 222$)
X	$u = 1.0389$ $\hat{\beta} = 0.5565$ $\hat{\xi} = 0.1060$	$u = 1.4182$ $\hat{\beta} = 0.6129$ $\hat{\xi} = 0.1033$
Y	$u = 1.1395$ $\hat{\beta} = 0.5127$ $\hat{\xi} = 0.1047$	$u = 1.5071$ $\hat{\beta} = 0.5688$ $\hat{\xi} = 0.0818$
Z	$u = 1.1078$ $\hat{\beta} = 0.6459$ $\hat{\xi} = 0.0966$	$u = 1.6037$ $\hat{\beta} = 0.5996$ $\hat{\xi} = 0.1704$

Table 12: Values of the parameters of the GPD adjusted for the three markets for different thresholds.

In Table 12, we give the values of the estimation for the parameters ξ and β of the GPD distributions adjusted for the tail of each of the three markets for the different thresholds. We provide in Table 13, the bootstrap confidence intervals for these estimations using 100 replications of length 443 (for the 90% level) and of length 222 (for the 95% level).

	90% ($N_u = 443$)	95% ($N_u = 222$)
X	$\hat{\beta} \in [0.5370, 0.5944]$, $\hat{\xi} \in [0.0635, 0.1321]$	$\hat{\beta} \in [0.5891, 0.6714]$, $\hat{\xi} \in [0.0454, 0.1306]$
Y	$\hat{\beta} \in [0.4865, 0.5424]$, $\hat{\xi} \in [0.0607, 0.1337]$	$\hat{\beta} \in [0.5396, 0.6143]$, $\hat{\xi} \in [0.0205, 0.1185]$
Z	$\hat{\beta} \in [0.6237, 0.6821]$, $\hat{\xi} \in [0.0522, 0.1359]$	$\hat{\beta} \in [0.5564, 0.6442]$, $\hat{\xi} \in [0.0844, 0.2064]$

Table 13: Bootstrap confidence intervals for the estimation of ξ and β.

Whatever the case, the parameter ξ is positive and significant, thus a Pareto distribution can be fitted for the tail of all the markets.

Now, using the estimator (22) with the values of the parameters $\hat{\xi}$ and $\hat{\beta}$ given in Table 12, we can compute the tail of the marginal distribution of each market for $x > u$, where u corresponds to a chosen threshold. In the following, $\hat{F}$, $\hat{G}$ and $\hat{J}$ will denote the tail distributions of the American market, the French market and the Japanese market respectively.

2. In the second step, we compute the empirical Kendall's tau $\hat{\tau}$ between the different markets in the tails. To do so, we use the points that are beyond the 0.9-quantile ($N_u = 443$ points) of the distribution of each market, i.e. the points for which we adjusted the GPD in the first step. We do the same for the 0.95-quantile ($N_u = 222$ points). The results are given in Table 14.

3. In the third step, using the values of Kendall's tau, we compute the parameters α of the different Archimedean copulas which enable us to approximate the joint distribution of two markets. We begin by recalling the fundamental result of Sklar used in this session.

	90%	95%
$\hat{\tau}(X_T, Y_T)$	0.0588	0.0588
$\hat{\tau}(X_T, Z_T)$	0.0090	-0.0136
$\hat{\tau}(Y_T, Z_T)$	-0.0271	-0.0679

Table 14: Empirical Kendall's tau relative to the quantiles for the three markets considered in the tails (the tails of X, Y and Z are denoted X_T, Y_T and Z_T respectively).

Let us consider a general random vector $Z = (X, Y)^T$ and assume that it has a joint distribution function $H(x, y) = \mathbb{P}[X \leq x, Y \leq y]$ and that each random variable X and Y has a *continuous* marginal distribution function denoted F and G respectively. It has been shown by Sklar (1959) that every 2-dimensional distribution function H with margins F and G can be written as $H(x, y) = C(F(x), G(y))$ for an *unique* (because the marginals are continuous) function C that is known as the copula of H. Like the previous section, we will use the notation C_α for Archimedean copulas and the notation C for a general copula. In the case of the Archimedean copulas, we then obtain the following relationship:

$$H(x,y) = C_\alpha\big(F(x), G(y)\big). \tag{23}$$

A copula C is a bivariate distribution with uniform marginals and it has the significant property of not changing under strictly increasing transformations of the random variables X and Y. Moreover, it makes sense to interpret C as the dependence structure of the vector Z. In the literature, this function has been called "dependence function" by Deheuvels (1978), "uniform representation" by Kimelsdorf and Sampson (1975) and "copula" by Sklar (1959). This sometimes makes reading the papers on this topic difficult. The last denomination is now the most popular, in particular in financial circles, and we use it here.

Practically, to obtain the joint distribution H of the random vector $Z = (X,Y)^T$ given the marginal distribution functions F and G of X and Y respectively, we have to choose a copula to apply to these margins.

Now, using the expression (22) for the empirical tail of the marginal distribution of two markets X and Y defined for $x > u_X$ and $y > u_Y$, and also using the relationship (23), we obtain:

$$\hat{H}(x,y) = C_{\hat{\alpha}}(\hat{F}(x), \hat{G}(y)),\ x > u_X,\ y > u_Y. \tag{24}$$

In particular, this expression models the dependence structure of observations exceeding the thresholds u_X and u_Y using Archimedean copulas $C_{\hat{\alpha}}$ for some estimated values $\hat{\alpha}$ of the dependence parameter α.

Using the values obtained for $\hat{\tau}$ in Table 14, we can compute the parameter $\hat{\alpha}$ for different Archimedean laws. For the Gumbel law (G_α), the Cook and Johnson law (J_α) and the Dependent law (D_α) we obtain the parameters using an inversion of the formula which gives the expression of the copula. We will now recall these simple relations between α and τ:

- For the Gumbel law: $\alpha = \frac{\tau}{1-\tau}$,

- For the Cook and Johnson law: $\alpha = \frac{2\tau}{1-\tau}$
- For the Dependent law: $\alpha = \frac{2}{3(1-\tau)}$.
- For the Ali-Mikhail-Haq (A_α) law and for the Frank law (F_α) we use a numerical resolution. In Tables 15, 16 and 17 we specify the values of the parameter α for these different laws.

4. Fourth step. In this part, we specify the methods that can be used to assess the approximation of the empirical tail of the joint distribution of two markets via the use of copulas. We consider the random vector $Z = (X, Y)^T$ of two markets X and Y. To estimate the tail of their joint distribution function, we use the expression (24), where C_α denotes the particular choice of Archimedean copulas.

5. Now, we will try to determine the best Archimedean copulas C_α, amongst G_α, J_α, D_α, A_α and F_α, for adjusting the empirical tail of the joint distribution function H of the random vector $(X, Y)^T$, with X and Y having empirical margins $\hat{F}$ and $\hat{G}$ respectively.

   To obtain this result we will use two different diagnoses: a numerical method and a graphical method. Both methods will enable us to decide on the best approximation from the range of the previous Archimedean copulas.

   First, we use a numerical criterion that we denote D_2 which corresponds to:
$$D_2 = \sum_{x,y} \left| C_{\hat{\alpha}}\big(\hat{F}(x), \hat{G}(y)\big) - \hat{H}(x,y) \right|^2.$$
   Then, copulas $C_{\hat{\alpha}}$, for which we obtain the minimum distance D_2, will be chosen as the best approximation. For the various copulas, the quantities D_2 are given in Tables 15, 16 and 17 relative to the different couples of markets.

90%	G_α	J_α	A_α	F_α	D_α
$\hat{\alpha}$	1.0625	0.1250	0.2476	0.5357	0.7083
D_2	2.8929	0.4158	0.4912	0.5758	103.9952
95%	G_α	J_α	A_α	F_α	D_α
$\hat{\alpha}$	1.0625	0.1250	0.2476	0.5357	0.7083
D_2	0.3034	0.0743	0.0794	0.0838	8.8490

Table 15: Values of $\hat{\alpha}$ and of D_2 for the couple (X=American market, Y=French market) relative to the various copulas.

For the couple (X, Y), we obtain the best approximation using the Cook and Johnson law for both thresholds.

90%	G_α	J_α	A_α	F_α	D_α
$\hat{\alpha}$	1.0091	0.0183	0.0403	0.6978	0.6728
D_2	0.7109	0.5268	0.5434	0.7000	151.1397
95%	G_α	J_α	A_α	F_α	D_α
$\hat{\alpha}$	0.9866	-0.0268	-0.0620	0.7452	0.6577
D_2	0.0551	0.0530	0.0526	0.0567	16.0837

Table 16: Values of $\hat{\alpha}$ and of D_2 for the couple (X=American market, Z=Japanese market) relative to the various copulas.

For the couple (X, Z), we obtain the best approximation using the Cook and Johnson law for the 0.9-quantile threshold and for the threshold that corresponds to the 0.95-quantile, which we obtain using the Ali-Mikhail-Haq law.

90%	G_α	J_α	A_α	F_α	D_α
$\hat{\alpha}$	0.9736	-0.0529	-0.1259	0.7666	0.6490
D_2	0.8158	0.3933	0.4066	0.4346	203.4705
95%	G_α	J_α	A_α	F_α	D_α
$\hat{\alpha}$	0.9364	-0.1271	-0.3295	0.8110	0.6243
D_2	0.2289	0.0455	0.0425	0.0533	22.8012

Table 17: Values of $\hat{\alpha}$ and of D_2 for the couple (Y=French market, Z=Japanese market) relative to the various copulas.

For the couple (Y, Z), we obtain the best approximation using the Cook and Johnson law for the 0.9-quantile threshold and for the threshold that corresponds to the 0.95-quantile, which we obtain using the Ali-Mikhail-Haq law .

Now, we will use a graphical criterion. From the definition of a copula C, we know that if U and V are two uniform random variables then the random variables

$$C(V|U) = \frac{\partial C}{\partial U}(U,V)$$

and

$$C(U|V) = \frac{\partial C}{\partial V}(U,V)$$

are also uniformly distributed. We use this property to quantify the adjustment between the empirical joint distribution and the different copulas, using the classical QQ-plot method. For that, we need to calculate the partial derivatives of the various Archimedean copulas under consideration. Since the Archimedean copulas C_α are symmetric, we only investigate $C_\alpha(V|U)$.

These partial derivatives are the following:

- For the Gumbel copulas:

$$G_\alpha(v|u) = G_\alpha(u,v)\frac{|\log u|^\alpha}{u}\Big(|\log u|^{\alpha+1} + |\log v|^{\alpha+1}\Big).$$

- For the Cook and Johnson copulas:

$$J_\alpha(v|u) = \Big(1 + u^\alpha(v^{-\alpha} - 1)\Big)^{-(1+\alpha)/\alpha}.$$

- For the Ali-Mikhail-Haq copulas:

$$A_\alpha(v|u) = \frac{v(1-\alpha(1-u)(1-v)) - uv\alpha(1-v)}{(1-\alpha(1-u)(1-v))^2}.$$

- For the Frank copulas:

$$F_\alpha(v|u) = \Big(1 + \frac{(\alpha^u - 1)(\alpha^v - 1)}{\alpha+1}\Big)^{-1}\frac{\alpha^u}{\alpha+1}(\alpha^v - 1).$$

- For the dependent copulas:

$$D_\alpha(v|u) = (D_\alpha(u,v))^{-2}\Big((u^{-1}-1)^\alpha + (v^{-1}-1)^\alpha\Big)^{(1-\alpha)/\alpha}(u^{-1}-1)^{\alpha-1}u^{-2}.$$

Thus, as the distribution functions of $\hat{F}(X)$ and $\hat{G}(Y)$ are uniform, we plot, for each copula C_α, the empirical distribution $C_{\hat{\alpha}}(\hat{G}(Y)|\hat{F}(X))$ against the uniform distribution. The straighter the line, the better the adjustment of the joint distribution $\hat{H}$ by the copulas $C_{\hat{\alpha}}$.

We note that we obtain similar results using this graphical method and the first numerical method.

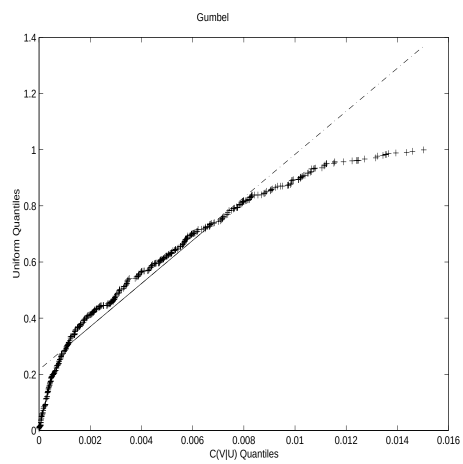

Figure 3: QQ-plot for the Gumbel copulas

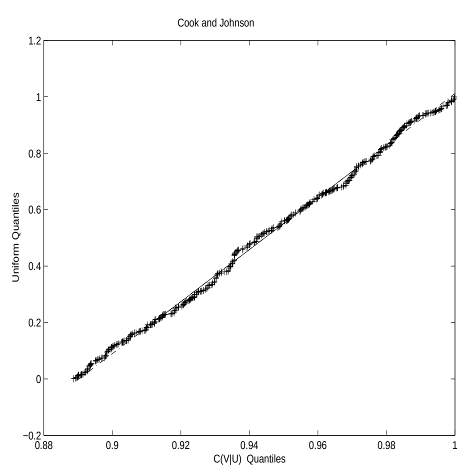

Figure 4: QQ-plot for the Cook and Johnson copulas

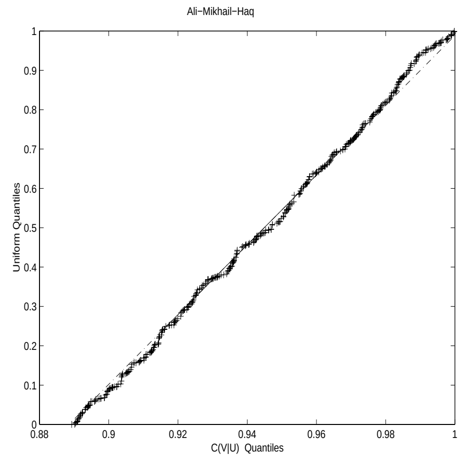

Figure 5: QQ-plot for the Ali-Mikhail-Haq copulas

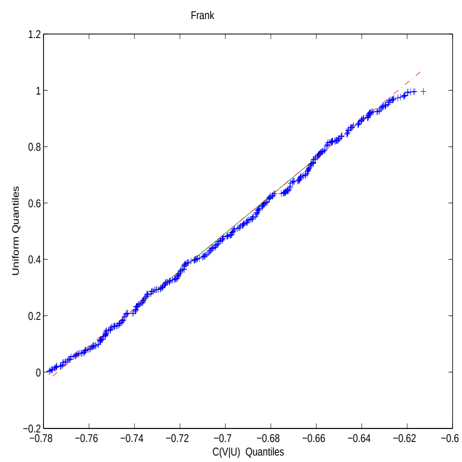

Figure 6: QQ-plot for the Frank copulas

To illustrate this graphical method, we only give the results for one pair of markets by considering the series X and Y of the American market and the French market defined in their tail by the 0.9-quantile threshold. The QQ-plots that correspond to the five Archimedean copulas G_α, J_α, D_α, A_α and F_α are proposed in Figures 3, 4, 5, 6 and 7 respectively. We observe that we obtain the straightest line with the Cook and Johnson copulas, see Figure 4.

With regards to Kendall's tau that we obtained in Table 14, we ob-

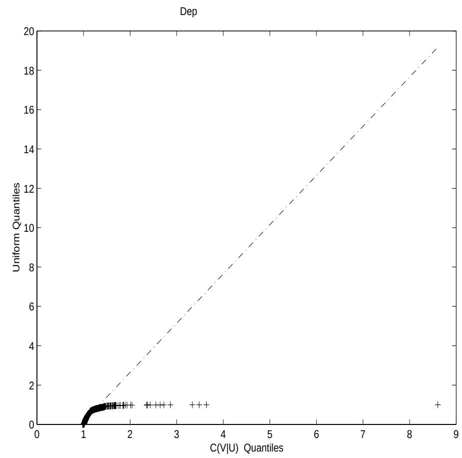

Figure 7: QQ-plot for the Dependent copulas

serve that the tail structures of the various couples of indices are close to independent structures. This explains the poor results we obtain with the Gumbel copulas and the Dependent copulas since they have upper tail dependence (due to a strictly positive λ_U) as we have seen previously. We recall that we obtained the best approximation by using either the Cook and Johnson copulas or the Ali-Mikhail-Haq copulas that have no upper tail dependence, and this fact does not contradict the independence structure obtained from the computation of Kendall's tau.

6.2 Multivariate Archimedean Copulas

An m-variate family of Archimedean copulas is an extension of a bivariate Archimedean family if all bivariate marginal copulas of the multivariate copulas are in the given bivariate family and if all multivariate marginal copulas of order 3 to $m-1$ have the same multivariate form. It is important to note that there is no natural multivariate extension of a bivariate family.

To extend the notion of bivariate Archimedean copulas C_α to the m dimensional framework, there is a degree of constraint on the dependence parameters α. For instance, let us consider the case $m = 3$ and assume that the (i, j) bivariate margins $(i \neq j \in \{1, 2, 3\})$ have dependence parameter $\alpha_{i,j}$. If $\alpha_2 > \alpha_1$ with $\alpha_{1,2} = \alpha_2$ and $\alpha_{1,3} = \alpha_{2,3} = \alpha_1$, then 3-variate Archimedean copulas can be expressed as follows:

$$C_{\alpha_1,\alpha_2}(u_1,u_2,u_3) = \phi_{\alpha_1}^{-1}(\phi_{\alpha_1}\ o\ \phi_{\alpha_2}^{-1}(\phi_{\alpha_2}(u_1)+\phi_{\alpha_2}(u_2))+\phi_{\alpha_1}(u_3)). \quad (25)$$

In the sequel, for two random variables X and Y, $\alpha(X,Y)$ denotes the dependence parameter deduced from Kendall's tau $\tau(X,Y)$ by means of the formula (11). For a random vector $(X,Y,Z)^T$ with joint three dimensional distribution H and margins F, G and J, we obtain from (25) the following extension of Sklar Theorem in the trivariate case:

$$\begin{aligned} H(x,y,z) &= C_{\alpha_1,\alpha_2}\big(F(x),G(y),J(z)\big) \\ &= C_{\alpha_1}\Big(C_{\alpha_2}\big(F(x),G(y)\big),J(z)\Big) \end{aligned} \quad (26)$$

if $\alpha_2 \geq \alpha_1$ with $\alpha_2 = \alpha(X,Y)$ and $\alpha_1 = \alpha(X,Z) = \alpha(Y,Z)$.

We aim to apply this theory to the three markets in order to adjust the tail of their three dimensional joint distribution H by means of Archimedean copulas. As previously, X, Y and Z denote the series of the log returns of the MSCI indices of the American market, the French market and the Japanese market respectively.

For the three markets, we choose to define the tails with the thresholds that correspond to the 0.95-quantiles. In order to be able to use (26), we recall that the dependence parameters of two couples of markets defined in the tails have to be equal. In Table 18, we recall the empirical dependence parameters that we obtained for each couple of markets with the various Archimedean laws.

Couple	G_α	J_α	A_α	F_α	D_α
(X,Y)	1.0625	0.1250	0.2476	0.5357	0.7083
(X,Z)	0.9866	-0.0268	-0.0620	0.7452	0.6577
(Y,Z)	0.9364	-0.1271	-0.3295	0.8110	0.6243

Table 18: Parameters $\hat{\alpha}$ for the different couples of markets (X=American market, Y=French market, Z=Japanese market) considered in the tails that correspond to the 0.95 quantiles relative to the various copulas.

For the Archimedean laws G_α, J_α, A_α and D_α, we note that the bigger dependence parameter is obtained for the couple (X,Y) and that the parameters are not equal for the two other couples (X,Z) and (Y,Z). For the

Archimedean law F_α, we obtain the higher parameter for the couple (Y, Z). In spite of these results, we nevertheless decided to continue our empirical study.

For each Archimedean law, $\hat{\alpha}_2$ equals the dependence parameter $\hat{\alpha}$ computed for the couple (X, Y) and $\hat{\alpha}_1$ equals the dependence parameter $\hat{\alpha}$ computed for the couple (X, Z) (we make the same choice for the law F_α so that we can compare the results). Then, using these parameters, we compute the 3-variate Archimedean copulas (25) for each Archimedean law. For determining the best Archimedean copulas C_{α_1,α_2} for adjusting the empirical tail of the joint distribution function H of $(X, Y, Z)^T$, we use the numerical criteria D_2 that corresponds to:

$$D_2 = \sum_{x,y,z} \left| C_{\hat{\alpha}_1}\Big(C_{\hat{\alpha}_2}\big(\hat{F}(x), \hat{G}(y)\big), \hat{J}(z)\Big) - \hat{H}(x, y, z) \right|^2.$$

We report the results in Table 19.

	G_{α_1,α_2}	J_{α_1,α_2}	A_{α_1,α_2}	F_{α_1,α_2}	D_{α_1,α_2}
D_2	26.5131	2.1215	2.1410	2.2540	914.7512

Table 19: Results for the distance D_2 relative to the various copulas (0.95-quantile)

Looking at the results, we obtain the best results using the 3-variate Cook and Johnson copulas which are then chosen for modelizing the tail of the joint distribution of the three markets.

7 Conclusion

In the literature, a question often raised is whether the correlation between international markets increases in periods of high turbulence. However, we have seen that international markets have recently grown more independent. In this paper, we have tried to answer a number of questions linked to the following issues:

- Are markets more highly correlated in periods of high volatility? Sections 4 and 5 show that it is very difficult to answer. Using conditional correlation implies a contradiction compared with the notion of concordance.

- Is the correlation higher when markets fall, rather than rise? Using the transfer function in Section 3, we can distinguish the importance of the correlation by examining the values of the first coefficients, but if the correlation appears higher after a shock, this is not true for all the markets, and in particular when we compare the American market and the Asian market.
- What is the influence of the business cycle? This question is not considered here.

In this paper, using daily data, we show that the different measures of interdependence do not always give identical information concerning the evolution of the markets and their interdependence. If the notion of correlation and cross correlation is restricted because it only takes into account the linear characteristics of the data, we have seen that the conditional correlation can introduce some mistakes in the interpretation. The result greatly depends on the choice of the conditioning. Kendall's tau appears as an interesting measure when we compare the general evolution of two markets over different periods, nevertheless it is quite difficult to use Kendall's tau with more than two markets. In this case, it is more useful to consider Archimedean copulas. However, the difficulty with the copula is to obtain the best adjustment. In this paper, we try to propose a method illustrated by the data sets under consideration here.

Sections 5 and 6 discuss properties and characteristics of copulas. We explain how to determine a copula and how the association structure of copulas can be summarized in terms of familiar measures of dependence. Recall that copulas are useful in examining the dependence structure of multivariate random vectors. Thus, we used copulas in Section 6 in order to compare the three markets under consideration here. Note that other measures of association only allow us to compare the markets by pair. This is one of the main advantages of working with copulas.

Another problem that we have not addressed in this paper concerns the same investigation with high frequency data. Some papers have already tried to consider the notion of interdependence for this kind of data sets, see for instance King and Wadhwani (1990) and Bertero and Mayer (1990). We will discuss our approaches in a companion paper for the same data sets observed with high frequency.

8 Appendix: transfer function's models between the three markets

8.1 Full period: 01/01/1985-31/12/2001 (4434 points)

- X_t and Y_t.

Over the full period, we obtain the following models:

$$X_t = -0.05X_{t-2} - 0.04X_{t-3} + \varepsilon_t$$

and

$$Y_t = 0.06Y_{t-1} + \eta_t.$$

For all the parameters the standard deviation is equal to 0.01. We obtain the following model for the cross-correlation between η_t and ε_t:

$$\eta_t = 0.34\varepsilon_t + 0.24\varepsilon_{t-1} + e_t^1.$$

The standard deviation for the both parameters is equal to 0.01. We adjust the following model for e_t^1:

$$e_t^1 = -0.06e_{t-1}^1 + e_t^2.$$

The standard deviation of the parameter is equal to 0.01. Finally, using these different modelizations, we obtain the model relating Y_t, X_t and e_t^2:

$$Y_t = 0.34X_t + 0.26X_{t-1} + 0.03X_{t-2} + 0.02X_{t-3} + 0.01X_{t-4} + e_t^2.$$

Series	mean	standard deviation	skewness	kurtosis
X_t	$4.34\ 10^{-4}$	$1.04\ 10^{-2}$	-2.67	59.93
ε_t	$2.77\ 10^{-7}$	$1.00\ 10^{-2}$	-2.88	62.07
Y_t	$5.27\ 10^{-4}$	$1.23\ 10^{-2}$	-0.37	7.03
η_t	$1.34\ 10^{-7}$	$1.20\ 10^{-2}$	-0.33	7.05
e_t^1	$8.03\ 10^{-7}$	$1.14\ 10^{-2}$	-0.18	6.33
e_t^2	$6.79\ 10^{-7}$	$1.10\ 10^{-2}$	-0.21	6.32

Table A1: Statistics on the returns X_t and Y_t and their residuals, and on the two residuals e_t^1 and e_t^2

- X_t and Z_t.

Over the full period, we obtain the following models:

$$X_t = -0.05X_{t-2} - 0.04X_{t-3} + \varepsilon_t$$

and

$$Z_t = -0.05Z_{t-6} + \beta_t.$$

For all the parameters the standard deviation is equal to 0.01. We obtain the following model for the cross-correlation between ε_t and β_t:

$$\beta_t = 0.36\varepsilon_{t-1} + e_t^1.$$

The standard deviation for the parameter is equal to 0.02. Finally, using these different modelizations, we obtain the model relating Z_t, X_t and e_t^1:

$$Z_t = 0.36X_{t-1} + 0.02X_{t-3} + 0.01X_{t-4} - 0.02X_{t-7} + e_t^1 - 0.05e_{t-6}^1.$$

Series	mean	standard deviation	skewness	kurtosis
X_t	$4.34\ \ 10^{-4}$	$1.04\ \ 10^{-2}$	-2.67	59.93
ε_t	$2.77\ \ 10^{-7}$	$1.00\ \ 10^{-2}$	-2.88	62.07
Z_t	$1.94\ \ 10^{-4}$	$1.47\ \ 10^{-2}$	-0.10	12.69
β_t	$-2.57\ \ 10^{-7}$	$1.43\ \ 10^{-2}$	-0.08	12.74
e_t^1	$-6.35\ \ 10^{-7}$	$1.40\ \ 10^{-2}$	0.32	7.93

Table A2: Statistics on the returns X_t and Z_t and on the residuals ε_t, β_t and e_t^1

- Y_t and Z_t.

Over the full period, we obtain the following models:

$$Y_t = 0.06Y_{t-1} + \eta_t$$

and

$$Z_t = -0.05Z_{t-6} + \beta_t.$$

For all the parameters the standard deviation is equal to 0.01. We obtain the following model for the cross-correlation between η_t and β_t:

$$\beta_t = 0.30\eta_t + 0.18\eta_{t-1} + e_t^1.$$

The standard deviation for the both parameters is equal to 0.02. Finally, using these different modelizations, we obtain the model relating Z_t, Y_t and e_t^1:

$$Z_t = 0.30Y_t + 0.16Y_{t-1} - 0.01Y_{t-2} - 0.02Y_{t-6} - 0.01Y_{t-7} + e_t^1 - 0.05e_{t-6}^1.$$

Series	mean	standard deviation	skewness	kurtosis
Y_t	$5.27\ 10^{-4}$	$1.23\ 10^{-2}$	-0.37	7.03
η_t	$1.34\ 10^{-7}$	$1.20\ 10^{-2}$	-0.33	7.05
Z_t	$1.94\ 10^{-4}$	$1.47\ 10^{-2}$	-0.10	12.69
β_t	$-2.57\ 10^{-7}$	$1.43\ 10^{-2}$	-0.08	12.74
e^1_t	$-7.05\ 10^{-9}$	$1.40\ 10^{-2}$	0	10.90

Table A3: Statistics on the returns Y_t and Z_t and on the residuals η_t, β_t and e^1_t

8.1.1 Subperiods

a) October, 1987

- Before the crash X_t and Y_t (22/07/1987 - 13/10/1987 (60 points)).

We obtain the following models:

$$X_t = -0.30X_{t-5} + 0.31X_{t-7} + \varepsilon_t$$

and

$$Y_t = 0.44Y_{t-1} + \eta_t.$$

The standard deviation of both parameters is equal to 0.13. We obtain the following model for the cross-correlation between ε_t and η_t:

$$\eta_t = -0.28\varepsilon_{t-2} + e^1_t.$$

The standard deviation of the parameter is equal to 0.11. The process e^1_t is a white noise process.
Finally, using these different modelizations, we obtain the model relating Y_t, X_t and e^1_t:

$$\begin{aligned} Y_t \;=\;& 0.28X_{t-2} + 0.12X_{t-3} - 0.05X_{t-4} + 0.03X_{t-5} + 0.01X_{t-6} \\ &+0.08X_{t-7} + 0.04X_{t-8} - 0.11X_{t-9} - 0.04X_{t-10} + 0.02X_{t-11} \\ &+e^1_t + 0.44e^1_{t-1} - 0.19e^1_{t-2} + 0.09e^1_{t-3} + 0.04e^1_{t-4} + 0.01e^1_{t-5}. \end{aligned}$$

Series	mean	standard deviation	skewness	kurtosis
X_t	3.63 10^{-4}	1.03 10^{-2}	0	3.28
ε_t	-7.58 10^{-4}	9.80 10^{-3}	-0.38	2.97
Y_t	-3.50 10^{-4}	9.78 10^{-3}	0	3.37
η_t	-2.44 10^{-4}	8.80 10^{-3}	0.17	2.94
e_t^1	-2.39 10^{-4}	8.36 10^{-3}	0	3.64

Table A4: Statistics on the series X_t, Y_t, η_t and e_t^1

- After the crash X_t and Y_t (23/10/1987 - 14/01/1988 (60 points)).

We obtain the following model for the cross-correlation between the two series:

$$Y_t = 0.55X_t + e_t^1.$$

The standard deviation of the parameter is equal to 0.12. The process e_t^1 is a white noise.

Series	mean	standard deviation	skewness	kurtosis
X_t	-2.49 10^{-4}	2.29 10^{-2}	-1.11	5.32
Y_t	-2.07 10^{-3}	2.50 10^{-2}	-0.21	5.38
e_t^1	0	2.16 10^{-2}	-0.43	5.03

Table A5: Statistics on the returns X_t and Y_t and on the residual e_t^1

- Including the crash X_t and Y_t (22/07/1987 - 14/01/1988 (127 points)).

We obtain the following model for the cross-correlation between the two series:

$$Y_t = 0.37X_t + e_t^1.$$

The standard deviation of the parameter is equal to 0.05. The process e_t^1 is a white noise.

Series	mean	standard deviation	skewness	kurtosis
X_t	-1.80 10^{-3}	2.88 10^{-2}	-3.85	32.20
Y_t	-2.15 10^{-3}	2.07 10^{-2}	-0.63	7.62
e_t^1	0	1.77 10^{-2}	-0.39	6.55

Table A6: Statistics on the returns X_t and Y_t and on the residual e_t^1

- Before the crash X_t and Z_t (22/07/1987 - 13/10/1987 (60 points)).

  We obtain an independent evolution between both series.

- After the crash X_t and Z_t (23/10/1987 - 14/01/1988 (60 points)).

  We obtain the following model on Z_t:

$$Z_t = -0.31 Z_{t-2} + \beta_t.$$

The standard deviation of the parameter is equal to 0.13. We obtain the following model for the cross-correlation between X_t and β_t:

$$\beta_t = 0.20 X_{t-1} + e_t^1.$$

The standard deviation of the parameter is equal to 0.12. The process e_t^1 is a white noise. Finally, using these different modelizations, we obtain the model relating Z_t, X_t and e_t^1:

$$\begin{aligned} Z_t &= 0.20 X_{t-1} - 0.06 X_{t-3} + 0.02 X_{t-5} \\ &\quad + e_t^1 - 0.31 e_{t-2}^1 + 0.10 e_{t-4}^1 - 0.03 e_{t-6}^1. \end{aligned}$$

Series	mean	standard deviation	skewness	kurtosis
X_t	-2.49 10^{-4}	2.29 10^{-2}	-1.11	5.32
Z_t	1.31 10^{-3}	1.74 10^{-2}	0.45	3.55
β_t	-2.93 10^{-5}	1.60 10^{-2}	0.30	3.82
e_t^1	5.68 10^{-4}	1.54 10^{-2}	0.28	3.72

Table A7: Statistics on the returns X_t, Z_t, β_t and on the residual e_t^1

- Including the crash X_t and Z_t (22/07/1987 - 14/01/1988 (127 points)).

  We obtain the following model for the cross-correlation between the two series:

$$Z_t = 0.58 X_{t-1} + e_t^1.$$

The standard deviation of the parameter is equal to 0.06. The process e_t^1 is not a white noise. Then, we obtain the following model on e_t^1:

$$e_t^1 = -0.21 e_{t-1}^1 + e_t^2$$

where the standard deviation of the parameter is equal to 0.08. Finally, using these different modelizations, we obtain the model relating Z_t, X_t and e_t^2:

$$Z_t \quad = \quad 0.58X_{t-1} + e_t^2 - 0.21e_{t-1}^2 + 0.04e_{t-2}^2 - 0.01e_{t-3}^2.$$

Series	mean	standard deviation	skewness	kurtosis
X_t	-1.80 10^{-3}	2.88 10^{-2}	-3.85	32.20
Z_t	1.12 10^{-3}	2.45 10^{-2}	-2.84	28.97
e_t^1	-1.92 10^{-4}	1.80 10^{-2}	0.58	6.06
e_t^2	-2.49 10^{-4}	1.76 10^{-2}	0.25	5.05

Table A8: Statistics on the returns X_t, Z_t, e_t^1 and e_t^2

- Before the crash Y_t and Z_t (22/07/1987 - 13/10/1987 (60 points)).

We obtain the following model on Y_t:

$$Y_t = 0.44Y_{t-1} + \eta_t.$$

The standard deviation of the parameter is equal to 0.13. We obtain the following model for the cross-correlation between η_t and Z_t:

$$Z_t = 0.46\eta_t + e_t^1.$$

The standard deviation of the parameter is equal to 0.19. Finally, using these different modelizations, we obtain the model relating Y_t, Z_t and e_t^1:

$$Z_t \quad = \quad 0.46Y_t - 0.20Y_{t-1} + e_t^1.$$

Series	mean	standard deviation	skewness	kurtosis
Y_t	-3.50 10^{-4}	9.78 10^{-3}	0	3.37
η_t	-2.44 10^{-4}	8.80 10^{-3}	0.17	2.94
Z_t	3.41 10^{-3}	1.35 10^{-2}	0.77	5.20
e_t^1	-1.23 10^{-4}	1.28 10^{-2}	0.65	4.51

Table A9: Statistics on the series Y_t, η_t, Z_t and e_t^1

- After the crash Y_t and Z_t (23/10/1987 - 14/01/1988 (60 points)).

We obtain the following model on Z_t:

$$Z_t = -0.31 Z_{t-2} + \beta_t.$$

The standard deviation of the parameter is equal to 0.13. We obtain the following model for the cross-correlation between Y_t and β_t:

$$\beta_t = 0.20 Y_t + 0.29 Y_{t-1} + e_t^1.$$

The standard deviation of the both parameters is equal to 0.07. The process e_t^1 is a white noise. Finally, using these different modelizations, we can give the model relating Y_t, Z_t and e_t^1:

$$\begin{aligned} Z_t &= 0.20 Y_t + 0.29 Y_{t-1} - 0.06 Y_{t-2} - 0.09 Y_{t-3} + 0.02 Y_{t-4} + 0.03 Y_{t-5} \\ &\quad + e_t^1 - 0.31 e_{t-2}^1 + 0.10 e_{t-4}^1 - 0.03 e_{t-6}^1. \end{aligned}$$

Series	mean	standard deviation	skewness	kurtosis
Y_t	-2.07 10^{-3}	2.50 10^{-2}	-0.21	5.38
Z_t	1.31 10^{-3}	1.74 10^{-2}	0.45	3.55
β_t	-2.93 10^{-5}	1.60 10^{-2}	0.30	3.82
e_t^1	3.47 10^{-4}	1.36 10^{-2}	0.60	3.51

Table A10: Statistics on the returns Y_t, Z_t, β_t and on the residual e_t^1

- Including the crash Y_t and Z_t (22/07/1987 - 14/01/1988 (127 points)).

We obtain the following model for the cross-correlation between the two series:

$$Z_t = 0.38 Y_t + 0.50 Y_{t-1} + e_t^1.$$

The standard deviation of the parameter is equal to 0.06. The process e_t^1 is a white noise.

Series	mean	standard deviation	skewness	kurtosis
Y_t	-2.15 10^{-3}	2.07 10^{-2}	-0.63	7.62
Z_t	1.12 10^{-3}	2.45 10^{-2}	-2.84	28.97
e_t^1	-1.47 10^{-4}	2.09 10^{-2}	-1.27	18.44

Table A11: Statistics on the series Y_t, Z_t and e_t^1

b) Asian crisis 1997

• Before the crash X_t and Y_t (25/07/1997 - 16/10/1997 (60 points)).

Since, X_t and Y_t are white series, we obtain the following model for the cross-correlation between these two series:

$$Y_t = 0.36X_{t-1} + e_t^1.$$

The standard deviation of the parameter is equal to 0.14. The process e_t^1 is a white noise.

Series	mean	standard deviation	skewness	kurtosis
X_t	$2.21\ \ 10^{-4}$	$9.84\ \ 10^{-3}$	0.27	4.01
Y_t	$7.36\ \ 10^{-4}$	$1.07\ \ 10^{-2}$	0.18	2.99
e_t^1	$2.84\ \ 10^{-5}$	$1.02\ \ 10^{-2}$	0.30	2.91

Table A12: Statistics on the series X_t, Y_t and e_t^1

• After the crash X_t and Y_t (28/10/1997 - 19/01/1998 (60 points)).

Since, X_t and Y_t are white series, we obtain the following model for the cross-correlation between these two series:

$$Y_t = 0.52X_t + 0.33X_{t-1} + e_t^1.$$

The standard deviation of the both parameters is equal to 0.10. The process e_t^1 is a white noise.

Series	mean	standard deviation	skewness	kurtosis
X_t	$7.54\ \ 10^{-4}$	$1.07\ \ 10^{-2}$	-0.20	3.04
Y_t	$1.12\ \ 10^{-3}$	$1.23\ \ 10^{-2}$	0.81	6.58
e_t^1	$-8.96\ \ 10^{-4}$	$7.68\ \ 10^{-3}$	-0.27	2.45

Table A13: Statistics on the series X_t, Y_t and e_t^1

• Including the crash X_t and Y_t (25/07/1997 - 19/01/1998 (127 points)).

We obtain the following model on X_t:

$$X_t = 0.28X_{t-10} + \varepsilon_t.$$

The standard deviation of the parameter is equal to 0.09. We obtain the following model for the cross-correlation between ε_t and Y_t:

$$Y_t = 0.35\varepsilon_t + 0.50\varepsilon_{t-1} + e_t^1.$$

The standard deviation for the both parameters is equal to 0.07, and e_t^1 is a white noise.
Finally, using these different modelizations, we obtain the model relating Y_t, X_t and e_t^1:

$$Y_t = 0.35X_t + 0.50X_{t-1} - 0.10X_{t-10} - 0.14X_{t-11} + e_t^1.$$

Series	mean	standard deviation	skewness	kurtosis
X_t	1.91 10^{-4}	1.27 10^{-2}	-0.83	9.79
ε_t	-4.01 10^{-5}	1.23 10^{-2}	-1.09	11.00
Y_t	-4.48 10^{-5}	1.20 10^{-2}	0.38	5.03
e_t^1	5.80 10^{-5}	9.86 10^{-3}	0.25	3.05

Table A14: Statistics on the series X_t, ε_t, Y_t, η_t and e_t^1

- Before the crash X_t and Z_t (25/07/1997 - 16/10/1997 (60 points)).

Since, X_t and Z_t are white series, we obtain the following model for the cross-correlation between these two series:

$$Z_t = 0.58X_{t-1} + e_t^1.$$

The standard deviation of the parameter is equal to 0.18. The process e_t^1 is a white noise.

Series	mean	standard deviation	skewness	kurtosis
X_t	2.21 10^{-4}	9.84 10^{-3}	0.27	4.01
Z_t	-2.10 10^{-3}	1.45 10^{-2}	0.33	2.66
e_t^1	-1.27 10^{-4}	1.35 10^{-2}	0.68	3.32

Table A15: Statistics on the series X_t, Z_t and e_t^1

- After the crash X_t and Z_t (28/10/1997 - 19/01/1998 (60 points)).

We obtain the following model on Z_t:

$$Z_t = -0.31 Z_{t-2} + \beta_t.$$

The standard deviation of the parameter is equal to 0.13. We obtain the following model for the cross-correlation between the series X_t and β_t:

$$\beta_t = 0.78 X_{t-1} + e_t^1.$$

The standard deviation of the both parameters is equal to 0.20. The process e_t^1 is a white noise. Finally, using these different modelizations, we obtain the model relating X_t, Z_t and e_t^1:

$$\begin{aligned} Z_t &= 0.78X_{t-1} - 0.24X_{t-3} + 0.08X_{t-5} - 0.02X_{t-7} \\ &\quad + e_t^1 - 0.31e_{t-2}^1 + 0.10e_{t-4}^1 - 0.03e_{t-6}^1. \end{aligned}$$

Series	mean	standard deviation	skewness	kurtosis
X_t	7.54 10^{-4}	1.07 10^{-2}	-0.20	3.04
Z_t	-1.17 10^{-3}	2.56 10^{-2}	0.34	3.81
β_t	-4.19 10^{-4}	2.44 10^{-2}	0.71	4.27
e_t^1	-8.83 10^{-4}	2.29 10^{-2}	0.88	4.66

Table A16: Statistics on the series X_t, Z_t, β_t and e_t^1

- Including the crash X_t and Z_t (25/07/1997 - 19/01/1998 (127 points)).

We obtain the following models on X_t and Z_t:

$$X_t = 0.28 X_{t-10} + \varepsilon_t$$

and

$$Z_t = -0.26 Z_{t-2} + \beta_t.$$

The standard deviation of the both parameters is equal to 0.09. We obtain the following model for the cross-correlation between ε_t and β_t:

$$\beta_t = 0.56 \varepsilon_{t-1} + e_t^1.$$

The standard deviation for the parameter is equal to 0.14, and e_t^1 is a white noise.
Finally, using these different modelizations, we obtain the model relating Y_t, Z_t and e_t^1:

$$\begin{aligned} Z_t &= 0.56X_{t-1} - 0.15X_{t-3} + 0.04X_{t-5} - 0.01X_{t-7} + 0.16X_{t-11} - 0.04X_{t-13} \\ &\quad + e_t^1 - 0.26e_{t-2}^1 + 0.07e_{t-4}^1 - 0.02e_{t-6}^1. \end{aligned}$$

Series	mean	standard deviation	skewness	kurtosis
X_t	1.91 10^{-4}	1.27 10^{-2}	-0.83	9.79
ε_t	-4.01 10^{-5}	1.23 10^{-2}	-1.09	11.00
Z_t	-2.26 10^{-3}	2.06 10^{-2}	0.43	4.75
β_t	-1.70 10^{-4}	1.99 10^{-2}	0.71	5.13
e_t^1	5.80 10^{-5}	9.86 10^{-3}	0.25	3.05

Table A17: Statistics on the series X_t, ε_t, Z_t, β_t and e_t^1

- Before the crash Y_t and Z_t (25/07/1997 - 16/10/1997 (60 points)).

Since, Y_t and Z_t are white series, we obtain the following model for the cross-correlation between these two series:

$$Z_t = 0.35Y_{t-1} + e_t^1.$$

The standard deviation of the parameter is equal to 0.17. The process e_t^1 is a white noise.

Series	mean	standard deviation	skewness	kurtosis
Y_t	7.36 10^{-4}	1.07 10^{-2}	0.18	2.99
Z_t	-2.10 10^{-3}	1.45 10^{-2}	0.33	2.66
e_t^1	-8.53 10^{-6}	1.41 10^{-2}	0.46	2.60

Table A18: Statistics on the series Y_t, Z_t and e_t^1

- After the crash Y_t and Z_t (28/10/1997 - 19/01/1998 (60 points)).

We obtain the following model on Z_t:

$$Z_t = -0.31Z_{t-2} + \beta_t.$$

The standard deviation of the parameter is equal to 0.13. We obtain the following model for the cross-correlation between the series Y_t and β_t:

$$\beta_t = 1.01Y_t + e_t^1.$$

The standard deviation of the parameter is equal to 0.22. The process e_t^1 is a white noise. Finally, using these different modelizations, we obtain the model relating Y_t, Z_t and e_t^1:

$$\begin{aligned} Z_t &= 1.01Y_t - 0.31Y_{t-2} + 0.10Y_{t-4} - 0.03X_{t-6} + 0.01X_{t-8} \\ &\quad + e_t^1 - 0.31e_{t-2}^1 + 0.10e_{t-4}^1 - 0.03e_{t-6}^1 + 0.01e_{t-8}^1. \end{aligned}$$

Series	mean	standard deviation	skewness	kurtosis
Y_t	1.12 10^{-3}	1.23 10^{-2}	0.81	6.58
Z_t	-1.17 10^{-3}	2.56 10^{-2}	0.34	3.81
β_t	-4.19 10^{-4}	2.44 10^{-2}	0.71	4.27
e_t^1	-4.19 10^{-4}	2.10 10^{-2}	0.37	3.42

Table A19: Statistics on the series Y_t, Z_t, β_t and e_t^1

- Including the crash Y_t and Z_t (25/07/1997 - 19/01/1998 (127 points)).

We obtain the following model on Z_t:

$$Z_t = -0.26 Z_{t-2} + \beta_t.$$

The standard deviation of the parameter is equal to 0.09. We obtain the following model for the cross-correlation between Y_t and β_t:

$$\beta_t = 0.70 Y_t + e_t^1.$$

The standard deviation for the parameter is equal to 0.13, and e_t^1 is a white noise.
Finally, using these different modelizations, we obtain the model relating Y_t, Z_t and e_t^1:

$$Z_t = 0.70Y_t - 0.18Y_{t-2} + 0.05Y_{t-4} - 0.01Y_{t-6} + e_t^1 - 0.26e_{t-2}^1 + 0.07e_{t-4}^1 - 0.02e_{t-6}^1.$$

Series	mean	standard deviation	skewness	kurtosis
Y_t	-4.48 10^{-5}	1.20 10^{-2}	0.38	5.03
Z_t	-2.26 10^{-3}	2.06 10^{-2}	0.43	4.75
β_t	-1.70 10^{-4}	1.99 10^{-2}	0.71	5.13
e_t^1	-1.70 10^{-4}	1.80 10^{-2}	0.50	4.30

Table A20: Statistics on the series Y_t, Z_t, β_t and e_t^1

c) Russian crisis 1998

- Before the crash X_t and Y_t (04/06/1998 - 26/08/1998 (60 points)).

We obtain the following model on X_t:

$$X_t = 0.32 X_{t-8} + \varepsilon_t.$$

The standard deviation of the parameter is equal to 0.13. We obtain the following model for the cross-correlation between ε_t and Y_t:

$$Y_t = 0.67\varepsilon_t + e_t^1.$$

The standard deviation of the parameter is equal to 0.16. The process e_t^1 is a white noise. Finally, using these different modelizations, we obtain the model relating Y_t, X_t and e_t^1:

$$Y_t = 0.67X_t - 0.21X_{t-8} + e_t^1.$$

Series	mean	standard deviation	skewness	kurtosis
X_t	-2.57 10^{-5}	1.10 10^{-2}	-0.63	3.61
ε_t	1.11 10^{-4}	1.04 10^{-2}	-0.39	3.09
Y_t	-1.46 10^{-3}	1.44 10^{-2}	-0.18	2.81
e_t^1	-7.44 10^{-5}	1.26 10^{-2}	-0.44	2.83

Table A21: Statistics on the returns X_t and Y_t and on the residual e_t^1

- After the crash X_t and Y_t (08/09/1998 - 30/11/1998 (60 points)).

We obtain the following model for the cross-correlation between X_t and Y_t:

$$Y_t = 0.74X_t + e_t^1.$$

The standard deviation of the parameter is equal to 0.14. The process e_t^1 is a white noise process.

Series	mean	standard deviation	skewness	kurtosis
X_t	2.31 10^{-3}	1.43 10^{-2}	-0.16	3.38
Y_t	5.99 10^{-4}	1.86 10^{-2}	-0.22	2.96
e_t^1	0	1.53 10^{-2}	0.33	3.62

Table A22: Statistics on the returns X_t, Y_t and e_t^1

- Including the crash X_t and Y_t (04/06/1998 - 30/11/1998 (127 points)).

We obtain the following model for the cross-correlation between X_t and Y_t:

$$Y_t = 0.55X_t + e_t^1.$$

The standard deviation for the parameter is equal to 0.08, and e_t^1 is a white noise.

Series	mean	standard deviation	skewness	kurtosis
X_t	5.99 10^{-4}	1.53 10^{-2}	-0.61	6.48
Y_t	-2.69 10^{-4}	1.67 10^{-2}	-0.25	3.18
e_t^1	0	1.44 10^{-2}	0.11	3.31

Table A23: Statistics on the series X_t, Y_t, ε_t, η_t and e_t^1

- Before the crash X_t and Z_t (04/06/1998 - 26/08/1998 (60 points)).

  We obtain an independent evolution between both series.

- After the crash X_t and Z_t (08/09/1998 - 30/11/1998 (60 points)).

  We obtain an independent evolution between both series.

- Including the crash X_t and Z_t (04/06/1998 - 30/11/1998 (127 points)).

  We obtain an independent evolution between both series.

- Before the crash Y_t and Z_t (04/06/1998 - 26/08/1998 (60 points)).

  We obtain the following model on Z_t:

$$Z_t = 0.31 Z_{t-1} + \beta_t.$$

The standard deviation of the parameter is equal to 0.13. We obtain the following model for the cross-correlation between Y_t and β_t:

$$\beta_t = 0.61 Y_t + e_t^1.$$

The standard deviation of the parameter is equal to 0.14. Finally, using these different modelizations, we obtain the model relating Z_t, Y_t and e_t^1:

$$\begin{aligned} Z_t &= 0.61 Y_t + 0.19 Y_{t-1} - 0.06 Y_{t-2} + 0.02 Y_{t-3} \\ &\quad + e_t^1 + 0.31 e_{t-1}^1 - 0.10 e_{t-2}^1 + 0.03 e_{t-3}^1. \end{aligned}$$

Series	mean	standard deviation	skewness	kurtosis
Y_t	-1.46 10^{-3}	1.44 10^{-2}	-0.18	2.81
Z_t	-1.69 10^{-3}	1.85 10^{-2}	0.76	3.30
β_t	-4.31 10^{-5}	1.76 10^{-2}	0.36	2.81
e_t^1	-4.31 10^{-5}	1.53 10^{-2}	0.59	3.30

Table A24: Statistics on the returns Y_t, Z_t and on the residuals β_t and e_t^1

- After the crash Y_t and Z_t (08/09/1998 - 30/11/1998 (60 points)).

We obtain the following models on Z_t:

$$Z_t = -0.29 Z_{t-5} + \beta_t.$$

The standard deviation of the parameter is equal to 0.13. We obtain the following model for the cross-correlation between Y_t and β_t:

$$\beta_t = 0.38 Y_t + 0.40 Y_{t-1} + e_t^1.$$

The standard deviation of the both parameters is equal to 0.16. Moreover, we adjust the following model for e_t^1:

$$e_t^1 = -0.31 e_{t-4}^1 + e_t^2.$$

The standard deviation of the parameter is equal to 0.13. Using these modelizations, we obtain the following model between Y_t and Z_t:

$$\begin{aligned} Z_t &= 0.38 Y_t + 0.40 Y_{t-1} - 0.11 Y_{t-5} - 0.12 Y_{t-6} + 0.03 Y_{t-10} + 0.03 Y_{t-11} \\ &\quad + e_t^2 - 0.31 e_{t-4}^2 - 0.29 e_{t-5}^2 + 0.10 e_{t-8}^2 + 0.09 e_{t-9}^2. \end{aligned}$$

Series	mean	standard deviation	skewness	kurtosis
Y_t	5.99 10^{-4}	1.86 10^{-2}	-0.22	2.96
Z_t	1.33 10^{-3}	2.71 10^{-2}	1.48	7.90
β_t	9.35 10^{-5}	2.60 10^{-2}	1.16	6.49
e_t^1	8.28 10^{-4}	2.27 10^{-2}	0.91	5.53
e_t^2	1.36 10^{-3}	2.16 10^{-2}	1.45	7.35

Table A25: Statistics on the series Y_t, Z_t, β_t, e_t^1 and e_t^2

- Including the crash Y_t and Z_t (04/06/1998 - 30/11/1998 (127 points)).

We obtain the following model for the cross-correlation between Y_t and Z_t:

$$Z_t = 0.44 Y_t + 0.38 Y_{t-1} + e_t^1.$$

The standard deviation of the both parameters is equal to 0.11.

Series	mean	standard deviation	skewness	kurtosis
Y_t	-2.69 10^{-4}	1.67 10^{-2}	-0.25	3.18
Z_t	4.31 10^{-4}	2.36 10^{-2}	1.33	7.38
e_t^1	9.83 10^{-5}	2.13 10^{-2}	1.00	6.58

Table A26: Statistics on the series Y_t, Z_t and e_t^1

References

[1] Ali M.M., Mikhail N.N., Haq M.S. (1978), "A class of bivariate distributions including the bivariate logistics, *J. Multivariate Anal.*, **8**, 405 - 412.

[2] Avouyi-Dovi S., Guégan D., Ladoucette S. (2002), "Applications des processus de longue mémoire à l'analyse des indices boursiers", NER 94, Banque de France, France.

[3] Bertero E., Mayer C. (1990), "Structure and performance: Global interdependence od stock markets around the crash of october 1987", *The European Economic Review*, **34**, 1155 - 1180.

[4] Boyer B.H., Gibson M.S., Loretan M. (1999), "Pitfalls in tests for change in correlations", *Preprint* n^0 *597*, Board of Governors of the Federal Reserve System, United States.

[5] Brockwell P.J., Davis R.A. (1996), *Introduction to time series and forecasting*, Springer texts in statistics, Springer, New York.

[6] Brummelhuis R., Guégan D. (2000), "Multi-period conditional distribution functions for heteroscedastic models with applications to VaR", *Preprint University of Reims*, **00-13**, France.

[7] Burns A. Mitchell W.C. (1946), "Measuring business cycles", *NBER*, New York.

[8] Cook R.D., Johnson M.E. (1981), "A family of distributions for modelling nonelliptical symmetric multivariate data", *J.R.S.S. B*, **43**, 210 - 218.

[9] Deheuvels P. (1978), "Caractérisation complète des lois extrêmes multivariées et de la convergence des types extrêmes", *Publications de l'Institut de Statistique de l'Université de Paris*, **23**, 1 - 36.

[10] Embrechts P., McNeil A., Straumann D. (1999), "Correlation and dependence in risk managment: properties and pitfalls", *Preprint ETH*, Zurich.

[11] Fama E.F., French K. (1986), "Permanent and transitory components of stock prices", *CRSP Working paper*, **178**, mimeo, Chicago.

[12] Frank M.S. (1979), "On the simultaneous associativity of $F(X,Y)$ and $x-y-F(X,Y)$", *Aequationes Math.*, **19**, 1964 - 1976.

[13] Frey R, McNeil A. (2000), "Estimation of tail-related risk measures for heteroscedastic financial time series: an extreme value approach", *Journal of Empirical Finance*, **7**, 271 - 300.

[14] Gray H.L., Zhang N.F., Woodward W.A (1989), "On generalized fractional processes", *Journal of Time Series Analysis*, **10**, 233 - 257.

[15] Guégan D., Ladoucette S. (2001 b), "Non-mixing properties of long memory processes", *C.R.A.S.*, Série I, **333**, 373 - 376.

[16] Gumbel E.J. (1958), "Distributions à plusieurs variables dont les marges sont données", *Ann. Univ. Lyon*, **3**, 53 - 77.

[17] Harding D., Pagan A. (1999), "Knowing the cycle", *Preprint*, University of Melbourne, Australia.

[18] Harding D., Pagan A. (2002), "Dissecting the cycle: A methodological investigation", *Journal of Monetary Economics*, **49**, 365-381.

[19] Kimeldorf G., Sampson A.R. (1975), " Uniform representations of bivariate distributions", *Comm. Stat.*, **4**, 617 - 627.

[20] King M.A., Wadhwani S. (1990), "Transmission of volatility between stock markets", *The Review of Financial Studies*, **3**, 5 - 33.

[21] King R.G., Plosser C.I. (1994), "Real business cycles and the test of the Adelmans", *Journal of Monetary Economics*, **33**, 405 - 438.

[22] Lehmann E.L. (1966), " Some concepts of dependence", *Ann. Math. Stat.*, **37**, 1137 - 1153.

[23] Ling C.H. (1965), "Representation of associative functions", *Publ. Math. Debrecen*, **12**, 189 - 212.

[24] Longin F., Solnik B. (1995), "Is the correlation in international equity returns constant: 1969 - 1990?", *Journal of International Money and Finance*, **14**, 3 - 26.

[25] Longin F., Solnik B. (2001), "Extreme correlation of international equity markets", *The Journal of Finance*, **LVI**, 649 - 676.

[26] Mardia K.V. (1962), " Multivariate Pareto distributions", *Ann. Math. Stat.*, **33**, 1008 - 1015.

[27] Mardia K.V. (1970), *Family of bivariate distributions*, Griffin, London.

[28] McDermott C.J., Scott A. (1999), "Concordance in Business cycles", Preprint G99/7, Reserve Bank of New Zealand.

[29] Oakes D. (1982), "A model for association in bivariate survival data", *J.R.S.S. B*, **60**, 516 - 522.

[30] Plackett R.L. (1965), "A class of bivariate distributions", *JASA*, **60**, 516 - 522.

[31] Poterba J., Summers L.H. (1987), "Mean reversion in stock returns: evidence and implications", *Journal of Financial Economics*, **22**, 27 - 60.

[32] Satterhwaite S.P., Hutchinson T.P. (1978), "A generalization of Gumbel's bivariate logistic distribution", *Metrika*, **25**, 163 - 170.

[33] Schweizer B., Sklar A. (1983), *Probabilistic metric spaces*, New York, North Holland.

[34] Sklar A. (1959), "Fonctions de répartition à n dimensions et leurs marges", *Publications de l'Institut de Statistique de l'Université de Paris*, **8**, 229 - 231.

[35] Takahasi K (1965), "Note on the multivariate Burr's distribution", *Ann. Inst. Stat. Math.*, **17**, 257 - 260.

[36] Yanagimoto T., Okamoto M. (1969), " Partial orderings of permutations and monotonicity of a rank correlation statistic", *Ann. Inst. Stat. Math.*, **21**, 489 - 506.

Notes d'Études et de Recherche

1. C. Huang and H. Pagès, “Optimal Consumption and Portfolio Policies with an Infinite Horizon: Existence and Convergence,” May 1990.

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38. Banque de France - CEPREMAP - Direction de la Prévision - Erasme - INSEE - OFCE, « Structures et propriétés de cinq modèles macroéconomiques français », juin 1996.

39. F. Rosenwald, « L'influence des montants émis sur le taux des certificats de dépôts », octobre 1996.

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46. E. Jondeau, « Représentation VAR et test de la théorie des anticipations de la structure par terme », septembre 1997.

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