WORKING paper

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The Dynamic IS Curve when there is both Investment and Savings

Stéphane Dupraz[1]

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Abstract

The dynamic IS curve of New-Keynesian models captures the dependence of aggregate demand on future interest rates, but only in the case where there is no investment and the interest rate channel only originates in the savings decisions of households. The paper derives the dynamic IS curve analytically in a model with investment, where the interest rate channel originates both in the the savings decisions of households and the investment decisions of firms. This generalized dynamic IS curve sheds light on several new factors that shape the dependence of aggregate demand on interest rates. In particular, interest rates are discounted in investment and aggregate demand if and only if the intertemporal elasticity of substitution in consumption (IES) is low enough, and compounded if it is higher. The addition of household heterogeneity can generate discounting in aggregate consumption as well, in a new way that does not rely on precautionary savings. Instead, household heterogeneity creates discounting by making consumption respond to interest rates primarily as a ripple effect of the response of investment, not through intertemporal substitution.[2]

Keywords: Investment, Intertemporal Substitution, Household Heterogeneity.

JEL classification: E22, E52, E21.

[1]Banque de France, stephane.dupraz@banque-france.fr.

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NON-TECHNICAL SUMMARY

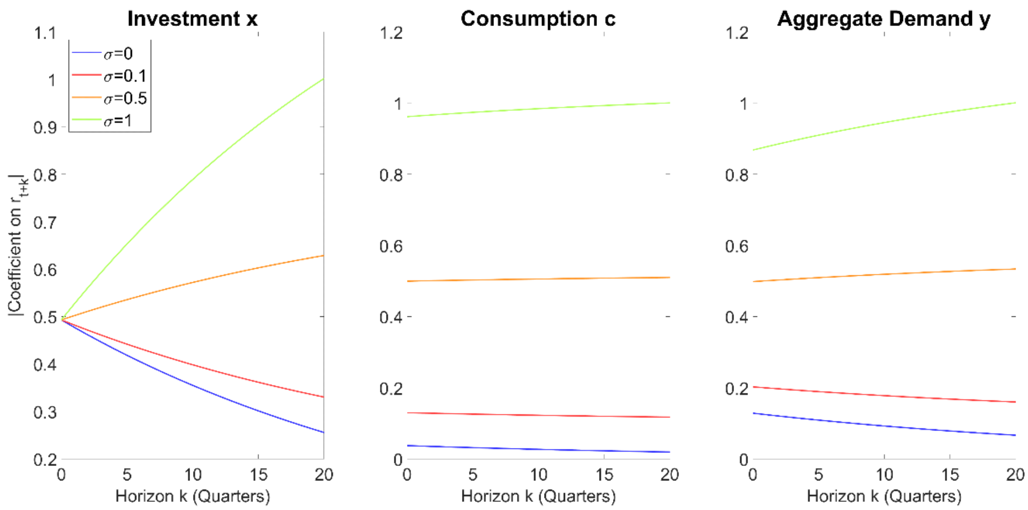
The interest rate channel is key to the transmission of monetary policy, yet whether it is properly captured in baseline models of aggregate demand has been widely debated in recent years. In baseline models, the dependence of aggregate demand on interest rates is captured by the Euler equation, which originates in households' desire to substitute consumption across periods. The Euler equation improves on the static IS curve of IS-LM by being a dynamic IS curve, capturing the dependence of aggregate demand on all future interest rates, not just the current one. But the precise dependence it assigns seems unrealistic. Because it puts the same weight on all interest rates, the Euler equation puts an unrealistically high weight on future interest rates. This leads to the forward-guidance puzzle (Del Negro, Giannoni, and Patterson, 2012; Carlstrom, Fuerst, and Paustian, 2015), and questions the realism of the Euler equation in capturing the intertemporal dimension of the IS curve.

The recent literature on aggregate demand has considered several assumptions on the modeling of households' consumption—such as heterogeneity or bounded rationality—that can add discounting to the dynamic IS curve (i.e. the effect of interest rate cuts is weaker, the higher the horizon of the interest rate cut). While such models often show that intertemporal substitution is then only the first step in a transmission channel that goes through several amplification mechanisms, they retain the underlying assumption that the interest rate channel originates in (at least some) households' desire to intertemporally substitute consumption.

What interest rates have an initial lever on, however, is not only households' consumption, but also—perhaps primarily—firms' investment. In order to capture the dependence of aggregate demand on future interest rates, considering the dynamic IS curve in a model where the interest rate channel originates in both the savings decisions of households and the investment decisions of firms—like in the original Investment-Savings curve of IS-LM—can be better suited.

This paper derives an analytical expression of the dynamic IS curve in a model with both consumption and investment, capturing both the savings and the investment components of the interest rate channel. It shows that the weight on future interest rates in the generalized dynamic IS curve is shaped by several new factors absent in the simple IS curve that abstracts from the investment component. In particular, a key determinant of the extent of discounting in aggregate demand is then the intertemporal elasticity of substitution in consumption (IES). For a low IES—below 0.3 in the main calibration I use—interest rates are discounted in the dynamic IS curve, while for a higher IES they are instead compounded (i.e. the effect of interest rate cuts is stronger, the higher the horizon of the interest rate). Intuitively, this is because the IES determines the relative importance of the two components of the interest rate channel, and the one that originates in investment demand discounts future interest rates more heavily than the one that originates in intertemporal substitution.

The Effect of the Elasticity of Substitution on Discounting/Compounding



Note: The figure gives the (absolute value of the) coefficients of future interest rates in aggregate investment (left panel) and in aggregate demand (right panel) for three different calibration of the intertemporal elasticity of substitution σ. There is discounting for σ=0.1, weak compounding for σ=0.5, and compounding for σ=1.

La courbe IS dynamique lorsqu'il y a à la fois investissement et épargne

RÉSUMÉ

La courbe IS dynamique des modèles nouveaux-keynésiens donne la dépendance de la demande agrégée vis-à-vis des taux d'intérêt futurs, mais uniquement dans le cas où il n'y a pas d'investissement et où le canal des taux d'intérêt n'a son origine que dans les décisions d'épargne des ménages. Cet article dérive la courbe IS dynamique de manière analytique dans un modèle avec investissement, où le canal des taux d'intérêt a son origine à la fois dans les décisions d'épargne des ménages et dans les décisions d'investissement des entreprises. Cette courbe IS dynamique généralisée met en lumière plusieurs nouveaux facteurs qui déterminent la dépendance de la demande agrégée vis-à-vis des taux d'intérêt. En particulier, les taux d'intérêt sont escomptés dans l'investissement et la demande agrégée si et seulement si l'élasticité intertemporelle de substitution de la consommation (IES) est suffisamment faible. L'hypothèse additionnelle d'hétérogénéité des ménages peut également générer une escompte des taux d'intérêt dans la consommation agrégée, à travers un nouveau mécanisme qui ne repose pas sur l'épargne de précaution. L'hétérogénéité des ménages génère une escompte des taux d'intérêt en faisant réagir la consommation aux taux d'intérêt principalement comme un effet d'entraînement de la réponse de l'investissement, et non via la substitution intertemporelle.

Mots-clés : investissement, substitution intertemporelle, hétérogénéité des ménages.

Introduction

The interest rate channel is key to the transmission of monetary policy, yet whether it is properly captured in baseline models of aggregate demand has been widely debated in recent years. In baseline models, the dependence of aggregate demand on interest rates is captured by the Euler equation, which originates in households' desire to substitute consumption across periods. The Euler equation improves on the static IS curve of IS-LM by being a dynamic IS curve, capturing the dependence of aggregate demand on all future interest rates, not just the current one. But the precise dependence it assigns seems unrealistic. Because it puts the same weight on all interest rates, the Euler equation puts an unrealistically high weight on future interest rates. This leads to the forward-guidance puzzle (Del Negro, Giannoni, and Patterson, 2012; Carlstrom, Fuerst, and Paustian, 2015), and questions the realism of the Euler equation in capturing the intertemporal dimension of the IS curve.

The recent literature on aggregate demand has considered several assumptions on the modeling of households' consumption—such as heterogeneity or bounded rationality—that can add discounting to the dynamic IS curve.[1] While such models often show that intertemporal substitution is then only the first step in a transmission channel that goes through several amplification mechanisms, they retain the underlying assumption that the interest rate channel originates in (at least some) households' desire to intertemporally substitute consumption.

What interest rates have an initial lever on however, is not only households' consumption, but also—perhaps primarily—firms' investment.[2] In order to capture the dependence of aggregate demand on future interest rates, considering the dynamic IS curve in a model where the interest rate channel originates in both the savings decisions of households and the investment decisions of firms—like in the original Investment-Savings curve of IS-LM—can be better suited.[3]

This paper derives an analytical expression of the dynamic IS curve in a model with both consumption and investment, capturing both the savings and the investment components of the interest rate channel. It shows that the weight on future interest rates in the generalized dynamic IS curve is shaped by several new factors absent in the simple IS curve that abstracts from the investment component. In particular, a key determinant of the extent of discounting in aggregate demand is then the intertemporal elasticity of substitution in consumption (IES). For a low IES—below 0.3 in the main calibration I use—interest rates are discounted in the dynamic IS curve, while for a higher IES they are instead compounded. Intuitively, this is

[1] On household heterogeneity, see e.g. McKay, Nakamura, and Steinsson (2016), Bilbiie (2020, 2018), Werning (2015) and Acharya and Dogra (2020). On bounded rationality, see e.g. Woodford (2019), Farhi and Werning (2019), Gabaix (2020), Angeletos and Lian (2018), Dupraz, Le Bihan, and Matheron (2022).

[2] A common view is that in a simple model without investment, the Euler equation is better seen as a stand-in for the dependence of the whole of aggregate demand—including aggregate investment—on interest rates. For instance,Woodford (2003), p.352: *"I have suggested that the basic neo-Wicksellian model ought not be "calibrated" on the basis of studies of intertemporal substitution of consumer expenditures, but should be taken instead to refer to the degree of intertemporal substitutability of overall private expenditure, largely as a result of intertemporal substitution in investment spending."*

[3] A note on vocabulary: investment demand also amounts to intertemporal substitution. Since the term intertemporal substitution is widely associated to intertemporal substitution in consumption however, in the paper I restrict the use of the term to intertemporal substitution *in consumption*.

because the IES determines the relative importance of the two components of the interest rate channel, and the one that originates in investment demand discounts future interest rates more heavily than the one that originates in intertemporal substitution.

The paper proceeds in three steps, through three main analytical results on the effects of future interest rates in a model with investment.[4] Following the distinction between partial and general equilibrium in the aggregate-consumption literature, it first analyzes partial-equilibrium (decision-theoretic) mechanisms. It derives the investment function—the equivalent of the consumption function for investment, taking all aggregate variables as exogenous—and shows that interest rates are substantially discounted there. In particular, the discounting of future rates is stronger in the investment function than it is in the standard consumption function. While the discounting of future rates in the baseline consumption function is determined by the preference discount factor alone and is therefore negligible, in the investment function it is at least as strong as the combined effect of the preference discount factor and the depreciation rate of capital, making discounting substantially stronger. In addition, discounting is stronger the smaller adjustment costs are, and can be arbitrarily strong for arbitrarily small adjustment costs.

Does the discounting in investment survive in general equilibrium though? I show, second, that it critically depends on the value of the intertemporal elasticity of substitution in consumption. A small enough IES—below 0.3 in the main calibration I use—is necessary to preserve discounting. With a high IES, future interest rates are instead compounded. To explain why the IES *in consumption* plays such a large role on the effect of interest rates *on investment*, I distinguish between two sets of general-equilibrium channels at play in a model with both consumption and investment. First is the general-equilibrium amplification of the initial response of investment: Once lower interest rates have kick-started an increase in investment through the investment function, present and expected future aggregate demand increase, which feedbacks on investment demand, and so on. This feedback loop is the equivalent for investment of the feedback loop of the Keynesian cross for consumption.[5] In reference to it, I call it the investment cross. Second is the general-equilibrium collateral effect on investment of the part of the interest-rate channel that originates in consumption: Once lower interest rates have kick-started an increase in consumption through intertemporal substitution, present and expected future aggregate demand increase, which impacts investment demand, and so on.

I show that the investment cross is not responsible for a possible lack of discounting in general equilibrium. The extent of discounting when taking only the investment cross into account can be conveniently captured by one of the roots of the general-equilibrium system, which I call the investment root. This investment root does deliver less discounting than there is in the partial-equilibrium investment function: Just like the Keynesian cross weakens the discounting present in the consumption function, the investment cross weakens

[4]Throughout the paper, I focus on the effect of future real interest rates. The effect of future nominal interest rates in equilibrium also depends on aggregate supply. Focusing on the effect of real interest rates allows to focus on aggregate demand alone.

[5]This feedback loop on investment is absent in IS-LM. While Hicks (1937) assumes that the consumption function depends on aggregate income in addition to interest rates—the root of the Keynesian cross—he assumes that the investment function depends on the interest rate alone. Assuming so is not inconsistent in a static model: what investment today depends on is aggregate demand tomorrow, a feedback loop best captured in a dynamic model.

the discounting present in the investment function. But except under extreme calibrations, the investment root remains below one, so that discounting survives. In addition, the investment root does not depend on the IES. Of independent interest, I show that the determinants of the investment root differ markedly from the determinants of discounting in the partial-equilibrium investment function. In particular, whereas adjustment costs are a primary determinant of the extent of discounting in the partial-equilibrium investment function, they have very little effect on the general-equilibrium investment root. Instead, the investment root depends importantly on the specification of labor supply. A more inelastic labor supply and/or a stronger income effect on labor supply increase the value of the investment root and so diminishes the extent of discounting. This is because an important part of the investment cross goes through the reaction of real wages: after an interest rate cut, the stronger the response of real wages to the increase in aggregate demand, the more firms want to substitute labor with capital, amplifying the initial increase in investment.

Instead, the IES matters because, whenever the IES is non-zero, investment in general equilibrium also depends on interest rates as a ripple effect of consumption's initial reaction to interest rates through intertemporal substitution. If interest rates increase consumption through intertemporal substitution, it increases aggregate demand, giving firms more incentives to invest. Discounting of future interest rates through this other part of the interest rate channel is captured by the other root of the general-equilibrium system, which I call the consumption root. Under standard specifications of aggregate consumption, it is simply the unit root of the Euler equation. The consumption root depends on the IES no more than the investment root does. But the IES matters because it weights the relative importance of the two components of the interest-rate channel: the investment-demand component captured by the investment root, and the intertemporal-substitution component captured by the consumption root. I show that for a low enough IES—so that the interest rate channel originates primarily in investment demand—interest rates are discounted in investment, but that for a high enough IES—so that the interest rate channel originates primarily in intertemporal substitution in consumption—interest rates are *compounded*. This is even though the investment root is less than 1 and the consumption root is equal to 1 and occurs because, with a high IES, investment can load negatively on the investment root.

Third, I turn to the impact of investment on the dependence on interest rates of aggregate demand and aggregate consumption. I first show that the result on investment carries over to aggregate demand, i.e. the dynamic IS curve: the effect of interest rates in the dynamic IS curve is shaped by both the consumption and investment roots, and interest rates are discounted in aggregate demand if and only if the IES is below the same threshold as defined for investment. Second, I show that, under a form of household heterogeneity, investment can introduce discounting in aggregate consumption as well. The way heterogeneity introduces discounting here is thoroughly different from the way heterogeneity can introduce discounting in models with consumption only (McKay, Nakamura, and Steinsson, 2016; Bilbiie, 2020, 2018; Werning, 2015; Acharya and Dogra, 2020). There, discounting arises from precautionary savings, while in the present model with investment it arises from heterogeneity in marginal propensities to consume (MPCs). To make this clear, I introduce household heterogeneity through a baseline Two-Agent New-Keynesian (TANK) model without precautionary savings (as in, e.g. Bilbiie, 2008) which gives back the baseline Euler equation without discounting in a model without investment.[6] I show that with investment, whenever hand-to-mouth households with a high MPC have a larger share of aggregate labor income than of aggregate capital income net of investment, aggregate consumption depends positively on aggregate investment. This is because, in this case, investment punctures predominantly the income of permanent-income households with a low MPC, so that high-MPC hand-to-mouth households get a larger share of household income (aggregate income net of investment). Consumption can therefore respond to interest rate changes in general equilibrium even if intertemporal substitution is absent (the IES is zero) and the interest rate channel originates in investment demand only. Consumption then only reacts to interest rates as a ripple effect of the response of investment. Provided the investment root is less than one—which, as mentioned above, is easily the case—there is then discounting in consumption.

The discounting of future interest rates in aggregate consumption is robust to a small yet non-zero IES. It survives whenever the IES remains below the same threshold necessary for discounting in investment and aggregate demand—0.3 in the main calibration I use. This implies that interest rates are discounted in investment, consumption, and aggregate demand with an IES of 0.1, the value recently estimated by Best, Cloyne, Ilzetzki, and Kleven (2020), but compounded for higher, more common calibrations of the IES.

The paper first builds on the recent literature on the determinants of aggregate consumption. Although it shifts focus from consumption to investment, it shares the emphasis of this literature on the distinction between partial-equilibrium (decision-theoretic) and general-equilibrium effects, as well as on the role of households heterogeneity (e.g. Auclert, 2019; Kaplan, Moll, and Violante, 2018; Acharya and Dogra, 2020; Patterson, 2019). Within this literature, it shares the analytical, pen-and-paper approach of papers that rely on first-generation TANK models (Campbell and Mankiw, 1989; Mankiw, 2000; Galí, Lopez-Salido, and Vallés, 2004, 2007; Bilbiie, 2008; Debortoli and Gali, 2018) and second-generation TANK models with precautionary savings at the zero-liquidity limit (McKay, Nakamura, and Steinsson, 2016; Bilbiie, 2020, 2018; Werning, 2015).

Within the literature on household heterogeneity, the present paper connects most closely to a set of recent papers that consider the role of investment in the transmission channel of monetary policy. Although Kaplan, Moll, and Violante (2018)—see also Alves, Kaplan, Moll, and Violante (2020)—use a simple analytical model without investment to explain the indirect channels of labor income and taxes that they emphasize, through numerical results from their full-fledged HANK model with investment they emphasize the role of the portfolio rebalancing channel as an amplifying channel of monetary policy. Since this channel requires the existence of two assets—one liquid, one illiquid—it is absent in the present papers where bonds and capital trade at the same price. Luetticke (2021) also use numerical results from a HANK model with investment to document

[6] This is the case of acyclical risk (Werning, 2015) or acyclical inequality (Bilbiie, 2020, 2018). The roles of precautionary savings and heterogeneity in MPCs in this mechanism is therefore the exact opposite to their roles in the usual mechanism: as Acharya and Dogra (2020) show by considering a model where all agents have the same MPC, through the usual mechanism only precautionary savings is necessary to generate discounting.

the role of the portfolio rebalancing channel, as well as the Fisher channel, from which the present paper also abstracts. Ottonello and Winberry (2020) focus on the investment component of the interest rate channel in a model where firms are financially constrained, highlighting the heterogeneity between the response of financially constrained and unconstrained firms.[7] The present paper shares most with Auclert, Rognlie, and Straub (2020), who find that a HANK model with inattentive households assigns a much larger role to investment in the transmission of monetary policy. Their assumption of inattentive households, which effectively mutes intertemporal substitution in their model, connects to the low IES needed in the present paper to make the interest-rate channel originate mostly in investment demand. Bilbiie, Kanzig, and Surico (2022) may be the paper closest in spirit to the present one, through its emphasis on analytical, pen-and-paper results. However, their analytical results assume an interest-rate inelastic investment function—in the tradition of Hansen-Samuelson's accelerator—while the present paper considers a neoclassical investment function that is interest rate elastic. As a consequence, the capital inequality channel they emphasize works to amplify the response of intertemporal substitution in consumption, while the present paper emphasizes instead the part of the interest rate channel that does not originate in intertemporal substitution.[8] Finally, McKay and Wieland (2022) consider the effect of forward guidance in a model without capital investment but with durable consumption goods, which is intuitively related. They find their model is not subject to the forward-guidance puzzle. Whether the results of the present paper—e.g. the importance of the value of IES and the ripple effects of investment to (non-durable) consumption—apply to durables as well is an open question.[9]

Finally, the paper also connects to an older literature on whether long-run interest rates or only short-term interest rates matter for investment demand (see, e.g. Hall, 1977). Relative to this literature, the present paper highlights that the result that only short-term interest rates matter when adjustment costs are small holds in partial equilibrium, but not in general equilibrium.

The paper is organized as follow. Section 1 derives the decision-theoretic investment function and studies how much it discounts future interest rates. Section 2 introduces a simple TANK model of household heterogeneity that makes aggregate consumption depend on aggregate investment. Section 3 derives the extent of discounting in aggregate investment after taking into account general-equilibrium amplifying effects. Section 4 derives the extent of discounting in aggregate consumption and aggregate demand.

[7] See also Jungherr, Meier, Reinelt, and Schott (2022) on the role of firms' debt maturity in the response of investment to monetary policy.

[8] Another difference with Kaplan, Moll, and Violante (2018); Luetticke (2021); Bilbiie, Kanzig, and Surico (2022) is that these papers assume that households own the capital stock and make investment decisions, while the present paper assumes that firms do. Since households are financially constrained in their models, the distinction matters. Auclert, Rognlie, and Straub (2020) assume investment is done by financially unconstrained firms, like in the present paper.

[9] In their model, McKay and Wieland (2022) calibrate the IES to the relatively low value of 0.25, which is for instance lower than the threshold for discounting of interest rates in the present paper (0.3).

1 The Investment Function

This section derives the decision-theoretic investment function: the investment decision of an individual firm taking all the aggregate variables beyond its control (wages, interest rates, and the level of demand for its good) as given. It then looks at how much the investment function discounts future interest rates, and compare it to the extent of discounting in the standard consumption function.

1.1 Production Function and Capital Adjustment Costs

There is a single good in the economy whose price is normalized to 1. There is a continuum $i \in [0,1]$ of identical firms which produce from capital K_t^i and labor L_t^i using the Cobb-Douglas production function[10]

$$Y_t^i = F(K_{t-1}^i, L_t^i) = K_{t-1}^{i\,\alpha} L_t^{i\,1-\alpha}. \tag{1}$$

Capital depreciates at rate δ. Firm i owns its capital stock K_t^i and therefore decides of its accumulation.[11] Firm i's capital accumulation is subject to quadratic adjustment costs. The capital expenditures X_t needed to move the capital stock from $(1-\delta)K_{t-1}^i$ to K_t^i are given by:

$$X_t^i = (K_t^i - (1-\delta)K_{t-1}^i) + \frac{\kappa}{2}\frac{(K_t^i - (1-\delta)K_{t-1}^i - \delta K^*)^2}{\delta K^*}. \tag{2}$$

Note that the denominator in the quadratic term of (2) is the time-invariant steady-state level of capital K^*. An alternative specification of quadratic adjustment costs is to assume that the denominator is the previous level of capital K_{t-1}^i. As derived in details in appendix B however, this alternative specification implies that firm i decides of its capital stock today in part to affect the cost of adjusting capital tomorrow. The specification (2) avoids this effect which is less economically meaningful.

1.2 Investment Decision

Throughout the paper, I focus on the determinants of aggregate demand and leave aggregate supply—i.e. price-setting and the determinants of inflation—unspecified. Consider therefore the cost-minimization program of firm i. Firm i decides of its investment and labor demands in order to minimize the costs of producing a quantity Y_t^i of goods. Its present-period costs are labor costs $W_t L_t^i$, where W_t is the real wage taken as given, plus investment expenditures X_t^i. Because of adjustment costs, the investment decision today impacts profits at all future periods and firm i minimizes the sum of present and expected future costs, using

[10] The decision-theoretic results on the investment function generalize without difficulty to a CES production function. The restrictions on capital and labor shares imposed by the Cobb-Douglas production function bring simplifications in the generalized consumption bloc introduced in section 2.

[11] Whether capital accumulation is done by individual firms or a representative household makes a difference in monetary models with capital adjustment costs, even absent households' financial constraints (e.g. Woodford, 2003). When capital is accumulated by individual firms, adjustment costs bear on firm-specific capital stocks and cannot be reallocated across firms costlessly.

the stochastic discount factor $M_{t,t+k}$ (which it takes as given as well) to value costs k periods ahead:

$$\min_{K_t^i, X_t^i, L_t^i} E_t \sum_{k=0}^{\infty} M_{t,t+k} \left(W_{t+k} L_{t+k}^i + X_{t+k}^i \right), \tag{3}$$

subject to (2), and the constraint of producing Y_t^i each period (1).

Appendix A shows that firm i's optimal investment decision is to equate the marginal cost of more installed capital to the marginal value of installed capital

$$Q_t^i = 1 + \kappa \left(\frac{K_t - (1-\delta)K_{t-1}}{\delta K^*} - 1 \right), \tag{4}$$

where the marginal value of installed capital is

$$Q_t^i = E_t \left(\sum_{k=1}^{\infty} M_{t,t+k} (1-\delta)^{k-1} \frac{F_K^i(t+k)}{F_L^i(t+k)} W_{t+k} \right). \tag{5}$$

For a given level of production, the benefits of more capital is that it substitutes for labor in production. The marginal benefit of more capital is therefore a function of the marginal rate of transformation between capital and labor, and the real wage.

Because of adjustment costs, the benefits are not just current benefits, but present and expected future benefits. The firm discounts future benefits with a discount rate that includes not only the time and risk preferences embedded in the stochastic discount factor $M_{t,t+k}$, but also the depreciation rate of capital $(1-\delta)^{k-1}$. Indeed, the faster capital depreciates, the more discounted the future benefits of installing capital are, since capital will have evaporated more by then.

1.3 Interest Rate Discounting and Capital Depreciation

How do future expected interest rates affect present investment? Combining and loglinearizing equations (4) and (5) gives investment expenditures as

$$x_t^i = -\frac{1}{\kappa} \left(E_t \left(\sum_{k=0}^{\infty} (\beta(1-\delta))^k r_{t+k} \right) + E_t \left([1-\beta(1-\delta)] \sum_{k=0}^{\infty} (\beta(1-\delta))^k \left[\frac{1}{1-\alpha} (y_{t+k+1}^i - k_{t+k}^i) + w_{t+k+1} \right] \right) \right), \tag{6}$$

where r_t is the real interest rate.

Equation (6) stresses the role of capital depreciation in shaping the extent of interest rate discounting. Future interest rates enter with a discount factor $\beta(1-\delta)$. The role of capital depreciation in discounting future interest rates is intuitive. The faster capital depreciates, the more discounted the future benefits of investing in capital today are. When capital depreciation is high so that future benefits are already much discounted, variations in discounting due to variations in future interest rates matter less. Hence they are more discounted.

With a yearly depreciation rate of 10%, a discount factor of $\beta(1-\delta)$ is substantial. As a comparison point, compare it to the extent of discounting in the consumption function of a permanent-income household j, which is derived in appendix C:

$$c_t^j = (1-\beta)b_{t-1}^j - \beta\sigma \sum_{k=0}^{\infty} \beta^k E_t(r_{t+k}) + (1-\beta)\sum_{k=0}^{\infty} \beta^k E_t(\omega_{t+k}^j), \tag{7}$$

where σ is the intertemporal elasticity of substitution (IES), ω_t^j is household j's income and b_t^i is its bonds holdings.[12] Discounting only occurs at the rate of the time-preferences discount factor $\beta >> \beta(1-\delta)$.

1.4 Interest Rate Discounting and Adjustment Costs

Equation (6) does not give a full account of the determinants of firm i's investment decision however. The right-hand-side of equation (6) features firm i's future capital stock k_{t+k}^i, which depends on firm i's future investment decisions. To get firm i's investment function—the function giving firm i's investment as a function of the variables it takes as given—we need to solve for the expected path of firm i's own capital stock. It is given by equation (2), which loglinearizes into:

$$k_t^i = \delta x_t^i + (1-\delta)k_{t-1}^i. \tag{8}$$

Appendix D solves (6) and (8) forward to obtain the investment function of firm i.

Proposition 1 *Firm i's investment function is:*

$$x_t^i = -\mu k_{t-1}^i + \theta\left(-E_t\left(\sum_{k=0}^{\infty} \lambda^k r_{t+k}\right) + E_t\left([1-\beta(1-\delta)]\sum_{k=0}^{\infty}\lambda^k\left(\frac{1}{1-\alpha}y_{t+k+1}^i + w_{t+k+1}\right)\right)\right), \tag{9}$$

where λ is the root of the polynomial

$$P(\lambda) = \lambda^2 - \left(\beta(1-\delta) + \frac{1}{1-\delta}\left(1+\delta\frac{1-\beta(1-\delta)}{(1-\alpha)\kappa}\right)\right)\lambda + \beta \tag{10}$$

that is smaller than 1 and

$$\mu = \frac{\beta(1-\delta)-\lambda}{\beta\delta} > 0, \tag{11}$$

$$\theta = \frac{1}{\kappa}\frac{\lambda}{\beta(1-\delta)}. \tag{12}$$

The extent of discounting in the investment function differs from $\beta(1-\delta)$. Appendix E shows that this extent of discounting is always greater than $\beta(1-\delta)$ and crucially depends on the extent of adjustment costs κ.

[12] Income is taken to include both labor income and dividends from the possible ownership of firms—see appendix C.

Table 1: Default Calibration

β	0.9925
δ	2.5%
α	1/3
κ	2
s_x	20%
ψ	1
$1/\sigma_2$	1
s_L^H	0.35
s_K^H	0
σ	[various values]

Note: The table gives the default calibration used in the paper whenever not explicitly stated otherwise. The calibration is quarterly.

Corollary 1 *The discount factor λ in firm i's investment function lies between 0 and $\beta(1-\delta)$ and is increasing in the level of adjustment costs κ.*

- *At the limit without adjustment costs $\kappa \to 0$, interest rates past tomorrow are fully disregarded $\lambda \to 0$.*
- *At the limit where adjustment costs are infinite $\kappa \to +\infty$, the discount factor tends to $\lambda \to \beta(1-\delta)$.*

The dependence of the extent of discounting on adjustment costs is illustrated on Figure 1. On the figure, the other parameters are calibrated, on a quarterly basis, to $\beta = 0.9925$ (a 1% annualized risk-free interest rate), $\delta = 2.5\%$ (a 10% yearly depreciation rate), and $\alpha = 1/3$ for the steady-state labor share, as summed up in Table 1. A way to think of the magnitude of κ is as the inverse of the elasticity of investment to Tobin's Q. Figure 1 considers values of κ from 0 to 10, i.e. an elasticity of investment to Tobin's Q from 0.1 and infinity (no adjustment costs). In the rest of the paper, unless otherwise stated, the default calibration will be $\kappa = 2$, i.e. an elasticity of 0.5 close to what Cummins, Hassett, and Hubbard (1994) estimate.

Since $\lambda < \beta(1-\delta)$, there is even more discounting of the future benefits of capital and of future interest rates in the decision-theoretic investment function than appears when reasoning with the firm's future capital stock as given as in equation (6). The stronger discounting of future benefits occurs because, in response to future interest rate cuts, the firm anticipates that it will invest tomorrow. As a result, future marginal rates of substitution between capital and labor will be lower due to decreasing returns to capital, decreasing the incentive to invest today. Put otherwise, the firm does not invest as much today because it intends to invest tomorrow. In turn, the stronger discounting of future benefits implies that future interest rates are more

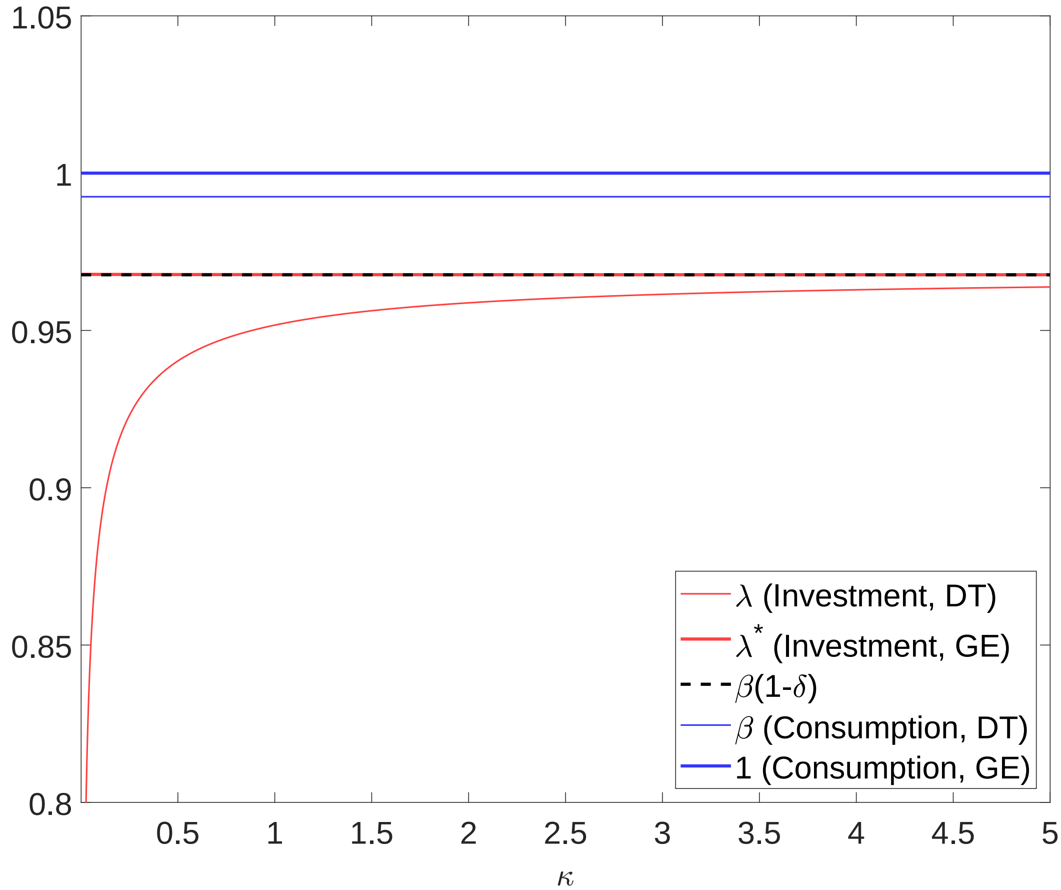


Figure 1: Interest Rate Discounting as a Function of Adjustment Costs κ

Note: The figure gives the discounting coefficients of future interest rates as a function of the level of adjustment costs κ. In red are the coefficients related to investment. The coefficient λ is the discounting coefficient of the decision-theoretic investment function of an individual firm, keeping aggregate variables constant. The coefficient λ^* is the discounting coefficient in investment taking into account general-equilibrium amplifying effects. In blue are, for comparison, the coefficients related to consumption. The discounting of future interest rates is β in the decision-theoretic consumption function of the permanent incomer. Taking into account general-equilibrium effects, there is no discounting in the consumption block based on the permanent incomer. All other parameters are calibrated according to Table 1.

discounted as well, because stronger overall discounting makes variations in discounting due to variations in future interest rates matter less.

This reinforcing effect of expected future investment on discounting is all the stronger that capital adjustment costs are smaller: The lower the adjustment costs, the easier it is to adjust capital and the less important the future becomes. At the limit with no adjustment costs $\kappa \to 0$, there is full discounting of the future $\lambda \to 0$, since capital is then a jump variable that only depends on the current short-term interest rate:

$$k_t = -\frac{1-\alpha}{1-\beta(1-\delta)} r_t + E_t\Big(y_{t+1} + (1-\alpha) w_{t+1}\Big). \tag{13}$$

In contrast, at the limit with infinite adjustment costs $\kappa \to +\infty$, the additional discounting from expected future investment disappears and λ tends to $\beta(1-\delta)$.[13]

1.5 Aggregation

Assume all firms are identical. Denote aggregate output, aggregate investment and the aggregate capital stock by $y_t = \int_i y_t^i di, x_t = \int_i x_t^i di, k_t = \int_i k_t^i di$. The individual investment demand function (D.4) easily aggregates into the aggregate investment function

$$x_t = -\mu k_{t-1} + \theta \left(-E_t \left(\sum_{k=0}^{\infty} \lambda^k r_{t+k} \right) + E_t \left([1-\beta(1-\delta)] \sum_{k=0}^{\infty} \lambda^k \left(\frac{1}{1-\alpha} y_{t+k+1} + w_{t+k+1} \right) \right) \right). \quad (14)$$

2 Consumption-Investment Interplays

The aggregate investment function (14) reflects only the decision-theoretic (or partial equilibrium) response of aggregate investment to interest rates, taking the level of future aggregate demands y_{t+k} and future wages w_{t+k} as fixed. In equilibrium however, aggregate demand and wages are affected by investment, which feedbacks on investment demand and so on. The rest of this paper analyzes these general-equilibrium amplifying effects.

Because future aggregate demand y_{t+k} also depends on future consumption demand c_{t+k}, the equilibrium dependence of aggregate investment on interest rates cannot be solved without first specifying the determinants of consumption demand. This section introduces such a consumption bloc. It features household heterogeneity in a way that can make aggregate consumption in turn depend on aggregate investment, a dependence absent in standard specifications of the consumption bloc.

2.1 Dependence of Investment on Consumption

Investment in the aggregate investment function (14) depends on future marginal rates of transformation between capital and labor, and future wages—the term $\frac{1}{1-\alpha} y_{t+k+1} + w_{t+k+1}$. This makes investment depend on future aggregate demand y_t, both directly because aggregate demand affects the marginal rate of transformation, and indirectly because aggregate demand affects wages. Specifying the latter dependence requires specifying the labor supply schedule. I assume workers have the following separable preferences:

$$U(C_t^j, L_t^j) = \frac{(C_t^j)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \frac{(L_t^j)^{1+\psi}}{1+\psi} \bar{C}_t^{\frac{1}{\sigma_2}-\frac{1}{\sigma}}, \quad (15)$$

where $\bar{C}_t$ is a preference shock that the household takes as exogenous, but which is equal to aggregate consumption C_t in equilibrium. Its purpose is to allow to parameterize the income effect on labor supply

[13] Of course, when adjustment costs become larger, investment also responds less to all interest rates of all horizons. But the response to short-term and long-term interest rates become more alike.

$1/\sigma_2$ independently from the intertemporal elasticity of substitution σ. This approach to disentangling the two is taken from Gali (2011). Disentangling intertemporal substitution and the income effect on labor supply will help disentangling the different general-equilibrium amplifying effects. It is not necessary however: it is possible to restrict the model to the calibration $\sigma = \sigma_2$.

I assume wages are flexible, so that workers are on their labor supply. Given workers' preferences (15) and aggregating across households, aggregate labor supply is

$$w_t = \frac{1}{\sigma_2} c_t + \frac{\psi}{1-\alpha}(y_t - \alpha k_{t-1}). \tag{16}$$

using the production function (1) to replace aggregate labor l_t. Higher consumption and a higher level of activity increase the wage required to convince workers to work. More capital decreases the wage, since less labor is then required to produce the same amount.

The labor-supply schedule (16) makes apparent that the wage—and therefore investment—depends on aggregate consumption through the income effect on labor supply. In addition, investment depends on consumption because consumption is a large part of aggregate demand. Market-clearing imposes $Y_t = C_t + X_t$, or in log-linear form:

$$y_t = s_c c_t + s_x x_t, \tag{17}$$

where $s_c = C^*/Y^*$ and $s_x = X^*/Y^* = 1 - s_c$ are the consumption and investment shares in steady-state.

Combining equations (16) and (17), the term $\frac{1}{1-\alpha} y_{t+k+1} + w_{t+k+1}$ in equation (14) can be written

$$\frac{1}{1-\alpha} y_{t+k} + w_{t+k} = \left(\frac{1+\psi}{1-\alpha} s_c + \frac{1}{\sigma_2} \right) c_{t+k} + \left(\frac{1+\psi}{1-\alpha} s_x \right) x_{t+k} - \frac{\alpha\psi}{1-\alpha} k_{t+k-1}. \tag{18}$$

All in all, investment today depends on future investment—including the effect of future investment on the future capital stock—and future consumption.[14]

2.2 Independence of Consumption from Investment in Standard Models of Aggregate Consumption

Conversely, does aggregate consumption depend on aggregate investment? As is well known, it does not in an economy composed of identical permanent income households. In this case, the aggregate consumption behavior of households, captured by the consumption function (7), results in the aggregate Euler equation (see appendix F):

$$c_t = -\sigma r_t + E_t(c_{t+1}). \tag{19}$$

[14] Investment today does not depend on current consumption and current investment (except through the effect of current investment on the future capital stock), because of the one-period delay it takes for purchased capital to be used in production.

Iterating (19) forward gives

$$c_t = -\sigma E_t \sum_{k=0}^{\infty} r_{t+k}, \tag{20}$$

which solves for the dependence of aggregate consumption on real interest rates regardless of the specification of investment. This implies that whatever the dependence of investment on interest rates, the presence of investment will not change the dependence of consumption on interest rates.

The possibility to solve for the dependence of aggregate consumption on real interest rates independently of aggregate investment is somewhat more general than the case of permanent-income households. In particular, it can survive the introduction of household heterogeneity. If individual incomes are a function of aggregate household income only, as in the models of Werning (2015), Bilbiie (2020, 2018) and Acharya and Dogra (2020), aggregate consumption can still be solved independently of investment, even though precautionary savings can introduce discounting or compounding of future interest rates depending on the cyclicality of income risk, as these papers show.[15]

2.3 Household Heterogeneity and Dependence of Consumption on Investment

That consumption is the same function of real interest rates regardless of the specification of investment no longer obtains when heterogeneity in marginal propensities to consume interact with heterogeneity in the share of labor income in household income.

Consider the following Two-Agents-New-Keynesian (TANK) model. Households are divided between permanent-income households, whose consumption behavior is given by the consumption function (7), and hand-to-mouth households, who consume their incomes each period,

$$C_t^H = \Omega_t^H. \tag{21}$$

Permanent-income and hand-to-mouth households do not oscillate between the two types, like in the first-generation TANK models (e.g. Bilbiie, 2008).

Households' income, i.e. aggregate income net of aggregate investment $\Omega_t = Y_t - X_t$, can be divided between aggregate labor income Ω_t^L and aggregate capital income net of aggregate investment, or aggregate dividends, $\Omega_t^K = Y_t - \Omega_t^L - X_t$. I allow for the possibility of profits due to firms pricing a markup $\mathcal{M}_t$ over their marginal costs, and include profits in capital income. For the assumed Cobb-Douglas function (1), labor incomes are given by:

$$\Omega_t^L = w_t L_t = \frac{1-\alpha}{\mathcal{M}_t} Y_t. \tag{22}$$

[15] On the role of idiosyncratic risk and precautionary savings, see also Debortoli and Galí (2022).

Table 2: Summary of Income Distribution

	Household Income		Labor Income		Capital Income
	$\Omega = Y - X$		$\Omega^L = w.L$		$\Omega^K = Y - w.L - X$
Hand-to-Mouth Households	Ω^H	$=$	$s_L^H \Omega^L$	$+$	$s_K^H \Omega^K$
Permanent Income Households	Ω^P	$=$	$(1 - s_L^H)\Omega^L$	$+$	$(1 - s_K^H)\Omega^K$

Note: The table sums up the assumed distribution of capital and labor incomes between hand-to-mouth and permanent-income households.

Aggregate capital income net of investment is then:

$$\Omega_t^K = \left(1 - \frac{1-\alpha}{\mathcal{M}_t}\right) Y_t - X_t, \tag{23}$$

I allow the incomes of permanent-income households and hand-to-mouth households to load differently on labor and capital incomes. Specifically, hand-to-mouth households receive a share s_H^L of aggregate labor income, and a share s_H^K of aggregate capital income. (Permanent-income households receive the remaining share $1 - s_H^L$ of aggregate labor income, and the remaining share $1 - s_H^K$ of aggregate capital income.) The assumption that hand-to-mouth and permanent-income households receive a constant share of capital income is equivalent to assuming that they hold a constant share of firms' stocks.[16] In turn, this assumption implicitly assumes that hand-to-mouth households and permanent-income households do not trade firms' stocks across the two groups. They may however still trade firms' shares within each group. The assumption is consistent with the idea that hand-to-mouth households are hand-to-mouth because they face important costs in trading illiquid assets (Kaplan and Violante, 2014). Table 2 sums up the distribution of income between hand-to-mouth and permanent-income households.

Under these assumptions, hand-to-mouth households' income is:

$$\Omega_t^H = \left(\left(\frac{1-\alpha}{\mathcal{M}_t}\right) s_L^H + \left(1 - \frac{1-\alpha}{\mathcal{M}_t}\right) s_K^H \right)\Omega_t + \left(\frac{1-\alpha}{\mathcal{M}_t}\right)(s_L^H - s_K^H) X_t, \tag{24}$$

while permanent-income households' income is:

$$\Omega_t^P = \left(\left(\frac{1-\alpha}{\mathcal{M}_t}\right)(1 - s_L^H) + \left(1 - \frac{1-\alpha}{\mathcal{M}_t}\right)(1 - s_K^H) \right)\Omega_t - \left(\frac{1-\alpha}{\mathcal{M}_t}\right)(s_L^H - s_K^H) X_t. \tag{25}$$

Whenever hand-to-mouth households' income loads differently on labor and capital incomes, $s_L^H \neq s_K^H$, the share of hand-to-mouth's incomes Ω_t^H in total household income Ω_t depends on aggregate investment

[16] See appendix C for the equivalence for permanent-income households. For hand-to-mouth households, the equivalence holds under the assumption that dividends are liquid incomes so that they are part of the flow incomes that the hand-to-mouth households consume each period.

X_t. This is because investment expenditures punctures capital income and not labor income. For instance, if hand-to-mouth households receive a larger share of aggregate labor income than of aggregate capital income $s_L^H > s_K^H$, then an increase in investment expenditures increases the share of hand-to-mouth's incomes in total household's income Ω_t. Symmetrically, the share of permanent-income's incomes in total household's income decreases.

Because hand-to-mouth households and permanent-income households have different marginal propensities to consume (MPCs), how investment shifts this distribution of household incomes matters for aggregate consumption. This can be seen first by looking at the aggregate consumption function, derived in appendix G.

Lemma 1 *The aggregate consumption function can be written, in log-linear form:*

$$c_t = \left(1 - \beta(1 - ((1-\alpha)s_L^H + \alpha s_K^H))\right)\omega_t + \left(\beta(1-\alpha)(s_L^H - s_K^H)\frac{1}{1-s_x}\right)(s_x x_t - \mu_t) + \frac{\Omega^{P*}}{\Omega^*}\left((1-\beta)\sum_{k=1}^{\infty}\beta^k\omega_{t+k}^P - \sigma\sum_{k=0}^{\infty}\beta^{k+1}r_{t+k}\right), \quad (26)$$

where Ω^{P^*}/Ω^* is the share of household income going to permanent-income households in steady-state and μ_t is the log-deviation of profits from a steady-state with no markup $\mathcal{M}^* = 1$.

Unless $s_L^H = s_K^H$, investment x_t enters the aggregate consumption function (26). When hand-to-mouth households have a larger share of aggregate labor income than of aggregate capital income $s_L^H > s_K^H$, aggregate consumption increases with aggregate investment, because higher investment increases the share of hand-to-mouth's incomes in total household income, and hand-to-mouth households have a higher MPC (1) than permanent-income households $(1-\beta)$. The point captured in the present TANK model is more general. Whenever the shift in the distribution of household income induced by investment happens to shift incomes between households with different MPCs, investment affects aggregate consumption.

The expression of the consumption function (26) still includes the future incomes of permanent-income households ω_{t+k}^P, which itself depends on future investment. The multiple occurrences of investment do not cancel out in equilibrium however. Imposing market-clearing $c_t = \omega_t$ to endogenize aggregate income ω_t through the Keynesian cross gives the following consumption bloc that generalizes the Euler equation (19).

Lemma 2 *Aggregate consumption solves:*

$$c_t = \xi\left(x_t - \frac{\mu_t}{s_x}\right) - \sigma(1-\xi)r_t + E_t\left((c_{t+1} - \xi\left(x_{t+1} - \frac{\mu_{t+1}}{s_x}\right)\right), \quad (27)$$

$$\textit{where } \xi = \frac{(1-\alpha)(s_L^H - s_K^H)\frac{s_x}{1-s_x}}{1 - ((1-\alpha)s_L^H + \alpha s_K^H)}. \quad (28)$$

When $s_L^H = s_K^H$, then $\xi = 0$ and the consumption bloc reduces to the standard Euler equation (19) where consumption does not depend on investment. This irrelevance result is the same as the one that obtains in baseline TANK models without investment (Bilbiie, 2008): when hand-to-mouth households' income Ω_t^H and permanent-income households' income Ω_t^P are both proportional to aggregate household income Ω_t, aggregate consumption still obeys the Euler equation of the representative-agent model, even though the decomposition between partial-equilibrium and general-equilibrium effects differs. Lemma 2 first emphasizes that the *as if* result can survive the introduction of investment: heterogeneity in MPCs and the resulting higher average MPC are not sufficient to make aggregate consumption depend on aggregate investment. Indeed, even though an increase in investment generates new labor income, it also punctures capital income. If all households get the same share of labor income and capital income, this is a wash, regardless of the heterogeneity in MPCs.

Whenever heterogeneity in MPCs interacts with heterogeneity in the source of household income between labor and capital however, $s_L^H \neq s_K^H$, the *as if* result is lost. When hand-to-mouth households have a larger share of aggregate labor income than of aggregate capital income $s_L^H > s_K^H$, higher investment punctures predominantly the income of households with low MPCs. As a result, the share of household income—aggregate income net of investment—that goes to high-MPC households increases. Consumption consequently increases.

Note that equation (27) maintains the unit root of the standard Euler equation. This features of the simple TANK model without idiosyncratic risk proves robust to the introduction of investment. Considering idiosyncratic risk—e.g. a risk of shifting between the two types—would add a precautionary-saving motive that could add discounting or compounding in the consumption bloc, depending on the cyclicality of income risk (Werning, 2015; Bilbiie, 2020, 2018; Acharya and Dogra, 2020). Throughout the paper I stick to the baseline TANK model without precautionary savings to emphasize that the way household heterogeneity can introduce discounting of interest rates in the present model is thoroughly distinct form the precautionary-savings mechanism emphasized in the previous literature.

In the numerical illustrations that follow, I calibrate the share s_K^H and s_K^L as follows. Hand-to-mouth households are assumed to hold no shares of the firms and therefore to receive no capital income $s_K^H = 0$, as assumed in Bilbiie, Kanzig, and Surico (2022). Besides, hand-to-mouth households are assumed to represent 35% of the population, in line with Kaplan and Violante (2014), and so (assuming a hand-to-mouth household and a permanent-income households have the same labor income on average) to receive 35% of labor income, $s_L^H = 0.35$.[17]

2.4 Profits

In addition to investment, the markup μ_t also affects consumption when $\xi \neq 0$, because the markup determines profits, and profits affect capital incomes just as investment does. It implies that the cyclicality of markups also has important consequences when there is household heterogeneity. The importance of markup

[17] In the model, the share of hand-to-mouth households is also the average MPC out of labor income. An average MPC of 35% is well within the range of empirical estimates (e.g. Johnson, Parker, and Souleles, 2006; Parker, Souleles, Johnson, and McClelland, 2013; Misra and Surico, 2014; Commault, 2022).

cyclicality—and of the way they are distributed—in models with heterogeneous households has been emphasized in other papers however (e.g. Debortoli and Gali, 2018) and is orthogonal to the mechanisms of the present paper, which focuses on the role of investment. In addition, the cyclicality of markups—equivalently of the labor share—remains an open question empirically (e.g. Nekarda and Ramey, 2021).[18]

Therefore, to avoid burdening equations, in what follows I abstract from the markup term in equation (27). Effects from pro or countercyclical markups would simply come in addition to the effect of investment studied in this paper.

3 Discounting in Investment in General Equilibrium

The aggregate investment function (14) reflects only the decision-theoretic (or partial equilibrium) response of aggregate investment to interest rates, taking all future demands y_{t+k} and future wages w_{t+k} as fixed. Yet the higher investment initially triggered by lower rates in the investment function—and the higher consumption in the consumption function—increase aggregate demand, which further increase investment, and so on. This section analyzes the general-equilibrium amplifying loops at play in determining equilibrium investment, and how it affects the discounting of interest rates in aggregate investment.

3.1 The Investment Cross

What are the general-equilibrium channels that amplify the initial decision-theoretic response of investment to interest rates captured in the investment function? Plugging in the determinants of future aggregate demand and future wages (18) into the aggregate investment function (14) gives investment as

$$x_t = -\mu k_{t-1} - \theta E_t \left(\sum_{k=0}^{\infty} \lambda^k r_{t+k} \right) + \frac{\lambda}{\beta(1-\delta)} E_t \left(\sum_{k=0}^{\infty} \lambda^k \left(a^* c_{t+k+1} + b^* x_{t+k+1} - c^* k_{t+k} \right) \right), \tag{29}$$

where

$$a^* = \frac{1-\beta(1-\delta)}{\kappa} \left(s_c \frac{1+\psi}{1-\alpha} + \frac{1}{\sigma_2} \right), \tag{30}$$

$$b^* = \frac{1-\beta(1-\delta)}{\kappa} \left(s_x \frac{1+\psi}{1-\alpha} \right), \tag{31}$$

$$c^* = \frac{1-\beta(1-\delta)}{\kappa} \left(\frac{\alpha\psi}{1-\alpha} \right). \tag{32}$$

This expression takes note of the ways in which aggregate investment depends on future aggregate investment, future aggregate capital and future aggregate consumption. An initial increase in investment potentially affects all three, which feedbacks on investment and so on, creating an amplifying loop. These

[18] The basic New-Keynesian supply-side with price rigidity makes markups procyclical, but wage rigidity can make them countercyclical.

general-equilibrium amplifying loops are the analogues for investment of the general-equilibrium loop of the Keynesian cross for consumption. In reference to it, I call them the investment cross.[19]

What are the channels at play in the investment cross? Consider first the coefficient $b^* \geq 0$ on x_{t+k+1} in equation (29). Higher investment tomorrow increases the demand addressed to firm i tomorrow, increasing the benefit for firm i of investing today. This effect is compounded if labor-supply is not infinitely elastic ($\psi > 0$): in this case, higher aggregate demand pushes wages up, creating incentives for firm i to invest even more to substitute labor with capital.

Second, by the law of motion of aggregate capital

$$k_t = (1 - \delta)k_{t-1} + \delta x_t, \tag{33}$$

higher investment brings about a higher capital stock, which has effects on its own.[20] Indeed, whenever labor supply is not infinitely elastic a higher capital stock creates a counter-weighing effect, as reflected by the negative coefficient $-c^*$ on k_{t+k} in equation (29). This is because a higher capital stock shifts labor demand down, pushing wages down if labor-supply is elastic ($\psi > 0$). This lowers investment as firms have then less incentives to substitute labor with capital.

When $\xi = 0$ so that aggregate consumption is independent of investment, these are the only amplifying loops at play. But whenever $\xi > 0$, the initial increase in investment also increases consumption (and decreases it if $\xi < 0$), as captured by equation (27). This kick-starts a new array of amplifying channels captured by the coefficient $a^* \geq 0$ on c_{t+k+1} in equation (29). Higher aggregate consumption tomorrow first increases the demand addressed to firm i tomorrow, just like higher aggregate investment does. This increases the benefit of investing today, once again all the more that labor-demand is elastic. In addition, higher consumption tomorrow further increases firm i's incentive to invest today through the income effect on labor supply (if $1/\sigma_2 > 0$): higher consumption shifts labor supply up, increasing wages and creating further incentives for firms to substitute labor with capital.

3.2 The Effect on Investment of Intertemporal Substitution in Consumption

The investment cross captures the ways through which the initial effect of interest rates in the investment function gets amplified. But the investment function is not the only source of the interest rate channel. Interest rates also have an initial lever in the consumption function—the intertemporal substitution channel that the aggregate consumption literature has studied extensively. While this part of the interest rate channel originates in consumption, since aggregate investment depends on aggregate consumption, it also affects investment in general equilibrium. If lower interest rates increase consumption, this increases aggregate

[19] The term investment cross is intended to designate the feedback loop from higher investment *at all periods* to higher aggregate demand *at all periods*, similar to the intertemporal Keynesian cross of Auclert, Rognlie, and Straub (2018). Aggregate demand today has actually no effect on investment today, due to the one-period lag between installing new capital and getting it to produce.

[20] The aggregate law of motion (33) is the aggregation of the individual law of motion (8) across firms.

demand, which increases investment, which further increase aggregate demand, and so on.[21]

Keeping track of what part of the general-equilibrium dependence of investment on interest rates is due to which of the two parts of the interest rate channel is potentially challenging. However, isolating the two can be done in a simple way. Setting the intertemporal elasticity of substitution to $\sigma = 0$ shuts down the part of the interest rate channel that originates in intertemporal substitution. It allows to assess how aggregate investment would load on interest rates if the interest rate channel originated in the investment function only, then amplified by the investment cross. The rest of this section gives the general dependence of investment on interest rates when both parts of the interest rate channel are present, then proceed to disentangle the effects of each part.

3.3 Overall Discounting of Interest Rates in Investment

Taking into account both components of the interest rate channels, how do general-equilibrium amplifying effects change the discounting of future interest rates present in the investment function (14)? Appendix H solves the dependence of investment in general equilibrium to show the following result.

Proposition 2 *Consider the economy defined by the investment function (29), the capital-accumulation equation (33) and the consumption equation (27).*

(i) The two forward-looking roots of the economy are the unit root coming from intertemporal substitution in consumption in equation (27), and λ^*, the smaller root of the quadratic polynomial

$$Q(\lambda^*) = (\lambda^*)^2 - \left(\beta(1-\delta) + \frac{1}{1-\delta}\left(1 + \delta\frac{1-\beta(1-\delta)}{(1-\alpha)\kappa}\right) + b^* + \xi a^* + \frac{\delta}{1-\delta}c^*\right)\lambda^* + \left(\beta + \frac{1}{1-\delta}(b^* + \xi a^*)\right). \tag{34}$$

(ii) Aggregate investment is given by :

$$x_t = -\mu^* k_{t-1} - \theta^* \sum_{k=0}^{\infty}\left(\zeta 1 + (1-\zeta){\lambda^*}^k\right) E_t(r_{t+k}), \tag{35}$$

where

$$\zeta = \frac{a^*}{1-\lambda^*}\kappa(1-\xi)\sigma \tag{36}$$

and θ^* and μ^* are positive coefficients whose expression is given in the appendix.

Equation (35) highlights that in general equilibrium, investment depends on interest rates through two roots, the unit root of the Euler equation and a second root λ^*. The two roots conveniently map into the two parts of the interest rate channel. Indeed, when intertemporal substitution originating in consumption is absent and only the part of the interest rate channel originating in investment is at play ($\sigma = 0$), then $\xi = 0$

[21] The higher consumption also shifts labor supply down through the income effect on labor supply. This puts upward pressure on wages, which gives further incentives to substitute labor with capital, and therefore invest more.

and investment loads on interest rates only through the root λ^*. Consistently, λ^*, μ^* and θ^* do not depend on the intertemporal elasticity of substitution σ.[22] They depend only on the parameters in the decision-theoretic investment function (14), and on the parameters of the amplifying channels of the investment cross listed in section 3.1.[23] The root λ^* therefore captures the discounting due to this part of the interest rate channel—the investment function and the investment cross. Accordingly, I call λ^* the investment root.

Symmetrically, the unit root of the Euler equation is tied to the intertemporal susbtitution part of the interest rate channel. Were investment constant and the interest rate channel to originate in intertemporal substitution only, it would be the only root shaping the dependence of aggregate demand on interest rates. Accordingly, I call it the consumption root.[24]

3.4 The Investment Root

To focus on the part of the interest rate channel that originates in investment demand, consider the case when intertemporal substitution is shut down, $\sigma = 0$. Proposition 2 then reduces to the following expression for investment.

Corollary 2 *If there is no intertemporal substitution in consumption $\sigma = 0$, then $\zeta = 0$ and aggregate investment in general equilibrium is given by:*

$$x_t = -\mu^* k_{t-1} - \theta^* \sum_{k=0}^{\infty} {\lambda^*}^k E_t(r_{t+k}), \tag{37}$$

The difference between the discounting at rate λ^* in equation (37) and the discounting of interest rates at rate λ in the investment function (9) captures how the general-equilibrium amplifying effects of the investment cross modify the extent of discounting present in partial equilibrium. Appendix I shows the following results on the extent of discounting of interest rates in investment once the investment cross in taken into account.

Corollary 3 *The root λ^* satisfies the following properties:*

1. *Provided the condition*
$$\left(\frac{1}{1-\delta} - \lambda\right)(b^* + \xi a^*) > \frac{\delta}{1-\delta} c^* \lambda \tag{38}$$
is satisfied, then $\lambda^* > \lambda$.

2. *Provided the condition*
$$b^* + \xi a^* - c^* < (1 - \beta(1-\delta))\left(1 + \frac{1}{\kappa(1-\alpha)}\right) \tag{39}$$

[22] That λ^* does not depend on σ follows from the fact that a, b, and c in the quadratic equation (34) do not depend on σ. For μ^* and θ^*, see their expressions in appendix H.

[23] They can depend on σ_2, which parameterizes the strength of the income effect on labor supply. This highlights the relevance of distinguishing between the role of σ as the intertemporal elasticity of substitution and its role in parameterizing the strength of the income effect on labor supply, à la Gali (2011).

[24] The consumption root could of course take a value different from 1 under a different specification of the consumption bloc. It would be the case in a model with precautionary savings induced by idiosyncratic income shocks for instance.

is satisfied, then $\lambda^* < 1$.

3. *In all cases,* $\lambda^* \leq \frac{1}{1-\delta}$.

The first item states that, provided the general-equilibrium amplifying channels are strong enough—if the amplifying effects measured by $b^* + \xi a^*$ are sufficiently larger than the dampening effects measured by c^*—then future interest rates are less discounted once the investment cross is taken into account than they are in the decision-theoretic investment function. While condition (38) can fail for extreme parameter values, it is easily satisfied for realistic parameterizations: in practice, the investment cross decreases the extent of discounting of future interest rates.

The second item states that, provided the general-equilibrium amplifying channels are not too strong—if the amplifying effects measured by $b^* + \xi a^*$, net of the dampening effects measured by c^*, are not too large—then future interest rates remain discounted after the investment cross is taken into account. While condition (39) can fail for extreme parameter values, it is easily satisfied for realistic parameterizations: in practice, the investment cross does not eliminate the discounting of future interest rates arising from the investment function. For the extreme parameterizations in which $\lambda^* \geq 1$, the third item provides an upper bound on the degree of compounding of interest rates.

While the investment cross typically does not eliminate the discounting of future interest rates, to which extent it diminishes it depends on the strength of the general-equilibrium amplifying effects, as captured by $b^* + \xi a^*$ and c^*. One can show that, keeping all the decision-theoretic parameters β, κ, δ fixed,

$$d\lambda^* = \left(\frac{1}{1-\delta} - \lambda^*\right) d(b^* + \xi a^*) - \lambda^* \frac{\delta}{1-\delta} dc^*, \tag{40}$$

which shows that an increase in $b^* + \xi a^*$ increases λ^*, while an increase in c^* decreases it.

These general-equilibrium effects depend in turn on the general-equilibrium parameters. Among them, the specification of labor supply, through the calibration of its elasticity $1/\psi$ and of the strength on the income effect $1/\sigma_2$ plays a key role. Except for unrealistically extreme calibrations, a more inelastic labor supply—a higher ψ—increases the strength of the amplifying effects and increase λ^*. Similarly, provided $\xi > 0$—i.e. provided household heterogeneity—a larger income effect on labor supply $1/\sigma_2$ increases the strength of the amplifying effects and increases λ^*. Both a higher ψ and a higher $1/\sigma_2$ make wages increase more strongly in reaction to an increase in investment, increasing incentives for firms to further substitute labor with capital, and invest.

Figure 2 illustrates the dependence of λ^* in these labor-supply parameters. On the figure, the decision-theoretic parameters are calibrated as discussed in section 1. The shares of capital and labor incomes going to hand-to-mouth households s_K^H and s_L^H (in the case of heterogeneous households) are calibrated as discussed in section 2. As for the other general-equilibrium parameters, the share of investment in GDP is calibrated to $s_x = 20\%$, the inverse of the elasticity of labor supply (in the right panel) to $\psi = 1$, and the strength of the income effect on labor supply (in the left panel) to $1/\sigma_2 = 1$. This is summed up in Table 1. Note that

λ^* does not depend on the elasticity of substitution σ.

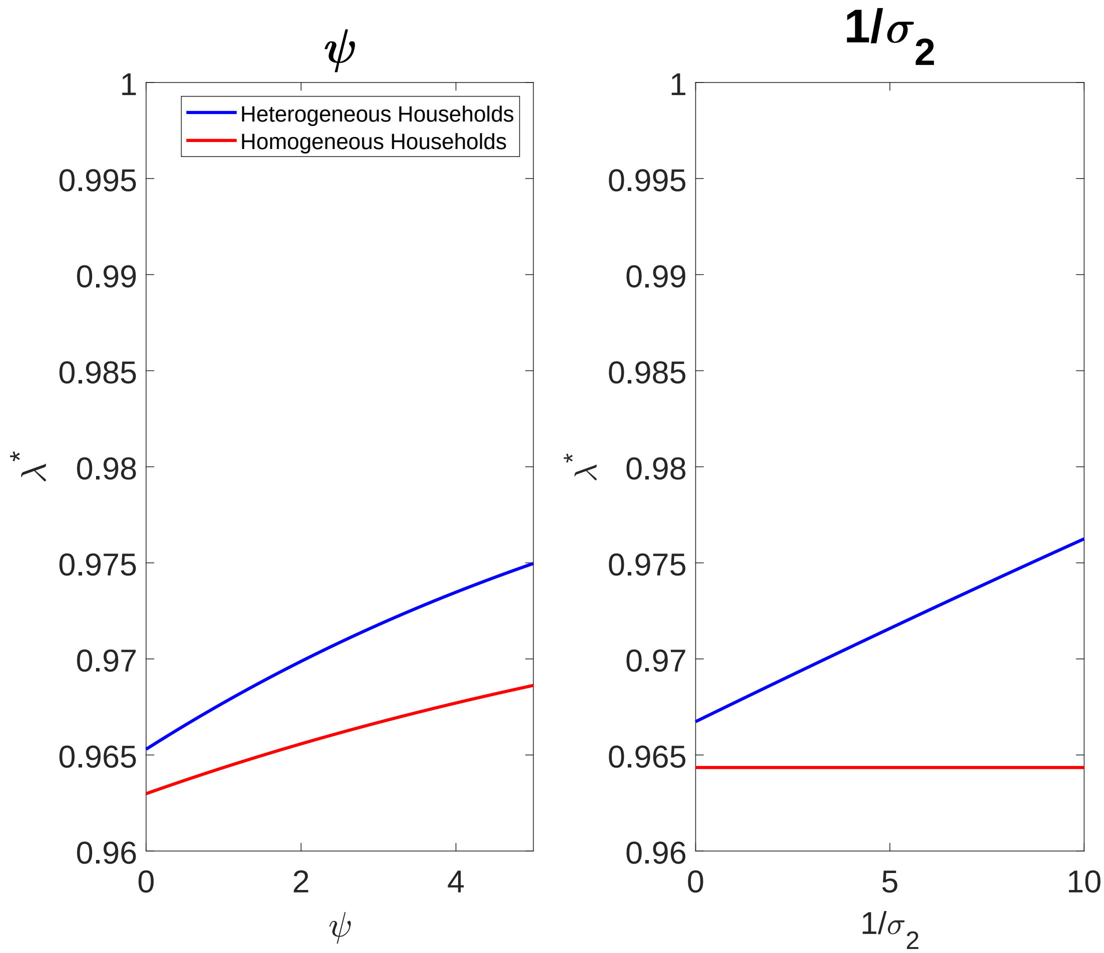


Figure 2: Interest Rate Discounting λ^* as a Function of the Labor Supply Parameters ψ and $1/\sigma_2$

Note: The figure gives the root λ^* as a function of the inverse of the Frisch elasticity of labor supply ψ (left panel) and the income effect on labor supply $1/\sigma_2$ (right panel). In each panel, all other parameters are calibrated according to Table 1 (for homogeneous households $s_K^H = s_L^H$).

While general-equilibrium parameters such as the ones that specify labor supply become important determinant of the extent of discounting in general equilibrium, some partial-equilibrium parameters lose much importance. This is particularly the case of the extent of adjustment costs. In the partial-equilibrium investment function, adjustment costs had an important impact on discounting factor of the investment function λ (cf. corollary 1). The following corollary gives the limiting results on the dependence of λ^* on adjustment costs in general equilibrium.

Corollary 4 *Limits of λ^* for low and large adjustment costs.*

- *At the limit without adjustment costs $\kappa \to 0$,*

$$\lim_{\kappa \to 0} \lambda^* = \frac{(s_x + \xi s_c)\frac{1+\psi}{1-\alpha} + \xi \frac{1}{\sigma_2}}{(1-\delta)\left((s_x + \xi s_c)\frac{1+\psi}{1-\alpha} + \xi \frac{1}{\sigma_2}\right) + \delta \frac{1+\alpha\psi}{1-\alpha}}. \tag{41}$$

- *At the limit where adjustment costs are infinite $\kappa \to +\infty$,*

$$\lim_{\kappa \to +\infty} \lambda^* = \beta(1-\delta). \tag{42}$$

For large adjustment costs, λ^* tends toward $\beta(1-\delta)$ just as λ did. For small adjustment costs however, λ^* no longer tends toward 0 but instead to a positive limit. Beyond this limiting analytical result, the dependence of λ^* on adjustment costs in general equilibrium is illustrated on Figure 1 for the main calibration of Table 1, where it is superimposed on the decision-theoretic dependence of λ on adjustment costs. The general-equilibrium discounting factor λ^* varies very little with the level of adjustment costs κ and is very close to $\beta(1-\delta)$ for all levels of adjustment costs.

While the overall level of discounting depends little on κ, the nature of the dependence of investment on future interest rates is radically different at different levels of adjustment costs. At high levels of adjustment costs, the dependence on future interest rates mostly comes from partial equilibrium effects, while at low levels of adjustment costs it mostly comes from general equilibrium effects. Interest rate cuts tomorrow increase investment demand tomorrow, and investing today allows firms to serve this demand tomorrow. The model therefore brings a new light to the question of whether only short-term interest rates matter for investment when adjustment costs are low (e.g. Hall, 1977). Only short-term rates matter in partial equilibrium, but not in general equilibrium.

3.5 Intertemporal Substitution and the Consumption Root

Come back now to the general case of equation (35) with $\sigma > 0$, to see how the part of the interest rate channel that originates in households' intertemporal substitution affects the dependence of aggregate investment on interest rates. When both components of the interest rate channel are present, the consumption root inherited from the consumption equation (27) also shapes the dependence of investment on interest rates. Since the investment root typically provides discounting while the consumption root from intertemporal substitution does not, the dependence of investment on the consumption root mitigates the discounting due to the investment root λ^*. In particular, whenever $\sigma > 0$, the effect of future interest rates r_{t+k} does not converges to zero as the maturity k tends to infinity.

The intertemporal elasticity of substitution (IES) is therefore a key determinant of the extent of discounting of interest rates in investment, as it weights the importance of the intertemporal substitution part of the interest rate channel. This is in contrast to the response of consumption in standard TANK models without investment, in which the extent of discounting of future rates does not depend on the IES. As is intuitive, the higher the IES, the higher the weight ζ on the consumption root, and the weaker the discounting of future interest rates. But a high IES can actually even lead to *compounding* instead of discounting of future interest rates, as nothing restricts ζ from being greater than 1. The following corollary spells out the condition for discounting.

Corollary 5 *The coefficient on the interest rate r_{t+k} varies from $-\theta^*$ to $-\zeta\theta^*$ as the horizon k increases. The (absolute value of the) coefficient on r_{t+k} decreases with k if and only if $\zeta > 1$, i.e. if and only if:*

$$\sigma < \sigma^* = \frac{1-\lambda^*}{\kappa(1-\xi)a^*}. \tag{43}$$

When condition (43) is not satisfied, there is compounding of future interest rates: the effect of interest rate cuts is all the stronger the higher the horizon of the interest rate cut.[25] When the intertemporal susbtitution motive is large, the net effect of the presence of investment is actually to worsen the absence of discounting in the Euler equation into compounding. For the main calibration of Table 1, the threshold value for compounding to occur is only $\sigma^* = 0.28$. An IES of $\sigma = 1$ or even $\sigma = 0.5$ therefore generates compounding. However, a low IES of $\sigma = 0.1$, as recently estimated by Best, Cloyne, Ilzetzki, and Kleven (2020) delivers discounting.

The left panel of Figure 3 illustrates the cases of discounting and compounding graphically, by plotting the (absolute value of) the coefficient of the interest rate r_{t+k} as a function of the horizon k, for various values of σ.

Do other parameters beyond σ affect the relative weight ζ on the investment and consumption roots? Figure 4 plots the dependence of the threshold σ^* on the IES as a function of the other parameters of the model. The extent of adjustment costs κ, the depreciation rate of capital δ, the investment share s_x, and the difference in the share of labor and capital incomes of hand-to-mouth income $S_L^H - s_K^H$ make virtually no change to σ^*.[26] Once again, the labor-supply parameters ψ and $1/\sigma_2$ have more of an effect. With a more inelastic labor supply or a strong income effect on labor supply, wages react more strongly to aggregate demand, amplifying general equilibrium effects and therefore the impact of future interest rates, making compounding more likely. A lower IES is then required for there to be discounting. While the labor-supply parameters can lower the threshold σ^* however, they cannot increase it much: For an perfectly elastic labor supply $\psi = 0$ or no income effect $1/\sigma_2 = 0$, the threshold σ^* does not exceed 0.6.

[25] Note that when there is compounding, the coefficient does not diverge to $+\infty$. It only converges to the larger value (in absolute value).

[26] That the extent of adjustment costs κ has very little impact on ζ does not mean that adjustments costs do not matter for the response of investment to interest rates. The coefficients μ^* and θ^* are strongly decreasing in κ. Yet, since both λ^* and ζ depend little on κ, κ has little effect on the relative importance for investment of interest rates of different maturities.

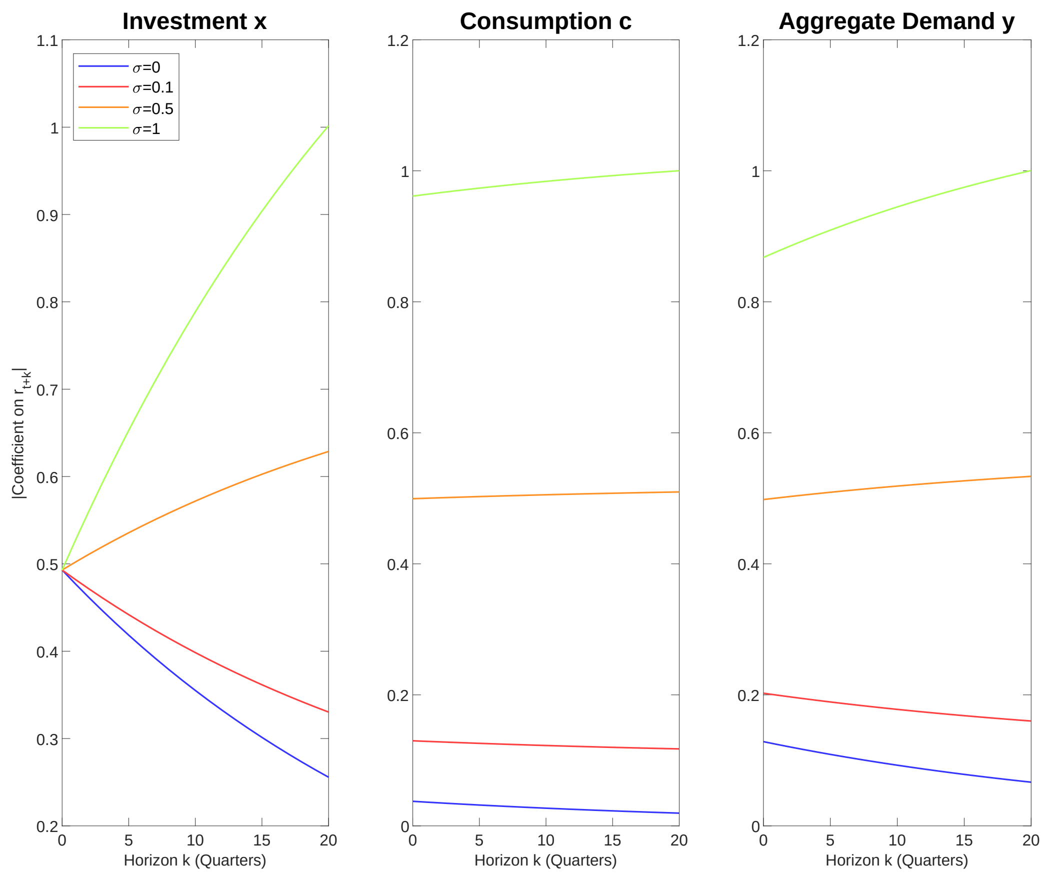


Figure 3: The Effect of the Elasticity of Substitution on Discounting/Compounding

Note: The figure gives the (absolute value of the) coefficients of future interest rates in aggregate investment (left panel) and in aggregate demand (right panel) for three different calibration of the intertemporal elasticity of substitution σ. There is discounting for $\sigma = 0.1$, weak compounding for $\sigma = 0.5$, and compounding for $\sigma = 1$. The rest of the calibration is according to Table 1.

4 The Dynamic IS Curve beyond Intertemporal Substitution in Consumption

Provided a low enough IES, interest rates can be discounted in investment even after taking into account general-equilibrium effects. Can it generate discounting in aggregate demand and aggregate consumption? If the consumption bloc can be reduced to the standard Euler equation (20), interest rates can be discounted in aggregate demand provided they are so in aggregate investment. But the remained undiscounted in aggregate consumption. This section shows that the form of household heterogeneity introduced in section 2 can introduce discounting in consumption as well, provided again a low enough IES.

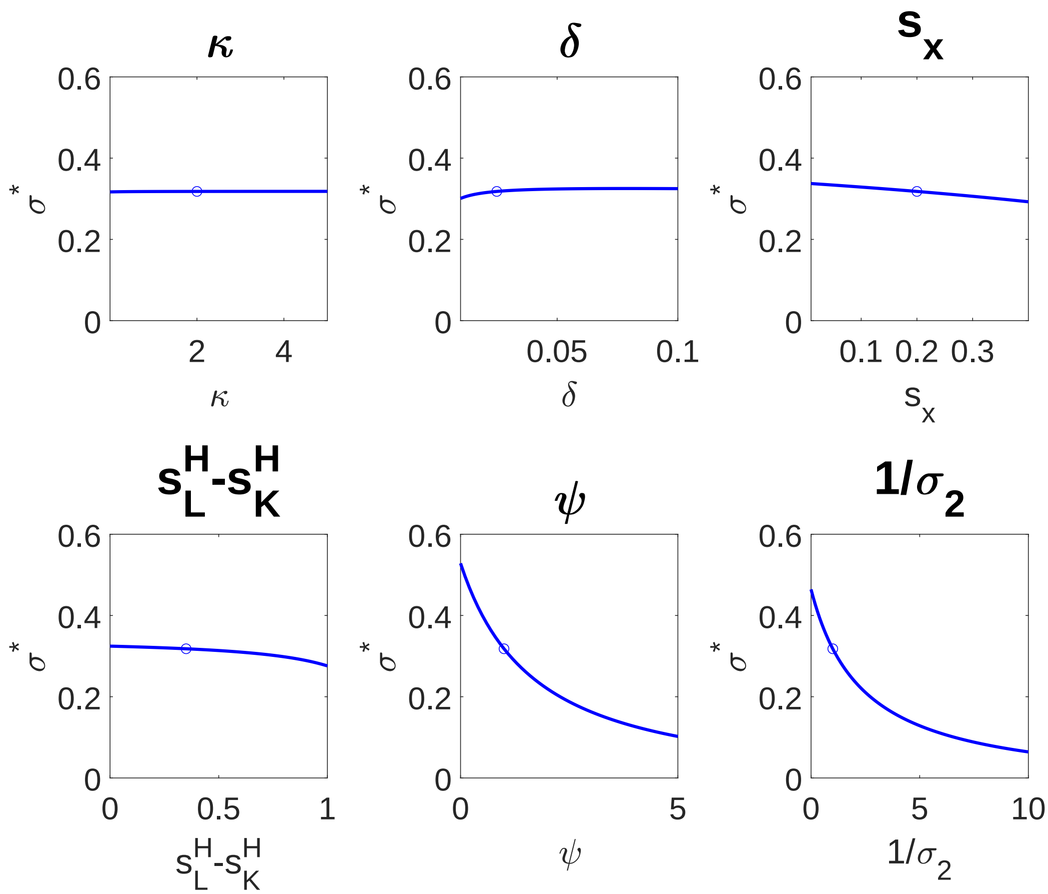


Figure 4: Dependence of the Threshold σ^* on Other Parameters

Note: The figure gives the dependence of the threshold value σ^* on the IES σ as a function of the extent of adjustment costs κ, the depreciation rate of capital δ, the investment share s_x, the difference in the share of labor and capital incomes of hand-to-mouth income $S_L^H - s_K^H$, the inverse of the Frisch elasticity of labor supply ψ, and the income effect on labor supply $1/\sigma_2$. On each panel, the parameters that do not vary are set to their value in Table 1, and the dot corresponds to the baseline calibration of Table 1 for the parameter that varies in the panel.

4.1 Consumption Response to Interest Rates without Intertemporal Substitution

When $\xi = 0$—which includes in particular the case of homogeneous permanent-income households—the consumption bloc (27) reduces to the standard Euler equation (20) and real rates enter aggregate consumption with no discounting. Anything that occurs on investment has no effect on consumption since interest rates continue to affect consumption through intertemporal substitution only. However, whenever $\xi > 0$, interest rates also affect consumption through the ripple effect of the impact of interest rates on investment demand. The discounting of interest rates in consumption is then shaped in part by the investment root λ^*.

How much discounting of interest rates there is in consumption is then determined by the relative weight of the two parts of the interest rate channel, just like for investment. It is easy to see that in the extreme case with no intertemporal substitution $\sigma = 0$, discounting in consumption is equivalent to discounting in investment, since $c_t = \xi x_t$. In this case, the dependence of consumption on interest rates is then completely independent from intertemporal substitution. Instead, consumption respond to interest rates solely as a ripple effect of the response of investment in the investment function, amplified by the investment cross. Provided the investment root λ^* is less than 1—which as we saw in section 3 is easily the case—there is discounting of interest rates in consumption.

4.2 Condition for Discounting in Consumption

Beyond the extreme case without intertemporal substitution $\sigma = 0$, appendix K shows that, when $\xi > 0$, a low enough IES is enough to deliver discounting of interest rates in consumption.

Proposition 3 *Aggregate consumption is given by:*

$$c_t = -\xi\mu^* k_{t-1} - \sum_{k=0}^{\infty} \left(\Big(\xi\theta^*\zeta + \sigma(1-\xi)\Big) 1 + \Big(\xi\theta^*(1-\zeta)\Big) {\lambda^*}^k \right) E_t(r_{t+k}). \tag{44}$$

When $\xi > 0$, interest rates are discounted in consumption iff $\lambda^* < 1$ and $\zeta < 1$ i.e. iff condition (43) $\sigma < \sigma^*$ is satisfied.

The condition for discounting of future interest rates in consumption is the same as the one for discounting in investment: an IES below the threshold value in (43). Just like for investment, a low enough IES makes the channel originating in investment demand dominate the intertemporal substitution channel, which—provided $\lambda^* < 1$—delivers discounting. Therefore, with household heterogeneity, a low enough IES guarantees discounting of future interest rates not only in investment but also in consumption. The middle panel of Figure 3 illustrates the cases of discounting and compounding for consumption graphically, in the same way as for investment in the left panel.

4.3 The Dynamic IS Curve with Investment

From the expressions of aggregate investment and aggregate consumption, it is straightforward to obtain the expression of aggregate demand as a function of present and future interest rates, i.e. the dynamic IS curve of the model. This dynamic IS curve generalizes the standard dynamic IS curve—the Euler equation—of the model with consumption only.

Proposition 4 *The dynamic IS curve giving aggregate demand as a function of interest rates is:*

$$y_t = -(s_x + \xi s_c)\mu^* k_{t-1} - \sum_{k=0}^{\infty} \left(\Big((s_x + \xi s_c)\theta^*\zeta + s_c\sigma(1-\xi)\Big) 1 + \Big((s_x + \xi s_c)\theta^*(1-\zeta)\Big) {\lambda^*}^k \right) E_t(r_{t+k}), \tag{45}$$

Interest rates are discounted in the dynamic IS curve iff $\lambda^* < 1$ and $\zeta < 1$ i.e. iff equation (43) is satisfied.

As in aggregate investment and aggregate consumption, the dependence of aggregate investment on interest rates now depends on both the consumption root and the investment root, as the dynamic IS curve now captures both components of the interest rate channel. The discounting or compounding of future interest rates in the dynamic IS curve is determined by both roots. The condition for discounting is the same as the one for discounting in investment and consumption: an IES below the threshold value in (43). This makes the interest rate channel originate predominantly in investment demand rather than intertemporal substitution in consumption. The right panel of Figure 3 illustrates the cases of discounting and compounding for aggregate demand.

Conclusion

The paper derived the dynamic IS curve in a model with investment, where the interest-rate channel originates not only in intertemporal substitution in consumption but also in investment demand. The overall conclusion is that the part of the interest rate channel that originates in investment demand discounts future interest rates more than the part that originates in intertemporal substitution. As a consequence, investment can deliver discounting in aggregate demand, but only under the assumption that the interest rate channel originates predominantly in the response of investment, not in the response of consumption. The importance of the intertemporal substitution part of the interest rate channel is ultimately an empirical question, largely dependent on the value of the intertemporal elasticity of substitution. The role of the IES in determining the importance of long-term rates for aggregate demand adds to the importance of good estimates of the IES—a long-debated empirical issue—to get an accurate picture of the interest rate channel.

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A Derivation of Firm i's Investment Decision

Define

$$I_t^i = K_t^i - (1-\delta)K_{t-1}^i \tag{A.1}$$

the new capital installed, i.e. investment (as opposed to investment expenditures X_t). Capital expenditures rewrite

$$X_t^i = I_t^i + \frac{\kappa}{2}\frac{(I_t^i - \delta K^*)^2}{\delta K^*}. \tag{A.2}$$

Denote λ_t^i the Lagrange multiplier on the production function constraint (1) and Q_t^i the Lagrange multiplier on equation (A.2), so that Q_t^i can be interpreted as the marginal value of installed capital to the firm. The Lagrangian of the firm's cost minimization program writes:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} M_{0,t}\left(W_t L_t^i + I_t^i + \frac{\kappa}{2}\frac{(I_t^i - \delta K^*)^2}{\delta K^*} + Q_t^i\Big(K_t^i - (1-\delta)K_{t-1}^i - I_t^i\Big) + \lambda_t^i\Big(Y_t^i - F(K_{t-1}^i, L_t^i)\Big)\right). \tag{A.3}$$

The first-order conditions with respect to L_t^i, K_t^i and I_t^i are:

$$W_t = \lambda_t^i F_L^i(t), \tag{A.4}$$

$$Q_t^i = E_t\left(M_{t,t+1}\lambda_{t+1}^i F_K^i(t+1) W_{t+1}\right) + E_t\left(M_{t,t,+1}(1-\delta)Q_{t+1}^i\right), \tag{A.5}$$

$$Q_t^i = 1 + \kappa\left(\frac{I_t^i}{\delta K^*} - 1\right). \tag{A.6}$$

Replacing the value of λ_t^i from (A.4) in (A.5)-(A.6) gives:

$$Q_t^i = E_t\left(M_{t,t+1}\frac{F_K^i(t+1)}{F_L^i(t+1)}W_{t+1}\right) + E_t\left(M_{t,t,+1}\Big(1-\delta\Big)Q_{t+1}^i\right), \tag{A.7}$$

$$Q_t^i = 1 + \kappa\left(\frac{K_t - (1-\delta)K_{t-1}}{\delta K^*} - 1\right). \tag{A.8}$$

The first condition expresses the marginal benefit Q_t^i of installing one more unit of capital, while the second condition equates it to the marginal cost of increasing the capital stock. Iterating equation (A.7) forward gives equation (5) in the text.

In log-linear form, equations (A.7)-(A.8) write:

$$q_t^i = -r_t + [1-\beta(1-\delta)]E_t\left(\frac{1}{1-\alpha}(y_{t+1}^i - k_t^i) + w_{t+1}\right) + \beta(1-\delta)E_t(q_{t+1}^i), \tag{A.9}$$

$$q_t^i = \kappa x_t^i, \tag{A.10}$$

where we denoted r_t the real interest rate, $\beta = M^*$ the steady-state value of the SDF (the inverse of the steady-state real interest rate), and used the relationship $r_t = -E_t(\widehat{M_{t,t+1}})$ obtained from pricing a riskless asset. Combining equations (A.9) and (A.10) gives

$$x_t^i = \frac{1}{\kappa}\left(-r_t + [1-\beta(1-\delta)]E_t\left(\frac{1}{1-\alpha}(y_{t+1}^i - k_t^i) + w_{t+1}\right)\right) + \beta(1-\delta)E_t(x_{t+1}^i), \qquad \text{(A.11)}$$

Iterating it forward gives equation (6) in the text.

B Alternative Specification of Adjustment Costs

An alternative common specification of quadratic capital adjustment costs is to replace K^* in (2) by K_{t-1}^i:

$$X_t^i = \phi(K_t^i, K_{t-1}^i) = (K_t^i - (1-\delta)K_{t-1}^i) + \frac{\kappa}{2}\frac{(K_t^i - (1-\delta)K_{t-1}^i - \delta K_{t-1}^i)^2}{\delta K_{t-1}^i}, \qquad \text{(B.1)}$$

which can also be written as:

$$X_t^i = \phi(I_t^i/K_{t-1}^i) = I_t^i + \frac{\kappa}{2}\left(\frac{I_t^i}{\delta K_{t-1}^i} - 1\right)^2 \delta K_{t-1}^i, \qquad \text{(B.2)}$$

where $I_t^i = K_t^i - (1-\delta)K_{t-1}^i$ is the new capital installed, i.e. investment (as opposed to investment expenditures X_t). An advantage of this assumption is that investment expenditures are homogeneous of degree 1 in (K_t, K_{t-1}).[27] However, because it makes the adjustment costs at t depend on the capital stock at $t-1$, it implies that firm i has an incentive to choose its capital stock at t also in view of reducing its investment costs at $t+1$—a mechanism which may be less economically meaningful. Indeed, under specification (B.2) firm i's first-order conditions are:

$$Q_t^i = E_t\left(M_{t,t+1}\frac{F_K^i(t+1)}{F_L^i(t+1)}W_{t+1}\right) + E_t\left(M_{t,t,+1}\left(1-\delta\right)Q_{t+1}^i\right) + E_t\left(M_{t,t+1}\frac{\kappa}{2}\left(2\left(\frac{I_{t+1}-\delta K_t}{K_t}\right) + \left(\frac{I_{t+1}-\delta K_t}{K_t}\right)^2\right)\right), \qquad \text{(B.3)}$$

$$Q_t^i = 1 + \kappa\left(\frac{I_t}{\delta K_{t-1}} - 1\right). \qquad \text{(B.4)}$$

The new term in equation (B.3) adds a new term to the marginal value of installed capital: how one more unit of capital affects to cost of adjusting capital tomorrow. This new term survives at first order:

$$q_t^i = -r_t + [1-\beta(1-\delta)]E_t\left(\frac{1}{1-\alpha}(y_{t+1}^i - k_t^i) + w_{t+1}\right) + \beta(1-\delta)E_t(q_{t+1}^i) + \beta\kappa\delta E_t(x_{t+1}^i - k_t^i), \qquad \text{(B.5)}$$

$$q_t^i = \kappa(x_t^i - k_{t-1}^i). \qquad \text{(B.6)}$$

[27] Both specifications (2) and (B.1) guarantee that the steady-state level of investment is the same ratio of steady-state capital regardless of the value of steady-state capital, $X^*/K^* = \delta$.

The specification of adjustment costs in (2) therefore offers a more economically meaningful interpretation.

C Derivation of the Permanent Income Consumption Function

The permanent-income household j maximizes utility:

$$\max_{C_t^j, B_t^j} E_t \sum_{k=0}^{\infty} \frac{(C_{t+k}^j)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}, \tag{C.1}$$

subject to a no-Ponzi scheme constraint, and the flow budget constraint

$$\frac{B_t^j}{R_t} + s^j E_t(M_{t,t+1}V_{t+1}) + C_t^j = B_{t-1}^j + s^j V_t + \Omega_t^{L,i}, \tag{C.2}$$

where B_t^j is household j's holding of bonds, $\Omega_t^{L,j}$ is its labor income, and s^j is its holding of shares of the firms. Since all firms are ultimately identical, the composition of household j's portfolio across firms is irrelevant. V_t is the value of shares at the beginning of the period, before the payment of dividends, and $E_t(M_{t,t+1}V_{t+1})$ is their value at the end of the period after the payment of dividends. Aggregate dividends are equal to firms' profits net of investment expenditures

$$\Omega_t^K = Y_t - \Omega_t^L - X_t, \tag{C.3}$$

so that the value of shares satisfies

$$V_t = \Omega_t^K + E_t(M_{t,t+1}V_{t+1}) = E_t \sum_{k=0}^{\infty} M_{t,t+k}\Omega_{t+k}^K. \tag{C.4}$$

The flow budget constraint (C.2) can be rewritten

$$\frac{B_t^j}{R_t} + C_t^j = B_{t-1}^j + \Omega_t^j, \tag{C.5}$$

where

$$\Omega_t^j = \Omega_t^{K,j} + \Omega_t^{L,j}, \tag{C.6}$$

$$\Omega_t^{K,j} = s_j \Omega_t^K. \tag{C.7}$$

The household's intertemporal budget constraint writes:

$$\sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}} C_{t+k}^j = B_{t-1}^j + \sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}} \Omega_{t+k}^j, \tag{C.8}$$

where

$$\mathcal{R}_{t,t+k} = \prod_{n=0}^{k-1} R_{t+n}. \tag{C.9}$$

In log-linear form, in a steady-state where $\Omega^{i,*} = C^{i,*}$

$$\sum_{k=0}^{\infty} c_{t+k}^{j} = b_{t-1}^{j} + \sum_{k=0}^{\infty} \omega_{t+k}^{j}, \tag{C.10}$$

where $b_{t-1}^{j} = dB_{t-1}^{j}/C^{i,*}$, and

$$\widehat{\mathcal{R}}_{t,t+k} = \sum_{n=0}^{k-1} r_{t+n}. \tag{C.11}$$

Maximization of utility gives the Euler equation, which writes in log-linear form

$$c_t^j = -\sigma r_t + E_t(c_{t+1}^j). \tag{C.12}$$

Injecting it in the intertemporal budget constraint gives the consumption function (7) in the text.

D Derivation of Firm i's Investment Function

Equations (A.11) and (8) write in matrix form:

$$\begin{bmatrix} x_t^i \\ \\ k_{t-1}^i \end{bmatrix} = AE_t \begin{bmatrix} x_{t+1}^i \\ \\ k_t^i \end{bmatrix} + \begin{bmatrix} \frac{1}{\kappa} \\ \\ -\frac{\delta}{\kappa(1-\delta)} \end{bmatrix} \left(-r_t + [1-\beta(1-\delta)] \left(\frac{1}{1-\alpha} E_t(y_{t+1}^i) + E_t(w_{t+1}) \right) \right), \tag{D.1}$$

where:

$$A = \begin{bmatrix} \beta(1-\delta) & -\frac{1-\beta(1-\delta)}{\kappa(1-\alpha)} \\ \\ -\beta\delta & \frac{1}{1-\delta}\left(1+\delta\frac{1-\beta(1-\delta)}{\kappa(1-\alpha)}\right) \end{bmatrix}. \tag{D.2}$$

The roots of the system are the solutions to the quadratic equation (10). The polynomial has two positive real roots. Since $P(1) < 0$, one root is greater than 1 and one smaller than 1. Denote the latter root λ, and $[1, \mu]'$ the associated left eigenvector, where μ is given by

$$\mu = \frac{\beta(1-\delta) - \lambda}{\beta\delta} > 0. \tag{D.3}$$

The forward-looking equation along the λ root is:

$$x_t^i + \mu k_{t-1}^i = \frac{1}{\kappa}\frac{\lambda}{\beta(1-\delta)}\left(-r_t + [1-\beta(1-\delta)]E_t\left(\frac{1}{1-\alpha}y_{t+1}^i + w_{t+1}\right)\right) + \lambda E_t(x_{t+1}^i + \mu k_t^i), \tag{D.4}$$

which iterated forward gives equation (9).

E Dependence of λ in Adjustment Costs κ

Since $P(\beta(1-\delta)) < 0$, we have that $\lambda < \beta(1-\delta)$. Consider now the dependence of λ in κ. Differentiating equation (10) gives $\frac{\partial \lambda}{\partial \kappa} > 0$.

As for the limits, when $\kappa \to 0$, equation (10) is equivalent to

$$\frac{-\delta(1-\beta(1-\delta))}{(1-\delta)(1-\alpha)}\lambda = 0, \tag{E.1}$$

whose unique solution is $\lambda = 0$.

When $\kappa \to +\infty$, equation (10) is equivalent to

$$\lambda^2 - \left(\beta(1-\delta) + \frac{1}{1-\delta}\right)\lambda + \beta = \left(\lambda - \beta(1-\delta)\right)\left(\lambda - \frac{1}{1-\delta}\right) = 0, \tag{E.2}$$

whose smaller root is $\beta(1-\delta)$.

F Derivation of the Baseline Consumption Bloc

Aggregating the individual consumption function (7) across households gives the aggregate consumption function:

$$c_t = (1-\beta)b_{t-1} - \beta\sigma\sum_{k=0}^{\infty}\beta^k E_t(r_{t+k}) + (1-\beta)\sum_{k=0}^{\infty}\beta^k E_t(\omega_{t+k}). \tag{F.1}$$

Using the flow budget constraint (C.5), it can be written recursively as

$$c_t = -\sigma\beta r_t + (1-\beta)\omega_t + \beta E_t(c_{t+1}). \tag{F.2}$$

Imposing the equilibrium requirement that aggregate household income is equal to aggregate consumption $\omega_t = c_t$ gives the Euler equation (19) in the text. The general-equilibrium amplifying mechanisms of the Keynesian cross—higher consumption leading to higher production leading to higher income, which further increases consumption, etc.—has removed the discounting of interest rates present in the aggregate consumption function (F.1).

G Proofs of Lemmas 1 and 2

Around a steady-state with no markup $\mathcal{M}^* = 1$, equations (24) and (25) write in log-linear form

$$\frac{\Omega^{H*}}{\Omega^*}\omega_t^H = \Big((1-\alpha)s_L^H + \alpha s_K^H\Big)\omega_t + \Big((1-\alpha)(s_L^H - s_K^H)\frac{1}{1-s_x}\Big)(s_x x_t - \mu_t), \tag{G.1}$$

$$\frac{\Omega^{P*}}{\Omega^*}\omega_t^P = \Big((1-\alpha)(1-s_L^H) + \alpha(1-s_K^H)\Big)\omega_t - \Big((1-\alpha)(s_L^H - s_K^H)\frac{1}{1-s_x}\Big)(s_x x_t - \mu_t). \tag{G.2}$$

Aggregate consumption is $C_t = C_t^H + C_t^P$, or in log-linear form:

$$c_t = \frac{\Omega^{H*}}{\Omega^*}c_t^H + \frac{\Omega^{P*}}{\Omega^*}c_t^P. \tag{G.3}$$

The consumption function of permanent-income households is, aggregating their individual consumption function (7):

$$c_t^P = -\beta\sigma\sum_{k=0}^{\infty}\beta^k E_t(r_{t+k}) + (1-\beta)\sum_{k=0}^{\infty}\beta^k E_t(\omega_{t+k}^P). \tag{G.4}$$

which uses the fact that, since the derivation abstracts from taxes for convenience, debt (traded among permanent-income households only) is in zero net supply. Plugging in the consumption function of permanent-income households (G.4) and of hand-to-mouth households (21) into (G.3), and the expressions of incomes (G.1)-(G.2) into the consumption functions gives equation (26).

H Proof of Proposition 2

To solve for the equilibrium dependence of investment on interest rates, it is possible to start from the investment function (29) derived in section 1, and to take into account the general-equilibrium amplifying loops by considering the system (29)-(33)-(27). This two-step procedure would be the analogue for investment of what we did in appendix C and F for consumption, deriving the consumption function (F.1), then taking into account the general-equilibrium amplifying loop of the Keynesian cross.

Yet, it is simpler to solve for the equilibrium dependence on interest rates by bypassing the investment function (29) and starting instead from equation (A.11). Writing it recursively, aggregating it across firms and using the expressions of future aggregate demadn and future wages (18) gives

$$x_t = -\frac{1}{\kappa}r_t + a^*E_t(c_{t+1}) + (b^* + \beta(1-\delta))E_t(x_{t+1}) - \left(c^* + \frac{1-\beta(1-\delta)}{\kappa(1-\alpha)}\right)k_t, \tag{H.1}$$

where a^*, b^*, c^* are given in equations (30)-(31)-(32). Equation (H.1) can then be combined with the law of motion of aggregate capital (33) and the consumption bloc (27) to give the system

$$\begin{bmatrix} 1 & -\xi & 0 \\ 0 & 1 & 0 \\ 0 & \delta & 1-\delta \end{bmatrix} \begin{bmatrix} c_t \\ x_t \\ k_{t-1} \end{bmatrix} = \begin{bmatrix} 1 & -\xi & 0 \\ a^* & b^* + \beta(1-\delta) & -\left(c^* + \frac{1-\beta(1-\delta)}{\kappa(1-\alpha)}\right) \\ 0 & 0 & 1 \end{bmatrix} E_t \begin{bmatrix} c_{t+1} \\ x_{t+1} \\ k_t \end{bmatrix} - \begin{bmatrix} (1-\xi)\sigma \\ \frac{1}{\kappa} \\ 0 \end{bmatrix} r_t. \tag{H.2}$$

This direct resolution approach is the analogue for investment of, for consumption, getting to the aggregate Euler equation (19) directly by aggregating the Euler equations of individual households (C.12), without going through writing the consumption function (F.1).

Pre-multiplying equation (H.2) by the inverse of the matrix on the left-hand side:

$$\begin{bmatrix} c_t \\ x_t \\ k_{t-1} \end{bmatrix} = AE_t \begin{bmatrix} c_{t+1} \\ x_{t+1} \\ k_t \end{bmatrix} + Br_t, \tag{H.3}$$

where

$$A = \begin{bmatrix} 1+\xi a^* & \xi(b^* + \beta(1-\delta) - 1) & -\xi\left(c^* + \frac{1-\beta(1-\delta)}{\kappa(1-\alpha)}\right) \\ a^* & b^* + \beta(1-\delta) & -\left(c^* + \frac{1-\beta(1-\delta)}{\kappa(1-\alpha)}\right) \\ \frac{-\delta}{1-\delta} a^* & \frac{-\delta}{1-\delta}(b^* + \beta(1-\delta)) & \frac{1}{1-\delta}\left(1 + \delta\left(c^* + \frac{1-\beta(1-\delta)}{\kappa(1-\alpha)}\right)\right) \end{bmatrix}, \tag{H.4}$$

$$B = \begin{bmatrix} -((1-\xi)\sigma + \xi\frac{1}{\kappa}) \\ \frac{-1}{\kappa} \\ \frac{\delta}{\kappa(1-\delta)} \end{bmatrix}. \tag{H.5}$$

(H.6)

From equation (27) we know that one root of the system is equal to one. The two other roots have their product equal to $det(A)$ and their sum equal to $tr(A) - 1$, so they are the two roots of the quadratic polynomial

$$Q(\lambda^*) = (\lambda^*)^2 + (1 - tr(A))\lambda^* + det(A). \tag{H.7}$$

It can be rewritten as (34). The polynomial has two positive real roots.

Denote the smaller root λ^*, and $[-\gamma_c, 1, \gamma_k]'$ the associated left eigenvector. The coefficients of the eigenvector can be written:

$$\gamma_k = \frac{\left(c^* + \frac{1-\beta(1-\delta)}{\kappa(1-\alpha)}\right)(1-\lambda^*)}{\frac{1}{1-\delta}\left((1 - b^* - \beta(1-\delta)) + \delta c^* + \delta\frac{1-\beta(1-\delta)}{\kappa(1-\alpha)}\right) - (1 - b^* - \beta(1-\delta))\lambda^*}, \tag{H.8}$$

$$\gamma_c = \frac{a^*}{1 + \xi a^* - \lambda^*}\left(1 - \gamma_k\frac{\delta}{1-\delta}\right). \tag{H.9}$$

The forward-looking equation along the λ^* root is:

$$x_t - \gamma_c c_t + \gamma_k k_{t-1} = -\eta r_t + \lambda^* E_t(x_{t+1} - \gamma_c c_{t+1} + \gamma_k k_t), \tag{H.10}$$

$$\text{where } \eta = -[-\gamma_c, 1, \gamma_k] \times B = \frac{1}{\kappa}\left(1 - \frac{\delta}{1-\delta}\gamma_k\right) - \gamma_c\left((1-\xi)\sigma + \xi\frac{1}{\kappa}\right). \tag{H.11}$$

Iterating it forward gives:

$$x_t = -\gamma_k k_{t-1} + \gamma_c c_t - \eta\sum_{k=0}^{\infty} {\lambda^*}^k E_t(r_{t+k}). \tag{H.12}$$

Equation (H.12) combines with the equation (27) to form the forward-component of the system. Iterated forward and written in matrix form, they give:

$$\begin{bmatrix} 1 & -\xi \\ -\gamma_c & 1 \end{bmatrix}\begin{bmatrix} c_t \\ x_t \end{bmatrix} = -\begin{bmatrix} 0 \\ \gamma_k \end{bmatrix} k_{t-1} - \sum_{k=0}^{\infty}\begin{bmatrix} 1 & 0 \\ 0 & \lambda^* \end{bmatrix}^k \begin{bmatrix} \sigma(1-\xi) \\ \eta \end{bmatrix} E_t(r_{t+k}). \tag{H.13}$$

Premultiplying by the inverse of the matrix on the left-hand side gives:

$$c_t = -\frac{\xi\gamma_k}{1-\xi\gamma_c}k_{t-1} - \frac{1}{1-\xi\gamma_c}\sum_{k=0}^{\infty}\left[\sigma(1-\xi)1 + \xi\eta{\lambda^*}^k\right]E_t(r_{t+k}), \tag{H.14}$$

$$x_t = -\frac{\gamma_k}{1-\xi\gamma_c}k_{t-1} - \frac{1}{1-\xi\gamma_c}\sum_{k=0}^{\infty}\left[\gamma_c\sigma(1-\xi)1 + \eta{\lambda^*}^k\right]E_t(r_{t+k}). \tag{H.15}$$

Expression (H.15) for x_t can be rewritten as expression (35) in the main text, with coefficients:

$$\mu^* = \frac{\gamma_k}{1 - \xi\gamma_c}, \tag{H.16}$$

$$\theta^* = \frac{1 - \frac{\delta}{1-\delta}\gamma_k}{\kappa} \frac{1 - \lambda^*}{1 - \lambda^* + \frac{\delta}{1-\delta}\gamma_k \xi a^*}. \tag{H.17}$$

I Proof of Corollary 3

1. Since λ^* solves the quadratic equation (34), $\lambda^* > \lambda$ is equivalent to $Q(\lambda) > 0$. Using the fact that λ solves the quadratic equation (10), this condition can be written as condition (38).
2. That $\lambda^* > 1$ is equivalent to the condition $Q(1) < 0$, which can be written as condition (39).
3. Since $Q(1/(1-\delta)) < 0$, we always have $\lambda^* < 1/(1-\delta)$.

Equation (40) is obtained by differentiating the quadratic equation (34).

J Proof of Corollary 4

When $\kappa \to 0$, equation (34) is equivalent to:

$$-\left(\frac{\delta}{1-\delta}\left(\frac{1-\beta(1-\delta)}{(1-\alpha)\kappa}\right) + b^* + \xi a^* + \frac{\delta}{1-\delta}c^*\right)\lambda^* + \left(\frac{1}{1-\delta}(b^* + \xi a^*)\right) = 0, \tag{J.1}$$

whose unique solution is (41).

When $\kappa \to +\infty$, equation (34) is equivalent to:

$$(\lambda^*)^2 - \left(\beta(1-\delta) + \frac{1}{1-\delta}\right)\lambda^* + \beta = 0, \tag{J.2}$$

whose smaller root is $\beta(1-\delta)$.

K Proof of Proposition 3

There is discounting in consumption and aggregate demand if and only if the limit coefficient on the interest rate in the infinite future $k \to \infty$ is lower than the coefficient on the contemporaneous interest rate $k = 0$. This occurs if and only if $\xi\theta^*(\zeta - 1) < 0$, i.e. for $\xi > 0$ if and only if $\zeta < 1$.