WORKING paper

BANQUE DE FRANCE
EUROSYSTÈME

Sovereign Debt and International Trade

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ABSTRACT

Evidence suggests that sovereign defaults disrupt international trade. As a consequence, countries that are more open have more to lose from a sovereign default and are less inclined to renege on their debt. In turn, lenders should trust more open countries and charge them with lower interest rate. In most cases, the country should also borrow more debt as it gets more open. This paper formalizes this idea in a sovereign debt model *à la* Eaton and Gersovitz (1981), proves these theoretical relations, and quantifies them in a calibrating model. We also provide evidence suggesting a causal relationship between trade and debt or CDS spreads, using gravitational instrumental variables from Frankel and Romer (1999) and Feyrer (2019) as a source for exogenous variation in trade openness.

Keywords: Sovereign Debt, International Trade and Finance, Economic Integration

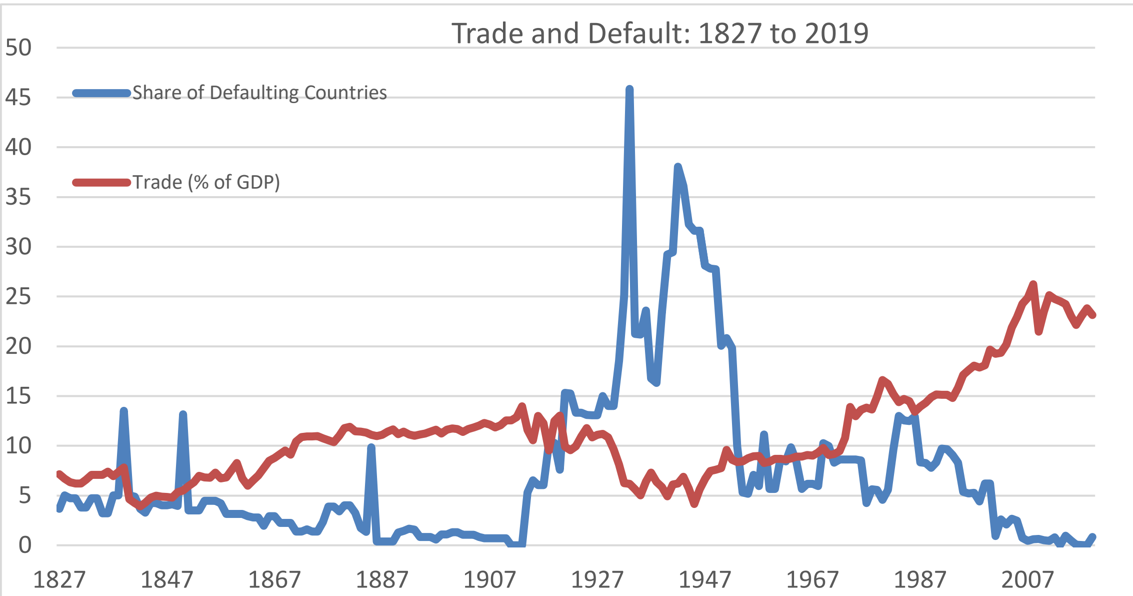
JEL classification: H63, B17, F15

[1] Banque de France, charles.serfaty@banque-france.fr. I am infinitely grateful to my PhD director from MIT, Arnaud Costinot, for his constant support and inventive suggestions, that went unabated by the 2020 crisis. I owe a great debt of gratitude to my PhD advisor Ivàn Werning for his incredibly helpful insights. I am also especially thankful to Ricardo Caballero and Daniel Cohen for insightful discussions about this paper. I want to thank Daron Acemoglu, Marios Angeletos, Kai Arvai, David Atkin, Antoine Berthou, Schwan Badirou Gafari, Agnès Bénassy-Quéré, Martin Beraja, Antoine Berthou, Nuno Coimbra, Dave Donaldson, Olivier de Bandt, Joel Flynn, Michele Fornino, Juliette Fournier, Masao Fukui, Clémence Idoux, Antoine Levy, Andrea Manera, Ugo Panizza, Parinitha Sastry, Julia Schmidt, Alp Simsek, Claudia Steinwender, John Sturm, Adrien Verdelhan as well as all other participants from Séminaire Fourgeaud, MIT macro lunch and trade tea seminars, Banque de France seminars for their very useful comments.

Non-Technical Summary

One of the consequences of a sovereign default is that it makes it more difficult in a country to borrow from foreigners but also to trade with foreigners. As a consequence, a sovereign default is more frightening for more open economies, which should make more open economies more reluctant to default. Consequently, more open economies should be able to sustain higher debt-to-GDP ratios because their willingness to pay their debt back should be higher. Moreover, this higher willingness to pay should be anticipated by the financial markets, which should translate into lower cost of financial borrowing. Finally, countries should be tempted to enjoy this lower cost of borrowing and borrow more debt – although not enough to make default more likely.

The starting point of the paper is the long-run relationship between sovereign defaults in the world (as measured by the share of countries in the world that default weighted by their GDP) and international trade (total volume of trade divided by world GDP). It is apparent on the data that waves of default coincide with decreases in commercial integration.



Moreover, a regression using historical data shows that, in the years following a sovereign default, imports decrease by 10% or more depending on the specification. The same is true for exports, though the magnitude tends to be lower, as defaulting countries are generally equilibrating their trade balance. Therefore, total gains from trade are lower after a default.

From these stylized facts, I write a sovereign debt model with strategic default and international trade. Not only do countries borrow from other countries to smooth their consumption over time, but they also trade with the rest of the world to enjoy the benefits associated with specialization. The gains from trade can be computed directly from the openness ratio in the model. If a country defaults, it loses its ability to borrow from financial markets and its trade costs increase, which reflects a disruption in trade market credits. Since imports and exports tend to decrease proportionally because of this increase in trade costs, more open countries find it more costly to default. I prove that, because rational markets anticipate that, more open countries benefit from lower default costs through lower interest rates. Impatient governments benefit from that discount to borrow more debt. In general equilibrium, these properties stay true. Calibrations of a rich model including realistic debt maturities and possibility to redeem from default as well as fluctuations from terms of trade shows that governments emit more debt, which they borrow at a lower interest rate, and are less likely to default.

Finally, I test these theoretical predictions empirically. To avoid reverse causality issues or spurious regressions driven by political cycles, I use geographic instruments from the international trade literature as proxies for trade. The Frankel-Romer instrument is an instrument that predicts total trade in a country based on its geographic proximity with other big countries: small countries surrounded by big populated countries tend to be more than isolated large countries. The Feyrer instrument is a similar instrument that is dynamic: it takes advantage of the technical transformations in transport costs: air transport, that was too expensive for trade, became a more common tool for trade in the last decades. As a consequence, the propension to trade of a country does not depend the same way on sea distance and air distance depending on the period we consider: air distance has become a more important determinant. Thanks to these instruments, I can estimate plausible causal estimates for the effect of trade openness on spreads and debt. Let us assume that a country suddenly trades twice as much relative to its GDP, as it happened in France between the 1960s and today. I find that such a country benefits from a 300 b.p. decrease in its spreads. Moreover, this country tends to borrow twice as much debt.

Dette souveraine et commerce international

RÉSUMÉ

Les défauts souverains perturbent le commerce international. Par conséquent, les pays les plus ouverts ont plus à perdre d'un défaut souverain et sont moins enclins à renier leur dette. En retour, les prêteurs devraient faire davantage confiance aux pays plus ouverts et leur offrir un taux d'intérêt plus faible. Dans la plupart des cas, le pays devrait également s'endetter davantage à mesure qu'il s'ouvre davantage. Cet article formalise cette idée dans un modèle de dette souveraine à la Eaton et Gersovitz (1981), prouve ces relations théoriques et les illustre dans un modèle quantitatif. Nous fournissons également des preuves suggérant une relation causale entre le commerce et la dette ou les *spreads* de CDS, en utilisant les variables instrumentales gravitationnelles de Frankel et Romer (1999) et Feyrer (2019) comme source de variation exogène de l'ouverture commerciale.

Mots-clés : dette souveraine, commerce international et finance, intégration économique

1 Introduction

The main peculiarity of sovereign debt contracts is that repayments are not easily enforceable by the lender: a sovereign country with a strong enough army and divided enough lenders can default without expecting dire consequences. In the absence of enforceable contracts, a good borrower is someone with whom the lender has frequent business relations, as suggested by the repeated game literature. A borrower afraid of paying the cost of losing those relationships would be incentivized to repay debt. From the point of view of sovereign borrowing, a form of relation with the outside world can serve as a commitment device. An obvious form of such a reputational cost is the interruption of sovereign borrowing. However, Bulow and Rogoff (1989) proved an theorem about impossibility of sovereign debt if the only cost of default is the impossibility of borrowing in the future. We must therefore assume that some kind of relation with the outside world gets interrupted after a sovereign default, making it worth for a sovereign debtor repaying its debt under normal circumstances.

What kind of relationship with the outside world gets interrupted after a sovereign default exactly? Does it get interrupted for external reasons (other countries deciding to sanction defaulters) or internal reasons (destruction of the financial local markets, relying on sovereign debt)? The answer is not entirely clear from historical precedents nor from the literature, but an obvious candidate is international trade, because it summarizes relations with the outside world from a static point of view. As we show in figure 1, periods of decreasing commercial integration have coincided with global default waves since 1800. It suggests that during defaults, international trade decreases or vice-versa.

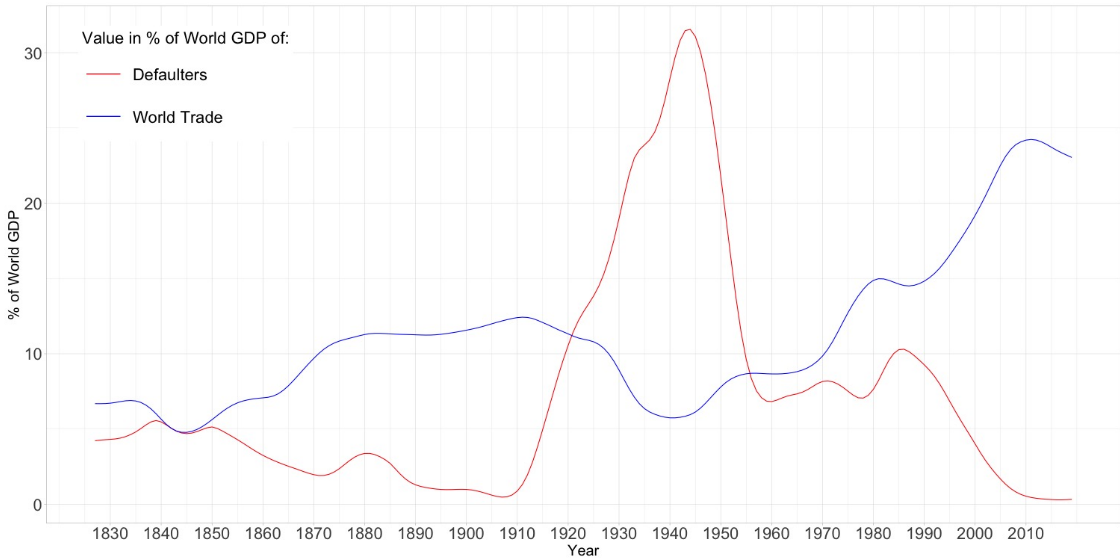


Figure 1.1: Trade and default from 1815 to 2007 (HP-filtered). Sources: Reinhart, Reinhart, and Trebesch (2016), Fouquin, Hugot, et al. (2016).

This paper argues that international trade is an important component of non-reputational default costs. It argues that trade gets interrupted partially in the wake of a sovereign default. As a consequence, default is more costly for large traders. Indeed, larger gross trade flows imply that more is at stake when a government decides to go into financial autarky and to default. The defaulting country's inhabitants and firms can face tighter international constraints or trust issues which can affect their ability to trade internationally. Thus, governments in countries more open to trade should find it easier to borrow from international lenders: trade acts as a commitment device for these borrowers. We argue in favor of this mechanism with a simplified Eaton and Gersovitz (1981) model and provide empirical evidence in favor of it with cross-country regressions using instrumental variables for trade inspired by Frankel and Romer (1999) and Feyrer (2019) .

This paper completes our understanding of sovereign default costs but it also has important normative consequences. Indeed, a direct consequence of this paper's argument is that protectionist policies restricting both imports and exports should deteriorate government's ability to borrow.[1] Moreover, the large decrease in transport costs that has been observed since World War 2 can explain the development of sovereign debt markets: easier transport means more trade and more sensitivity to autarky as a result, therefore more commitment.

Section 2 starts with motivating evidence that sovereign defaults lead to a shrinkage in trade. It revisits the findings in Rose (2005) with updated data and more general controls. It finds that periods of default seem to coincide with declines in trade. It also finds that during a sovereign default, bilateral trade between a defaulter and its partners decreases by 10-50% depending on the specification.

Section 3 formalizes the idea with a simple model inspired by Eaton and Gersovitz (1981) where trade autarky is the cost of sovereign default. It finds that openness, defined as lower trade costs, improves a government's ability to borrow and also its actual borrowings in most cases. A calibration exercise on a state-of-the-art sovereign debt model inspired by Aguiar et al. (2016) confirms these results quantitatively and finds a noticeable increase in debt levels as trade openness increases (due to lower trade costs or higher foreign demand).

Section 4 presents the data and instruments that we use for the empirical part. Section 5 presents the empirical results. Because of endogeneity concerns that trade variations depends on sovereign debt finance, we need to use instrumental variables, inspired by Frankel and Romer (1999) and Feyrer (2019). We define them using geographical predictors of trade and time variation in the relative importance of trade and sea distances and exploit them for regression analysis. They confirm the results Section 3: more openness leads to cheaper credit costs and to larger levels of debt as well. An increase in total

[1] Assuming these policies are suboptimal from a static point of view and there are no dynamic externalities: in this case, a protectionist policy is seen as a self-inflicted damage. Therefore, the cost of this kind of policies increases when one takes into account its effects on sovereign debt crises. The existence of an optimal positive tariff could change the direction of our claim. We do not explore these issues in this paper.

volume of trade by 10% is shown to lead up to a 30 b.p. decrease in CDS spreads, and to a 10% increase in debt. The orders of magnitude for the causal impact are the same as for the the results we obtain from the calibrated sovereign debt model.

Literature Review One of the questions in the sovereign debt literature is why a sovereign should repay when there is no clear mechanism to enforce either repayment or punishment from the point of view of investors. Private firms may be constrained to go bankrupt and their assets are then shared between their debtors when they default in developed financial markets. Direct invasions of defaulting countries by creditors have not been frequent since 1945, although they used to be frequent, as shown in Mitchener and Weidenmier (2010).[2] Government's assets cannot be seized easily. In that case, why should a sovereign borrower ever repay debt at any moment? Our suggestion to that old question is that international trade is a casualty of sovereign default, either because of sanctions or because of reliance of trade on sovereign debt finance (we stay agnostic about this mechanism).

Eaton and Gersovitz (1981), in a seminal paper, argue that reputation concerns may explain government's willingness to borrow. But Bulow and Rogoff (1989) that we cited earlier proved that the own model of Eaton and Gersovitz (1981) was not consistent with positive levels of borrowing, and one assume that there is direct cost for defaulting apart from financial autarky. Bulow and Rogoff (1989), like Kaletsky (1985) or Cole and Kehoe (1998), suggested the risk of trade wars or trade interruption, either because of retaliation, trade finance interruption or reputational spillovers was such a direct penalty of default. Mendoza and Yue (2012) directly used trade interruption as the cost of default, and they attributed it to trade credit, but focused on the dynamic implications of this assumption. In their model, trade finance deteriorates in bad times and the commercial interest rate is equal to the sovereign debt interest rate. As consequence, incentives to default in bad times get amplified. They did not study the effect of trade openness on sovereign debt finance as we do in this paper and focused more on the dynamic aspects of this assumption.

Most other sovereign debt papers took this cost of default as a given black box and rather focused on net trade flows rather than gross trade flows: current account and its relation to business cycles should indeed matter for sovereign debt, as underscored by Aguiar and Gopinath (2007) or Aguiar and Gopinath (2006). But relations between debtor country and the rest of the world is not summarized by debt or net trade flows. It also relies on *gross* trade flows. There are more general reputation concerns that are not about *intertemporal* trade, but also about *intratemporal* trade: Cole and Kehoe (1998) argued there might be reputation spillovers on other activities, such as trade, but they have not been studied widely. There has also been a trade literature focused on the links between intertemporal and intratemporal trade: Eaton et al. (2016), Reyes-Heroles et al. (2016). Kikkawa and Sasahara (2020) study more explicitly the relation between trade

[2]For example, the small state of Newfounland, as a consequence of its default in 1933, lost its sovereignty to Canada.

and sovereign default. In their model, default is associated with a negative productivity shock, as in the seminal paper of Arellano (2008). This negative productivity shock limits countries' incentives to default. In the presence of trade, Kikkawa and Sasahara (2020) note that the same productivity is also associated with terms-of-trade effects that affect both the value of a country's endowment as well as the value of its debt and, in turn, its probability to default. In contrast, default in our model is associated with a demand rather than a supply shock: countries that default lose foreign demand, while their endowments remain unchanged. This implies, in particular, that more openness to trade always creates less incentives to default in our model.

Fitzgerald (2012) also studied the link between risk sharing between countries and trade costs, following a suggestion made by Obstfeld and Rogoff (2000) that international macroeconomic puzzle might be attributed to trade costs. However, these papers do not feature defaultable sovereign debt.

The trade disruption occurring after sovereign default has been documented in several papers, prominently in Rose (2005), whose evidence we replicate later; similar contributions include Manasse and Roubini (2009). Martinez and Sandleris (2011), Kohlscheen and O'Connell (2008), Borensztein and Panizza (2009), Zymek (2012) found similar results, arguing that trade credit was the driver of this effect, rather than direct sanctions. On the microeconomic level, Gopinath and Neiman (2014), Borensztein and Panizza (2010), Arteta and Hale (2008) , Hébert and Schreger (2017) found in different contexts that exporting firms were disproportionately hurt by sovereign default, which is quite consistent with our hypothesis. Nevertheless, citetcaselli2021benefits found that Columbia, that did not default in Latin America in the 1980s, might have had lower exports and higher imports than other defaulting countries: this is not inconsistent with our framework and results as long as imports are assumed to decrease as a consequence of default.

Finally, this paper also has theoretical contributions that belong to general sovereign debt models: it exploits the recent theoretical breakthrough by Aguiar and Amador (2019) to derive certain analytical properties of sovereign debt models, such as comparative statics default threshold and borrowing when there is a variation of structural parameters. Analytical results are rare in this field, in spite of other recent contributions such as Auclert and Rognlie (2016), that are summarized in Aguiar and Amador (2021).

2 Motivational Evidence: Trade Collapse After Default

In this Section, we update findings in Rose (2005) including more recent years, with a different method: instead of defining default as an event, we are going to distinguish default phrases (from default to the end of restructuring) as in Reinhart and Rogoff (2009). We are also going to use more data points and to allow for more general controls: for example, a bilateral pair fixed effect and time-varying regional fixed effects, instead of geographical predictors of trade and simple year fixed effects.

2.1 Data and Specification

To define sovereign defaults, we use data by Reinhart and Rogoff (2009), available on their website and updated up to 2012. Their data starts in 1800 and allows use to include some early sovereign defaults. In their data, a country is considered defaulting as long as it did not find an agreement with creditors (on average, this period lasted 7 years). Therefore, restructuring to date the end of default has a broader end than Rose (2005) who used Paris debt renegotiations to define defaulting countries. Rose (2005) found lasting effects that were similar from one year to the other: however, the size of the effect of default on trade did not vary significantly in his findings, so that we do not study the dynamic effects of default. We also use CEPII data from Fouquin, Hugot, et al. (2016) that give historical series of bilateral trade data and allow us to go far as back as 1800 to estimate the effect of sovereign default on bilateral trade data.

We test the following equation with different sets of controls for all pairs of countries (i, j) and all years t:

$$\ln Exports_{i,j,t} = \gamma^e D_{i,t} + \gamma^i D_{j,t} + \beta Controls_{i,j,t} + \varepsilon_{i,j,s,t}, \qquad (2.1)$$

where $Exports_{i,j,t}$ is exports from country i to country j at year t, of which we take the log, except when we include null observations.[3] $D_{i,t}$ is a dummy variable indicating whether a country is still defaulting, $Controls_{i,j,t}$ is a set of controls including at least a pair fixed effect $\alpha_{i,j}$ taking into account all possible fixed predictors of trade and a year fixed effect α_t taking into account variation. We allow for several other types of controls, as a time varying pair fixed effect $\alpha_{i,j,c(t)}$ defined for different bins of data (every 20 years), regional year fixed effects $\alpha_{R(i),t}$ and $\alpha_{R(j),t}$ for large regions.[4] We also allow for more flexibility to the structure by including the possibility of time-varying bilateral trade functions: if p is a function that associates a period to each year (for example, decades, every 20 years), we can control for time-varying pair fixed effects $\alpha_{i,j,p(t)}$ and still find significant effect of default on imports.

2.2 Results

We run equation 2.1 with different covariates and specifications and show our results in table 1. We find results similar to those in Rose (2005). The decrease of imports after default is between 10% and 90% in the most pessimistic case. We observe the effect of

[3] When we allow for null observations, we use hyperbolic arcsine. Fouquin, Hugot, et al. (2016), who provide the Tradhist database from CEPII, claim that null bilateral trade data correspond when bilateral trade data could indeed be estimated to be 0, although it might in some cases also be due to lack of evidence. We allow both interpretations as we either include or exclude observations where bilateral trade flow is "null" in the results below. Including null observations lessens the effect of default but does not change our effect qualitatively. We include regressions with null observations to stay conservative.

[4] The regions we define are Europe, Asia, Middle East, Atlantic Ocean, Africa, North America, Latin America.

default on imports is larger than on exports but exports still decrease significantly after default, even in not favorable conditions (time-varying fixed effect). Our theoretical results are consistent with the assumption that imports are disproportionately hurt by default.

One important question for significance is whether we should include observations of 0 bilateral trade as literally meaning 0 trade or as a mistake. Not including these observations sensibly reduces the size of the effect, which would make sense if null observations indeed corresponded to no trade: defaults seem to impact the extensive margin of trade).[5] When we include null observations, we use the inverse hyperbolic sine of exports rather than the log, to include more easily null observations.

Table 1: Effect of sovereign default on bilateral trade

	Dependent variable:					
	Exports (log or hyperbolic arcsine)					
	(1)	(2)	(3)	(4)	(5)	(6)
Default (origin)	-0.643^{***}	-0.438^{***}	-0.447^{***}	-0.126^{***}	-0.117^{***}	-0.027^{***}
	(0.016)	(0.017)	(0.018)	(0.009)	(0.018)	(0.009)
Default (destination)	-0.904^{***}	-0.534^{***}	-0.521^{***}	-0.149^{***}	-0.195^{***}	-0.108^{***}
	(0.014)	(0.016)	(0.016)	(0.008)	(0.017)	(0.009)
		Controls				
GDP (log, destination)	No	Yes	Yes	Yes	Yes	Yes
GDP (log, origin)	No	Yes	Yes	Yes	Yes	Yes
Pair F.E.	Yes	Yes	Yes	Yes	Yes	Yes
Time-Varying Pair F.E.	No	No	No	No	Yes	Yes
Data Before 1950	Yes	Yes	No	No	No	No
Null=0	Yes	Yes	Yes	No	Yes	No
Observations	837,067	686,030	637,316	427,185	637,316	427,185
R^2	0.736	0.748	0.750	0.836	0.839	0.895
Note:						*p<0.1; **p<0.05; ***p<0.01

3 Two-Period Model

In this Section, we present a model where a small open economy that trades with the rest of the world and borrows from it. In case of default, the country enters into financial autarky (which should not matter in the two-period case that we also look at), and more

[5] This macroeconomic evidence would be the macroeconomic equivalent of what Gopinath and Neiman (2014) find at the firm level in Argentina after 2001 default in Argentina: a large number of firms completely stopped importing certain kinds of inputs. It would mean that defaulting countries stop importing from some trade partners with whom they were trading less before.

importantly, partial trade autarky: as a consequence of default, trade costs increase. To make the exposition simpler, we assume the cost of default is going to be complete trade autarky.

The timing of the model is as follows for a government that did not default:

- At the beginning of each period, government inherits past stock of debt.

- It learns the price of the foreign good $p(s)$ and of the value of domestic endowment $y(s)$.

- It decides whether to repay or to default.

- If government repays its debt, it chooses how much to borrow and how much domestic and foreign goods to consume.

If a government defaults, it consumes its own resources and enters into financial autarky.

Because some of our results will be available onlyin the case with a finite number of periods, we will not only present a stationary model with infinite horizon but also allow time horizon to be finite in the model with a slightly more general presentation.

3.1 Assumptions and Primitives

- **Underlying shocks** There is a finite set of states for the economy $\mathcal{S}$. There is an exogenous Markov process $(s_t)_{t\in\mathbb{N}}$ with transition probabilities given by the function $\pi(.|.)$: for every s and s', $\pi(s'|s)$ is the probability of transition from s to s'. We also define $(s^t)_{t\in\mathbb{N}} := (s_0, \ldots, s_t)_{t\in\mathbb{N}}$, the history of past Markov states.

This underlying Markov process determines the endowment of the economy at time t, $y(s_t)$, and the price of the domestic good in the international markets $p(s_t)$.

- **Static Consumption and Gains from Trade** There are $T \in \mathbb{N} \cup \{+\infty\}$ periods $t = 1, \ldots, T$. Aggregate consumption C_t at each period t is given by:

$$C_t := m(c_t, c_t^\star),$$

where m is an aggregator with constant returns to scale.[6] Trade is motivated by the inability of the country to produce foreign varieties of consumption goods as in Armington (1969).

We make this assumption for the sake of simplicity, but it does not matter: the only relevant point for our results is that gains from trade can be inferred from the variation in imports, as in Arkolakis, Costinot, and Rodriguez-Clare (2012). Therefore, all trade models embedded in the results of Arkolakis, Costinot, and Rodriguez-Clare (2012) would

[6]The most common example of such an aggregator would be CES: $A(c_t, c_t^\star) = (\alpha^{\frac{1}{\sigma}} c_{t,D}^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} c_{t,F}^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}$

work in our framework[7] for the first basic proposition: more open countries can borrow a larger amount of debt. As long as more openness implies larger gains from trade, our model could also encompass more recent models of trade that account for heterogenous elasticities and input-output networks, as for example Baqaee and Farhi (2019).

- Utility There are $T \in \mathbb{N} \cup \{+\infty\}$ periods $t = 1, \ldots, T$. The small open economy with a representative agent takes all prices as given and maximizes utility:

$$U = \mathbb{E} \sum_{t=1}^{T} \beta^t u(C_t),$$

given the budget constraints and default constraints we are defining below. We are going to assume for the sake of simplicity that the representative agent is also the government, which is the most common assumption in the sovereign debt literature. From a decentralized market perspective where the inhabitants of the country would take the quantity of sovereign debt as given, it is equivalent to assuming that government subsidizes or taxes borrowings to make agents internalize the impact of their savings, as it was proved in Na et al. (2018).

- Budget constraint and trade costs There are iceberg trade costs $\tau > 1$ for imports.[8] They are the main variable of interest for comparative statics: when we say an economy is "more open", we mean it has lower trade costs. Although we assume that τ might depend on s, we will generally omit to index τ on s in order to alleviate notations.[9]

The domestic good is produced with an inelastic effective labor supply (alternatively, endowment or output) $y_t = y(s_t)$ in t. Domestic good is assumed to be the *numéraire*, in which debt is labeled. This hypothesis is consistent with a recent trend of countries borrowing more in their own currencies, but it does not include the case of a country borrowing in dollars. We keep it for theoretical reasons that have to do with computational details in our proofs. We reverse it in the calibrated model, where we assume that foreign good is the *numéraire* and find this assumption does not affect our results.

The price of the foreign good in international markets is going to be determined exogenously as $p(s_t)$. In national markets, it is also going to depend on trade costs τ and be equal to $p(s_t, \tau) := \tau p(s_t)$. It is assumed to depend on the underlying Markov process s_t and trade costs. It can be interpreted as the outcome of an underlying trade model,

[7] It includes Eaton and Kortum (2002) and Melitz (2003) among others.

[8] We might also assume there are trade costs for exports without modifying any of the results. However, there are concerns that the results might be due to exports having more value on international markets as a result of the decrease in the trade costs associated with exports. To alleviate these concerns, we are going to assume that there are trade costs for imports only, knowing that this assumption is only *weakening* our results. It means that the value of the *numéraire* on which debt is indexed should not vary. It also implies that, in case of default, imports should be disproportionately hurt.

[9] If we consider general trade costs $\tau = (\tau(s))_{s \in \mathcal{S}}$, then we say that trade costs τ' is less than τ if and only if, for every $s \in \mathcal{S}$, $\tau(s) \geq \tau'(s)$.

where the variable s_t not only reflects the endowment for the domestic economy but also the endowments of foreign economies.

Prices are exogenous, which means we do not allow current account to have an impact on prices. The main reason we make this assumption is that it prevents the derivation of general analytical results by making price determination ambiguous. However, we allow it in calibrations and find that it does not affect the results of interest in the model - although it implies the existence of an optimal tariff. This assumption is equivalent to assuming that the economy is small relative to the rest of the world and can take the macroeconomic international conditions as given.

The budget constraint for government that repays debt at period t can be written:

$$c_t + p(s,\tau)c_t^\star + b_{t-1} = y(s_t) + q_t(s,b_t)b_t,$$

where b_{t-1} is debt inherited from the previous period, b_t is the face value of newly emitted bonds and q_t the actual price of those new bonds. The price of bonds q_t is a function of their face value b_t because the price of bonds depends on government's incentives to default in the next period and therefore on τ, which is a structural parameter of the analysis. Note that, because τ is a structural parameter, it plays no dynamic role in the model, so we should omit to mention it in functional forms until we do the comparative statics of the model.

- Timing of the Model and Default Decision Bond schedule q_t and inherited debt b_{t-1} are considered given at the beginning of period t by the government. Price q_t of government bonds is determined by the financial markets. If there are only 2 periods, $q_2 \equiv 0$: no debt should be left at the final period. However, the model can easily be extended to infinity.

At the beginning of period t, government learns the value of s_t. It chooses whether to default or to repay debt. If the government defaults, then it does not pay debt, but cannot borrow any more, and faces larger trade costs $\tau_D > \tau$. Budget constraint becomes:

$$p(s_t,\tau^D)c_t^\star + c_t = y(s_t).$$

The rest of this Section will consider the simplifying extreme case where $\tau^D = +\infty$. In this case, the country enters commercial trade autarky when it defaults and consumption should be given by:

$$\begin{aligned} c_t &= y(s_t) \\ c_t^\star &= 0. \end{aligned}$$

If the consequences of default are permanent, as we are going to assume they are in the theoretical results, then the value of default should be exogenous and it should not depend on τ either: we can define $V_t^D(s)$, the exogenous welfare that a country gets from defaulting at time t, which will be important for some of our proofs.

- Financial Markets We assume that investors are risk-neutral.[10] This assumption can be considered natural, if we assume that the economy is small and open: if so, then the economy's endogenous risks should be inconsequential to the lenders' portfolio risks. In other words, matters regarding risk diversification can be considered incorporated in an exogenous safe rate $r^\star$.

As a consequence of this assumption, the pricing of bonds should be equal in equilibrium to the probability of default.

In the case when T is finite, we can easily derive the value of bonds at period $T-1$. Because financial markets are competitive, the price $q(b_{T-1}, s_{T-1})$ of a government bond depends on new debt b_{T-1} and is computed according to the corrected probability of default:

$$q_{T-1}(b_{T-1}, s) = \frac{1 - \mathbb{P}(C^R(b_{T-1}, s_T) < C_2^{aut}(s_T) | s_{T-1} = s)}{1 + r^\star},$$

where C_2^{aut} is the aggregate consumption in case of default and $C^R(b_{T-1})$ is the aggregate consumption in case of repayment, that we will detail later. In the more general case, if we assume that $V_t^R(b, s)$ is the value function associated with repaying debt b at time t and at state $s \in \mathcal{S}$, then we can write:

$$q_{t-1}(b, s) = \frac{1 - \mathbb{P}(V_t^R(b, s_t) < V_t^D(s_t) | s_{t-1} = s)}{1 + r^\star},$$

where $V^D : s \in \mathcal{S} \mapsto V^D(s)$ is the value function associated with default.

- Equilibrium Definition With all the elements above, we can define Markovian equilibrium. As before, we present equilibrium with $T \in \mathbb{N} \cup \{+\infty\}$ periods. We assume that the commercial consequence of default is complete autarky below. We present another similar version of the equilibrium with more realistic assumptions in the Appendix.

Definition. Let $N \in \mathbb{N}$, $(s_t)_{t \in [0,N]}$ be a Markov process, τ be a value for trade costs and $p(s, \tau)$ be a corresponding price function. A competitive equilibrium associated with trade costs τ is given by a sequence of value functions $(V_t(B, s), V_t^R(B, s), V_t^D(s))_{t \in [0,N]}$, policy function for borrowing $(b_t^\star(b, Ys))_{t \in [0,N]}$, policy function for default $(D_t(b, s))_{t \in [0,N]}$, and lending functions $(q_t(b, s))_{t \in [0,N]}$ such that, for any $(b, s) \in \mathbb{R} \times \mathcal{S}$:

[10] Sovereign debt literature mostly considers risk-neutral investors. However, some models take into account time-varying risk-aversion. Since the effects of risk aversion can be considered GDP shocks or shocks on default cost, their inclusion is an issue for calibrations mostly. It should not matter too much for our study as we are interested in the local effect of a decrease in trade costs. In this case, risk aversion can be considered a simple multiplier φ to apply on the probability of default when we compute bonds' price.

- The value functions solve the recursive equations:

$$V_t^R(b,s) = \max_{b',c_t,c_t^\star} u(m(c_t, c_t^\star)) + \beta \mathbb{E}_s(V_{t+1}(b',s')|s_t = s)$$
$$\text{subject to } p(s,\tau)c_t^\star + c_t + b = y(s) + q_t(b',Y)b'$$

$$V_t(b,s) = \max\{V_t^R(b,s), V_t^D(s)\}$$
$$V_t^D(s) = u(Y \times m(1,0)) + \beta \mathbb{E}(V_{t+1}^D(s_{t+1})|s_t = s).$$

- Policy functions solve the government's optimization problem::

$$D_t(b,s) = \mathbb{I}\{V_t^R(b,s) < V_t^D(s)\}$$
$$b_t^\star(b,s) \in \arg\max_{b',c_t,c_t^\star} u(m(c_t,c_t^\star)) + \beta \mathbb{E}(V_{t+1}(b',s_{t+1})|s_t = s)$$
$$\text{subject to } p(\tau,Y)c_t^\star + c_t + v = y(s) + q_t(B',Y)B'.$$

- Financial markets are competitive:

$$q_t(b,s) = \frac{\mathbb{P}(D_t(b,s_{t+1}) = 0|s_t = s)}{1+r}.$$

We used the convention that $V_{T+1} \equiv V_{T+1}^R \equiv V_{T+1}^D \equiv 0$. If $T = +\infty$, the problem is stationary so that value functions and the competitive equilibrium functions do not depend on period t but only on state variables.

3.2 Trade Costs, Probability of Default and Debt

In this Section, we are going to look at the effect of trade costs (hence trade openness by contraposition) on the probability of default and on the level of debt. We prove that the probability of default is going to decrease in trade openness.[11] We easily show it to be true for a fixed level of debt, but also when we do not control for debt: countries should adopt a safer behavior as they get more open. We also discuss whether a similar kind of results can apply to debt levels: does face value of debt increase in the total value of debt? While we cannot conclude unambiguously on this level, we show that under some plausible technical conditions about the distribution of GDP shock, debt should increase as a country gets more open (that is, debt should decrease as trade costs increase).

3.2.1 Gains From Trade and Probability of Default

If T is finite, in the final period of the model, government has borrowed b_1 at the previous period (negative b_t would mean net savings). It now learns y_2, which was distributed according to a given probability distribution with density f.

[11] Equivalently, the probability of default should be increasing in trade costs.

After learning s, government chooses whether to default or not.[12] If the country chooses to default, the cost for defaulting is the interruption of international trade. A defaulting government is stuck in autarky and it can only consume its own good, so that:

$$\begin{aligned} C_T^{aut} &= m(y(s_T), 0) \\ &= y(s_T) m(1, 0). \end{aligned}$$

If government does not default, it can consume the foreign good but it has to bear the burden of debt, so that it gets:

$$C_T^R = (y(s_T) - b_T)\nu(s, \tau),$$

where $\nu(., .)$ is the following quantity summarizing gains from trade:

$$\begin{aligned} \nu(s, t) &= \max m(c, c^\star) \\ \text{subject to } & c + p(s, t)c^\star = 1. \end{aligned}$$

Let us assume for example that preferences are CES with elasticity of substitution σ. In this case, we can easily derive from Arkolakis, Costinot, and Rodriguez-Clare (2012) the following formula for gains from trade:

$$\frac{\nu(s, t)}{m(1, 0)} = (1 - IM^\star)^{\frac{1}{1-\sigma}},$$

where $IM^\star$ is the share of imports in the final consumption, or equivalently the share of imports in GDP that would correspond to balanced trade. It can be computed as follows:

$$IM^\star = IM/(1 - x),$$

where IM is imports in value as a share of GDP and x is trade balance (equivalently in the model, current account) at time t.[13] Consequently, government should default whenever debt is more than the default threshold b^D defined by the following equation::

$$b^D(s_2) = y(s_2)\big(1 - (1 - IM^\star(s_2))^{\frac{1}{\sigma - 1}}\big) \tag{3.1}$$

$$\implies b^D(s_2) = y(s_2)\frac{g(s_2)}{1 + g(s_2)}, \tag{3.2}$$

[12] Here, the large size and the low volatility of productivity in country F that faces no productivity shock guarantee that it will not default. Indeed, the economy is deterministic from its point of view. This assumption may be interpreted as the simplification of a world in which wealthy entrepreneurs who buy a lot of insurances invest in sovereign bonds. A default of this country would be problematic since it would entail a global disaster for world trade: in the absence of specialization between small islands, all indebted countries would immediately default if trade with the central country were interrupted. Another way to rule that possibility out would simply be to assume that the central country is more patient than all the islands, as measured by the discount rate: $\rho_F < \inf_{i \in [0,1]} \rho_i$. We explore collective incentives to default in a companion paper.

[13] Arkolakis, Costinot, and Rodriguez-Clare (2012) assumes the absence of trade imbalances, but we can still apply the ACR formula due to our assumption that trade imbalances do not affect relative prices. In any case, we can directly compute the equilibrium in this simple model and find the same thing as their formula.

where $g(s_2) = \nu(s,t)/m(1,0) - 1$ summarizes gains from trade in the model.

Note that this computation would be possible using only the result established in Arkolakis, Costinot, and Rodriguez-Clare (2012) that

$$1 + g = \varDelta \ln W = -\ln(1 - IM^{\star})/\varepsilon,$$

where $\varDelta \ln W$ is the difference between welfare in free trade and welfare in autarky, ε is the inverse elasticity of substitution. This result is equivalent for any aggregator m with a non-unitary elasticity of substitution: it would also apply to a non-Armington context, most notably to the class of Ricardian trade models developed in the wake of Eaton and Kortum (2002). Then, conditional on having a trade model embedded in Arkolakis, Costinot, and Rodriguez-Clare (2012), more open countries should be able to sustain a larger debt-to-GDP ratio, *ceteris paribus* (including the level of debt) in our model. The intuition that more open economies benefit from trade, once we correct each flow of sectoral trade by the relevant elasticities, could make it possible to generalize this kind of result to larger frameworks.

We can deduce from this result that more open countries should have lower costs of borrowing, everything else equal. Indeed, sovereign bond's price is determined by the probability of default. We can therefore write:

Theorem 1. *Assume that the economy is a sovereign debt model in $T \in \mathbb{N} \cup \{+\infty\}$ periods as described above where gains from trade can be computed as in Arkolakis, Costinot, and Rodriguez-Clare (2012). Conditional on the level of debt B, a more open country, that is a country with higher import share in its final consumption, should be charged with a lower interest rate. Equivalently, any change in trade costs that makes a country more open should also decrease the interest rate it faces.*

Proof. We just established the result when there are two periods. We can also establish it by induction when there are $T < +\infty$ periods. We prove the proposition for the general infinite horizon case in theorem 9, using a method inspired by Aguiar and Amador (2019). □

We should test this proposition later in the empirical part. One can ask a more general question: what would happen to default probability as trade cost vary, without fixing the level of debt? As we are going to see it in section 3.2.2, debt should also vary as the cost of default decreases, because policy functions also depend on trade costs. For example, if the government is very impatient, it could not care about next period consumption and borrow as much as possible in the current period. In this case, an increase in trade costs would allow this government to borrow more today, without reducing the probability to default: it could even increase it.

While this extreme case is possible in infinite horizon in Example 12, we can prove that in the two-period version of the model, the probability of default should not increase as trade costs decrease if utility is CRRA and concave, and if the discount factor is positive.

Proposition 2. *Assume that we are a two-period sovereign debt model as described above with fixed prices (i.e. gains from trade are fixed). Assume that GDP has a differentiable cumulative distribution function. Assume that instantaneous utility function is concave and homothetic. As a country gets more open, it should face a lower interest rate, unconditional on the level of debt B.*

Proof. See Appendix. □

3.2.2 Borrowing

The decision a government makes how much to borrow is an endogenous outcome and might have complex interactions with structural parameters.

Before presenting the next general result, we remind that the revenue raised from debt is the quantity $q(b_t)b_t$, while the quantity b_t is called face value debt.

Theorem 3. *Assume that u is concave. Let us consider the model with infinite time and assume that the utility function is concave. Let us compare two economies that are equivalent, except the first one is more open than the second one due to lower trade costs. Assume that the two economies have the same level of welfare at period t. Then the more open economy will raise more revenue from debt at period t.*

Proof. See Appendix, more specifically Theorem **??** (building on a few lemmas). □

The proof heavily relies on Aguiar and Amador (2019) and it is the first such general result about the comparative statics of a policy function in this class of sovereign debt models to our knowledge.

We prove in the appendix that this result is the most general one we could get in our framework. Indeed, if we do not specify how gains from trade can vary depending on the state we are considering, there is always a possibility to find *ad hoc* transition probabilities such that government would decide to decrease debt or to increase the level of risk - when it gets more open. We show two numerical examples in 12 and 11 where respectively bonds become riskier and face value debt decreases.

However, this result rules out the possibility that government does both at the same time: more open government raise more revenue from debt.

Also note that we compare two governments with similar levels of *welfare* and not two economies with similar levels of *debt*. We derive this kind of results by the use of the dual operator defined in Aguiar and Amador (2019). Because we need to use this kind of dual operator for proofs, we need to compare economies with similar level of welfares rather than similar levels of debt. Welfare being an endogenous variable in the model, the result might seem weaker at first hand. However, it can be approximated by spreads from the past period, and constraints related to welfare can appear in restructuring models: in this case, government can be brought to the case when it is indifferent between default and repayment if its creditors have enough bargaining power. In such a case, the more open government would borrow more.

There is one limit to this result: it does not tell us whether the face value of debt will increase or not, and face value debt is the most available empirical variable to test. We will show in Section 3.3 that reasonable calibrations lead to such results.

In the remainder of this Section, we study whether face value debt increases as default gets costlier in a two-period model. We assume that the price of the foreign good is fixed for the sake of simplicity. At the beginning of the model, government chooses how much to borrow so as to maximize utility after inheriting debt b_0:

$$\max_{c_1, c_1^\star, c_2, c_2^\star, b} u\big(m(c_1, c_1^\star)\big) + \beta \mathbb{E} u\big(m(c_2, c_2^\star)\big)$$

$$\text{subject to } p(\tau)c_1^\star + c_1 + b_0 = y_1 + q(b)b$$

$$p(\tau)c_2^\star + c_2 + b = y_2 \text{ or } \Big\{c_2 = y_2 \text{ and } c_2^\star = 0\Big\}.$$

In a deterministic model, government should be interested in borrowing if and only if we assume $\frac{\beta-1}{\beta} =: \rho < r + \gamma\frac{L_2}{L_1}$ where γ is local relative risk aversion. In quantitative exercises, authors always assume that $\rho < r$. In other words, emerging countries' governments are assumed to be impatient:[14] otherwise, they would prefer to save at a better safe rate. We therefore assume that β is low enough to create positive borrowing.

To simplify notations, we note gains from trade $1 + g(\tau)$ as in the previous function, and also assume that terms of trade are not affected by the domestic country's GDP. The problem then writes:

$$\max_b V(b, g(\tau)) := u\Big((1 + g(\tau))(1 + q(b, g(\tau))b)\Big) + \beta \mathbb{E} u\Big(\max((1 + g(\tau))(y - b); y)\Big).$$

We want to see what happens to the level of debt as trade costs decrease. Given the framing of the model, a decrease in trade cost is equivalent to an increase in default cost if the utility function has constant relative risk aversion. In standard calibrations of the infinite-period version of this problem such as Arellano (2008), the average level of debt increases when the cost of default increases. This problem does not allow a simple analytic characterization of solutions without further specification. Using the implicit function theorem, we give a local condition for debt to be decreasing in trade costs at the optimum in the appendix but it does not allow straightforward conclusions. However, if we assume that utility function is linear, we can establish the following result.

Proposition 4. *Let B_τ be the optimal level of borrowing corresponding to a given level of trade costs τ.*

B_τ is decreasing in τ in the following cases:

[14] Or, alternatively, governments are assumed to expect high enough future growth so that consumption smoothing would imply borrowing.

- If the distribution of y_2 is uniform or exponential.

- If the distribution of y_2 is log-normal and the default probability is less than 50% at equilibrium.

More generally, if f is the density associated with GDP shock and if we define default threshold for GDP $x := \frac{B_\tau g(\tau)}{1+g(\tau)}$, then if f is locally continuously differentiable around x and:

$$(2 - (1+r)\beta)f(x) + xf'(x) > 0. \tag{3.3}$$

Then a local decrease in trade costs τ involves an increase in the optimal level of debt B_τ.

Proof. See appendix. □

The equation (3.3) above, although it looks technical, makes sense from an economic perspective. Indeed, one can prove that revenue function $B \mapsto q(B, \delta) \times B$ has the following double derivative:

$$\frac{\partial^2 (q(B,\tau)B)}{\partial B \partial \tau} =^{\text{sign}} -(2f(x) + xf'(x)).$$

It means that the condition above is simply related to the Laffer curve of bond supply: does the revenue-maximizing level of bonds increase or decrease in τ? In the proposition above, this is simply corrected by discount factor, because more debt today implies less consumption tomorrow (in cases when debt are repaid at least). Then, this parametric assumption seems natural: it simply states that the revenue-maximizing level of debt decreases in the cost of default.

In the more general case with non-linear utility, formulas are more tedious (see the discussion of Proposition 4 in the appendix). Overall, whether debt increases as trade costs decrease depends on the interaction between three effects:

- The direct price effect or substitution effect, the same as in the linear utility case: it is $\frac{\partial^2 q(B,\tau)}{\partial B \partial \tau}$ times marginal utility. Under the same kind of technical assumption as in the proposition, this term should be positive and push debt to be decreasing in trade costs.

- Contemporaneous consumption smoothing or income effect: larger price of government bonds increases consumption. Thus it reduces contemporaneous marginal utility and encourages more savings for tomorrow.

- Future consumption smoothing : this term is the marginal utility in the second period discounted by the discount factor. If default cost increases, there are more states of the world where government repays debt tomorrow, therefore government should be more reluctant to borrow. This is the effect we would get if default cost increased but the borrowing function stood the same.

The negative effects of trade costs on debt should be stronger as the government is risk-averse or values future consumption (high β) or has low growth expectations. Overall, which effects dominate is an open empirical question, although we will see that simulations suggest that debt decrease in trade costs in most cases, because substitution effect should dominate the other effects.

3.3 Calibration Results

In this Section, we illustrate certain of our results with a more complete and less stylized sovereign debt model. Because analytical results are hard to derive in sovereign debt models, we could allow the specifications that are common in more complete calibrations, whose properties have been summarized in a survey by Aguiar et al. (2016). We leave the detailed description of the model in the appendix. We shortly explain the changes without equations in the next paragraph, before showing some quantitative results.

3.3.1 Models' Properties

In this Section, we briefly present the properties of our calibration model that distinguish it from the framework under which we have established our theoretical results. The stochastic structure and details of the model are presented in the appendix.

We introduce for countries the ability to borrow and trade again as they normally did after default: at the end of each period after default, they have a probability λ to return back to financial markets and to regain access to the same trade facilities as before. This feature is common in sovereign debt calibrations and was already introduced in Aguiar and Gopinath (2006). We also allow debt to have longer duration, as in Chatterjee and Eyigungor (2012), which allows more volatile spreads in the quantitative model. If government has debt B_t, a share ψ of this debt matures at period t.

Regarding trade, there are two differences between the calibration model and the one we presented above.

The first difference is that we do not make default as extreme as in our theoretical results. We assume that, when government defaults, trade costs increase from τ to $\tau^D(\tau) = c_D \times \tau$ where $c_D > 1$.

The second difference is that the government's decisions impact its terms of trade. Government's trade deficit or surplus modifies the demand for its own good, hence its price, and prices are not completely exogenous any more. They are influenced by exogenous factors, such as foreign demand for domestic goods.

These changes have an important consequence in terms of optimal policy: the monopoly power that a country has over its own good makes it optimal to impose a tariff. Therefore, gains from trade should increase with an optimal tariff compared to a situation where the tariff is zero. In the previous model, because prices were assumed to be exogenous, the optimal tariff policy was to set tariffs to 0. This is not the case any more. Because of optimal tariff, it might happen that more closed economies can commit more to repaying debt if increasing tariff also rises their gains from trade. We assume that the

Table 2: Comparisons between two similar economies with different levels of openness.

Variable	Closed economy	Open Economy
Mean Imports (in % of GDP)	34.3	43
Debt to GDP ratio	24.1	35.4
Mean Spread	0.112	0.102
S.d. of spread S.d. of spread	0.076	0.064
Spread diff., 95th percentile	0.062	0.052
Frequency of Defaults	0.038	0.031
Mean Current Account (CA) (in % of GDP)	0.013	0.017
Corr(Y,CA)	-0.186	-0.239
Corr(Δ Y, CA)	-0.255	-0.244
Corr(CA, Δ Spread)	0.012	0.027
Corr(Imports, Δ Spread)	-0.427	-0.483
Terms of Trade	0.863	0.952

economies already apply the optimal tariff and consider comparative statics where trade costs decrease or where demand for domestic goods increase.

3.3.2 Calibration Results

We can compute the equilibrium value functions and policy functions through classic value function iteration. We are going to compare two economies with similar characteristics, except that one is more open to trade than the other.

In

In table 2, we show what happens when the openness ratio of an economy increases, everything else equal: we use the same parameters for both economies, except for the general level of foreign demand that is calibrated to be higher, and we simulate the model 100 times for $T = 10\,000$ periods each time. In this second economy, the level of exports-to-GDP is roughly 25% higher: imports represent 43% of GDP, rather than 34%. This reduction in trade openness has a positive effect on terms of trade (that increase by 10%), on debt levels (that go from 24.1% of GDP to 35.4%) and on the likelihood of crises, that decreases from 3.8% to 3.1%. The effects are rather large, and give the same orders of magnitude as our empirical results below.

We show the result of another calibration in figure 3.1, with a different choice of parameters ($\sigma = 2$, and lower average foreign demand) and compare two economies with two different levels of openness. While the average size of debt changes, it is still the case that the economy that is more open (with an import penetration ratio close to 20%) borrows twice as much debt as the more closed economy. We can observe this is apparent not only in the long run, but in the first few periods: the figure compares debt build-up over a few periods when the open and the closed economies face the same shocks.In this calibration, as trade openness doubles, so does total quantity of debt.

The results of our calibrations show that face value debt increases in most states when the economy gets more open. Intuitively, the reason why it is so is that impatience makes countries avid to borrow as large quantities of money as possible in the calibration, and any additional leverage on commitment should therefore be exploited.

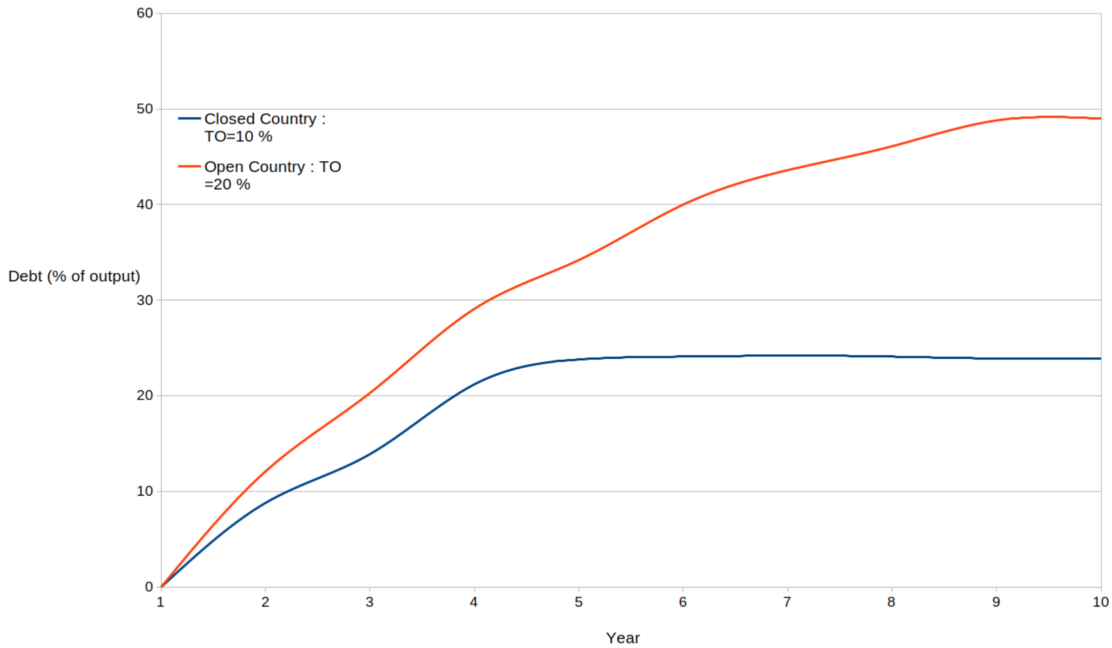


Figure 3.1: Debt build-up in an open and a closed economies

4 Data and Instruments

In this Section, we present the data and the instrumental strategies that we are going to use for our Empirical Results. In Section 4.1, we present our data. In Section 4.2, we define two instruments that we will for trade, based on gravity equations and coming from the empirical literature on gains from trade.

4.1 Data

In the following empirical analysis, we use sovereign Credit Default Swap (CDS) data collected for 88 countries from two sources from 1994 to 2019:

- World Bank for spreads from 1994 to 2019 for 60 emerging countries - it is downloadable on a monthly basis but we annualized the data by averaging it.

- Datastream collects them on a daily basis, for 69 countries, 19 of which were not already included in the World Bank database.

CDS are an insurance for bond holders against default. It covers losses associated to default or restructuring of debt. CDS holders pay an insurance fee, called CDS premium every semester and, if the corresponding entity defaults, CDS sellers pay back the bonds, up to what the entity does not pay back (the less the haircut of a precise bond, the more the CDS pays back). More precisely, CDS give insurance against "credit events", more general than debt, as defined by the International Swaps & Derivatives Association. For example, when Greece restructured its debt in 2011 and 2012, Greece did not officially default but holders of former bonds lost some of their value and the CDS holders got reimbursed after a period of institutional hesitation in 2012. In this case, CDS covered 3.2 billion dollars insured against a Greek default (to compare to more than 400 billion dollars of Greek debt). When they are activated, CDS take into account partial repayments from government, that they do not cover.

The interest of using CDS data, besides its large availability, is that CDS markets are more liquid and more precise indicators or risks perceived in financial markets. We excluded from the data a few suspicious time series with very low availability of data: Iraq, Ukraine, Malaysia and Singapore (two of them being involved in a military conflict over the period). For these countries, certain observations relied on only one day of transaction over several years. Including the spare available data from these countries did not change our results.

Thanks to CDS wide availability, we can successfully average spreads over each year and get good estimates of risks. The corresponding estimate of the associated sovereign risk should be more precise. While the CDS is priced on secondary market and may not reflect the cost of borrowing the country faces, due to maturation mismatch and strategic timing of borrowing, it reflects the probability of default. If the probability of default of the sovereign is constant and equal to P, and with a null recovery rate, then the relation between CDS premium λ and the instantaneous probability of default should be given by:

$$\lambda = -\ln(1 - P).$$

This is a simplification and more sophisticated models take into account maturity and more complicated risk functions. We abstract from them as we sense they should not affect our results: using 1-year or 10-year maturities did not affect our results.

We exclude from the analysis observations where CDS spreads went above a threshold because they are synonymous with default: very high CDS spreads are synonymous with very likely devaluations and large and noisy movements in both trade-to-GDP and CDS spreads. In our regressions, we alternatively used a 1,000 b.p. and a 500 b.p. threshold and got similar results.

Total flows of trade (including services) and debt are collected on yearly basis by the IMF and World Penn in publicly available data. We use World Penn Database for other general macroeconomic indicators. World Penn includes all countries and years included in IMF Global Debt Database, collected by Mbaye, Badia, and Chae (2018). IMF Global Debt Database, that we completed with other debt indicators from IMF and World Bank for years when data was missing, includes data for 175 countries for years spanning from 1950 to 2018, including most countries from the CDS database. Bilateral flows of trade come bilateral trade database from CEPII, given in Fouquin, Hugot, et al. (2016). As the series stops in 2014, we complement it after 2019 with bilateral trade series computed from Comtrade as given in the BACI database from Gaulier and Zignago (2010).

Because the mechanism at stake in the cost is assumed to be channeled by finance, we take into account total government debt rather than just debt owed to foreigners. Indeed, even a purely internal default might disrupt external finance and we do not attempt to discriminate both experiences.

4.2 Instruments

In Section 5, we want to show that more trade openness leads to a decrease in CDS premium and we need empirical strategies to overcome potential omitted variable bias. We construct a gravitational instrument for trade, inspired by Frankel and Romer (1999) as described below, in Section 4.2.1. It should help us avoid some of the most obvious issues with the direct use of trade in the regression, although it does not allow us to include country fixed effects in order to study time variations. To do so we will also introduce the instrument by Feyrer (2019) in Section 4.2.2.

4.2.1 Frankel-Romer Instrument

To reconstruct the Frankel-Romer and Feyrer modified instruments, we used CEPII's data with bilateral trade in merchandises between countries from Fouquin, Hugot, et al. (2016) and from Gaulier and Zignago (2010). We also used geographical (distance between countries, area, borders, language) and demographic data from Head, Mayer, and Ries (2010) to reconstitute the geographical variables.

As we want to directly address the question to whether a change in trade policy in the long run would affect a country's ability to borrow funds from the sovereign markets, we use the same instrument as in Frankel and Romer (1999) to evaluate the impact of trade on the terms of direct borrowing as measured by CDS spreads. This instrument relies on the intuition, given by gravitational models and almost universally observed in trade data, that bilateral trade between two countries depends on their distance and on their size. As a consequence, a small country surrounded by large and rich neighbors such as the Netherlands should trade more than a large country in an isolated island such as Australia, although both countries are rich. Frankel and Romer (1999) build their geographical instrument based on gravitational theories, prevalent in trade models.

More precisely, bilateral trade $T_{i,j}$ (as measured by the sum of imports and exports) between country i and j is assumed to behave that way:

$$\ln(\frac{T_{i,j}}{Y_i}) = a_0 + a_1 \ln d_{i,j} + a_2 \ln N_i + a_3 \ln N_j + a_4 B_{i,j} + a_5 B_{i,j} \ln d_{i,j} + a_6 B_{i,j} \ln N_i + a_7 B_{i,j} \ln N_j + a_8 l_{i,j} + a_9 A_i + a_{10} A_j + \varepsilon_{i,j},$$

where $d_{i,j}$ is the bilateral distance between countries i and j[15], N_i the population of country i, A_i the area of country i, $B_{i,j}$ a dummy indicating whether they share a common border, $l_{i,j}$ a dummy indicating that countries i and j share a common language. To precise the variables we used, the results of this regression are summarized in table 9 (see appendix). Without surprise, distance matters a lot to explain bilateral trade. The total R^2 is less than 50%, in part because we did not include GDP of trade partners in the regression to avoid potential biases in the next regressions, since level of development may be an explanatory variable.

Using the predictors given by this last regression, one can therefore predict the total trade level of one country using only the geographical variables:

$$\widehat{Trade}_i^{FR} := \sum_{j \neq i} \frac{\widehat{Trade}_{i,j,t_0}^{FR}}{GDP_{i,t_0}} = \sum_{j \neq i} \exp(\hat{a}_0 \log d_{i,j} + \hat{\beta} X_{i,j}).$$

To compute the instrument, we use only one year: we use 2007 as a reference point, before the beginning of our CDS series to ensure the instrument is exogenous.

4.2.2 Feyrer Instrument

The problem of the previous instrument is that it is fixed for each country. Then, it cannot be used for diff-in-diff regressions or with country fixed effects. To avoid that issue, we will also use the same time-varying gravitational instrument as in Feyrer (2019): this instrument is based on the idea that there are changes over time in the importance of geographical variables for trade. Indeed, sea distance matters relatively less today than in 1950, at least relative to air distance: the greater availability of planes for trade changes the impact of geography over time, as Feyrer (2019) explains in his paper: some goods, especially electronics and luxury leather goods, are often exchanged through air distance, which can represent 20% of the trade for some countries. This change in the importance of air trade heterogeneously impacted countries over time. A country such as the United States did not greatly benefit from air travel from the point of view of trade: to give the most salient example, sea distance between the US and most countries in the world coincides with air distance, while this would not be true between Europe and Eastern Asia: there are also significant variations within large regions.

[15] Distance between countries i and j is measured as the distance between the capitals of the two countries.

We exclude neighbor countries to compute bilateral trade, and the distance from any country to a country with no maritime borders is computed through the closest port. Using total bilateral trade flows in goods, we estimate the following panel regression:

$$\log(Trade_{i,j,t}) = a_{i,j} + a_t + \beta_t^{sea} dist_{i,j}^{sea} + \beta_t^{air} dist_{i,j}^{air} + u_{i,j},$$

where $dist_{i,j}^{sea}$ is sea distance as computed by Feyrer (2019), $dist_{i,j}^{air}$ is the air population-weighted distance between countries (see Mayer and Zignago (2011)). The bilateral fixed effect $a_{i,j}$ takes into account all the constant determinants of trade a gravitational equation would normally control for, while a_t controls for time changes. The time-varying parameters on sea distance and air distance give some variation to the instrument. We can compute the instrument:

$$\widehat{Trade}_{i,t}^{Feyrer} = \sum_{j \neq i} Weight_{i,j,t_0} \exp(\hat{\beta}_{c(t)}^{sea} dist_{i,j}^{sea} + \hat{\beta}_{c(t)}^{air} dist_{i,j}^{air}), \tag{4.1}$$

where $c(t)$ defines a time bin (periods of 5 years in our examples), and where $Weight_{i,j,t_0}$ is a ponderation weight for which we allow several specifications: population or GDP of country j at a time t_0 divided by the sea distance between countries i and j, or simply exports between i and j at time t_0.[16] The idea is that the weight should bear no information regarding the evolution of trade after t_0. We usually did not select exports as a weight, because the quality of bilateral trade data made it too noisy: the quality of instrument would be dependent on how representative trade in a reference year (1970, 1980 or 1994) is of long-run trade evolution.

The instrument exhibits some time variance for each country, which can be attributed to partial shift of trade from sea to air travel. As a consequence, this instrument is compatible with country fixed effects, unlike the previous one, which makes it robust to critics as the ones in Rodrik, Subramanian, and Trebbi (2004).

We also allowed the coefficients to vary exogenously over time by fixing the values they took over time, assuming that: we define the instrument the same way as in equation 4.1, except that we use predetermined values of $\hat{\beta}_{c(t)}^{sea}$ and $\hat{\beta}_{c(t)}^{air}$, where typically the first one begins at 0. In this case, we define a beginning time t_0 and a final time $t_1 = 2019$ and we define:

$$\hat{\beta}_t^{air} = -1 - \hat{\beta}_t^{sea} = -\frac{t - t_0}{t_1 - t_0},$$

which simply means that $\hat{\beta}_t^{air}$ is equal to 0 in year t_0 and to -1 in year t_1, and to the linear interpolation between the two at year t, while $\hat{\beta}_t^{sea}$ does the opposite, which means that sea distance matters less than air distance over time. Overall, the different specifications chosen give similar results.

[16] Because of the bilateral fixed effect in the regression equation, the absolute value of the coefficients β has no meaning by itself, only its variations can be interpreted. We then fix the value of the coefficients at time t_0 to 1 by dividing the other values for the coefficients by $\hat{\beta}_{t_0}$,hence the necessity to take distance into account when defining certain weights.

We replicated Feyrer's results independently, using air distance as measured in the TRADHIST database from Fouquin, Hugot, et al. (2016), but using a different variation for bilateral sea distance, as defined in Bertoli, Goujon, and Santoni (2016); we included in the sea distance the distance between the closest port and the capital of a country when it was relevant. As some of our observations are related to changes in the number of countries in the world, for which the different databases available were not always precise, we added some computations. For example, the sea distance between France and USSR is equal to the average distance between France and each new country formed out of the USSR, weighted by their population the year after dissolution in 1991.

5 Results

In this part, we are going to test empirically Theorem 1 and Proposition 2 as well as Proposition 4 using the data and the instruments presented in the previous Section. In Section 5.1, using data from 1994 to 2019, we find some evidence suggestive that probability of default comoves with trade openness in the short run and in the long run: we use direct regression and instrumental regression with geographical variation in the relevant distance (air sea and trade sea) as an instrument for series on debt. In Section 5.2, we find some evidence suggesting a causal link between trade openness and debt, using variation in the effects of geography.

5.1 Trade and CDS Spreads

In this Section, we test for the result of Theorem 1 and Proposition 2 to test the following fact: *whether we condition on the level of debt or not, a more open country should face better a lower interest rate* and of proposition 2 as well: *a more open country should face a lower interest rate.* As a proxy for interest rates, we are going to use CDS premia. More precisely, we are going to test for:

$$CDS_{i,t} = -\gamma \log Trade_{i,t} + \psi X_{i,t} + u_{i,t}, \quad (5.1)$$

where i is an index for country i, t for year t, $Trade_{i,t}$ trade openness (as a percentage of GDP, in log), $X_{i,t}$ a set of controls including fixed effects and possibly debt-to-GDP ratio, $u_{i,t}$ an error term. We want to prove that γ , the coefficient of interest, is positive (which means that our estimates should give a negative coefficient). This precise functional form can be derived from a two-period model with specific assumptions regarding the distribution (see Appendix).

Because we are concerned with potential omitted variable bias, we should use Frankel-Romer's trade predictor, computed on 2007 bilateral trade flows, as an instrument for trade. More precisely, we will run the following regressions:

$$CDS_{i,t} = \gamma \log Trade_{i,t} + \delta \log Debt_{i,t} + \beta GDP_{i,t} + \alpha_t + \varepsilon_{i,t}$$
$$Trade_{i,t} = c. \log \widehat{Trade}_i^{FR} + d \log Debt_{i,t} + b.GDP_{i,t} + a_t + u_{i,t},$$

where the first equation is the reduced form of the IV and the second equation is the first stage, $D_{i,t}$ is debt-to-GDP ratio of country i at time t, $GDP_{i,t}$ is the real output of country i at time t.

If one assumes that the geographical variables determining trade affect financial institutions and countries' credibility only through their effect of trade, this predicted trade share can therefore be an instrument for trade in this paper's analysis. The identification assumption is that variations in $\hat{Trade}_i^{FR}$ are not correlated with institutional quality otherwise than through GDP (and other covariates). Frankel and Romer (1999) used this instrument to evaluate the benefits of trade on growth[17]. We show the results of this IV regression in table 3. We cannot include country fixed effects because the instrument is time-invariant for each country, as for the original Frankel-Romer instrument. For the same reason, we cluster by year only, and not by countries[18].

In columns 1, 2 and 4, we add a control for oil countries, thanks to specific time fixed effects for oil-producing countries, as listed by the Direction of Trade of Statistics (DOTS) of the International Monetary Fund (IMF)[19]. We do it to deal with variations in commodities' prices that affects gains from trade in a (oil is an easy to trade good that might be affected by default differently from non-commodity goods). We also control for trade balance in some specifications.

In our estimation, the effect of trade is more important than the effect of debt (when we measure it), which is striking as debt is the first motive invoked in sovereign debt crises, for example by rating agencies. In these estimations, a 10% increase in trade leads tp a 30 b.p. (basis points) decrease in spreads: the doubling of trade-to-GDP ratio through trade agreements could have large effects on sovereign borrowing according to this estimation; around 300 basis points. For example, in 2014, Italy trades twice as much of its GDP as Argentina. Then, according to our estimate, if Argentina traded as

[17] We will use the same proxy as Frankel and Romer (1999), substracting a few variables that we think may cause endogeneity issue such as regions, or the fact that a country is landlocked: indeed, they are likely to be directly correlated with financial institutions. Also, unlike Frankel and Romer (1999), we include area in the bilateral trade regression and not in our direct regressions. we run the regressions defining the proxy in 2007, which is the beginning of the period for the rest of the empirical analysis. Therefore, the proxy should not capture any variation posterior to 2007.

[18] However, clustering by year and country gave significant results at the 5% threshold.

[19] These countries are Algeria, Angola, the Republic of Azerbaijan, the Kingdom of Bahrain, Brunewe Darussalam, Chad, the Republic of Congo, Ecuador, Equatorial Guinea, Gabon, the Islamic Republic of Iran, Iraq, Kazakhstan, Kuwait, Libya, Nigeria, Oman, Qatar, Russian Federation, Saudwe Arabia, the Republic of South Sudan, Timor-Leste, Trinidad and Tobago, Turkmenistan, United Arab Emirates, the RepúbliCurrent AccountBolivariana de Venezuela and the Republic of Yemen. See http://datahelp.imf.org/knowledgebase/articles/516096-which-countries-comprise-export-earnings-fuel-a. This is a classification from the World Economic Outlook from the IMF.

much as Italy in the beginning of 2014, its CDS premium could have been 300 b.p. lower, which is a very significant difference: it is more than twice the maximal spread between Germany and Italy in 2022.

These results are also close to the calibration results we presented earlier, which implied a coefficient γ close to 300 (if we measure interest rates in terms of b.p.).[20]

Table 3: CDS and Frankel-Romer's instrument: OLS and reduced form IV. Standard errors clustered by year.

	Dependent variable:			
	CDS Spread			
	(1)	(2)	(3)	(4)
Trade (Percent of GDP, log)	−108.185***	−367.370***	−323.289***	−308.364***
	(39.784)	(55.022)	(51.406)	(52.245)
Debt (percent of GDP, log)	94.036***			−19.007
	(18.765)			(13.075)
Real GDP (log)	−200.046***	−104.947***	−89.533***	−90.830***
	(41.065)	(13.507)	(10.990)	(12.654)
Current Account	Yes	Yes	No	Yes
Instrument for Trade	No	Yes	Yes	Yes
Year Fixed Effects	Yes	Yes	Yes	Yes
Country Fixed Effects	Yes	No	No	No
Year and Oil Fixed Effects	Yes	No	Yes	Yes
Observations	704	703	541	537
418				
R^2	0.216	0.680	0.161	0.150
0.233				

Note:
*p<0.1; **p<0.05; ***p<0.01

Because the collection of our data only starts in 1994 and the number of counties with stable data is relatively weak, we can only use limited prospect for our data. Adding a country fixed effect was not always possible. However, although the Feyrer instrument was conceived for long-run studies over a period of 50 years, the same principle to allow

[20]We obtained this number by computing the ratios of the decline in the interest rate over the log increase in trade in our calibrating exercise.

the coefficients of sea distance and air distance to vary over time still has predictive power: from 1994 to 2019, the predictive power of sea distance for bilateral trade declined again, suggesting that shipping costs mattered even less, while the negative impact of air distance on bilateral trade staid roughly constant. However, the predictive power and the stability of the first stage regression is much less important than for long-run periods. We however found a negative significant effect of trade on CDS spreads in the period if we use the Feyrer instrument based on the period from 1994 to 2019, as we show in table (5) in the appendix.

5.2 Debt and Trade

In this Section, we want to show that, consistently with Proposition 4 and quantitative results, *debt increases in trade openness.* Calibration results suggest this effect should be quite important quantitatively. More precisely, we want to test the following model:

$$\log Debt_{i,t} = \gamma \log Trade_{i,t} + \psi X_{i,t} + u_{i,t}.$$

This equation is derived from a simplified model presented in the appendix, where shocks follow a Pareto distribution. We want to prove that γ, the coefficient of interest, is positive.

Because of endogeneity concerns, we will not use OLS equations for this relation either. We are going to use the Feyrer instrument and run the following IV regression for all countries i and years t.

$$\begin{aligned} \log Debt_{i,t} &= \boldsymbol{\gamma} \log Trade_{i,t} + \alpha_{GDP} \log GDP_{i,t} + \alpha_i + \alpha_t + \varepsilon_{i,t} \\ \log Trade_{i,t} &= c \log \widehat{Trade}_{i,t}^{Feyrer} + a_{GDP} \log GDP_{i,t} + a_i + a_t + e_{i,t}, \end{aligned}$$

with the different specifications for the Feyrer instruments defined as above.

Unlike CDS data, debt data are available over the long run: a lot of countries have debt data from 1950, and even further for certain Western countries. Moreover, debt is typically slow-moving, so that long-run relations make more sense. As a consequence, it is possible to use efficiently the Feyrer instrument in order to estimate debt. The exclusion restriction hypothesis under which our regression delivers a causal estimate is that long-run variations between countries in trade accessibility due to technological development in airplanes did affect debt only through its effect on trade and GDP.[21]

By construction, the first stage is very robust. We computed confidence intervals and estimated p-value using bootstrap. We give these results in appendix for 14 different computations of the Feyrer instrument alongside coefficient estimates in table (4). The estimate is positive in all cases. For 12 out of 14 different estimation strategies, the

[21] It is necessary to control for real GDP in the regression. Indeed, Feyrer (2019) found a significant relationship between growth and trade through this air trade instrument. Another way to formulate the exclusion-restriction hypothesis is that the change of geographical factors affected the macroeconomy only through changes in trade and the implied changes in GDP.

estimate has a p-value less than 5%, and the p-value is less than 6% in the two other cases. The results of the regression clearly confirm the theoretical results of our paper: more openness is causally associated with larger debt stocks. This stays true even if we restrict our observations to more recent periods (such as years after 1980 or years after 1994).

As we can see from in table (4), the implied estimate for γ is close to 1 in all specifications. Given the specification in logarithm, this estimate means that a 10% increase in trade leads to a 10% increase in debt-to-GDP ratio, which is a sizable effect. Since our other results suggest that more trade openness does not increase the level of risk, this implies a quite sizable effect of trade on debt. According to our estimates, a country whose trade openness doubles could double its debt-to-GDP ratio without facing negative consequences in terms of spreads. This estimate is completely consistent with the effect we found in our calibration, which implied a coefficient γ close to 1.3.[22]

[22] As in the precedent Section, we obtained this number by computing the ratios of the log increases in debt and trade in our calibrating exercise.

Table 4: Confidence intervals and p-values through bootstrap for the impact of trade on debt

Instrument Determination	Weights	Coefficient estimate	p-value (in %)	95% Confidence Interval
Regression 1950-2019	Population 1970	1.42	0.0	[0.77, 2.60]
Regression 1950-2019	Population 1980	1.44	0.0	[0.79, 2.39]
Regression 1950-2019	GDP 1970	1.45	0.7	[0.56, 2.51]
Regression 1950-2019	GDP 1980	1.34	0.5	[0.53, 2.35]
Regression 1980-2019	Population 1980	1.33	0.4	[0.53, 2.62]
Regression 1980-2019	GDP 1980	1.52	0.4	[0.55, 2.81]
Regression 1994-2019	Population 1994	1.05	5.4	[-0.03, 2.78]
Regression 1994-2019	GDP 1994	1.48	2.9	[0.17, 3.98]
Exogenous 1950-2019	Population 1970	0.80	0.6	[0.30, 1.29]
Exogenous 1950-2019	GDP 1970	0.66	5.3	[-0.02, 1.25]
Exogenous 1980-2019	Population 1980	1.07	0.0	[0.45, 1.88]
Exogenous 1980-2019	GDP 1980	1.21	0.0	[0.48, 1.99]
Exogenous 1994-2019	Population 1994	1.19	1.7	[0.26, 2.67]
Exogenous 1994-2019	GDP 1994	1.18	0.8	[0.32, 2.38]

6 Conclusion

In this paper, we argued that more open countries should be able to commit more easily to repay debt. After showing that defaulting countries seem to trade less as a consequence of default, we argued that the trade interruption was a realistic representation of what default cost could be. We investigated two consequences: more open countries are considered safer by markets, and more open countries seem to borrow more in the long run, and we have shown empirically that they were plausible. Our results suggest that a 10% increase in the total volume of trade to GDP should lead up to a 40 b.p. (basis points) decrease in CDS premia, for a given level of debt. Moreover, it should lead to a 10% increase in debt-to-GDP ratio. With those estimates, we can argue that if Argentina had been trading as much as Italy relative to its GDP in early 2014, its CDS premium would have been up to 400 b.p. lower, which is quite large: to give an example, increase in trade volume between China and Argentina might have played an important role in the build-up or Argentinian debt before 2000: according to our theory, it might have created favorable terms of credit for Argentina and led the country to borrow more. The empirical results are consistent not only with theoretical predictions but also with our calibration results, which gave results within the range of our estimates.

Roos (2019) noticed that, as of 2019, the share of world defaulters was surprisingly low (defaulting countries were 0.2% of world GDP only), and that even very fragile countries preferred to repay large debt burdens rather than to default. He argued that it was because the power of lenders and financial systems from rich countries had dramatically increased. Our theory can be considered a complementary explanation: the fear of an interruption of trade may have become much stronger today, after the deep international integration of goods' markets. This paper gives hints at trade as an important commitment device for sovereign international finance. Countries with anti-tariff policies do not only send a signal to markets about their economic management: they tie their hands with their gains from trade. Larger dependence on trade means that sovereign debt crises might be less likely but also more dramatic. This phenomenon could also explain the covariation of CDS sovereign premia observed by Longstaff et al. (2011) and Pan and Singleton (2008): the cycle of world trade could partly determine comovement of spreads.

After the accumulation of debt due to Covid pandemic and the geopolitical instability that followed the Russian invasion of Ukraine, two pessimistic scenarios related to this paper could turn into reality: a wave of sovereign debt crises and the formation of trade blocs, potentially restricting the access to certain markets for certain emerging countries. This paper stresses that these two developments could have ramifications on each other and potentially amplify the potential emerging crisis, hampering the difficult recovery from the Covid crisis in emerging economies.

We think two topics might be worth investigating for future research.The first is how trade channels might impact financial systems. A country's international liabilities can be paid back because of the country's dependence on international trade: how would a crisis in neighbor countries affect trade of other countries? The second is the extent to which trade depends on finance: did it change over time, is there a way to increase it? For example, if default interrupts trade only through an interruption of trade finance, dependence on trade finance is double-edged: it increases the ability to borrow ex ante, but hurts defaulting countries ex post. What has been the evolution of trade finance? Is it the reason why some countries chose not to default in recent years?

The study of sovereign debt could more generally give us a better understanding of the gains from trade: some countries accept to pay very large debt burdens inherited from previous years. If we assume these governments are rational, the size of these burdens should give us an insight of what the real gains from trade and financial integration are.

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7 Appendix: Proofs for Theoretical Results of Section 3

7.1 Two-Period Models

Proof of Proposition 2

In this Section, we prove that the probability of default should increase in trade costs (equivalently, decrease in trade openness or default costs). We use the same notation as in the appendix Section above, and prove that when δ increases, the probability of default decrease. We assume that the cumulative distribution function of the GDP in the final period is increasing. The price of the good is constant as well.

To prove it, we define an equivalent dual maximization problem where government maximizes its utility as a function of the probability of default and apply theorem 1 in Topkis (1978). Let P be the probability of default.

To keep exposition as simple as possible, we suppose $r = 0$ so that:

$$q(b,\delta) = 1 - P = \mathbb{P}(Y \geq \frac{b}{\delta}) = 1 - F(\frac{b}{\delta}),$$

where $\delta = \frac{g}{1+g}$ is a representation of the cost of default through the gains from trade. Then we can write B as:

$$B = \delta F^{-1}(P)$$

As long as F^{-1} is uniquely defined. If it is not uniquely defined, it means a local increase in debt B should lead to a no impact on the probability of default, so that the proposition would still hold. For now, we assume F^{-1} is uniquely defined and differentiable.

We write the new maximization problem of the government which maximizes its utility as a function of the probability of default at the next period, depending on the default cost:

$$V(P,\delta) := u\big(1 + \delta F^{-1}(P)(1-P)\big) + \beta \int_0^{+\infty} f(y) \max\{u(y - \delta F^{-1}(P)), u((1-\delta)y)\} dy.$$

First order condition implies:

$$\frac{\partial V(P,\delta)}{\partial P} = 0,$$

which can also be written:

$$\delta\Big(F^{(-1)\prime}(P)(1-P) - F^{-1}(P)\Big)u'\big(1+\delta F^{-1}(P)(1-P)\big) - \beta\delta \int_{F^{-1}(P)}^{+\infty} F^{(-1)\prime}(P) f(y) u'(y - \delta F^{-1}(P)) dy = 0,$$

where $F^{(-1)\prime}$ is the derivative of F^{-1}. Finally, one can compute:

$$\begin{aligned}\frac{\partial^2 V(P,\delta)}{\partial P \partial \delta} &= ((F^{(-1)\prime}(P)(1-P) - F^{-1}(P))u'\big(1+\delta F^{-1}(P)(1-P)\big) \\ &- \beta \int_{F^{-1}(P)}^{+\infty} F^{(-1)\prime}(P) f(y) u'(y - \delta F^{-1}(P)) dy \\ &+ \delta F^{-1}(P)(1-P)\Big(F^{(-1)\prime}(P)(1-P) - F^{-1}(P)\Big)u''\big(1+\delta F^{-1}(P)(1-P)\big) \\ &+ \beta F^{-1}(P) \int_{F^{-1}(P)}^{+\infty} F^{(-1)\prime}(P) f(y) u''(y - \delta F^{-1}(P)) dy.\end{aligned}$$

From the first order condition, one can observe that, at the optimum, the two first terms should cancel out:

$$\begin{aligned}\frac{\partial^2 V(P,\delta)}{\partial P \partial \delta} &= \delta F^{-1}(P)(1-P)\Big(F^{(-1)\prime}(P)(1-P) - F^{-1}(P)\Big)u''\big(1+\delta F^{-1}(P)(1-P)\big) \\ &+ \beta F^{-1}(P) \int_{F^{-1}(P)}^{+\infty} F^{(-1)\prime}(P) f(Y) u''(Y - \delta F^{-1}(P)) dy.\end{aligned}$$

We know that the term $F^{(-1)\prime}(P)(1-P) - F^{-1}(P)$ cannot be negative: otherwise, a decrease in the default probability (equivalent to a decrease in borrowing) would imply more revenues today: this option would be improving consumption today and tomorrow. The integral on the right-hand side is negative because u is concave. As a consequence the term $\frac{\partial^2 V(P,\delta)}{\partial P \partial \delta}$ is negative at the optimum if the utility function is concave. Hence, as consequence of Topkis' theorem, if the utility function is concave, the probability of default should be decreasing in default cost.

Proof and Discussion of Proposition 4

In this paragraph, we derive the formula from proposition 3 and also show the more general formula and detail the discussion about forces in motion to know whether the face value of debt increases as trade openness increases in the model.

More generally, if one assumes that preferences only consist in an aggregator with constant returns, then define gains from trade $1+g$ as in the body of the paper.

With that notation, government defaults if and only if:

$$\begin{aligned} y &\leq (1+g)(y-b) \\ y &\leq \frac{b}{g}(1+g) \\ \iff (1-\delta)L &\geq L-b \\ \iff y &\leq \frac{b}{\delta}, \end{aligned}$$

where $\delta := \frac{g}{1+g}$ is the cost of default in standard models. δ depends on τ. We use this simpler notation. One obviously finds that δ increases in τ, because g decreases in τ. Is the optimal borrowing quantity larger in this case? To answer that question, one only needs to compute what happens when δ increases. We assume that utility is homothetic so that we can consider default cost δ instead of gains from trade g (we also assume that international prices are constant in this result).

At the first period, government maximizes (after normalizing the GDP of the first period to 1 and the interest rate to 0):

$$\begin{aligned} V(b,\delta) &= u(1+q_\delta(b)b) + \beta\mathbb{E}u\big(\max((1-\delta)y; y-b)\big) \\ &= u(1+\mathbb{P}(y>\frac{b}{\delta})\times b) + \beta\int_0^{\frac{B}{\delta}} u((1-\delta)y)f(y)dy + \beta\int_{\frac{B}{\delta}}^{+\infty} u(y-b)f(y)dy, \end{aligned}$$

To prove this, we are going to use Topkis' theorem. Since we assume the distributional function is smooth enough, we can compute cross-derivatives, using the equilibrium condition to compute the function q. We compute

$$\begin{aligned} \frac{\partial}{\partial b}V(b,\delta) &= \frac{\partial(q(b,\delta)y)}{\partial y}u'(1+q(y,\delta)y) - \beta\int_{\frac{b}{\delta}}^{+\infty} u'(y-b)f(y)dy \\ \theta(b,\delta) := \frac{\partial^2}{\partial b\partial\delta}V(b,\delta) &= \frac{\partial^2(q(b,\delta)b)}{\partial b\partial\delta}u'(1+q(b,\delta)b) + \frac{\partial(q(b,\delta)b)}{\partial b}\frac{\partial(q(b,\delta)b)}{\partial\delta}u''(1+q(b,\delta)b) \\ &\quad - \beta\frac{\partial(q(b,\delta)b)}{\partial\delta}u'((1-\delta)\frac{b}{\delta}). \end{aligned}$$

One can notice that this quantity is equal to 0 whenever debt is negative or when b/δ is strictly less than the lower bound of the support of the distribution of GDP. In such a case, a change in the cost of default (equivalently, a change in trade costs) should not affect the will to borrow. Indeed, if the optimal level of borrowing is negative or strictly below the threshold for a positive probability of default, it means that the government is not constrained by default risk: it could happen for example if β is large enough, or, equivalently, if the government expects low GDP growth at the next period. The cost of default is irrelevant in this case: in an economy with pure commitment, government would borrow the same quantities.

Back to the general case, let A be absolute risk-aversion for a given level of consumption:

$$\begin{aligned} \theta(b,\delta) &= \frac{b}{\delta^2}u'(1+q(b,\delta)b) \times \Big((\frac{b}{\delta}f'(\frac{b}{\delta}) + 2f(\frac{b}{\delta}) - \beta\frac{u'(\frac{b}{\delta})}{u'(1+q(b,\delta)b)}f(\frac{b}{\delta})) \\ &\quad - (q(b) - \frac{b}{\delta}f(\frac{b}{\delta}))f(\frac{b}{\delta})A(1+q(b,\delta)b)\Big). \end{aligned}$$

If u is linear, u' is constant and positive and $A = u'' = 0$, so that θ is positive if and only if:

$$(2-\beta)f(x) + xf'(x) > 0,$$

where x is the default threshold for GDP. This is equation of the proposition, modulo the normalization of interest rate. To finish the proof on the two special distributions, it is enough to notice that, because we assume that $\beta(1+r) < 1$, density functions f such that $x \mapsto f(x)+xf'(x)$ is nonnegative always respect the condition. It is true for a log-normal distribution with parameters 0 and σ,[23] whose density is $f : x \mapsto \frac{e^{\frac{-(\log x)^2}{2\sigma^2}}}{x\sqrt{2\pi}\sigma}$,because:

$$\begin{aligned} xf'(x) &= -(1 + \frac{\log x}{\sigma^2})\frac{e^{\frac{-(\log x)^2}{2\sigma^2}}}{x\sqrt{2\pi}\sigma} \\ &= -(1 + \frac{\log x}{\sigma^2})f(x), \end{aligned}$$

and we deduce that $f(x) + xf'(x) = -\frac{\log x}{\sigma^2}f(x)$, which is positive when x is less than 1. If the default threshold for income y_2 is such that default probability is less than 50%, then the default threshold should be less than 1, and local condition to apply Topkis' theorem is satisfied. For a uniform distribution, $f' = 0$ so that the result holds trivially.

If f is the density of an exponential distribution with parameter λ, then we have:

$$f(x) + xf'(x) = \lambda e^{-\lambda x}(1 - \lambda x),$$

which is positive as long as x is less than $\frac{1}{\lambda}$. Now, we only need to show that government would never choose an amount of debt such that the default threshold for GDP is more than $\frac{1}{\lambda}$. Indeed, the amount of debt revenue corresponding to a default threshold of x is $Cxe^{-\lambda x}$, where C is a positive constant. It is easy to show that this quantity needs to be locally nondecreasing at the optimum: otherwise, government would be able to reduce its debt and to increase its revenue by decreasing face value debt and default threshold for GDP; by doing so, it would increase its consumption in first and second periods alike, which would mean that the amount of debt is not optimal. But this quantity is decreasing whenever $x \geq \frac{1}{\lambda}$, so that the default threshold has to be less than $\frac{1}{\lambda}$ in equilibrium, and the condition to apply Topkis' theorem will then be satisfied.

For the decomposition of the effect in the more general case, one can notice that:

$$\begin{aligned} \frac{\partial^2}{\partial b \partial \delta} q(b)b &= 2\frac{b}{\delta^2} f(\frac{b}{\delta}) + \frac{b^2}{\delta^3} f'(b) \\ &= \frac{b}{\delta^2}(f(\frac{b}{\delta}) + \frac{b}{\delta} f'(\frac{b}{\delta})). \end{aligned}$$

A more general local condition guaranteeing that a larger default cost (which is equivalent to lower trade costs) implies more debt is the equation:

$$\theta(b, \delta) > 0.$$

[23]The reasoning generalizes to $\mu \neq 0$, just replace x by xe^{μ} in the following reasoning.

One can note that, in an infinite-time model, as β goes to 0, the solution converges to the one of the two-period model. Therefore, when β is low enough, this result should extend to infinite time period.

In the expression:

$$\frac{\partial(q(b,\delta)b)}{\partial b}u'(1+q(b,\delta)b) - \beta\int_{\frac{B}{\delta}}^{+\infty} u'(y-b)f(y)dy = 0,$$

default cost appears in 3 different ways:

- As a factor impacting price of debt today in $\frac{\partial(q(b,\delta)b)}{\partial b}$. As long as $\frac{\partial^2(q(b,\delta)b)}{\partial b\partial\delta}$ is positive, this effect should increase debt. All the standard distributions we have tested are such that this assumption is true for the range of relevant welfare-maximizing debt levels. This is the meaning of the term $\frac{b}{\delta}f'(\frac{b}{\delta}) + 2f(\frac{b}{\delta})$ in the formula for $\theta(b,\delta)$. This is the substitution effect.

- It appears in the final period's consumption: larger default costs mean that there are more cases where debt should be repaid. This is the meaning for the term $\beta\frac{u'(\frac{b}{\delta})}{u'(1+q(b,\delta)b)}f(\frac{b}{\delta})$ in the formula for $\theta(b,\delta)$.

- Inside the marginal utility $u'(1+q(b,\delta)b)$ with an unambiguous negative effect on debt: better borrowing conditions today increase consumption today, and therefore lead to a decrease in marginal utility today and shift the consumption smoothing towards more consumption tomorrow. This is the meaning of the term $-(q(b) - \frac{b}{\delta}f(\frac{b}{\delta}))f(\frac{b}{\delta})A(1+q(b,\delta)b)$

-The second and third effects are clearly negative: if the cost of default of default increases, it means debt should have to be repaid in more states of the world in the next period. Then, borrower with a larger default cost should pay more attention to debt regarding its future consumption. The size of that effect should naturally get multiplied by β. This is the meaning of the term $\beta\frac{u'(\frac{b}{\delta})}{u'(1+q(b,\delta)b)}f(\frac{b}{\delta})$ in the formula for $\theta(b,\delta)$.

7.2 Proofs using the Dual Operator From Aguiar and Amador (2019)

Although Eaton-Gersovitz models have existed since 1981, Aguiar and Amador (2019) was the first to prove that equilibrium existed under certain conditions, defining a contraction converging to the equilibrium. The contraction they defined will allow us to prove certain of the results we presented.

We assume there is a countable number of states $s \in \mathbb{S}$. Their paper defines the contraction that should be useful to us. We resume the assumptions of the model: foreign prices are fixed and exogenous, so that gains from trade depend on the state of the world $g(s)$. In other words, debt does not affect relative prices, and effective consumption is the amount spent in consumption multiplied by gains from trade $1+g(s)$ in state s, consistently with our earlier presentation of the model. We assume that consumption is bounded as well. To prove our theorem, we conveniently assume that total consumption is bounded by a certain c', so that, as long as long as interest rate is small enough, we have: $V(b,s) \leq \bar{V}$ for every b and s, where $\bar{V}$ is pre-determined exogenously thanks to preference parameters.

We are interested in the following problem: we assume that $(V^D(s))_{s\in\mathbb{S}}$ is given and fixed, we take $(g(s))_{s\in\mathbb{S}}$ as given, and $(V^D(s))_{s\in\mathbb{S}}$ is independent of $(g(s))_{s\in\mathbb{S}}$. The Bellman equation is:

$$
\begin{aligned}
V^R(b,s) &= \max_{c,b'} u((1+g(s))c) + \beta \mathbb{E}_s V(b',s') \\
\text{subject to } y + b' &\sum_{s':V(b',s')\geq V^D(s')} \pi(s'|s) = c + b \\
V(b,s) &= \max\{V(b,s), V^D(s)\}.
\end{aligned}
$$

The operator associated with this Bellman equation can be written:

$$
\begin{aligned}
T^V f(b,s) &= \max_{c,b'} u((1+g(s))c) + \beta \mathbb{E} V(b',s') \\
\text{subject to } y + b' &\sum_{s':f(b',s')\geq V^D(s')} \pi(s'|s) = c + b \\
V(b,s) &= \max\{f(b,s), V^D(s)\}.
\end{aligned}
$$

It is not a contraction unfortunately, unlike the dual operator that we obtain by inverting the Bellman equation:

$$
\begin{aligned}
\hat{B}(v,s) = &\sup_{c\in[0,\bar{c}],b'} \Big\{ y(s) - c + R^{-1}\Big[\sum_{s'w(s')\geq V^D(s')} \pi(s'|s) \Big] b' \Big\} \\
\text{subject to } v \leq u((1+g(s))c) + \beta &\sum_{s':V^R(b',s')\geq V^D(s')} \pi(s'|s) \max\{V^R(s',b^{'}), V^D(s')\},
\end{aligned}
$$

from which we derive the following operator:

$$
\begin{aligned}
Tf(v,s) = &\sup_{c\in[0,\bar{c}],b',(w(s'))_{s'\in\mathbb{S}}} \Big\{ y(s) - c + R^{-1}\Big[\sum_{s':w(s')\geq V^D(s')} \pi(s'|s) \Big] b' \Big\} \qquad (7.1) \\
\text{subject to } &v \leq u((1+g(s))c) + \beta \sum_{s'\in\mathcal{S}} \pi(s'|s) \max\{w(s'), V^D(s')\} \\
&b' \leq f(w(s'), s') \text{ if } w(s') \geq V^D(s') \\
&w(s') \leq \bar{V} \, \forall s',
\end{aligned}
$$

The operator T depends on gains from trade g, and we should note it T_g when there is an ambiguity about the value of the vector g. However, in the absence of such ambiguity, we should denote it T.

This operator T is a contraction, such that, for any initial function f, $T^n f$ converges to the true solution of the Bellman equation as n goes to infinity. We can apply the conservation principle to this transformation. We define the associated solution of bonds:

$$B_g^{f\star}(v,s) = \text{argmax}^b_{c\in[0,\bar{c}],b',(w(s'))_{s'\in\mathbb{S}}} \Big\{ y(s) - c + R^{-1}\Big[\sum_{s':w(s')\geq V^D(s')} \pi(s'|s)\Big] b' \Big\}$$

$$\text{subject to: } v \leq u((1+g(s))c) + \beta \sum_{s'\in S} \pi(s'|s) \max\{w(s'), V^D(s')\}$$
$$b' \leq f(w(s'),s') \text{ if } w(s') \geq V^D(s')$$
$$w(s') \leq \bar{V}\, \forall s',$$

where argmax^b refers to the b component of the argmax.

Lemma 5. *T is monotonic: for any $f \leq h$, $Tf \leq Th$, where $f \leq h$ means that, for every $(v,s) \in \mathbb{V} \times S$*

Proof. It is a simple consequence of the fact that an increase in the function f is equivalent to a relaxing of the constraint:[24] indeed, for any b, s and $(w(s'))_{s'\in S}$ such that:

$$v \leq u((1+g(s))c) + \beta \sum_{s':w(s')\geq V^D(s')} \pi(s'|s) \max\{w(s'), V^D(s')\}$$
$$b' \leq f(w(s'),s') \text{ if } w(s') \geq V^D(s'),$$

we also have:

$$v \leq u((1+g(s))c) + \beta \sum_{s':w(s')\geq V^D(s')} \pi(s'|s) \max\{w(s'), V^D(s')\}$$
$$b' \leq h(w(s'),s') \text{ if } w(s') \geq V^D(s'),$$

simply because $h(w(s'),s') \geq f(w(s'),s')$ for every s'. As a consequence, for any (v,s), $Tg(v,s)$ and $Tf(v,s)$ are the maximum of the same function over two different sets, but the set over which we maximize the function is larger for the constraints associated with g, so that we can conclude $Tg(v,s) \geq Tf(v,s)$. □

Lemma 6. *$T_g f$ is monotonic in g: for every $g' \geq g$, we have:*

$$T_{g'} f \geq T_g f.$$

Proof. This is almost a matter of definition. We can rewrite the operators the following way, for any $\gamma \in \mathbb{R}^{|S|}$:

$$\forall (v,s) \in \mathbb{V} \times S,\ Tf(v,s) = \sup y(s) - c + R^{-1}\Big[\sum_{s':w(s')\geq V^D(s')} \pi(s'|s)\Big] b$$
$$(c, b, (w(s')_{s'\in S}) \in \mathbb{X}_\gamma(v,s),$$

[24] Note that the proof already exists in Aguiar and Amador (2019).

where $\mathbb{X}_\gamma$ is simply the set of variables defined by the constraints when gains from trade are equal to the vector γ. If we can prove that, whenever $g' \geq g$, we have for any s and v $\mathbb{X}_g(v,s) \subseteq \mathbb{X}_{g'}(v,s)$, then the proof is over. It is actually quite straightforward. Let $(b, c, (w(s')_{s' \in S}) \in \mathbb{X}_g(v,s)$ for some given v and s. Then we have:

$$\begin{aligned} u((1+g'(s))c) + \beta \sum_{s':w(s') \geq V^D(s')} \pi(s'|s) \max\{w(s'), V^D(s')\} &\geq u((1+g'(s))c) \\ &+ \beta \sum_{s':w(s') \geq V^D(s')} \pi(s'|s) \max\{w(s'), V^D(s')\} \\ &\geq v \end{aligned}$$

and the other constraints obviously hold as they are not impacted by the change in the value in g. As a consequence, $(b, c, (w(s'))_{s' \in S} \in \mathbb{X}_{g'}(v,s)$.

□

Lemma 7. *T preserves monotonicity: ket $f : \mathbb{V} \times S$ be a function decreasing in its first term (i.e., for every $s \in S$, $v \mapsto f(.,s)$ is a decreasing function). Then Tf is decreasing in its first term. Moreover, if u is continuous, T preserves continuity for decreasing functions: if $f(.,s)$ is continuous and decreasing in its first term for any s, $Tf(.,s)$ is also continuous in its first term for any s.*

Proof. Let us consider a given s, and two different values $(v, v') \in \mathbb{V}^2$ such that $v \leq v'$. The proof is exactly the same as before: we notice that, for any s, $Tf(v,s)$ and $Tf(v',s)$ are the maximum of the same function over two different constrained sets $\mathbb{X}(v,s)$ and $\mathbb{X}(v',s)$. If we can prove that $\mathbb{X}(v',s) \subseteq \mathbb{X}(v,s)$ for any given s, then the proof is done. Let s be given. Let $(b, c, (w(s')_{s' \in S}) \in \mathbb{X}_g(v',s)$. We want to prove that $(b, c, (w(s')_{s' \in S}) \in \mathbb{X}_g(v,s)$. The second and the third constraints defining $\mathbb{X}_g(v,s)$ do not depend on v, so that they are satisfied for any $(b, c, (w(s')_{s' \in S})$. Moreover, we have:

$$\begin{aligned} u((1+g(s))c) + \beta \sum_{s':w(s') \geq V^D(s')} \pi(s'|s) \max\{w(s'), V^D(s')\} &\geq v' \\ &\geq v. \end{aligned}$$

where the first inequality is simply a consequence of $(b, c, (w(s')_{s' \in S}) \in \mathbb{X}_g(v',s)$ and the second one a consequence of $v' \geq v$. We can therefore conclude.

Regarding continuity, we can go back to the definition of $Tf(v,s)$ and define optimal values $c^\star > 0$, $b^\star$ that solve the maximization problem for a given (v, s):[25]

$$Tf(v,s) = y(s) - c^\star + R^{-1}\Big[\sum_{s':w^\star(s')\geq V^D(s')} \pi(s'|s)\Big]b^\star.$$

Let $\epsilon > 0$. Let us define:

$$\phi : x \mapsto u^{-1}(x - \sum_{s'} \pi(s'|s)\max\{V^D(s'), w^\star(s')\}),$$

which is continuous in v because u is continuous and because $v - \sum_{s'} \max\{V^D(s'), w^\star(s')\} = c > 0$. Then there is $\eta > 0$ such that, for any $x \in [v - \eta, v + \eta]$, we have:

$$|\phi(x) - \phi(v)| \leq \epsilon.$$

Let us consider $x \in [v, v + \eta]$. We can prove, by definition of $Tf(x, s)$ as a maximal operator and by definition of ϕ, that:

$$\begin{aligned} Tf(x,s) &\geq y(s) - \phi(x) + R^{-1}\Big[\sum_{s':w^\star(s')\geq V^D(s')} \pi(s'|s)\Big]b^\star \\ &\geq Tf(v,s) - \epsilon, \end{aligned}$$

If $x \geq v$,we know from the first part of the lemma that Tf is decreasing so that $Tf(x,s) \in [Tf(v,s) - \epsilon, Tf(v,s)]$, which implies that $Tf(.,s)$ is continuous on the right as the previous reasoning is valid for any positive ϵ.

The reasoning to prove continuity on the left is slightly harder.

Define for any x the hat notations $\hat{c}_x$, $\hat{b}_x$ and $\hat{w}_x(s')$ the following way:

$$Tf(x,s) := y(s) - \hat{c}_x + R^{-1}\Big[\sum_{s':\hat{w}_x(s')\geq V^D(s')} \pi(s'|s)\Big]\hat{b}_x,$$

where the values also satisfy the constraints associated with the maximization problem defined above.

Assume that $\lim_{x\to v, x<v} Tf(x,s) = Tf(v,s) + \alpha$, and $\alpha > 0$. Consider some $x < v$. Then we define:

$$Tf(x,s) := y(s) - \hat{c}_x + R^{-1}\Big[\sum_{s':\hat{w}_x(s')\geq V^D(s')} \pi(s'|s)\Big]\hat{b}_x \geq Tf(v,s) + \alpha,$$

[25] We leave it to the reader to show that, as long as f is continuous, the supremum is always a maximum: indeed, we can find a sequence of elements in the constrained set such that $y(s) - c_n + R^{-1}\Big[\sum_{s':w_n(s')\geq V^D(s')} \pi(s'|s)\Big]b'_n\Big\}$ converges to $Tf(v,s)$, we can exploit Bolzano-Weierstrass theorem and the fact these elements should be bounded to find a convergent subsequence for all these elements, and finally show that the limit defined this way gives the maximal value and satisfies the constraints. We do all the steps in a slightly different context at the end of proof of Theorem 10.

and we have:

$$Tf(v,s) \geq y(s) - c^{\$} + R^{-1}\Big[\sum_{s':\hat{w}_x(s')\geq V^D(s')} \pi(s'|s)\Big]\hat{b}_x > \alpha + Tf(v,s) + \hat{c}_x - c_x^{\$},$$

$$\implies \hat{c}_x + \alpha < c_x^{\$}$$

where $c^{\$}(x) = u^{-1}(v - \pi(s'|s)\max\{V^D(s'), \hat{w}_x(s')\})$. We know that $\hat{c}_x = u^{-1}(x - \pi(s'|s)\max\{V^D(s'), \hat{w}_x(s')\})$. This is true for any $x < v$. We can prove that $(w_x(s'))$ is bounded as x approaches v: otherwise, we can find a sequence $w_n(s')$ diverging to infinity solving problems defining a sequence $Tf(x_n, s)$ for x_n contained in a small neighborhood, which would imply that the function $Tf(x_n, s)$ tends to $-\infty$ as one would need infinite savings to reach such a level. Because $(w_x(s'))$ needs to be bounded, so does $\sum_{s'} \pi(s'|s)\max\{V^D(s'), \hat{w}_x(s')\}$. As a consequence, we can extract a sequence (x_n) converging to v such that that $\sum_{s'} \pi(s'|s)\max\{V^D(s'), \hat{w}_x(s')\}$ converges to a value l. As a consequence, using the notation above, we should have, by continuity of u^{-1}: $\lim c_{x_n} - c_{x_n}^{\$} = 0$, which is a contradiction with the inequality above. We conclude $Tf(v,s)$ is also continuous in v on the left.

□

Lemma 8. *Consider an operator T_g as above. For any $g' \geq g$, any function f decreasing in its first term, and any s, v, we have*

$$B_{g'}^{f\star}(v,s) \geq B_g^{f\star}(v,s).$$

Proof. Let $b_1 = B_g^{f\star}(v,s)$ and $b_2 = B_{g'}^{f\star}(v,s)$, and similarly c_2, c_1 and $(w_2(s'))_{s'\in\mathbb{S}}$ and $(w_1(s'))_{s'\in\mathbb{S}}$ be the relevant solutions. We will proceed by contradiction. Assume that $b_2 < b_1$.

We have:

$$\begin{aligned} T_{g'}f(v,s) = \Big\{y(s) - c_2 + R^{-1}\Big[\sum_{s':w(s')\geq V^D(s')} \pi(s'|s)\Big]b_2\Big\} \\ \text{and: } v = u((1+g'(s))c) + \beta\sum_{s'} \pi(s'|s)\max\{w(s'), V^D(s')\} \\ b_2 \leq f(w(s'), s') \text{ if } w(s') \geq V^D(s') \\ w(s') \leq \bar{V}\,\forall s', \end{aligned}$$

We know that $T_g f(v,s)$ is increasing in g. As a consequence, we have:

$$T_{g'}f(v,s) \geq T_g f(v,s).$$

In particular, it implies:

$$R^{-1}\Big[\sum_{s':w_2(s')\geq V^D(s')} \pi(s'|s)\Big]b_2 - R^{-1}\Big[\sum_{s':w_1(s')\geq V^D(s')} \pi(s'|s)\Big]b_1 \geq c_2 - c_1.$$

But we know that:

$$\forall s', f(w_2(s'), s') = b_2 < b_1 = f(w_1(s'), s'),$$

where the last equality comes from the fact this constraint should be binding at the optimum because f is continuous (it is a consequence of the fact T preserves continuity). As a consequence, $w_2(s') > w_1(s')$ for every s', so that:

$$\sum_{s':w_2(s')\geq V^D(s')} \pi(s'|s)\max\{w_2(s'), V^D(s')\} \geq \sum_{s':w_1(s')\geq V^D(s')} \pi(s'|s)\max\{w_1(s'), V^D(s')\},$$

the inequality being strict whenever the sum is not zero. Therefore, we have, because of the first constraint:

$$(1+g'(s))c_2 < (1+g(s))c_1.$$

We can define the alternative eligible points for the problem that b_1, c_1 and $(w_1(s'))_{s'\in S}$ are supposed to solve: $b_1^{\text{alt}} = b_2$, $w_1^{\text{alt}} = w_2$ and $c_1^{\text{alt}} = c_2 \times \frac{1+g'(s)}{1+g(s)} < c_1$. Because $u((1+g(s))c_1^{\text{alt}}) = u((1+g'(s))c_2)$, we can check that the constraints are verified for this alternative consumption proposition. Because the initial solution is the optimal point, we should have:

$$-c_1 + R^{-1}\Big[\sum_{s':w_1(s')\geq V^D(s')} \pi(s'|s)\Big]b_1 \geq -c_1^{\text{alt}} + R^{-1}\Big[\sum_{s':w_1^{\text{alt}}(s')\geq V^D(s')} \pi(s'|s)\Big]b_1^{\text{alt}},$$

which can be written with the previous notations:

$$-c_1 + R^{-1}\Big[\sum_{s':w_1(s')\geq V^D(s')} \pi(s'|s)\Big]b_1 \geq -c_2 \times \frac{1+g'(s)}{1+g(s)} + R^{-1}\Big[\sum_{s':w_2(s')\geq V^D(s')} \pi(s'|s)\Big]b_2$$

We can find an alternative eligible point for the second problem: $b_2^{\text{alt}} = b_1$, $w_2^{\text{alt}} = w_2$, $c_2^{\text{alt}} = c_1\frac{1+g(s)}{1+g'(s)}$:

$$-c_2 + R^{-1}\Big[\sum_{s':w_2(s')\geq V^D(s')} \pi(s'|s)\Big]b_2 \geq -c_1\frac{1+g(s)}{1+g'(s)} + R^{-1}\Big[\sum_{s':w_1(s')\geq V^D(s')} \pi(s'|s)\Big]b_1.$$

Combining everything, we have:

$$c_2\frac{1+g'(s)}{1+g(s)} - c_1 \geq \Big[\sum_{s':w_2(s')\geq V^D(s')} \pi(s'|s)\Big]b_2 - \Big[\sum_{s':w_1(s')\geq V^D(s')} \pi(s'|s)\Big]b_1 \geq c_2 - c_1\frac{1+g(s)}{1+g'(s)}.$$

However, we know that $c_2\frac{1+g'(s)}{1+g(s)} - c_1$ is negative, and as a consequence, because $\frac{1+g(s)}{1+g'(s)} \leq 1$, we should have:

$$c_2\frac{1+g'(s)}{1+g(s)} - c_1 \leq (c_2\frac{1+g'(s)}{1+g(s)} - c_1) \times \frac{1+g(s)}{1+g'(s)}.$$

The only way for this inequality not to be contradictory is for $\frac{1+g(s)}{1+g'(s)}$ to be equal to 1. However, in this case, we can check that the two problems are equivalent and should have the same solutions. This leads to a contradiction is $g(s) \neq g'(s)$.

□

Theorem 9. *Let f be the fixed point associated with T_g and h the fixed point associated with $T_{g'}$, and $g \leq g'$. Then $h \leq h$. Moreover, $(T^n_{g'}f)_{n\in\mathbb{N}}$ defines a sequence of increasing functions. As a consequence, default debt threshold is higher at equilibrium for the more open economy.*

Proof. Let f be the fixed point for T_g , h the fixed point for $T_{g'}$. Because $T_{g'}$ is a contraction, $T_{g'}f$ converges to h for the supremum norm, and consequently pointwise as well.

we also have:

$$T_{g'}f \geq T_g f = f.$$

because $T_{g'} \geq T_g$, we can prove that for every $n \in \mathbb{N}^\star$, we have:

$$T^{n+1}_{g'}f \geq T_g T^n_{g'}f \geq T^2_g T^{n-1}_{g'}f \geq \dots T^n_g T_{g'}f \geq T^{n+1}_g f,$$

where $T^{n+1}_{g'}f \geq T_g T^n_{g'}f$ is just the lemma 8 applied to function $T^n_{g'}f$. We can prove $T^k_g T^{n+1-k}_{g'}f \geq T^{k+1}_g T^{n-k}_{g'}f$ for any $n \geq k$ and $k \geq 1$ in two steps:

- $T^{n+1-k}_{g'}f \geq T_g T^{n-k}_{g'}f$ derives from applying lemma 8 to the function $T^{n-k}_{g'}f$

-$T^k_g T^{n+1-k}_{g'}f \geq T^{k+1}_g T^{n-k}_{g'}f$ follows from using the monotonicity of the operator T^k_g (which is monotonous because T_g is monotonous).

This is enough to prove that for any n, we have:

$$T^{n+1}_{g'}f \geq T^{n+1}_g f = T^n_g(T_g f) = T^n_g f = \dots = f,$$

which implies, by taking the limit when n tends to infinity for every value $(v, s) \in \mathbb{V} \times \mathbb{S}$:

$$h \geq f.$$

This result allows us to derive directly the result we announced in Theorem **??** : higher debt-to-GDP should be sustainable at any state for a more open economy. Indeed, debt-to-GDP ratio of a government at state $s \in \mathcal{S}$ is going to be defined as:

$$b^D_g(s) = f(V^D(s), s),$$

where f is the fixed point of the operator T_g defined above. Indeed, this is direct consequence of the definition of f as the fixed point of the operator. Hence, debt-to-GDP ratio for a more open government with gains from trade g' is going to be at state $s \in \mathcal{S}$:

$$b^D_{g'}(s) = h(V^D(s), s),$$

where h is as before the fixed point of $T_{g'}$. Because $h \geq f$, we can immediately conclude that every $s \in \mathcal{S}$, we have:

$$b^D_{g'}(s) \geq b^D_g(s).$$

The next result is the most difficult one in the paper, as it forces us to compare policy functions. □

Theorem 10. *Assume u is concave. Let $Q^\star_g(v, s)$ be the amount of revenue raised by a government with value v and in state s given gains from trade g. Then we can prove that, if $g' \geq g$, we have:*

$$\forall (v, s) \in \mathbb{V} \times \mathcal{S},\ Q^\star_{g'}(v, s) \geq Q^\star_g(v, s).$$

As a consequence, if a country gets more open while keeping its spread constant, we should expect this country to raise more revenues from financial markets. This is equivalent to Theorem 3.

Proof. Let $g' \geq g$ be given. Let f be the fixed point of the operator T_g, and h the fixed point of $T_{g'}$.

Before going further, we introduce a notation. For every function $r : \mathbb{V} \times \mathcal{S}$, we define $b^{v,s}_r$, $(w^{v,s}_r(s'))_{s' \in \mathcal{S}}$, $c^{v,s}_r$ as the solutions to the maximization problem define in (7.1); if there are several solutions to a given problem, we allow these notations to denote a particular solution to be specified in case of need. Let us note $T = T_{g'}$.

Let us define the sequence of function $f_n = T^n_{g'} f$. f_0 is equal to f, and f_n converges to h. Moreover, we know that, because $g' \geq g$, the sequence of functions $(f_n)_{n \in \mathbb{N}}$ is an increasing sequence. Because f is a decreasing function and continuous in its first term (for any given second term), and because T conserves these properties, we can conclude that all functions from the sequence $(f_n)_{n \in \mathbb{N}}$ have such properties.

We are going to use, for some fixed v and s, the sequence $(Q_n(v, s))_{n \in \mathbb{N}^\star}$, where $(Q_n(v, s))_{n \in \mathbb{N}^\star} = (b^{v,s}_{f_n} \sum_{s' : w^{v,s}_{f_n}(s') \geq V_D(s')} \pi(s'|s))_{n \in \mathbb{N}^\star}$ and where $Q_0(v, s)$ is defined as the optimal policy function in the equilibrium defined by T_g and f. First, we are going to prove it is an increasing function. Then, we are going to prove that the limit of $(Q_n(v, s))_{n \in \mathbb{N}^\star}$ is indeed the revenue raised by the government in the new equilibrium: this just means that the iterative policy functions converge to the actual policy functions, which, although it is intuitive, needs to be proved formally.

We know from lemma 8 that $Q_1(v, s) \geq Q_0(v, s)$ for any eligible v and s. Now, let us consider any $n \geq 1$. We want to prove that, for any v and s, $Q_{n+1} \geq Q_n$ at least for one solution to the problem associated with the function f_{n+1}.

Let (v, s) be fixed and let us note $(w_2(s'))_{s' \in \mathcal{S}} = (w^{v,s}_{f_{n+1}}(s'))_{s' \in \mathcal{S}}$ and $(w_1(s'))_{s' \in \mathcal{S}} = (w^{v,s}_{f_n}(s'))_{s' \in \mathcal{S}}$, $b_2 = b^{v,s}_{f_{n+1}}$ and $b_1 = b^{v,s}_{f_n}$, $c_2 = c^{v,s}_{f_{n+1}}$ and $c_1 = c^{v,s}_{f_n}$. We also define $q_i = \sum_{s' : w_i(s') \geq V^D(s')} \pi(s'|s)$.

We want to prove that $q_2b_2 \geq q_1b_1$ for a solution b_2 . We will prove it by the contrary and assume that $q_2b_2 < q_1b_1$ for all solutions b_1 and q_1 to the problem associated with f_n.

We can first see that this implies that $b_2 < b_1$. Indeed, if $b_2 \geq b_1$ and $q_2b_2 < q_1b_1$, we can come back to the definition of the optimization problem associated with the definition of $T_{g'}(T_{g'}^{n+1})(v, s)$ and see that setting $b = b_1$ is going to deliver an eligible set that will have better results: lower debt implies higher reservation utility and more consumption if the revenue raised is higher.

Then we have $b_2 < b_1$. We know that:

$$q_2b_2 - c_2 \geq q_1b_1 - c_1 \implies c_2 < c_1,$$

which implies in turn:

$$v = u((1+g(s))c_1)+\sum_{s'} \pi(s'|s) \max\{V^D(s'), w_1(s')) = u((1+g(s))c_2)+\sum_{s'} \pi(s'|s) \max\{V^D(s'), w_2(s')).$$

One can prove that $w_2(s') > w_1(s')$ for at least one s' such that $w_2(s') \geq V^D(s')$. Because $b_2 = Tf^{n+1}(w_2(s'), s') \geq Tf^n(w_2(s'), s')$ and $Tf^n(w_1(s'), s') = b_1$ if $V^D(s') \leq w_1(s')$, we can conclude that, without loss of generality, that $w_2(s') > w_1(s')$ for every s' (for values of s' such that $V^D(s') > w_1(s')$, the definition of $w_1(s')$ does not matter as long as it is inferior to $V^D(s')$, so that we can without loss of generality set it lower, for example equal to $\inf_{s''} V^D(s'')$-1).

Now, we know by definition of w_2, b_2 and c_2 that they solve:

$$\begin{aligned} &\max_{c\in[0,\bar{c}],b',(w(s'))_{s'\in\mathbb{S}}} \Big\{ y(s) - c + R^{-1}\Big[\sum_{s':w(s')\geq V^D(s')} \pi(s'|s)\Big]b'\Big\} \\ &\text{subject to } v \leq u((1+g'(s))c) + \beta \sum_{s'\in\mathcal{S}} \pi(s'|s)\max\{w(s'), V^D(s')\} \\ &\qquad b' \leq T^{n+1}f(w(s'), s') \text{ if } w(s') \geq V^D(s') \\ &\qquad w(s') \leq \bar{V}\, \forall s', \end{aligned}$$

We can define alternative eligible values for this maximization problems, and we know that applying the objective function to these values should give us a lower result. Let us consider the alternative candidate $b_2^{\text{alt}} = b_1$, $(w_2^{\text{alt}}(s'))_{s'\in\mathcal{S}}$, and c_2^{alt}, where $(w_2^{\text{alt}}(s'))_{s'\in\mathcal{S}}$ is defined implicitly by:

$$b_1 = T^{n+1}f(w_2(s'), s') \text{ if } T^{n+1}f(b_1, s') \geq V^D(s'),$$

and equal to $\min V^D - 1$ otherwise, and c_2^{alt} is then defined by:

$$v = u((1+g'(s))c_2^{\text{alt}}) + \beta \sum_{s'} \max\{V^d(s'), w_2^{\text{alt}}(s')\}.$$

Because these alternative solution satisfies the constraints, and by definition of the initial points c_2, b_2 and $(w_2(s'))_{s'\in\mathcal{S}}$ as the maximal solution of a problem, we have:

$$\begin{aligned}-c_2 + R^{-1}\Big[\sum_{s':w_2(s')\geq V^D(s')} \pi(s'|s)\Big]b_2 &\geq -c_2^{\text{alt}} + R^{-1}\Big[\sum_{s':w_2^{\text{alt}}(s')\geq V^D(s')} \pi(s'|s)\Big]b_1 \\ &\geq -c_1 + R^{-1}\Big[\sum_{s':w_1(s')\geq V^D(s')} \pi(s'|s)\Big]b_1,\end{aligned}$$

where the second inequality stems from the fact that $f_{n+1} \geq f_n$. *Mutatis mutandis*, we can define an equivalent alternative solution for the first problem with $b_1^{\text{alt}} = b_2$:

$$-c_1 + R^{-1}\Big[\sum_{s':w_1(s')\geq V^D(s')} \pi(s'|s)\Big]b_1 \geq -c_1^{\text{alt}} + R^{-1}\Big[\sum_{s':w_1^{\text{alt}}(s')\geq V^D(s')} \pi(s'|s)\Big]b_2.$$

We know that $c_2 > c_1$. Using the definitions above and the fact that $f_{n+1} \geq f_n$, we can easily prove:

$$c_2^{\text{alt}} \geq c_1 > c_2 \geq c_1^{\text{alt}},$$

as well as:

$$w_2 \geq w_2^{\text{alt}} \geq w_1,$$

and

$$w_2 \geq w_1^{\text{alt}} \geq w_1.$$

Moreover, because the participation constraint should be binding at the optimum, and by definition of the alternative solutions, we have:

$$\begin{aligned}u((1+g'(s))c_1) + \beta\sum_{s'}\max\{V^d(s'), w_1(s')\} &= u((1+g'(s)c_2) + \beta\sum_{s'}\max\{V^d(s'), w_2(s')\} \\ &= u((1+g'(s))c_2^{\text{alt}}) + \beta\sum_{s'}\max\{V^d(s'), w_2^{\text{alt}}(s')\} \\ &= u((1+g'(s))c_1^{\text{alt}}) + \beta\sum_{s'}\max\{V^d(s'), w_1^{\text{alt}}(s')\} \\ &= v.\end{aligned}$$

In particular, this implies:

$$\begin{aligned}u(c_1) - u(c_2) &= \beta\sum_{s'}\max\{V^d(s'), w_2(s')\} - \beta\sum_{s'}\max\{V^d(s'), w_1(s')\} \\ u((1+g'(s))c_2^{\text{alt}}) - u((1+g'(s))c_1^{\text{alt}}) &= \beta\sum_{s'}\max\{V^d(s'), w_1^{\text{alt}}(s')\} - \beta\sum_{s'}\max\{V^d(s'), w_2^{\text{alt}}(s')\}.\end{aligned}$$

As we noticed that $w_1^{\text{alt}} \leq w_2$ and $w_2^{\text{alt}} \geq w_1$, we have:

$$\begin{aligned}u((1+g'(s))c_2^{\text{alt}}) - u((1+g'(s))c_1^{\text{alt}}) &= \beta\sum_{s'}\max\{V^d(s'), w_1^{\text{alt}}(s')\} - \beta\sum_{s'}\max\{V^d(s'), w_2^{\text{alt}}(s')\} \\ &\leq \beta\sum_{s'}\max\{V^d(s'), w_2(s')\} - \beta\sum_{s'}\max\{V^d(s'), w_1(s')\} \\ &\leq u((1+g'(s))c_1) - u((1+g'(s))c_2).\end{aligned}$$

Here, we can use concavity of u to deduce that:

$$c_2^{\text{alt}} - c_1^{\text{alt}} \leq c_1 - c_2.$$

Indeed, if we had the opposite inequality, we would have the following contradiction:

$$\begin{aligned}
u((1+g'(s))c_1) - u((1+g'(s))c_2) &= \frac{1}{1+g'(s)} \int_{c_2}^{c_2+(c_1-c_2)} u'((1+g'(s))x)dx \\
&\leq \frac{1}{1+g'(s)} \int_{c_1^{\text{alt}}}^{c_1^{\text{alt}}+(c_1-c_2)} u'((1+g'(s))x)dx \\
&< \frac{1}{1+g'(s)} \int_{c_1^{\text{alt}}}^{c_1^{\text{alt}}+(c_2^{\text{alt}}-c_1^{\text{alt}})} u'((1+g'(s))x)dx \\
&< u((1+g'(s))c_2^{\text{alt}}) - u((1+g'(s))c_1^{\text{alt}}),
\end{aligned}$$

where the second inequality simply stems from the fact that u' is increasing and $c_1^{\text{alt}} > c_2$ and the third inequality results from the assumption and the fact that u' is positive.

Because we have $c_2^{\text{alt}} - c_1^{\text{alt}} \leq c_1 - c_2$ and $c_2^{\text{alt}} \geq c_1 > c_2 \geq c_1^{\text{alt}}$, then $c_2^{\text{alt}} = c_1$ and $c_1^{\text{alt}} = c_2$. Moreover, we can deduce $w_2^{\text{alt}} = w_1$ and $w_1^{\text{alt}} = w_2$. Indeed, we have:

$$\begin{aligned}
v &= u((1+g'(s))c_2) + \beta \sum_{s'} \max\{V^d(s'), w_2(s')e\} \\
&= u((1+g'(s))c_2) + \beta \sum_{s'} \max\{V^d(s'), w_1^{\text{alt}}(s')\},
\end{aligned}$$

and moreover we have the constraint:

$$\begin{aligned}
w_1^{\text{alt}}(s') \geq V^D(s') &\implies T^n f(w_1^{\text{alt}}(s'), s') = b_2 \\
w_2(s') \geq V^D(s') &\implies T^{n+1} f(w_2(s'), s') = b_2 \\
\sum_{s'} \max\{V^d(s'), w_1^{\text{alt}}(s')\} &= \sum_{s'} \max\{V^d(s'), w_2(s')\}
\end{aligned}$$

We now prove that the vectors w_1^{alt} and w_2 must be identical, up to the values below $V^D(s')$, that are irrelevant (we can modify vectors w_1 and w_2 by giving them a value equal to $v_{\min} - 1 = \inf_{s' \in \mathcal{S}} V^D(s') - 1$ whenever they are strictly inferior to $V^d(s')$. This modification does not change the other properties or the eligibility of w_1 and w_2, or the fact they maximize the problem).

Assume that $\left(w_1^{\text{alt}}(s')\right)_{s' \in \mathcal{S}} \neq \left(w_2(s')\right)_{s' \in \mathcal{S}}$. Then if there is s' such that $w_2(s') < w_1^{\text{alt}}(s')$, one can notice that this is a contradiction: indeed, this would imply $T^{n+1} f(w_2(s'), s') < T^n f(w_1^{\text{alt}}(s'), s') = b_2$, so that $w_2(s')$ could increase which would allow to have a better optimum for the original problem. If $w_2(s') > w_1^{\text{alt}}(s')$, then because

$$\sum_{s'} \max\{V^d(s'), w_1^{\text{alt}}(s')\} = \sum_{s'} \max\{V^d(s'), w_2(s')\},$$

there should be another $s^\star$ such that $w_1^{\text{alt}}(s^\star) > w_2(s^\star)$ (this is true because of the transformation we applied to both vectors) and we are back to the previous case with the contradiction it implies. As a consequence, we have:

$$-c_2 + q_2 b_2 \geq -c_1 + R^{-1} q_1 c_1 \geq -c_2 + R^{-1} q_2 b_2.$$

It implies that (b_1, c_1, w_1) also solves the problem associated with the function $f_{n+1} = T^{n+1} f$ and the operator T. It is a contradiction with our initial assumption.

Now, we have to prove that the limit of our operator is indeed a solution. There is a sequence of solutions to the problem associated with f_n such that (b_n, c_n, w_n) such that (Q_n) is an increasing sequence. We have:

$$f_{n+1}(v, s) = -c_n + Q_n.$$

The sequences $(f_{n+1}(v,s))_{n\geq 1}$ and $(Q_n)_{n\geq 1}$ are increasing, so that $(c_n)_{n\geq 1} = (f_{n+1}(v,s) + Q_n)_{n\geq 1}$ is also increasing. We know that $(f_n(v,s))$ is bounded because it is converges to $h(v,s)$, and we can establish that Q_n is bounded coming back to the definition of the problem and using the fact that in the equilibrium associated with operator $T_{g'}$ and function h, the maximal borrowed amount is bounded. As a consequence, $(Q_n)_{n\in\mathbb{N}} = (q_n b_n)_{n\in\mathbb{N}}$ converges to a finite limit Q, while c_n converges to a limit c, such that:

$$h(v, s) = -c + Q.$$

Because the sequence (q_n) is bounded by R^{-1} and 0, we can extract a subsequence $(q_{\phi(n)})_{n\in\mathbb{N}}$ converging to a value q, where $\phi : \mathbb{N} \to \mathbb{N}$ is a strictly increasing function. We can prove that q is positive under certain conditions such as a debt ceiling. Hence, the extracted sequence $(b_{\phi(n)})_{n\in\mathbb{N}}$ converges to Q/q. We now need to prove that b and c indeed define an eligible point for our functions and define a maximum. We can define the candidate w such that:

$$\forall s' \in \mathcal{S}, h(V^D(s'), s') \geq b \implies h(w(s'), s') = b,$$

and as before, for other values of s' we set $w(s') = v_{\min} - 1$ without loss of generality. Let $\epsilon > 0$. We want to prove two things in order to conclude:

$$q^b := R^{-1} \sum_{s' : w(s') \geq V^D(s')} \pi(s'|s) = q = \lim_{n\to+\infty} q_{\phi(n)} = \lim_{n\to+\infty} R^{-1} \sum_{s' : w_{\phi(n)}(s') \geq V^D(s')} \pi(s'|s),$$

and:

$$v = u((1 + g'(s))c) + \beta \sum_{s'} \pi(s'|s) \max\{V^D(s'), w(s')\}.$$

Consider the first equality.

We can prove first that $q^b \geq q = \lim q_{\phi(n)}$. Indeed, define the set $\mathcal{S}^D$ of states s' such that $h(V^D(s'), s') < b$ and define $\alpha = \min_{s' \in \mathcal{S}^D} b - h(V^D(s'), s')$ which is finite as the number of states is finite (this part of the proof is robust to this assumption of finiteness). There is n_0 such that $|b_{\phi(n)} - b| < \frac{\alpha}{3}$ and $|f_{\phi(n)} - h| < \frac{\alpha}{3}$ for every $n \geq n_0$, which then implies for every $n \geq n_0$:

$$\lim_n R^{-1} - q_n = R^{-1} \lim_n \sum_{s': w_{\phi(n)}(s') < V^D(s')} \pi(s'|s) \geq R^{-1} \sum_{s': w(s') < V^D(s')} \pi(s'|s) = R^{-1} - R^{-1} q^b,$$

because for every $n \geq n_0$, we have:

$$b_{\phi(n)} - f_n(V^D(s'), s') \geq b - \frac{\alpha}{3} - h(V^D(s'), s') - \frac{\alpha}{3} \geq \frac{\alpha}{3} > 0 \implies w(s') < V^D(s'),$$

by definition of α. We conclude $q^b \geq q$. The same reasoning also implies that:

$$u((1 + g'(s))c) + \beta \sum_{s'} \pi(s'|s) \max\{V^D(s'), w(s')\} \geq v.$$

Let us assume that at least one of these inequalities is strict. Then (b, c, w) is an eligible point for the maximization problem that defines $Th(v, s)$:

$$Th(v, s) = \max_{w', c', b'} -c' + R^{-1} \sum_{s': w'(s') \geq V^D(s')} \pi(s'|s) b'$$
$$\text{subject to } v \leq u((1 + g'(s))c') + \beta \sum_{s'} \pi(s'|s) \max\{V^D(s'), w'(s')\}$$
$$\forall s' \in \mathcal{S}, V^D(s') < w(s') \implies b' \leq h(w'(s'), s').$$

If $q^b > q$, it means that:

$$Th(v, s) \geq -c + q_b b > -c + qb = Th(v, s).$$

In the same way, if $u((1 + g'(s))c) + \beta \sum_{s'} \pi(s'|s) \max\{V^D(s'), w(s')\} > v$, it is possible to slightly decrease c to c^b while still satisfying the constraint, and then we have:

$$Th(v, s) = -c^b + qb > -c + qb = Th(v, s).$$

In both cases, this gives a contradiction. We can conclude that b and q are the quantities for borrowing and spread in the new equilibrium, and this finishes the proof. □

The result is less strong than the empirical results we test in two ways. Because we have to rely on the dual operator, the theorem tells us how a more open government borrows more when comparing two governments with the same level of initial spreads or risks of default - indeed, the theorem allows us to compare two governments with the same level of welfare at a given moment, and this level of welfare can be equated with the level of the interest rates given the structure of the model.

We would like to prove one the following two propositions in this set-up: face value debt increases or government's debt gets safer. However, such propositions are not correct. It should be enough to provide counter-examples, which we are going to with extremely stark assumptions (two states and an absorbing one, infinite discount factor). These counter-examples are extreme cases that do not represent the results a more seriously calibrated model would give, but we expose them to show that our framework did not allow more general results than the ones we have exposed.

Example 11. *Face value debt can decrease when a government gets more open.* Assume there are two states of the world (s_1, s_2), and we have $y(s_1) = y(s_2) = 1$, $g(s_1) = 1$, $g(s_2) = 0$, $\pi(s_2|s_2) \approx 1$, $\pi(s_2|s_1) = \alpha > 0$, $\beta = 0$, $R^{-1} = 1 - \epsilon$.[26] In this case, government would like to maximize their current income. Government should default at state s_2 for any amount of debt and default at state s_1 for a level of debt higher than b defined the following way:

$$(1-\alpha)(1-\epsilon)b - b = -g(s_1) \implies b = \frac{1}{\alpha + \epsilon - \alpha\epsilon}.$$

This should also be the observed level of debt in equilibrium starting at state s_1, as long as government stays in it. Now assume that government gets more open and $g'(s_2) = 1-\eta$, $g'(s_1) = 1$, with $\eta \in (0,1)$. As a consequence, government can either keep the same level of debt (with the same revenue) as before or play it safer and keep debt repayable in case state 2 happens, which would give the following result:

$$(1-\epsilon)b' - b' = -(1-\eta)g(s_1) \implies b' = \frac{1-\eta}{\epsilon}.$$

Government should choose that policy as long as it receives more income from it (it does not care about the future):

$$\frac{1-\alpha}{\alpha + \epsilon - \alpha\epsilon} < \frac{1-\eta}{\epsilon}.$$

Whatever happens, the level of raised revenue should not decline. However, face value debt is going to be lower for that policy as long as:

$$\frac{1-\eta}{\epsilon} < \frac{1}{\alpha + \epsilon - \alpha\epsilon}.$$

We only need to find values α, η and ϵ for which this is true: this is the case for $\alpha = 0.3$, $\eta = 0.6$ and $\epsilon = 0.2$.

Example 12. Debt can get riskier when government gets more open. Let us assume that $\beta = 0$, $R^{-1} = 1 - \epsilon$. There are two states s_1 and s_2, with $y(s_1) = g(s_1) = 1 = y(s_2) = g(s_2)$, and $g'(s_2) = 1$, $g'(s_1) = 2$. We assume s_2 is still an absorbing state: $\pi(s_2|s_2) \approx 1$, and $\pi(s_2|s_1) = \alpha \in (0,1)$. When gains from trade are g, government is going to borrow

[26] The general results in Aguiar and Amador (2019) assume that the transition probability from one state to another is always positive; as our results depend on them, we could think this is a mistake. We can circumvent this problem by simply assuming that this probability is positive but vanishingly small, which should force the government to default in the second state in equilibrium for almost any plausible debt level.

the maximal quantity of debt it can commit to repay (because it wants to benefit from a non-zero price on its bonds):

$$g(s_1) = b - (1-\epsilon)b \implies b = \frac{1}{\epsilon}.$$

When g increases to g', the government can either keep this strategy and get the same revenue and debt, or it can make a riskier bet and borrow as much as possible to reimburse in state 1 only:

$$\begin{aligned} g'(s_1) &= b' - (1-\epsilon)(1-\alpha)b' \\ \implies b' &= \frac{2}{\epsilon + \alpha - \epsilon\alpha}. \end{aligned}$$

This is the bet government should do whenever $b' > b$, which is true for example if $\alpha \in (0, \epsilon)$. Then, default probability becomes positive (α) in this case for the more open government with gains from trade defined by g', whereas it was zero for the less open government with gains from trade g.

In both cases, our previous result holds: the government raises more revenue from debt when they get more open. There are two ways to raise more revenues: decrease the interest rate or borrow more debt in face value. We cannot guarantee that government would do a combination of both in every case and it might choose only of the two ways, although numerical calibrations result that government is more likely to benefit from lower rates and increase its face value debt at the same time.

8 Appendix: Model for Calibration

In this Section, we present the calibration model that we use for numerical results in Section 3.3.

8.1 Preferences, Technology and Stochastic Shocks

Domestic economy has random endowment $(Y_t)_{t\geq 0}$ that is a stochastic process. The endowment is given in terms of the country's specific good that it will trade with the rest of the world. We are interested in a small open economy trading with the rest of the world. We assume that the rest of the world is a large economy behaving like a single country. This assumption serves as a simplification for a world economy with a large number of countries: the assumption restricts it to mean that what happens in the domestic economy should not affect the relative prices of goods between 2 foreign countries.

This is an Armington economy: each country produces its own good. Besides the domestic good, there is foreign good. The price of foreign goods is considered given by the domestic economy: it is the *numéraire*, which should matter as there will be debt in the model.

There are trade costs τ (for both imports and exports) and export taxes δ that we consider fixed and given. However, the behavior of the economy can modify the trade costs through default, as we will see below.

- Static Consumption and gains from trade

As in the rest of the paper, the country wants to maximize its utility:

$$\mathbb{E}_0 \sum_{t\geq 0} \beta^t u(C_t),$$

where total aggregate consumption $(C_t)_{t\geq 0}$ is defined, for every period t, as:

$$C_t = m_0(c_t, c_t^\star),$$

which means it is an aggregate mix of domestic good's consumption $(c_t)_{t\geq 0}$ and foreign consumption $(c_t^\star)_{t\geq 0}$.[27]

There is an intratemporal resource constraint and an intertemporal budget constraint. To simplify the presentation, we first present the intratemporal resource constraint taking as given transfers and then present the borrowing conditions in this sovereign debt model.

Foreign demand

There is a foreign demand for the domestic good that is going to determine its price - hence the welfare of the domestic economy. The domestic economy is a small open economy that does not affect relative prices from one country to another. However, it can affect through tariff the price of its own good and manipulate its terms of trade. Indeed, it exerts a monopoly power over its domestic good.

We also assume that all foreign economies have CES preferences with the same elasticity of substitution σ. As a consequence, total foreign demand for the domestic good should be equal to:

$$d_t^\star p^{-\sigma}.$$

[27]It is easy to see how this equation can include the more general case with any number of countries N. Assume each country $i \in [1, N]$ produces a good $(c_{1,t}, ..., c_{N,t})_{t\geq 0}$. We assume that preferences are weakly separable so that aggregate consumption can also be written:

$$C_t = m(c_t, c_t^\star)$$

where $c_t^\star = m^\star(c_{1,t}, ..., c_{N,t})$ is an aggregate of consumption of foreign goods. If the relative price of all foreign countries is assumed to be fixed, then, up to a normalization, the trade model will be equivalent to a 2-country trade model.

Moreover, the specific choice of Armington trade should not matter as we are mostly interested in gains from trade. Arkolakis, Costinot, and Rodriguez-Clare (2012) prove that, conditional on similar imports and a constant elasticity of substitution, using a Ricardian or an Armington trade model should not matter to estimate the gains from trade.

This functional form stems from our assumption that our economy is small relative to the rest of the world.[28] In a model with several countries, each one having its output $Y_{i,t}$, its own-good price level $p_{i,t}$ and aggregate price level $P_{i,t}$, we would have:

$$d_t^\star = \sum_{i=1}^{N} d_{i,t}(p) = \sum_{i=1}^{N} \frac{p_{i,t} Y_{i,t}}{P_{i,t}} \tau_i^{1-\sigma}.$$

Because of the intertemporal nature of the model, we would need to assume that government keeps track of the foreign demand of each country to predict the evolution of terms of trade.

For the sake of tractability, we are going to assume that aggregate demand $(d_t^\star)_{t\geq 0}$ is a Markov process. This is not always true for the sum of Markov processes,[29] but it is a reasonable assumption that is going to allow significant progresses in terms of tractability. We are also going to assume that, in the long run, $(d_t)_{t\geq 0}$ follows the tracks of the output process. We assume that there is an underlying process $(\xi_t)_{t\geq 0}$ such that:

$$d_t^\star = \xi_t Y_t,$$

and $(\xi_t)_{t\geq 0}$ is stationary (while Y_t is not necessarily stationary). This assumption is necessary in order to be able to solve the model numerically.[30] Furthermore, we make it because we are interested in the relation between debt and total openness of a country: any other assumption would modify the long-run value of trade openness.[31]

Stochastic Structure

[28] Under standard CES assumptions, the functional form $d_{i,t}$ of the demand of foreign country i for the domestic good should be given by:

$$d_{i,t}(p, \tau_{i,t}, \tau_{i,t}^F) := \frac{p_{i,t} Y_{i,t}}{P_{i,t}} (1 + \tau_{i,t}^d)^{1-\sigma^\star} (1 + \tau_{i,t}^F)^{-\sigma^\star} p^{-\sigma^\star}$$

where $p_{i,t}$ is the price of the Armington good of economy i - potentially including a tariff from the domestic economy, $Y_{i,t}$ is GDP, $\tau_{i,t}^d$ is an iceberg trade cost, $\tau_{t,i}^F$ is an import tariff imposed by country i, $\sigma^\star$ is the elasticity of substitution between different goods' consumption for country i, $P_{i,t}$ is the aggregate price index faced by country i at time t. We included as variables in the demand function parameters that depend on the domestic economy: its price, the tariff the domestic economy faces, which can decrease thanks to free-trade agreements, and the trade costs, which might increase if the economy defaults. Variable $d_t^\star$ is then an aggregate weighted by standard gravitational determinants of trade. It includes potentially relevant political variable, such as the import tariff imposed by foreign countries.

[29] In this case, learning past observations of total demand can be useful to predict which countries have low or high demand at the moment, if they have different sizes. As a consequence, it might impact predictions about which shocks are going to evolve.

[30] The assumption is not necessary strictly speaking to solve the model numerically but it sensibly reduces the number of states we have to consider for state variables.

[31] If we introduce long-run growth, this assumption means that $(d_t^\star)$ has the same long-run trends as (Y_t). When (Y_t) is stationary, this assumption only implies that $(d_t^\star)$ should be stationary. Additional state variables have a very heavy computational costs and we are interested in the aggregate foreign demand in this paper.

As we have seen, there are two exogenous state variables $(Y_t)_{t\in\mathbb{N}}$ and $(d^\star_t)_{t\in\mathbb{N}}$. The stochastic structure is going to rely on a Markov process $(s_t)_{t\in\mathbb{N}} \in \mathcal{S}^{\mathbb{N}}$ where S would typically be a subset of $\mathbb{R}^d$. For calibrations, we will assume that S is finite.

We simply assume there are functions Y and $d^\star$ such that: $(Y_t)_{t\in\mathbb{N}} = (Y(s_t))_{t\in\mathbb{N}}$ and $(d^\star_t)_{t\in\mathbb{N}} = (d^\star(s_t))_{t\in\mathbb{N}}$. As a consequence of our assumption, we will be able to write value functions as a function of s_t directly.

A specific case of this structure is when $(Y_t, d^\star_t)$ is a Markov process.[32]

8.2 Intratemporal Trade

Trade Costs and Tariff There are trade costs τ that are assumed to be constant over time, except when there is a default, as we will explain below.

We also assume that the government chooses a tariff level on exports equal to δ. As a consequence, foreign demand for exports should be defined by the perceived price δp:

$$D_t(p) = d^\star_t \tau^{1-\sigma}\delta^{-\sigma}p^{-\sigma}.$$

Using a tariff on exports is, for our purpose, equivalent to a tariff for imports, as it has been demonstrated in Costinot and Werning (2019). The only thing that could break the equivalence would be if foreign assets were labeled in domestic currency. It turns out that we assume that debt is monetized in the foreign currency. However, if foreign debt were labeled in domestic currency rather than in the reserve currency, import tariff and export tax would not be equivalent any more. Indeed, in such a case, government would have a tendency to favor export taxes or import tariff depending on whether it would be borrowing or lending money: the policy choice affects the value of the current account in this case. However interesting such an asymmetry might be, we exclude it from the analysis: the model is a better fit for countries borrowing their money in foreign currencies.

Budget Constraint and Resource Constraint Let T be the size of financial transfers from others countries in the world to country 1 at time t - equivalently, T is the current account deficit, that should depend on inherited debt, new debt emission and the price of new debt emissions as we will discuss in the next section. The budget constraint is:

$$p_t c_t + \tau c^\star_t = p_t Y_t + R_t + T,$$

where $R_t = (\delta - 1)p_t(Y_t - c_t)$ is the revenue from tariff. The resource constraint for the economy's own good can be written:

$$D(\tau p_t) + c_t = Y_t$$
$$\iff p_t D(\tau p_t) = \tau c^\star_t - R_t - T.$$

[32]The purpose of this general definition is to include all cases that would usually appear in a calibration exercise. For example, when we consider persistent growth shocks similar to those modeled by Aguiar and Gopinath (2006), (Y_t) is not a Markov chain so that we need a more general definition for a Markov equilibrium.

In the model, the price of other goods is considered exogenous. We consider our economy is small enough not to have any effect on relative prices between foreign countries. As a consequence of this assumption, we do not need to take into account resource constraints in foreign countries. If we fix the level of intertemporal transfers T and the revenue from tariff $R(p_t)$, the domestic demand for the domestic good should be equal to the following function of p:

$$D^d(p) = \frac{p^{-\sigma}}{p^{1-\sigma} + \tau^{D\,1-\sigma}} \Big[pY + T + R\Big].$$

Summing these two demands and computing the revenue from tariff should give us the complete price equation that defines the implicit price equation $p(s,T)$.

8.3 Default and Financial Markets

As a reminder, government maximizes the expected of utility of its future aggregate consumption:

$$\max \sum_{t\geq 0} \beta^t u(C_t),$$

where C_t is a CES aggregate of domestic consumption and foreign good's consumption:

$$C_t = \big(\alpha^{\frac{1}{\sigma}} c_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} c_t^{\star\frac{\sigma-1}{\sigma}}\big)^{\frac{\sigma}{\sigma-1}}.$$

Default Utility

In case of default, government gets temporarily out of financial markets and has to finance its own trade which results in an increase in trade costs from τ to $\tau^D \geq \tau$ until the end of default. Moreover, the country faces a temporary loss of productivity $x \in [0,1)$.

Hence, the default budget constraint becomes:

$$p^D(s)c + \tau^D c^\star = p^D(s)(1-x)Y,$$

where function $p^D(Y, d^\star)$ is the value of p that solves:

$$\frac{p^{-\sigma}}{p^{1-\sigma} + \tau^{D\,1-\sigma}}(p(1-x)Y + d^\star(\delta-1)\delta^{-\sigma}\tau^{1-\sigma}p^{1-\sigma^\star}) + d^\star p^{-\sigma}\tau^{D\,-\sigma} = (1-x)Y. \quad (8.1)$$

At each period during default, government can get out of default with probability λ - which happens independently with the evolution of other variables. From that, we conclude that total aggregate consumption should be equal to:

$$\frac{(p^D(Y,d^\star)(1-x)Y + T + d^\star(\delta-1)\delta^{-\sigma}(p\tau^D)^{1-\sigma})}{P^D(Y,d^\star)},$$

and $P^D(Y,d^\star) := \big(p^D(Y,d^\star)^{1-\sigma} + \tau^{D\,1-\sigma}\big)^{\frac{1}{1-\sigma}}$.

Indeed, the trade cost increase applies to both imports and exports. Utility from default is then given by solving this recursive equation:

$$V^D(Y_t, d_t^\star) = \max_{s.t.} u(C_t) + \beta\mathbb{E}\big[(1-\lambda)V^D(Y_{t+1}, d_t^\star) + \lambda V(Y_{t+1}, d_{t+1}^\star, \bar{B})|Y_t, d_t^\star\big].$$

Because of the possibility to reenter into financial markets, this value function depends on the value function associated with debt repayment, V,that shall be defined recursively in the following paragraph.

Repayment Utility

The domestic economy inherits debt from the previous period, B_{t-1}, indexed on the price of foreign goods (which is the *numéraire*). Given this level of debt, government either defaults or emits new debt B_t taking into account debt schedule price. Government has to choose B_t so has to maximize its expected utility. In case of repayment, it solves the following recursive equation:

$$\begin{aligned} V^R(B_{t-1}, s_t) = & \max_{B_t, C_t, T \text{s.t.}} u(C_t) + \beta\mathbb{E}_t V(B_t, s_{t+1}), \\ \text{s.t.} C_t = & \frac{p(s_t, T)Y(s_t) + T + d^\star(s_t)(\delta-1)\delta^{-\sigma}(\tau p)^{1-\sigma}}{P(s_t, T)} \\ T = & q(B_t, Y_t, d_t^\star)(B_t - (1-\psi)B_{t+1}) - (\psi + r)B_{t-1} \end{aligned}$$

and we define the price function implicitly as the solution to:

$$Y_t = \frac{p^{-\sigma}\Big[pY_t + q(B_t, s_t)(B_t - (1-\psi)B_{t-1}) - (\psi + r)B_{t-1} + d_t^\star(\delta-1)\delta^{-\sigma}p^{1-\sigma}\tau^{1-\sigma})\Big]}{p^{1-\sigma} + \tau^{D\,1-\sigma}} \tag{8.2}$$
$$+ d_t^\star p^{-\sigma^\star}(1+\tau)^{-\sigma^\star}.$$

Default Decision

The government should choose to default whenever the value of default is more than the value of repayment. Let $D(B_{t-1}, Y_t, d_t^\star)$ be the default decision. Then we should have:

$$D(B_{t-1}, Y_t, d_t^\star) = \mathbb{I}\{V^R(B_{t-1}, Y_t, d_t^\star) < V^D(Y_t, d_t^\star)\}.$$

Price of Bonds

Government takes the price schedule of bonds as given. We assume all bonds have the same maturity, indexed by parameter ψ. The period after borrowing, government has to pay back the interests of its past debt as well as a fraction $\psi \in [0, 1]$ of it. When $\psi = 1$, we have a standard sovereign debt model with one-period bonds. When $\psi = 0$, all bonds consist in perpetuities.

Because defaulting government cannot reenter into financial markets, we have $q = 0$ when after a government's default until it reenters into financial markets. We also assume that financial markets are competitive and risk-neutral. There is a fixed global interest rate r, and the price of bonds should be determined by the following dynamic equation:

$$q(B_t, s_t) = (\psi + r)\mathbb{P}(D(B_t, s_{t+1}) = 0) + \frac{1-\psi}{1+r}\mathbb{E}_t((1 - D(B_t, s_{t+1}))q^D(B_{t+1}, s_{t+1})|B_t, s_t),$$

where the term inside the expectation is assumed to be equal to 0 when $D(B_t, s_{t+1}) = 0$.[33] When $\psi = 1$, this equation simply means that lenders correctly forecast default risk in the next period. When $\psi \in [0, 1)$, it also implies that lenders correctly forecast long-run default risks and the borrowing behavior of the government.

Intertemporal Budget Constraint

Combining all the previous elements, we can write the intertemporal budget constraint in case of repayment:

$$\begin{aligned} P(s_t, T)C_t &= p(s_t, T)Y(s_t) \\ &+ T \\ &+ (\delta - 1)d^\star \delta^{-\sigma}(\tau p)^{1-\sigma}(s_t, T), \\ \text{where}\, T &= q(B_t, s_t)(B_t - (1-\psi)B_{t-1}) - (\psi + r)B_{t-1}. \end{aligned}$$

The variable T is the sum of financial transfers received by the domestic economy, or the capital account. Because there is only one kind of assets in our economy - sovereign bonds, these financial transfers are the opposite of the trade balance (which is also equal to the current account) in the model.

8.4 Equilibrium definition

Definition 13. Let ψ, λ, σ, $\tau < \tau^D$ be fixed parameters. Let $(s_t)_{t\geq 0} \in (\mathbb{R}^d)^{\mathbb{N}}$ be a Markov process on space $\mathcal{S}$ and $Y, d^\star$ two functions $\mathcal{S} \to \mathbb{R}_+^\star$. Define $Y_t = Y(s_t)$ and $d_t^\star := d(s_t)$. Let p and p^D price functions $p : Y \times d^\star \times T \in (\mathbb{R}_+^\star)^2 \times \mathbb{R} \mapsto p \in \mathbb{R}_+$ and $p^D : Y \times d^\star \in (\mathbb{R}_+^\star)^2$ that solve for 8.2 and 8.1, and let P and P^D be the associated aggregate price functions.

A Markov equilibrium associated with these parameters and functions is given by (i) value functions $V^R(B_{t-1}, s_t)$, $V^D(s_t)$ (ii) policy function $b(B_{t-1}, s_t)$, (iii) Default decision $D(B_{t-1}, s_t)$ and (iv) bond price schedule $q(B_t, s_t)$ such that:

[33] Another equivalent way to write this recursive equation without this additional notation is to use conditional expectation:

$$q(B_t, s_t) = \mathbb{P}(D(B_t, s_{t+1}) = 0)\Big((\psi + r) + \frac{1-\psi}{1+r}\mathbb{E}_t(q(B_{t+1}, s_{t+1})|B_t, s_t, D(B_t, s_t) = 1)\Big)$$

We have to write it that way because the function q is not assumed to depend on default decisions but only on state variables that would be relevant for a default.

- When the government takes function q as given, V^D and V^R solve the following Bellman equation, for every $t \in \mathbb{N}, s_t \in \mathcal{S}$:

$$\begin{aligned} V^D(s_t) = &\max_{c_t, c_t^\star} u(C_t) + \beta \mathbb{E}\big[(1-\lambda)V^D(s_{t+1}) + \lambda V(s_{t+1}, 0)|s_t\big], \\ &\text{s.t.} P^D(s_t)C_t = p^D(s_t)(1-x)Y \end{aligned} \tag{8.3}$$

and:

$$\begin{aligned} V^R(B_{t-1}, s_t) = &\max_{B_t, C_t \text{s.t.}} u(C_t) + \beta \mathbb{E} V(B_t, s_{t+1}|s_t), \\ \text{s.t.} P(s_t, T)C_t = &p(s_t, T)Y(s_t) + T + (\delta - 1)d^\star \delta^{-\sigma}(\tau p)^{1-\sigma}(s_t, T) \\ &T = q(B_t, s_t)(B_t - (1-\psi)B_{t-1}) - (r+\psi)B_{t-1} \end{aligned} \tag{8.4}$$

where $V(B_{t-1}, s_t) := \max\{V^R(B_{t-1}, s_t), V^D(s_t\})$.

- Optimal saving policy $b(B_{t-1}, s_t)$ solves the maximization in the Bellman equation 8.4.

- Default decision $D(B_{t-1}, s_t)$ is equal to 1 if and only if:

$$V^R(B_{t-1}, s_t) < V^D(s_t),$$

and equal to 0 otherwise.

- Price schedule function correctly predicts the future likelihood of default:

$$\forall B_t, q(B_{t-1}, s_t) = \mathbb{P}\big(D(B_t, s_{t+1}) = 0\big)\Big(\frac{\psi + r}{1+r} + \frac{1-\psi}{1+r}\mathbb{E}\big[q(b(B_t, s_{t+1}), s_{t+1})|D(B_t, s_{t+1}) = 0\big]\Big).$$

8.5 Shocks, State Variables and Numerical Solutions

Exogenous State Variables For the sake of generality, we have not fully described shocks to GDP and foreign demand in the previous section. We are going to make the same assumption as Aguiar and Gopinath (2006) and Aguiar et al. (2016) and assume that output Y_t at time t can be written:

$$Y_t = e^{z_t} \prod_{i=0}^{t} e^{\tilde{g}_t},$$

where $(z_t)_{t\in\mathbb{N}}$ and $(\tilde{g}_t)_{t\in\mathbb{N}}$ are two independent AR(1) processes, such that, for every $t \in \mathbb{N}$:

$$\begin{aligned} z_t &= \rho_z z_{t-1} + \varepsilon_t^z \\ \varepsilon_t^z &\sim \mathcal{N}(0, \sigma_z^2) \\ \tilde{g}_t &= m_g + \rho_g \tilde{g}_{t-1} + \varepsilon_t^g \\ \varepsilon_t^g &\sim \mathcal{N}(0, \sigma_g^2), \end{aligned}$$

where $(\rho_z, \rho_g) \in [0,1)^2$ and $\sigma_z, \sigma_g \geq 0$ are four parameters that can be determined by observing GDP processes, and m_g is the average long-run growth rate. z_t is the stationary component, or the cyclical component of GDP, while g_t is the trend. As long as $\rho_g > 0$, a shock to the trend ε_t^g has much more persistent effects on GDP than a shock on the cyclical component. Indeed, past growth affects the level of GDP, but, more importantly, a shock on growth persists through higher distribution of future growth.

We will assume that foreign demand follows the same trend as GDP growth. Foreign demand also has a stationary component that follows an AR(1) process as well:

$$
\begin{aligned}
D_t &= e^{d_t} \prod_{i=0}^{t} e^{\tilde{g}_t} \\
d_t &= m_d + \rho_d d_{t-1} + \varepsilon_t^d \\
\varepsilon_t^d &\sim \mathcal{N}(0, \sigma_d^2),
\end{aligned}
$$

where ρ_d and σ_d are parameters determining the stationary component of foreign demand. As a consequence of our assumption, growth shocks should have little effect on the price of commodities.[34] We assume that growth shocks are the same for tractability and consistency. With independently but identically distributed growth rates between D_t and Y_t, the ratio D_t/Y_t can land anywhere between 0 and infinity - which compels us to add more grid points than we could. Moreover, if the growth rate distributions of Y and D are different, the ratio might converge to 0 and $+\infty$ which is not realistic and makes the model non-stationary. We study in the model what happens when foreign demand for domestic goods increases relative to GDP.

Parametric Restrictions For numerical resolution, we use a CRRA utility function u with relative risk aversion parameter γ as in the rest of the literature:

$$\forall C \geq 0,\ u(C) = \frac{C^{1-\gamma}}{1-\gamma}.$$

Because this utility function is homogeneous of degree $1-\gamma$, we can simplify the terms inside and divide the terms in the Bellman equation by $\prod_{i=0}^{t} e^{\tilde{g}_t}$.

As a consequence, we have three exogenous state variables to follow: z_t, $\tilde{g}_t$ and d_t. 3 is usually the maximal possible number of exogenous state variables in endogenous sovereign debt models. There is an additional endogenous state variable: the stock of sovereign debt B_t.

To solve for the model, we first discretize the auto-regressive continuous processes for each variable, using the algorithm developed by Tauchen (1986).[35] We are restricting

[34] In the absence of intertemporal trade, this model clearly implies that any shock on $\tilde{g}_t$ has no effect on the price. However, growth affects intertemporal trade and changes the weight of inherited debt relative to current GDP and demand, so it still plays an indirect although second-order role.

[35] We used for several functions, including the discretization, the Python library QuantEcon from the https://quantecon.org/, which, among things.

to 7 states per variable, a number for which the Tauchen algorithm is generally deemed precise enough. We allow 101 different values for debt on a grid scaled according to other variables in the model.

8.6 Calibration Parameters

We have to assign a value to each parameter through estimation. We discuss the stakes associated with problematic variables by the same token. In order to compare our model with the quantitative sovereign debt literature, we align our estimates with Aguiar et al. (2016) when it is possible.

- $\gamma, r,$: We use the standard value that is equal to 2 in most models, in the sovereign debt literature as well as in many DSGE simulations. For the safe interest rate r, we use the value 1.5%, in line with average Treasury yield bonds in the last few decades.

- The elasticity of substitution σ is usually estimated before 5 and 10 as it is reminded by Anderson and Van Wincoop (2004) and we keep the lower estimate. In our model, Armington trade means that gains from trade, absent any intertemporal trade, have the general form described by Arkolakis, Costinot, and Rodriguez-Clare (2012): they depend on the level of imports' penetration ratio and on the elasticity of substitution. However, in the presence of net trade, the results can change, and we show numerically in the appendix that the ACR formula gives a lower bound for gains from trade in the presence of net trade (although the ACR formula gives the right order of magnitude). The ACR formula is usually considered a "disappointing" result, because it infers low gains from trade relative to empirical estimates (Frankel and Romer (1999) and Feyrer (2009)) and relative to what other input-output network models suggest. As a consequence, we use the lower value of the parameter: $\sigma = 5$.

- τ and m_d: we assign to this variable a value that is more than 1, typically 1.1. With this variable, The decision is partly arbitrary because the mean value m_d also affects the total volume of trade from the point of view of the domestic economy. Because the foreign good is the *numéraire*, a change in τ should affect the real value of debt from the point of view of the domestic economy, which can access the foreign good only after paying the iceberg costs τ. We also have to take into account the role of the elasticity of substitution σ to see the total impact of trade costs on trade, while the effect of the parameter m_d on foreign demand function does not depend on σ. More generally, τ gives an assessment of the gains from trade. To avoid relying on some nominal effects of trade, we will explore the role of trade openness using m_d. We want to target the average gross trade flows thanks to this parameter. We use Mexico as an example, with an openness ratio around 40% in 2019.

- x, τ^D: when default costs are linear, the value usually chosen is 2%. Because we rely on default mechanism, we choose a proportional default loss that is 4 times less, equal to 0.5%. For τ^D, we choose $\tau^D = (1 + f)\tau$, where $f > 0$ is chosen as a function of other parameters to obtain a decrease in trade after default similar to what we observe in the data, in Rose (2005) or in our own findings: 20%. We choose $f = 1.1$ in the following regressions. The relative importance of x and f matter a lot and we will study how making them vary affects the dynamics of the paper.

- We fix other parameters such as ψ or θ the same way as it is suggested in the literature.

Parameter	Description	Value	Source
g	Average growth rate	2.42%	Mexico
σ	Elasticity of substitution	5.0	Anderson and Van Wincoop (2004)
γ	Risk aversion	2.0	SD literature
r	Safe interest rate	1.5%	Treasury Rates
τ	Trade Costs	-	-
θ	Probability of Redemption	0.1	Wright (2012)
x	Effect of default on prod.	0.005	Assumption
ψ	(Inverse) Maturity	0.125	Broner, Lorenzoni, and Schmukler (2013)
τ_D	Increase in trade costs	5%	Regressions

- Parameters for the distribution of GDP (cycles and growth) m_g, ρ_y, ρ_g, σ_y, σ_g can be estimated directly from GDP series. We use the same values as in Aguiar et al. (2016) that is a useful benchmark.

Parameter	Description	Value	Source
ρ_g	Autocorrelation of g	0.45	Based on GDP series
σ_g	Variance of shocks to g	0.011	-
ρ_y	Autocorrelation of y	0.85	-
σ_y	Variance of y	0.05	-

- Parameters about the distribution of demand shocks ρ_d and σ_g can be estimated from the data with a structural approach. Indeed, in our model, the value of exports-to-GDP (which is observable) should be equal to:

$$X_t = \frac{d_t^\star \delta^{-\sigma} \tau^{1-\sigma} p_t^{-\sigma}}{Y}$$
$$\implies d_t^\star \propto X_t p_t^\sigma Y_t$$

where Y_t is real GDP, X_t is exports to GDP ratio, and p_t is equal to terms of trade. The relation is proportional with a fixed factor that does not move because we assume that δ and τ are fixed over time. Using data on terms of trade from the World Bank and real GDP series, we run this regression for Mexico between 1980 and 2018 and find $\rho_d = 0.89$ and $\sigma_d = 0.20$, which makes this term more volatile than GDP.

Parameter	Description	Value	Source
ρ_D	Autocorrelation of Foreign Demand	0.89	Computations on Mexico
σ_D	Variance of D	0.20	-

9 Appendix: Empirical Analysis

In this Section, we present a simplifier model with Pareto shocks that can be used to justify our specification for the empirical regressions we run. We also present the definition regression of the Frankel-Romer instrument.

Default Risk and Trade: a Log Formula

In this paragraph, we present hypothesis under which the equation tested in Sections 5.1 and 5.2 becomes a structural regression.

As seen earlier, the gains from trade are a good summary of each government's willingness to repay its debt in the model, and they can also be computed indirectly thanks to a sufficient statistics approach. We use a simplifying assumption (local Pareto) to derive an approximation that we can directly test in the data.

Let $L_{j,t}$ be the GDP of country j at time t. The probability of default of a government that borrower $B_{j,t}$ at the next period should then be:

$$P^D = \mathbb{P}(\frac{B_{j,t}}{Y_{j,t+1}} > 1 - (1 - IM^\star)^\varepsilon)$$

If you assume that $\varepsilon = 1$ [36], then the probability of default is simply given by:

$$P^D = \mathbb{P}(\tilde{Y}_{j,t+1} < \frac{b_{j,t}}{IM^\star})$$

where $b_{j,t} = B_{j,t}/L_{j,t}$. Combining this with previous assumption, the CDS premium for risky countries should be given by:

$$CDS = -\log(1 - \mathbb{P}(\tilde{Y}_{j,t+1} < \frac{b_{j,t}}{IM^\star}))$$

Assume now that $\tilde{L}_{j,t+1}$ is distributed according to a Pareto distribution (at least locally) with parameters C_jC_t and γ, then:

$$CDS = \gamma \log b_{j,t} - \gamma \log IM^\star + \log C_j + \log C_t$$

The fact the coefficients for $b_{j,t}$ and $IM^\star_{j,t}$ are the same stem from our assumptions. With different functional forms and different elasticity of substitution for, one can find different results.

Frankel-Romer's definition Regression

Impact of Trade Openness on CDS: Feyrer regression

[36]This assumption is equivalent to assuming that the elasticity of substitution between international goods is $\sigma = 2$, a lower bound of the estimates, generally between 4 and 10.

	Dependent variable:
	Trade between Reporter and Partner (over reporter's GDP)
Distance (log)	−0.700***
	(0.016)
Common Border	3.920***
	(1.229)
Distance if Common Borser	0.519***
	(0.143)
Common official language	0.381***
	(0.056)
Common language	0.453***
	(0.056)
Population (log) of partner	0.474***
	(0.006)
Population (log) of reporter	−0.386***
	(0.006)
Area (reporter)	−39.936***
	(5.756)
Area (partner)	73.696***
	(5.843)
Population if common border (partner)	−0.198***
	(0.060)
Population if common border (reporter)	−0.216***
	(0.060)
	(0.202)
Observations	25,129
R^2	0.426
Note:	*p<0.1; **p<0.05; ***p<0.01

Table 5: CDS and Frankel-Romer's instrument: OLS and reduced form IV. Standard errors clustered by year.

	Dependent variable:			
	CDS Spread			
	(1)	(2)	(3)	(4)
Trade Openness	−763.542***	−601.455**	−368.750**	−626.498***
(in log)	(231.589)	(236.383)	(145.270)	(166.320)
Real GDP	−409.140***	−327.412***	−288.626***	−383.283***
(in log)	(86.532)	(76.461)	(52.280)	(58.504)
Weight for the Instrument	Population in 1970	Trade in 1994	GDP in 1994	GDP in 1980
Year Fixed Effects	Yes	Yes	Yes	Yes
Year and oil Fixed Effects	Yes	Yes	Yes	No
Country Fixed Effects	Yes	Yes	Yes	Yes
Year and Oil Fixed Effects	Yes	Yes	Yes	No
Controls	Debt	Debt, Population and Current account	Debt and current account	No
Observations	466	462	462	466
R^2	0.718	0.782	0.828	0.740

Note: *p<0.1; **p<0.05; ***p<0.01